

Sudakov Suppression in the Soft Heavy to Light Meson Transition Form Factor.

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We calculate hard gluon contribution to the decay vertex that determines the heavy-to-light meson transition form factor at large recoil. It is found that resulting Sudakov suppression significantly decreases the soft, wave function dependent contribution to the form factor. Phenomenological implications of these findings are discussed.

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I. INTRODUCTION

Weak decays of heavy mesons give a unique opportunity for studying strong interactions. Due to interference between strong and weak processes, hadronic matrix elements may be extracted with a much higher precision than from purely strong decays. Furthermore, the presence of a large scale provided by the heavy quark mass, $m_Q \gg \Lambda_{QCD}$, results in significant simplifications of the theoretical analysis. For example, the decays in which another heavy quark with small momentum is produced in the final state can be studied using a nonrelativistic approximation, via an expansion in powers of Λ_{QCD}/m_Q . On the other hand, highly relativistic decays, with small mass hadrons produced in the final state, are amenable to perturbative QCD methods as the running coupling constant, $\alpha_s \sim \alpha(m_Q)$ becomes small for highly virtual relativistic quarks. In this paper, we discuss the Sudakov suppression of the pseudoscalar heavy-to-light meson transition amplitude associated with the production of a relativistic light quark which is near energy shell. If one neglects final state interactions, then the strong interaction contribution to a two body decay of a heavy meson into two light mesons (say, $B \rightarrow \pi\pi$), or to a semileptonic decay (such as $B \rightarrow \pi l\bar{\nu}$) is given in terms of the heavy-to-light meson transition matrix element of the weak current.

For a $J^P = 0^- \rightarrow 0^-$ transition, one needs to know (both for the two-body hadronic decay and for the semileptonic decay at large recoil) the magnitude of $f_+ = f_+(0)$, the form factor $f_+(Q^2)$ in the limit when the momentum transfer squared vanishes. The definition of $f_+(Q^2)$ is given by

$$\langle P_H | \bar{Q} \gamma^\mu q | P_L \rangle = f_+(Q^2) (P_H + P_L)^\mu + f_-(Q^2) (P_H - P_L)^\mu. \quad (1.1)$$

Here P_H and P_L are the momenta of the heavy and of the light meson in the final state and $Q = P_H - P_L$ is the four momentum transfer. The momenta P_H , Q and P_L satisfy,

$$0 \sim M_L^2 = P_L^2, \quad Q^2 \ll P_H^2 = M_H^2 = m_Q^2 \left(1 + O\left(\frac{\Lambda_{QCD}}{m_Q}\right) \right), \quad (1.2)$$

with M_L and M_H representing the light and the heavy meson masses, respectively and m_Q being the mass of the decaying heavy quark. The discussion of the form factor in the kinematic region of Eq. (1.2) was given in Ref. [1]. In analogy with exclusive amplitudes at high momentum transfer, it was shown that exchange of hard gluons between the decaying heavy quark and the light valence spectator may be required to correlate the produced light quark and the light spectator in order to enhance the probability for hadronization into a single light meson. Even though perturbative QCD analysis of the heavy-to-light meson transition form factor $f_+(Q^2)$ for $Q^2 \lesssim \Lambda_{QCD}^2$ and an exclusive amplitude at high momentum transfer, $Q^2 \gg \Lambda_{QCD}^2$ (e.g., the pion elastic form factor) are very similar at first sight, there is a fundamental difference between the two. In the latter case, the soft contributions are Λ_{QCD}^2/Q^2 suppressed in the asymptotic region compared to the leading perturbative QCD amplitude. On the other hand, in the case of the heavy-to-light meson transitions, the two contributions are of same order in Λ_{QCD}/m_Q . For this reason, when studying f_+ , it is crucial to employ a scheme in which both soft and hard contributions are addressed simultaneously[†]. To express f_+ entirely in terms of relativistic hadronic wave functions, it is necessary to quantize the system on the light front where boosts are kinematical. These goals may be achieved using the method of Ref. [3,4]. The method is close in spirit to both QCD sum rules [5] and to the quark model approach [6]. It is based on the analysis of vacuum current correlators. For a given correlator expressed as a sum of perturbative covariant Feynman amplitudes, the integration over the light cone energy, *i.e.* the “minus” component of the loop momenta is performed analytically. The Borel transformation is then used to model the soft part of the meson-quark vertex (wave function) with the perturbative gluon exchange kernels explicitly generated in higher-loop diagrams. This is a novel approach for handling perturbative amplitudes in the off-energy shell environment with nonperturbative hadronic bound states in the asymptotic states. Using this method it was found [3] that the gluon exchange contribution to f_+ may be enhanced beyond the collinear approximation [1,7] and that the one loop correction to the decay vertex may significantly reduce the $O(1)$ soft contribution. In this letter we extend the analysis of Ref. [3] to account for the full Sudakov suppression of the decay vertex and calculate the relevant corrections to f_+ .

[†]Sudakov corrections to the hard contribution were considered in Ref. [2]

II. SOFT CONTRIBUTION.

Consider the three-point function defined by the correlator

$$\Pi(p_L^2, p_H^2) = (-i)^2 \int dx dy e^{ip_L x - ip_H y} \langle 0 | T \bar{q}(x) \gamma^+ \gamma_5 q(x), \bar{q}(0) \gamma^+ Q(0), \bar{Q}(y) \gamma^+ \gamma_5 q(y) | 0 \rangle, \quad (2.1)$$

for $p_L^+ = p_H^+ \equiv p^+$ where q and Q are the light and heavy quark fields, respectively. The $O(1)$ contribution to Π is shown in Fig. 1a. After a double Borel transformation, we get

$$\Pi(p_L^2, p_H^2) \rightarrow \Pi(\beta_L, \beta_H) = \left[\frac{1}{2\pi i} \right]^2 \oint dp_L^2 dp_H^2 \Pi^0(p_L^2, p_H^2) \left[\frac{e^{-p_L^2/2\beta_L^2}}{f_L} \right] \left[\frac{e^{-(p_H^2 - M_H^2)/2m_Q \beta_H}}{f_H} \right], \quad (2.2)$$

where $M_H \sim m_Q + O(\beta_H)$ is the heavy meson mass, $f_{L(H)}$ are the light and heavy meson decay constants and we have neglected the light quark and meson masses, To $O(1)$, the transformed correlator can be written as

$$\Pi = 2(P^+)^3 \int \frac{d^2 \mathbf{1}_\perp dy}{16\pi^3} \Psi_H(y, \mathbf{1}_\perp) \Psi_L(y, \mathbf{1}_\perp). \quad (2.3)$$

Here $\Psi_{H(L)}$ may be interpreted as the heavy (light) meson light cone wave functions. The Borel transformation is equivalent to the Gaussian model and corresponds to

$$\Psi_L = \frac{2\sqrt{6}}{f_L} \exp\left(-\frac{1}{2\beta_L^2} \frac{\mathbf{1}_\perp^2}{y(1-y)}\right), \quad \Psi_H = \frac{2\sqrt{6}}{f_H} \exp\left(\frac{1}{2m_Q \beta_H} \left[M_H^2 - \frac{\mathbf{1}_\perp^2}{y(1-y)} - \frac{m_Q^2}{1-y} \right]\right), \quad (2.4)$$

with $-\mathbf{1}_\perp$, $(1-y)$ and $\mathbf{1}_\perp$, y being the relative light cone momenta of the struck quark and the spectator, respectively. Truncating the phenomenological spectral representation of the correlator to a single contribution from $J^P = 0^-$, Qq heavy and $\bar{q}q$, light meson ground states and comparing with Eq. (2.3) leads to the standard light cone representation for $f_+ = f_+(0)$,

$$f_+ = \Psi_H \otimes \Psi_L \equiv \int \frac{d^2 \mathbf{1}_\perp dy}{16\pi^3} \Psi_H(y, \mathbf{1}_\perp) \Psi_L(y, \mathbf{1}_\perp). \quad (2.5)$$

If the heavy and light meson interpolating fields in Eq. (2.1) are replaced by two identical ones, i.e., by either two $\bar{Q}\gamma^+\gamma_5 Q$ or two $\bar{q}\gamma^+\gamma_5 q$ currents, then a similar analysis would result in a normalization condition for the meson wave functions,

$$1 = \Psi_{H(L)} \otimes \Psi_{H(L)}. \quad (2.6)$$

With $f_L = f_\pi \sim f_H = f_B \sim 130$ MeV this gives $\beta_H = 300$ MeV, and $\beta_L = 400$ MeV, respectively and from Eq. (2.5) we obtain

$$f_+ \sim 0.22, \quad (2.7)$$

which is close to the standard value of the soft contribution to the $B \rightarrow \pi$ transition form factor, $f_+ \sim 0.3$ obtained using QCD sum rules or more sophisticated quark model wave functions [5,6].

III. ONE LOOP CORRECTIONS.

The one loop correction to Π from the dressing of the decay vertex is shown in Fig. 1b. In order to ensure current conservation, we also consider one loop corrections to the two propagators connected to this vertex. Integration over the ‘‘minus’’ components of loop momenta picks up poles in the spectator quark and gluon propagators *i.e.* puts these two on the mass-shell. The Borel transformation of Eq. (2.2) combined with the the $O(1)$ contribution from the bare triangle then yields

$$f_+ = \Psi_H \otimes [I + T] \otimes \Psi_L + \Psi_{gH} \otimes T_{gH} \otimes \Psi_L + \Psi_H \otimes T_{gL} \otimes \Psi_{gL} + \Psi_{gH} \otimes I_{gg} \otimes \Psi_{gL}. \quad (3.1)$$

The term $T = T(y, \mathbf{1}_\perp; x, \mathbf{k}_\perp)$ involving a one gluon exchange between the valence wave functions is given by

$$T = 16\pi^3 \delta(y-x) \delta(\mathbf{1}_\perp - \mathbf{k}_\perp) \times \int \frac{dz d\mathbf{p}_\perp}{16\pi^3} \frac{8\pi\alpha_s C_F}{z(1-z)} \left[\frac{N(z, \mathbf{p}_\perp)}{D_H(z, \mathbf{p}_\perp) D_L(z, \mathbf{p}_\perp)} - \frac{z}{2} \frac{1}{D_H(z, \mathbf{p}_\perp)} - \frac{z}{2} \frac{1}{D_L(z, \mathbf{p}_\perp)} \right], \quad (3.2)$$

with

$$N(z, \mathbf{p}_\perp) \equiv \frac{\mathbf{p}_\perp^2}{1-z} - m_Q^2, \quad D_H(z, \mathbf{p}_\perp) \equiv \frac{\mathbf{p}_\perp^2}{z(1-z)} + \frac{zm_Q^2}{1-z}, \quad D_L(z, \mathbf{p}_\perp) \equiv \frac{\mathbf{p}_\perp^2}{z(1-z)}. \quad (3.3)$$

The last two terms in the square bracket in Eq. (3.2) come from the loops involving quark propagators. The remaining, three contributions to f_+ in Eq. (3.1) involve nonvalence wave functions $\Psi_g = \Psi_g(y, \mathbf{1}_\perp; x, \mathbf{k}_\perp)$, which contain an extra gluon in addition to the two valence quarks. In our approach these are given by,

$$\begin{aligned} \Psi_{gL} &= \frac{\sqrt{8\pi\alpha_s C_F}}{D_L(x, \mathbf{k}_\perp)} \frac{2\sqrt{6}}{f_L} \exp\left(-\frac{1}{2\beta_L^2} \left[\frac{\mathbf{1}_\perp^2}{y(1-y)} + \frac{\mathbf{k}_\perp^2}{x(1-x)(1-y)} \right]\right), \\ \Psi_{gH} &= \frac{\sqrt{8\pi\alpha_s C_F}}{D_H(x, \mathbf{k}_\perp)} \frac{2\sqrt{6}}{f_H} \exp\left(\frac{1}{2m_Q\beta_H} \left[M_H^2 - \frac{\mathbf{1}_\perp^2}{y(1-y)} - \frac{\mathbf{k}_\perp^2}{x(1-x)(1-y)} - \frac{xm_Q^2}{(1-x)(1-y)} \right]\right), \end{aligned} \quad (3.4)$$

for the light and heavy meson, respectively. The arguments in the exponents are given by the invariant masses of the three body ‘‘valence plus gluon’’ configurations. Finally, the corresponding current matrix elements are given by

$$T_{gH} = \frac{\sqrt{8\pi\alpha_s C_F}}{x(1-x)} \left[\frac{x}{2} - \frac{N(x, \mathbf{k}_\perp)}{D_L(x, \mathbf{k}_\perp)} \right], \quad T_{gL} = \frac{\sqrt{8\pi\alpha_s C_F}}{x(1-x)} \left[\frac{x}{2} - \frac{N(x, \mathbf{k}_\perp)}{D_H(x, \mathbf{k}_\perp)} \right], \quad I_{gg} = \frac{N(x, \mathbf{k}_\perp)}{x(1-x)}. \quad (3.5)$$

The expression for f_+ in Eq. (3.1) is both IR and UV finite even though each individual contribution is divergent. In particular, the first term involving the valence quark wave function has an IR double logarithmic divergence coming from the region $z, \mathbf{k}_\perp \rightarrow 0$, in which the two quarks in the vertex loop go on mass-shell. To identify contributions from the purely hard gluons, we introduce cut-off functions, $\Theta_i(\mu_i) = \Theta(D_i, \mu_i)$ ($i = H, L$) where D_i is either one of the two denominators in Eq. (3.3) such that for small D_i , $D_i \ll \mu_i^2$, $\Theta_i(\mu_i) = D_i$ while for $D_i \gg \mu_i^2$, $\Theta_i(\mu_i) \rightarrow 1$. We may then rewrite Eq. (3.1) as

$$f_+ = \Psi_H \otimes [I + T^{hard}] \otimes \Psi_L + \hat{\Psi}_{gH} \otimes T_{gH}^{hard} \otimes \hat{\Psi}_L + \hat{\Psi}_H \otimes T_{gL}^{hard} \otimes \hat{\Psi}_{gL} + \hat{\Psi}_{gH} \otimes I_{gg} \otimes \hat{\Psi}_{gL}, \quad (3.6)$$

where the gluon exchange kernel staying in between the valence wave functions becomes $T \rightarrow T^{hard}$

$$T^{hard} = 16\pi^3 \delta(y-x) \delta(\mathbf{1}_\perp - \mathbf{k}_\perp) \times \int \frac{dz d\mathbf{p}_\perp}{16\pi^3} \frac{8\pi\alpha_s C_F}{z(1-z)} \left[N(z, \mathbf{p}_\perp) \frac{\Theta_H(\mu_H) \Theta_L(\mu_L)}{D_H(z, \mathbf{p}_\perp) D_L(z, \mathbf{p}_\perp)} - \frac{z}{2} \frac{\Theta_H(\mu_H)}{D_H(z, \mathbf{p}_\perp)} - \frac{z}{2} \frac{\Theta_L(\mu_L)}{D_L(z, \mathbf{p}_\perp)} \right], \quad (3.7)$$

with the remaining terms modified accordingly,

$$T_{gH}^{hard} = \frac{\sqrt{8\pi\alpha_s C_F}}{x(1-x)} \left[\frac{x}{2} - N(x, \mathbf{k}_\perp) \frac{\Theta_L(\mu_L)}{D_L(x, \mathbf{k}_\perp)} \right], \quad T_{gL}^{hard} = \frac{\sqrt{8\pi\alpha_s C_F}}{x(1-x)} \left[\frac{x}{2} - N(x, \mathbf{k}_\perp) \frac{\Theta_H(\mu_H)}{D_H(x, \mathbf{k}_\perp)} \right]. \quad (3.8)$$

The modified μ_i -dependent nonvalence wave functions, $\hat{\Psi}_{gi}$ are given by

$$\Psi_{gi} \rightarrow \hat{\Psi}_{gi} = \Psi_{gi}(y, \mathbf{1}_\perp; x, \mathbf{k}_\perp) - \frac{\sqrt{8\pi\alpha_s C_F} [1 - \Theta_i(\mu_i)]}{D_i(x, \mathbf{k}_\perp)} \Psi_i(y, \mathbf{1}_\perp). \quad (3.9)$$

Choosing $\Theta_i(\mu_i) = D_i/(D_i + \mu_i^2)$ amounts to replacing D_i 's with $D_i + \mu_i^2$ in the current matrix elements. Even though each individual term in Eq. (3.6) becomes now μ_i -dependent, after summation the μ_i dependence disappears. Furthermore, for $\mu_L^2 \sim 2\beta_L^2$ and $\mu_H^2 \sim 2m_Q\beta_H$, the second term in Eq. (3.9) strongly reduces the magnitude of the nonvalence amplitudes $\hat{\Psi}_{gi}$ and, therefore,

$$f_+ \sim \Psi_H \otimes [I + T^{hard}(\mu_i)] \otimes \Psi_L. \quad (3.10)$$

With the above choice of the cut-off functions, the contributions to f_+ from nonvalence sectors have been effectively absorbed by the valence sector through mass terms of the order μ_i^2 added into the free propagators. Having isolated the hard contribution to f_+ , we may sum to all orders in α_s the leading single and double logarithms. From Eq. (3.10) it follows that these are given by

$$f_+ \sim \Psi_H \otimes S(\mu_i) \otimes \Psi_L, \quad (3.11)$$

with

$$S(\mu_i) = 16\pi^3 \delta(x-y) \delta(\mathbf{k}_\perp - \mathbf{l}_\perp) \left[1 + \frac{\alpha_s}{2\pi} C_F \left(\frac{3}{4} \log \frac{m_Q^2}{\mu_L^2} - \frac{1}{2} \log^2 \frac{m_Q^2}{\mu_L^2} + \frac{1}{2} \log^2 \frac{\mu_H^2}{\mu_L^2} \right) \right]. \quad (3.12)$$

Taking $\mu_L^2 = 2\mu^2$, $\mu_H^2 = 2m_Q\mu$, after exponentiation this results in

$$f_+ = \left(\frac{\alpha(m_Q^2)}{\alpha(\mu^2)} \right)^{-\frac{6}{33-2N_F}} \left(\frac{m_Q}{\mu} \right)^{-\frac{6}{33-2N_F} \log \frac{\alpha(\mu^2)}{\alpha(m_Q^2)}} \Psi_H \otimes \Psi_L. \quad (3.13)$$

IV. CONCLUSIONS.

The first factor in Eq. (3.13) comes from the evolution of the leading, UV logarithm which is due to a large difference between the light and heavy quark virtualities [8]. The second term is the Sudakov form factor which suppresses the interaction in the kinematic region where the heavy and light quark go on-mass shell. The important feature of Eq. (3.13) is that it factorizes the hard and the soft (i.e., wave function dominated) contributions. Taking the average value for the factorization scale, $\mu = 500$ MeV and $m_Q = m_b = 4.8$ GeV, together with $N_F = 4$ and $\Lambda_{QCD} = 230$ MeV we obtain that the net result due to hard gluon exchange is a suppression by approximately

$$S \sim 0.66 \quad (4.1)$$

corresponding to a 34% correction to $O(\alpha^0)$ form factor. The standard value for the soft form factor, $f_+ \sim 0.33$ is in agreement with the $B \rightarrow \pi l \bar{\nu}$ branching ratio as measured recently by CLEO [9]. It also predicts the $B \rightarrow \pi\pi$ branching ratio to be approximately two times smaller than the current upper limit [10]. Reduction of the form factor by a $\sim 30\%$, as calculated here, indicates that contribution from the hard gluon exchange between the decaying heavy quark and the light spectator may be comparable to the soft form factor reduced by the Sudakov term. In Ref. [3] the hard gluon exchange contribution was calculated and indeed found to be approximately $5.3(\alpha_s/\pi)$ times the soft form factor. For $\alpha_s \sim 0.3$ this would give additional 40-50%. Therefore combining the hard gluon exchange contribution with the Sudakov-suppressed soft one increases f_+ back to approximately ~ 0.3 and agreement with the semileptonic data is recovered. In other, words, Sudakov effects are compensated by the hard gluon exchange and the net $O(\alpha_s)$ correction is rather small. The QCD corrections to f_+ have also been studied using light cone sum rules applied to an off diagonal correlator of heavy-light currents taken between vacuum and the light meson state [11,12]. In these approaches the soft contribution is expressed as a series over collinear terms from operator matrix elements of increasing twist. There the total $O(\alpha_s)$ correction to the twist-2 piece is also small: it does not exceed 20%. Our results seem to be somewhat larger. One possible reason being that our approach sums up subleading twist contributions in both soft and gluon exchange terms. This may be relevant since in the light cone sum rules it is found that in the α_s^0 order higher twist (3 and 4) contributions are as large as the leading one. A more detailed comparison of the two approaches should be undertaken.

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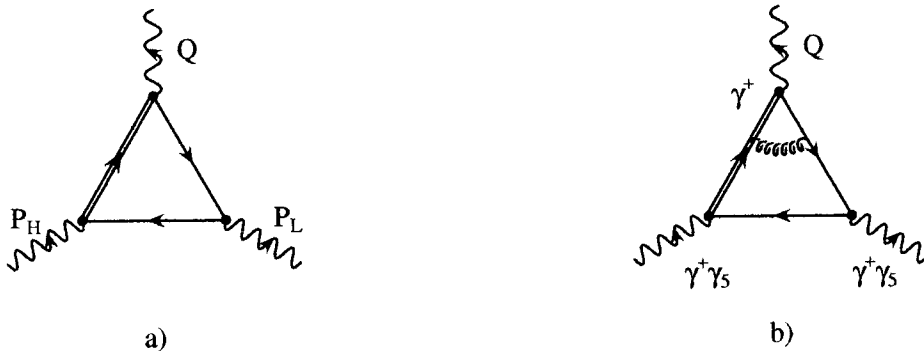


FIG. 1. Perturbative expansion of the three point function used to calculate f_+ .