Nucleon Elastic Form Factors and Local Duality

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Utilizing quark-hadron duality, a relation can be derived between the ratio of longitudinal to transverse deep inelastic electron-proton scattering cross sections and the ratio of the electric to magnetic proton elastic form factors. This relation allows the extraction of the elastic form factor ratio from purely inelastic data, and the extracted ratio agrees surprisingly well with the existing world data. This agreement is a first experimental verification of duality in the longitudinal channel, and suggests that effects of higher-twist operators are suppressed on average in the longitudinal electron-nucleon scattering process.

Nearly three decades ago, Bloom and Gilman observed that the prominent resonance enhancement region observed in inclusive electron-proton scattering averages to, and tracks with changing momentum transfer, the smooth scaling curve of the deep inelastic structure function, if expressed in terms of a scaling variable connecting the two different kinematic regimes [1,2]. This relationship between resonance electroproduction and the scaling behavior observed in deep inelastic scattering, termed local duality, suggests a common origin for both phenomena. Duality shows that the single-quark scattering process determines the scale of the reaction, even in the nucleon resonance region. Additional interactions between the struck quark and the spectator quarks (higher-twist effects) will nonetheless occur in this region, inducing much or most of the final state to produce a given resonance. However, if one averages over a reasonably wide region of kinematics, these additional interactions appear to cancel out and the reaction process still mimics the single-quark scattering process. A quantitative Quantum Chromodynamics (QCD) analysis of this empirical observation was given by De Rujula, Georgi, and Politzer [3,4]. They showed that local duality holds, if averaged over a large kinematic region, as the higher-twist effects are not large. Such QCD explanations of quark-hadron duality apply as well to the longitudinal structure function as the transverse [5].

Experimentally, higher-twist terms in the deep inelastic F_2 data have been found to be small for Bjorken $x < 0.40$ [6]. Recently, a reanalysis of deep inelastic F_2 data led to modified parton distribution functions [7]. Starting from these modified distribution functions the authors conclude that, in next-to-leading order (NLO) analysis, only minor higher-twist effects are needed to describe the deep inelastic F_2 data and $R = \sigma_L/\sigma_T$ (the ratio of longitudinal to transverse deep inelastic lepton-nucleon scattering), up to large Bjorken x and down to four-momentum transfer squared $Q^2 = 1$ (GeV/c)^2. Furthermore, when this analysis of R was repeated in next-to-next-to-leading order (NNLO), the higher-twist contributions were found to be even smaller [7]. In this analysis, the Georgi-Politzer calculation [8] was used to take target-mass corrections into account. These target-mass corrections, in terms of the Nachtmann variable $\xi = 2x/(1+\sqrt{1+4M^2x^2/Q^2})$ [9], where M is the proton mass, are necessary as the quarks can not be treated as massless partons for low to moderate momentum transfers.

Local duality between resonance electroproduction and deep inelastic scattering has been recently shown to hold surprisingly well down to momentum transfers squared, Q^2, as low as 1.0 (GeV/c)^2 [10]. It was shown that, if integrating over local nucleon resonance regions, the average strength in this region and the deep inelastic scaling curves agree to better than 10%, down to $Q^2 = 1.0$ (GeV/c)^2. Furthermore, the proton magnetic form factor could be reasonably well extracted (to better than 30%) from purely inelastic data, assuming duality, down to $Q^2 = 0.2$ (GeV/c)^2 [10].

The latter two observations, i.e. that higher-twist contributions to R in deep inelastic scattering data are small up to large x and down to $Q^2 = 1.0$ (GeV/c)^2, and that local duality appears to work to better than 10% down to $Q^2 = 1.0$ (GeV/c)^2, beg the question how well the proton electric form factor can be determined from duality arguments.

The definitions of the electric and magnetic form factor of the proton (G_E and G_M, respectively) lead to a straightforward relation between their ratio G_E/G_M and R [11,8]:

$$\frac{\mu G_E}{G_M} = \frac{\mu Q}{2M}\sqrt{\frac{\int d\xi \sigma_L}{\int d\xi \sigma_T}}, \tag{1}$$

where we have included μ, the proton anomalous magnetic moment, on both sides of the Equation. We calculate σ_L as $R \times \sigma_T$, and calculate [11] σ_T from the resonance-averaged scaling curve for F_2 from Ref. [10]. The integrals are performed over the region between ξ corresponding to pion threshold and $\xi = 1.0$. Note that this implies that the lower integration limit varies as a function of Q^2. For R we choose two different forms, one given by the best fit to the world's deep inelastic data [12], the other as derived from a QCD calculation including target-mass corrections, using parton distribution functions as input [8]. Note that the kinematic effects due to target mass dominate at small Q^2 and large x, the region where we can relate the deep inelastic R data to the elastic form factors by duality.

In Fig. 1 we show a sample of the world's data on R from deep inelastic scattering. The data have been extracted from Refs. [12–14]. The data at $x = 0.100$ extend down to low Q^2, approximately 0.5 $(\mathrm{GeV/c})^2$, whereas the data at $x = 0.625$ initiate at $Q^2 \approx 4$ $(\mathrm{GeV/c})^2$. Note that equating the elastic data in terms of Nachtmann ξ to the deep inelastic data at the same ξ corresponds to, for example, comparing deep inelastic data at ($Q^2 = 5$ $(\mathrm{GeV/c})^2$, $W^2 = 4$ $(\mathrm{GeV})^2$) to elastic data at ($Q^2 = 0.7$ $(\mathrm{GeV/c})^2$, $W^2 = M^2$), where W is the invariant mass of the hadronic system. Thus, in order to perform the duality-based derivation of $\mu G_E/G_M$ from R, we need to extend the models/calculations for R into a region where they are not constrained by measurements. In principle a similar extrapolation has to be made for the local duality witnessed in resonance electroproduction data and deep inelastic data [1,2], but as the world's data on F_2 at small to intermediate Q^2 and intermediate to large x show predominantly an x dependence, and less of a Q^2 dependence, this is less of an issue. In contrast, the world's data for R in these regions show more of a Q^2 dependence, and hardly an x dependence [12–14]. This can be seen from Fig. 1 in that the fit [12] to the world's data on R (solid curve) varies little between $x = 0.100$ (Fig. 1a) and $x = 0.625$ (Fig. 1b).

We expand on this in Fig. 2, where we show a sample of the world's data on R as a function of ξ, now at *fixed* values of $Q^2 = 2.5$ $(\mathrm{GeV/c})^2$ (Fig. 2a) and of $Q^2 = 8.0$ $(\mathrm{GeV/c})^2$ (Fig. 2b). The solid curve is again the fit to the world's data on R. The open circles are data obtained in the deep inelastic scattering region [12–14]. The squares are data in the nucleon resonance region, *averaged* over the full region [15]. The triangles are elastic data from Refs. [16–18]. The R data show a smooth ξ dependence, regardless whether the data are from the deep inelastic, the resonance, or the elastic region. At first order this confirms the local duality picture in the ratio R. The solid circles indicate the expected values for R in the elastic case assuming form factor scaling, i.e. assuming that the charge and magnetic moment distributions have the same spatial dependence. This underlines the fact that the fit [12] to the world's data on R smoothly links to available elastic data. Note, however, that the fit would not appear to go through the form factor scaling point at $x = 1$ at a lower Q^2 [12,14].

To estimate model dependencies in extrapolations from the kinematics space in x and Q^2 of the measured deep inelastic data, we also calculated R following the formalism of Georgi and Politzer [8,13]. Here, starting with input parton distribution functions, target mass effects are included in the framework of the operator product expansion and QCD moment analysis. The results of such a calculation are indicated by the dotted curves in Figs. 1 and 2. These calculations have no inherent elastic constraints and do not seem to go through the form factor scaling points at $x = 1$ (in terms of ξ, the elastic scaling points are at $\xi = 0.78$ and 0.91, respectively). The solid and dotted curves vary drastically, especially in the region of small Q^2 and large x (see Fig. 1b). However, they both show a rising trend in R towards smaller Q^2.

In Fig. 3, we show the world's data on the ratio $\mu G_E/G_M$ of proton elastic form factor data. Most of these data [16–20] have been determined using the conventional Rosenbluth separation technique [21], and may be prone to large systematic uncertainties. This is evident in the large scatter of the data (note that the uncertainties for the data points from Ref. [18] include the published systematic uncertainties). More recently, a different technique has been used to reduce the systematic uncertainties by measuring the $^1\mathrm{H}(\vec{e},e'\vec{p})$ polarization transfer reaction [22,23]. In this method a direct ratio of polarization transfer observables is measured, directly proportional to the ratio $\mu G_E/G_M$, precluding the necessity to have two kinematically independent points as needed for a Rosenbluth separation. The dashed curve (constant at unity) in Fig. 3 indicates the expected behavior of $\mu G_E/G_M$ according to form factor scaling.

The remaining curves utilize the duality approach. The solid curve in Fig. 3 is the extraction of the elastic form factor from deep inelastic data using the fit [12], and the dotted curve is the extraction using the QCD (including target mass effects) calculation following the formalism of Georgi and Politzer [8]. Above we indicated observations [7,10] leading us to believe that higher-twist effects (apart from target-mass corrections) are reduced at $Q^2 > 1$ $(\mathrm{GeV/c})^2$, which is where we start our duality-motivated form factor extractions. The calculations would indicate form factor scaling to occur at approximately $Q^2 = 20$ $(\mathrm{GeV/c})^2$. Although the fit to the world's data and the QCD including target mass effects calculation rendered a drastically different value for R at small Q^2 (see Fig. 1b), they agree very well in their extraction of $\mu G_E/G_M$ using duality arguments. The calculations indicate only a slight Q^2 dependence, which is caused by countering effects of the linear dependence on Q in Eqn. 1 and the Q dependence of the integral ratio. The latter has two dependencies on Q, one in the integration area as the ξ belonging to pion threshold increases as a function of Q^2, thus decreasing the integration area, and one in the Q dependence of R. Apparently the absolute value of R at low Q^2 is of less relevance. We have included in Fig. 3 a dot-dashed curve assuming a *fixed* R of 0.15, which shows that obtaining the Q^2 dependence of this quantity by duality-extracted predictions for the ratio of elastic form factors is a non-trivial property of the nucleon, requiring the Q^2 dependence of R. The agreement between the two (non-constant R) duality-based extractions and the world's data on $\mu G_M/G_E$ is in our opinion amazingly good (we stress that no normalization was necessary).

The above extraction will be even more constrained when more data on R in the nucleon resonance region will become available. Again, using (local) duality arguments, this would render experimental values of R closer to the kinematic region where we use them for the extraction of $\mu G_E/G_M$. As mentioned above, the magnetic form factor was extracted from purely inelastic data using duality arguments to better than 30%, down to $Q^2 = 0.2$ (GeV/c)^2 [10]. This may indicate a similar absolute uncertainty on this duality-based extraction of $\mu G_M/G_E$. Still, we feel this result may be interpreted as a strong signature that higher-twist effects are also reduced in the longitudinal response of the electron-proton scattering process, if the data are averaged over a reasonably large kinematic region.

In summary, we have extracted the ratio of the proton elastic electric and magnetic form factors from purely deep inelastic data using quark-hadron duality arguments. The result agrees well with experimental data, and suggests that higher-twist contributions are suppressed in the longitudinal part of the electron-proton scattering process, if averaged over an extended kinematic region. This is an experimental indication that duality should hold on average in the longitudinal channel, as has been observed in the transverse, i.e. both the longitudinal and transverse parts of electron-proton scattering seem to resemble on average a single-quark scattering process.

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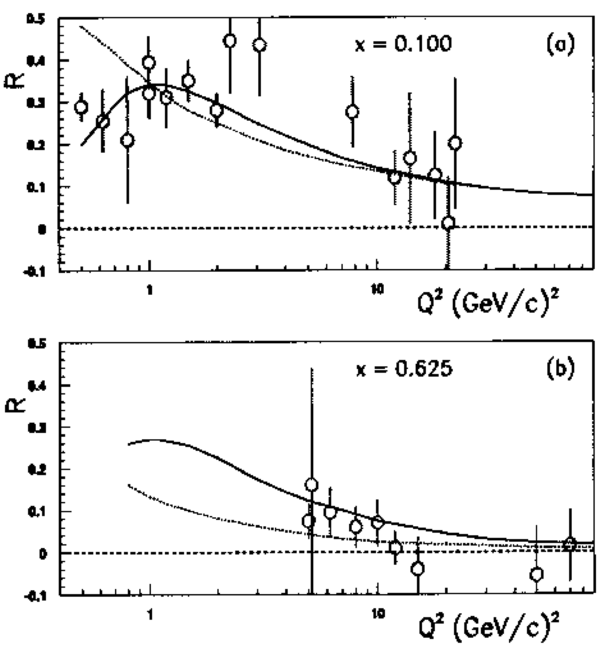

FIG. 1. Experimental values for R at $x = 0.100$ (a) and $x = 0.625$ (b). The data (circles) have been extracted from Ref. [14]. The solid line indicates the global fit to the world's data of R from Refs. [12]. The dotted line is the result from a QCD calculation including target-mass effects following Georgi and Politzer [8].

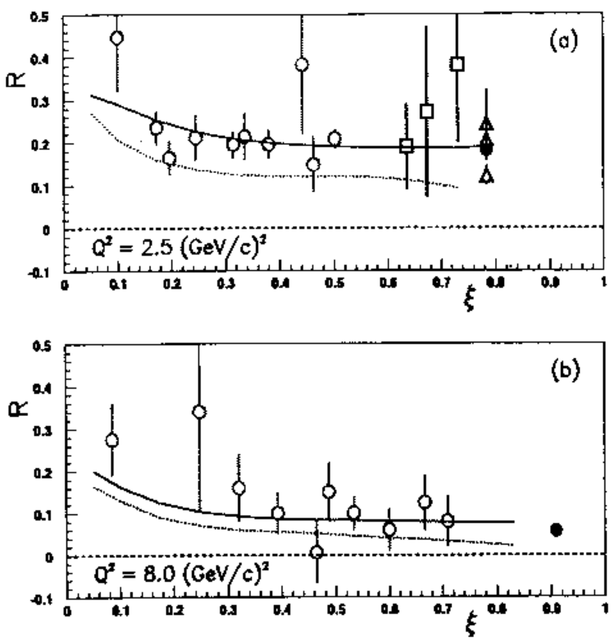

FIG. 2. Experimental values for R as a function of ξ, at fixed $Q^2 = 2.5\ (\mathrm{GeV/c})^2$ (a) and $Q^2 = 8.0\ (\mathrm{GeV/c})^2$ (b). Open circles are data obtained in the deep inelastic scattering region [12,14]. Squares are from data averaged over the full resonance region extracted from Ref. [15]. Triangles are elastic data from Refs. [17,18]. The solid lines indicate the global fit to the world's data of R from Refs. [12]. The dotted line is the result from a QCD calculation including target-mass effects following Georgi and Politzer [8]. The solid circles indicate the expected value at $x = 1$ for elastic form factor scaling.

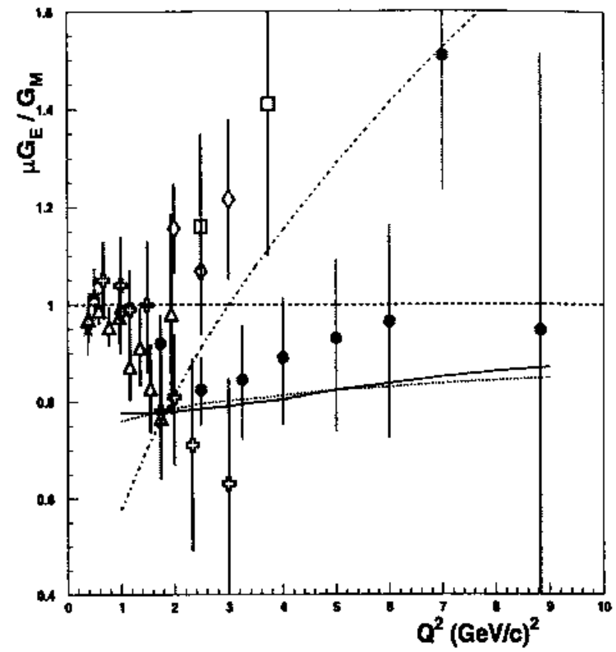

FIG. 3. The derived values for the ratio of the proton electric and magnetic form factor using local duality and (solid curve) the Whitlow fit to the ratio of longitudinal to transverse deep inelastic scattering, (dotted curve) a QCD calculation including target-mass corrections following Georgi and Politzer [8], and (dot-dashed curve) a constant value of $R = 0.15$. The experimental data are from Ref. [19] (pluses), Ref. [20] (triangles), Ref. [16] (squares), Ref. [17] (diamonds), Ref. [18] (circles), and Ref. [22] (stars).