Experimental Verification of Quark-Hadron Duality

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A newly-obtained sample of inclusive electron-nucleon scattering data has been analyzed for precision tests of quark-hadron duality. The data are in the nucleon resonance region, and span the range $0.3 < Q^2 < 5.0$ (GeV/c)^2. Duality is observed both in limited and extended regions around the prominent resonance enhancements. Higher twist contributions to the F_2 structure function are found to be minimal on average, even in the low Q^2 regime of ≈ 0.5 (GeV/c)^2. The proton magnetic form factor, extracted from purely inelastic data assuming duality, is in good agreement with direct measurements. In all cases, duality appears to be a non-trivial property of the nucleon structure function.

The interpretation of the resonance region in inclusive electron-proton scattering and its possible connection with deep inelastic scattering has been a subject of interest for nearly three decades since quark-hadron duality ideas, which successfully described hadron-hadron scattering [1], were first extended to electroproduction. Bloom and Gilman [2] showed that it was possible to equate the nucleon resonance region structure function $\nu W_2(\nu, Q^2)$ (at some typically low Q^2 value) to the structure function F_2 in the deep inelastic regime of electron-quark scattering (at some higher value of Q^2). These structure functions are obtained from inclusive electron-nucleon scattering where the substructure of the nucleon is probed with virtual photons of mass $-Q^2$ and energy ν. The resonance structure function was demonstrated to be equivalent in average to the deep inelastic one, with these averages obtained over the same range in a scaling variable $\omega' = 1 + W^2/Q^2$, where W is the invariant mass. Bloom and Gilman's quark-hadron duality qualitatively explained the data in the range $1 \leq Q^2 \leq 10$ (GeV/c)^2.

This relationship between resonance electroproduction and the scaling behavior observed in deep inelastic scattering suggests a common origin for both phenomena. Inclusive deep inelastic scattering on nucleons is a firmly-established tool for the investigation of the quark-parton model. At large enough values of W and Q^2, Quantum Chromodynamics (QCD) provides a rigorous description of the physics that generates the Q^2 behavior of the nucleon structure function $F_2 = \nu W_2$. The well-known logarithmic scaling violations in the F_2 structure function of the nucleon, predicted by asymptotic freedom, played a crucial role in establishing QCD as the accepted theory of strong interactions [3,4]. However, as Q^2 decreases, the description of the nucleon's structure cannot be expressed in terms of single parton densities with simple logarithmic behavior in Q^2. Inverse power violations in Q^2, physically representing initial and final state interactions between the struck quark and the remnants of the target (termed higher twist effects), must be taken into account as well.

An analysis of the resonance region in terms of QCD was first presented in [5,6], where Bloom and Gilman's approach was re-interpreted, and the integrals of the average scaling curves were equated to the non-singlet n =2 QCD moments of the F_2 structure function. The Cornwall-Norton moments of the structure function may be expressed as $\int_0^1 x^{n-2} F_2(x) dx$ [7], where $x = Q^2/2M\nu$ is the Bjorken scaling variable, M is the nucleon mass, and n is an integer index. The moments can be expanded, according to the operator product expansion, in powers of $1/Q^2$, and the fall of the resonances along a smooth scaling curve with increasing Q^2 was explained in terms of this QCD twist expansion of the structure function. The conclusion of [5] was that changes in the lower moments of the F_2 structure function due to higher twist effects are small, so that averages of this function over a sufficient range in x at moderate and high Q^2 are approximately the same. Duality is expected to hold so long as $O(1/Q^2)$ or higher inverse power scaling violations are small.

Substantial progress has been made both theoretically in understanding QCD in the past twenty years and experimentally in determining the scaling behavior of the F_2 structure function. Combining the latter with the new precision resonance data here presented [8], it is now possible to revisit quark-hadron duality with a more quantitative approach, addressing the recent theoretical interest in the topic (see, for example, [9–15]).

The data were obtained in Hall C, using the CEBAF accelerator at Jefferson Lab (JLab). Electron beam energies between 2.4 and 4 GeV, with currents between 20 and 100 μAmps were incident on 4 and 15 ($\pm$ 0.01) cm long liquid hydrogen and deuterium targets. Scattered electrons were detected in both the High Momentum Spectrometer (HMS) and the Short Orbit Spectrometer (SOS), each utilized in a single arm mode to measure the inclusive cross sections.

Nine spectra were obtained for hydrogen and eight for deuterium, covering the invariant mass range $0.88 < W^2 < 4.0$ GeV^2, with central four-momenta in the range $0.3 \leq Q^2 \leq 5.0$ (GeV/c)^2. The structure function $F_2 = \nu W_2$ was extracted from the measured differential cross section σ, using

$$\frac{\sigma \nu Q^4}{4\alpha^2 E'^2} = F_2 \left[\cos^2(\frac{\theta}{2}) + 2\sin^2(\frac{\theta}{2}) \frac{1+\nu^2/Q^2}{R+1} \right]. \quad (1)$$

Here, α is the fine structure constant, θ is the electron scattering angle, and E' is the scattered electron energy. $R = \sigma_L/\sigma_T$ is the ratio of longitudinal to transverse cross sections. This quantity will be measured at JLab [16], but is currently unknown at the $\pm$100% level in the resonance region for $Q^2 \geq 1$ (GeV/c)^2 [17]. The possible variation of R effects a 2% systematic uncertainty in the extracted F_2 data.

Sample νW_2 spectra extracted from the measured differential cross sections from hydrogen are plotted in Fig. 1 as a function of the Nachtmann scaling variable $\xi = 2x/(1 + 4M^2x^2/Q^2)$. It has been shown that ξ is the correct variable to use in studying QCD scaling violations in the nucleon [18,5]. The arrows indicate values of $x = 1$ (elastic scattering) for the three values of Q^2 shown. The solid and dashed curves are from a parameterization [19] of deep inelastic proton structure function data at $Q^2 = 10$ and 5 (GeV/c)^2, respectively. Notice that the resonance spectra at different Q^2 appear at different ξ on the deep inelastic scaling curve, but that the curve generally represents an average of the data at the disparate kinematics. This is a manifestation of the original Bloom and Gilman observation.

Because the data were obtained at fixed spectrometer angles, with a few overlapping fixed central momenta, the raw spectra in missing mass (W^2) cover a range in Q^2. The spectra in Fig. 1 have been adjusted to the Q^2 value at $W^2 = 2.5$ GeV^2 for each using a global fit [17] to inclusive resonance region spectra. The statistical uncertainty in the data is $\approx 1\%$, smaller than the symbols plotted. The overall systematic uncertainty in the cross sections due to experimental considerations such as target density, beam charge, beam energy, spectrometer acceptance, radiative corrections, detection efficiency, and the value of R, is 3% and is not depicted.

To quantify the observed duality, we show in Fig. 2 the Q^2 dependence of the ratio of the νW_2 structure function obtained from the resonance data, integrated in the region of ξ from pion threshold to the onset of the deep inelastic regime ($W^2 = 4$ GeV^2), compared to the F_2 deep inelastic structure function integrated in the same region of ξ. The uncertainties shown represent the experimental uncertainty in the numerator integral only, obtained from the correlated systematic uncertainties in the resonance data. The high Q^2 (above 4 (GeV/c)^2) values are generated from a global fit to inclusive SLAC resonance region data [17] and are assigned an uncertainty representing a combination of experimental uncertainty and normalization considerations involved in utilizing the older data sets [20]. Also shown are very low Q^2 (< 0.5 (GeV/c)^2) data from SLAC [21].

When used to obtain the deep inelastic denominator of the ratios in Fig. 2, the well-known CTEQ4 LQ [22] and MRS(G) LQ [19] scaling curves display a marked deviation from unity which increases with Q^2. This is not due necessarily to higher twist effects, but rather to the difficulty in accurately modeling the large ξ behavior of the structure function. With increasing Q^2, the moments are determined by a smaller and smaller region near $\xi = 1$. Unfortunately, there is a limited amount of deep inelastic F_2 data currently available at large ξ and x and these curves fall below the average resonance data for $\xi \gtrsim 0.7$.

The points on Fig. 2 labeled NMC represent ratios obtained using a parameterization of lower x deep inelastic F_2 data from CERN [23], which links smoothly to a global fit [20] to higher x deep inelastic data from SLAC, in the denominator integral. Because this is a fit to the data, it may implicitly contain higher twist effects. For the NMC parameterization, and for the two deep inelastic models discussed above, the integral used to form the ratio shown in the figure uses the parameterization at $Q^2 = 10$ (GeV/c)^2. Using the NMC parameterization, the ratio (Res/DIS) is above unity, but constant at about 1.2. The ratio decreases to about 1.1 when the appropriate logarthmic Q^2 dependence is included to obtain the parameterization at $Q^2 = 5$ (GeV/c)^2. This effect is indicated by the dashed line in the figure.

To obtain the points labeled JLab in Fig. 2, a fit to the *average* strength of *all* the hydrogen resonance spectra was obtained and utilized as a scaling curve. This approach assumes duality, and therefore that the average of the resonance data is equivalent to a proper scaling curve. This scaling curve, a function of ξ only, was obtained using a form similar to the x-dependent part of the NMC parameterization [23]. To constrain the fit in the kinematic regions beyond the scope of the data, data obtained from [24] were also used below $Q^2 = 4$ (GeV/c)^2 and $\xi = 0.3$. Resonance data in the range $5 < Q^2 < 8$ (GeV/c)^2 from [17] were generated at Q^2 values similar to our data and used to constrain the fit at large ξ. Note that the ratio comparing the resonance strength to the scaling curve need not be unity here even though the scaling curve was extracted from the resonance data. The average strength at any given value of ξ was obtained from a kinematic range of data at variant values of W^2 and Q^2.

In all cases in Fig. 2, the ratio apparently drops for the $Q^2 \leq 2$ (GeV/c)^2 points. This effect is largely negated, however, if the elastic cross section is also included in the integral. The elastic cross section is small at the larger Q^2 values and has minimal effect, but it is important at smaller Q^2. The total strength in the region below $W^2 = 4$ GeV^2, including the elastic and nucleon resonance regions, is equivalent within 10% to the JLab scaling curve, even at Q^2 values as low as 0.2. This indicates that higher twist effects are negligible if the data are integrated over the *full* region.

If the higher twist contributions are small in the region of the data, then duality allows for the determination of the nucleon form factor from data obtained in the purely inelastic region. Fig. 3 depicts the proton magnetic form factor, G_M^p, extracted using the NMC and JLab scaling curves integrated over the *entire* range in ξ, i.e. from 1 to the deep inelastic. The integral obtained from the resonance data (which starts at pion threshold rather than $\xi = 1$) is then subtracted from the scaling integrals. The magnetic form factor is then extracted from the remaining integrated strength using the prescription of [5]. In both cases, the extracted integrals are in remarkable agreement with the Gari-Krümpelmann model [25] of the world's magnetic form factor data. For the JLab fit worst case, we reproduce the proton magnetic form factor to within 30% of the accepted value.

Fig. 4 shows the same duality integral ratio as in Fig. 2, but here obtained locally in restricted ξ ranges around the three prominent resonance enhancement regions observed in inclusive nucleon resonance electroproduction, i.e. around the masses of the Δ $P_{33}(1232)$ ($1.3 \leq W^2 \leq 1.9$ GeV^2), the $S_{11}(1535)$ ($1.9 \leq W^2 \leq 2.5$ GeV^2), and the $F_{15}(1680)$ ($2.5 \leq W^2 \leq 3.1$ GeV^2) resonances, and in the higher W^2 region above these ($3.1 \leq W^2 \leq 3.9$ GeV^2). The uncertainties shown were computed as in Fig. 2. The latter higher mass ratio, which compares near deep inelastic data to deep inelastic data is essentially one and similar to the results in Fig. 2. Additionally, the lower mass resonances also appear to average to the deep inelastic strength, manifesting duality behavior even in these limited ranges of ξ at low Q^2 where higher twist effects might be expected to be large. A possible exception is the S_{11}, which appears systematically higher than the others. It has been pointed out [26] that the Δ resonance form factor decreases faster in Q^2 than the leading order perturbative QCD Q^{-4} behavior which the scaling curve should reflect. A similar observation may be made from Fig. 4 where the ratio (Res/DIS) drops below unity in the region $1 < Q^2 < 3.5$ (GeV/c)^2.

By utilizing new inclusive data in the resonance region at large x, it has been possible to revisit quark-hadron duality experimentally for the first time in nearly three decades. The original duality observations are verified, and the QCD moment explanation indicates that higher twist contributions to the $n = 2$ QCD moment of the F_2 structure function are small or cancelling, even in the low Q^2 regime of $Q^2 \approx 0.5$ (GeV/c)^2. The magnetic form factor has been extracted from the inelastic data using duality techniques and is in good agreement with global measurements. Additionally, duality is observed to hold for local resonance enhancements. In all cases, duality appears to be a non-trivial dynamic property of the nucleon structure function.

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[1] P.D.B. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press, Cambridge, 1977)
[2] E.D. Bloom and F.J. Gilman, Phys. Rev. D **4** (1970) 2901
[3] G. Altarelli, Phys. Rep. **81** (1982) 1
[4] A.J. Buras, Rev. Mod. Phys. **52** (1980) 199
[5] A. De Rujula, H. Georgi, and H.D. Politzer, Ann. Phys. **103** (1977) 315
[6] H. Georgi and H.D. Politzer, Phys. Rev. D **14** (1976) 1829
[7] R. Roberts, *The Structure of the Proton* (Cambridge University Press, Cambridge, 1990)
[8] I. Niculescu, Ph.D. Thesis, Hampton University, 1999
[9] X. Ji and P. Unrau, Phys. Rev. D **52** (1995) 73
[10] V.M. Belyaev and A. Radyushkin, Phys. Rev. B **359** (1995) 194
[11] G. West, hep-ph/9612403 (1996)
[12] G. Ricco *et al.*, Phys. Rev. C **57** (1998) 356
[13] C. Coriano, H-N. Li, and C. Savkli, J. High En. Phys. 9807 (1998) 008
[14] C. Carlson and N. Mukhopadhyay, Phys. Rev. Lett. **74** (1995) 1288
[15] M. Anghinolfi *et al.*, Nucl. Phys. **A602** (1996) 405
[16] Jefferson Lab Experiment E94-110
[17] C. Keppel, Proceedings of the Workshop on CEBAF at Higher Energies, Eds. N. Isgur and P. Stoler (1994) 237
[18] O. Nachtmann, Nucl. Phys. **B63** (1975) 237
[19] A.D. Martin, R.G. Roberts, and W.J. Stirling, Phys. Rev. D **50** (1994) 6734
[20] L.W. Whitlow *et al.*, Phys. Lett. **B282** (1992) 475
[21] S. Stein *et al.*, Phys. Rev. D **12** (1975) 1884
[22] H.L. Lai *et al.*, Phys. Rev. D **55** (1997) 1280
[23] M. Arneodo *et al.*, Phys. Lett. **B364** (1995) 107
[24] M. Arneodo *et al.*, Nucl. Phys. **B483** (1997) 3

[25] M. Gari and W. Krümpelmann, Phys. Lett. B141 (1984) 295

[26] P. Stoler, Phys. Rev. Lett. 66 (1991) 1003

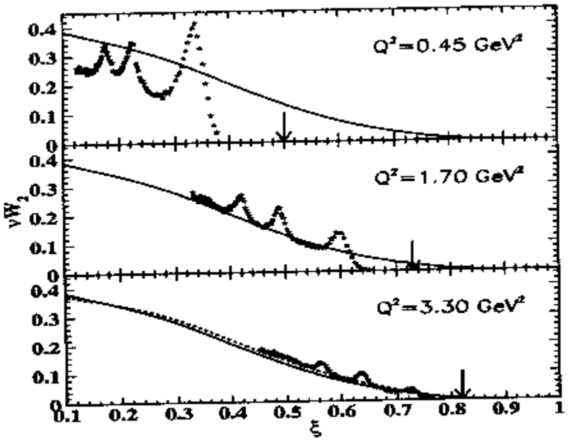

FIG. 1. Sample hydrogen νW_2 structure function spectra obtained at $Q^2 = 0.45, 1.70$, and 3.30 $(\mathrm{GeV/c})^2$ and plotted as a function of the Nachtmann scaling variable ξ. Arrows indicate elastic kinematics. The solid (dashed) line represents the NMC fit [23] of deep inelastic structure function data at $Q^2 = 10$ $(\mathrm{GeV/c})^2$ ($Q^2 = 5$ $(\mathrm{GeV/c})^2$).

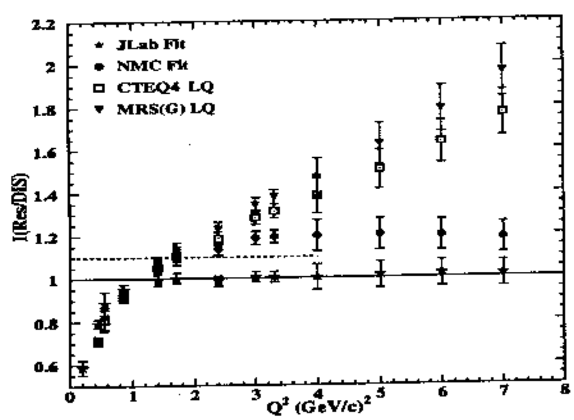

FIG. 2. The ratios of integrals obtained over the hydrogen resonance structure function in the ξ range corresponding to invariant masses between $1.14 < W^2 < 4$ GeV^2 (Res) to integrals of the deep inelastic structure functions obtained from parameterizations (stars are JLab fit; circles are NMC at $Q^2 = 10$ $(\mathrm{GeV/c})^2$; squares are CTEQ4 low Q^2; triangles are MRS(G) low Q^2) for the same range in ξ (DIS). The dashed line indicates what this ratio would be if the NMC curve were obtained at $Q^2 = 5$ $(\mathrm{GeV/c})^2$.

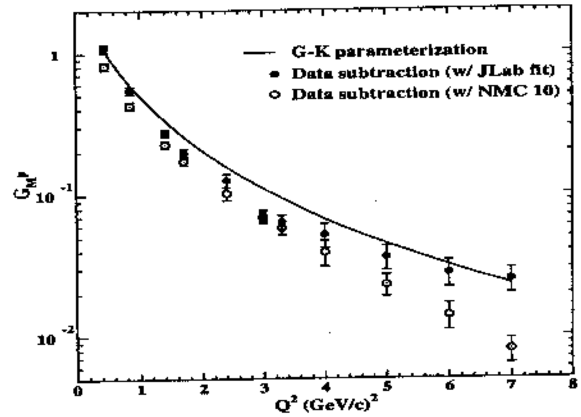

FIG. 3. The proton magnetic form factor extracted from the inelastic data using duality assumptions as described in the text. The extracted data are compared to the model curve of [25], which agrees with the global data set.

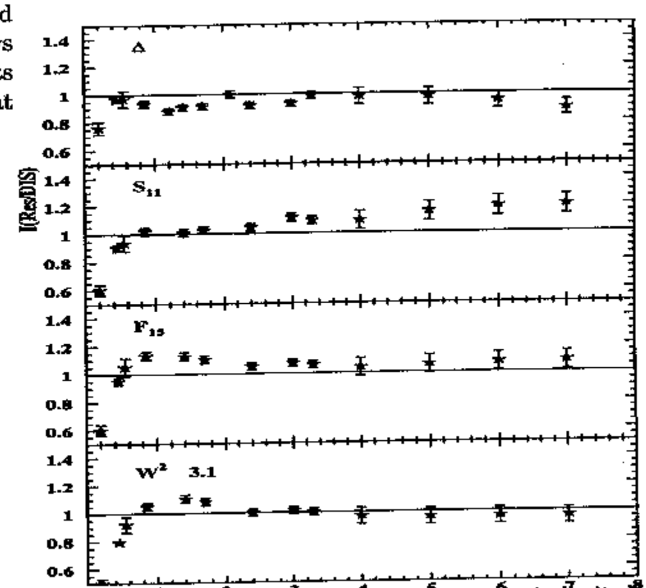

FIG. 4. The ratio of integrated strengths in limited ranges of ξ around the prominent resonance enhancement mass regions, to the strengths obtained from the average scaling curves in the same ξ regions.