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Pauli Interference in the 't Hooft Model: Heavy Quark Expansion and Quark-Hadron Duality

Ikaros Bigi [a] and Nikolai Uraltsev [a–c]

[a] *Dept. of Physics, Univ. of Notre Dame du Lac, Notre Dame, IN 46556, U.S.A.*
[b] *Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606*
[c] *Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350, Russia*

Abstract

Pauli Interference in decays of heavy flavor mesons, a genuinely nonleptonic nonperturbative effect, is considered in the 't Hooft model. Analytic summation of the exclusive decay widths yields the same expression as obtained in the OPE-based approach. Threshold-related violations of local duality in PI are found to be suppressed. A novel case is identified where the OPE effect is completely saturated by a single final state.

The dynamics driving heavy flavor transitions are concisely expressed on the quark level in terms of CKM parameters and masses. Yet their impact on the observable weak decays of heavy flavor hadrons H_Q is obscured by nonperturbative features of strong interactions. The principal elements of the model-independent treatment of inclusive decay probabilities based on Wilson's operator product expansion (OPE) [1], were laid down almost 15 years ago [2, 3]. Significant progress has later been achieved in developing this technique and establishing theoretical control over non-leptonic, semileptonic and radiative channels of practical interest (for a recent review, see Refs. [4]). The nonperturbative corrections which would scale like $1/m_Q$ are absent from the truly inclusive widths; the leading nonperturbative corrections arise at order $1/m_Q^2$ and are "flavor-independent" corrections insensitive to the flavor of the spectator(s) [5, 6]. Effects sensitive to the flavor of the spectator(s) emerge at the $1/m_Q^3$ level [2, 7, 5]. They are conventionally called Weak Annihilation (WA) in mesons [8], Weak Scattering (WS) [9] in baryons and Pauli Interference (PI) [10] in both systems. OPE relates their contributions to the expectation values of local four-quark operators.

The OPE is typically constructed as an asymptotic power expansion in Euclidean space, and neglects the terms $\sim \exp(-\sqrt{Q^2}/\Lambda_{\rm QCD})$ suppressed for high momentum scales $\sqrt{Q^2}$. Yet in Minkowski space these can lead to terms which oscillate and have only power suppression. For heavy quark decays the practical OPE *a priori* does not forbid, for instance, dependence of the type $\sin(m_Q \Lambda_{\rm QCD})/m_Q^k$ [11, 12]. To which degree they are suppressed by powers of $1/m_Q$ depends on details of the strong forces and the specifics of the process under study. This problem is behind the violation of local quark-hadron duality. Since WA and PI represent power suppressed effects, one might expect numerically larger corrections to duality at finite values of m_Q than for the leading terms in the OPE. Even a more radical concern had been expressed in the past that the OPE is not applicable to nonleptonic heavy flavor decays at all, even at arbitrary large heavy quark mass.

Implementation of duality in nonleptonic decays is not always obvious at first glance, in particular for PI. The underlying quark diagram for PI in B^- mesons is shown in Fig. 1. The $\bar{u}$ antiquark produced in the decay must be slow to interfere with the valence $\bar{u}$. Tracing the color flow one is then faced with a dichotomy. The large momentum flows through the diquark loop (cd) which represents the "hard core" of the process. In practical OPE one effectively replaces the propagation of this diquark by a nearly free di-fermion loop; the absorptive part is evaluated – at least to leading order – via the production of free quarks. Such identification cannot be justified by itself even at arbitrarily large momentum transfer, since the spectrum of QCD does not contain colored states.

Alternatively, one can combine a hard quark q from the loop with the slow valence antiquark to arrive at a color singlet configuration possibly dual to the hadronic spectrum. That comes at a price, though: this $q\bar{q}_{sp}$ system is no longer a hard one. For at least in a partonic picture with $p_{\rm sp} \sim m_{sp} \to 0$ the $q\bar{q}_{sp}$ invariant mass vanishes irrespective of m_b.

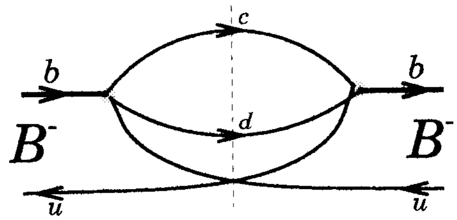

Figure 1: Quark diagram for PI in B^- decays. The weak vertices are broken to show the color flow for the leading-N_c contribution.

An intriguing example that interference might be delicate was noted in Ref. [13], where an inclusive and exclusive treatment of B decays seemed to yield different results for the interference of two separate color operators.

A dedicated consideration suggests that such apparent puzzles can be resolved without invoking new paradigms or assuming a higher onset of duality, growing with N_c. Here we will present an explicit analysis of PI in a soluble field theory, the 't Hooft model [14] which is QCD in 1+1 dimensions in the limit $N_c \to \infty$. We have found that the OPE expression matches the result obtained when summing over the hadronic resonances; the onset of duality is largely independent of the details. Moreover, the parton-deduced OPE expression appears to be *exact* in the chiral limit when all involved quarks except Q are massless. In this case the considered effect is saturated by a single final state. From this perspective it is complementary to the classical small velocity limit in semileptonic decays [3] where a similar saturation holds in the actual $D=4$ QCD for heavy enough b and c quarks, up to power corrections.

The 't Hooft model has already been employed recently for studying inclusive heavy flavor decays, both analytically [15] and numerically [16]. The large-N_c limit harnessed in the analyses, however, washed out the difference between semileptonic and nonleptonic decays. In the case of weak annihilation, additionally, it equates the validity of the OPE for the heavy flavor width with its applicability to the vacuum current-current correlators, and thus can be thought not to be representative for the problems encountered in the nonleptonic decays. PI is a genuinely nonleptonic effect. Invoking the spectator quark makes this a leading-N_c effect and allows a consistent evaluation in the framework of the 't Hooft eigenstate problem. Presentation of full analysis and details of the heavy quark expansion in the model is left for a separate dedicated publication.

The 't Hooft model and heavy quark decays

The 't Hooft model – QCD in 1+1 dimensions with $N_c \to \infty$ – has been described in many papers [14, 17]. While the Lagrangian has the usual form

$$\mathcal{L}_{1+1} = -\frac{1}{4g_s^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum \bar{\psi}_i (i \not{D} - m_i)\psi_i \,, \quad iD_\mu = i\partial_\mu + A_\mu^a T^a \,, \qquad (1)$$

the coupling g_s has dimension of mass making the theory superrenormalizable. A_μ has dimension of mass as in $D=4$ in the adopted normalization; however the fermion fields $\psi(x)$ carry dimension of $m^{1/2}$. In the large N_c limit strong interaction effects are driven by the parameter

$$\beta^2 = \frac{g_s^2}{2\pi}\left(N_c - \frac{1}{N_c}\right) \tag{2}$$

playing the role of the nonperturbative scale $\Lambda_{\rm QCD}$. Additional benefit of the $D=2$ theory is that the observables can be expressed in terms of *bare* masses and couplings, and that the perturbative corrections for hard quantities generate power series. More relevant details and discussions can be found in Refs. [15, 18].

In the limit $N_c \to \infty$ the spectrum of 1+1 QCD consists of mesonic quark-antiquark bound states which are stable under strong interactions. The meson masses are given by eigenvalues of the 't Hooft equation

$$M_n^2 \varphi_n(x) = \left[\frac{m_1^2-\beta^2}{x} + \frac{m_2^2-\beta^2}{1-x}\right]\varphi_n(x) - \beta^2 \int_0^1 {\rm d}y\, \frac{\varphi_n(y)}{(y-x)^2}\,, \tag{3}$$

where $m_{1,2}$ are the masses of the quark constituents, and the integral is understood in the principal value prescription. The solutions are the light-cone wave functions $\varphi_n(x)$.

Polarization operator of the vector $\bar{d}\gamma_\mu u$ current

$$\Pi_{\mu\nu}(q^2) = \frac{1}{\pi}\,\Pi(q^2)\left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\,, \qquad \rho(q^2) \equiv -\frac{1}{\pi}\,{\rm Im}\,\Pi(q^2) \tag{4}$$

(assuming that $m_u = m_d$ and the current is strictly conserved) has a resonance form

$$\Pi(q^2) = \pi \sum_n \frac{f_n^2}{q^2 - M_n^2}\,, \qquad \rho(q^2) = \pi \sum_n f_n^2\, \delta\left(q^2 - M_n^2\right)\,, \tag{5}$$

where the decay constant f_n of a particular meson n is given by

$$f_n = \sqrt{\frac{N_c}{\pi}} \int_0^1 {\rm d}x\, \varphi_n(x)\,. \tag{6}$$

In $D=2$ the exact correlator of vector currents for *massless* quarks is very simple:

$$\Pi(q^2) = \frac{N_c}{q^2}\,, \qquad \rho(q^2) = N_c\,\delta(q^2)\,. \tag{7}$$

In the 't Hooft model at $m_{u,d}=0$ one has $f_0 = \sqrt{N_c/\pi}$ and $M_0 = 0$; for all excitations $f_n = 0$. With nonzero quark masses the spectral density shifts upward, to the mass scale $\sim \beta m_{u,d}$ or $m_{u,d}^2$, and a high-energy tail in ρ appears $\sim N_c(m_u^2 + m_d^2)/q^4$ [18].

We choose the weak decay interaction of the current-current form. Since in 1+1 dimensions the axial current is related to the vector one, $J_\mu^A = \epsilon_{\mu\nu} J_\nu^V$, we simply consider the $V \times V$ interaction, and include both charge- and neutral-current vertices:

$$\mathcal{L}_{\text{weak}} = -\frac{G}{\sqrt{2}} \left(a_1 (\bar{c}\gamma_\mu b)(\bar{d}\gamma^\mu u) + a_2 (\bar{d}\gamma_\mu b)(\bar{c}\gamma^\mu u) \right) + \text{H.c.} \,. \tag{8}$$

While G is an analogue of the Fermi constant, it is dimensionless here. In the example of the parton width the interference term $\propto 2a_1a_2$ is non-leading in $1/N_c$:

$$\Gamma^{\text{quark}} = -\frac{G^2}{2} N_c \left(a_1^2 \Gamma_1^{\text{quark}} + a_2^2 \Gamma_2^{\text{quark}} + 2a_1a_2 \Gamma_{12}^{\text{quark}} \right) \,, \tag{9}$$

where $\Gamma_{1,2}^{\text{quark}}$ are $\sim \mathcal{O}(N_c^0)$, while $\Gamma_{12}^{\text{quark}} \sim \mathcal{O}(1/N_c)$. Yet also the interference term can be leading in $1/N_c$ [19] – if the contributions are non-leading in $1/m_b$. Specifically, considering B^- decays affected by PI, see Figs. 1 and 2, we find $\Gamma_{12}^{\text{PI}} \sim \mathcal{O}(N_c^0)$; the three terms Γ_1, Γ_2 and Γ_{12}^{PI} are all of the same order in $1/N_c$. This allows one to retain only the leading-N_c decay amplitudes.

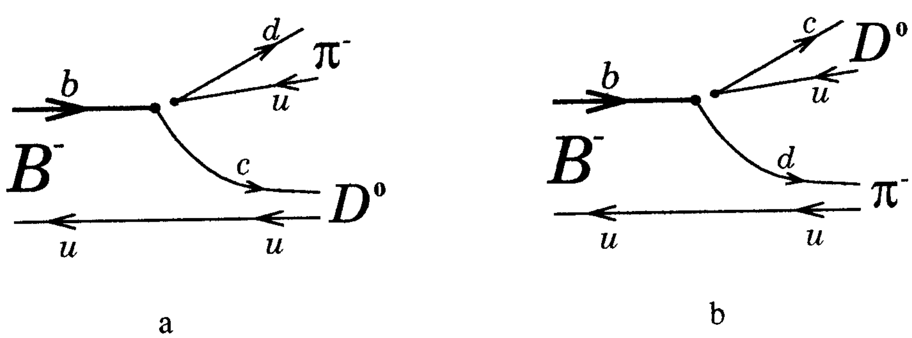

Figure 2: Large-N_c decay amplitudes induced by charge-current (a) and neutral-current (b) terms in the weak decay Lagrangian.

There is something else we can read off from Fig. 2. The two quark-antiquark clusters in the final state are produced with quite different characteristics: while the upper one is produced by a "pointlike" source, the lower is not, since it combines a decay quark with the valence quark; we will refer to the latter as "multiperipheral" production. On general grounds one expects the typical mass distribution for hadrons from the upper vertex to be considerably different from that from the lower one. Normally, the former is much wider spanning over the whole accessible range up to m_b. However, for the special case of vector-like interactions in $D = 2$ the bulk of pointlike-produced hadrons has smaller masses comparable to the corresponding quark masses. Yet interference between the two diagrams in Fig. 2 requires the $d\bar{u}$ and $c\bar{u}$ clusters to have the same mass in both cases. This explains why PI is power suppressed – a property that emerges automatically in the OPE.

We turn now to the 't Hooft model to transform these qualitative observations into specific expressions. Fig. 1 generates – to leading order in the OPE – the operator

$$\hat{\Gamma}^{\rm PI} = -a_1 a_2 \frac{G^2}{K} \left\{ \left(1 - \frac{m_c^2 + m_d^2}{m_b^2}\right) (\bar{b}\gamma_\mu\gamma_5 u)(\bar{u}\gamma_\mu\gamma_5 b) - \frac{2m_c m_d}{m_b^2}\left[(\bar{b}u)(\bar{u}b) + (\bar{b}i\gamma_5 u)(\bar{u}i\gamma_5 b)\right]\right\},$$

$$K = \left[\left(1 - \frac{(m_c + m_d)^2}{m_b^2}\right)\left(1 - \frac{(m_c - m_d)^2}{m_b^2}\right)\right]^{1/2} . \qquad (10)$$

This contribution remains finite for $m_c = m_d = m_u = 0$ and can conveniently be analyzed in this chiral limit. The B^- expectation value of the operators in Eq. (10) has a N_c-favorable color structure and is, therefore given by vacuum factorization:

$$\frac{1}{2M_B}\langle B^-|(\bar{b}\gamma_\mu\gamma_5 u)\,(\bar{u}\gamma_\mu\gamma_5 b)|B^-\rangle \;=\; \frac{1}{2}\, f_B^2 M_B \;, \qquad (11)$$

thus yielding in the chiral limit

$$\Gamma^{\rm PI} \;=\; -2a_1 a_2 \,\frac{G^2}{4}\, f_B^2 M_B \;. \qquad (12)$$

We note that $\Gamma^{\rm PI}$ approaches a constant for $m_b \to \infty$ since $f_B \propto 1/\sqrt{m_b}$. It constitutes a $1/m_Q$ nonperturbative correction to the decay width, in contrast to $1/m_Q^3$ in four dimension. This follows from different canonical dimension of quark fields [15].

There are no $1/m_b$ corrections to this result. This is a peculiarity of two dimensions where the absorptive part of the (di)quark loop in Fig. 1 scales as the momentum to the zeroth power and, thus does not depend on whether quark or meson momentum flows through it. The corrections to the Wilson coefficient or other higher-order operators can induce only terms suppressed by at least two powers of $1/m_b$, if $m_{c,u,d} \ll m_b$.

Let us now consider the decays in terms of hadrons. To leading order in $1/N_c$ the final states are pairs of mesons generically referred to as D_k^0 and π_n^-. The partial decay width $B \to D_k^0 \pi_n^-$ takes the form

$$\Gamma_{kn} \;=\; \frac{G^2}{8M_B^2|\vec{p}\,|}\,\left[a_1^2|\mathcal{A}_k\mathcal{B}_n|^2 \,+\, a_2^2|\mathcal{A}_n\mathcal{B}_k|^2 \,+\, 2a_1a_2\,{\rm Re}\,\mathcal{A}_k\mathcal{A}_n^*\mathcal{B}_k^*\mathcal{B}_n\right] \;, \qquad (13)$$

where $\mathcal{A}$ and $\mathcal{B}$ schematically denote the "multiperipheral" transition amplitudes and the "pointlike" meson creation amplitudes, respectively:

$$\mathcal{A}_k \;\sim\; \langle k|J_\mu|B\rangle \;, \qquad \mathcal{B}_n \;\sim\; \langle n|J_\mu|0\rangle \;. \qquad (14)$$

With $\vec{p}$ denoting the rest-frame momentum of the final state mesons, the PI width is then given by the sum

$$\Gamma^{\rm PI} \;=\; 2a_1a_2\,\frac{G^2}{8M_B^2}\,\sum_{k,n}\,\frac{1}{|\vec{p}\,|}\,\mathcal{A}_k\mathcal{A}_n^*\mathcal{B}_k^*\mathcal{B}_n \;. \qquad (15)$$

Eqs. (13,15) reflect the peculiar two-body phase space $\propto 1/|\vec{p}|$ in two dimensions.

The quantitative matching between the OPE-based calculation and the hadronic saturation of the interference width is most easily seen when all final state quarks u, d, c are massless. While not affecting the OPE analysis, this limit significantly simplifies the expressions for the individual hadronic amplitudes, as explained in Refs. [15, 18]. In the case at hand only $n = 0$ survives for the decay amplitude $\sim a_1$ (Fig. 2a) and $k = 0$ for the amplitude $\sim a_2$ (Fig. 2b). The interference then resides in the single final state containing the lowest lying massless D^0 and π^-. Moreover, the transition amplitudes between the two mesons take a particularly simple form at $q^2 = 0$ [15] in terms of their 't Hooft wavefunctions:

$$q_\mu \frac{1}{2M_l}\langle k|\epsilon_{\mu\nu}J_\nu|H_Q\rangle = -q_z \int_0^1 \mathrm{d}x\, \varphi_k(x)\varphi_B(x)\,, \tag{16}$$

with the 'pointlike' amplitude given by $\mathcal{B}_0 \sim i\epsilon_{\mu\nu} f_0 P_\nu^{(0)}$ yielding

$$\Gamma^{\rm PI} = -2a_1a_2 \frac{G^2 M_B N_c}{4\pi} \left|\int_0^1 \mathrm{d}x\, \varphi_B(x)\right|^2 = -2a_1a_2 \frac{G^2}{4} f_B^2 M_B\,, \tag{17}$$

where we have used the fact that $\phi_0(x) = 1$ for massless quarks. The minus sign emerges since the direction of the vector playing the role of $\vec{q}$ is opposite for the two interfering amplitudes.

Thus, the OPE prediction Eq. (8) is exactly reproduced. Apparently, there is no violation of local duality at all for PI when $m_u = m_d = m_c = 0$! It could be anticipated, for in this limit the only threshold in $\Gamma^{\rm PI}$ occurs at $m_b = 0$. Then the OPE series can have the same convergence properties in Minkowskian as in Euclidean space [11].

With $m_{u,d,c} \neq 0$ the situation becomes more complex since more production thresholds arise. Those exhibit singularities due to the singular two-body phase space in $D = 2$. As explained in detail elsewhere, the predictions of the practical OPE must be compared to the decay probabilities where the threshold singularities are averaged, and for heavy quark decays this procedure is naturally done by smearing over the interval of the heavy quark mass, exceeding the distance between the successive principal thresholds [11, 18]. Since this phase space factor is integrable, the threshold spikes do not affect the width smeared over a mass interval $\Delta m_b \sim 1/m_b$.

The calculation uses the detailed kinematic duality between the partonic and hadronic probabilities. Details of the derivation for the massive case can be found in Ref. [18]. Here we only sketch the basic steps. First, one takes into account that for the bulk of the decays the masses of both 'pointlike' i and 'multiperipheral' j mesons are small compared to m_b:

$$M_i^2 \lesssim m_q^2,\, \beta m_q\,; \qquad M_j^2 \lesssim \beta m_b\,, \tag{18}$$

where m_q generically denotes the final state quark masses. These features anticipated from the quark-level consideration has been demonstrated directly in the 't Hooft model in Refs. [15, 18], where their accuracy was also quantified. It is sufficient for calculating $\Gamma^{\rm PI}$ up to terms $1/m_b^2$. Then we can expand momentum $|\vec{p}\,|$ and transition amplitudes around the massless kinematics:

$$\frac{1}{|\vec{p}\,|} = \frac{2}{M_B}\left(1+2\frac{M_k^2+M_n^2}{M_B}+\,...\right)\;, \quad \mathcal{A}_k(M_n^2) \simeq \mathcal{A}_k(0) + \frac{M_n^2}{m_b^2}\,m_b^2\,\frac{{\rm d}\mathcal{A}_k}{{\rm d}q^2}\bigg|_{q^2=0}\;. \tag{19}$$

The set of sum rules derived in Ref. [15] allows to calculate the sum of the decay probabilities and the leading corrections associated with nonzero meson masses; the latter cancel the increase in $1/|\vec{p}\,|$ in Eq. (15).

For example, neglecting deviation of q^2 from 0 in calculating the 'multiperipheral' amplitudes, we get for the two interfering amplitudes [15, 18]

$$\begin{aligned}
\mathcal{A}_k\mathcal{B}_n \;&=\; \sqrt{\frac{N_c}{2\pi}}\,(M_B^2-M_k^2)\,\int_0^1 {\rm d}x\,\varphi_B(x)\varphi_k(x)\cdot\int_0^1 {\rm d}y\,\varphi_n(y)\\
\mathcal{A}_n\mathcal{B}_k \;&=-\; \sqrt{\frac{N_c}{2\pi}}\,(M_B^2-M_n^2)\,\int_0^1 {\rm d}x\,\varphi_B(x)\varphi_n(x)\cdot\int_0^1 {\rm d}y\,\varphi_k(y)\,.
\end{aligned} \tag{20}$$

Extending summation over all k and n and using the completeness of the 't Hooft eigenstates, we obtain

$$\Gamma^{\rm PI} \;=\; \sum_{k,n}\Gamma^{\rm PI}_{kn} \;\simeq\; -2a_1a_2\,\frac{G^2N_c}{4\pi M_B^2}\left|\int_0^1 {\rm d}x\,\varphi_B(x)\right|^2 \;=\; -2a_1a_2\,\frac{G^2}{4}\,f_B^2\,. \tag{21}$$

This expression is valid up to $\mathcal{O}(1/m_b^2)$ corrections. The leftover effect of the slope of the transition formfactor is quadratic in m_q/m_b [18].

Thus, $\Gamma^{\rm PI}$ *agrees with the expression given by the free quark loop* to the accuracy suggested by the OPE.

For small, yet nonvanishing final state quark masses one can estimate the limitation in local duality for PI as it relates to thresholds. Close to the opening of such a threshold the hadronic width is not exactly reproduced by the OPE. These effects are characterized by the nature of the threshold singularity, the scaling of its residue with m_b and the distance between principal thresholds.

It turns out that these effects essentially depend on the relation between the final state masses. The strongest violation of local duality occurs when one of the mesons is a low excitation while the other has a large mass close to m_b.

The case with $m_c\neq 0$, $m_u=m_d=0$ is somewhat special. For one of the interfering decay amplitudes vanishes right at threshold and $\Gamma^{\rm PI}$ undergoes a finite jump there (details can be found in Ref. [18]):

$$|\delta\Gamma^{\rm PI}_{k0}| \propto \frac{G^2}{2}\,2a_1a_2\,f_B\,m_c\frac{\beta^{9/2}}{M_B^{9/2}}\,\theta(M_B-M_k)\,, \qquad M_{k+1}-M_k \simeq \frac{\pi^2\beta^2}{2M_B}\,. \tag{22}$$

A more elaborate analysis suggests that the relative sign of the two interfering amplitudes alternates for successive thresholds, and we arrive at the following ansatz:

$$\delta\Gamma^{\rm PI}_{\rm osc} \;\propto\; \frac{G^2}{2}\,2a_1a_2\;f_B\,m_c\frac{\beta^{9/2}}{M_B^{9/2}}\,\sum_k(-1)^k\,\theta\left(M_B-\pi\beta\sqrt{k}\right)\,. \tag{23}$$

The oscillating term in PI thus gets damped by at least $1/m_b^5$.

For $m_{u,d}$ nonzero, the picture changes essentially in two respects: neither decay amplitude vanishes at threshold, however the phase space factor softens to $[2m_\pi(M_B - M_{\rm thr}^{(k)})]^{-1/2}$ for the decays $B \to D^{(k)} + \pi$ at $m_\pi \neq 0$. One then finds

$$\delta\Gamma^{\rm PI}_{\rm osc} \propto \frac{G^2}{2}\, 2a_1a_2\, \frac{m_c m_\pi^{1/2}\beta^5}{M_B^5} \sum_k (-1)^k \frac{\theta\left(M_B - M_{\rm thr}^{(k)}\right)}{\sqrt{M_B - M_{\rm thr}^{(k)}}}\,, \qquad M_{\rm thr}^{(k)} \simeq m_\pi + \pi\beta\sqrt{k}\,. \quad (24)$$

With all final-state masses non-zero, many additional thresholds open up and the dynamical landscape becomes in general rather complex. Yet one observes that the contributions from the final states where both meson masses constitute a finite fraction of m_b are suppressed by even higher powers of $1/m_b$ [18].

Conclusions

Let us first describe the hadronization picture suggested by the large-m_b analysis. We observe that duality applies to the quark-antiquark pairs where each energetic quark (c or d) is combined with the wee spectator antiquark or slow $\bar{u}$ produced in the weak vertex. The completeness of the hadronic states – or, in other words, the duality between the parton-level and mesonic states – is achieved already for a single fast moving decay quark when it picks up a slow spectator. The 'hardness' of these processes determining the applicability of the quasifree approximation, depends on the energy of the fast quark rather than on the invariant mass of the pair. For actual hadrons, the slow spectator(s) compensate the color of the initial static heavy quark. This initial distribution of the color field singles out the rest frame and introduces the new scale, energy *vs.* invariant mass.

In the full graph for the meson decay which would explicitly include propagation of the spectator, there is always a color mate for any quark in the "partonic" part of the diagram. Therefore, considering the colored diquark loop as nearly free yields the correct result for the properly formulated problem.

In this note we have analyzed PI as a spectator-dependent nonleptonic effect for heavy flavor decays within the 't Hooft model. We have found the OPE expression to match the result obtained when summing over exclusive final states at least up to terms $(\Lambda_{\rm QCD}/m_b)^3\,\Gamma_B$ which is the accuracy we adopted. This is the first analysis where the peculiarities of nonleptonic decays widths of heavy flavors emerge compared to their semileptonic decays or processes like e^+e^- annihilation. We did not encounter specific problems for applying the OPE, in accord with the general arguments justifying it for all types of fully inclusive decay widths [7, 12, 15, 18].

We estimated the effects of violation of local duality in PI in the heavy quark limit. It turned out to be strongly suppressed scaling like $1/m_b^5$. A novel case was identified where local parton-hadron duality is exact. Although applicable only in two dimension for large N_c, it complements the classical case of small velocity semileptonic $b \to c$ decays [3] which was instrumental in developing the modern applications of the heavy quark expansion.

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