Restless pions, nuclear forces and statistical noise in lattice QCD

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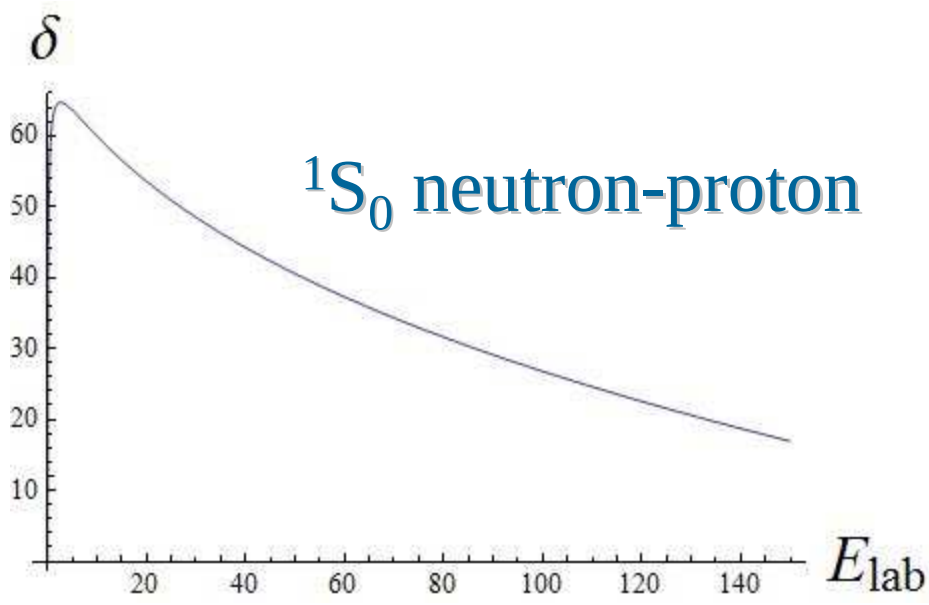
What do we know ?

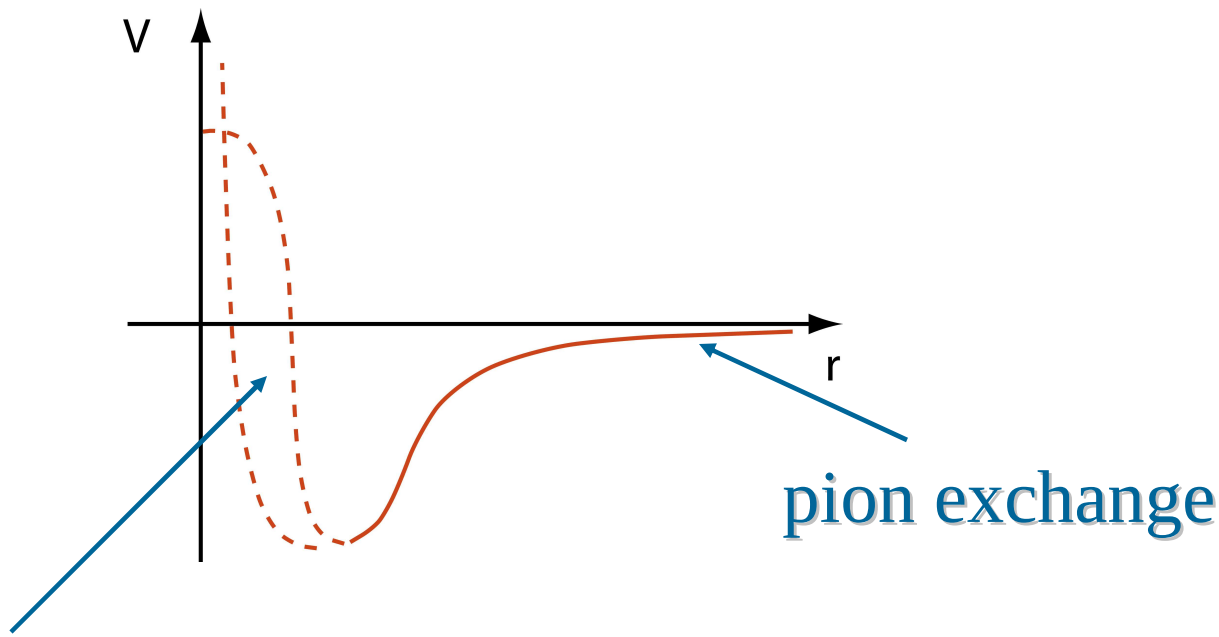
1) NN phase shifts



$^1S_0$ neutron-proton

What do we know ?

2) Several potentials that fit them



What do we know ?

3) These potentials explain a lot but not everything

- NNν, NNγ, couplings — few % on νd
- NNN forces — ~5% of nuclei binding
- NY forces — strangeness in neutron stars
- ...

The theory of strong nuclear forces does not describe (yet) nuclear forces …

Matter of principle and matter of practice

0, 1 baryon

- hadron masses
- decay constants
- weak matrix elements
- pion scattering
- "exotic" particles
- …

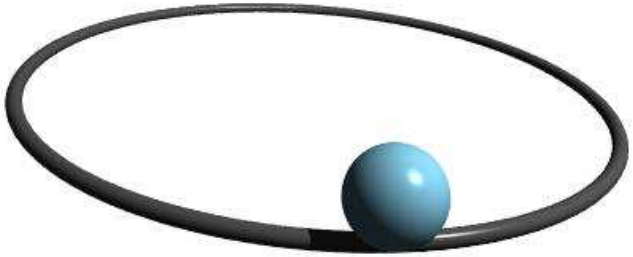
2 or more baryons

- NN phase shifts
- hyperon interactions
- pion-nucleon couplings
- electroweak “exchange currents”
- three-body forces
- …

Scattering through finite volumes: the Luscher method

(Marinari, Hamber, Parisi, Rebbi)

one particle



Periodic boundary conditions: box is a torus

Energy levels at $E_n = \sqrt{\left(\frac{2\pi n}{L}\right)^2 + m^2}$

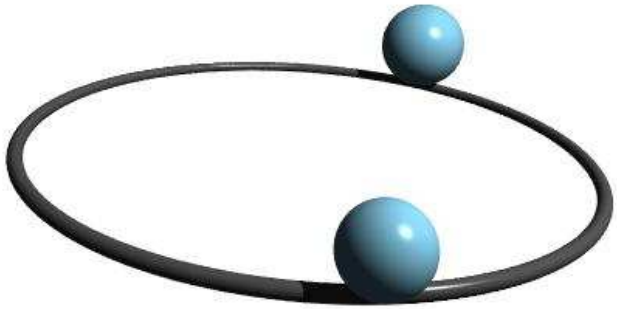
Scattering through finite volumes: the Luscher method

(Marinari, Hamber, Parisi, Rebbi)

two particles

$$\sqrt{M\Delta E}\,\cot\delta(\Delta E) = \frac{1}{\pi L}\,\mathcal{S}\left(\frac{M\Delta E L^2}{4\pi^2}\right)$$

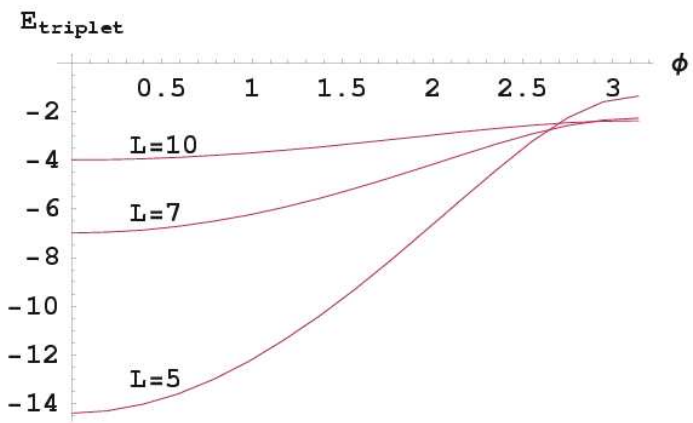
known function

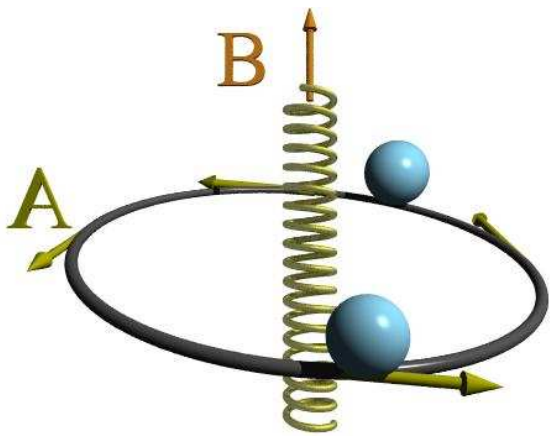


Learn about the deuteron in boxes smaller than the deuteron

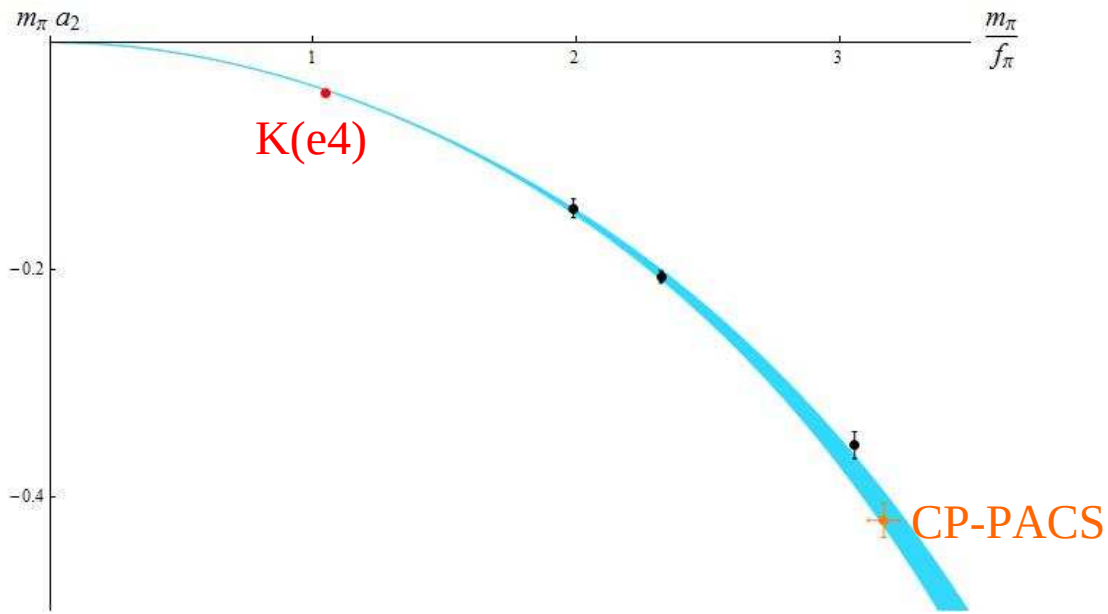
$\delta(\Delta E<0)$: unphysical region

Twisted boundary conditions = Aharonov-Bohm





“Gold-platted” scattering observable: I=2 ππ



$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1+\frac{3m_\pi^2}{16\pi^2 f_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2}+l_{\pi\pi}(\mu)\right)+\cdots\right]$$

$8(l_1+l_2)+2(l_3-l_4)$

weighted fit: $l_{\pi\pi} = 3.3(6)(3)$

different weigths

$m_\pi a_2 = -0.0426\ (6)(3)(18)$

$l_{\pi\pi}$

1-loop – 2-loop
w/o counterterm

K(e4): $m_\pi a_2 = -0.0454(31)(10)(8)$

theoretical

χPT predicts discretization errors (a^2) ~ 1% (D. O'Connel, A. Walker-Loud, R. V. Water, J. Chen)

Finite volume ($e^{-m_\pi L}$) ~ 1% (P.B. , I. Sato & Walker-Loud)

NEW $m_\pi a_2 = -0.04330(42)$

K(e4): $m_\pi a_2 = -0.0454(31)(10)(8)$

theoretical

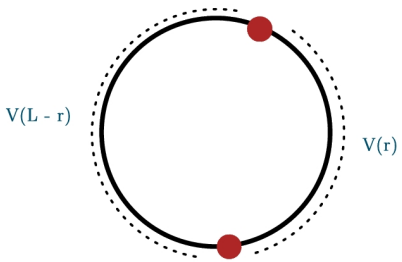
Other processes

πK:

$$m_\pi a_{3/2} = -0.0574 \pm 0.0026 \pm 0.0058$$
$$m_\pi a_{1/2} = \quad 0.1725 \pm 0.0017 \pm 0.0156$$

Fitting two nucleons inside a box

pions "around the world" are not much of a problem



$$\Delta E = \frac{1}{L^3}\int_{L^3} dr V_L(r) = \frac{1}{L^3}\int dr\, V(r) = \frac{4\pi a}{ML^3}$$

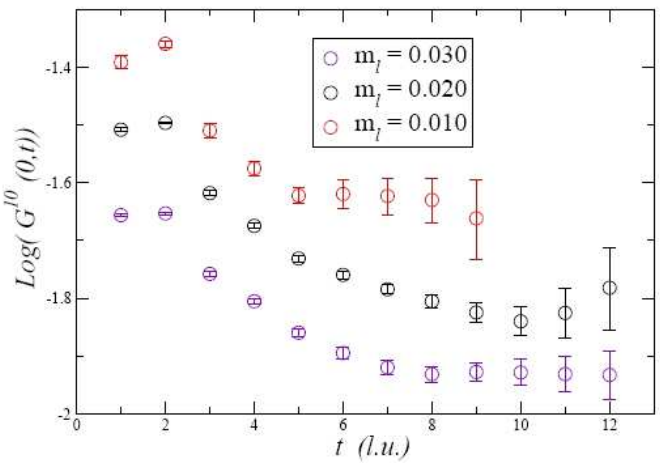
(P.B & I. Sato)

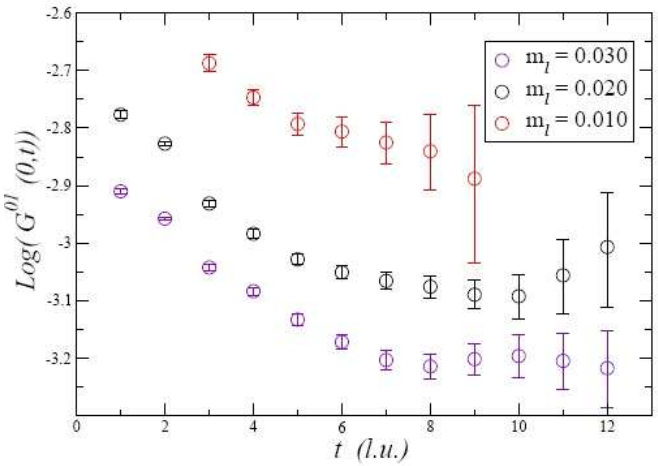
$$V_L(r) = V(r) + V(r+L) + V(r+2L) + \cdots$$

realistic m_π, L=5 fm ~ 10% error but **hard to estimate**

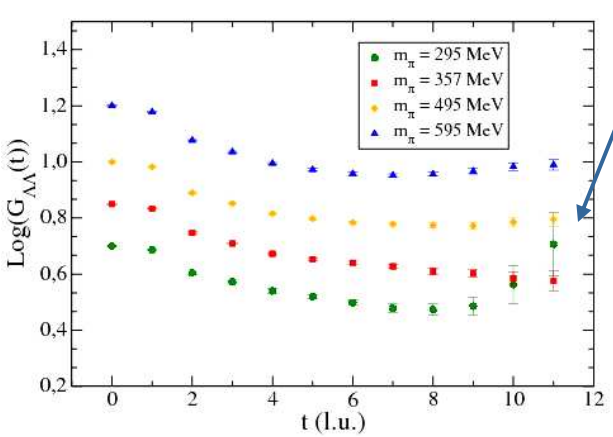
MUCH smaller errors with larger m_π

Nucleon-nucleon

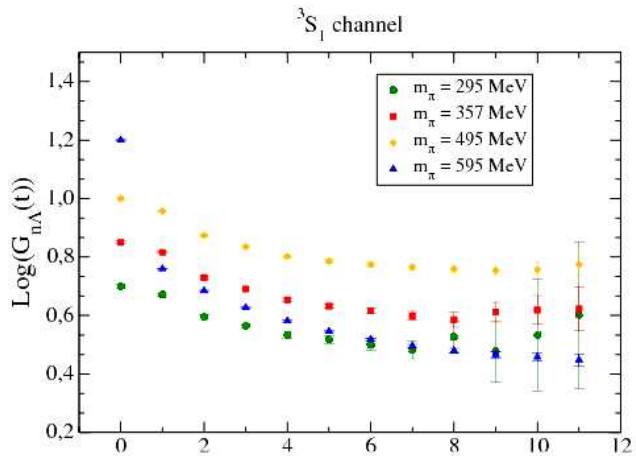




Similarly for hyperons



dibaryon ?



ΛΛ

NΛ triplet

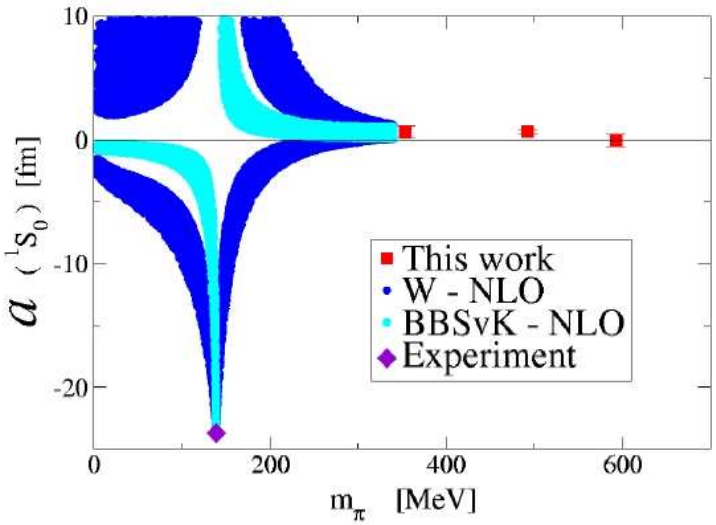
Nucleon-nucleon

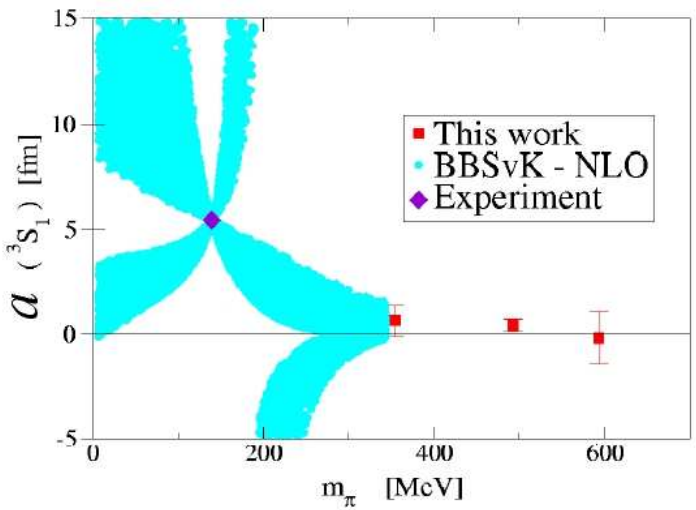
"natural" $|a| < 1$ fm for $350 < m_\pi < 600$ MeV

$\downarrow$

$a=5.4$ fm or 20 fm for $m_\pi=138$ MeV
is indeed fine tuned

Chiral "extrapolation"





- no anchor at $m_\pi = 0$
- wild behavior of the scattering length with m_q

The crucial problem is the large statistical errors

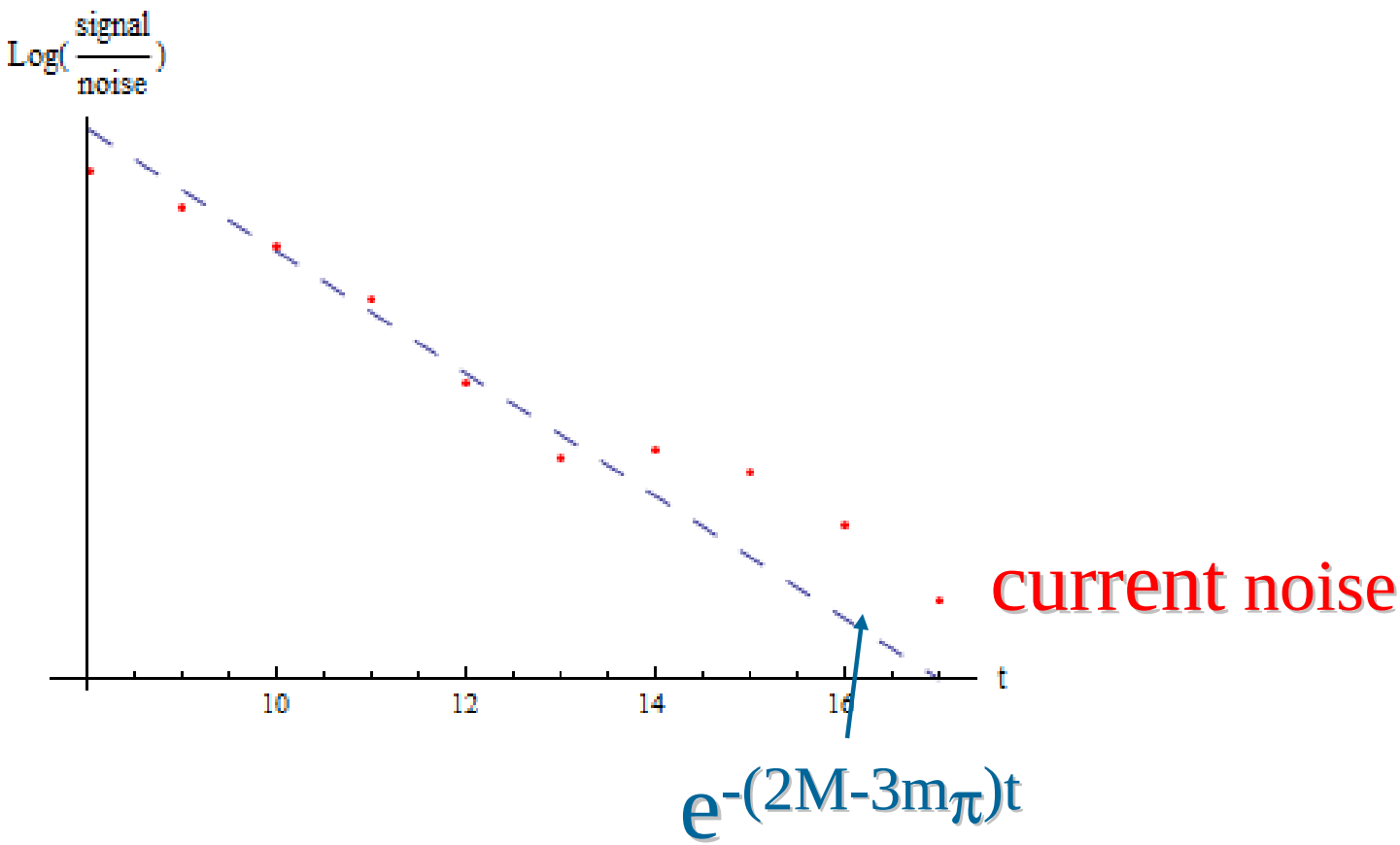
signal: $$C(t) = < q^6(t)\, \bar{q}^6(0) > \sim e^{-2Mt}$$

2 baryons

error: $$\sigma^2(t) = < q^6(t)\bar{q}^6(t)\, q^6(0)\bar{q}^6(0) > \sim e^{-6m_\pi t}$$

6 pions

$$\frac{\text{signal}}{\text{noise}} \sim \sqrt{N}\; e^{-(2M_N - 3m_\pi)t}$$

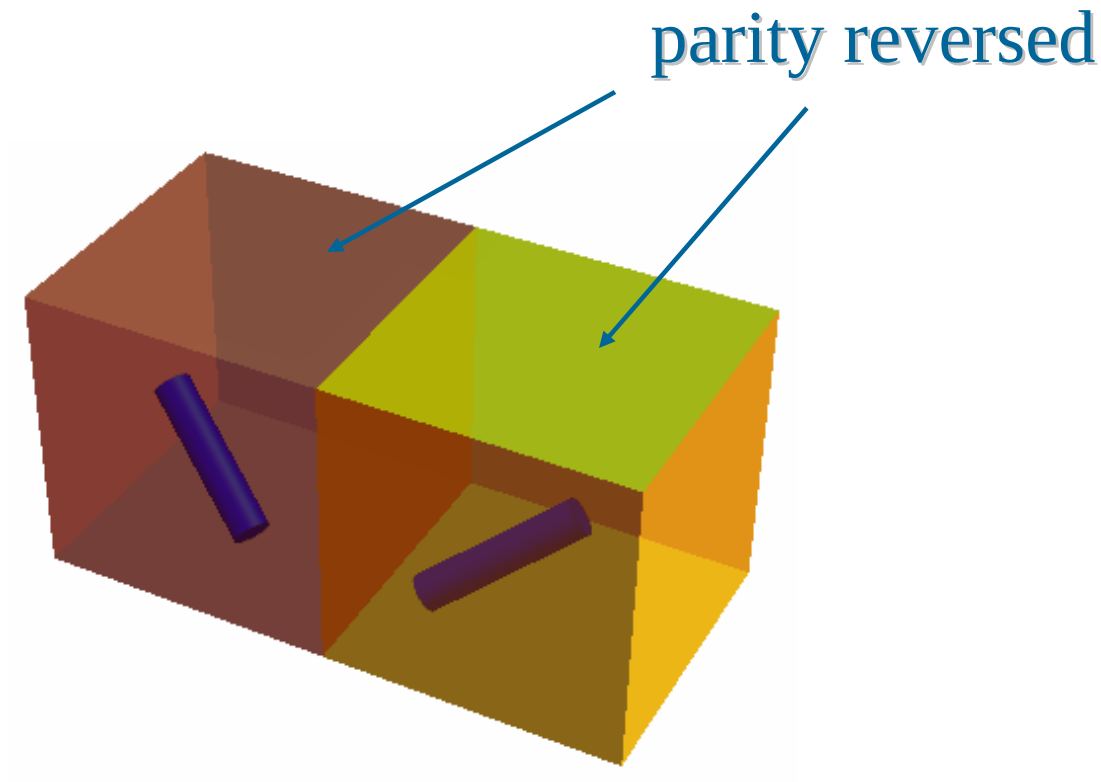

Log(signal / noise)
10
12
14
16
t
current noise
$e^{-(2M-3m_\pi)t}$

$$\frac{\text{signal}}{\text{noise}} \sim \sqrt{N}\, e^{-(2E_N - 3E_\pi)t}$$

If the minimum pion energy was larger m_π, the signal would be better

$\pi(-z) = -\pi(z)$?

Parity orbifold (P.B. +Walker-Loud)

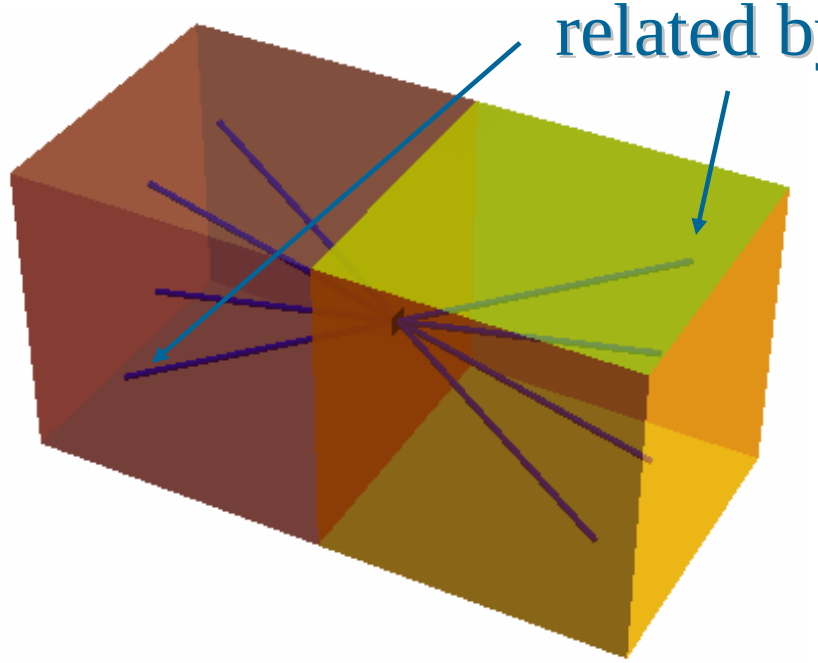


$\pi(-z) = -\pi(z)$ → minimum pion energy is

$$E_\pi = \sqrt{\left(\frac{\pi}{L}\right)^2 + m_\pi^2}$$

Parity orbifold: pinhole

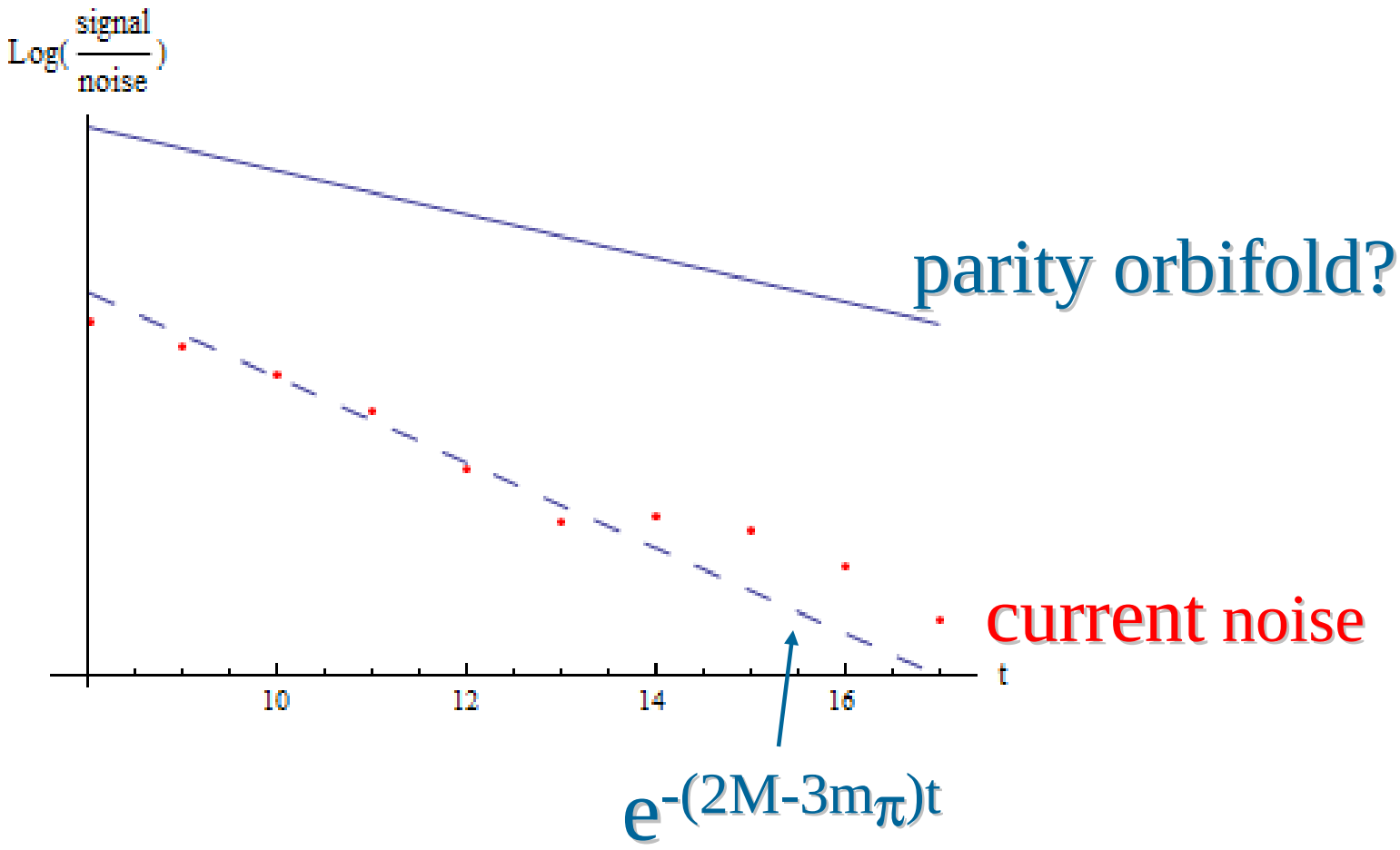
these points are related by parity



$\pi(-x,-y,-z) = -\pi(x,y,z)$ → minimum pion energy is

$$E_\pi = \sqrt{3\left(\frac{\pi}{L}\right)^2 + m_\pi^2}$$

Expected impact on noise



In the case of the simpler "mirror" orbifold

$$A_3(x,y,-z,t) = -A_3(x,y,z,t)$$

$$A_{0,1,2}(x,y,-z,t) = A_{0,1,2}(x,y,z,t)$$

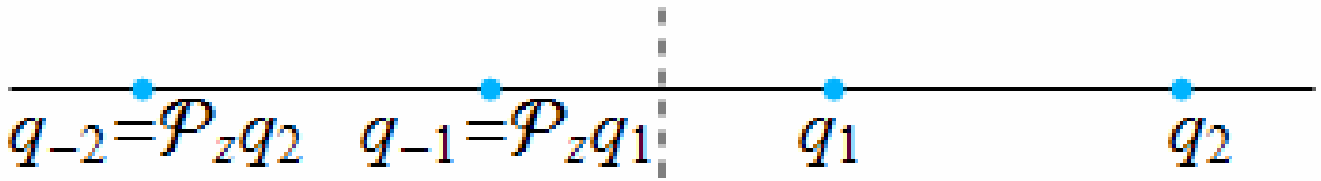
$$q(x,y,-z,t) = \wp_z\, q(x,y,-z,t)$$

$$\bar{q}(x,y,-z,t) = \bar{q}(x,y,-z,t)\wp_z$$

$$\wp_z = i\gamma_5\gamma_3$$

$$\pi(x,y,-z,t) = -\pi(x,y,z,t)$$

$$N(x,y,-z,t) = \sigma_3 N(x,y,z,t)$$



$$S = \kappa\left[\bar{q}_{-1}(\gamma_3 - r)\, q_1 - \bar{q}_1(\gamma_3 + r)\, q_{-1}\right] + a^4\left(\bar{q}_1 q_1 + \bar{q}_{-1} q_{-1}\right) + \cdots$$
$$= -2\kappa\left[\bar{q}_1(\gamma_3 + r)\, \wp_z q_1\right] + 2a^4 \bar{q}_1 q_1 + \cdots$$

Not γ_5 hermitian

In the case of the simpler "pinhole" orbifold

$$A_i(-\vec{r},t) = -A_i(\vec{r},t)$$

$$A_0(-\vec{r},t) = A_0(\vec{r},t)$$

$$q(-\vec{r},t) = \wp\, q(\vec{r},t)$$

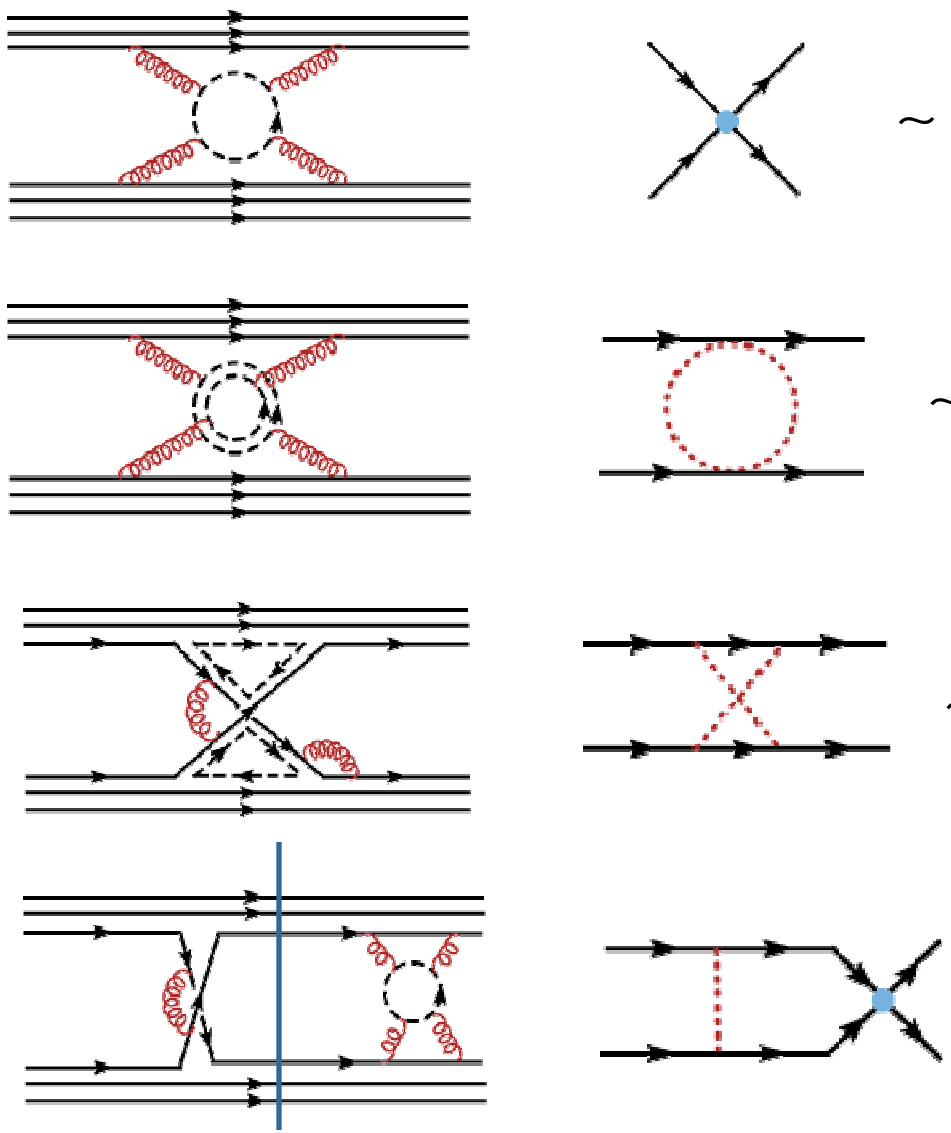
$$\bar{q}(-\vec{r},t) = \bar{q}(\vec{r},t)\wp$$

$$\wp = \gamma_0$$

$$\pi(x,y,-z,t) = -\pi(x,y,z,t)$$

$$N(x,y,-z,t) = N(x,y,z,t)$$

Twisting only the valence quarks (Sachrajda&Villadoro, P.B. & J. Chen)



$\sim e^{-\Lambda_{QCD} L}$

$\sim e^{-m_\pi L}$

$\sim e^{-m_\pi L}$

dependence on sea b.c. exponentially suppressed

Summary

- No nuclear physics on the lattice without new methods
- Anisotropic lattices, Luscher-Wolff and/or **ORBIFOLDS**
- A lot is hinging on this, it should be tried