JLab Theory Seminar, Oct 28 2007

Probing the SM and beyond with semi-leptonic meson decays

Vincenzo Cirigliano

Theoretical Division, Los Alamos National Laboratory

Outline

- Low-energy CC processes within and beyond the SM

- π (K) $\rightarrow e\nu$ / π (K) $\rightarrow \mu\nu$ to $O(e^2p^4)$ in ChPT

  - lepton universality

  VC, I.Rosell: JHEP 10 (2007) 005 [arXiv:0707.4464]
  arXiv:0707.3439 to appear in PRL

- $K \rightarrow \pi \ell \nu$ decays to $O(e^2p^2, p^4, p^6)$ in ChPT

  - lepton universality, quark mass ratios, CKM unitarity

  VC, M. Giannotti, H. Neufeld, in progress
  VC, G. Ecker, R. Kaiser, M. Knecht, H. Neufeld, A. Pich, H. Pichl, J. Portoles ['02- '05]

Low energy CC processes within and beyond the SM

CC processes and physics beyond the SM

- View SM as an EFT valid up to scale Λ [where new d.o.f. appear]

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \sum_{d\geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} O_n^{(d)}[\psi_i, A_k, H]$$

$$\downarrow$$

The effect of new d.o.f. at low E is parameterized
by gauge-invariant dim > 4 local operators

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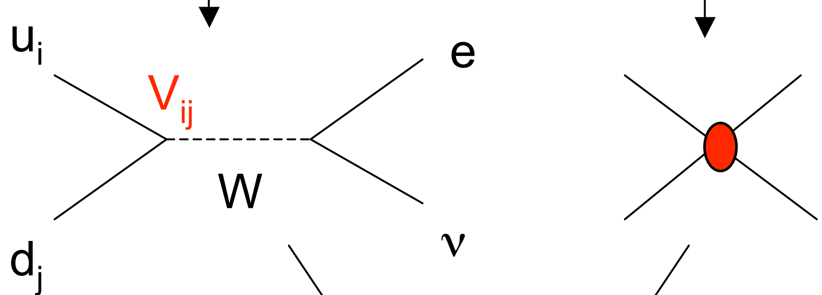

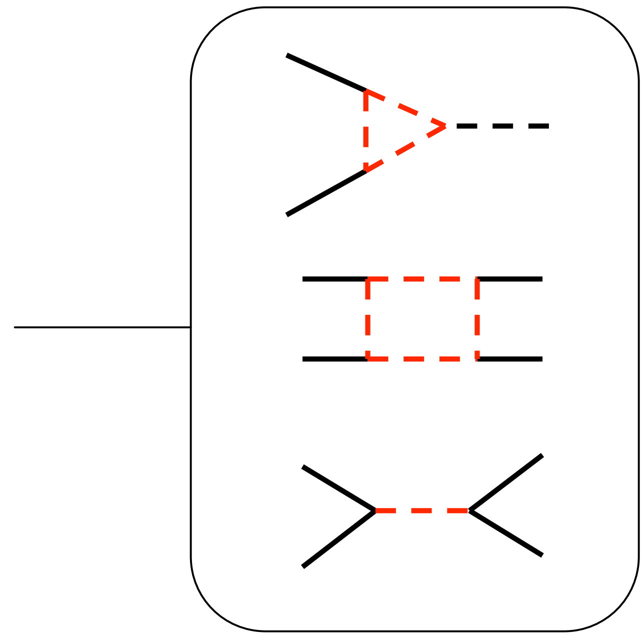

$$\boxed{G_F^{\text{eff}} \sim \frac{g^2\, V_{ij}}{M_W^2} + \frac{c_n}{\Lambda^2}}$$

CC processes and physics beyond the SM

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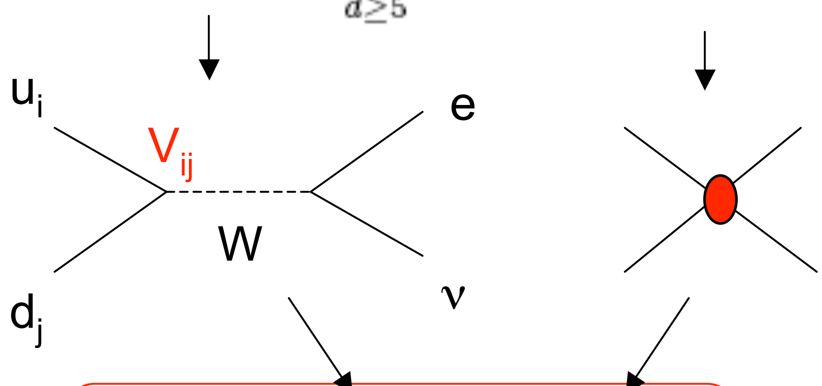
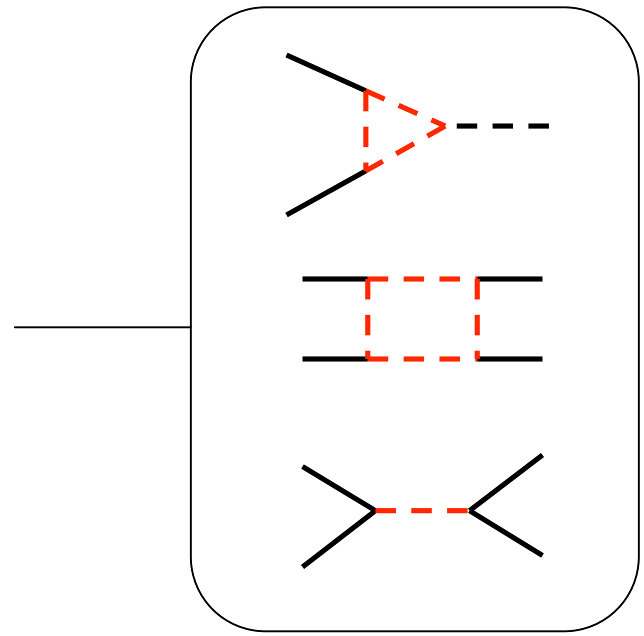




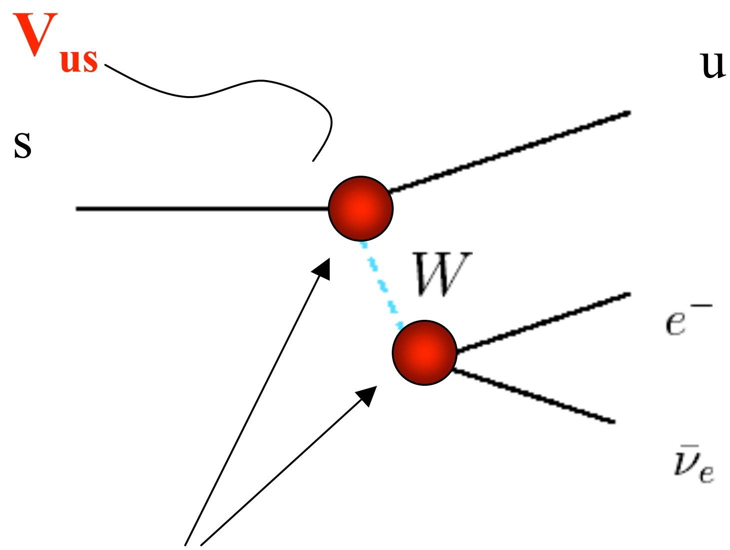
$$\boxed{G_F^{\text{eff}} \sim \frac{g^2\, V_{ij}}{M_W^2} + \frac{c_n}{\Lambda^2}}$$

- Expected size of deviations from SM? For new physics in the TeV range:

$$\frac{\delta G_F^{\text{eff}}}{G_F} \sim \frac{c_n}{g^2} \frac{M_W^2}{\Lambda^2}$$ $< 10^{-2} - 10^{-3}$ depending on size of c_n

Experimental and theoretical (SM) precision is reaching this level !!

What are we after:



- Probe nature of weak vertices

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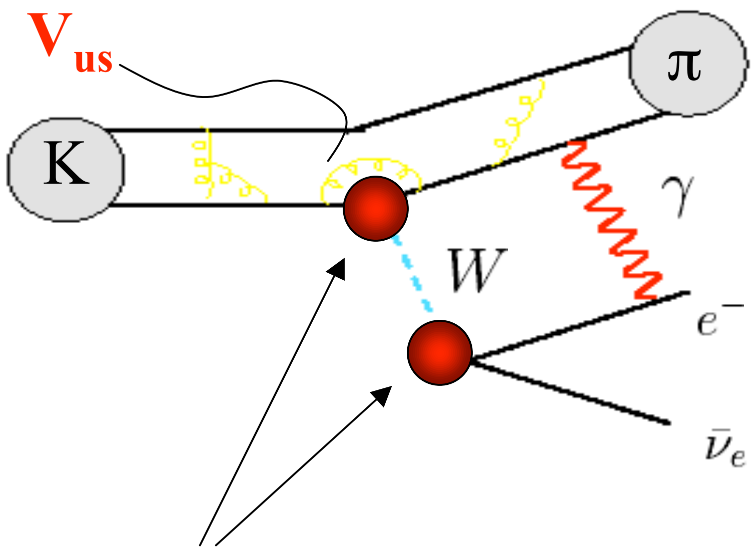
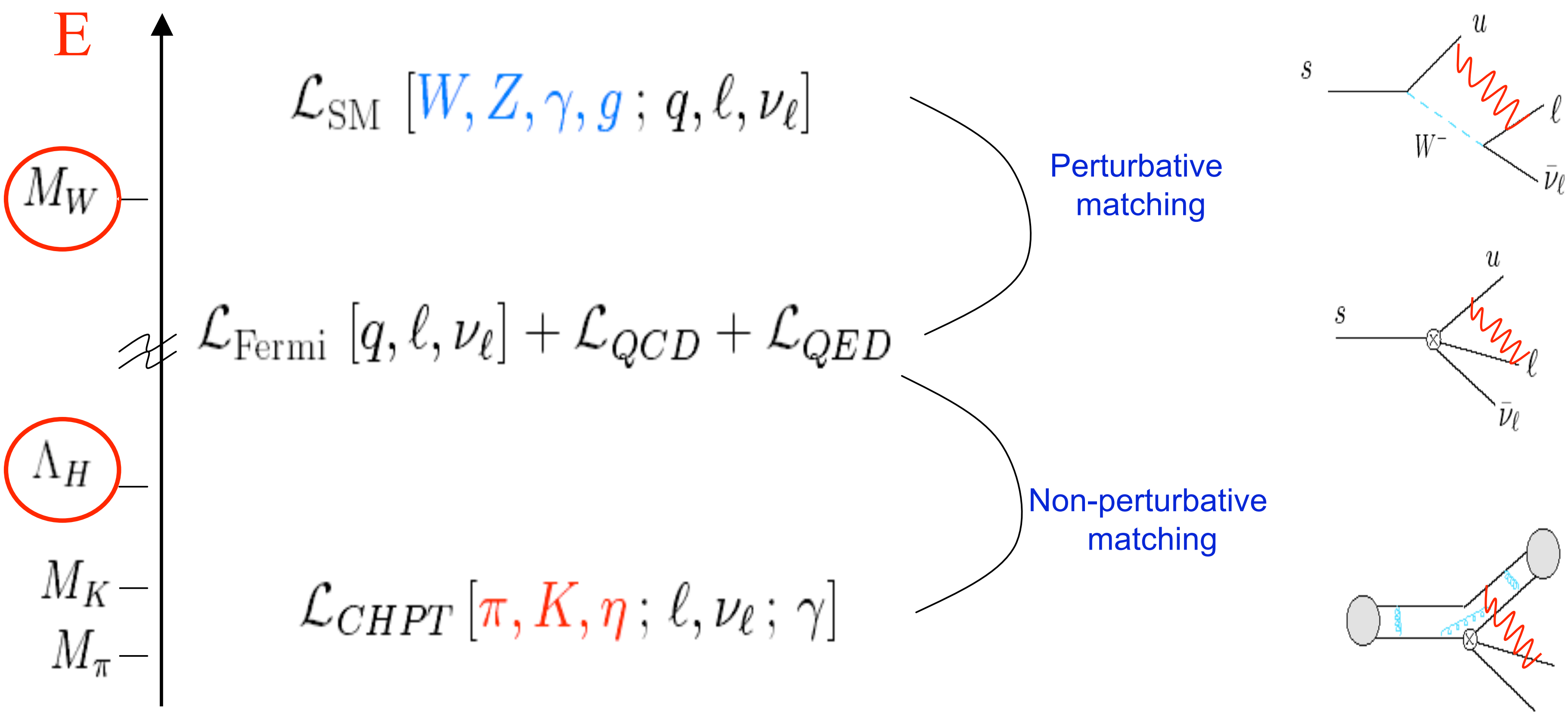


- Probe nature of weak vertices through (despite?) hadronic decays
- TH tools: ChPT, matching techniques ($1/N_C$, …), Lattice QCD
- I will discuss
  - π (K) $\rightarrow \ell\, \nu$ decays as probes of lepton universality;
  - K $\rightarrow \pi\, \ell\, \nu$ decays as probes of: (1) lepton universality; (2) V_{us} and "CKM unitarity"; (3) ratios of light quark masses.

Chiral Perturbation Theory

- Special role of π,K,η: GB of SχSB → lightest hadrons
- Effective theory: integrate out heavy states → local interactions dictated by symmetry considerations: $SU(3)_L \times SU(3)_R \to SU(3)_V$



- In the chiral EFT the lagrangian and amplitudes are systematically expanded in powes of:

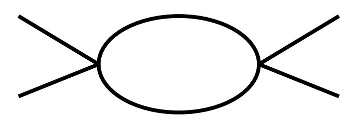
momenta [GB nature] , m_{quark} + ew couplings

$$\boxed{p^2 \sim \frac{p_{ext}^2}{\Lambda_H^2} \approx \frac{m_P^2}{\Lambda_H^2} \qquad m_q \sim m_P^2 \sim p^2 \qquad G_F\,, \quad e}$$

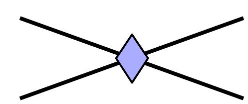
$$\Lambda_H \sim 1\, GeV$$

- To a given order:

- loops (leading IR singularities)



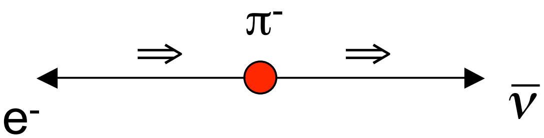
- "contact" terms, LECs (UV div.+ finite part, encoding short distance physics) to be determined from expt or estimated with non-perturbative technique



π (K) $\rightarrow e\nu$ BRs to O(e^2p^4)

The ratios $R_{e/\mu}^{(P)} = \Gamma(P \to e\nu) / \Gamma(P \to \mu\nu)$ $P = \pi, K$

- Helicity suppressed in the SM (V-A structure), zero if $m_e \to 0$



Sensitive probe of pseudo-scalar currents and lepton universality
Realistic chance to detect or highly constrain new physics:

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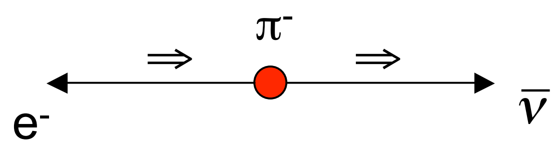

Sensitive probe of pseudo-scalar currents and lepton universality
Realistic chance to detect or highly constrain new physics:

- Experimental status: expect ~ 1 order of magnitude improvement

[TRIUMF, PSI 1993 average]   [TRIUMF, PSI]

$$R_{e/\mu}^{(\pi)} = (1.230 \pm 0.004) \cdot 10^{-4}$$ δR/R: $0.3\% \to < 0.05\%$

$$R_{e/\mu}^{(K)} = (2.457 \pm 0.032) \cdot 10^{-5}$$ δR/R: $1\% \to < 0.3\%$

[Expts from ‘70s, NA48/2, KLOE, 2007 average]   [NA48/3]

- Theory status:

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ldots\right]$$

- $F_{\pi,K}$ drops in the e/μ ratio
- Structure dependence appears only through EM correction $\Delta_{e^2p^4}^{(P)}$

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- $F_{\pi,K}$ drops in the e/μ ratio
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$$R_{e/\mu}^{(\pi)} = \begin{cases} (1.2352 \pm 0.0005) \cdot 10^{-4} \\ (1.2354 \pm 0.0002) \cdot 10^{-4} \end{cases}$$

Marciano-Sirlin 93

Finkemeier 96

$$R_{e/\mu}^{(K)} = (2.472 \pm 0.001) \cdot 10^{-5}$$

Finkemeier 96

δR/R ~0.04% in both π and K cases!

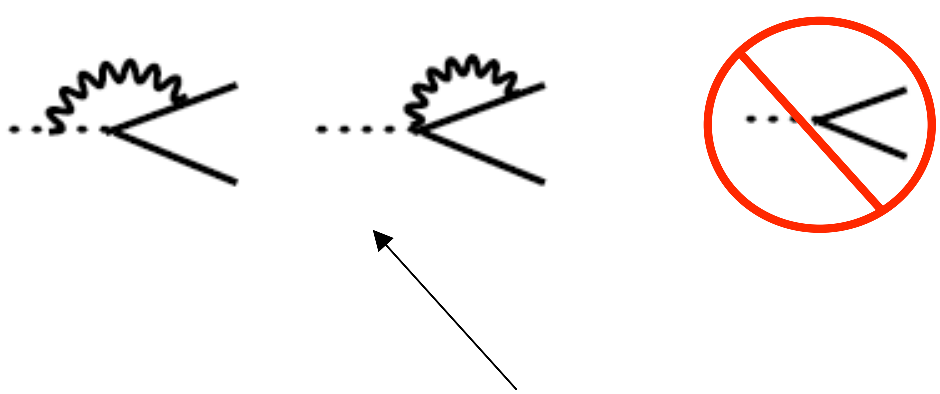
Goal: reproduce/improve these results using ChPT

ChPT analysis of $R_{e/\mu}^{(\pi,K)}$

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ldots\right]$$

- EFT to $O(e^2p^2)$: point-like pion and kaon [Kinoshita 1959]

  Example: corrections induced by virtual photons



No counterterm: its contribution cancels in the ratio !

Consider only diagrams with non-trivial dependence on the charged lepton mass

ChPT analysis of $R_{e/\mu}^{(\pi,K)}$

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} \ + \ \Delta_{e^2p^4}^{(P)} \ + \ ....\right]$$

- EFT to $O(e^2p^4)$: π (K) $\rightarrow \ell\, \nu\, \gamma$ to $O(p^4)$ + loops with virtual photons:

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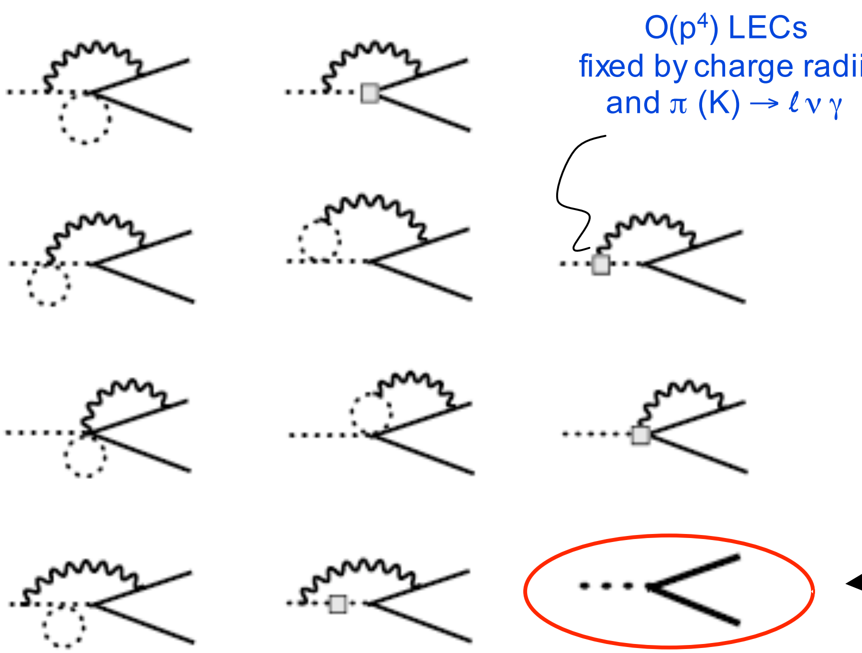


All vertices determined by chiral symmetry and gauge invariance

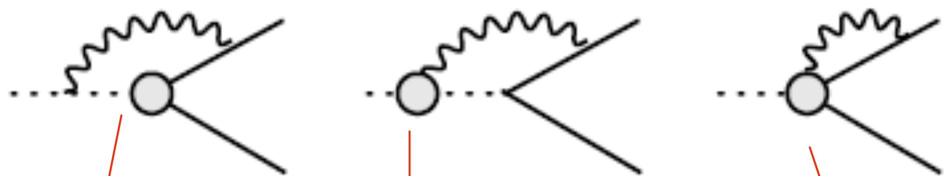
One- and two-loop diagrams ⇒ model-independent single and double logs

$O(e^2p^4)$ counterterm (same for pion and kaon)

The loops

- Effect of two loop diagrams:

  - Renormalization of $F_{\pi,K}$ and $m_{\pi,K}$ in 1-loop amplitude
  - Genuine correction to the amplitude takes the form of "effective" 1-loop diagrams



O(p^4) d-dimensional off-shell ChPT vertices (local and non-local)

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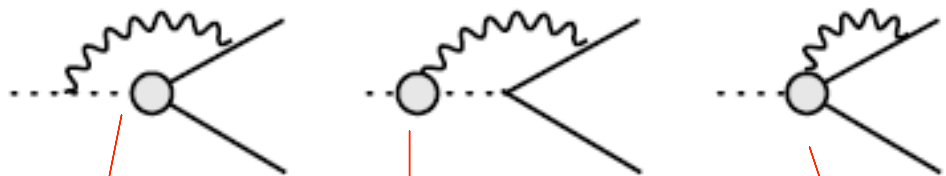
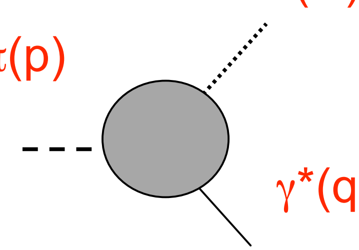


O(p^4) d-dimensional off-shell ChPT vertices (local and non-local)

- Convolution representation:

$$\int \frac{d^d q}{(2\pi)^d}\, K(p,q,p_\ell)\, \Pi^{ChPT}_{\pi W\gamma}(q^2,w^2)$$

O(p^4) w=p-q



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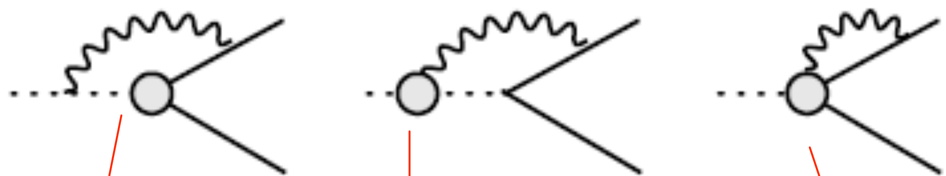

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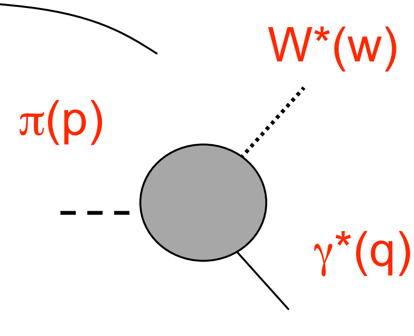

- Integrate, get single and double χ-logs, but also UV divergence

The counterterm

- Physical origin of counterterm: incorrect UV behavior of $\Pi^{\mathrm{ChPT}}_{\pi W \gamma}$

$$T^{ChPT} = \int \frac{d^d q}{(2\pi)^d} K(p, q, p_\ell)\, \Pi^{ChPT}_{\pi W \gamma}(q^2, w^2) + T^{\mathrm{CT}}$$

Removes UV divergence and restores "correct" finite part

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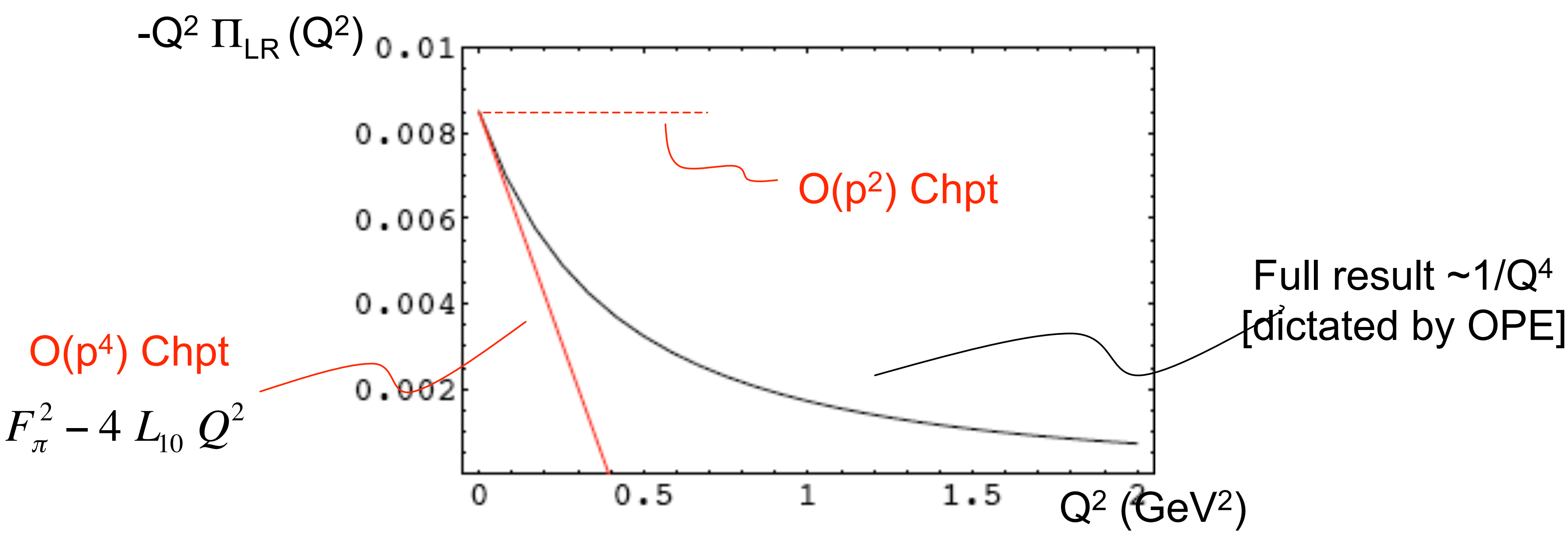
- Simplest example: $$(\Delta m_\pi^2)_{EM} = -\frac{3\alpha}{4\pi F^2}\int_0^\infty dQ^2 Q^2\, \Pi_{LR}(Q^2)$$



The counterterm

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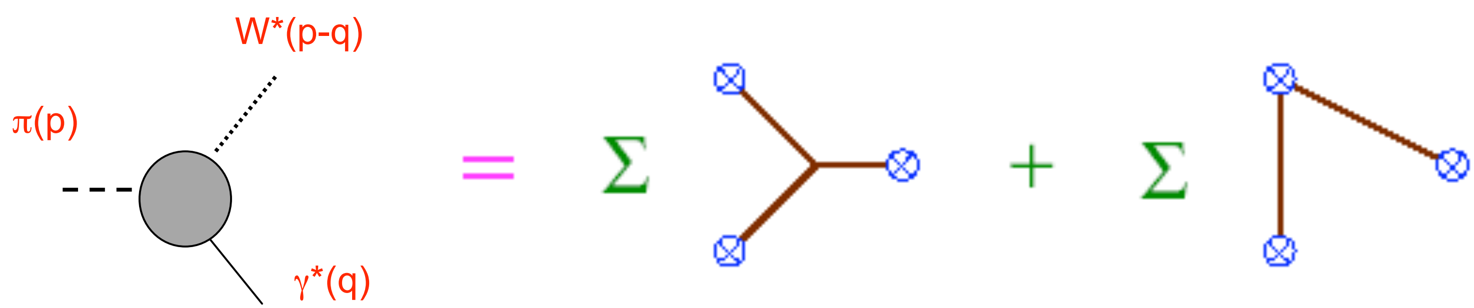
$$T^{ChPT} = \int \frac{d^d q}{(2\pi)^d} K(p,q,p_\ell)\, \Pi_{\pi W\gamma}^{ChPT}(q^2,w^2) + T^{\mathrm{CT}}$$

- To estimate T^{CT} need representation of $\Pi_{\pi W\gamma}$ valid for all Euclidean momenta. We use meromorphic form (truncated $1/N_C$ expansion):



- correct low Q^2 behavior is built-in
- fix resonance parameters by imposing correct QCD asymptotic behavior (OPE)

Finite number of narrow resonances

- Determine T^{CT} by matching QCD_∞ and ChPT

$$T^{\mathrm{CT}} = \int \frac{d^d q}{(2\pi)^d}\, K(p,q,p_\ell)\, \left[\Pi_{\pi W\gamma}^{QCD_\infty}(q^2,w^2) - \Pi_{\pi W\gamma}^{ChPT}(q^2,w^2)\right]$$

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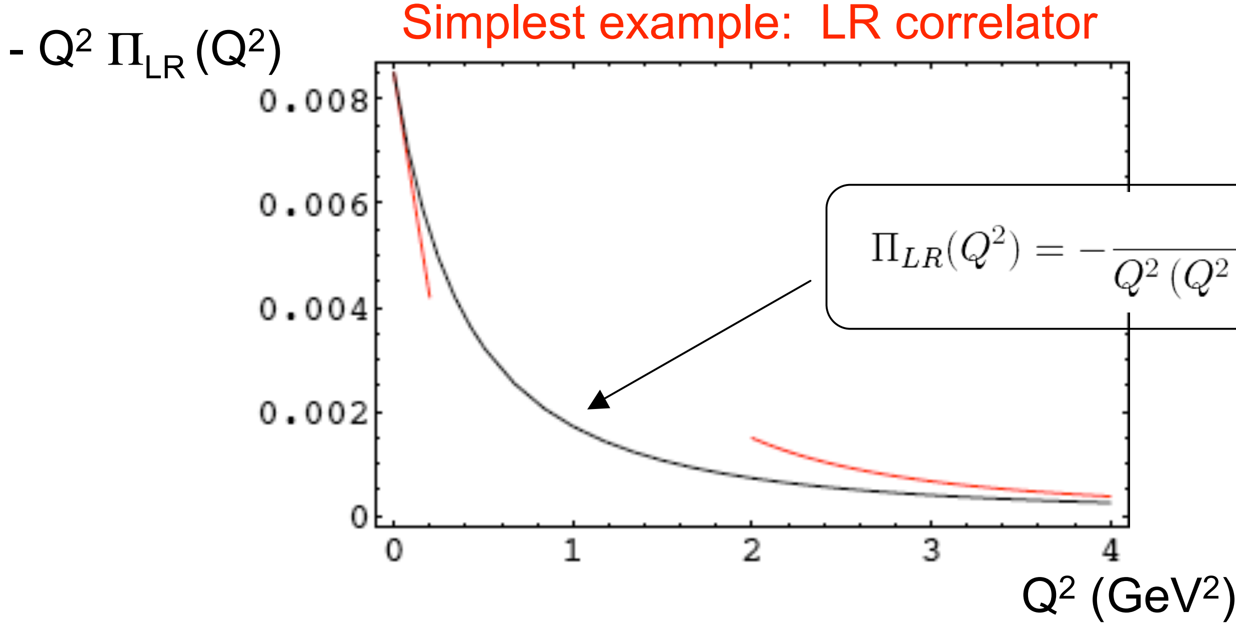
- Once fix low and high Q^2 behavior, *integral* is stable at the 30% level

Simplest example: LR correlator



$$\Pi_{LR}(Q^2) = -\frac{F^2\, M_V^2\, M_A^2}{Q^2\,(Q^2+M_V^2)(Q^2+M_A^2)}$$

- Determine T^{CT} by matching QCD_{∞} and ChPT

$$T^{\rm CT} = \int \frac{d^d q}{(2\pi)^d} \, K(p, q, p_\ell) \, \left[\Pi_{\pi W \gamma}^{QCD_\infty}(q^2, w^2) \; - \; \Pi_{\pi W \gamma}^{ChPT}(q^2, w^2) \right]$$

- Once fix low and high Q^2 behavior, *integral* is stable at the 30% level

- CT gives a small contribution [~10% of the full $O(e^2p^4)$ result] $\Rightarrow$ integral is dominated by low Q^2 region

- Result captures single-log renormalization scale dependence

- We conservatively assign 100% uncertainty to the CT contribution

$R_{e/\mu}^{(\pi,K)}$ results

$$R_{e/\mu}^{(P)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2 \times \left[1 + \Delta_{e^2p^2}^{(P)} + \Delta_{e^2p^4}^{(P)} + \ ....\right]\left[1 + \Delta_{LL}\right]$$

		$(P = \pi)$	$(P = K)$
	$\Delta_{e^2p^2}^{(P)}$ (%)	-3.929	-3.786
	$\Delta_{e^2p^4}^{(P)}$ (%)	0.053 ± 0.011	0.135 ± 0.011
*	$\Delta_{e^2p^6}^{(P)}$ (%)	0.073	
**	Δ_{LL} (%)	0.055	0.055

* SD contribution to $\pi \to e\,\nu\,\gamma$, unsuppressed by helicity argument [absent in K case due to different definition of inclusive observable]

** Contribution of higher order Leading Logarithms $O(e^{2n}\,p^2)$ [via RG, Marciano-Sirlin '93]

$R_{e/\mu}^{(\pi,K)}$ results

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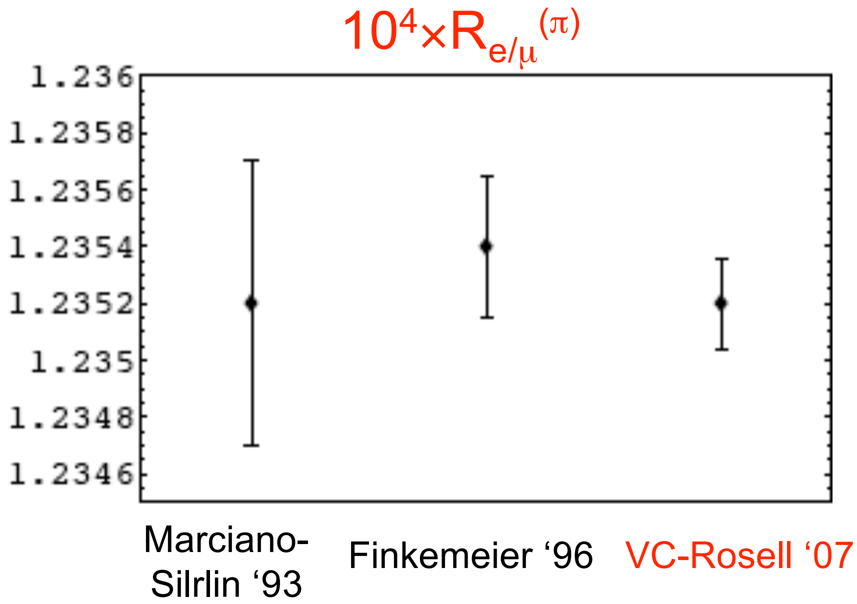
** Δ_{LL} (%)	0.055	0.055

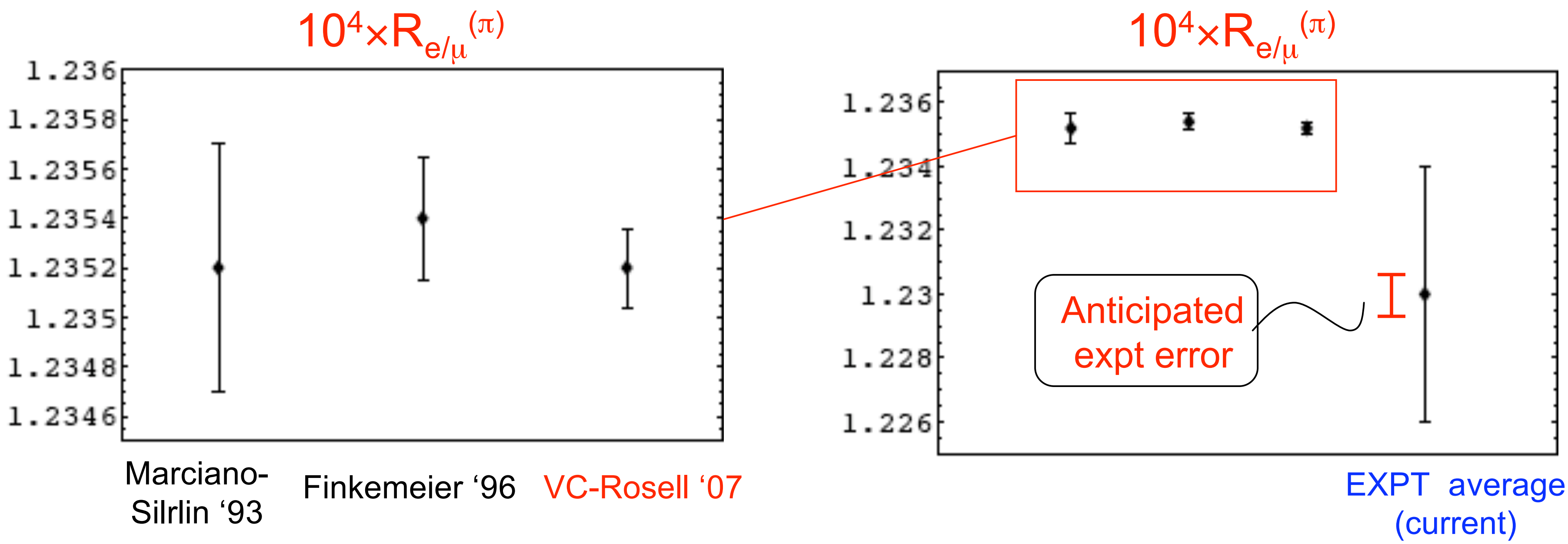
$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \times 10^{-4}$$
$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \times 10^{-5}$$

4 x matching uncertainty !
[estimate of e^2p^6 effect]

$R_{e/\mu}^{(\pi)}$ comparisons



$R_{e/\mu}^{(\pi)}$ comparisons



- Exciting prospects in view of expt. improvements (PSI, TRIUMF)

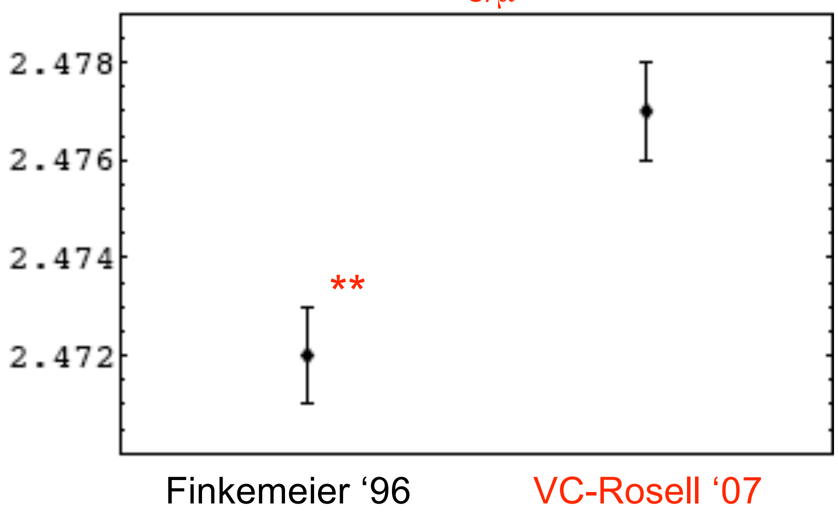
$$\frac{R_{e/\mu}^{EXP}}{R_{e/\mu}^{SM}}$$

$$\frac{G_e^2}{G_\mu^2} = 0.9966 \pm 0.0030\ (\mathrm{exp}) \pm 0.0004\ (\mathrm{th})$$ CURRENT

$$\pm\ 0.0005\ (\mathrm{exp}) \pm 0.0001\ (\mathrm{th})$$ FUTURE

$R_{e/\mu}^{(K)}$ comparisons

$10^5 \times R_{e/\mu}^{(K)}$

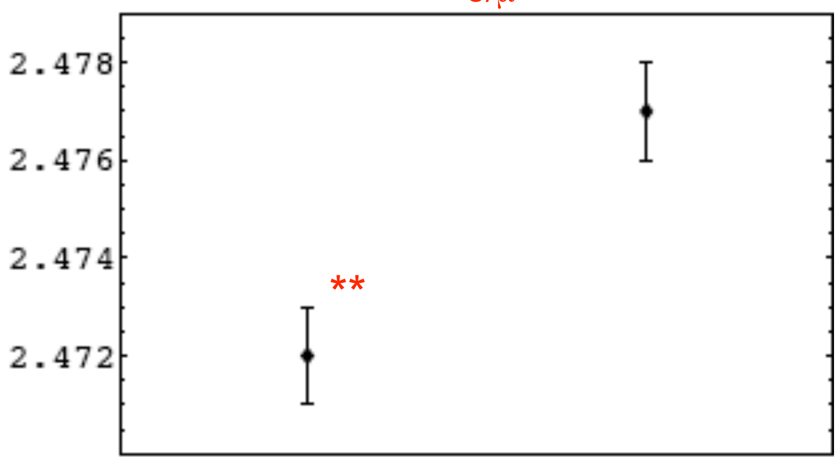


- Discrepancy traced back to sign mistake in Δ_{LL} and behavior of $\Pi_{\pi w \gamma}$

  Difference is 0.2%, comparable to anticipated expt. error 0.3%

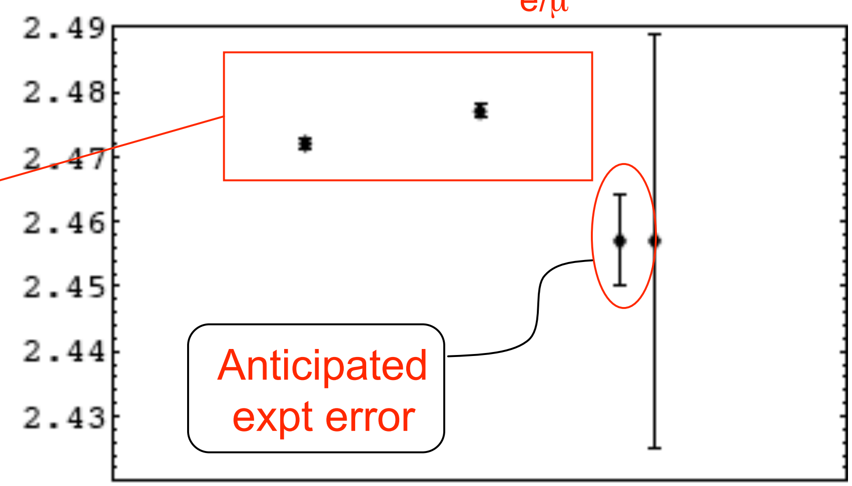
$R_{e/\mu}^{(K)}$ comparisons

$10^5 \times R_{e/\mu}^{(K)}$



Finkemeier '96 VC-Rosell '07

$10^5 \times R_{e/\mu}^{(K)}$



EXPT average (current)

- Discrepancy traced back to sign mistake in Δ_{LL} and behavior of $\Pi_{\pi w \gamma}$

Difference is 0.2%, comparable to anticipated expt. error 0.3%

$$\frac{G_e^2}{G_\mu^2} = 0.992 \;\pm\; 0.013\ (\mathrm{exp}) \;\pm\; 0.0004\ (\mathrm{th})$$ CURRENT

$$\pm\; 0.003\ (\mathrm{exp}) \;\pm\; 0.0004\ (\mathrm{th})$$ FUTURE

Precision SM tests with Kl3 decays

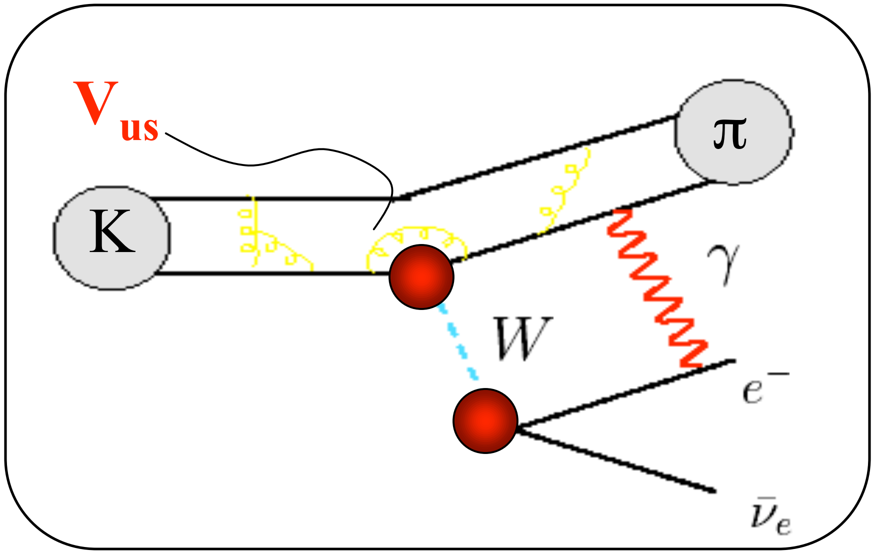
- EM corr. $\xrightarrow{+\text{ EXPT}}$ lepton universality
- (EM +) IB corr. $\rightarrow$ quark mass ratios
- (EM + IB +) SU(3) corr. $\rightarrow$ V_{us} and CKM unitarity

K → π ℓ ν master formula

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_\mu^2 S_{\rm ew} M_K^5}{192\pi^3} \cdot |V_{us}|^2 \, |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\,\Delta^K_{SU(2)} + 2\,\Delta^{K\ell}_{\rm EM}]$$

--- EXP input
--- TH input



$K \to \pi\, \ell\, \nu$ master formula

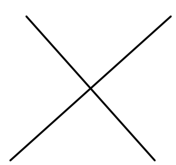
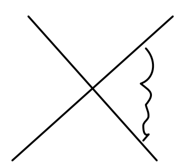
$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\boxed{\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_\mu^2\, S_{\mathrm{ew}}\, M_K^5}{192\pi^3} \cdot |V_{us}|^2\, |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\,\Delta_{SU(2)}^K + 2\,\Delta_{\mathrm{EM}}^{K\ell}]}$$

Short distance
electroweak correction:

Sirlin '82

$$\boxed{S_{\mathrm{ew}} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_s}{4\pi}\right) \log\frac{M_Z}{M_\rho} + O\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 1.0232}$$

 +  + … => $G_\mu V_{ij} \sqrt{S_{\mathrm{ew}}}$

$K \to \pi\,\ell\,\nu$ master formula

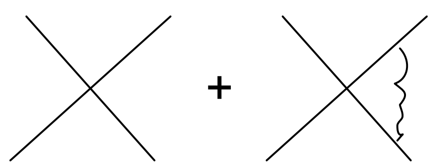
$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_\mu^2\, S_{\rm ew}\, M_K^5}{192\pi^3} \cdot |V_{us}|^2\, |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\,\Delta_{SU(2)}^{K} + 2\,\Delta_{\rm EM}^{K\ell}]$$

Short distance electroweak correction:

Sirlin '82

$$S_{\rm ew} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_s}{4\pi}\right)\log\frac{M_Z}{M_\rho} + O\left(\frac{\alpha\alpha_s}{\pi^2}\right) = 1.0232$$



+ … => $G_\mu V_{ij}\sqrt{S_{\rm ew}}$

Long distance EM correction, known to $O(e^2 p^2)$

	$\Delta_{\rm EM}^{K\ell}(\%)$
K_{e3}^+	$+0.08 \pm 0.15$
K_{e3}^0	$+0.57 \pm 0.15$
$K_{\mu 3}^+$	-0.12 ± 0.15
$K_{\mu 3}^0$	$+0.80 \pm 0.15$

VC et al '02-'04
Descotes-Moussallam '05
VC-Giannotti-Neufeld, preliminary

$K \to \pi\, \ell\, \nu$ master formula

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

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$$t = (p_K - p_\pi)^2$$

1 in $SU(3)_V$ symmetric limit
($m_u = m_d = m_s$)

$$\langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = f_+^{K^0\pi^-}(t)\,(p_K + p_\pi)_\mu + f_-^{K^0\pi^-}(t)\,(p_K - p_\pi)_\mu$$

$K \to \pi \ell \nu$ master formula

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\boxed{\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_\mu^2 S_{\rm ew} M_K^5}{192\pi^3} \cdot |V_{us}|^2 \, |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\,\Delta_{SU(2)}^K + 2\,\Delta_{\rm EM}^{K\ell}]}$$

$$t = (p_K - p_\pi)^2$$

$$f_{+,0}(t) = f_+(0) \left(1 + \lambda_{+,0} \frac{t}{M_\pi^2} + \lambda''_{+,0} \frac{t^2}{M_\pi^4} + \ldots \right)$$

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

$K \to \pi \ell \nu$ master formula

$$K = \{K^+, K^0\} \quad \ell = \{e, \mu\}$$

$$\boxed{\Gamma_{K_{\ell 3[\gamma]}} = C_K^2 \frac{G_\mu^2 S_{\rm ew} M_K^5}{192\pi^3} \cdot |V_{us}|^2 \, |f_+^{K^0\pi^-}(0)|^2 \cdot I^{K\ell}(\lambda_i) \cdot [1 + 2\,\Delta_{SU(2)}^K + 2\,\Delta_{\rm EM}^{K\ell}]}$$

$$t = (p_K - p_\pi)^2$$

$$\Delta_{SU(2)}^K \equiv \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1$$

ChPT to $O(p^4)$ relates this to ratios of the light quark masses:

$$\boxed{\Delta_{SU(2)}^K = F\left(R^{-1}, \Delta_M\right) = \tilde{F}\left(\frac{m_u}{m_d}, \frac{m_s}{m_d}\right)}$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u} \qquad \frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{m_u + m_d}\left(1 + \Delta_M + O(m_q^2)\right)$$

Testing lepton universality with Kl3

$$\frac{G_e^2}{G_\mu^2} = \frac{\Gamma_{K_{e3}}}{\Gamma_{K_{\mu 3}}} \cdot \frac{I^{K\mu}}{I^{Ke}} \left[1 + 2\,\Delta_{\mathrm{EM}}^{K\mu} - 2\,\Delta_{\mathrm{EM}}^{Ke} \right]$$

1 in the SM

Experimental input from FLAVIAnet
Kl3 working group fit (KAON 07)

$$\frac{G_e^2}{G_\mu^2} = \begin{cases} 0.994 \pm 0.009 & K^\pm \\ 0.996 \pm 0.006 & K_L \end{cases}$$

Theory error is 0.003

$$\boxed{\frac{G_e^2}{G_\mu^2} = 0.996 \pm 0.005}$$

Approaching *current* sensitivity in $\Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$ [0.003]

SU(2) breaking in K_{l3} and quark masses

- Predict $\Delta_{SU(2)}$ from quark mass ratios
- Leutwyler '96: three constraints on the m_u/m_d - m_s/m_d plane

Q = 22.7 ± 0.8
$\Delta_M > 0$
R < 44

$$Q = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{m_u + m_d}\left(1 + \Delta_M + O(m_q^2)\right)$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

$$Q^2 \equiv \frac{M_K^2}{M_\pi^2} \cdot \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2}$$

or from

$\Gamma(\eta \to \pi^+ \pi^- \pi^0)$

SU(2) breaking in K_{l3} and quark masses

- Predict $\Delta_{SU(2)}$ from quark mass ratios
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$Q = 22.7 \pm 0.8$
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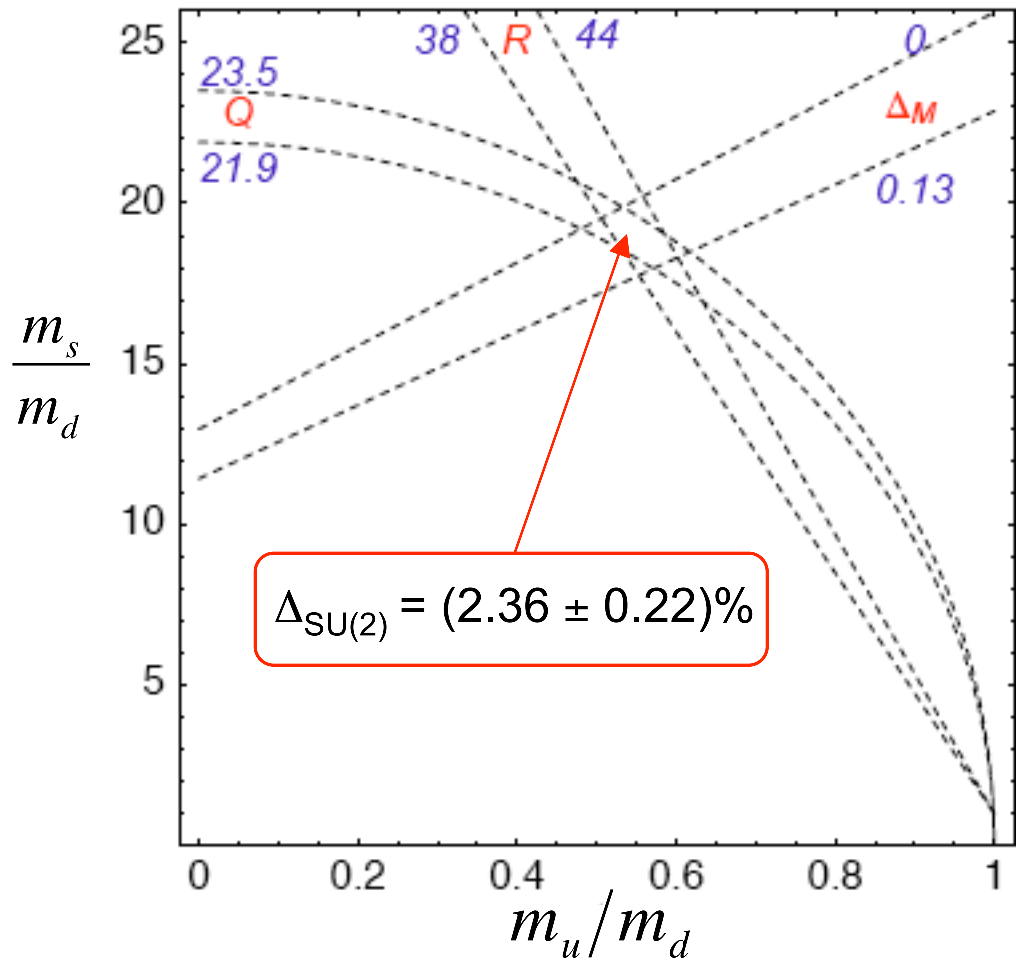


- Extract $\Delta_{SU(2)}$ from Kl3 data + EM corrections:

$$\Delta^{K}_{SU(2)} = \frac{\Gamma_{K^+_{\ell 3}}}{\Gamma_{K^0_{\ell 3}}} \cdot \frac{I^{K^0\ell}}{I^{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}}\right)^5 - \frac{1}{2} - \left[\Delta^{K^+\ell}_{\mathrm{EM}} - \Delta^{K^0\ell}_{\mathrm{EM}}\right] \longrightarrow (2.84 \pm 0.40)\ \%$$

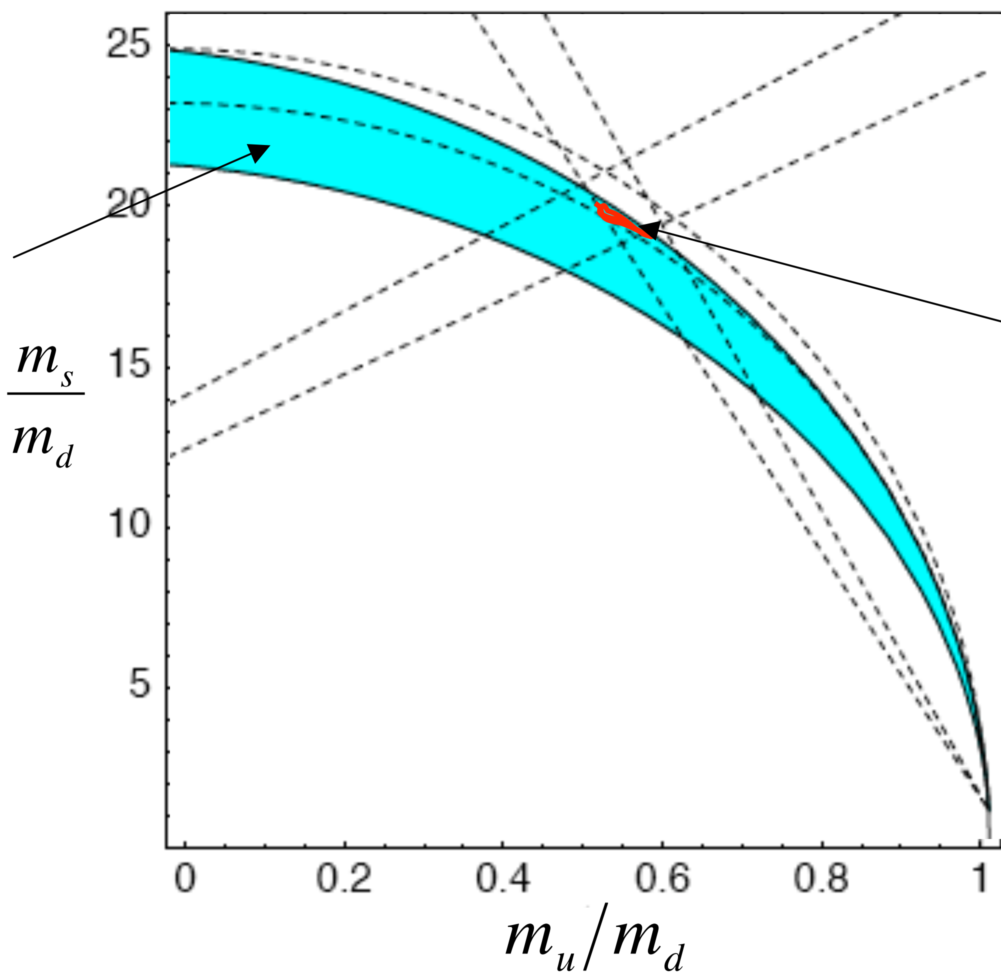
0.15%
from TH

Extract $\Delta_{SU(2)}$ from Kl3 data + EM corrections:

$$\Delta^K_{SU(2)} = \frac{\Gamma_{K^+_{\ell 3}}}{\Gamma_{K^0_{\ell 3}}} \cdot \frac{I^{K^0\ell}}{I^{K^+\ell}} \left(\frac{M_{K^0}}{M_{K^+}}\right)^5 - \frac{1}{2} - \left[\Delta^{K^+\ell}_{\mathrm{EM}} - \Delta^{K^0\ell}_{\mathrm{EM}}\right] \longrightarrow (2.84 \pm 0.40)\ \%$$

0.15% from TH

Kl3 constraint



Only marginally consistent with "standard" analysis

Kl3 constraint points to Q ~ 21 while Leutwyler uses Q = 22.7 ± 0.8

- “Kl3 constraint” begins to challenge “standard” picture and should be included in future analyses of light quark masses

- “Kl3 constraint” is almost degenerate with the “Q constraint” on the plane m_u/m_d vs m_s/m_d $\rightarrow$ does not provide independent information on the individual m_u/m_d and m_s/m_d ratios, but rather a consistency check on the value of Q

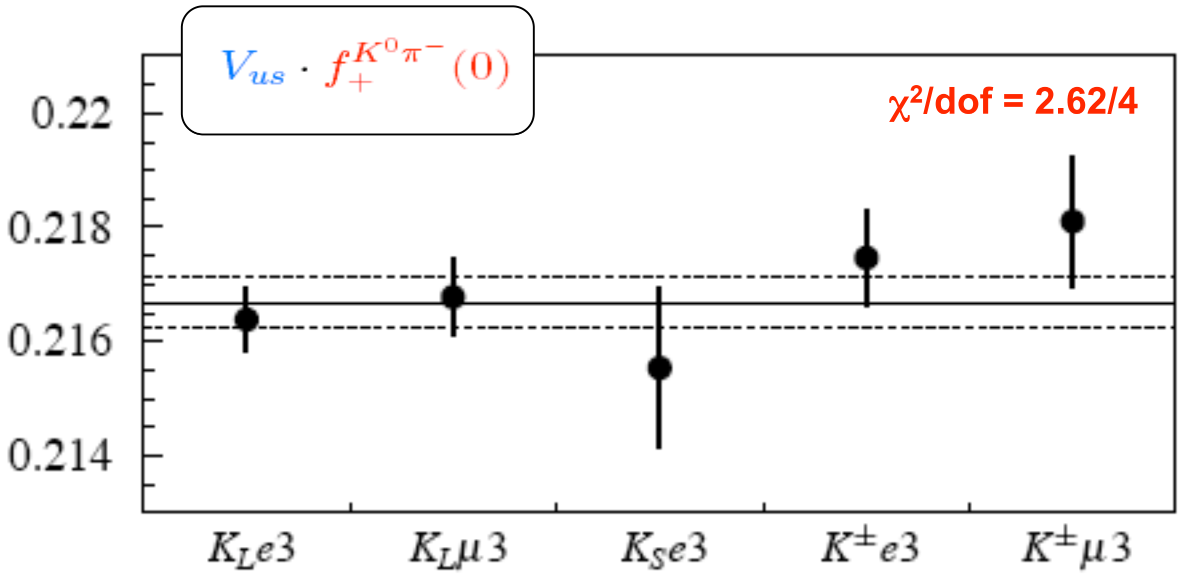
- Quantitatively, $\Delta_{SU(2)} \approx 2.8\%$ points to Q ~ 21 which in turn corresponds to large EM kaon mass splitting. This is consistent with post-1996 analyses of $(\Delta M_K^2)_{EM}$

$$(\Delta M_K^2)_{EM} = (2.5 \pm 1.0)\ (\Delta M_\pi^2)_{EM} \quad \leftrightarrow \quad Q=20.7\pm 1.2$$

[Ananthanarayan-Moussallam 04, Bijnens-Prades 97, Donoghue-Perez 97]

$\Delta_{EM} + \Delta_{SU(2)}$ + exp. data → $f_+(0)\ V_{us}$

$$V_{us} \cdot f_+^{K^0\pi^-}(0) = C_K^{-1} \left[\frac{192\pi^3 \cdot \Gamma_{K_{\ell 3[\gamma]}}}{G_\mu^2 S_{\rm ew} M_K^5 \cdot I^{K\ell}} \right]^{1/2} \cdot \frac{1}{1 + \Delta_{SU(2)}^K + \Delta_{\rm EM}^{K\ell}}$$



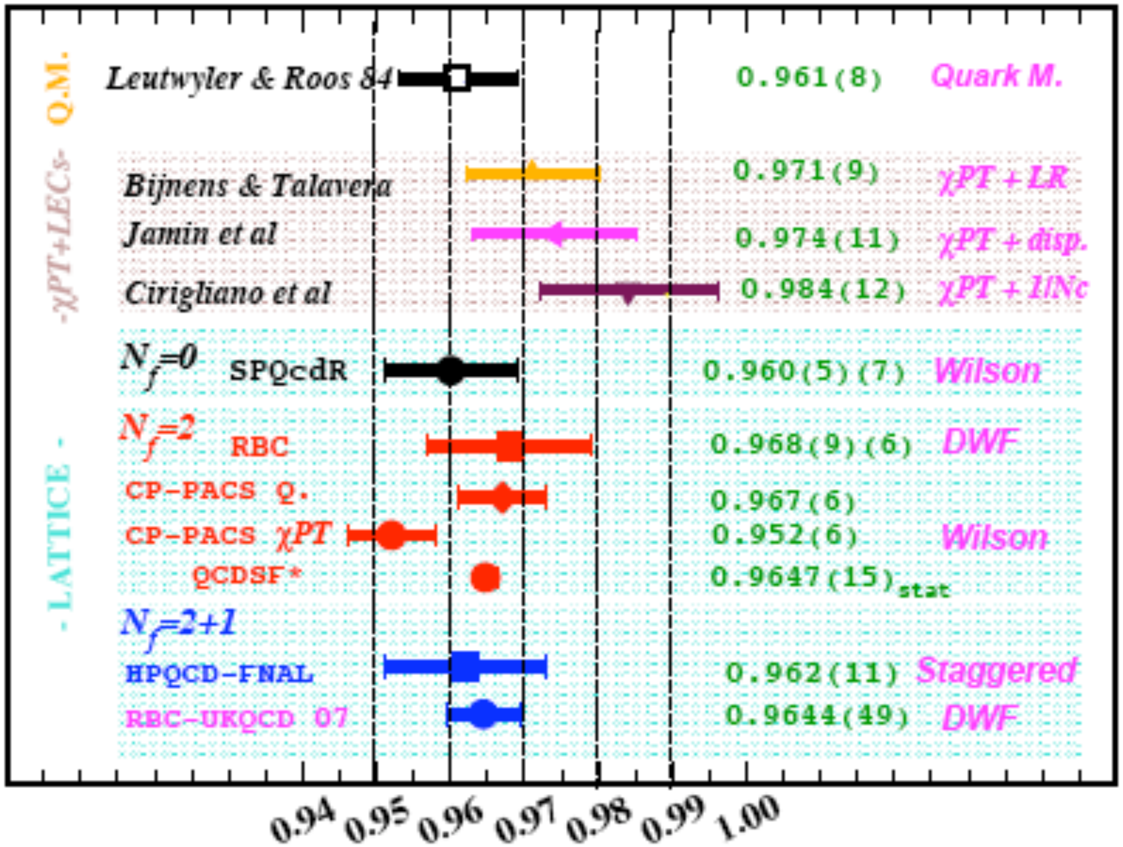
New results from KTeV, KLOE, NA48, ISTRA

I use FLAVIAnet fit presented at KAON07

$$V_{us} \cdot f_+^{K^0\pi^-}(0) = 0.2166 \pm 0.0005$$

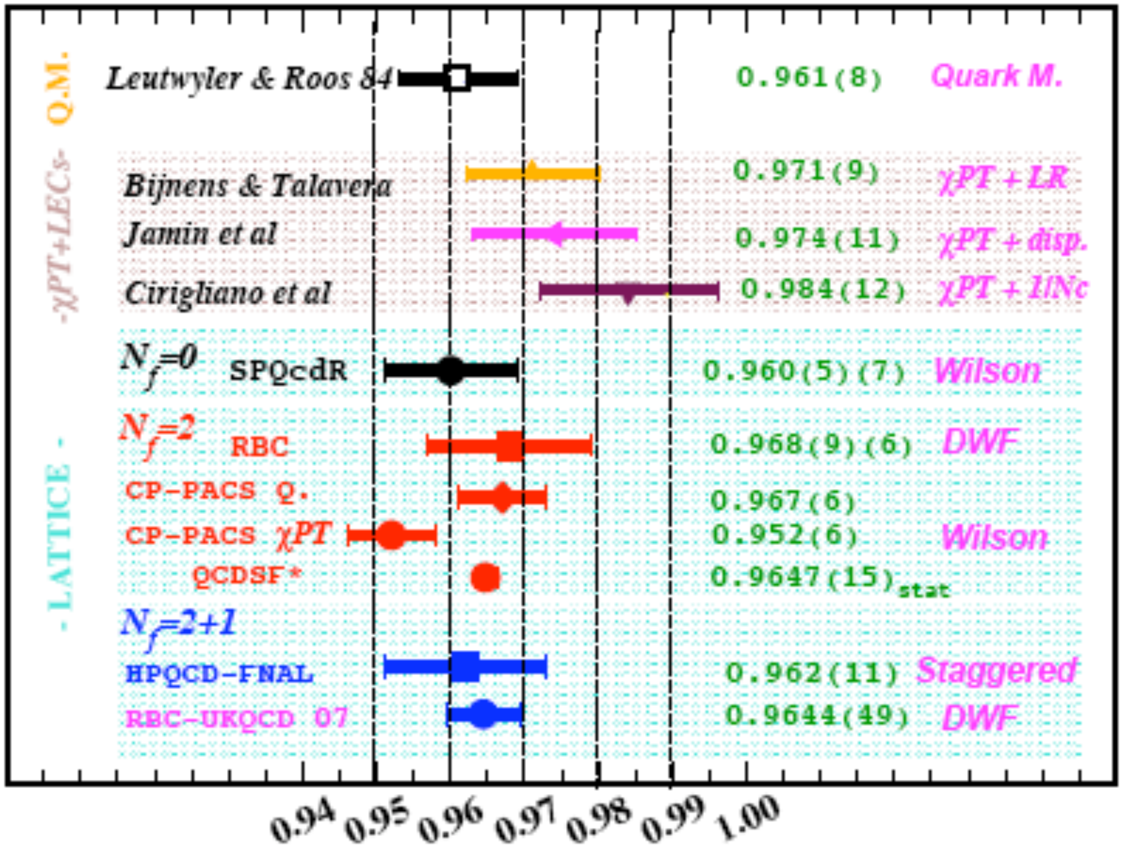
0.2%, dominated by K^0 modes

SU(3) breaking in $f_+^{K\pi}(0)$



- Inclusion of chiral logs increases analytic estimates over LR'84
- Key issue: understand role of (χ-logs)^2 both in chiral extrapolation of lattice data and in analytic estimates

SU(3) breaking in $f_+^{K\pi}(0)$



I will use use LR as reference value

$$V_{us}^{K\ell 3} = 0.2254\,(4)_{\rm exp}(19)_{\rm th} \cdot \frac{0.961}{f_+^{K^0\pi^-}(0)}$$

Implication for CKM unitarity test

V_{ud} = 0.97418 (26)

V_{us} = 0.2254 (19)

V_{ub} = 0.00431 (30)

Hardy-Towner
(last week!)

$$\left(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right)_{\text{pheno}} = 0.9998\,(5)_{V_{ud}}\,(8)_{V_{us}}$$

$$\equiv 1 + \Delta_{CKM}$$

$$\Delta_{CKM} = (-0.15 \;\pm\; 1)\cdot 10^{-3}$$

- What does this mean ?

$$\boxed{\frac{1}{1+BR^{\mu}_{\text{exotic}}} \cdot \frac{1-|V_{uD}|^2}{1-|U_{\mu N}|^2} \cdot \frac{G_{SL}^2}{G_\mu^2} \;=\; 1+\Delta_{CKM}}$$

$$\uparrow$$

$$\left(V_{ij}\right)_{pheno} = \frac{G_{SL}}{\tilde{G}_\mu}\, V_{ij}$$

- What does this mean ?

$$\frac{1}{1 + BR^{\mu}_{\text{exotic}}} \cdot \frac{1 - |V_{uD}|^2}{1 - |U_{\mu N}|^2} \cdot \frac{G^2_{SL}}{G^2_{\mu}} \; = \; 1 + \Delta_{CKM}$$

- Exotic muon decays (shift value of G_{Fermi} extracted from muon lifetime)

$$BR^{\mu}_{\text{exotic}} \; < \; 2 \cdot 10^{-3}$$

- Heavy fermion mixing: $|V_{uD}| \sim |U_{\mu N}| \; < \; 0.04$

- Quark-lepton universality => constraints on scale of effective operators

$$\frac{G^2_{SL}}{G^2_{\mu}} \; = \; 0.9998\,(10) \qquad\qquad \frac{\Lambda}{\sqrt{c}} \; > \; 5.5 \text{ TeV}$$

Summary

- Chiral EFT allows us to put SM theoretical uncertainty in π_{e2}, K_{e2}, K_{l3} decays on solid ground at 0.01%, 0.04%, ~ 1% level

- Together with current or anticipated experimental sensitivities this allows us to probe different aspects of the SM

lepton universality

$(G_e/G_\mu)^2$ @ 0.05% $(\pi_{\ell 2})$
$(G_e/G_\mu)^2$ @ 0.3% $(K_{\ell 2})$
$(G_e/G_\mu)^2$ @ 0.5% $(K_{\ell 3})$

Burden is on experiment

quark-lepton universality

δV_{us} ~ 1%, δV_{ud} ~0.03% =>
$(G_{SL}/G_\mu)^2$ @ 0.1%

Burden is on theory (V_{us})

quark mass ratios

Constraint from K^+ vs K_L on

$$Q = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

points to Q~21

Additional slides

Structure of $R_{e/\mu}^{(\pi,K)}$ to $O(e^2p^4)$

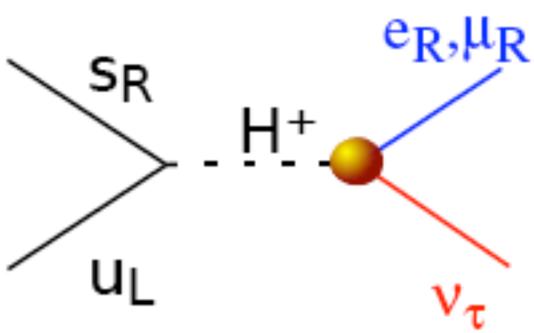
$$\Delta^{(P)}_{e^2p^4} = \frac{\alpha}{\pi}\frac{m_\mu^2}{m_\rho^2}\left(c_2^{(P)}\log\frac{m_\rho^2}{m_\mu^2} + c_3^{(P)} + c_4^{(P)}(m_\mu/m_P)\right) + \frac{\alpha}{\pi}\frac{m_P^2}{m_\rho^2}\,\tilde{c}_2^{(P)}\log\frac{m_\mu^2}{m_e^2}$$

	$(P=\pi)$	$(P=K)$
$\tilde{c}_2^{(P)}$	0	$(7.84 \pm 0.07_\gamma)\times 10^{-2}$
$c_2^{(P)}$	$5.2 \pm 0.4_{L_9} \pm 0.01_\gamma$	$4.3 \pm 0.4_{L_9} \pm 0.01_\gamma$
$c_3^{(P)}$	$-10.5 \pm 2.3_{\mathrm{m}} \pm 0.53_{L_9}$	$-4.73 \pm 2.3_{\mathrm{m}} \pm 0.28_{L_9}$
$c_4^{(P)}(m_\mu)$	$1.69 \pm 0.07_{L_9}$	$0.22 \pm 0.01_{L_9}$

$R_{e/\mu}^{(K)}$ in SUSY at large tan β

[Masiero-Paradisi-Petronzio '05]

$$R_K^{LFV} = \frac{\sum_i K \to e\nu_i}{\sum_i K \to \mu\nu_i} \simeq \frac{\Gamma_{SM}(K \to e\nu_e) + \Gamma(K \to e\nu_\tau)}{\Gamma_{SM}(K \to \mu\nu_\mu)} \ , \quad i = e, \mu, \tau$$



$$eH^\pm \nu_\tau \to \frac{g_2}{\sqrt{2}} \frac{m_\tau}{M_W} \Delta_R^{31} \tan^2\beta$$

$$\Delta_R^{31} \sim \frac{\alpha_2}{4\pi} \delta_{RR}^{31}$$

$$\Delta_R^{31} \sim 5 \cdot 10^{-4} \quad t_\beta = 40 \quad M_{H^\pm} = 500\mathrm{GeV}$$

$$\frac{R_{K,\pi}^{LFV}}{R_{K,\pi}^{SM}} \simeq \left[\left(1 - \frac{m_\tau}{m_e} \frac{m_{K,\pi}^2}{M_{H^\pm}^2} \Delta_{RL}^{11} \tan^3\beta \right)^2 + \frac{m_\tau^2}{m_e^2} \frac{m_{K,\pi}^4}{M_{H^\pm}^4} |\Delta_R^{31}|^2 \tan^6\beta \right]$$

$$R_K^{LFV} \simeq R_K^{SM}(1 - 0.032) \, , \qquad R_\pi^{LFV} \simeq R_\pi^{SM}(1 - 0.0021)$$

ChPT + truncated large N_C

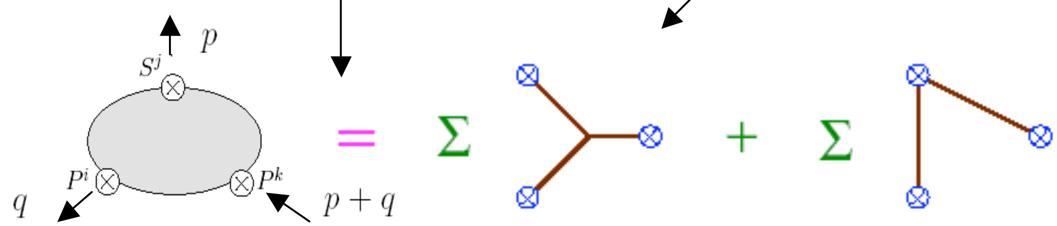
(Cirigliano-Ecker-Eidemuller-Kaiser-Pich-Portoles 2005)

- Obtain effective couplings by large-N inspired matching procedure:

Matching = impose correct QCD asymptotic behavior

Finite number of narrow resonances

$$P^a(x) = \bar{q}(x) i\gamma_5 \lambda^a q(x)$$
$$S^a(x) = \bar{q}(x) \lambda^a q(x)$$



$$\boxed{f_+^{K^0\pi^-}(0) \;=\; 0.984 \pm 0.012}$$

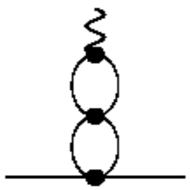
- Scale ambiguity (0.008)
- Resonance parameters

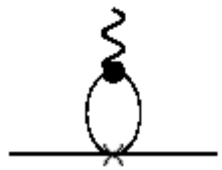
- Cross-checks: F_K/F_π and slope of scalar ff λ_0

Analytic calculation of $f_+(0)$ to $O(p^6)$

Post-Schilcher '02, Bijnens-Talavera '03

$$f_{p^6} = f_{p^6}^{2-\mathrm{loops}}(\mu) + f_{p^6}^{\mathrm{L_i}\times\mathrm{loop}}(\mu) + f_{p^6}^{\mathrm{tree}}(\mu)$$





$$\boxed{f_{p^6}^{L_i\times\mathrm{loop}}(M_\rho) \;=\; -0.0020}$$

$$\boxed{f_{p^6}^{2-\mathrm{loops}}(M_\rho) \;=\; 0.0113}$$

Large and positive
chiral loop contributions
@ $\mu = M_\rho$
(mildly scale dependent)

$$\boxed{8\frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2}\left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho)\right]}$$

Effective couplings not fixed by
Chiral Symmetry

- Identify this with result by Leutwyler-Roos
- Obtain LECs from <SPP> in truncated $1/N_C$ (VC et al 05)
- Calculate with Lattice QCD

Δ_{EM} to $O(e^2p^2)$

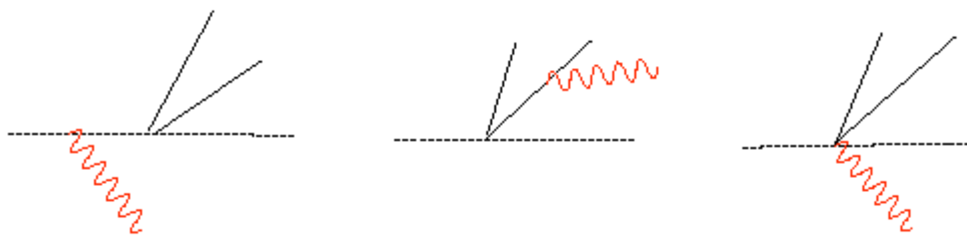
Virtual Photons

$O(p^2)$ vertices

X_n, K_n $O(e^2\,p^2)$
LECs @ large-N_C

Descotes
Moussallam '05

Real Photons



- Uncertainty accounts for neglected higher order effects

- Fully inclusive integration over 4-body phase space [expt.]

	$\Delta_{EM}^{K\ell}(\%)$
K_{e3}^{+}	$+0.08 \pm 0.15$
K_{e3}^{0}	$+0.57 \pm 0.15$
$K_{\mu3}^{+}$	-0.12 ± 0.15
$K_{\mu3}^{0}$	$+0.80 \pm 0.15$

Preliminary

VC-Giannotti-
Neufeld 07

SU(3) breaking in $f_+^{K\pi}(0)$

Ademollo-Gatto:

$$f_+^{K^0\pi^-}(0) = 1 + O(m_s - m_d)^2$$

SU(3)$_V$

Chiral Expansion:

$$f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4}}_{O(m_q)} + \underbrace{f_{p^6}}_{O(m_q^2)} + \ldots$$

SU(3)$_L$ x SU(3)$_R$

SU(3) breaking in $f_+^{K\pi}(0)$

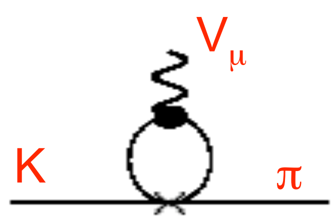
Ademollo-Gatto: $$f_+^{K^0\pi^-}(0) = 1 + O(m_s - m_d)^2$$ $SU(3)_V$

Chiral Expansion: $$f_+^{K^0\pi^-}(0) = 1 + \underbrace{f_{p^4}}_{O(m_q)} + \underbrace{f_{p^6}}_{O(m_q^2)} + \dots$$ $SU(3)_L \times SU(3)_R$

Gasser-Leutwyler'85

$$f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}$$

UV finite one loop diagrams in EFT:



$$\boxed{f_{p^4} = -0.0227}$$

SU(3) breaking in $f_+^{K\pi}(0)$

Ademollo-Gatto:

$$f_+^{K^0\pi^-}(0) = 1 + O(m_s - m_d)^2$$

$SU(3)_V$

Chiral Expansion:

$$f_+^{K^0\pi^-}(0) = 1 + f_{p^4} + f_{p^6} + \ldots$$

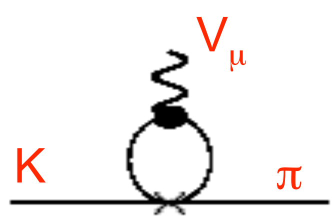
$SU(3)_L$ x $SU(3)_R$

f_{p^4}: $O(m_q)$ f_{p^6}: $O(m_q^2)$

Gasser-Leutwyler'85

$$f_{p^4} \sim \frac{(m_s - m_u)^2}{m_s}$$

UV finite one loop diagrams in EFT:



$$\boxed{f_{p^4} = -0.0227}$$

Up to two-loop graphs in EFT:

chiral logs and "local" terms

~ 1% , mild scale dep.

Post-Schilcher 02
Bijnens-Talavera 03

- Quark model
  Leutwyler-Roos 84
- <SPP> in $1/N_C$
  VC et al 05
- Lattice QCD