A Covariant model for the Nucleon and Δ

Jlab Cake Seminar

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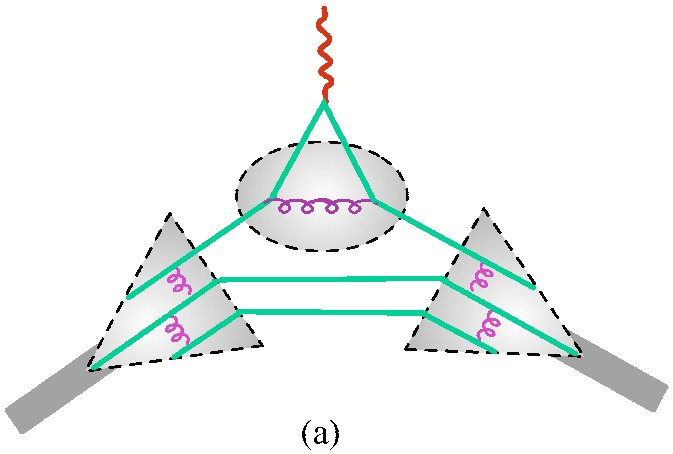
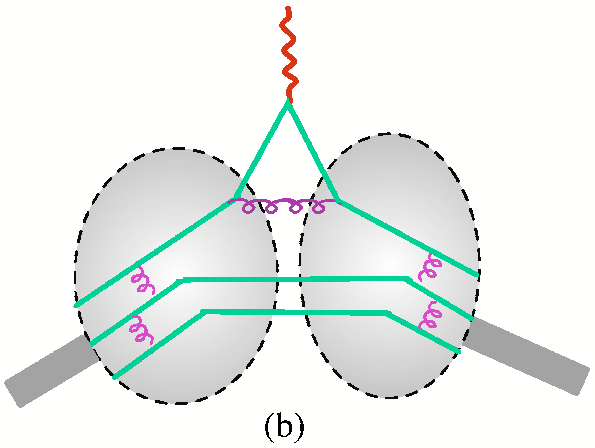
Outline

Background





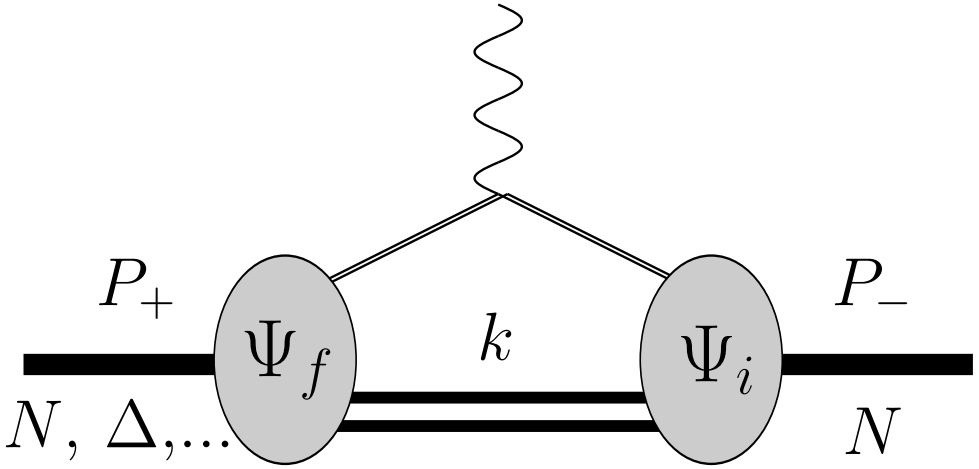
Constituent Quark Model view

- Quark dressed by gluons and $q\bar{q}$ interactions
- Gluon interactions between $q\bar{q} \Rightarrow$ quark form factors
- Quarks with anomalous moments κ_u, κ_d
- Nucleon FF can be explained without high angular momentum components

Light Front view

- Baryon states as a sum of Fock states: qqq, $qqqg$, $qqq(q\bar{q})$, ...
- Pointlike quarks
- No anomalous moments $\kappa_u, \kappa_d = 0$
- High angular momentum required to explain $\kappa_N \neq 0$

Spectator Quark Model



diquark on-mass-shell

Hadronic current

$$J^\mu = 3\sum_\lambda \int_k \bar{\Psi}_f(P_+,k) j_I^\mu \Psi_i(P_-,k)$$

Quark current

$$j_I^\mu = j_1\left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2}\right) + j_2 \frac{i\sigma^{\mu\nu}q_\nu}{2M}$$

$$j_i = \frac{1}{6} f_{i+} + \frac{1}{2} f_{i-}\tau_3$$

Vector Meson Dominance current

$$f_{1\pm} = \lambda_q + \frac{1-\lambda_q}{1+Q^2/\mathbf{m_V}^2} + c_\pm \frac{\mathbf{M_h}^2 Q^2}{(\mathbf{M_h}^2+Q^2)^2}$$

3 parameters $\left\{ \begin{array}{l} \lambda_q \Leftarrow \text{DIS} \\ c_\pm \Leftarrow \text{fit} \end{array} \right.$

$$f_{2\pm} = \kappa_\pm \left(\frac{d_\pm}{1+Q^2/\mathbf{m_V}^2} + \frac{1-d_\pm}{1+Q^2/\mathbf{M_h}^2} \right)$$

4 parameters $\left\{ \begin{array}{l} \kappa_\pm \Leftarrow G_M^{n,p}(0) \\ d_\pm \Leftarrow \text{fit} \end{array} \right.$

$\mathbf{m_V} = m_\rho$, $\mathbf{M_h} = 2M$

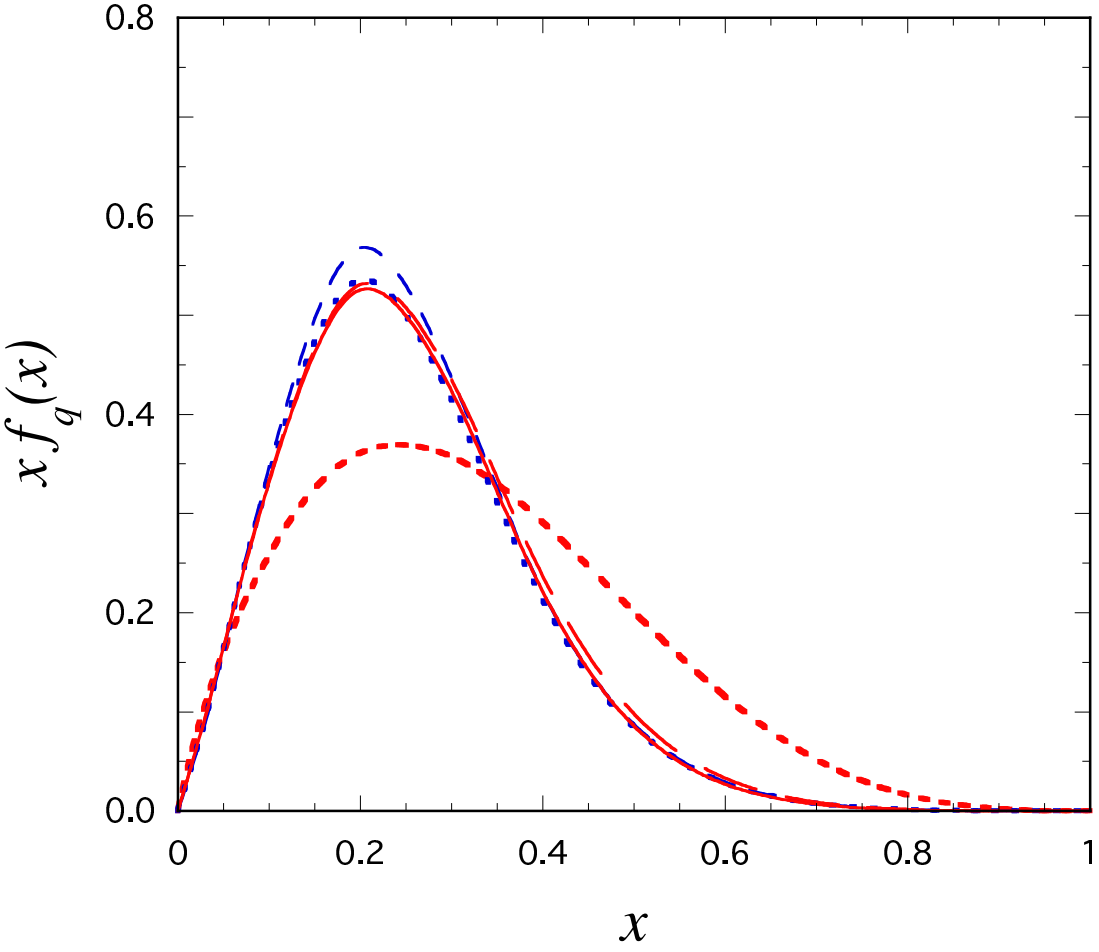
nucl-th/0606029v1

Deep Inelastic Scattering (Extra)

$$\frac{< x\ f >}{< f >} = 0.171$$

$\Rightarrow$ diquark mass

$$\int_{-\infty}^{1} f(x)dx = \lambda_q$$



Wave functions construction

Construction of a baryon wave function:

- Non Relativistic structure; baryon rest frame: $\mathbf{P} = 0$
  $\Rightarrow$ Relativistic form
- Consider a boost in the z-direction
  fixed-axis polarization states
- Initial and final state wave functions defined in a collinear frame
  diquark constraint
- Arbitrary Lorentz transformation Λ
  $\Rightarrow$ wave function defined in an arbitrary frame

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$\Longrightarrow$ Axial diquark with positive parity

Nucleon Wave Function (Non Relativistic form)

- Spin 1/2 NR spin-isospin S-state structure ($\mathbf{P} = 0$):

$$\Psi_{NR} = \frac{1}{\sqrt{2}} \left(\phi_I^0 \phi_S^0 + \phi_I^1 \phi_S^1 \right) \phi_N(P, k)$$

- ϕ_S^0 and ϕ_S^1 written in terms of the nucleon spin $\chi_N(s)$

$$S = 0: \qquad \phi_S^0(\pm) \equiv \frac{1}{\sqrt{2}} (\chi_+ \chi_- - \chi_- \chi_+) \chi_\pm \quad \alpha \quad \chi_N(\pm)$$

$$S = 1: \qquad \phi_S^1(s; \lambda) \equiv \sum (C^\lambda_{s1,s2,s3}) \chi_{s1} \chi_{s2} \chi_{s3} = \frac{1}{\sqrt{3}} (\sigma \cdot \varepsilon^*_\lambda) \chi_N(s)$$

Spin-1 polarization vectors

$$\varepsilon^\mu(\pm) = \mp \frac{1}{\sqrt{2}} (0, 1, \pm i, 0) \qquad \varepsilon^\mu(0) = (0, 0, 0, 1)$$

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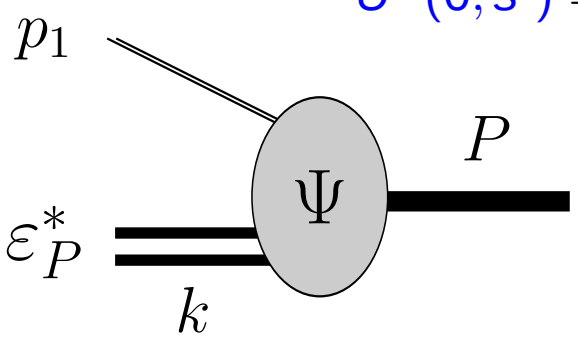
- Similar result for the isospin

Nucleon Wave Function (Relativistic Form)

- Relativist form with $\mathbf{P} = 0$; $S = 0$: $\phi_S^0 = u(0, s)$

$$S = 1: \quad \phi_S^1(s;\lambda) = -\frac{1}{\sqrt{3}}(\varepsilon_\lambda^*)_\alpha \gamma_5 \gamma^\alpha u(0,s) = -(\varepsilon_\lambda^*)_\alpha U^\alpha(0,s)$$

$$U^\alpha(0,s') = \sum_{s\lambda} \langle 1\lambda; \frac{1}{2}s | \frac{1}{2}s' \rangle \varepsilon_\lambda^\alpha u(0,s)$$



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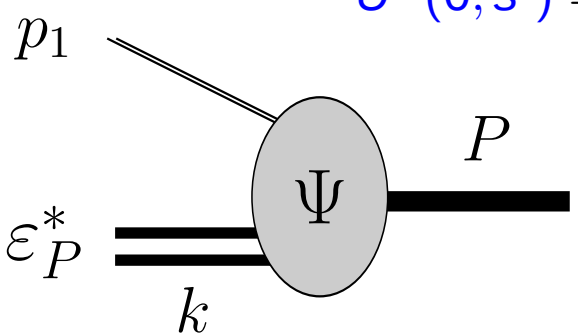

Spin wave function (S=1)

$$\phi_S^1(s;\lambda) = -\varepsilon_\alpha^* U^\alpha(P,s)$$

- Covariant wave function for a moving frame: $\mathbf{P} \neq 0$ boost

$$\Psi_N = \left\{ u(P,s)\chi^I - \frac{1}{\sqrt{3}}(\varepsilon_P^*)_\alpha U^\alpha(P,s)[(\xi^* \cdot \tau)\chi^I] \right\} \phi_N(P,k)$$

ξ^i cartesian rep. of isospin-1 states $\xi^{1,m}$ $(m = 0, \pm 1)$

Ψ_N satisfies Dirac eq $\quad \phi_N(P,k) = \dfrac{N_0}{(\beta_1 + (P-k)^2)(\beta_2 + (P-k)^2)}$

Diquark polarization states

- Helicity states are usually used to define polarization.
  In the $x - z$ plane: $k = (E_k, k\cos\theta, 0, k\sin\theta)$

$$\xi(0) = \frac{1}{m}\begin{bmatrix} k \\ E_k \sin\theta \\ 0 \\ E_k \cos\theta \end{bmatrix}, \qquad \xi(\pm) = \mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ \cos\theta \\ \pm i \\ -\sin\theta \end{bmatrix}$$

$\xi(\lambda)$ is $\theta-$ dependent; $\qquad k \cdot \xi = 0$

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- Fixed-axis: vector particle is bound to a system with $P = (P_0, 0, 0, \mathrm{P})$:

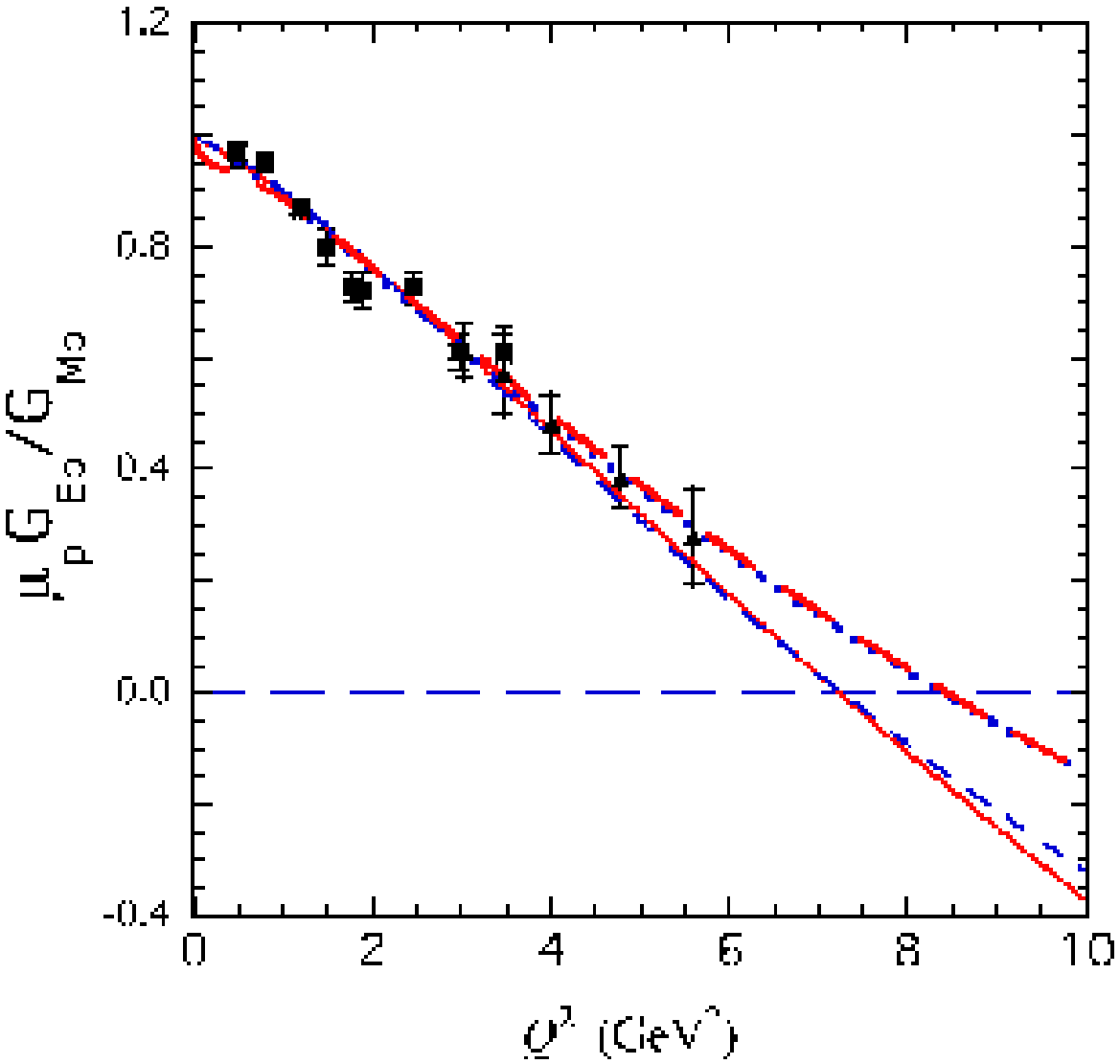
$$\varepsilon(0) = \frac{1}{M}\begin{bmatrix} P \\ 0 \\ 0 \\ P_0 \end{bmatrix}, \qquad \varepsilon(\pm) = \mp\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}$$

  No angular dependence; $\qquad P \cdot \varepsilon = 0$

arXiv:0708.0995 [nucl-th]

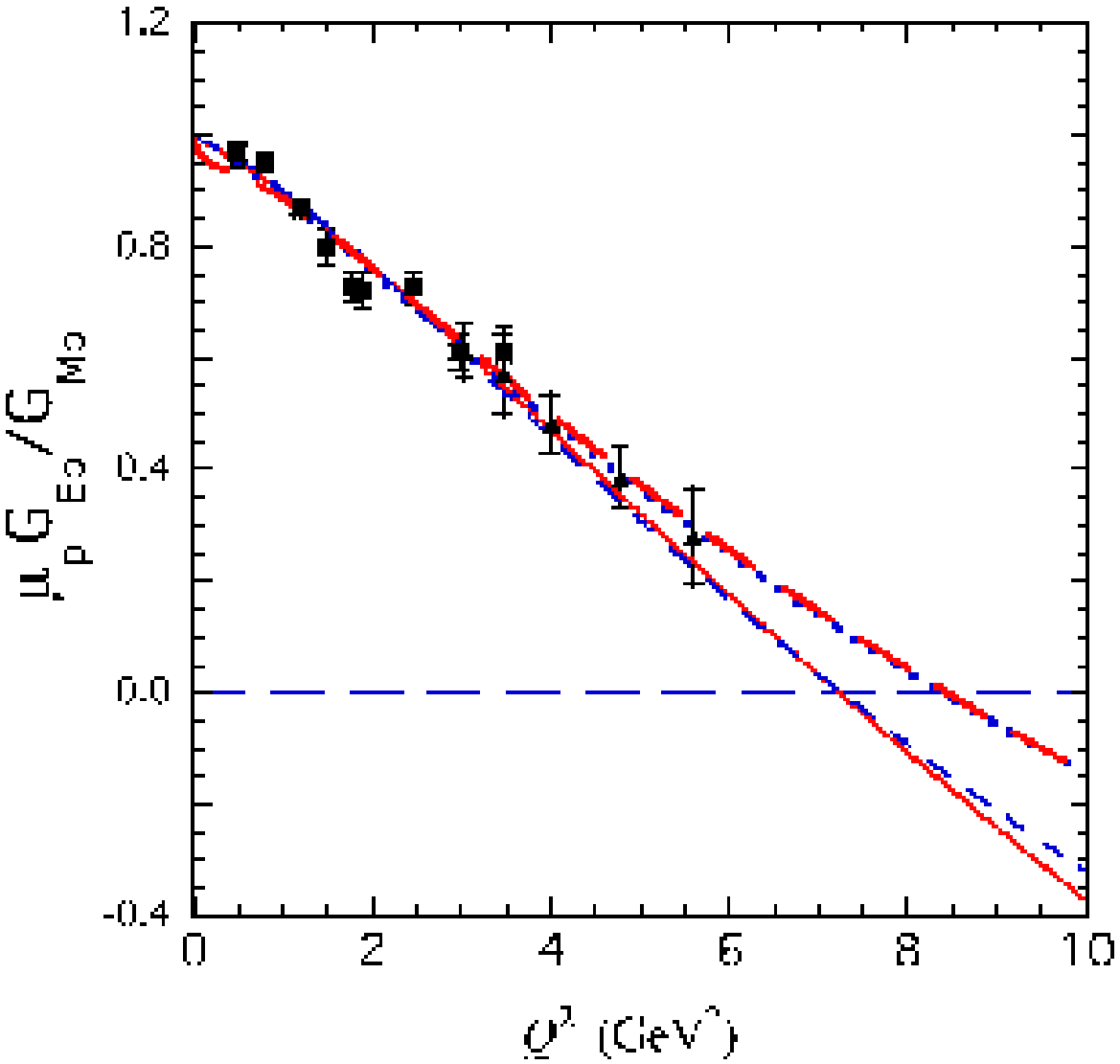
Nucleon Elastic Form Factors

- Good description of the Jlab G_{Ep}/G_{Mp} data with a S-state Nucleon rest frame



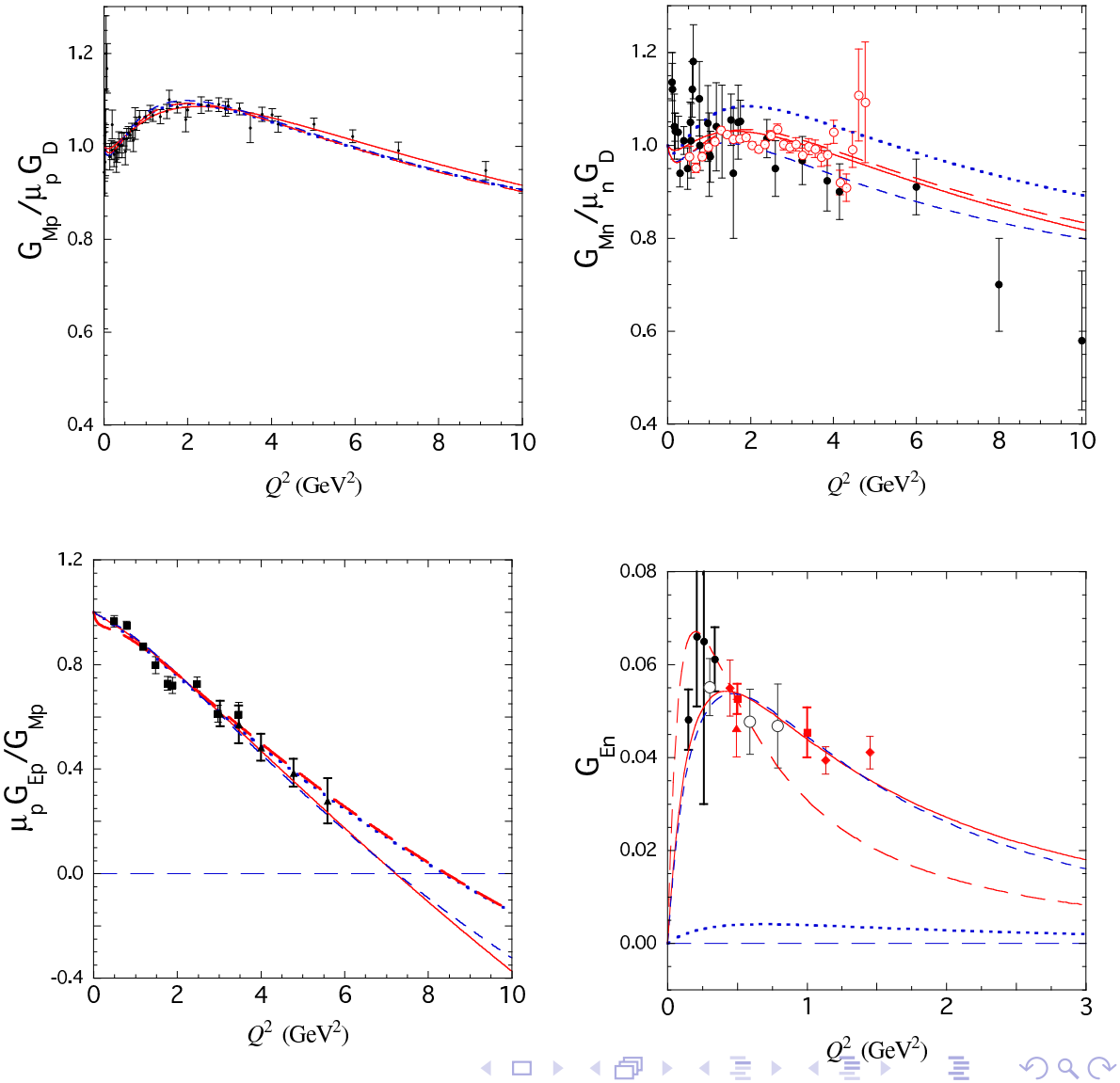
Nucleon Elastic Form Factors

- Good description of the Jlab G_{Ep}/G_{Mp} data with a S-state Nucleon rest frame
- $\Rightarrow$ G_{Ep}/G_{Mp} falloff is **not** a signature of high orbital momentum states



Nucleon Elastic Form Factors (2)

- Good description of the data using models with 5-9 parameters



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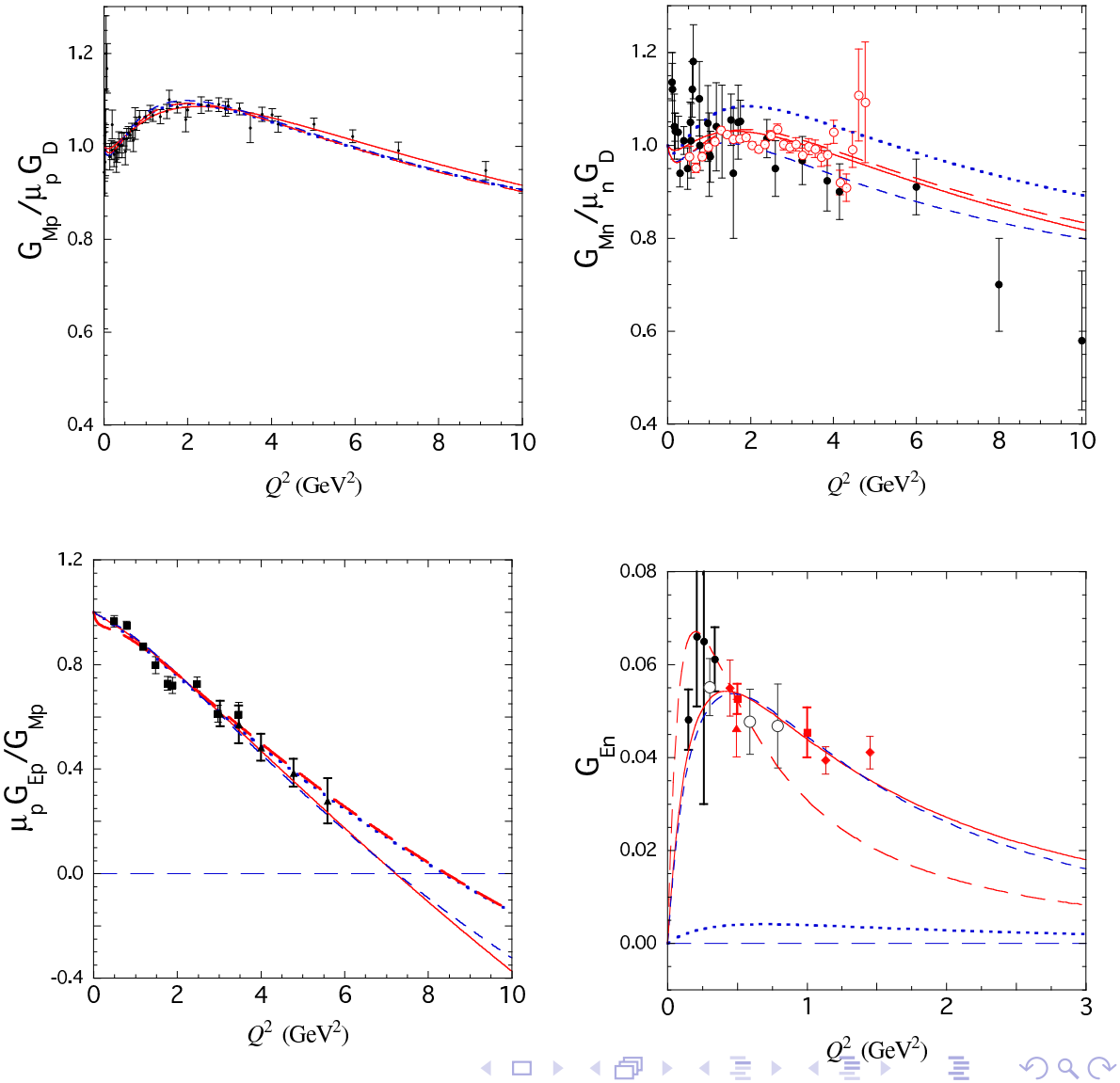
- Models with and without **explicit** pion cloud but VMD

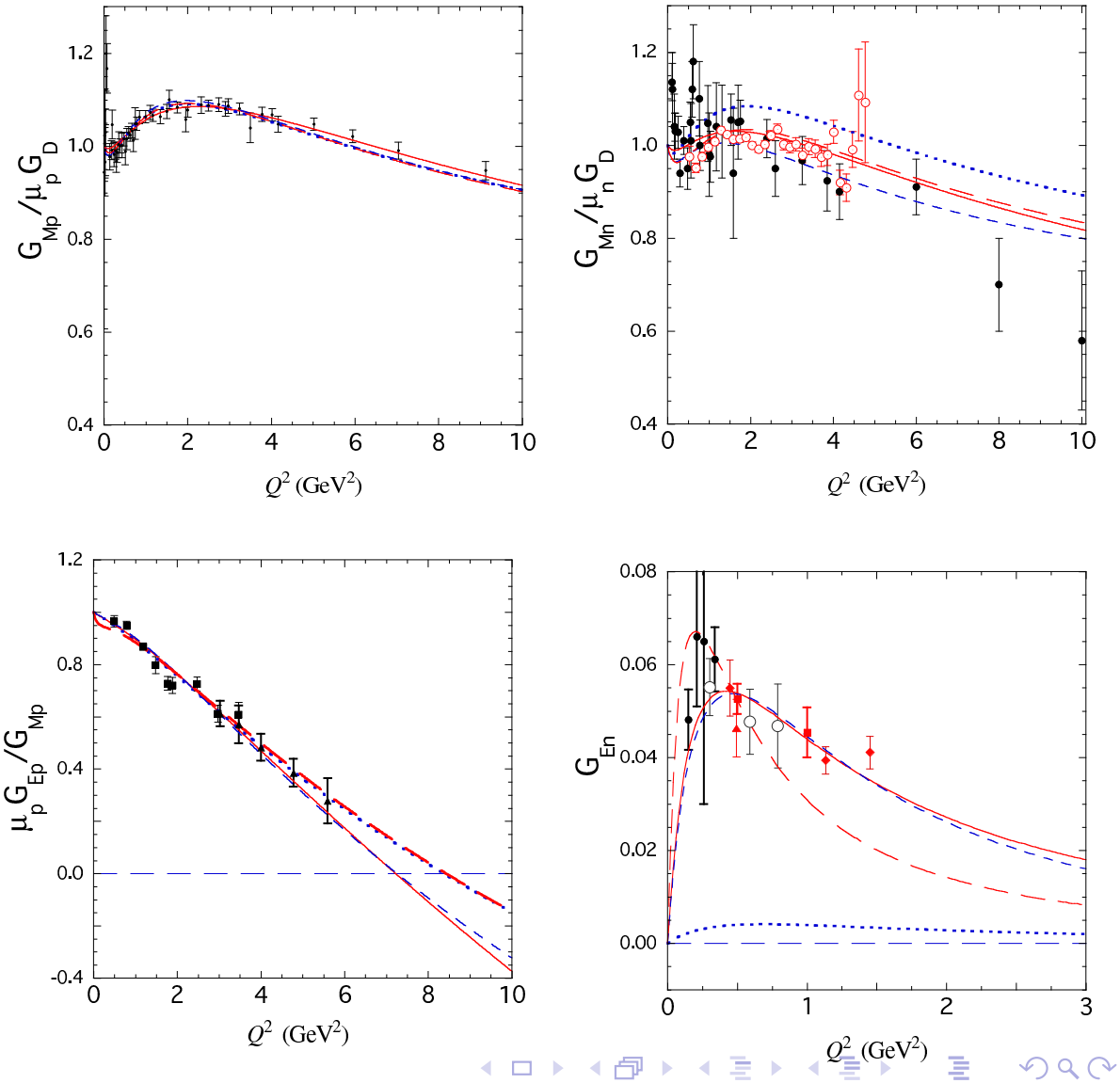


Nucleon Elastic Form Factors (2)

- Good description of the data using models with 5-9 parameters
- Models with and without **explicit** pion cloud but VMD
- pQCD asymptotic behavior

$$G_M, G_E \sim \frac{1}{Q^4} \times$$

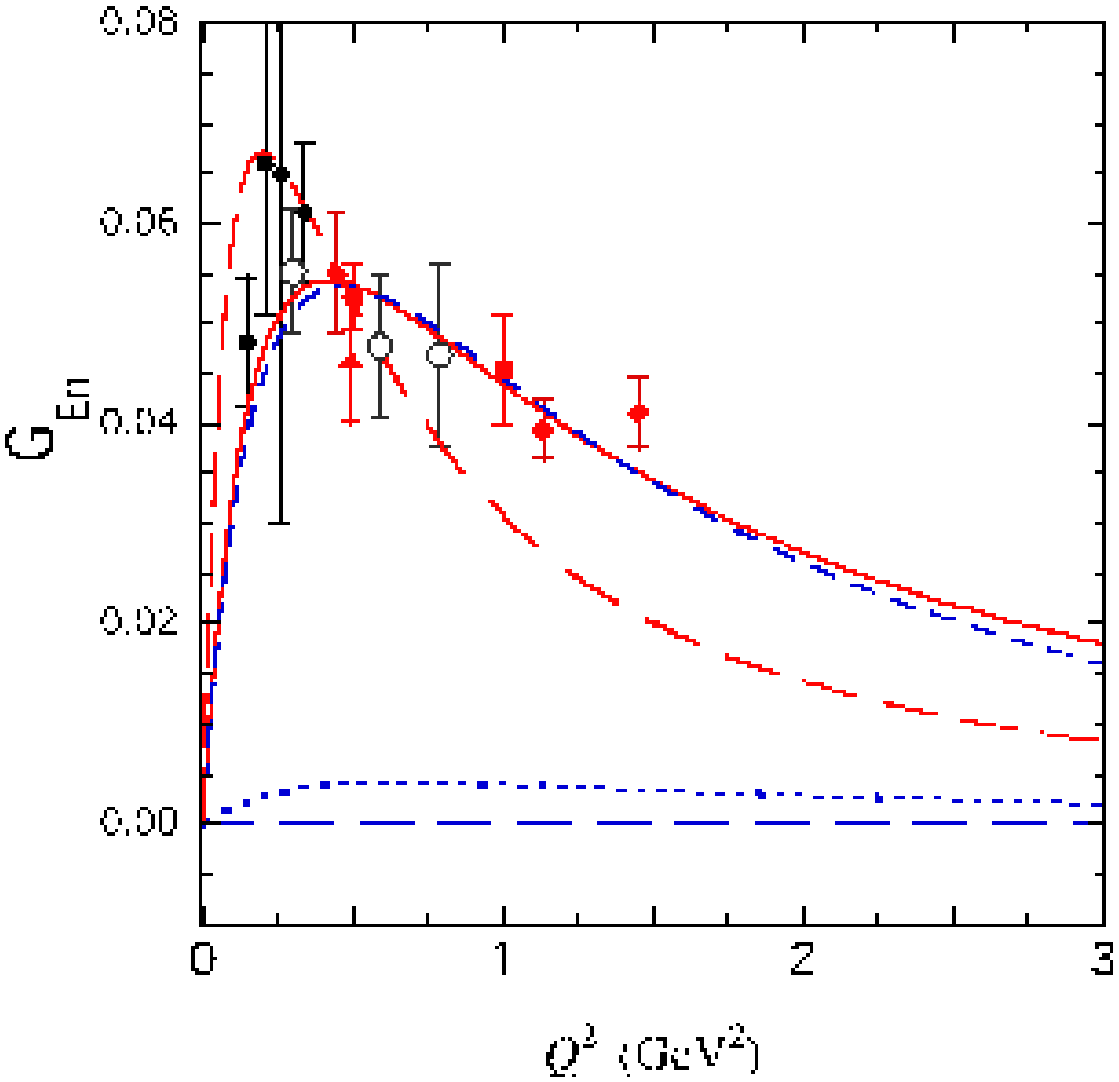
($\log Q^2$ corrections)



Nucleon Elastic Form Factors (3)

Models with **no** explicit pion cloud

- ····· Model I (4) : Isospin symmetry $f_{1+} = f_{1-}$ $\chi^2 =$9.26



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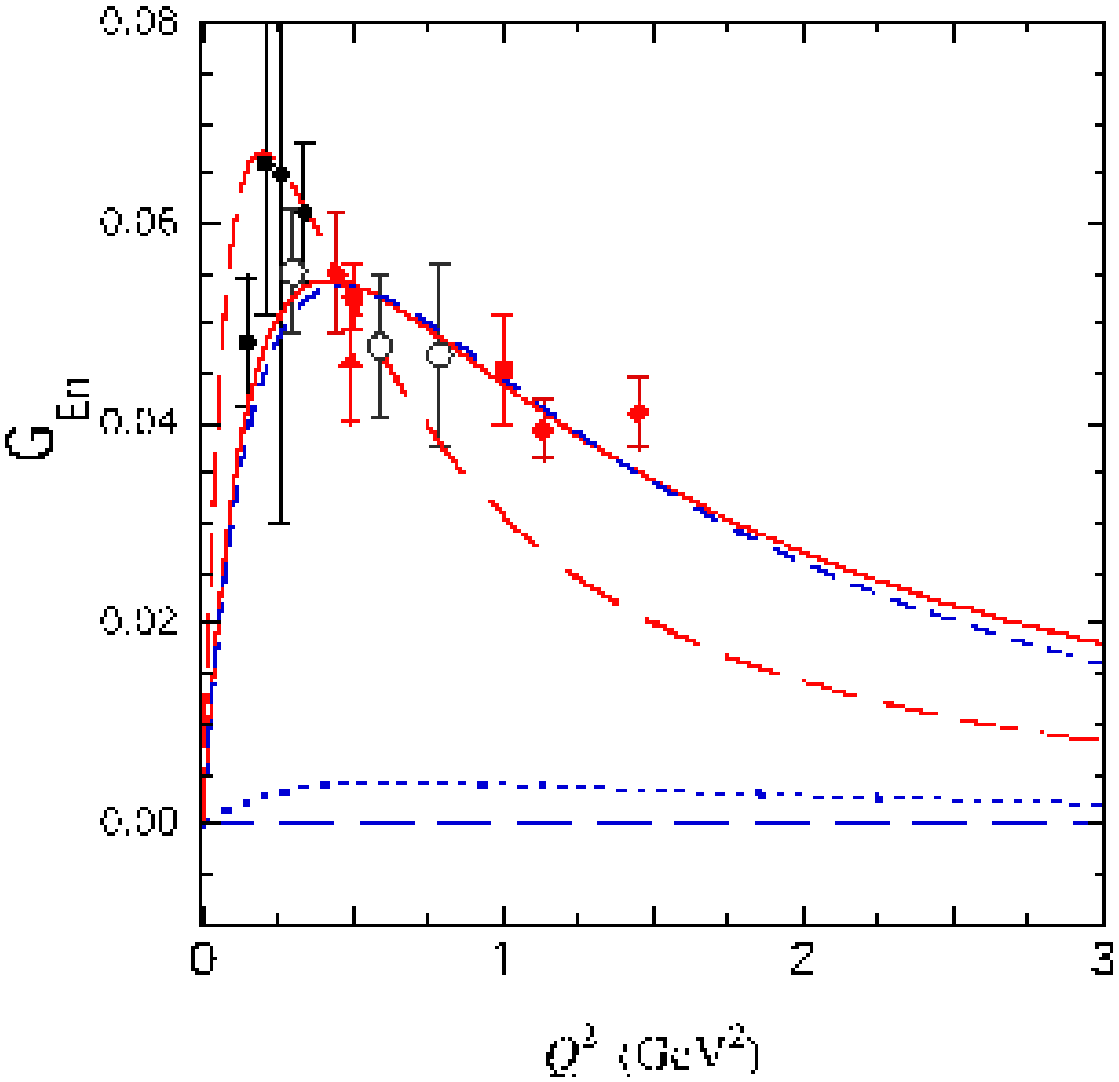
  $f_{1+} = f_{1-}$
  $\chi^2 =$9.26
- – – – Model II (5): No Isospin symmetry
  $f_{1+} \neq f_{1-}$
  $\chi^2 =$1.36



Delta wave function (S-state)

- Spin 3/2 NR form ($\mathbf{P} = 0$) S-state

$$\phi^i(s') = \sum_{s\lambda} \langle 1\lambda; \frac{1}{2}s | \frac{2}{2}s' \rangle \varepsilon^i_\lambda \chi_s = \begin{cases} \varepsilon^i_+ \chi_\uparrow & s' = +\frac{3}{2} \\ \sqrt{\frac{2}{3}}\varepsilon^i_0 \chi_\uparrow + \sqrt{\frac{2}{3}}\varepsilon^i_+ \chi_\downarrow & s' = +\frac{1}{2} \\ \sqrt{\frac{2}{3}}\varepsilon^i_0 \chi_\downarrow + \sqrt{\frac{2}{3}}\varepsilon^i_+ \chi_\uparrow & s' = -\frac{1}{2} \\ \varepsilon^i_- \chi_\downarrow & s' = -\frac{3}{2} \end{cases}$$

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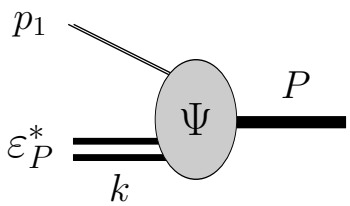
- States related with Rarita-Schwinger states w^i in rest frame

$$\phi^i(s) = \mathcal{T}^i \omega(s) \rightarrow \begin{bmatrix} \mathcal{T}^i \omega(s) \\ 0 \end{bmatrix} = w^i(0, s)$$

$\mathcal{T}^i$ **spin** 1/2$\rightarrow$ 3/2 transition matrix; $\omega(s)$ spin 3/2 state



Delta Spin wave function (S=1)

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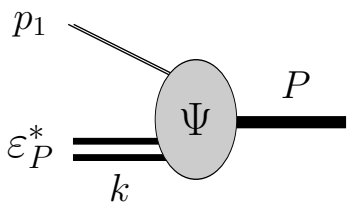

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- Covariant Δ wave function $\mathbf{P} \neq 0$

$$\Psi_\Delta = -(\varepsilon^*_P)^\beta_\lambda w_\beta(P, s) \left[(T \cdot \xi^*) \tilde{\chi}^I \right] \phi_\Delta(P, k)$$

T^i **isospin** 1/2$\rightarrow$ 3/2 transition matrix; $\tilde{\chi}^I$ Δ isospin states

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- Covariant form ($b = P_+ \cdot P_-$)

$$D^{\mu\nu} = \left(-g^{\mu\nu} + \frac{P_-^\mu P_+^\nu}{b}\right) + \frac{M_\Delta M}{b(M_\Delta M + b)} \left(P_- - \frac{b}{M_\Delta^2} P_+\right)^\mu \left(P_+ - \frac{b}{M^2} P_-\right)^\nu$$

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- The covariant expression of $D^{\mu\nu}$ holds for any frame

$$D^{\mu\nu}(P'_+, P'_-) = \Lambda^\mu_{\ \alpha} \Lambda^\nu_{\ \beta} D^{\alpha\beta}(P_+, P_-)$$

N-Δ transition (G_E^*, G_C^*)

Transition current

$$
\begin{aligned}
J^\mu &= 3\sum_\lambda \int_k \bar{\Psi}_\Delta \, j_I^\mu \, \Psi_N \\
&= \bar{w}_\beta [G_1 q^\beta \gamma^\mu + G_2 q^\beta P^\mu + G_3 q^\beta q^\mu - G_4 g^{\beta\mu}] \gamma_5 u
\end{aligned}
$$

Jones and Scadron, Annals Phys. 81, 1 (1973)

$$P = \tfrac{1}{2}(P_+ + P_-) \qquad q = P_+ - P_-$$

- $G_1, G_2, G_3 \rightarrow$ multipoles G_M^*, G_E^*, G_C^*

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$$P = \tfrac{1}{2}(P_+ + P_-) \qquad q = P_+ - P_-$$

- $G_1, G_2, G_3 \rightarrow$ multipoles G_M^*, G_E^*, G_C^*
- Nucleon and Delta S-states $\Rightarrow G_E^* = G_C^* = 0$
- With positive parity axial diquarks:
  $\Rightarrow$ Nucleon or Delta D-states needed to generate Quadrupole FF

N-Δ transition (G_M^*)

- Magnetic dipole FF

$$G_M^*(Q^2) = \frac{4}{3\sqrt{3}} \frac{M}{M + M_\Delta} j_- \int \phi_\Delta \phi_N = \overbrace{2.07 \int \phi_\Delta \phi_N}^{Q^2=0}$$

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- Cauchy-Schwartz inequality (for $Q^2 = 0$):

$$\int \phi_\Delta \phi_N \leq \sqrt{\int \phi_N^2} \sqrt{\int \phi_\Delta^2} = 1$$

$\Rightarrow G_M^*(0) \leq 2.07$

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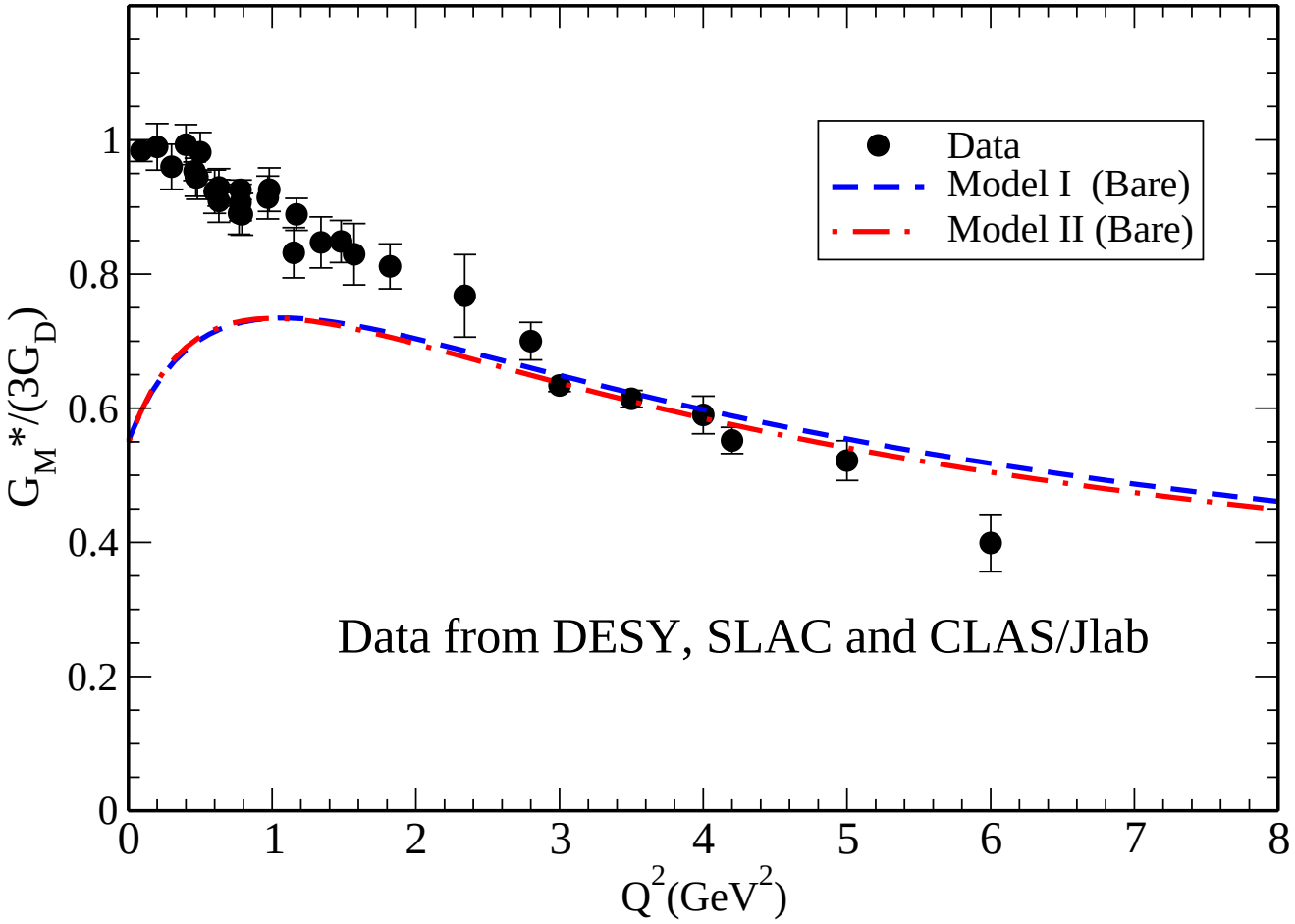
$$\int \phi_\Delta \phi_N \leq \sqrt{\int \phi_N^2} \sqrt{\int \phi_\Delta^2} = 1$$

$\Rightarrow G_M^*(0) \leq 2.07$

- $\Rightarrow$ Spectator QM can explain only 70% of the experimental $G_M^*(0)$

N-Δ transition (G_M^* form factor)



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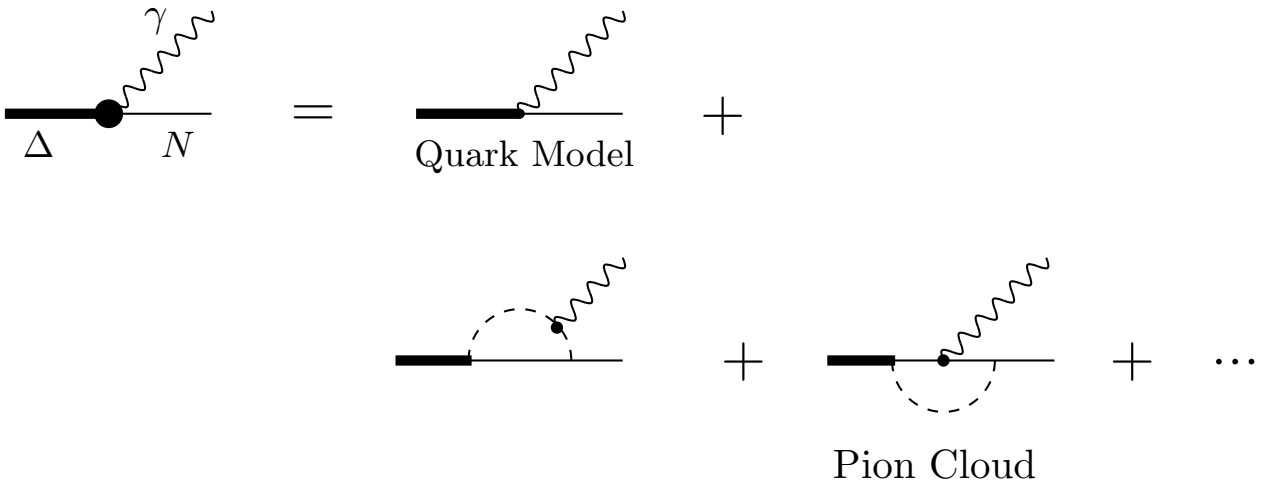
- Dynamical Models: Sato-Lee; Dubna-Mainz-Taipai (55-70%)
  Pion production dynamic: $\gamma N \to \pi N$, $\pi N \to \pi N$
  Sato-Lee PRC 54 2660 (1996), PRC 63 0552001 (2001);
  Kamalov et al PRC 64 032201 (2001), PRL 83 4494 (1999)

N-Δ transition (G_M^* form factor)



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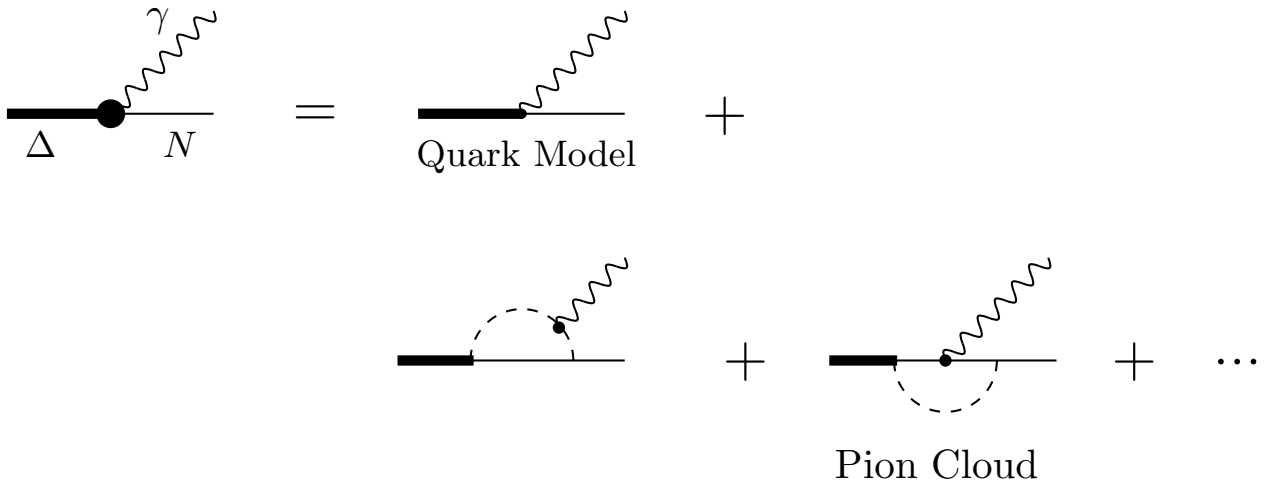

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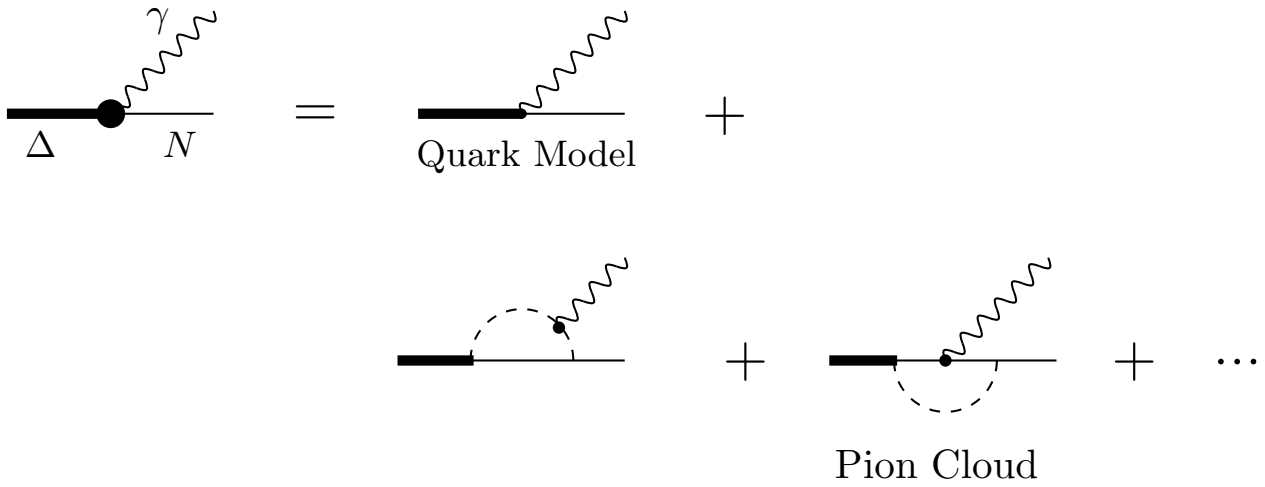

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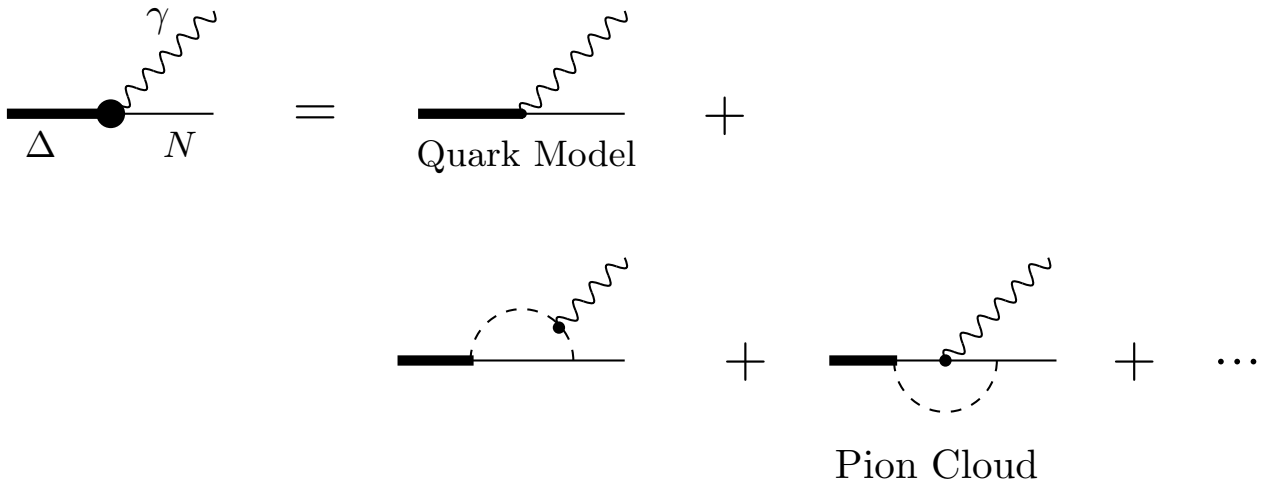

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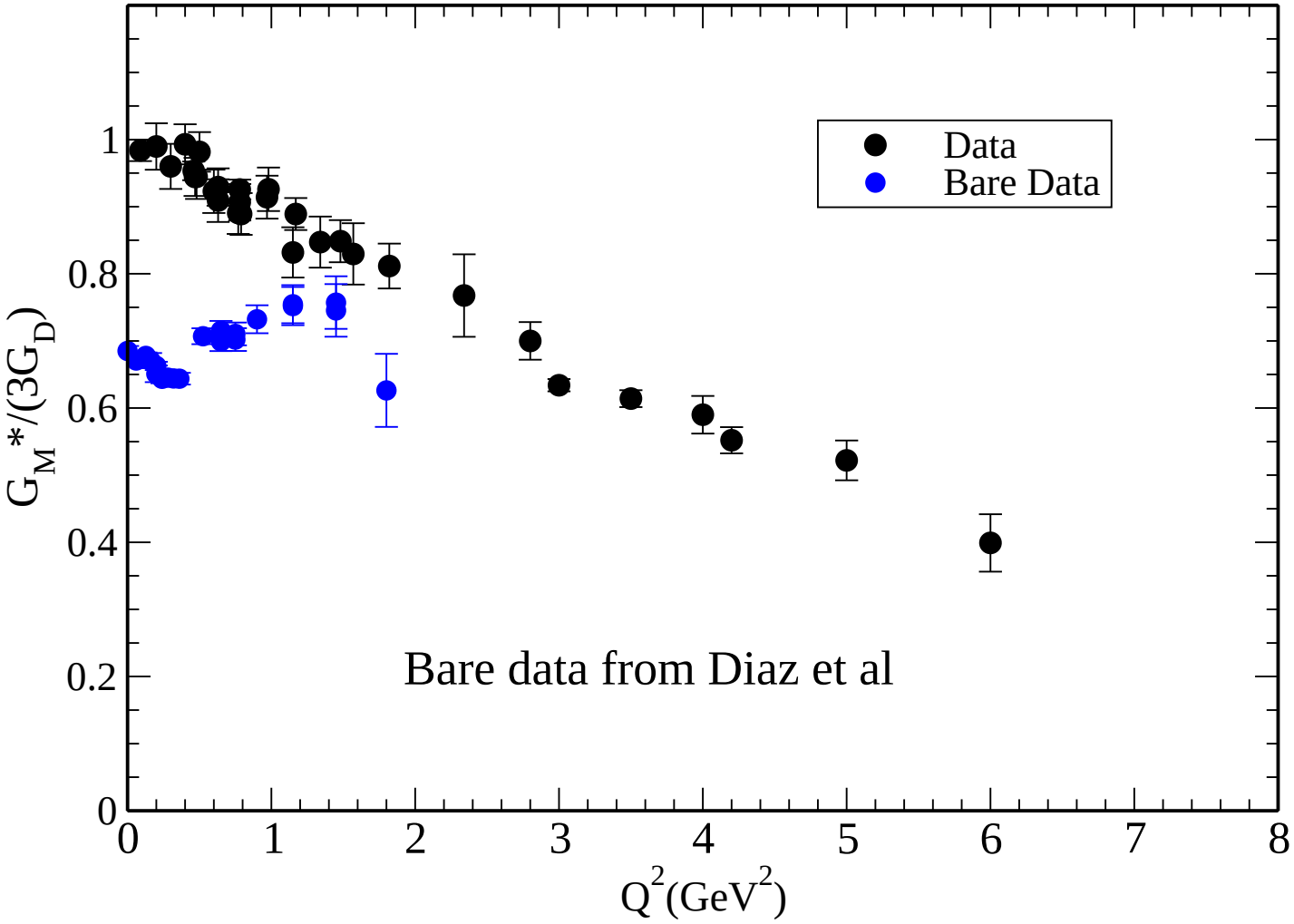
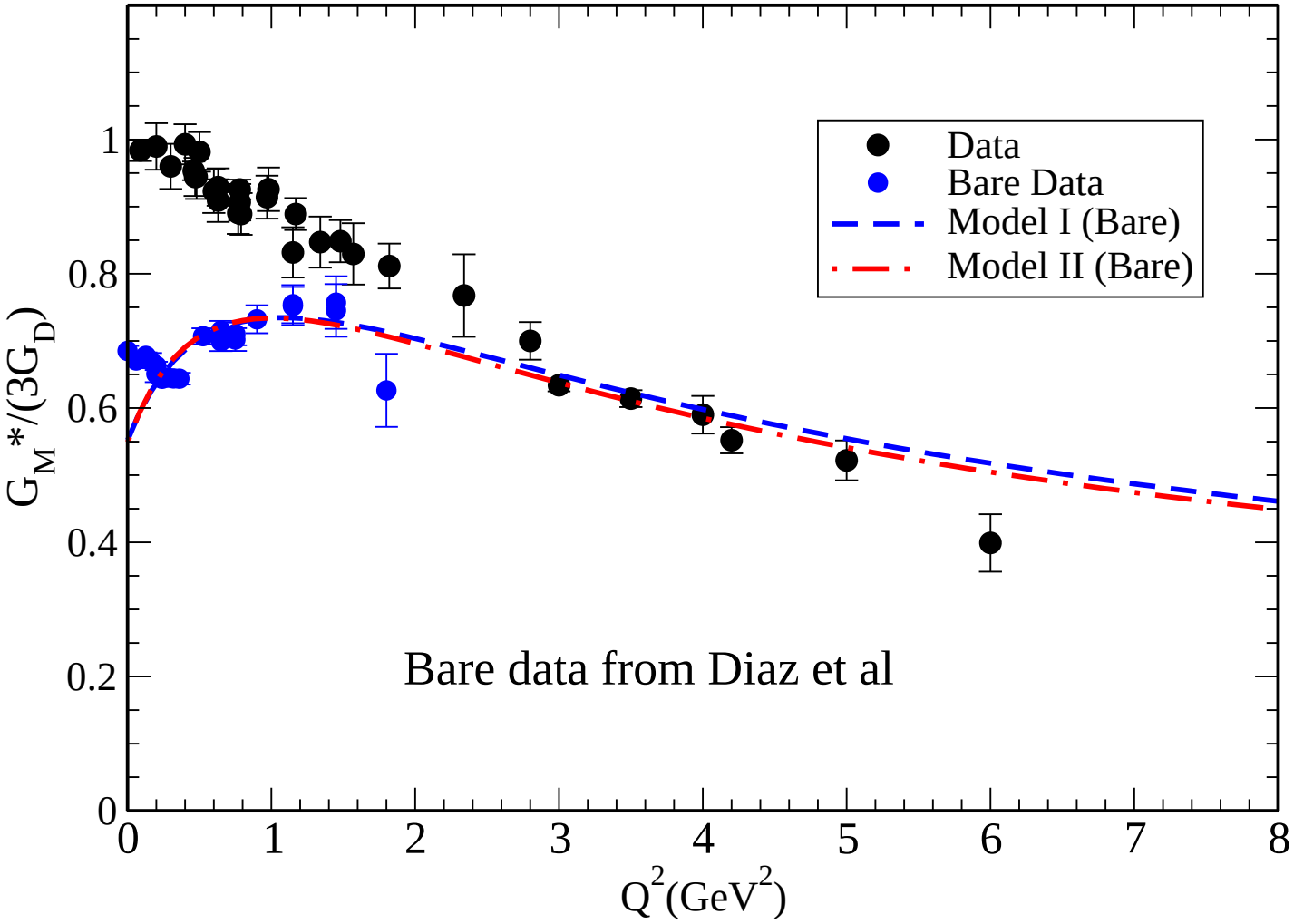
  Diaz, Sato, Lee and Smith PRC75, 015205 (2007)

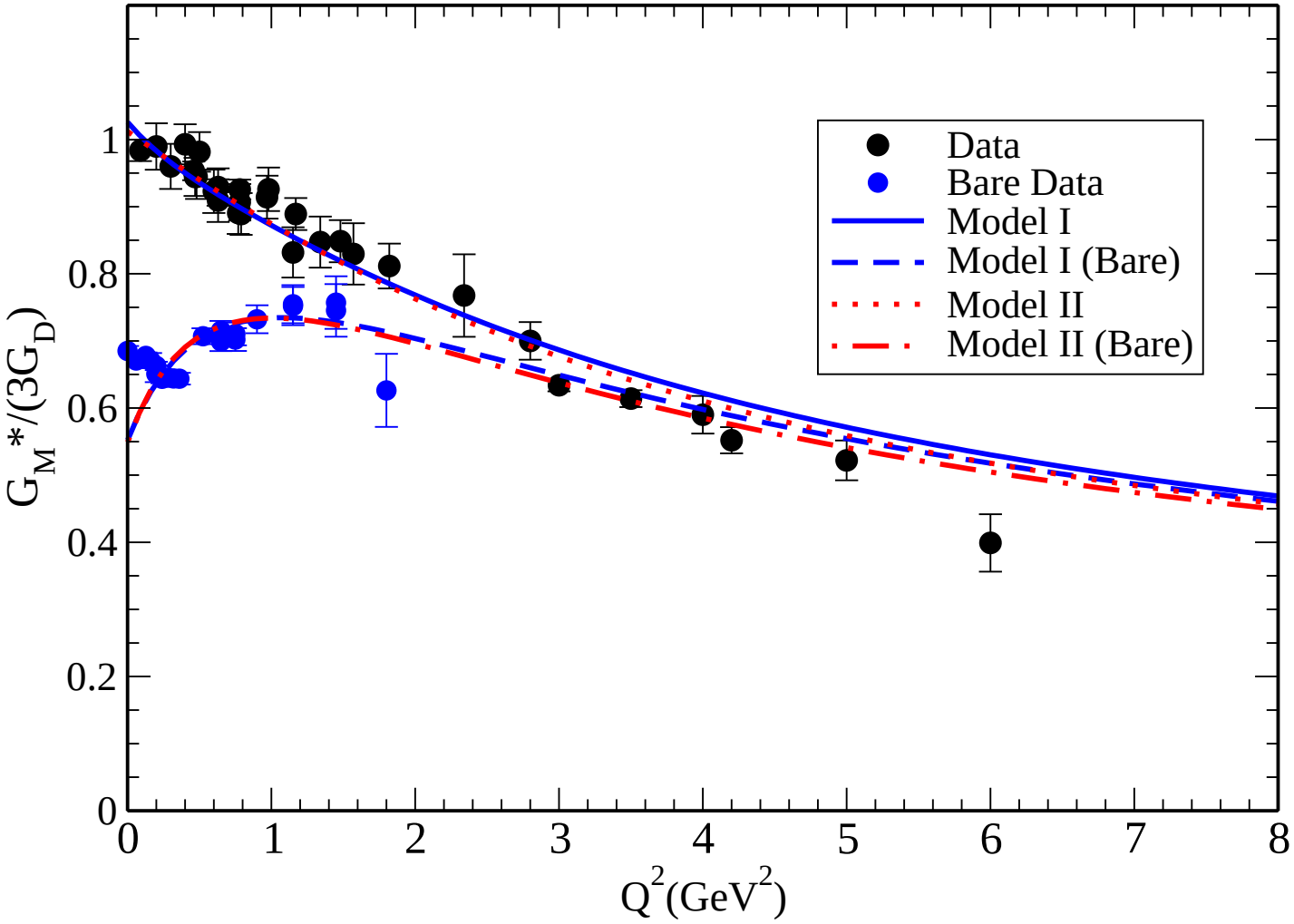
N-Δ transition: G_M^* form factor -Bare data



N-Δ transition: G_M^* form factor -Bare results



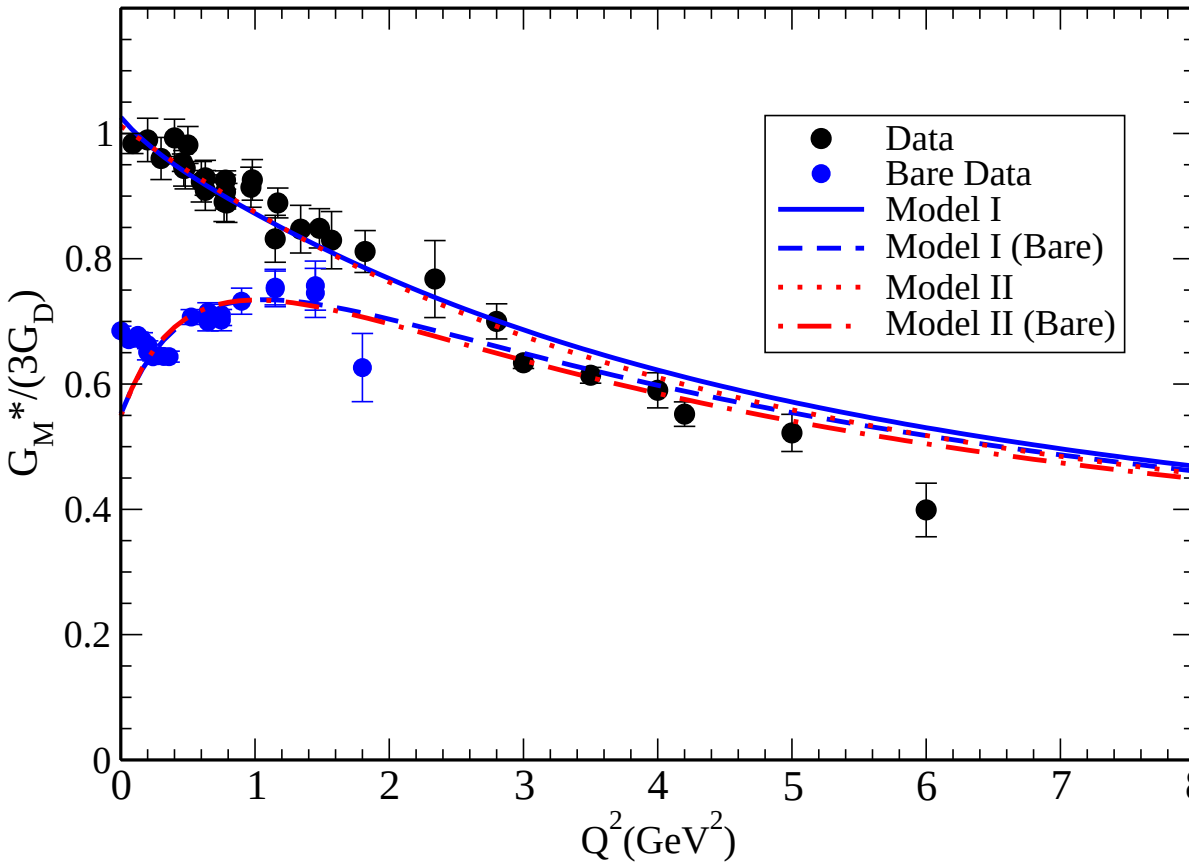
N-Δ transition G_M^* form factor - Results



N-Δ transition (Conclusions)

$$\phi_\Delta = \frac{N_\Delta}{[\alpha_1+(P-k)^2][\alpha_2+(P-k)^2]^2}$$

- Bare + Pion Cloud:
  Good description of the data $\chi^2 = 1.26$
  Explains falloff



2 + 2 parameters: (α_1, α_2), $(\lambda_\pi, \Lambda_\pi)$

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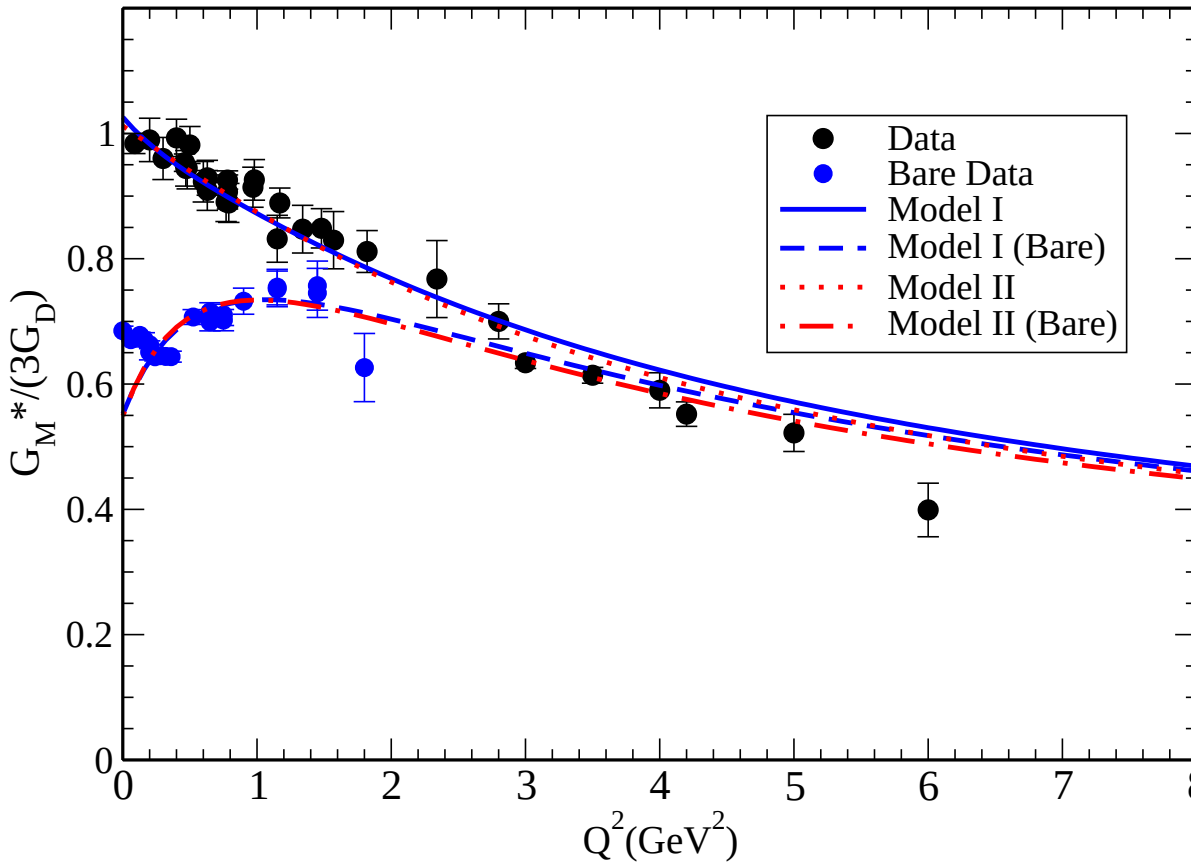

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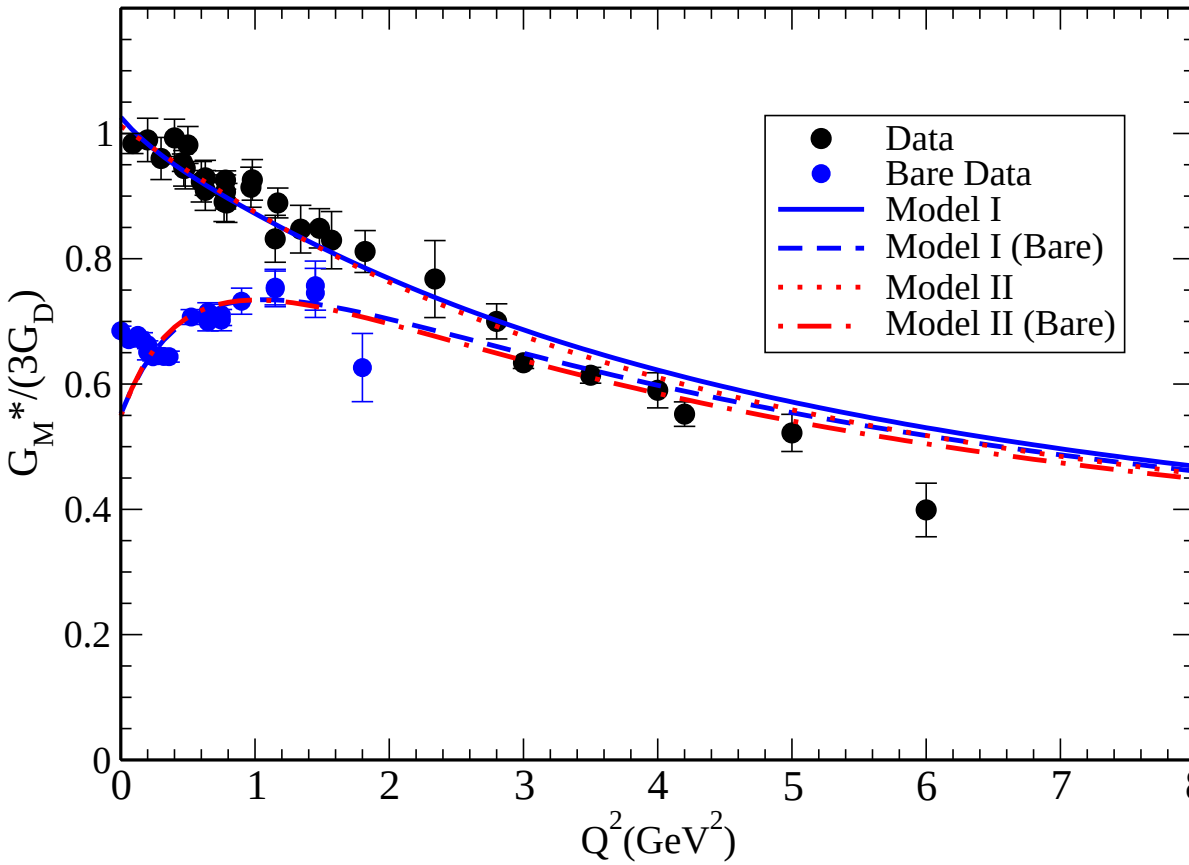

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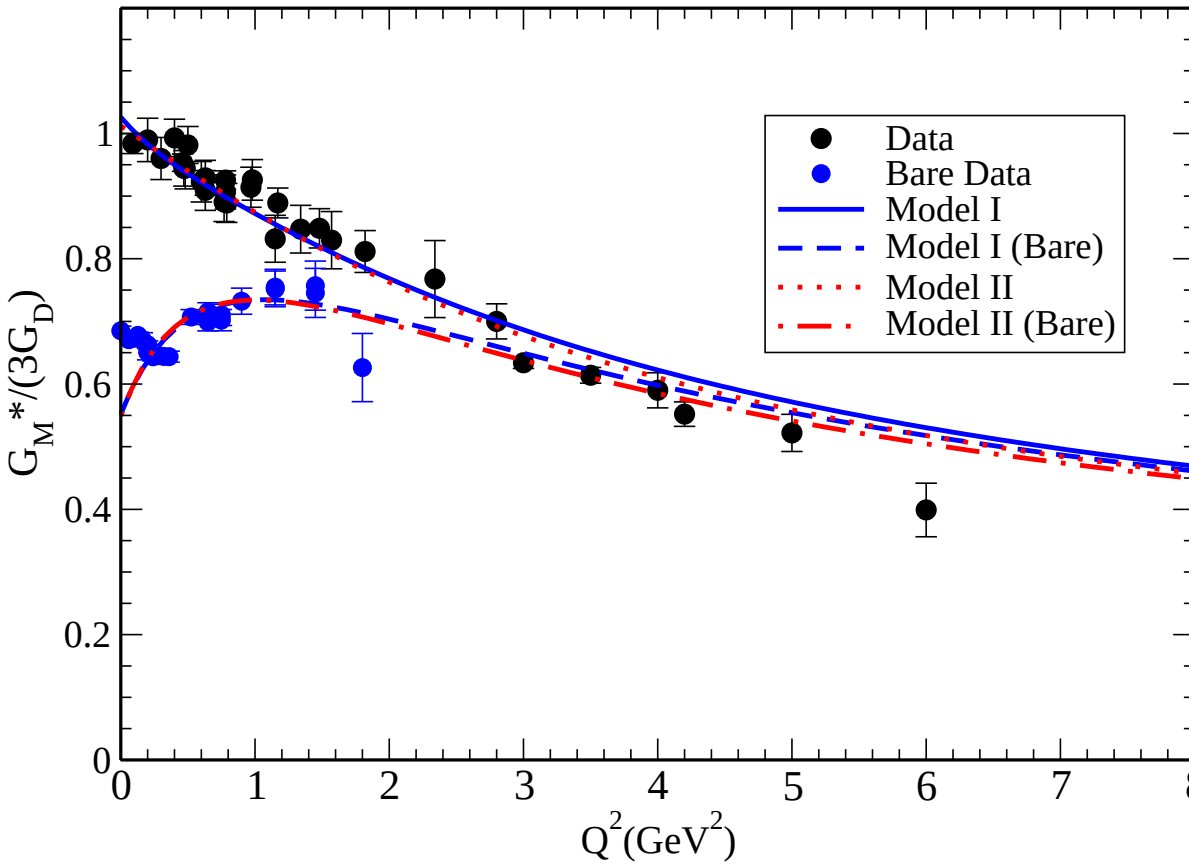

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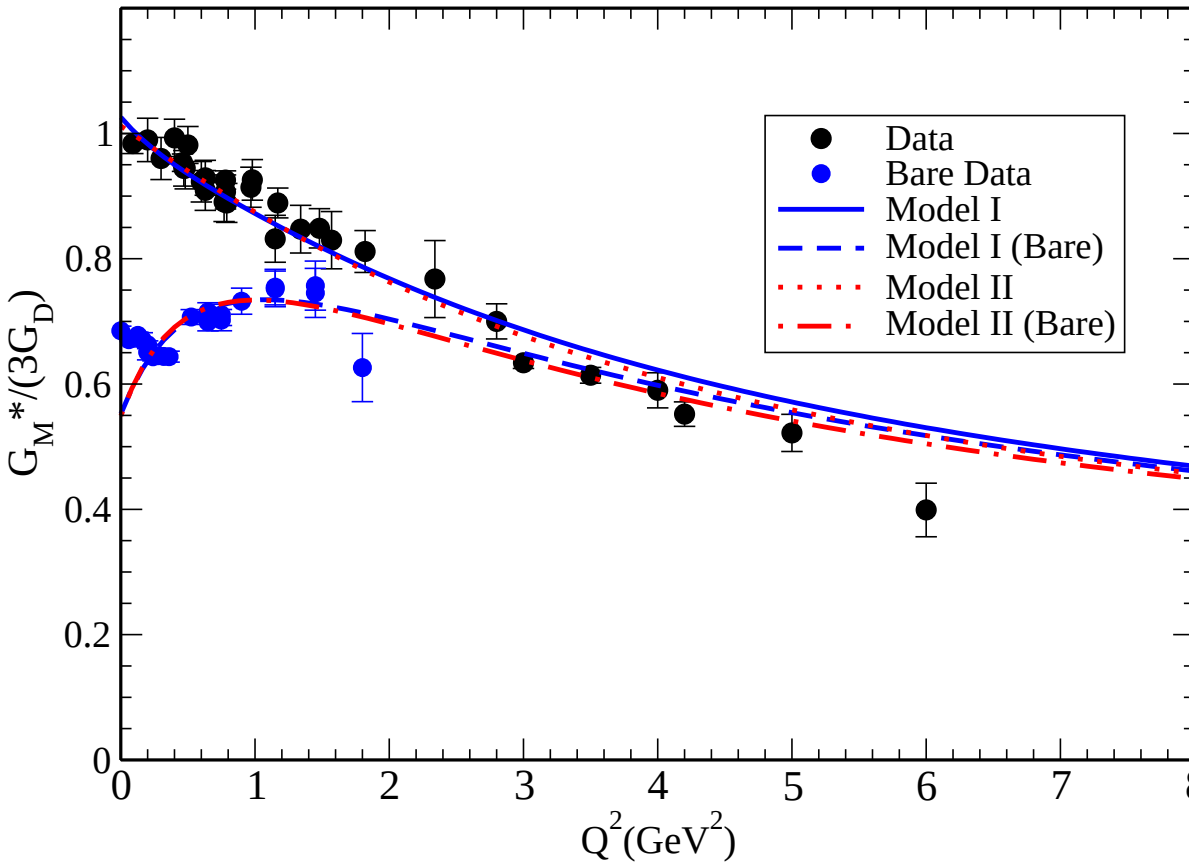

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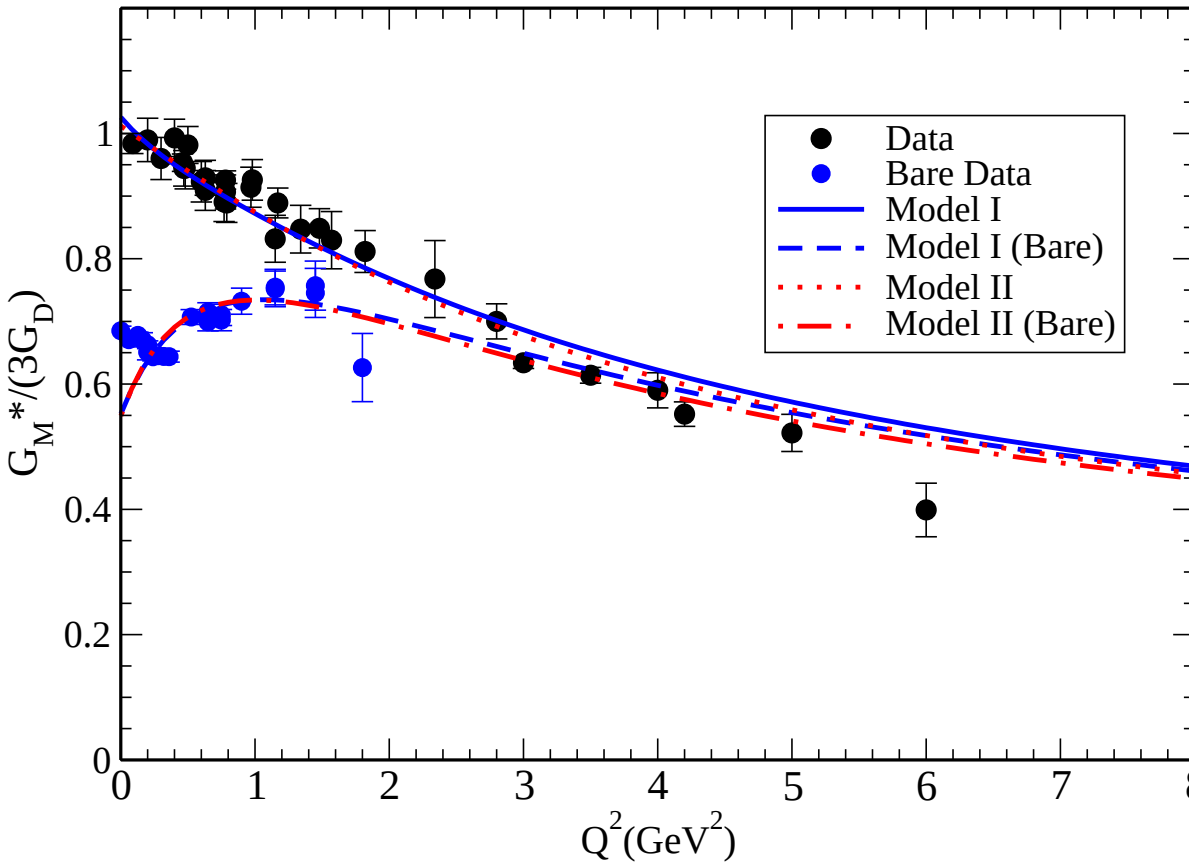
- $G_M^* \sim 1/Q^4$ like in pQCD



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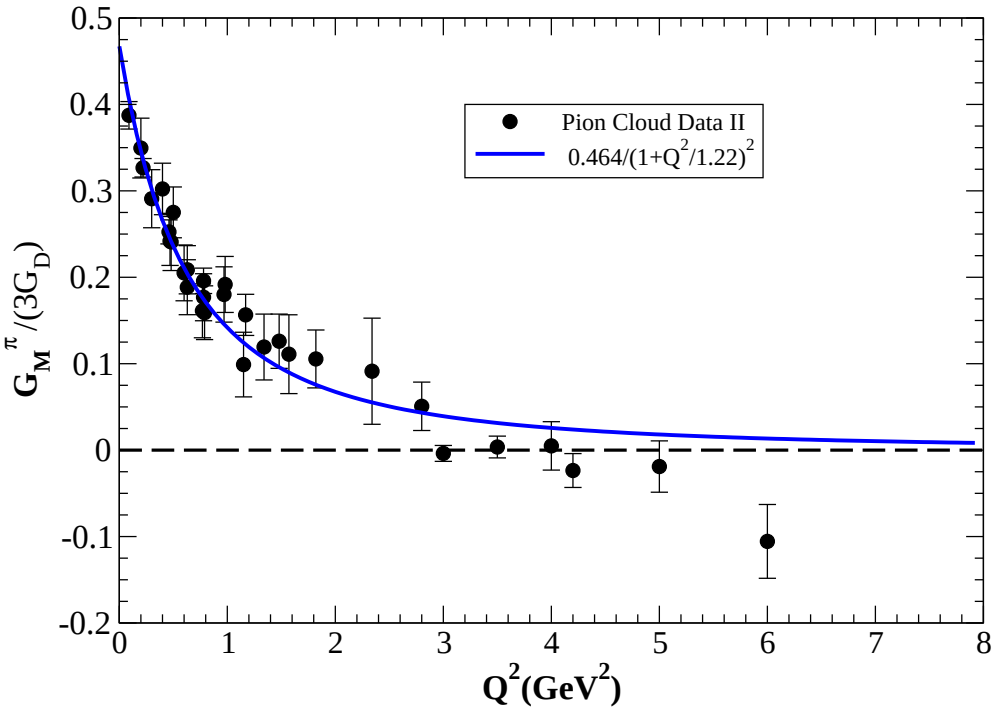
Pion Cloud

$$\frac{G_M^\pi}{3G_D} = \lambda_\pi \left(\frac{\Lambda_\pi^2}{\Lambda_\pi^2 + Q^2} \right)^2$$



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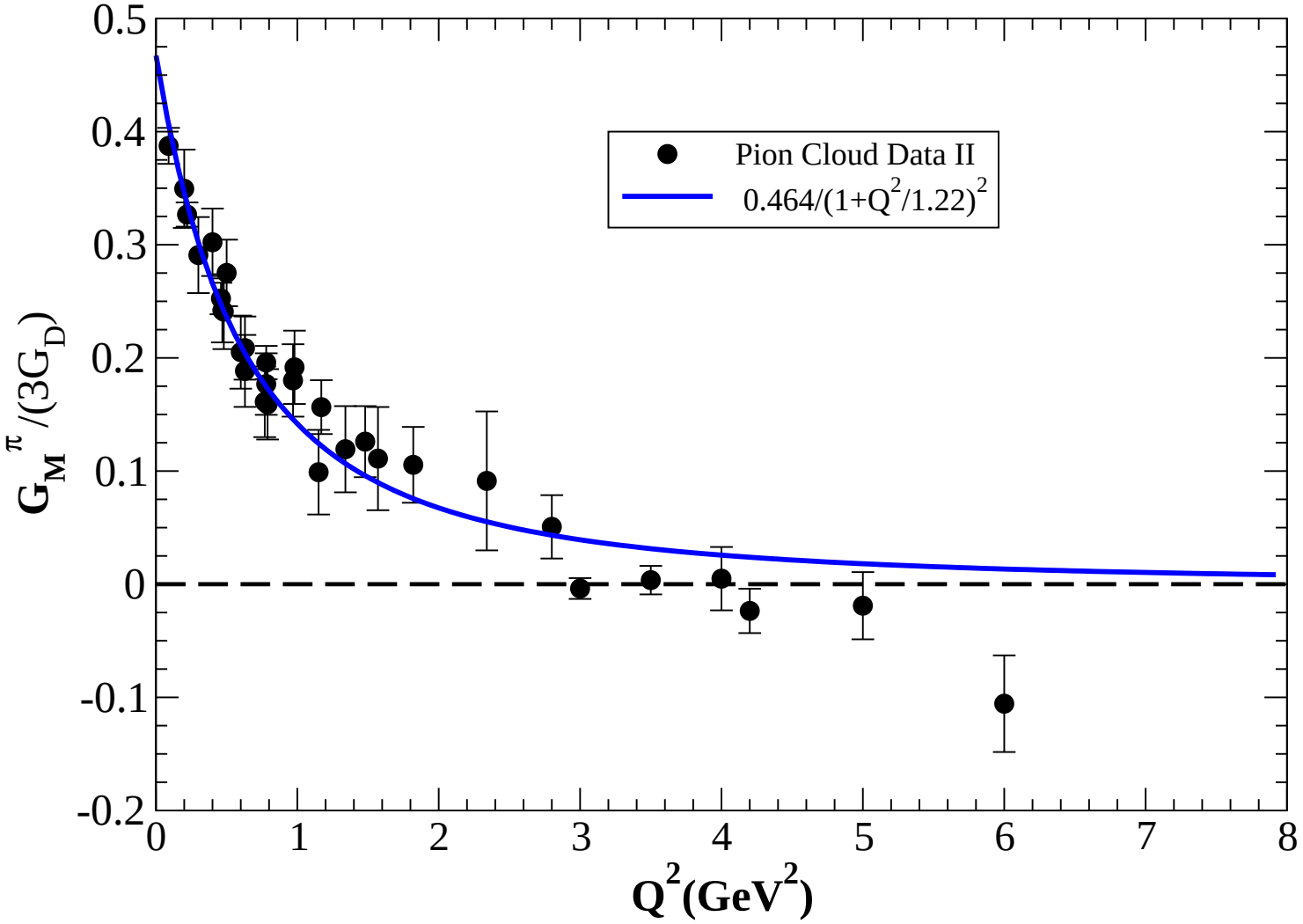
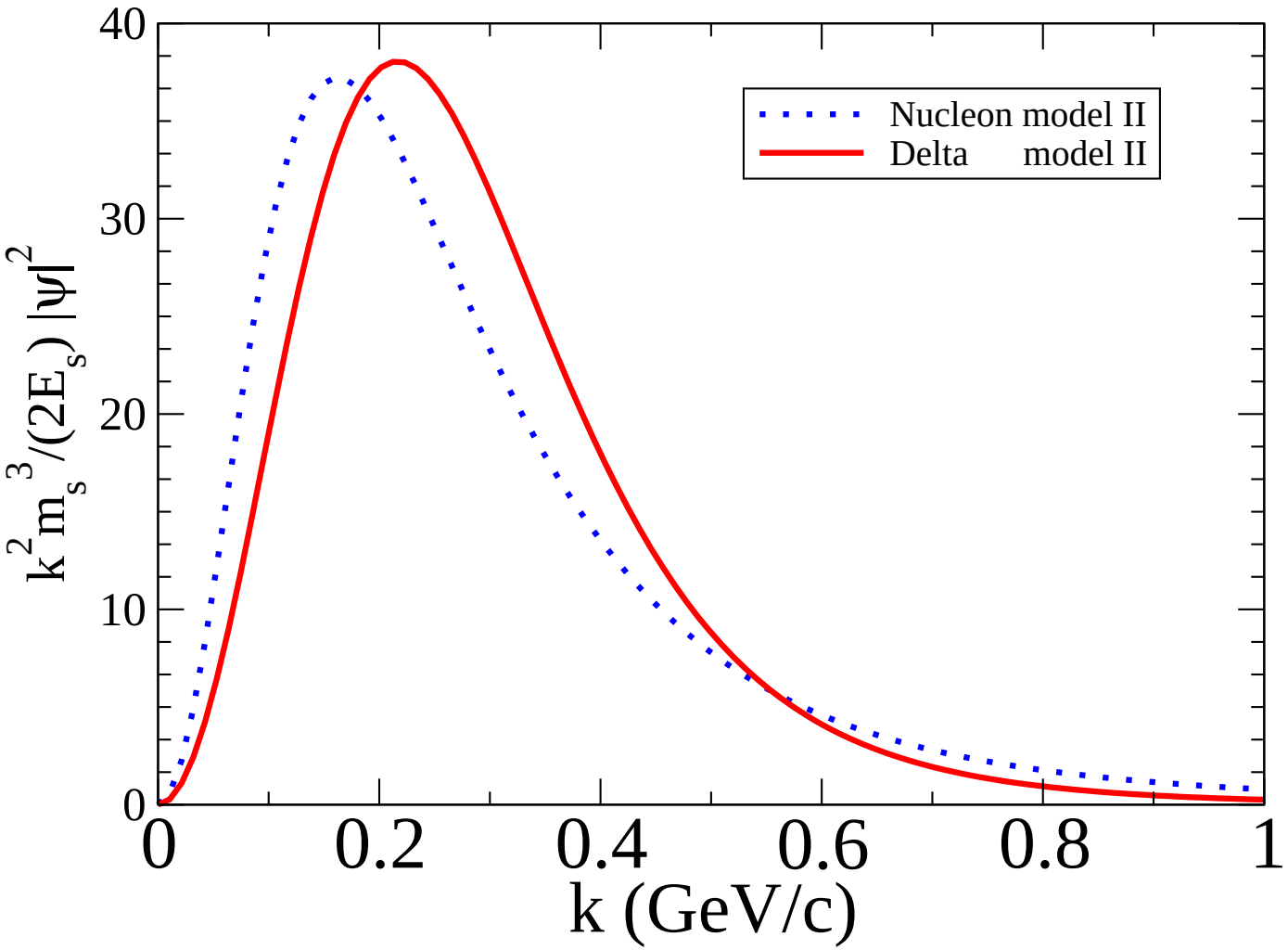

Pion Cloud Data II
0.464/(1+Q^2/1.22)^2
G_M^π/(3G_D)
Q^2(GeV^2)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
1
2
3
4
5
6
7
8

Comparing Nucleon and Δ wave functions



Classification of AM states: projectors

S-state Direct product of $1 \oplus \frac{1}{2}$ M_B=baryon mass

M. Benmerrouche et al PRC 39, 2339 (1989)

- $S = \frac{1}{2}\ \left(1, \frac{1}{2}\right)$: U^α $\mathcal{P}^{\alpha\beta}_{1/2} = \frac{1}{3}\left(\gamma^\alpha - \frac{P^\alpha \not{P}}{M_B^2}\right)\left(\gamma^\beta - \frac{P^\beta \not{P}}{M_B^2}\right)$
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$$\mathcal{P}^{\alpha\beta}_{1/2} + \mathcal{P}^{\alpha\beta}_{3/2} = g^{\alpha\beta} - \frac{P^\alpha P^\beta}{M_B^2} \xrightarrow{NR} I^{ij}(= \delta^{ij})$$

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- State projections:

$$\mathcal{P}_{1/2}U^\alpha = U^\alpha \quad \mathcal{P}_{3/2}U^\alpha = 0$$
$$\mathcal{P}_{1/2}w^\alpha = 0 \quad \mathcal{P}_{3/2}w^\alpha = w^\alpha$$

Classification of AM states: proprieties

Proprieties of the S-states

$$U^\alpha = \frac{1}{\sqrt{3}}\gamma_5\left(\gamma^\alpha - \frac{P^\alpha}{M_B}\right)u \quad \gamma^\alpha U_\alpha \neq 0, \quad P^\alpha U_\alpha = 0, \quad \not{P} U_\alpha = M_B U_\alpha$$

$$w^\alpha = \mathcal{T}^\alpha \omega \qquad \gamma^\alpha w_\alpha = 0, \quad P^\alpha w_\alpha = 0, \quad \not{P} w_\alpha = M_B w_\alpha$$

Example: Δ D-states

$$\begin{aligned} W_D^\alpha &= \mathcal{D}^{\alpha\beta}(P,k) w_\beta(P) \\ &= (\mathcal{P}_{1/2})^{\alpha\beta}\mathcal{D}_{\beta\sigma} w^\sigma + (\mathcal{P}_{3/2})^{\alpha\beta}\mathcal{D}_{\beta\sigma} w^\sigma \end{aligned}$$

$\mathcal{D}^{\alpha\beta}$: D-state operator

Delta wave function (Extension)

Adding angular momentum components to the Δ wave function

Configuration: (L, S)

$$\Psi_N(S\text{-state}) \rightarrow \Psi_\Delta \begin{cases} \left(0, \frac{3}{2}\right) & \rightarrow \quad G_M^* \\ \left(2, \frac{3}{2}\right) & \rightarrow \quad G_M^*, G_E^* \\ \left(2, \frac{1}{2}\right) & \rightarrow \quad \bar{G}_M^*, \bar{G}_E^*, G_C^* \end{cases}$$

$$\bar{G}_M^*, \bar{G}_E^* = 0 \quad \text{when} \quad Q^2 = 0$$

Delta wave function (Extension 2)

Adding angular momentum components to the Δ wave function

$$\left.\begin{array}{l} S\text{-state} \\ \\ D\text{-state} \\ \left(2, \frac{3}{2}\right)? \end{array}\right\} \Psi_N \longrightarrow \Psi_\Delta \left\{\begin{array}{lcl} \left(0, \frac{3}{2}\right) & \longrightarrow & G_M^* \\ \\ \left(2, \frac{3}{2}\right) & \longrightarrow & G_M^*, G_E^* \\ \\ \left(2, \frac{1}{2}\right) & \longrightarrow & \bar{G}_M^*, \bar{G}_E^*, G_C^* \end{array}\right.$$

Is the nucleon D-state important for the NΔ transition ?

Final Conclusions

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  - based in S-states; fixed-axis polarization vectors
  - satisfies Dirac equation and NR limit
  - gauge invariant currents
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- Systematic classification of the angular momentum components $\Rightarrow$
  - extension of the previous applications (N and Δ)
  - application to higher nucleon-resonance transition
  - $\Delta\Delta$ form factors (Δ magnetic moments), ...

References

- F. Gross, GR and M.T. Peña
  A Pure S-wave covariant model for the nucleon
  To appear in PRC; replace the previous nucl-th/0606029v1
- F. Gross, GR and M.T. Peña
  Fixed-axis polarization states: covariance and comparisons,
  arXiv:0708.0995 change of title; submitted to publication
- GR, M.T. Peña and F. Gross,
  A covariant model for the nucleon and Δ
  to be submitted very son ...