Parity Violation in Electron DIS

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The Program

- We shall investigate the feasibility of extracting PDF information from leptonic DIS with a proton target
- This will be done through systematic comparison of PDF effects with several effects associated with sub-leading Q^2 corrections (including the ratio of longitudinal to transverse cross-sections)
- In the deuteron, we shall investigate the accessibility of charge symmetry violating effects, again looking at sub-leading corrections
- For the future, we shall consider the possibility of a similar study of PV asymmetries with spin-polarized hadron targets, for which modern uncertainties are also large

The PV Asymmetry

The general form of the parity-violating asymmetry is

$$A^{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \tag{1}$$

in which the $+$ and $-$ are notational shorthand for anti-parallel lepton helicities in the unpolarized case.

Generically, these cross-sections are given by the current amplitudes of the lepton exchanges, i.e.

$$\frac{d^2\sigma_{nc}^{lN}}{d\Omega dE'} = \frac{\frac{E'}{E} \times |M_\gamma + M_Z|^2}{2M(4\pi)^2}, \tag{2}$$

where

$$|M_\gamma + M_Z|^2 = M_\gamma^2 + M_Z^2 + M_\gamma^* M_Z + M_\gamma M_Z^*. \tag{3}$$

As such, our main interest is with the interference terms of the cross-section, whence the parity violation originates. So,

$$A^{PV} \propto \frac{M_\gamma^* M_Z + M_\gamma M_Z^*}{|M_\gamma + M_Z|^2}. \tag{4}$$

Lagrangian and Current Couplings

The Lagrangian for the DIS interaction is

$$L = \frac{G_F}{\sqrt{2}} \cdot [\bar{e}\gamma^\lambda e[C_{2u}\bar{u}\gamma_5 u + C_{2d}\bar{d}\gamma_\lambda\gamma_5 d] \\ +\bar{e}\gamma^\lambda\gamma_5 e[C_{1u}\bar{u}\gamma_\lambda u + C_{1d}\bar{d}\gamma_\lambda d]]; \tag{1}$$

we know from Bjorken that in $SU(2) \times U(1)$, the couplings should assume the form

$$\begin{aligned} C_{1u} &= -\frac{1}{2}(1 - \frac{8}{3}\sin^2\theta_W) = 2g_A^e \cdot g_V^u \\ C_{1d} &= \frac{1}{2}(1 - \frac{4}{3}\sin^2\theta_W) = 2g_A^e \cdot g_V^d \\ C_{2u} &= -\frac{1}{2}(1 - 4\sin^2\theta_W) = 2g_V^e \cdot g_A^u \\ C_{2d} &= \frac{1}{2}(1 - 4\sin^2\theta_W) = 2g_V^e \cdot g_A^d. \end{aligned} \tag{2}$$

Interference Cross-Section

The most general form of the interference current cross-section is

$$\frac{d^3\sigma_{nc}^{lN}}{dxdyd\phi} = \frac{y\alpha^2}{2Q^2} L_{\mu\nu}^{\gamma Z} W_{\gamma Z}^{\mu\nu} \eta^{\gamma Z}, \tag{1}$$

in which $\eta^{\gamma Z} = (\frac{G_F M_Z^2}{\sqrt{2}\pi\alpha})(\frac{Q^2}{Q^2+M_Z^2})$ and the leptonic part may be written

$$L_{\mu\nu}^{\gamma Z} = [\overline{u}(l')\gamma_\mu(g_V^e - g_A^e\gamma_5)u(l)]^*[\overline{u}(l')\gamma_\nu u(l)]; \tag{2}$$

analogously, the hadronic tensor, which we average over $S = \frac{1}{2}$ and $S = -\frac{1}{2}$ is

$$W_{\gamma Z}^{\mu\nu} = \sum_X (\langle X|J_Z^\mu|N\rangle^* \langle X|J_\gamma^\nu|N\rangle + \langle X|J_\gamma^\mu|N\rangle^* \langle X|J_Z^\nu|N\rangle)(2\pi)^3\delta(p_x - p - q). \tag{3}$$

Written out in terms of structure functions, this is

$$\begin{aligned}\frac{1}{2M}W_{\mu\nu}^i = &\frac{-g_{\mu\nu}}{M}F_1^i + \frac{p_\mu p_\nu}{M(p\cdot q)}F_2^i + \frac{i\epsilon_{\mu\nu\alpha\beta}}{2(p\cdot q)}[\frac{p^\alpha q^\beta}{M}F_3^i + 2q^\alpha S^\beta g_1^i \\ &-4xp^\alpha S^\beta g_2^i] - \frac{p_\mu S_\nu + S_\mu p_\nu}{2(p\cdot q)}g_3^i + \frac{S\cdot q}{(p\cdot q)^2}p_\mu p_\nu g_4^i + \frac{S\cdot q}{p\cdot q}g_{\mu\nu}g_5^i,\end{aligned} \tag{4}$$

and again, $i = \gamma Z$.

Structure Functions in the QPM

As a simple reminder, the unpolarized structure functions have the following form in the parton model:

$$\begin{aligned} F_1^{\gamma} &= \frac{1}{2}\sum_q e_q^2(q+\bar{q}) \\ F_2^{\gamma} &= 2xF_1^{\gamma}, \end{aligned} \tag{1}$$

whereas the interference structure functions are

$$\begin{aligned} F_1^{\gamma Z} &= \sum_q e_q(G_V)_q(q+\bar{q}) \\ F_2^{\gamma Z} &= 2xF_1^{\gamma Z} \\ F_3^{\gamma Z} &= 2\sum_q e_q(g_A)_q(q-\bar{q}) \\ g_1^{\gamma Z} &= \sum_q e_q(g_V)_q(\Delta q+\Delta\bar{q}) \\ g_5^{\gamma Z} &= \sum_q e_q(g_A)q(\Delta q-\Delta\bar{q}). \end{aligned} \tag{2}$$

I have listed only those structure functions immediately germane to this study.

Basic Form, Proton PV Asymmetry

The full expression for the unpolarized proton asymmetry in terms of structure functions is

$$A^{PV} = \frac{G_F Q^2}{\sqrt{2}\pi\alpha} \frac{[g_A^e \cdot (2xyF_1^{\gamma Z} + \frac{2}{y}(1 - y - \frac{xyM}{E})F_2^{\gamma Z}) + g_V^e \cdot 2x(1 - \frac{y}{2})F_3^{\gamma Z}}{2xyF_1^{\gamma} + \frac{2}{y}(1 - y - \frac{xyM}{E})F_2^{\gamma}}. \tag{1}$$

Using the fact that

$$R(x, Q^2) = [1 + \frac{Q^2}{\nu^2}] \cdot \frac{F_2(x, Q^2)}{2xF_1(x, Q^2)} - 1, \tag{2}$$

we may write the full expression for the asymmetry as

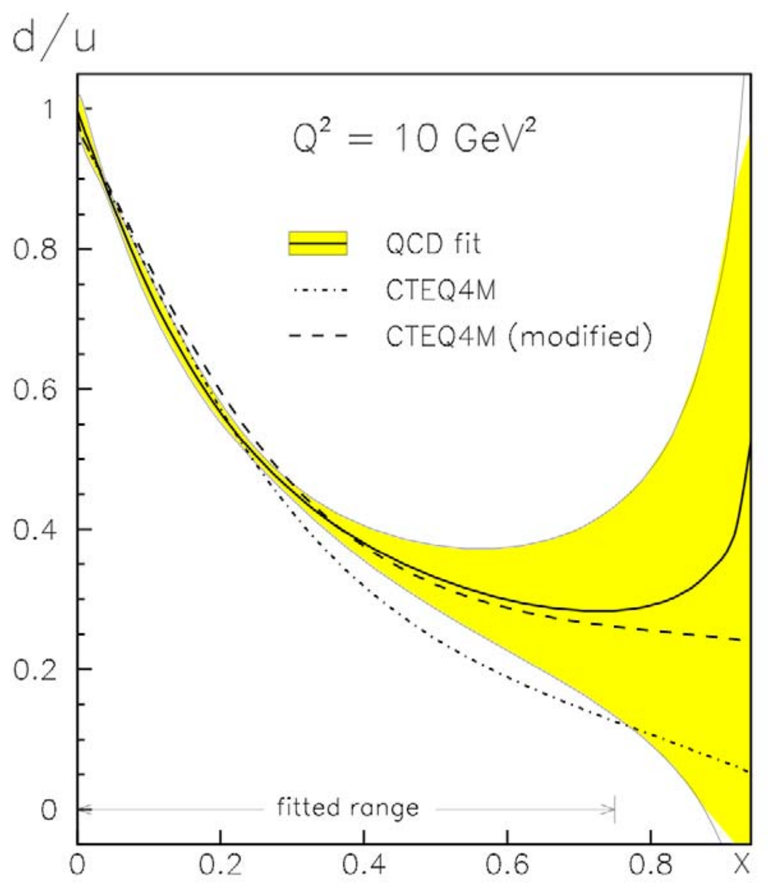
$$A^{PV} = (\frac{G_F Q^2}{\sqrt{2}\pi\alpha}) \cdot [g_A^e \frac{F_1^{\gamma Z}}{F_1^{\gamma}} f_1(y) + g_V^e \frac{F_3^{\gamma Z}}{F_1^{\gamma}} \frac{f_3(y)}{2}], \tag{3}$$

or

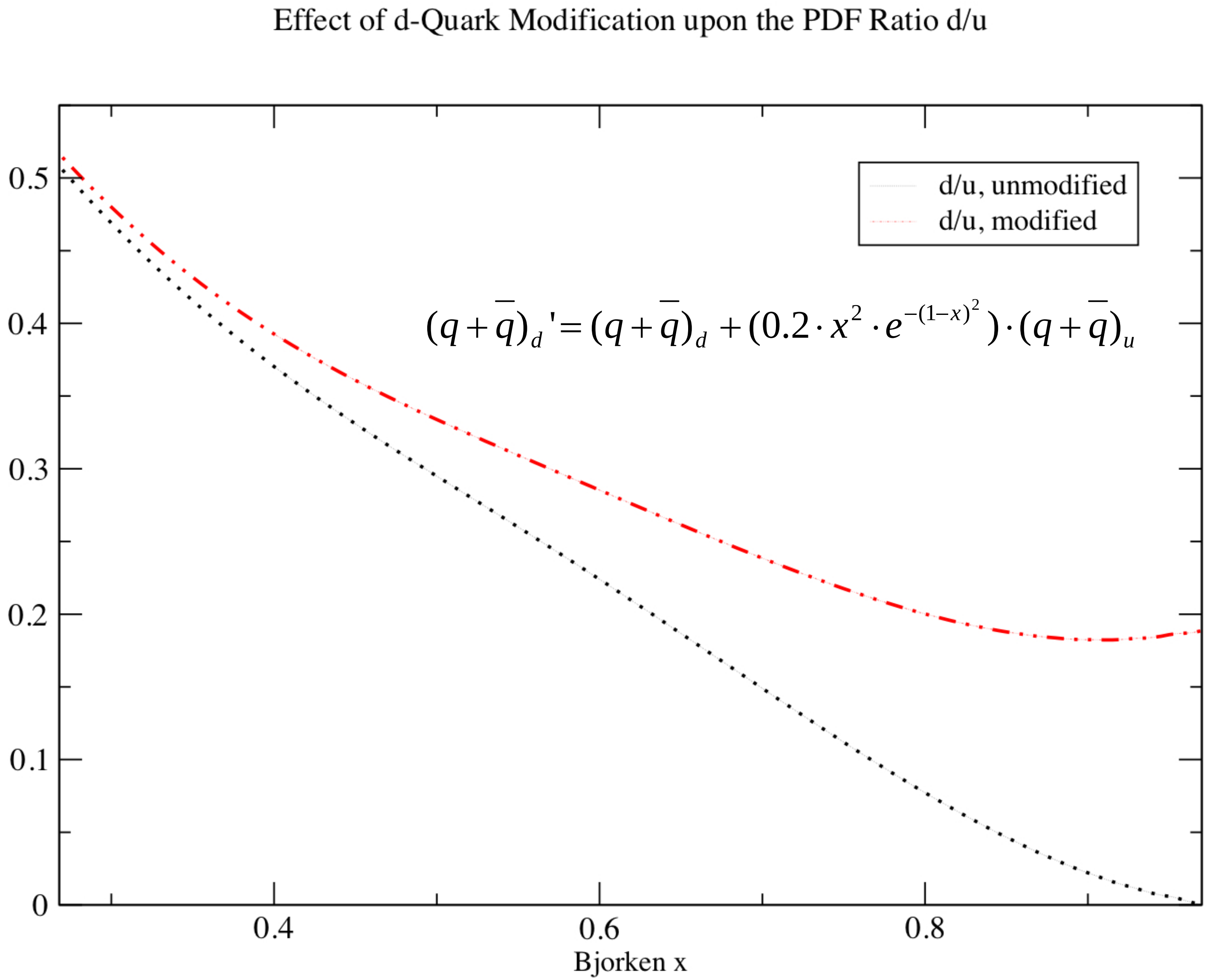
$$A^{PV} = (\frac{G_F Q^2}{\sqrt{2}\pi\alpha}) \cdot [g_A^e a(x) + g_V^e b(x)], \tag{4}$$

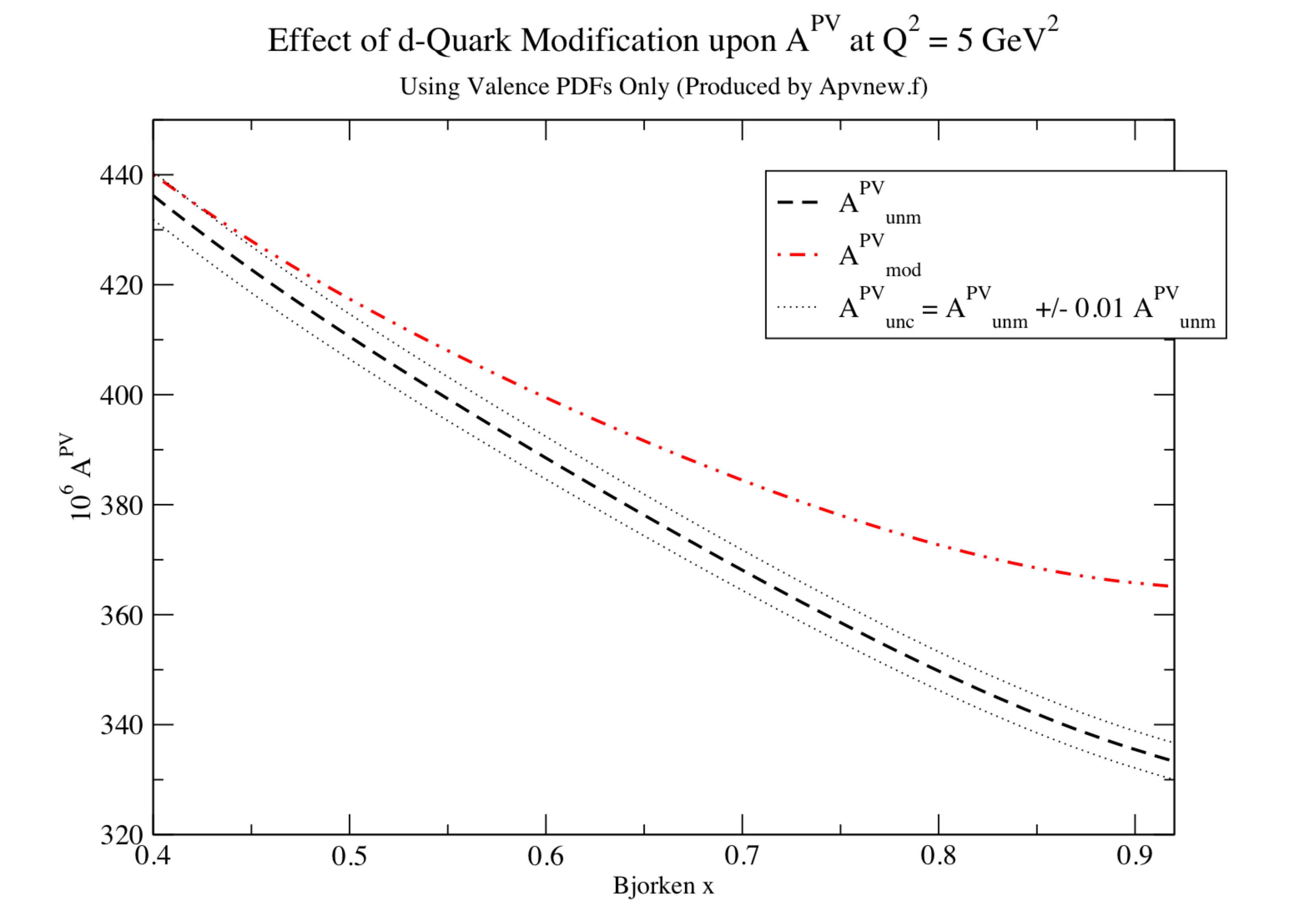
in which

$$\begin{aligned} f_1(y) &= \frac{[1 + (1-y)^2 - y^2(1 - \frac{r^2}{1+R^{\gamma Z}}) - \frac{2xyM}{E}]}{[1 + (1-y)^2 - y^2(1 - \frac{r^2}{1+R^{\gamma}}) - \frac{2xyM}{E}]} \cdot \frac{(1 + R^{\gamma Z})}{(1 + R^{\gamma})} \\ f_3(y) &= \frac{[1 - (1-y)^2]}{1 + (1-y)^2 - y^2(1 - \frac{r^2}{1+R^{\gamma}}) - \frac{2xyM}{E}]} \cdot \frac{r^2}{(1 + R^{\gamma})}. \end{aligned} \tag{5}$$

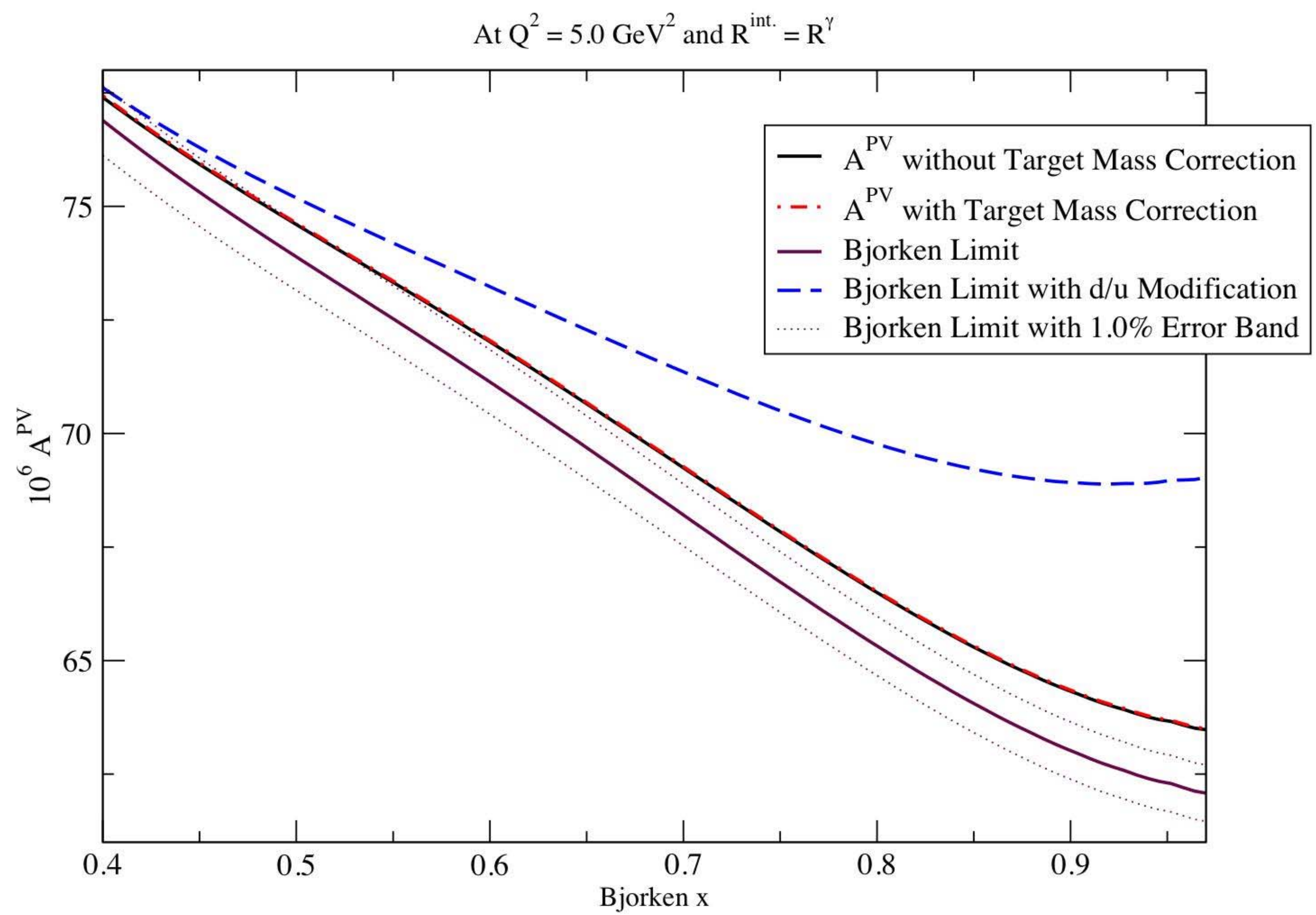


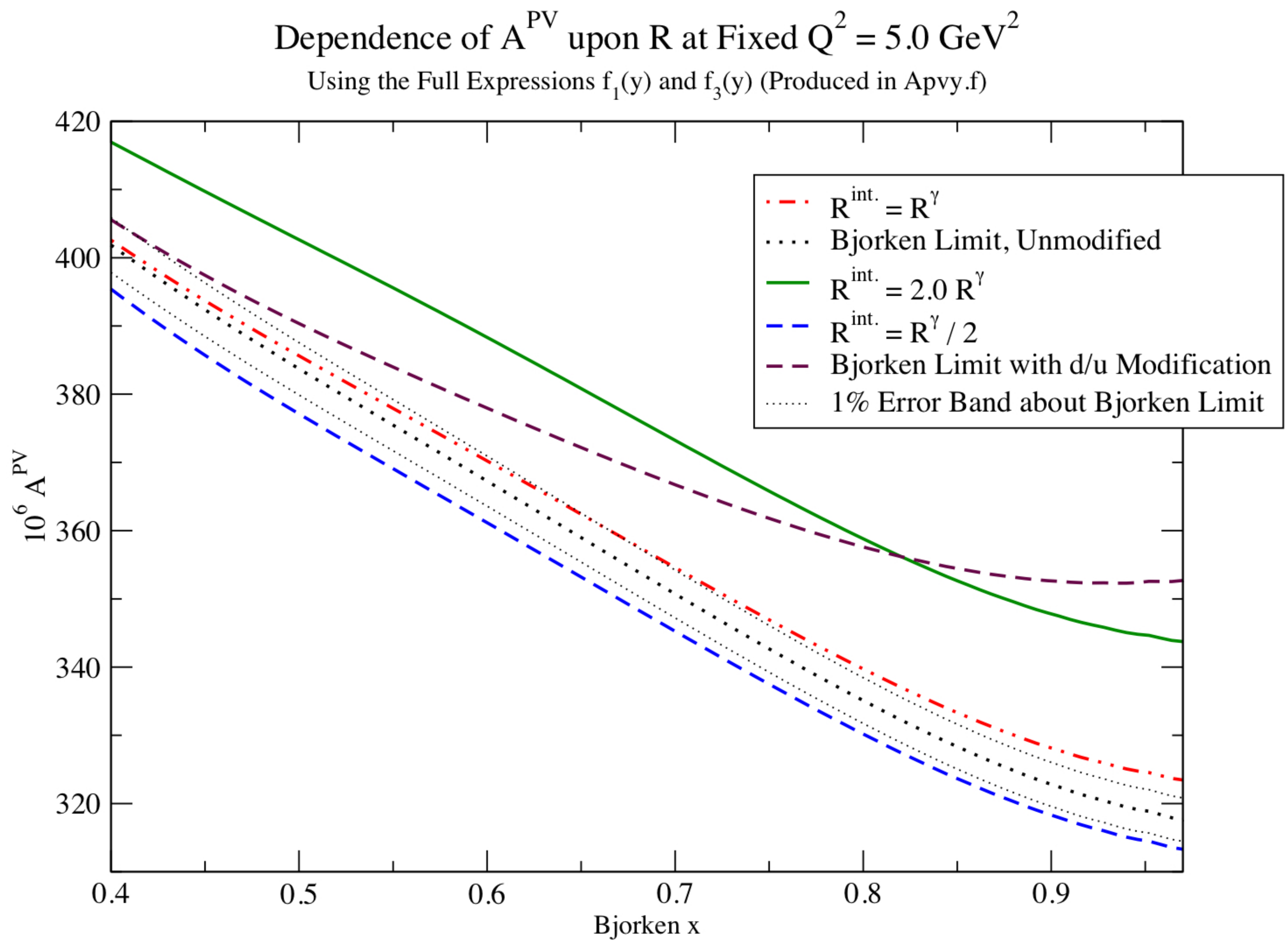
d/u
$Q^2 = 10 \text{ GeV}^2$
QCD fit
CTEQ4M
CTEQ4M (modified)
fitted range
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
X


Effect of d-Quark Modification upon the PDF Ratio d/u
d/u, unmodified
d/u, modified
$(q+\bar{q})_d' = (q+\bar{q})_d + (0.2 \cdot x^2 \cdot e^{-(1-x)^2}) \cdot (q+\bar{q})_u$
0.5
0.4
0.3
0.2
0.1
0
0.4
0.6
0.8
Bjorken x

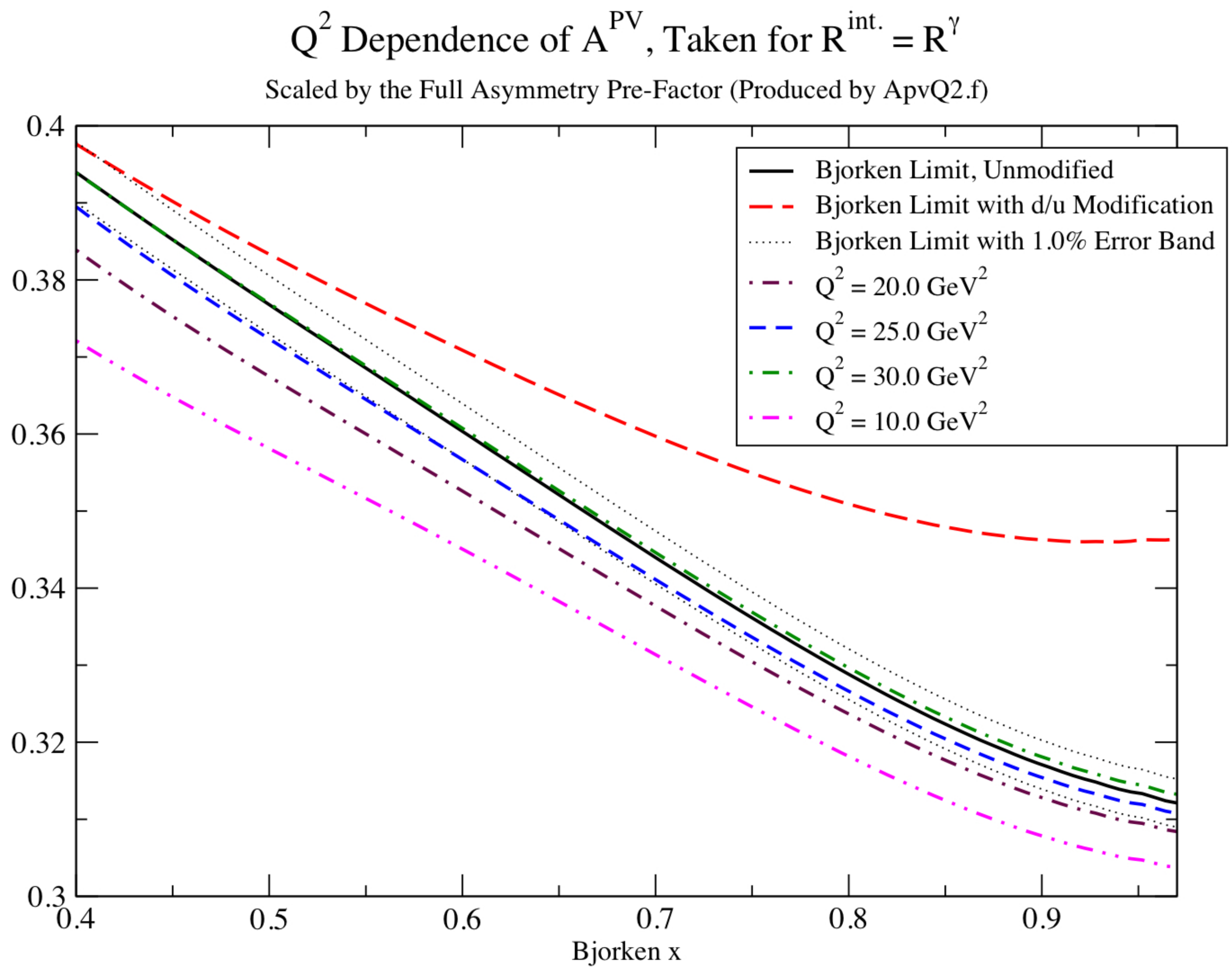

Effect of d-Quark Modification upon A^PV at Q^2 = 5 GeV^2
Using Valence PDFs Only (Produced by Apvnew.f)
$10^6 A^{PV}$
Bjorken x
440
420
400
380
360
340
320
0.4
0.5
0.6
0.7
0.8
0.9
A^{PV}_{unm}
A^{PV}_{mod}
$A^{PV}_{unc} = A^{PV}_{unm}$ +/- 0.01 A^{PV}_{unm}

Comparison of A^{PV} with and without M/E Correction





Dependence of A^PV upon R at Fixed Q^2 = 5.0 GeV^2
Using the Full Expressions f_1(y) and f_3(y) (Produced in Apvy.f)
420
400
380
360
340
320
10^6 A^PV
0.4
0.5
0.6
0.7
0.8
0.9
Bjorken x
R^int. = R^γ
Bjorken Limit, Unmodified
R^int. = 2.0 R^γ
R^int. = R^γ / 2
Bjorken Limit with d/u Modification
1% Error Band about Bjorken Limit



Q^2 Dependence of A^{PV}, Taken for $R^{int.} = R^{\gamma}$
Scaled by the Full Asymmetry Pre-Factor (Produced by ApvQ2.f)
Bjorken Limit, Unmodified
Bjorken Limit with d/u Modification
Bjorken Limit with 1.0% Error Band
Q^2 = 20.0 GeV^2
Q^2 = 25.0 GeV^2
Q^2 = 30.0 GeV^2
Q^2 = 10.0 GeV^2
0.3
0.32
0.34
0.36
0.38
0.4
0.4
0.5
0.6
0.7
0.8
0.9
Bjorken x

Observations on A^{PV}_{p}

- Encouragingly, A^{PV}_{p} appears sensitive to the d/u PDF ratio
- The effect of R^{γ} is important
- The effect of $R^{\gamma Z}$ may be important (although it seems reasonable to suspect $R^{\gamma Z} \approx R^{\gamma}$)

Charge Symmetry Violation in the Deuteron Asymmetry

The deuteron asymmetry retains the structure function form encountered in the proton.

Our main project is to write A^{PV}_d in a CSV-sensitive way.

We define the parameters δu and δd as measures of the deviation from charge symmetry in the $a(x)$ term of A_d^{PV} ; in particular, these assume the form

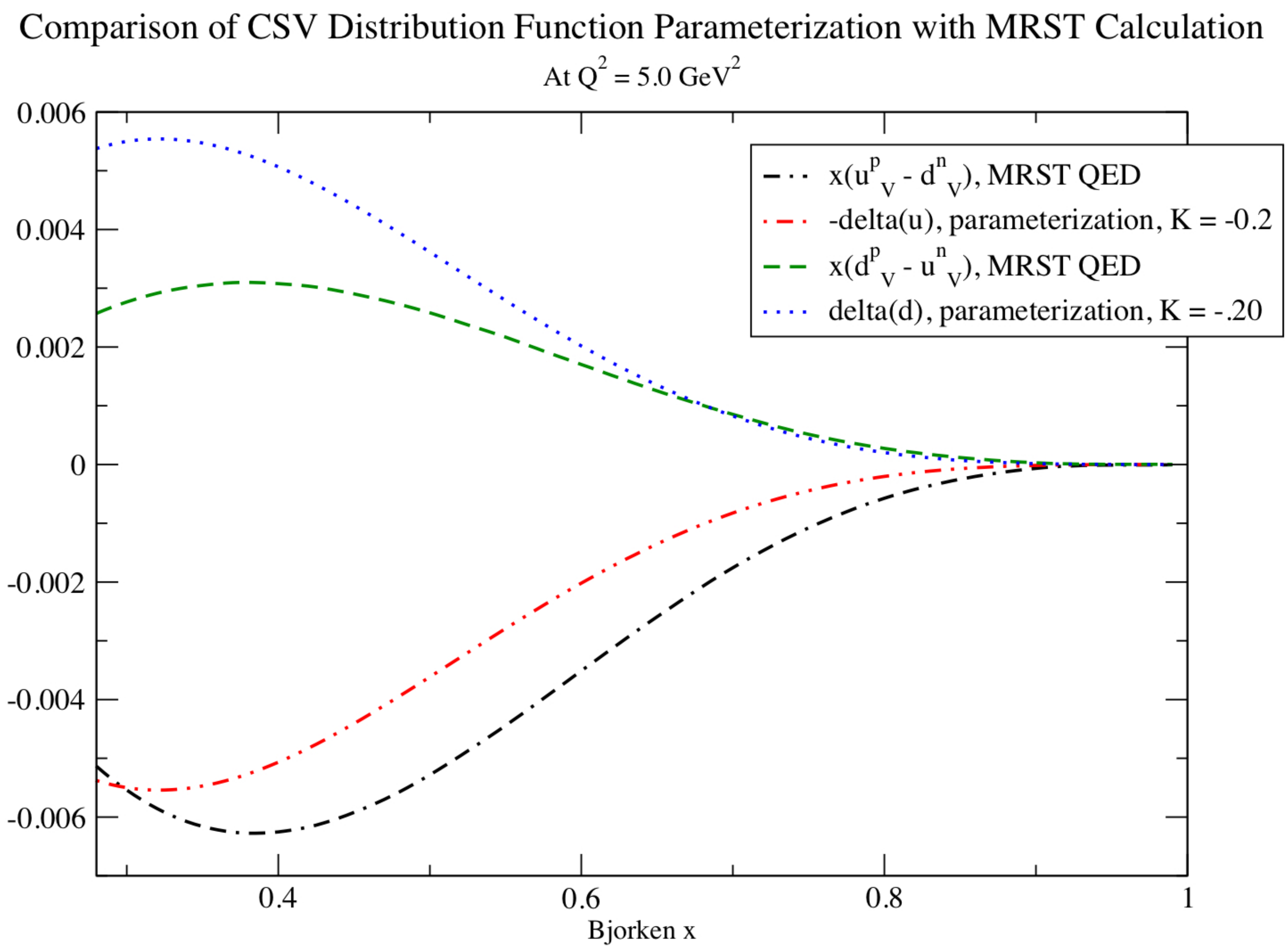
$$\begin{aligned} \delta u &= u^p - d^n \\ \delta d &= d^p - u^n. \end{aligned} \tag{1}$$

We notice that these definitions have the helpful property that

$$\begin{aligned} u^p - \frac{\delta u}{2} &= d^n + \frac{\delta u}{2} = u \\ d^p - \frac{\delta d}{2} &= u^n + \frac{\delta d}{2} = d \end{aligned} \tag{2}$$

– a trick due to Krishna Kumar. We also shall make use of a common parameterization:

$$\begin{aligned} \delta d_V &= -\kappa f(x) = -\delta u_V \\ f(x) &= x^{-\frac{1}{2}}(1-x)^4(x-0.0909). \end{aligned} \tag{3}$$


Comparison of CSV Distribution Function Parameterization with MRST Calculation
At Q2 = 5.0 GeV2
x(up V - dn V), MRST QED
-delta(u), parameterization, K = -0.2
x(dp V - un V), MRST QED
delta(d), parameterization, K = -.20
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
0.4
0.6
0.8
1
Bjorken x

Deviation from Charge Symmetry in a(x)

Using our definitions in the parton model expressions for the structure functions, we find

$$F_1^{\gamma Z} = -e_d \cdot [(2C_{1u} - C_{1d})(u+d) + \frac{(2C_{1u} - C_{1d})}{2}(\delta u - \delta d)]$$
$$F_1^{\gamma} = e_d^2 \cdot [5(u+d) + \frac{3}{2}(\delta u - \delta d)], \quad (3)$$

such that

$$\frac{\delta^{CSV} a(x)}{a_0(x)} = [\frac{3}{10} - \frac{2C_{1u} + C_{1d}}{2(2C_{1u} - C_{1d})}] \cdot \frac{\delta u - \delta d}{u+d} \quad (4)$$

and

$$a_0(x) = \frac{3}{5}(2C_{1u} - C_{1d}). \quad (5)$$

CSV in the b(x) term

These results are readily extended to the parity-violating $b(x)$ term. As we are ignoring effects of sea quarks in our numerics, the PDF structure of $b(x)$ does not differ from that of $a(x)$.

Given $b(x) = \frac{F_3^{\gamma Z}}{F_1^{\gamma}}$, and making use of the same CSV distribution functions, we find

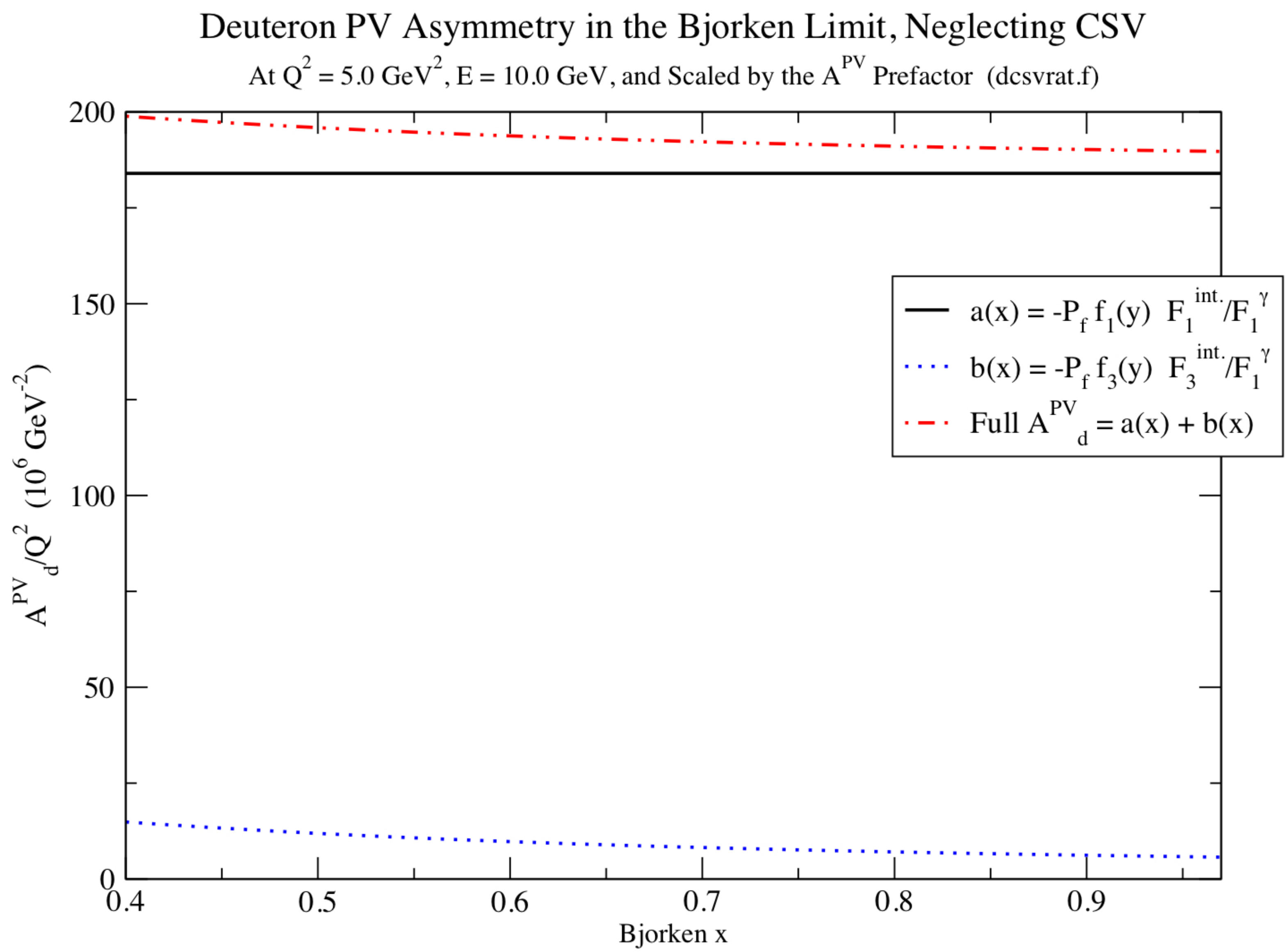
$$F_3^{\gamma Z} = -e_d \cdot [(2C_{2u} - C_{2d})(u+d) + \frac{(2C_{2u} - C_{2d})}{2}(\delta u - \delta d)]$$
$$F_1^{\gamma} = e_d^2 \cdot [5(u+d) + \frac{3}{2}(\delta u - \delta d)]. \quad (1)$$

Then,

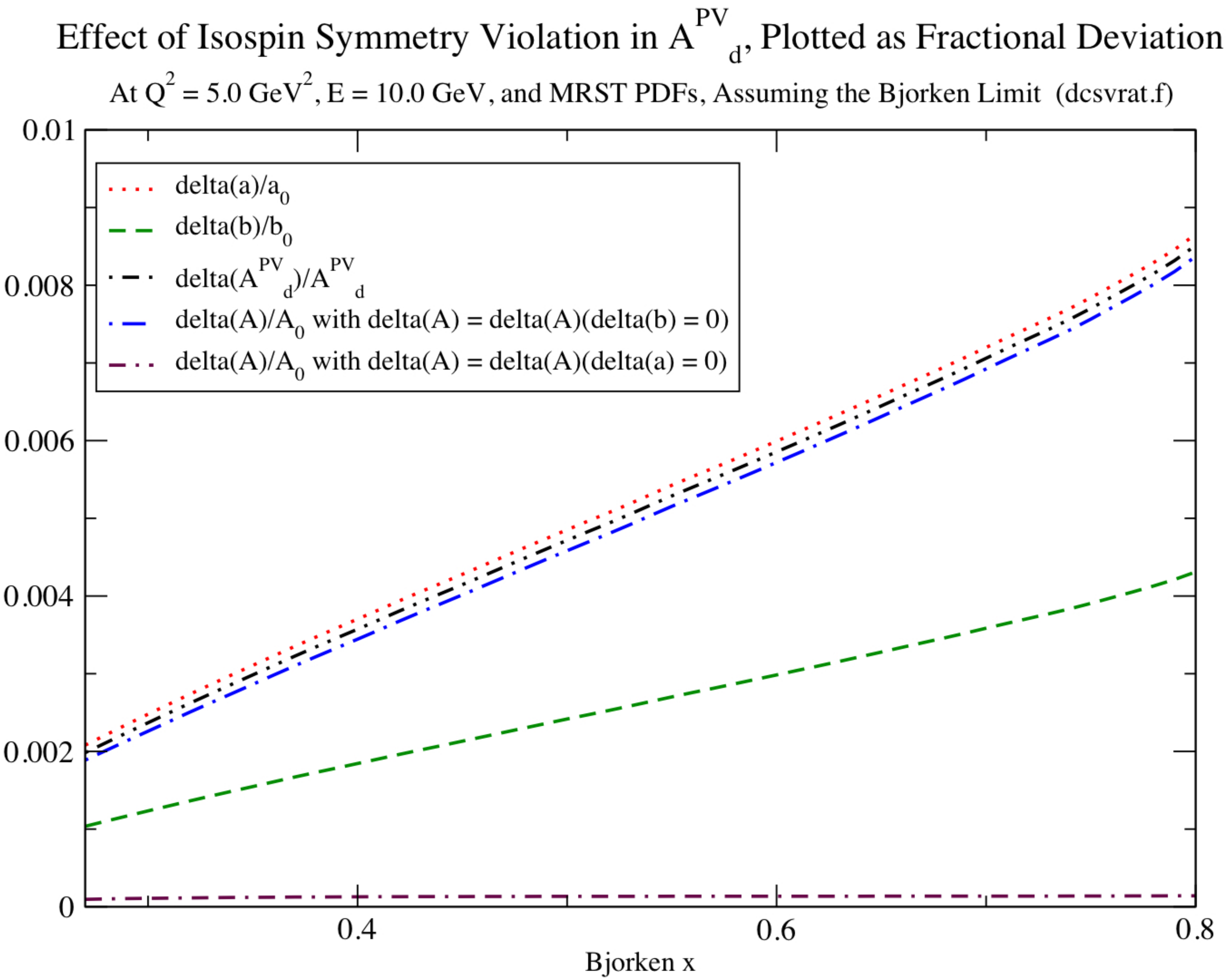
$$\frac{\delta^{CSV} b(x)}{b_0(x)} = [\frac{3}{10} - \frac{2C_{2u} + C_{2d}}{2(2C_{2u} - C_{2d})}] \cdot \frac{\delta u - \delta d}{u+d}, \quad (2)$$

and

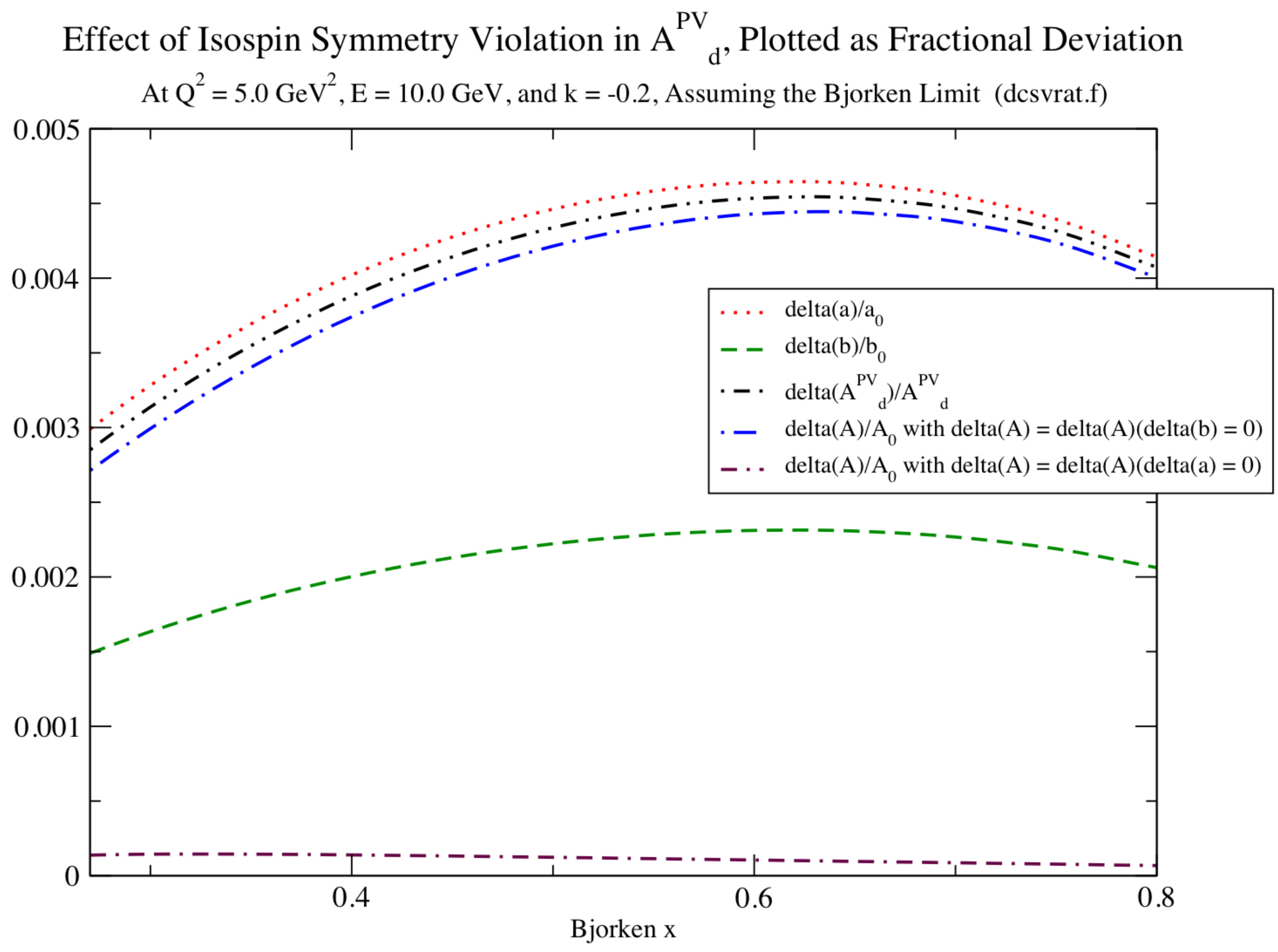
$$b_0(x) = \frac{3}{5}(2C_{2u} - C_{2d}). \quad (3)$$



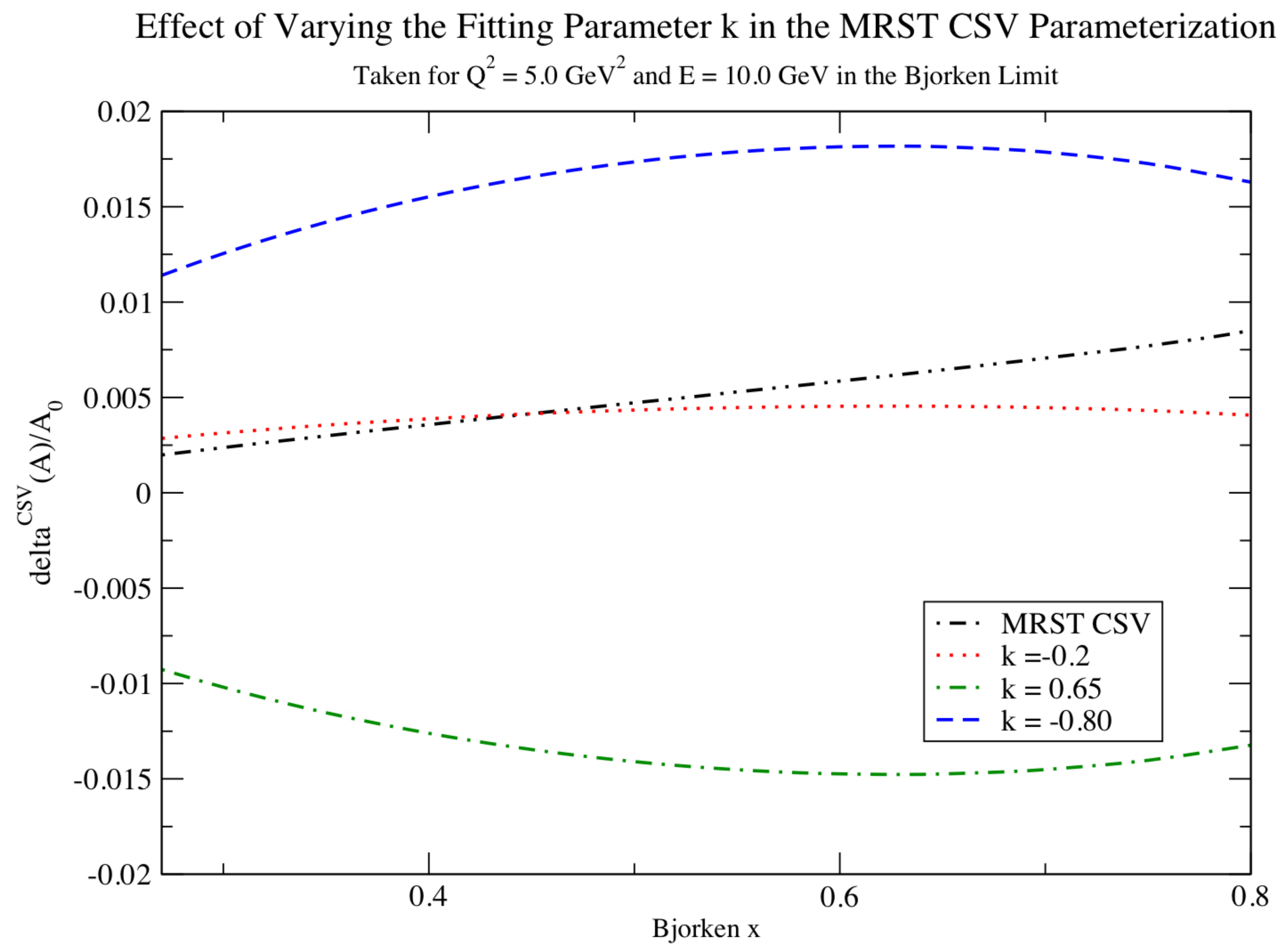
Deuteron PV Asymmetry in the Bjorken Limit, Neglecting CSV
At Q^2 = 5.0 GeV^2, E = 10.0 GeV, and Scaled by the A^{PV} Prefactor (dcsvrat.f)
A^{PV}_d/Q^2 (10^6 GeV^{-2})
200
150
100
50
0
0.4
0.5
0.6
0.7
0.8
0.9
Bjorken x
$a(x) = -P_f\, f_1(y)\ \ F_1^{int.}/F_1^{\gamma}$
$b(x) = -P_f\, f_3(y)\ \ F_3^{int.}/F_1^{\gamma}$
Full $A^{PV}_d = a(x) + b(x)$



Effect of Isospin Symmetry Violation in A^{PV}_d, Plotted as Fractional Deviation
At Q^2 = 5.0 GeV^2, E = 10.0 GeV, and MRST PDFs, Assuming the Bjorken Limit (dcsvrat.f)
delta(a)/a_0
delta(b)/b_0
delta(A^{PV}_d)/A^{PV}_d
delta(A)/A_0 with delta(A) = delta(A)(delta(b) = 0)
delta(A)/A_0 with delta(A) = delta(A)(delta(a) = 0)
0.01
0.008
0.006
0.004
0.002
0
0.4
0.6
0.8
Bjorken x



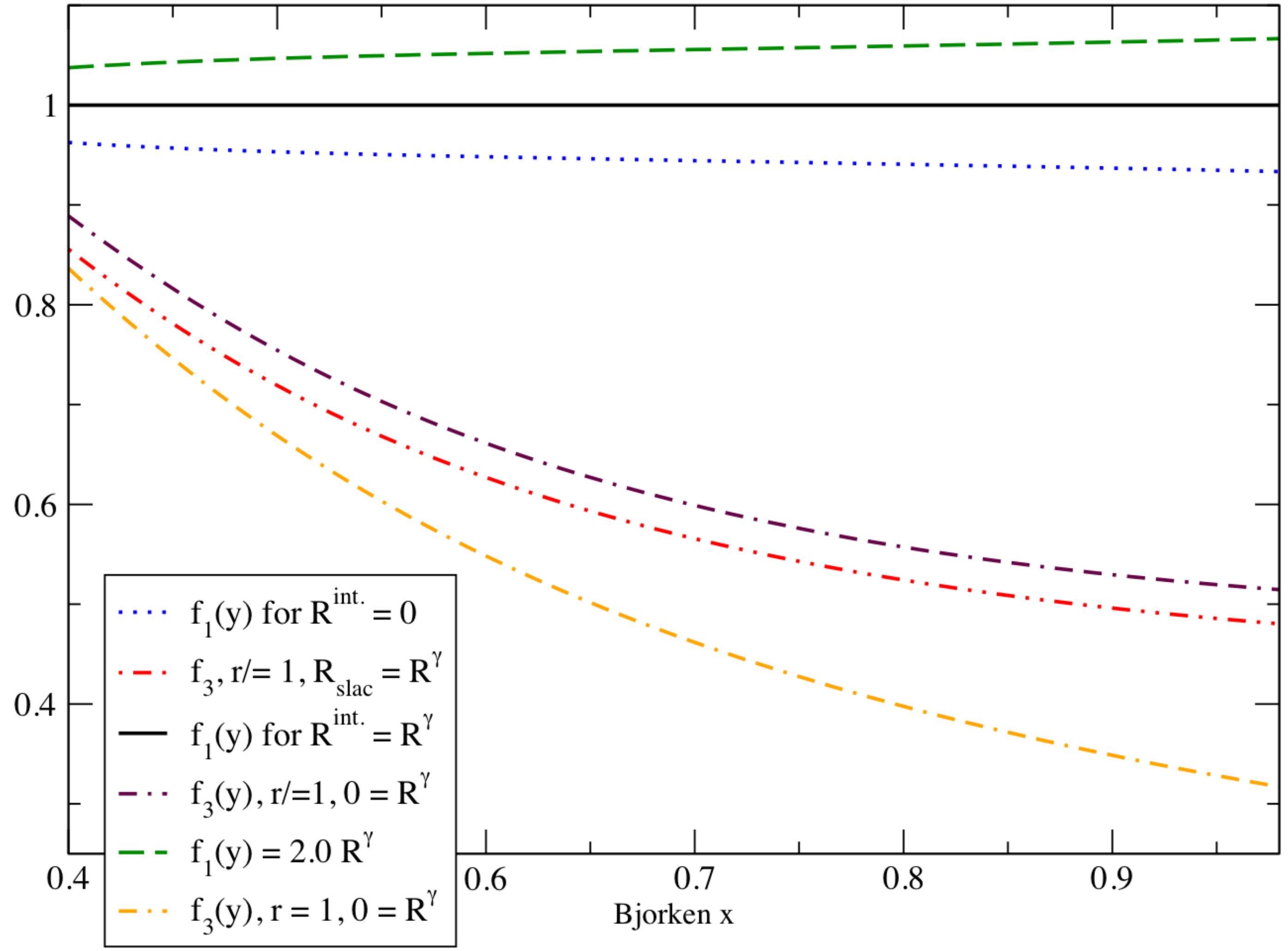
Effect of Isospin Symmetry Violation in A^{PV}_d, Plotted as Fractional Deviation
At Q^2 = 5.0 GeV^2, E = 10.0 GeV, and k = -0.2, Assuming the Bjorken Limit (dcsvrat.f)
0.005
0.004
0.003
0.002
0.001
0
0.4
0.6
0.8
Bjorken x
delta(a)/a_0
delta(b)/b_0
delta(A^{PV}_d)/A^{PV}_d
delta(A)/A_0 with delta(A) = delta(A)(delta(b) = 0)
delta(A)/A_0 with delta(A) = delta(A)(delta(a) = 0)

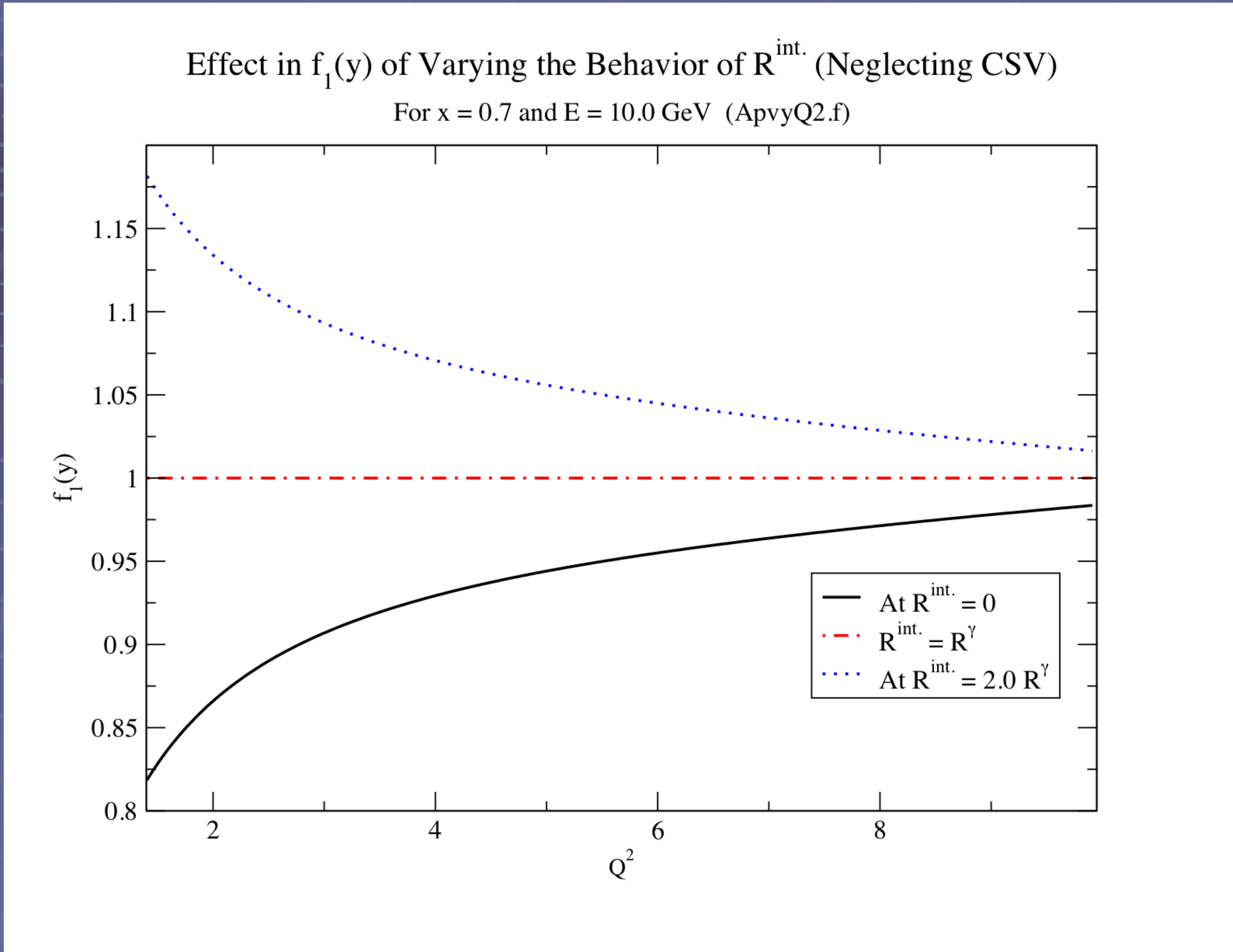


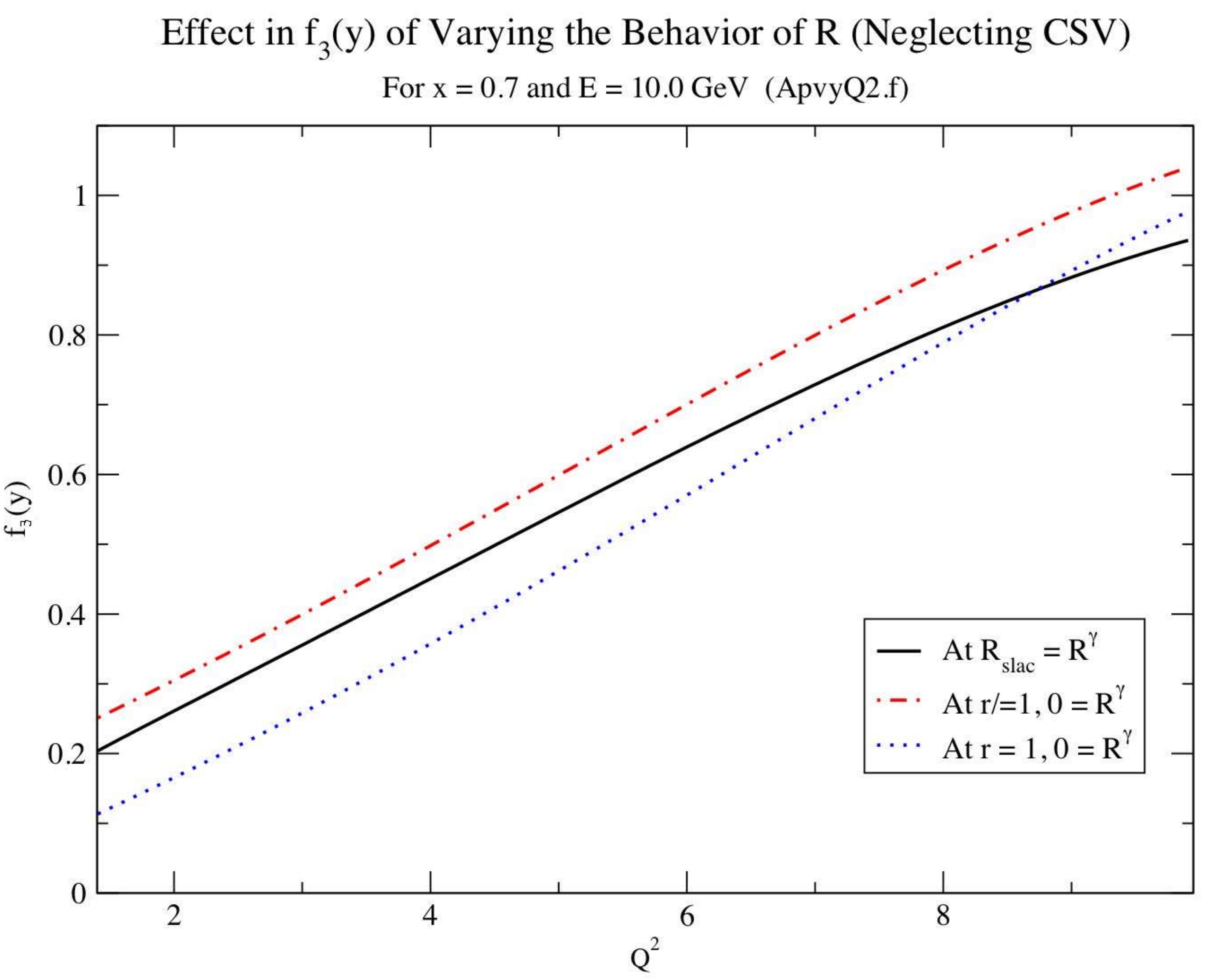
Effect of Varying the Fitting Parameter k in the MRST CSV Parameterization
Taken for Q^2 = 5.0 GeV^2 and E = 10.0 GeV in the Bjorken Limit
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
delta^CSV(A)/A_0
0.4
0.6
0.8
Bjorken x
MRST CSV
k =-0.2
k = 0.65
k = -0.80

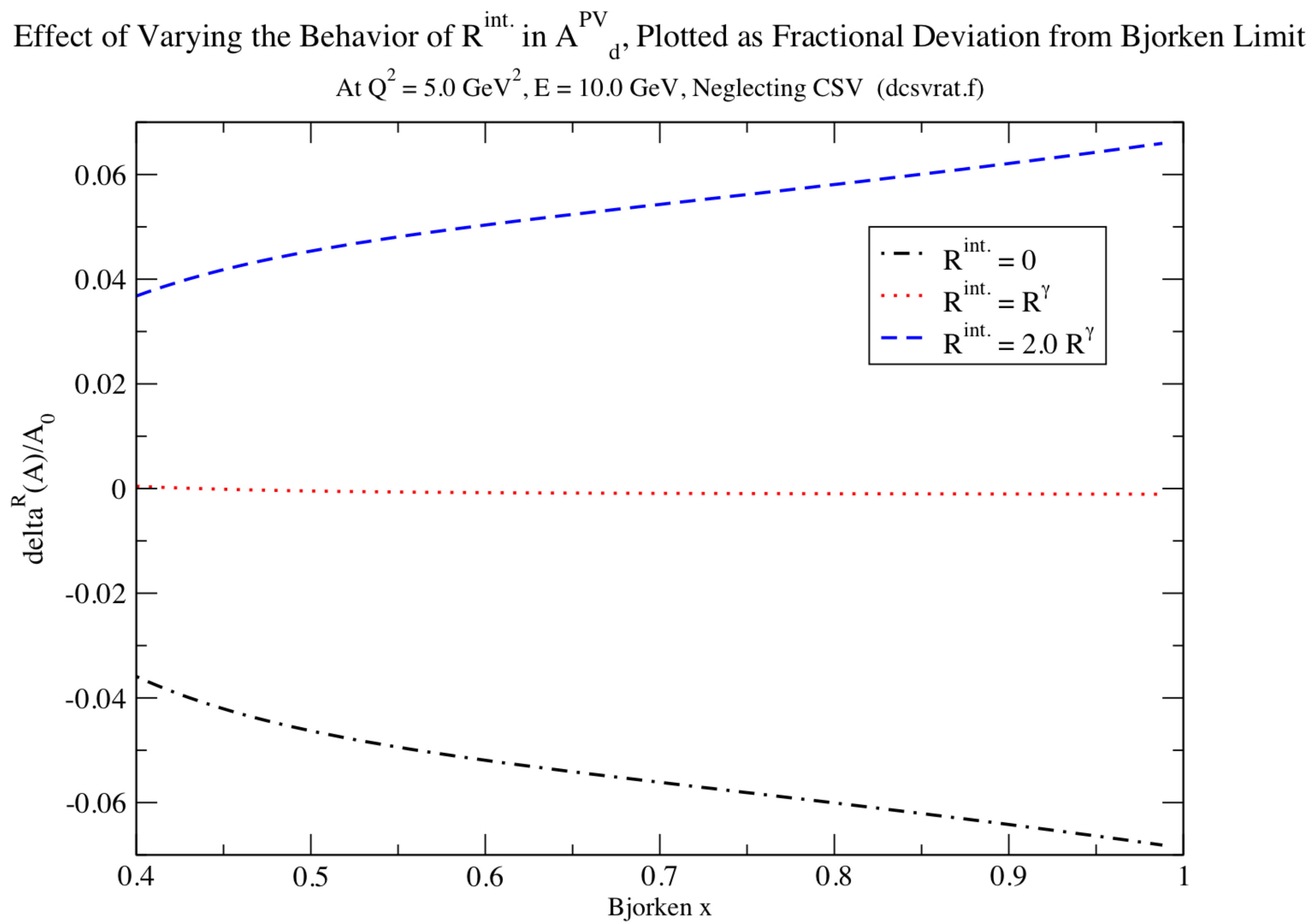
Effects in the Kinematical Parameters $f_i(y)$ of Varying the Behavior of R (Neglecting CSV)

At $Q^2 = 5.0\ \mathrm{GeV}^2$, E = 10.0 GeV (dcsvrat.f)

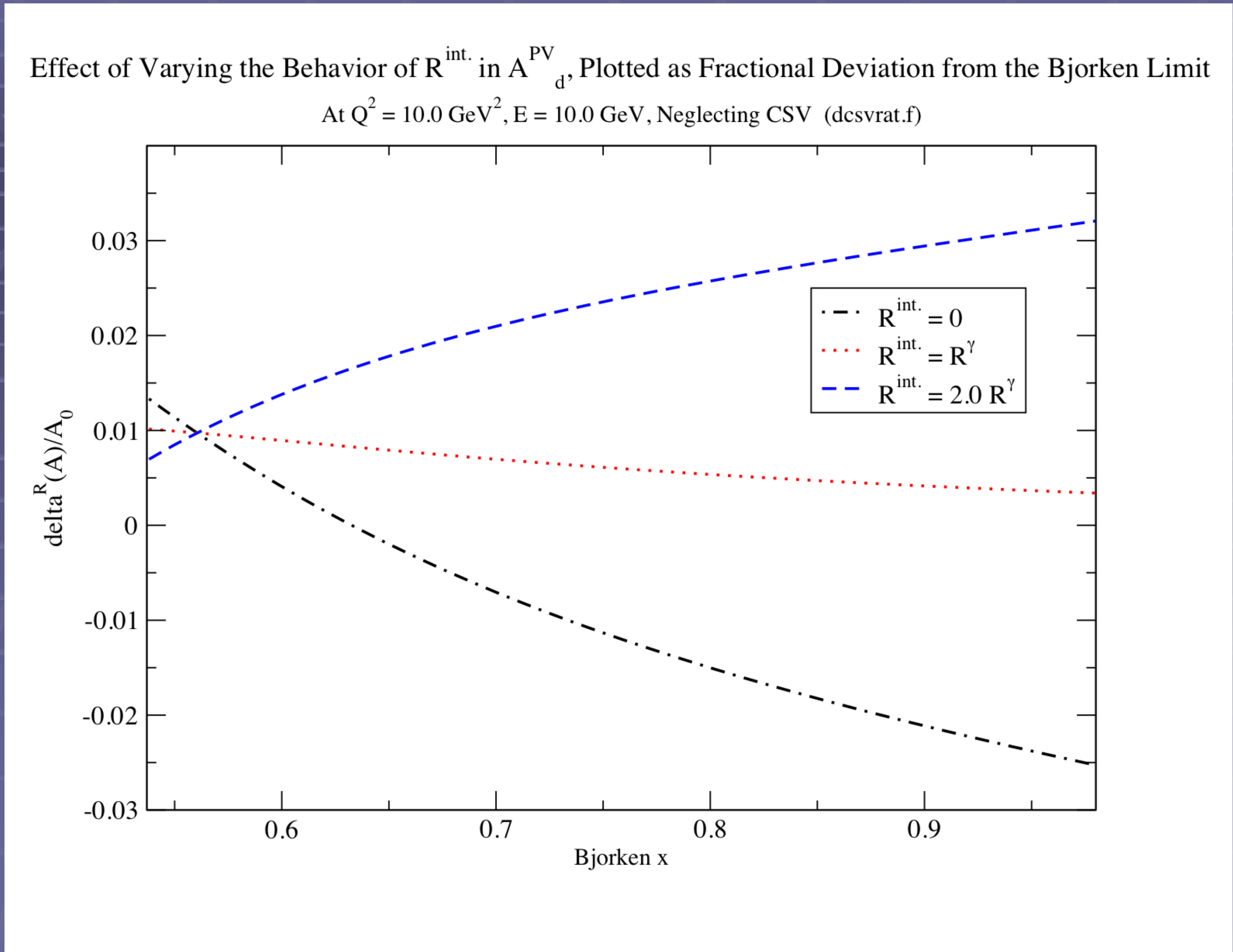



Effect in $f_1(y)$ of Varying the Behavior of $R^{int.}$ (Neglecting CSV)
For x = 0.7 and E = 10.0 GeV (ApvyQ2.f)
$f_1(y)$
Q^2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
2
4
6
8
At $R^{int.} = 0$
$R^{int.} = R^{\gamma}$
At $R^{int.} = 2.0\ R^{\gamma}$


Effect in $f_3(y)$ of Varying the Behavior of R (Neglecting CSV)
For x = 0.7 and E = 10.0 GeV (ApvyQ2.f)
$f_3(y)$
Q^2
0
0.2
0.4
0.6
0.8
1
2
4
6
8
At $R_{slac} = R^{\gamma}$
At r/=1, 0 = R^{γ}
At r = 1, 0 = R^{γ}



Effect of Varying the Behavior of $R^{int.}$ in A^{PV}_d, Plotted as Fractional Deviation from Bjorken Limit
At Q^2 = 5.0 GeV^2, E = 10.0 GeV, Neglecting CSV (dcsvrat.f)
$R^{int.} = 0$
$R^{int.} = R^{\gamma}$
$R^{int.} = 2.0\ R^{\gamma}$
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
$\mathrm{delta}^R(A)/A_0$
0.4
0.5
0.6
0.7
0.8
0.9
1
Bjorken x


Effect of Varying the Behavior of R^int. in A^PV_d, Plotted as Fractional Deviation from the Bjorken Limit
At Q^2 = 10.0 GeV^2, E = 10.0 GeV, Neglecting CSV (dcsvrat.f)
R^int. = 0
R^int. = R^γ
R^int. = 2.0 R^γ
delta^R(A)/A_0
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0.6
0.7
0.8
0.9
Bjorken x

Observations Concerning CSV and A^{PV}_{d}

- CSV effects in A^{PV}_{d} produced through the parameterized fit are qualitatively similar to the MRST QED Model
- $R^{\gamma Z}$ » or « R^{γ} could confound the accessibility of CSV effects in the deuteron

The Future: PV Asymmetries for Spin-Polarized Hadron Targets

In contrast to the unpolarized case, we average the general DIS scattering cross-section over anti-parallel lepton helicities to obtain

$$\frac{d^2\sigma_{cn}^{lN}}{dxdy}(\lambda_{avg}, S_L) = 2x(2 - y - \frac{xyM}{E})g_1^{\gamma Z} - \frac{4x^2M}{E}g_2^{\gamma Z} + \frac{2}{y}(1 - y - \frac{xyM}{2E})g_3^{\gamma Z}$$
$$-\frac{2}{y}(1 + \frac{xM}{E})(1 - y - \frac{xyM}{2E}g_4^{\gamma Z} + 2xy(1 + \frac{xM}{E})g_5^{\gamma Z}; \tag{1}$$

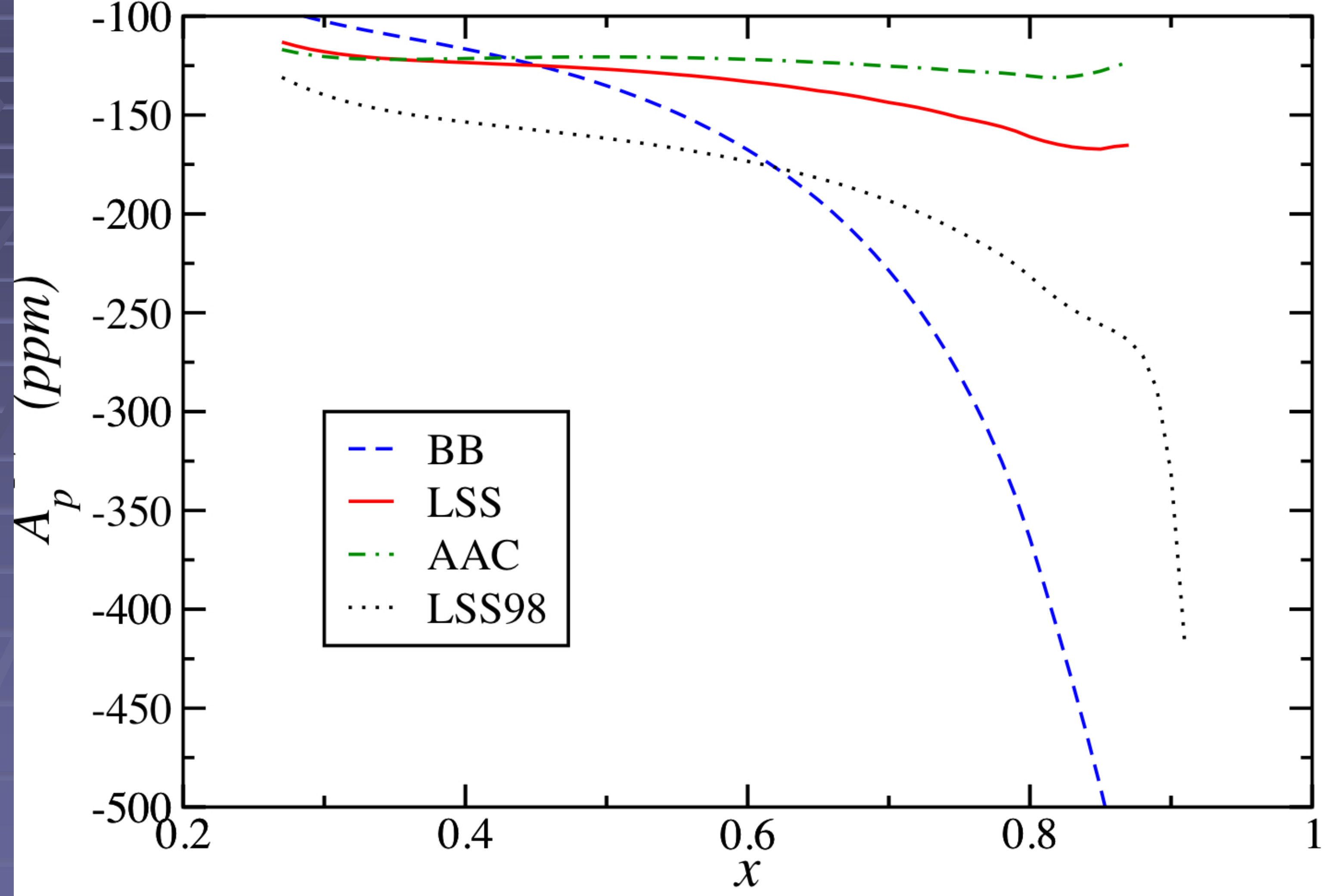
we make several assumptions:

$$\frac{M}{E} \to 0$$
$$g_3^{\gamma Z} - g_4^{\gamma Z} = 2xg_5^{\gamma Z} \tag{2}$$
$$F_2^{\gamma} = 2xF_1^{\gamma}.$$

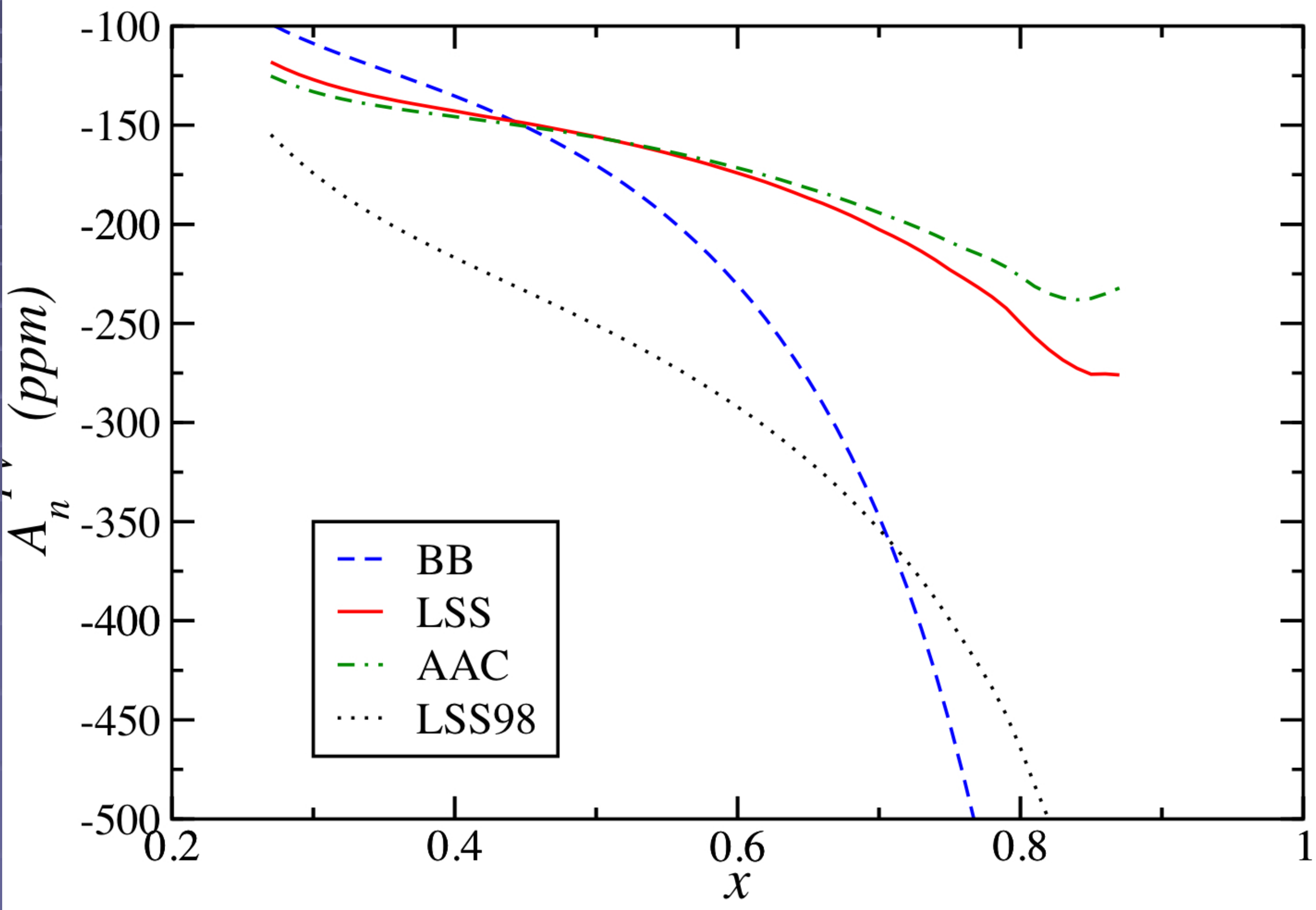
Then, $\Delta A^{PV} = \frac{d\sigma^{PV}(\lambda_{avg}, \Delta S_L)}{d\sigma(\lambda_{avg}, S_{avg})}$ implies

$$\Delta A^{PV} = \frac{G_F Q^2}{\sqrt{2}\pi\alpha} \cdot [g_A^e f_{Bj}(y)\frac{g_1^{\gamma Z}}{F_1^{\gamma}} + g_V^e \frac{g_5^{\gamma Z}}{F_1^{\gamma}}], \tag{3}$$

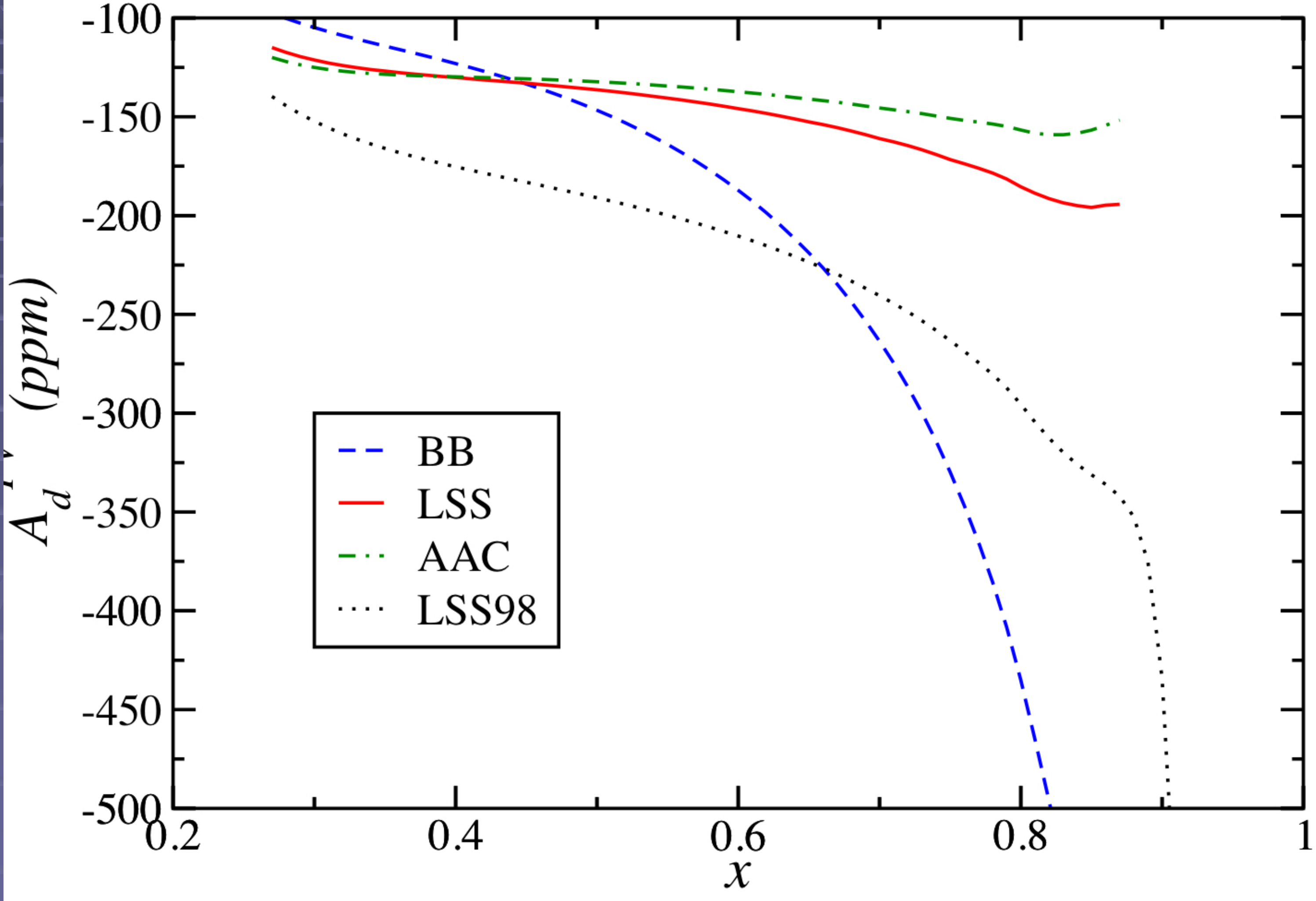
having neglected $g_2^{\gamma Z}$.



-100
-150
-200
-250
-300
-350
-400
-450
-500
0.2
0.4
0.6
0.8
1
x
A_p (ppm)
BB
LSS
AAC
LSS98



-100
-150
-200
-250
-300
-350
-400
-450
-500
0.2
0.4
0.6
0.8
1
x
(ppm)
BB
LSS
AAC
LSS98



-100
-150
-200
-250
-300
-350
-400
-450
-500
0.2
0.4
0.6
0.8
1
x
A_d^{PV} (ppm)
BB
LSS
AAC
LSS98

Conclusions

- Assuming a reasonable control of kinematical parameters and $R^{\gamma Z}$, extraction of quarks PDFs with good uncertainty seems possible in A^{PV}_p
- Similarly, measures of CSV in the deuteron are encouraging
- A^{PV} for spin-polarized nucleons remains a topic of interest, as model-dependent calculations push a 25% uncertainty