Electromagnetic two-body currents of one- and two-pion range $*$

S. Pastore - Cake seminar @ Jlab

OLD DOMINION UNIVERSITY

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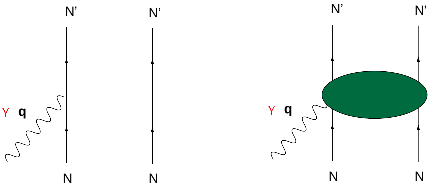
$*$ with R. Schiavilla and J.L. Goity - Phys. Rev. **C** in press; arXiv:0810.1941

- Nuclear two-body electromagnetic currents in non-relativistic χEFT
- Calculation up to one loop
- Nuclear electromagnetic observables: results
- Summary and outlook



- 1-body: describes the current of a free nucleon
- 2-body: includes the effect of the NN interaction on the currents of a nucleon pair

EM current operator related to the transition amplitute via

$$T_{fi} = \langle N'N' \mid T \mid NN;\gamma\rangle = -\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\,\omega_q}}\cdot\mathbf{j}$$

Relevant degrees of freedom:

- non relativistic nucleons (N)
- pions (π); mediators of the NN interaction at large interparticle distances
- non relativistic Delta-isobars (Δ)
  $m_\Delta \sim m_N + 2m_\pi$

Transition amplitude in time-ordered perturbation theory

$$T_{fi} = \langle N'N' \mid H_1 \sum_{n=1}^{\infty} \left(\frac{1}{E_i - H_0 + i\,\eta} H_1 \right)^{n-1} \mid NN;\gamma\rangle$$

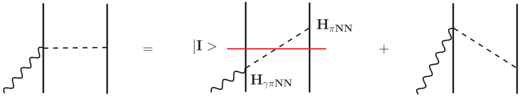
$H_0 =$ free π, N, Δ Hamiltonians
$H_1 =$ interacting π, N, Δ, γ Hamiltonians

In practice, insert complete sets of eigenstates of H_0 between successive terms of H_1

$$T_{f\,i} = \langle N'N' \mid H_1 \mid NN;\gamma\rangle + \sum_{|I\rangle} \langle N'N' \mid H_1 | \, I\rangle \frac{1}{E_i - E_I} \langle I \,| H_1 \mid NN;\gamma\rangle + \ldots$$

The contributions to the $T_{f\,i}$ are rapresented by time ordered diagrams

Example: seagull pion exchange current



N number of H_1's (vertices)

$\rightarrow$ N! time-ordered diagrams

χEFT Interacting Hamiltonians

H_1's are derived from the Chiral Effective Field Theory Lagrangians ($\mathscr{L}_{eff}$)

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- QCD is the underlying theory of strong interaction; on this basis π, N, and Δ interactions are completely determined by the underlying quark-gluon dynamics
- At low energies perturbative techniques (expansion in α_S) cannot be applied to solve QCD and we are far from a quantitative understanding of the low-energy physics by ab initio calculations from QCD
- χEFT exploits the χ symmetry exhibited by QCD at low energy to restrict the form of the interactions of pions among themselves and with other particles

χEFT Interacting Hamiltonians

- The pion couples by powers of its momentum $Q \rightarrow \mathscr{L}_{\mathit{eff}}$ can by systemtically expanded in powers of Q/M
$$\mathscr{L}_{\mathit{eff}} = \mathscr{L}^{(0)} + \mathscr{L}^{(1)} + \mathscr{L}^{(2)} + \ldots$$
$M \sim 1$ GeV is the hard scale where χEFT will break down and characterizes the convergence of the expansion $\rightarrow$ we are limited to kinematic regions with $Q \ll M$
- χEFT allows for a perturbative treatment in terms of Q - as opposed to a coupling constant - expansion
- The coefficients of the expansion, Low Energy Constants (LEC's) are unknown and need to be fixed by comparison with exp data

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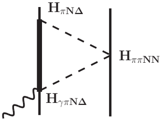
- The coefficients of the expansion, Low Energy Constants (LEC's) are unknown and need to be fixed by comparison with exp data
- Due to the chiral expansion, T_{fi} can be expanded as
$$T_{fi} = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots$$
$$\begin{aligned} T^{NLO} &\sim \frac{Q}{M} T^{LO} \\ T^{N^2LO} &\sim \left(\frac{Q}{M}\right)^2 T^{LO} \end{aligned}$$
- The power counting scheme allows us to arrange the contributions of T_{fi} in powers of a small momentum Q

Power Counting

Each contribution to the T_{fi} scales as

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$

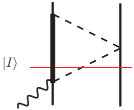
α_i = number of derivatives (momenta) in H_1
β_i = number of π's at each vertex
($Q^{-\beta_i/2}$ takes in account $\frac{1}{\sqrt{\omega_\pi}}$ energy factor in the π field)



$$H_1\ \text{scaling} \sim \underbrace{Q^1 \times Q^{-1/2}}_{H_{\pi N\Delta}} \times \underbrace{Q^1 \times Q^{-1}}_{H_{\pi\pi NN}} \times \underbrace{e\,Q^0 \times Q^{-1/2}}_{H_{\pi\gamma N\Delta}} \sim e\,Q^0$$

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{e\,Q^0} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$

$N =$ number of vertices
$N-1 =$ number of intermediate states
$L =$ number of loops
(Q^{3L} takes in account $\int d^3Q$)



- Energy denominator scales as Q^{-1} in the static limit

$E_N = m_N + \frac{p^2}{2\,m_N} \sim m_N$; $\qquad E_\Delta = m_\Delta + \frac{p^2}{2\,m_\Delta} \sim m_\Delta$
$m_\Delta - m_N \sim Q$
$\omega_\pi \sim Q$

$$\frac{1}{E_i - H_0}|I\rangle \sim \frac{1}{2\,m_N - (m_\Delta + m_N + \omega_\pi)}|I\rangle = -\frac{1}{m_\Delta - m_N + \omega_\pi}|I\rangle \sim \frac{1}{Q}|I\rangle$$

Power Counting

$$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i-\beta_i/2}\right)}_{e\,Q^0} \times \underbrace{Q^{-(N-1)}}_{Q^{-2}} \times \underbrace{Q^{3L}}_{Q^3} = e\,Q^1$$

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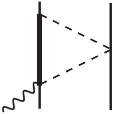

$\sim \quad \mathbf{e\,Q}$

- Energy denominator scales as Q^{-1} in the static limit
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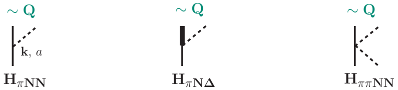
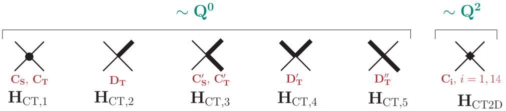
π, N and Δ Vertices



$$H_{\pi NN} = \frac{g_A}{F_\pi} \int \mathrm{d}\mathbf{x}\, N^\dagger(\mathbf{x})\, [\boldsymbol{\sigma} \cdot \nabla \pi_a(\mathbf{x})]\, \tau_a N(\mathbf{x})$$

$$\rightarrow V_{\pi NN} = -i \frac{g_A}{F_\pi} \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\sqrt{\omega_k}} \tau_a \sim Q^1 \times Q^{-1/2}$$

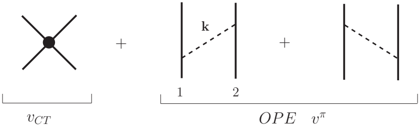
- $H_{\pi NN}$: $\left(\frac{m_\pi g_A}{F_\pi}\right)^2 \frac{1}{4\pi} = 0.075$ from Nijmegen analysis on NN scattering data
- $H_{\pi N\Delta}$: $h_A \sim 2.77$ fixed by reproducing the width of the Δ resonance



- $H_{\mathrm{CT},1}$: 4-nucleons contact terms, 2 LEC's
- $H_{\mathrm{CT},2-5}$: contact terms involving one or two Δ's, 5 LEC's
- H_{CT2D} : 4-nucleons contact terms with two derivatives acting on N, 14 LEC's

χEFT NN potential at LO

$v_{NN}^{LO} =$



$\sim Q^0$

$$T_{fi}^{LO} = \langle N'N' \mid H_{\mathrm{CT},1} \mid NN \rangle + \sum_{|I\rangle} \langle N'N' \mid H_{\pi NN} \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_{\pi NN} \mid NN \rangle$$

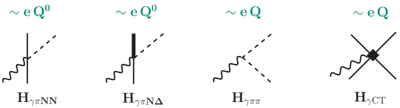
Leading order NN potential in χEFT

$$v_{NN}^{L0} = C_S + C_T\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{F_\pi^2} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

EM Vertices

- EM H_1 obtained by minimal substitution in the π- and N-derivative couplings

$$\begin{aligned}\nabla\pi_{\mp}(\mathbf{x}) &\rightarrow \left[\nabla \mp ie\mathbf{A}(\mathbf{x})\right]\pi_{\mp}(\mathbf{x}) \\ \nabla N(\mathbf{x}) &\rightarrow \left[\nabla - iee_N\mathbf{A}(\mathbf{x})\right]N(\mathbf{x})\,, \qquad e_N = (1+\tau_z)/2\end{aligned}$$



- EM H_1 of individual N's and Δ's obtained by non-relativistic reduction of the effective Hamiltonians

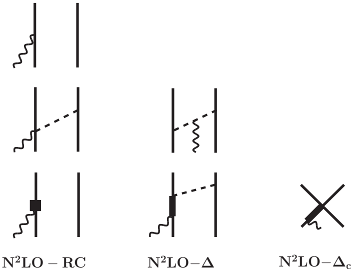
  - $H_{\gamma NN}$: $\mu_p = 2.793$ n.m. and $\mu_n = -1.913$ n.m. anomalous magnetic moments
  - $H_{\gamma N\Delta}$: $\mu^* \simeq 3$ n.m. from $\gamma N\Delta$ data

- Up to N^{2}LO

LO : eQ^{-2}

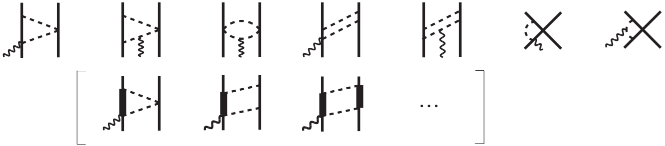
NLO : eQ^{-1}

N^{2}LO : eQ^{0}

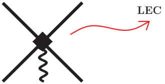


- One-loop corrections to the one-body current (absorbed into μ_N, $\langle r_N^2 \rangle$)

- One-loop corrections at N^{3}LO (eQ)

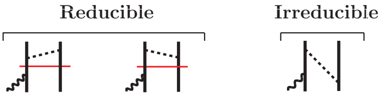


- Currents from $(NN)(NN)$ contact interactions with two gradients involving a number of LEC's



- One-loop renormalization of to the tree-level currents

- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$ 

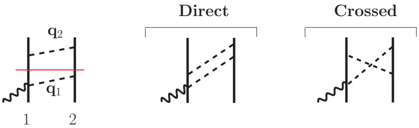
- Recoil corrections to the reducible contribution obtained by expanding in powers of E_N/ω_π the propagators

$$\simeq \; v^\pi \frac{1}{E_i - E_I} \mathbf{j}^{LO} + \frac{v^\pi}{2\,\omega_\pi} \mathbf{j}^{LO}$$

$$\equiv \; - \frac{v^\pi}{2\,\omega_\pi} \mathbf{j}^{LO}$$

- Recoil corrections to the reducible diagrams cancel irreducible contribution

$$\mathbf{j}^{\mathrm{N^3LO}} =$$



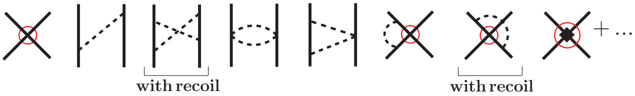
- Reducible contributions

$$\mathbf{j}_{\rm red} \sim \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\rm NLO}(\mathbf{q}_1)$$
$$- \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Irreducible contributions

$$\mathbf{j}_{\rm irr} = \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$
$$+ \int 2\,\frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}\,[V_{\pi NN}(2,\mathbf{q}_1), V_{\pi NN}(2,\mathbf{q}_2)]_{-}V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Observed partial cancellations at N^{3}LO between recoil corrections to the reducible diagrams and irreducible contributions

$v_{\mathrm{NN}}^{\text{up to N}^2\text{LO}} =$ 

- We constructed the NN potential υ_{ij} generated by the χEFT Hamiltonians and verified whether the continuity equation is satisfied

$$\mathbf{q}\cdot\mathbf{j} = \left[\frac{p_1^2}{2\,m_N} + \frac{p_2^2}{2\,m_N} + \upsilon_{12}\,,\,\rho\right]_-$$

- We found that the inclusion of recoil corrections (at N^{2}LO and N^{3}LO) in the evaluation of the NN potential and the currents ensures that the continuity equation is satisfied
- Expressions of NN up to N^{3}LO in agreement with those obtained with the method of unitary transformations

  -E. Epelbaum *et al.* Nucl.Phys. **A637**, 107 (1998)

- The calculation of EM observable is carried out in r-space
  $\longrightarrow$ we need configuration-space representation of the current operators
- At NLO and N^{2}LO the operator present $1/r^2$ and $1/r^3$ singularities
  $\longrightarrow$ regularize them by introducing a momentum cutoff

$$C_\Lambda(p) = \mathrm{e}^{-(p/\Lambda)^2}, \qquad \Lambda \leq M$$

- Λ=(500–800) MeV

- Hybrid approach

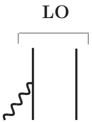
  The current operator is used in transition matrix elements between w.f.'s obtained from realistic Hamiltonians with two- and three-body potentials

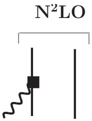
  - ▷ $A = 2$ w.f.'s from AV18 or CDB potentials
    - long-range NN interaction via OPE
    - fitted to reproduce NN scattering data
    - reproduce d properties
  - ▷ $A = 3$ w.f.'s (HH) from AV18-UIX or CDB-UIX* potentials
    - reproduce ^{3}H binging energy and a variety of N-d scattering data

- Isoscalar observables
  - μ_d deuteron magnetic moment
  - μ_S isoscalar combination of the trinucleon magnetic moments

$$\mu_S = \frac{1}{2}\left[\mu(^3\mathrm{He}) + \mu(^3\mathrm{H})\right]$$

Isoscalar currents :





- d magnetic moment (μ_d) and isoscalar combination (μ_S) of ^{3}H/^{3}He magnetic moments

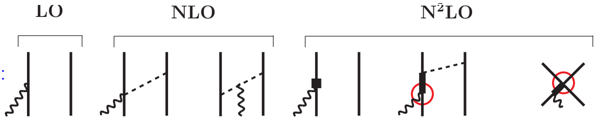
	μ_d (n.m.)		μ_S (n.m.)	
	AV18	CDB	AV18/UIX	CDB/UIX*
LO	+ 0.8469	+ 0.8521	+ 0.4104	+ 0.4183
N^{2}LO-RC	– 0.0082	– 0.0080	– 0.0045	– 0.0052
EXP	+0.8574		+0.426	

- N^{2}LO contribution (cutoff Λ independent) is 1% of LO, but of opposite sign

- Isovector observables
  - $n + p \longrightarrow d + \gamma$ cross-section at thermal neutron energies, $\upsilon_n \sim 2200$ m/s
  - μ_V isovector combination of the trinucleon magnetic moments

$$\mu_V = \frac{1}{2}\left[\mu(^3\mathrm{He}) - \mu(^3\mathrm{H})\right]$$

Isovector currents :



$\mathbf{C_\Delta = f\mu^* \frac{h_A g_A}{F_\pi^2}}$ $\mathbf{C_{\Delta_C} = f'\mu^* D_T}$

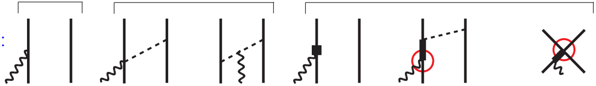
- ^{1}H(n,γ)^{2}H x-section
  - N^{2}LO-Δ_c gives no contribution to the x-section, ($\mu_{\Delta_c}^{\mathrm{N^2LO}}|\Psi_{NN}; l = \mathrm{even}\rangle = 0$)
  - C_Δ : h_A from Δ width, μ^* from $N\gamma$ data

	m.e. (mb$^{1/2}$) AV18
Λ (MeV)	600
LO	17.45
NLO	+ 0.42
N^{2}LO-RC	– 0.05
N^{2}LO-Δ	+ 0.16
Sum	17.99
EXP	18.24

- x-section is underpredicted by ~ 2.5 %: fix $C_\Delta(\Lambda)$ by reproducing the exp values of ^{1}H(n,γ)^{2}H x-section

Electromagnetic observables at N^2LO

LO NLO N^2LO

Isovector currents : 

$C_\Delta = f\mu^* \frac{h_A g_A}{F_\pi^2}$ $C_{\Delta_C} = f'\mu^* D_T$

- μ_V in $^3H/^3He$
  - with C_Δ (Λ) fixed to reproduce $\sigma(np \rightarrow d\gamma)$

	μ_V(n.m.)
	AV18
Λ (MeV)	600
LO	– 2.159
NLO	– 0.197
N^2LO-RC	+ 0.029
N^2LO-Δ	– 0.253
Sum	– 2.580
EXP	– 2.533

- N^2LO correction larger than NLO
- μ_V underpredicted by $\sim$ 2 %
- N^2LO current completely determined with C_{Δ_c} (Λ) fixed by reproducing μ_V

- $n + d \rightarrow {}^3\mathrm{H} + \gamma$ cross-section (σ_T) at thermal neutron energies
- $\vec{n} + d \rightarrow {}^3\mathrm{H} + \gamma$ photon circular polarization factor R_c

  $\mathbf{P}_N$ = neutron polarization
  P_γ = photon circular polarization

$$P_\Gamma = \frac{\sigma(\mathbf{P}_N, P_\gamma = 1) - \sigma(\mathbf{P}_N, P_\gamma = -1)}{2\,\sigma_T} = R_c\,\mathbf{P}_N \cdot \hat{\mathbf{q}}$$

Predictions for n-d capture at thermal energy

^{2}H(n,γ)^{3}H and ^{2}H($\vec{n}$,γ)^{3}H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

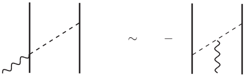
- LO $<$ 50% exp: 1-body currents suppressed due to pseudo-orthogonality between initial final and states (well known)

$$\mu_z^{LO} \mid \Psi(^3\mathrm{H})\rangle \simeq \mu_p \mid \Psi(^3\mathrm{H})\rangle \rightarrow \langle \Psi(^3\mathrm{H}) \mid \mu_z^{LO} \mid \Psi(n,d)\rangle \simeq \mu_p \langle \Psi(^3\mathrm{H}) \mid \Psi(n,d)\rangle$$

^{2}H(n,γ)^{3}H and ^{2}H($\vec{n},\gamma$)^{3}H radiative capture at thermal neutron energies (AV18/UIX)

	σ_T (mb)			R_c		
Λ (MeV)	500	600	800	500	600	800
LO	0.229	0.229	0.229	– 0.060	– 0.060	– 0.060
LO+NLO (seagull only)	0.425			– 0.425		
LO+NLO (full)	0.272	0.260	0.243	– 0.218	– 0.182	– 0.123
LO+NLO+N^{2}LO	0.450	0.382	0.315	– 0.437	– 0.398	– 0.331
EXP	0.508 ± 0.015			-0.42 ± 0.03		

- NLO seagull- and
  in-flight-contributions nearly cancel out



Predictions for n-d capture at thermal energy

^{2}H$(n,\gamma)^3$H and ^{2}H$(\vec{n},\gamma)^3$H radiative capture at thermal neutron energies (AV18/UIX)

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- N^{2}LO theory $\sim$ 25% smaller than exp: strong Λ-dependence
  - ▷ mainly due to short-range behavior of N^{2}LO-Δ_c contact current governed by a Gaussian of half-width $2/\Lambda$

^{2}H$(n,\gamma)^3$H and ^{2}H$(\vec{n},\gamma)^3$H radiative capture at thermal neutron energies (AV18/UIX)

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EXP	0.508 ± 0.015			-0.42 ± 0.03		

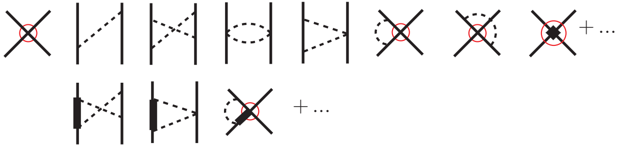
- N^{2}LO theory $\sim$ 25% smaller than exp: strong Λ-dependence
  - ▷ mainly due to short-range behavior of N^{2}LO-Δ_c contact current governed by a Gaussian of half-width $2/\Lambda$
- N^{2}LO contribution much larger than NLO
  - ▷ NLO cancellations
  - ▷ N^{2}LO makes up for missing loop corrections at N^{3}LO

Summary

- Currents up to N^{3}LO have been derived in χEFT
- Currents up to N^{2}LO have been completely determined by reproducing exp values of the ^{1}H$(n,\gamma)^2$H x-section and μ_V
- At N^{2}LO the ^{2}H$(n,\gamma)^3$H x-section and R_c are unpredicted by theory
- A strong cutoff dependence has been observed

- Incorporate the N^{3}LO operators into the calculations of the captures and the magnetic moments of light nuclei ($A < 8$)
- Fix the LEC's by fitting the NN potential

$v_{\rm NN} =$



- N^{3}LO 3-body currents also need to be derived