Transverse Charge Densities

Gerald A. Miller, U. of Washington

Theme- much data exist, interpret form factor as determining transverse charge and magnetization densities, nucleon transverse densities known now to high precision, pion known fairly well

Outline-

1. How not to and how to analyze electromagnetic form factors- transverse density

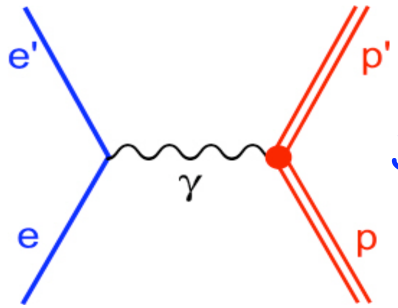
2. Model independent proton, neutron transverse charge density proton transverse magnetization density

3. Shape of proton.

4. Pion time-like data and transverse charge density

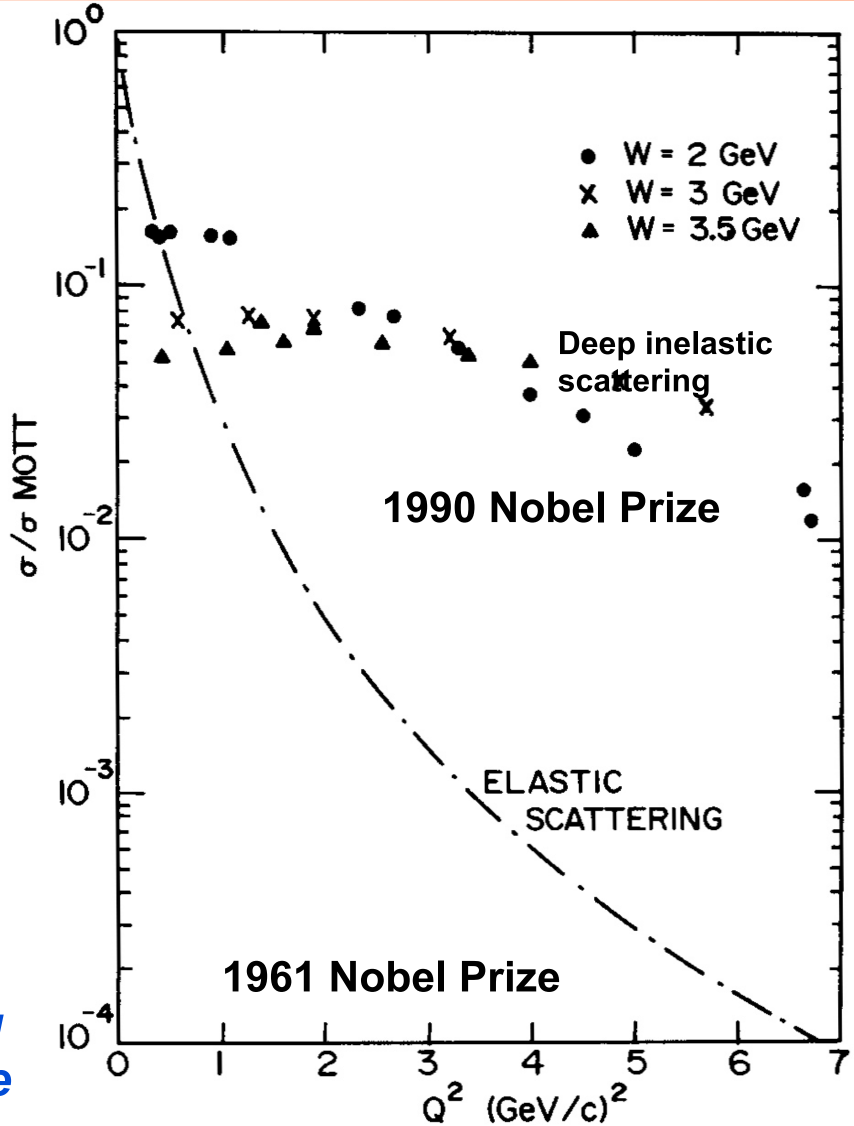
Transverse Charge Densities.
Gerald A. Miller, arXiv:1002.0355 [nucl-th] ARNPS

Electron-nucleon scattering



$j_\mu = \langle e'|\gamma_\mu|e\rangle$

$J_\mu = \langle p'|\Gamma_\mu|p\rangle$



Nucleon vertex:

$$\Gamma_\mu(p,p') = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M} F_2(Q^2)$$

Dirac Pauli

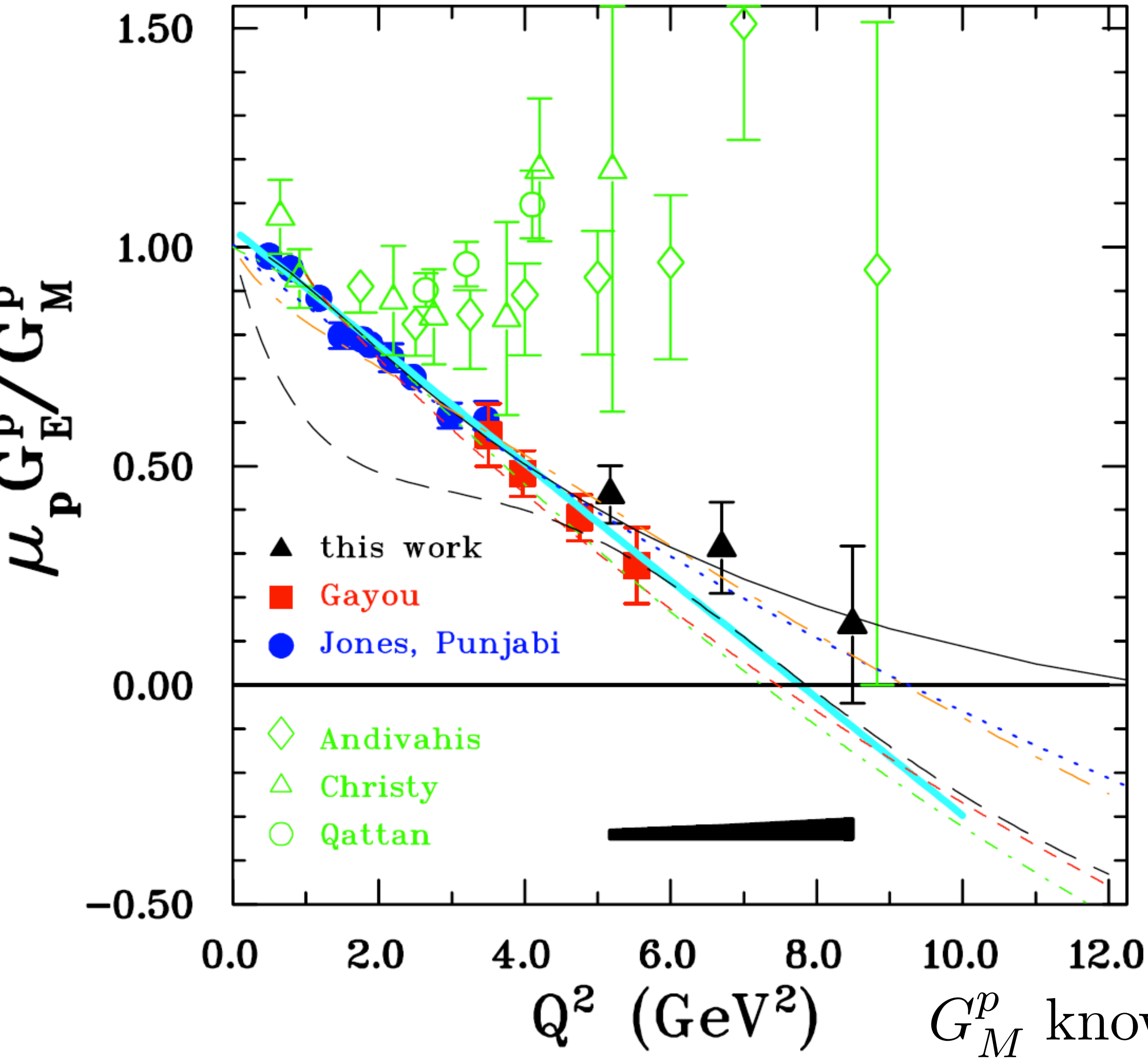
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(G_E^2 + \frac{\tau}{\epsilon} G_M^2\right) / (1+\tau)$$

Cross section for scattering from a point-like object

Form factors describing nucleon shape/structure

$$G_E(Q^2) \equiv F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \; G_M(Q^2) \equiv F_1(Q^2) + F_2(Q^2)$$

Proton



A.J.R. Puckett et al., Phys.Rev.Lett. 104:242301,2010.

G_M^p known $Q^2 \leq 31$ GeV^2

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

$$R^2 = -6\frac{dG_E(Q^2)}{dQ^2}|_{Q^2=0}$$

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

interpretation of Sachs FF is wrong

Final wave function is boosted from initial

Need relativistic treatment

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of charge density

$$R^2 = -6 \frac{dG_E(Q^2)}{dQ^2}|_{Q^2=0}$$

WRONG

Correct non-relativistic:

wave function invariant under Galilean transformation

Relativistic : wave function is frame dependent, initial and final states differ

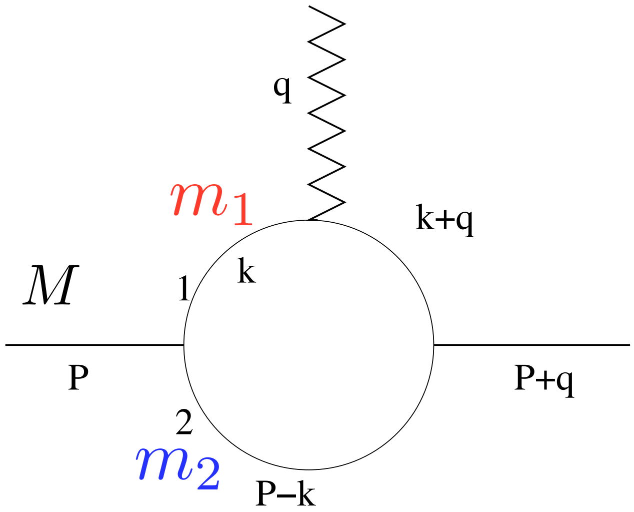
interpretation of Sachs FF is wrong

Final wave function is boosted from initial

Need relativistic treatment

Toy model GAM, **Phys.Rev.C80:045210,2009**.

- Scalar meson, mass M made of two scalars one neutral, $\mathbf{M = m_1 + m_2 - B,\ B > 0}$
- Exact covariant calculation of form factor

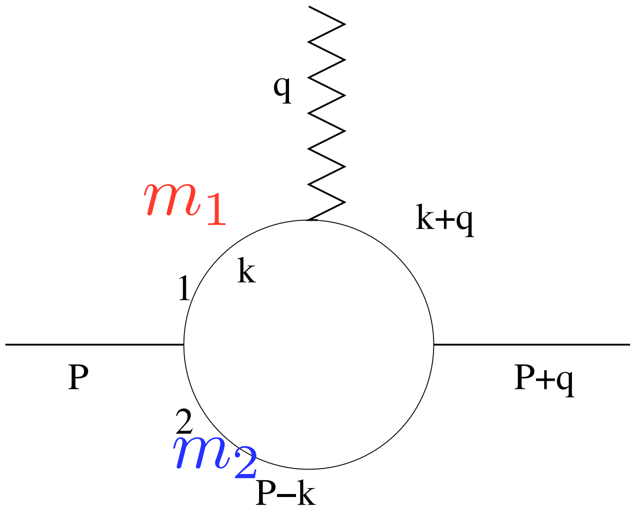


Formula exists
can be studied, vary masses
look for |wave functions|^2

$$F(Q^2)(2P^\mu + q^\mu) = 4\frac{g^2}{16\pi^2}(q^\mu + 2P^\mu)\int_0^1 dx \frac{x\mathrm{Tanh}^{-1}\left[\frac{\sqrt{Q^2}(1-x)}{\sqrt{4x\ m_2^2+4(1-x)m_1^2-x(1-x)M^2+(1-x)^2Q^2}}\right]}{\sqrt{Q^2}\sqrt{4x\ m_2^2+4(1-x)m_1^2-x(1-x)M^2+(1-x)^2Q^2}}$$

Toy model

- Exact covariant calculation of form factor
- Infinite momentum frame, same result
- Integrate over minus-component, same result
- $\mathbf{M = m_1 + m_2 - B,\ B > 0}$ If B is small, physics is non-relativistic



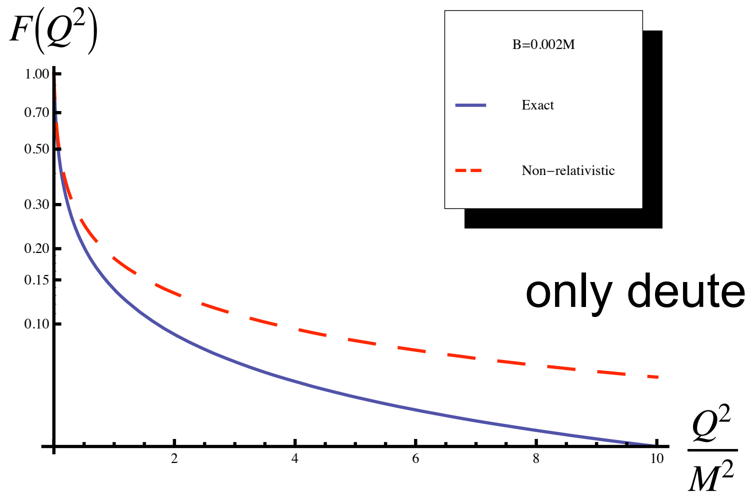
When non-relativistic approximation works, Form factor IS 3DFT of charge density.

How small must B be for non-relativistic approximation to work?

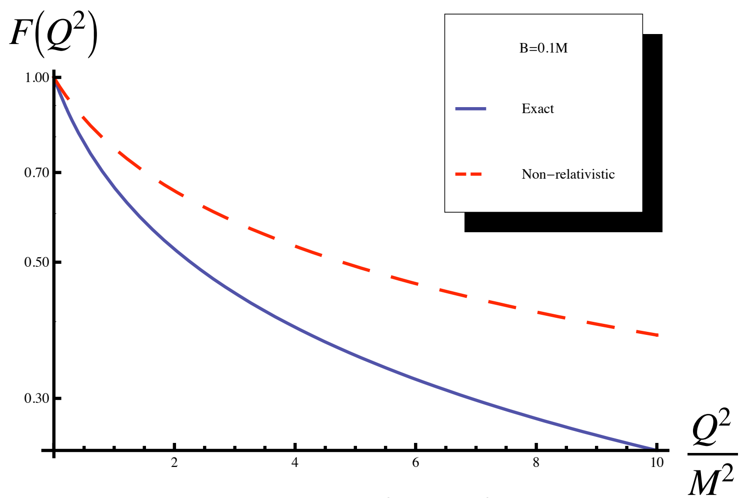
Validity of non-relativistic approximation:

M=2m-B, B=0.002 M, $Q^2 \leq 0.2M^2$

very limited



only deuteron kinematics are non-rel



Exact *vs* non-relativistic Form factors for the case $m_1 = m_2 = m$.

Validity of non-relativistic approximation:

M=2m-B, B=0.002 M, $Q^2 \leq 0.2M^2$

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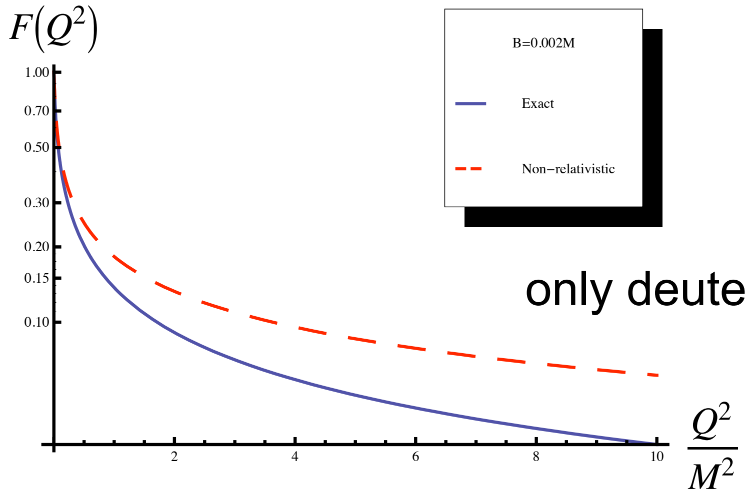
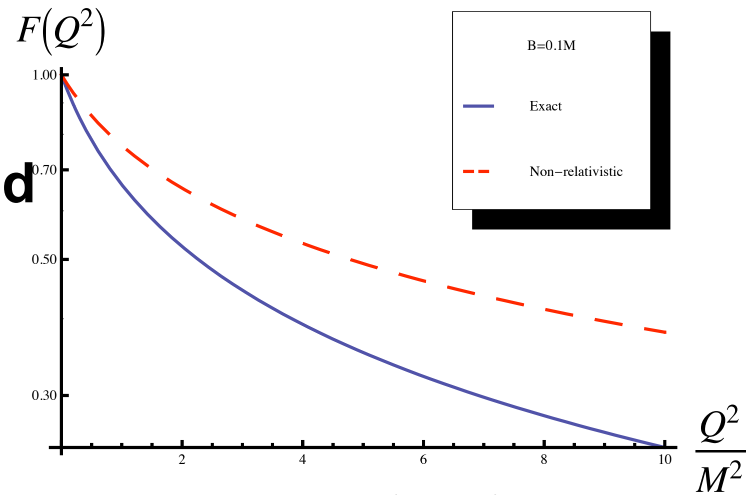


only deuteron kinematics are non-rel

Relativity needed



Exact *vs* non-relativistic Form factors for the case $m_1 = m_2 = m$.

Relation of this to current puzzle on proton radius

- The interpretation of G_E as density is not correct
- This has **no** impact on the puzzle
- Atomic physics depends on $G_{E,M}$

Light front, Infinite momentum frame

$$\text{“}Time\text{”},\ x^+ = x^0 + x^3, \quad \text{“}Evolve\text{”},\ p^- = p^0 - p^3$$

$$\text{“}Space\text{”},\ x^- = x^0 - x^3, \quad \text{“}Momentum\text{”},\ p^+ (\text{Bjorken})$$

$$Transverse\ position,\ momentum \quad \mathbf{b}, \mathbf{p}$$

These variables are used in GPDs, TMDs, standard variables

transverse boosts in kinematic subgroup

$$\mathbf{k} \to \mathbf{k} - k^+ \mathbf{v}$$

$$\text{space} - \text{like}\ q^\mu, q^+ = 0,$$

$$\text{momentum transfer in transverse direction}$$

then density is 2 Dimensional Fourier Transform

Model independent transverse charge density

$$J^+(x^-,\mathbf{b}) = \sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)$$ Charge Density

$$\rho_\infty(x^-,\mathbf{b}) = \langle p^+,\mathbf{R}=\mathbf{0},\lambda|\sum_q e_q q_+^\dagger(x^-,b)q_+(x^-,b)|p^+,\mathbf{R}=\mathbf{0},\lambda\rangle$$

$$F_1 = \langle p^+,\mathbf{p}',\lambda|J^+(0)|p^+,\mathbf{p},\lambda\rangle$$

$$\boxed{\rho(b) \equiv \int dx^- \rho_\infty(x^-,\mathbf{b}) = \int \frac{QdQ}{2\pi}F_1(Q^2)J_0(Qb)}$$

Soper '77

$$\text{Density is } u-\bar{u},\ d-\bar{d}$$

Impact parameter dependent GPD Burkardt

Probability that quark at b from CTM has long momentum fraction x

0 skewness, $\xi = 0$

$$\rho^q(b,x) = \int \frac{d^2q}{(2\pi)^2} e^{-i\mathbf{q}\cdot\mathbf{b}} H_q(x, t = \mathbf{q}^2)$$

Ji sum rule

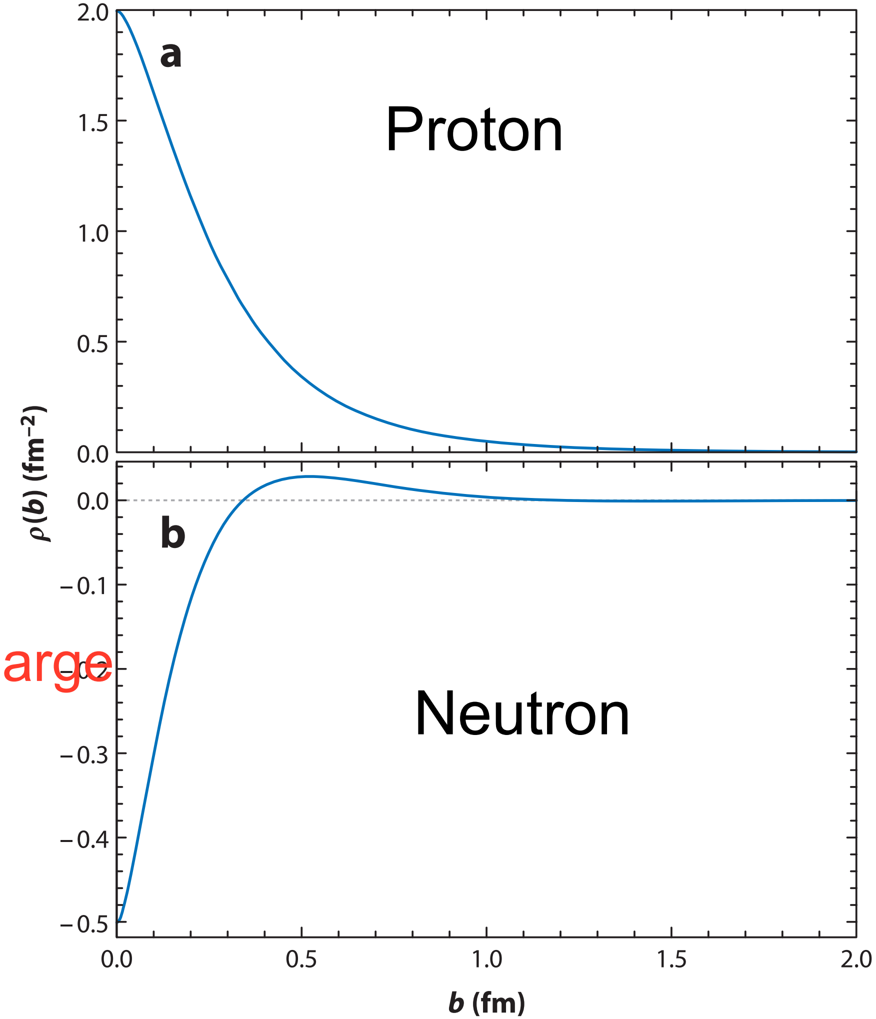
$$\rho(b) = \sum_q e_q \int dx \rho^q(b,x)$$

$$\mathbf{R} = \mathbf{0} = \sum_i^N x_i \mathbf{b}_i$$

Quark of x=1, must have b=0

Transverse density is integral over longitudinal position or momenta
example of Parseval's theorem

Transverse charge densities from parameterizations (Alberico)



Negative central charge density

Negative central density - GAM PRL '07

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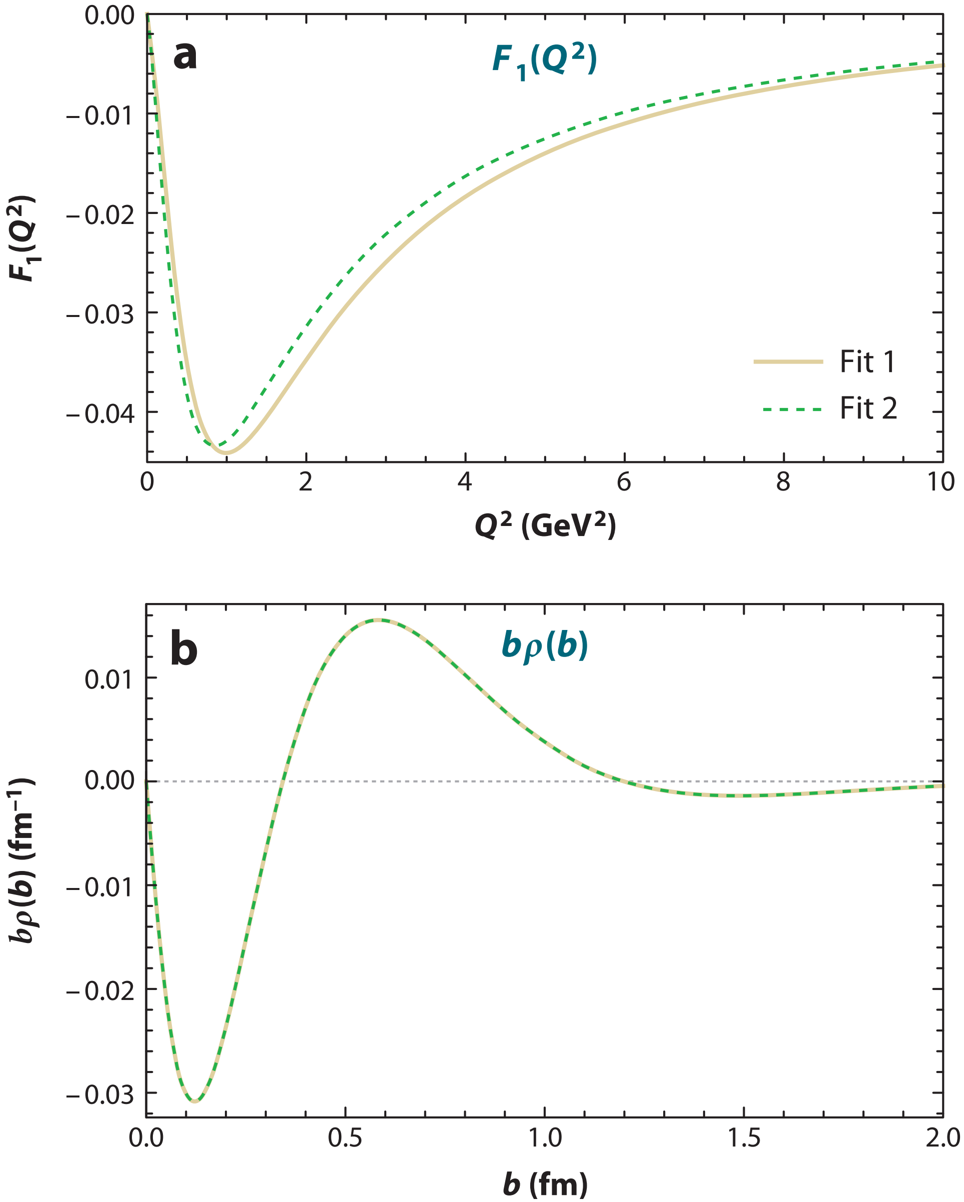
Neutron



F_1 is negative, so is central density

Negative at large b, pion cloud? see Strikman Weiss '10

arXiv:1004.3535

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Neutron interpretation

- Impact parameter gpd Burkardt $\rho(x,b)$
- Drell-Yan-West relation between high x DIS and high Q^2 elastic scattering
- High x related to low b, not uncertainty principle

$$\lim_{x\to 1}\nu W_2(x)=(1-x)^{2n-1}\leftrightarrow \lim_{Q^2\to\infty}F_1(Q^2)\sim\frac{1}{Q^{2n}},n=2$$

- d quarks dominate DIS from neutron at high x
- d quarks dominate at neutron center, or π^-

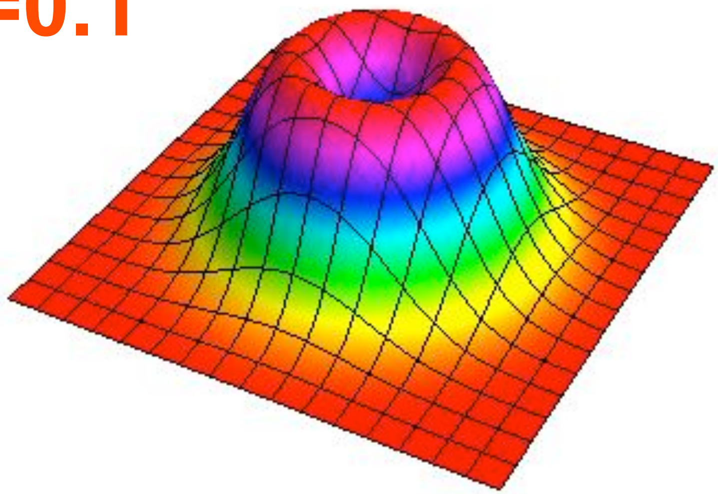
$$\text{Density is } u-\bar{u},\ d-\bar{d}$$
$$\pi^- \text{ is } \bar{u}d$$
$$\text{decreases } u \text{ contribution}$$
$$\text{enhances } d \text{ contribution}$$

Neutron interpretation ρ(x,b)

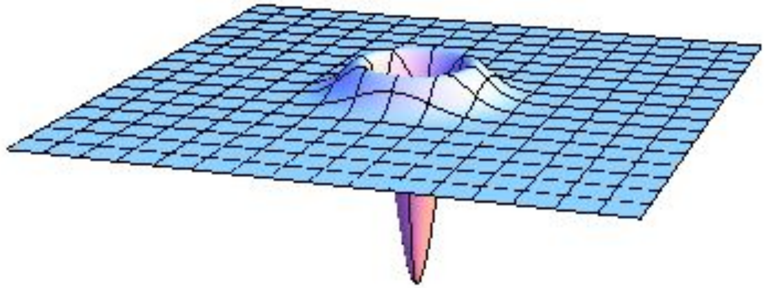
GAM, J. Arrington, PRC78,032201R '08

Using other people's models

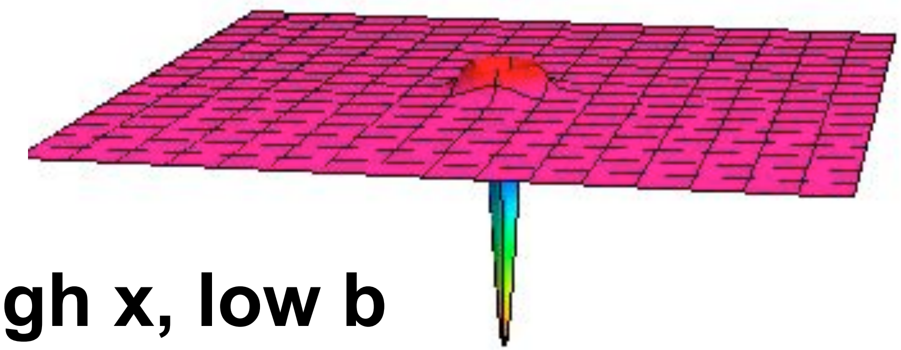
x=0.1



x=0.3



x=0.5



d or π^- dominates at high x, low b

Transverse Nucleon anomalous magnetization density

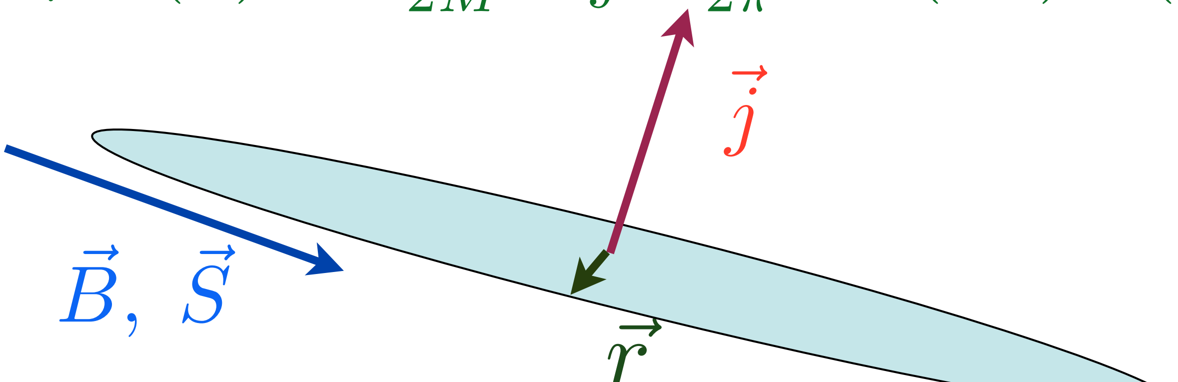
$$\vec{\mu}\cdot\vec{B} = \int d^3r\vec{j}\cdot\vec{A} = \frac{1}{2}\int d^3r\vec{j}\cdot(\vec{B}\times\vec{r}) = \frac{1}{2}\int d^3r(\vec{r}\times\vec{j})\cdot\vec{B}$$

$\frac{1}{2}\vec{r}\times\vec{j}$ is magnetization density (OAM) Spin included

$\vec{B}$ in x-direction, $\vec{J}$ in z-direction

Magnetization density

$$\rho_M(\mathbf{b}) = \frac{\sin^2\phi}{2M} b\int\frac{Q^2 dQ}{2\pi}F_2(Q^2)J_1(Qb)$$



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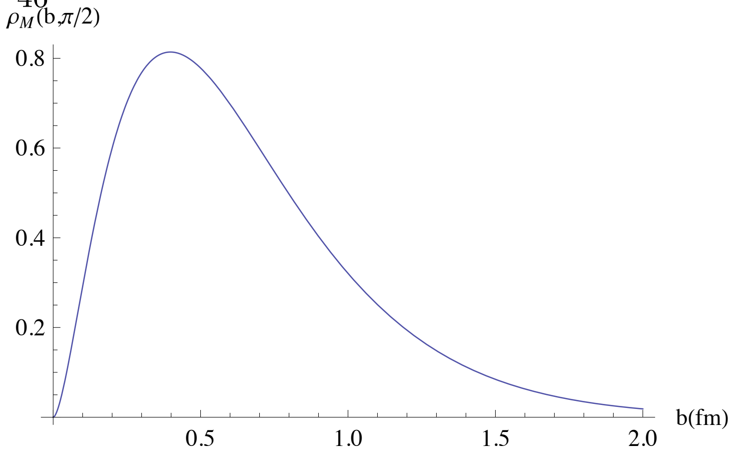
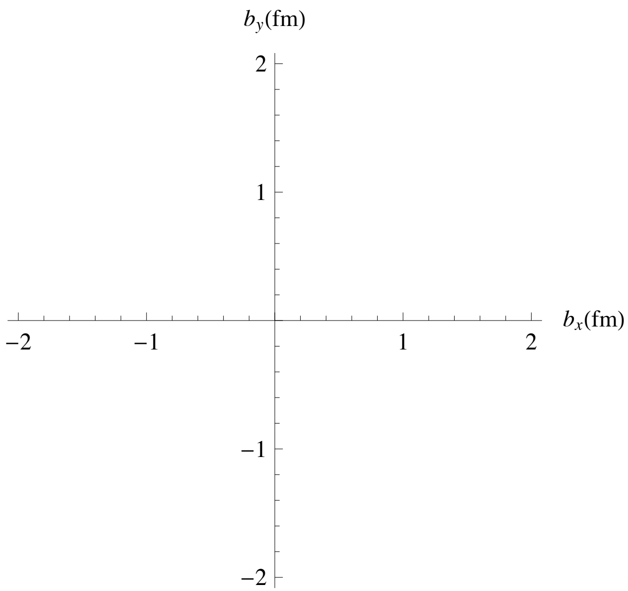
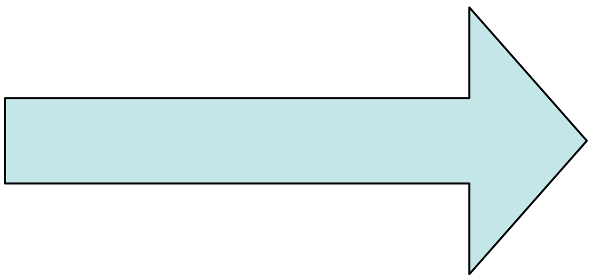


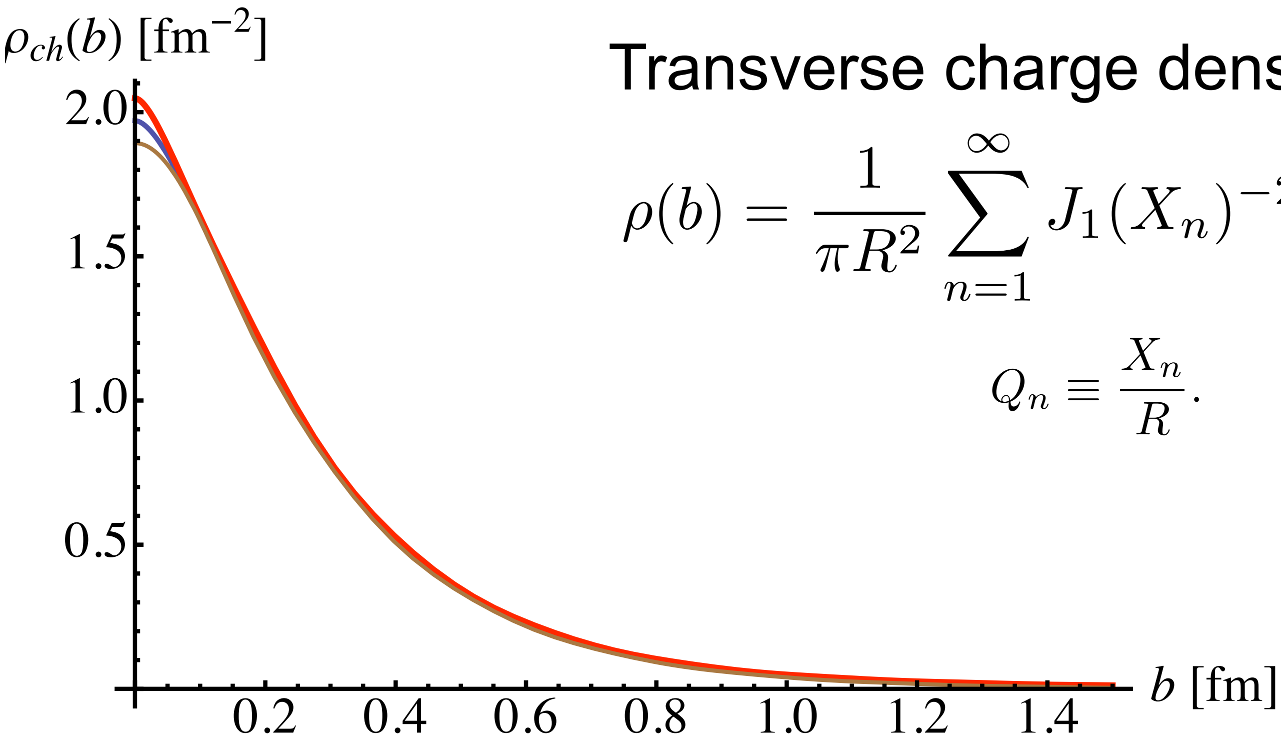


Direction of magnetic field



How well are these known now?

- Analyze effect of experimental errors and errors due to finite range of Q^2



Transverse charge density

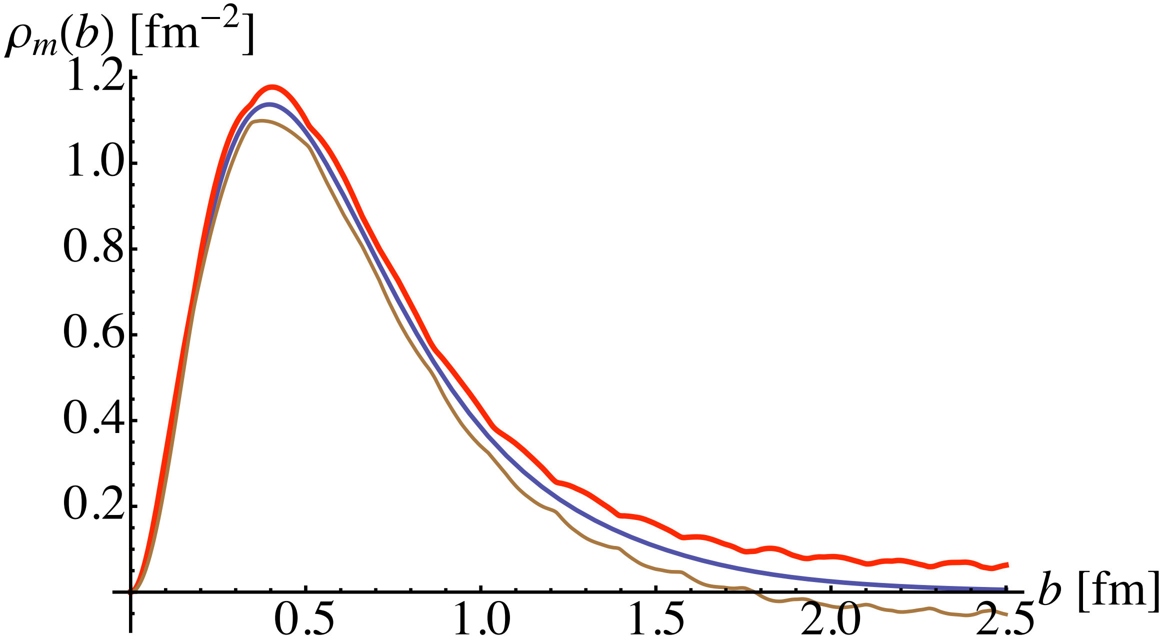
$$\rho(b) = \frac{1}{\pi R^2} \sum_{n=1}^{\infty} J_1(X_n)^{-2} F(Q_n^2) J_0(X_n \frac{b}{R}),$$

$$Q_n \equiv \frac{X_n}{R}.$$

Venkat, Arrington, Miller, Zhan new analysis-preliminary

Anomalous magnetization density

$$\rho_m^{FRA} = \frac{1}{\pi R_2^2} \sum_{n=1}^{\infty} J_2^{-2}(X_{1,n}) b Q_{1,n} F_2(Q_{1,n}^2) J_1(Q_{1,n} b), \; Q_{1,n} \equiv \frac{X_{1,n}}{R_2}$$



Generalized transverse densities-

$$\mathcal{O}_q^\Gamma(px, \mathbf{b}) = \int \frac{dx^- e^{ipxx^-}}{4\pi} q_+^\dagger(0, \mathbf{b})\Gamma q_+(x^-, \mathbf{b})$$

$$\rho^\Gamma(b) = \int dx \sum_q e_q \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \mathcal{O}_q^\Gamma(p^+x, \mathbf{b}) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$\int dx$ sets $x^- = 0$, get $q_+^\dagger(0, \mathbf{b})\Gamma q_+(0, \mathbf{b})$ **Density!**

$\Gamma = \frac{1}{2}(1 + \mathbf{n} \cdot \ \boldsymbol{\gamma}\gamma^5)$ gives spin-dependent density

Local operators calculable on lattice M. Göckeler et al PRL98,222001 $\widetilde{A}_{T10}'' \sim \mathrm{sdd}$ spin-dependent density

Schierholtz, 2009 -this quantity is not zero, proton is not round

Observing shape of proton

- Transverse coordinate space density is a GPD, observe on lattice
- Transverse momentum space density is a TMD, can be observed in

$$e, \uparrow p \to e' \pi \; X$$

Determination of F_π via Pion Electroproduction

At low Q^2<0.3 GeV², the π^+ form factor can be measured **exactly** using high energy π^+ scattering from atomic electrons.

$\Rightarrow$ 300 GeV pions at CERN SPS. *[Amendolia et al., NP **B277**(1986)168]*

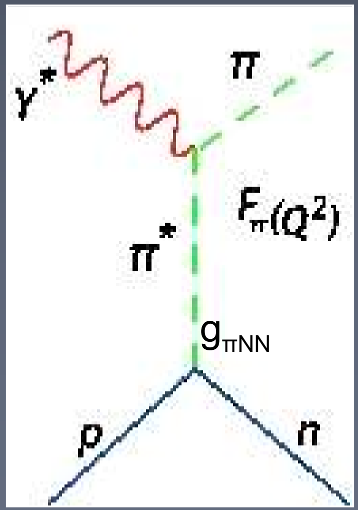
$\Rightarrow$ Provides an accurate measure of the π^+ charge radius.

$$r_\pi = 0.657 \pm 0.012 \text{ fm}$$

To access higher Q^2, one must employ the *p(e,e'π^+)n* reaction.

- *t*-channel process dominates σ_L at small $-t$.
- In the Born term model:

$$\frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t-m_\pi^2)} g_{\pi NN}^2(t) F_\pi^2(Q^2,t)$$



Pion Transverse Charge Density

━GAM **Phys.Rev.C79:055204,2009.**

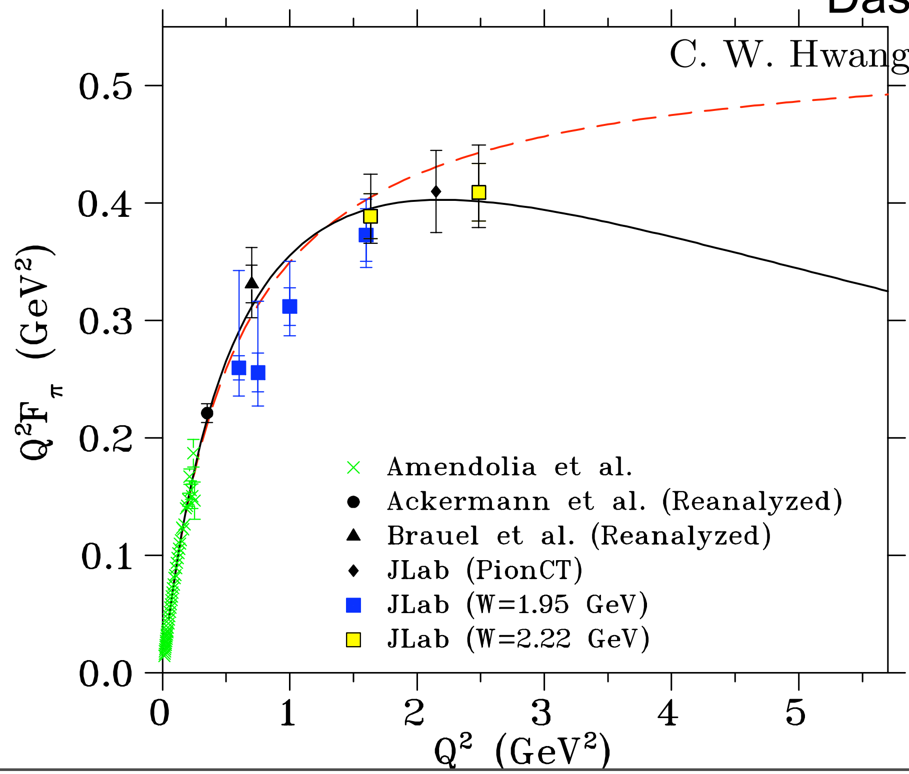
$$F_\pi(Q^2) = 1/(1 + R^2Q^2/6),$$

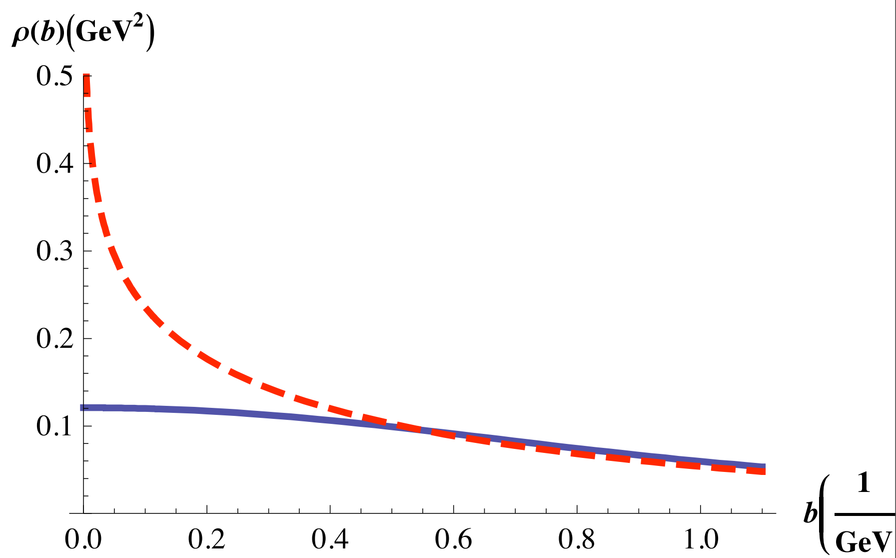
$$\rho(b) = \frac{3K_0\left(\frac{\sqrt{6}b}{R}\right)}{\pi R^2},$$

Singular - varies as log (b) small b, log(log(b)) in pQCD

Dashed- monopole fit, solid rel. cqm:

C. W. Hwang, Phys. Rev. D **64**, 034011 (2001).





Density not known too well

NT@UW-10-15

Pionic Transverse Density From Time-like and Space-Like Probes

Gerald A. Miller[1], Mark Strikman[2], Christian Weiss[3]

$$\rho(b) = \frac{1}{(2\pi)} \int_0^\infty dQ Q J_0(Qb) F_\pi(Q^2)$$

$$F_\pi(t) = \frac{1}{\pi} \int_{4m_\pi^2}^\infty dt' \frac{Im F_\pi(t')}{t' - t + i\epsilon}.$$

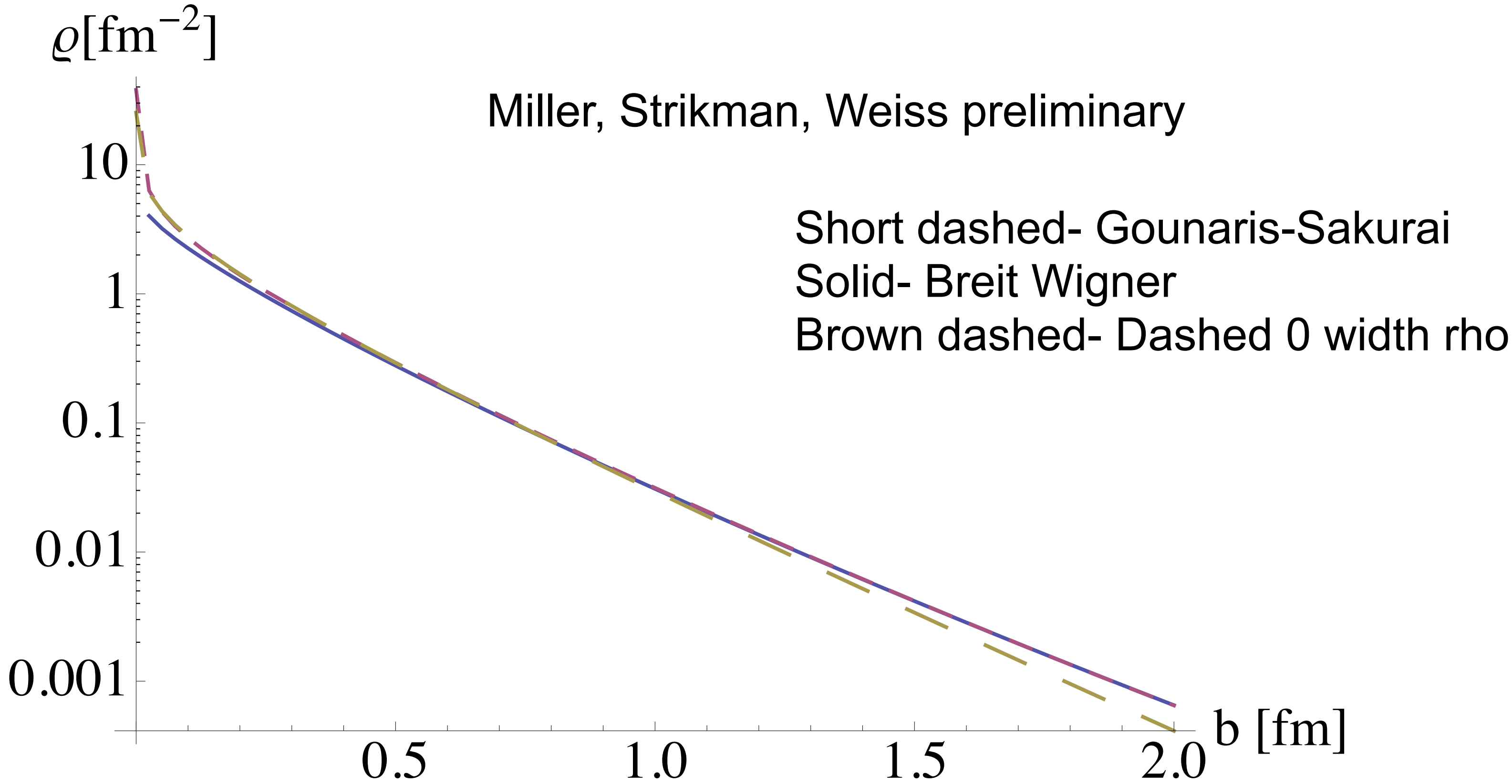
Dispersion relation

$$\rho(b) = \frac{1}{2\pi} \int_{4m_\pi^2}^\infty dt K_0(\sqrt{t} b) \frac{Im F_\pi(t)}{\pi}.$$

Low t dominates except for very small values of b

Model needed: C. Bruch et al E. J Phys.C39, 41

Pion transverse density



known for b greater than 0.3 fm (or less)

Summary

- Much data exist, Jlab12 will improve data set
- Charge density is not a 3 dimensional Fourier transform of G_E
- Interpret form factor as determining transverse charge and magnetization densities
- Nucleon transverse densities known now to high precision
- Spin-dependent density of proton is not round
- Pion transverse density known fairly well

Spares follow

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

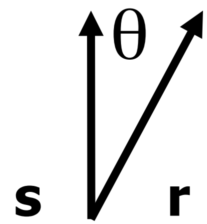
$$\langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = R(r_p) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}_p | s \rangle$$

$$\rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at $\mathbf{r}$ & spin direction $\mathbf{n}$:

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) \frac{(1+\boldsymbol{\sigma} \cdot \mathbf{n})}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} (1 + \boldsymbol{\sigma} \cdot \mathbf{n}) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} | s \rangle$$



$\mathbf{n} \parallel \hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r) \cos^2\theta$

$\mathbf{n} \parallel -\hat{\mathbf{s}}$: $\quad \rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r) \sin^2\theta$

non-spherical shape depends on spin direction

Summary

- **Much data exist, Jlab12 will improve data set**
- **Interpret form factor as determining transverse charge and magnetization densities**
- **Nucleon transverse densities known now to high precision,**
- **Pion known fairly well**
-

Relativistic formalism- kinematic subgroup of Poincare

- Lorentz transformation –transverse velocity v

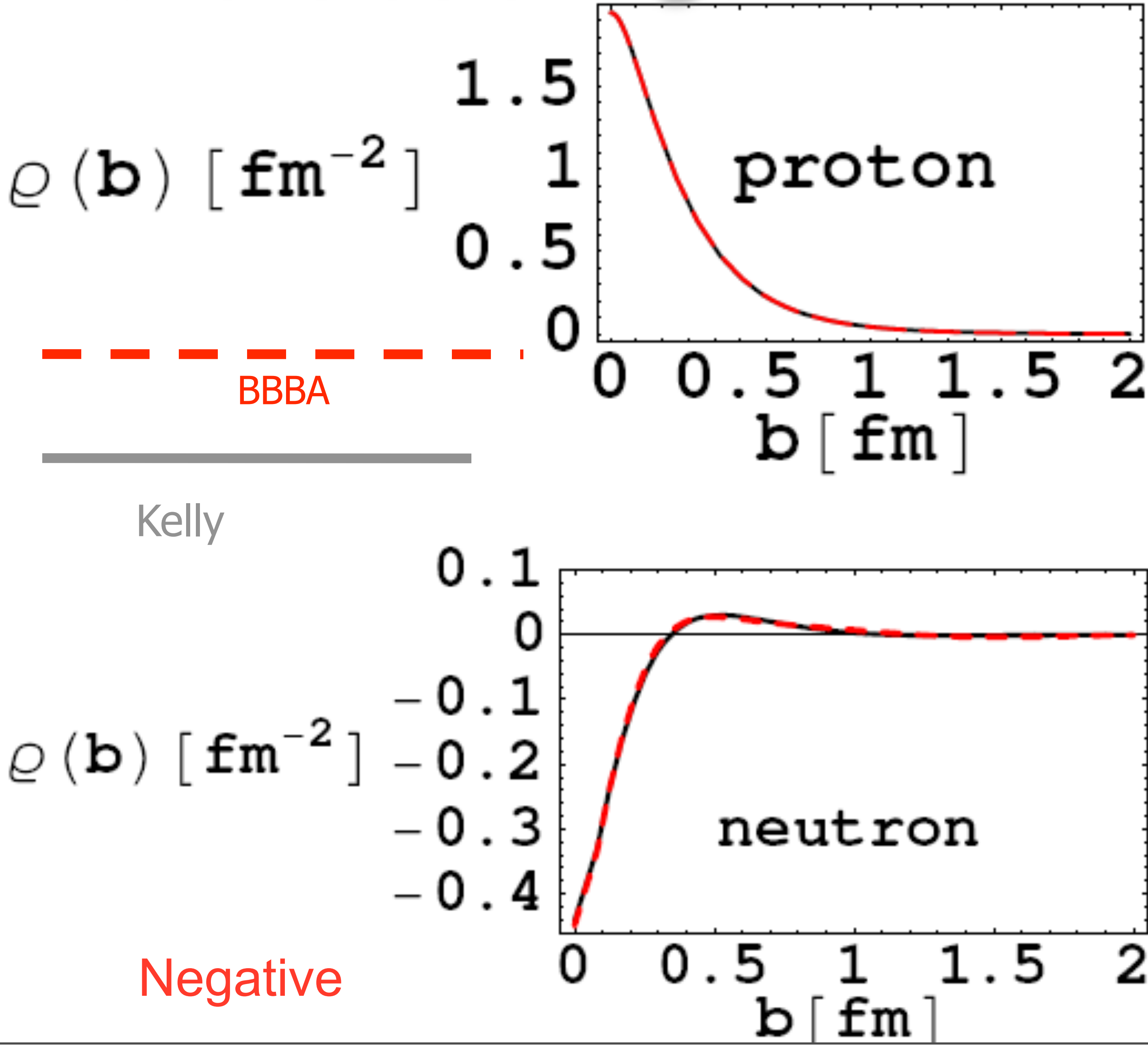
$$k^+ \rightarrow k^+, \ \mathbf{k} \rightarrow \mathbf{k} - k^+\mathbf{v}$$

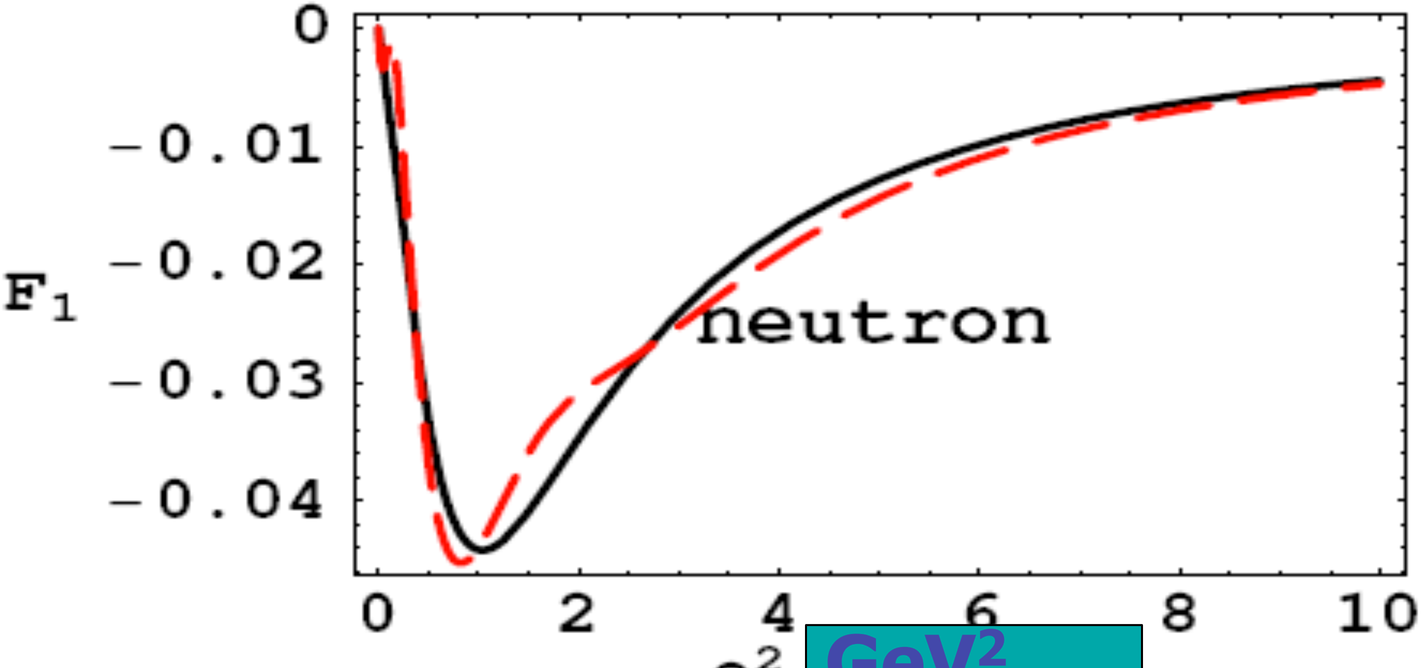
k^- such that k^2 not changed

Just like non-relativistic with k^+ as mass, take momentum transfer in perp direction, then density is 2 Dimensional Fourier Transform, also

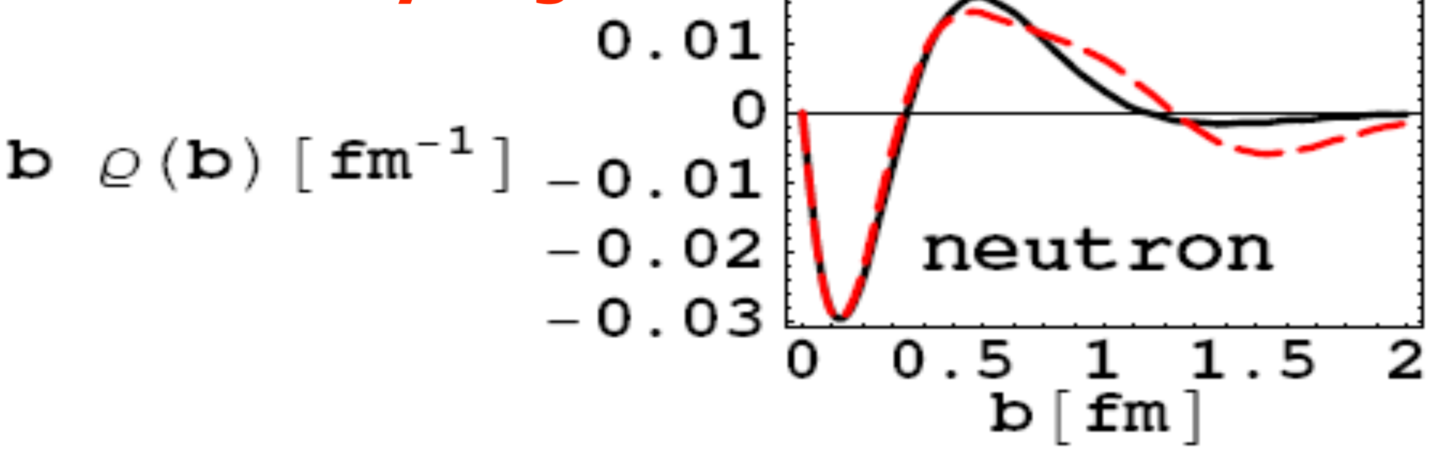
$$q^+ = q^0 + q^3 = 0, -q^2 = Q^2 = \mathbf{q}^2$$

Transverse charge densities





Negative F_1 means central density negative

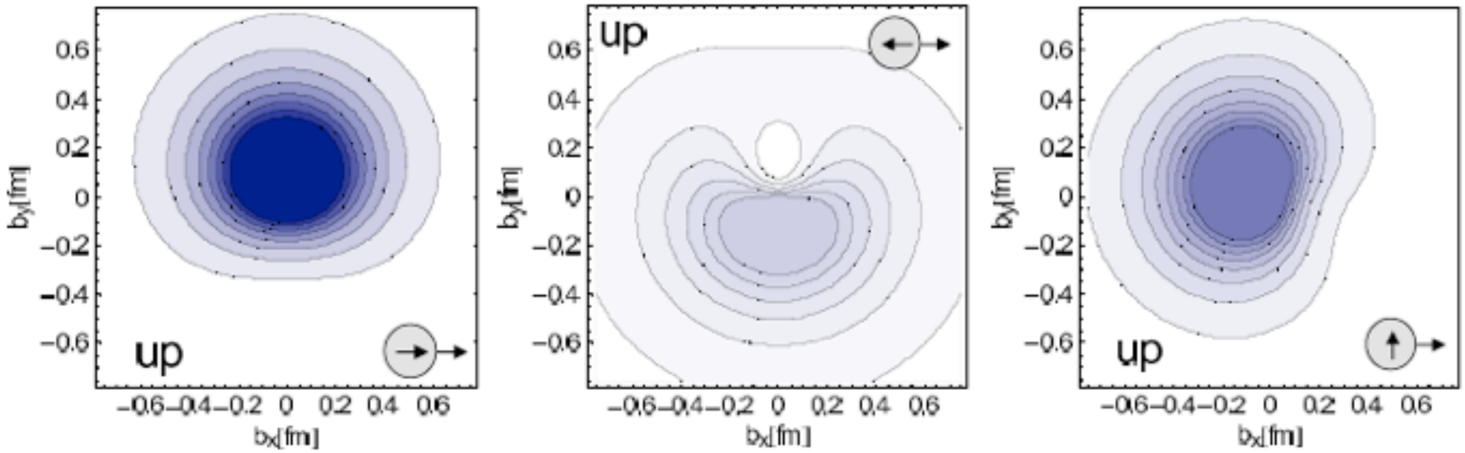


Return of the cloudy bag model

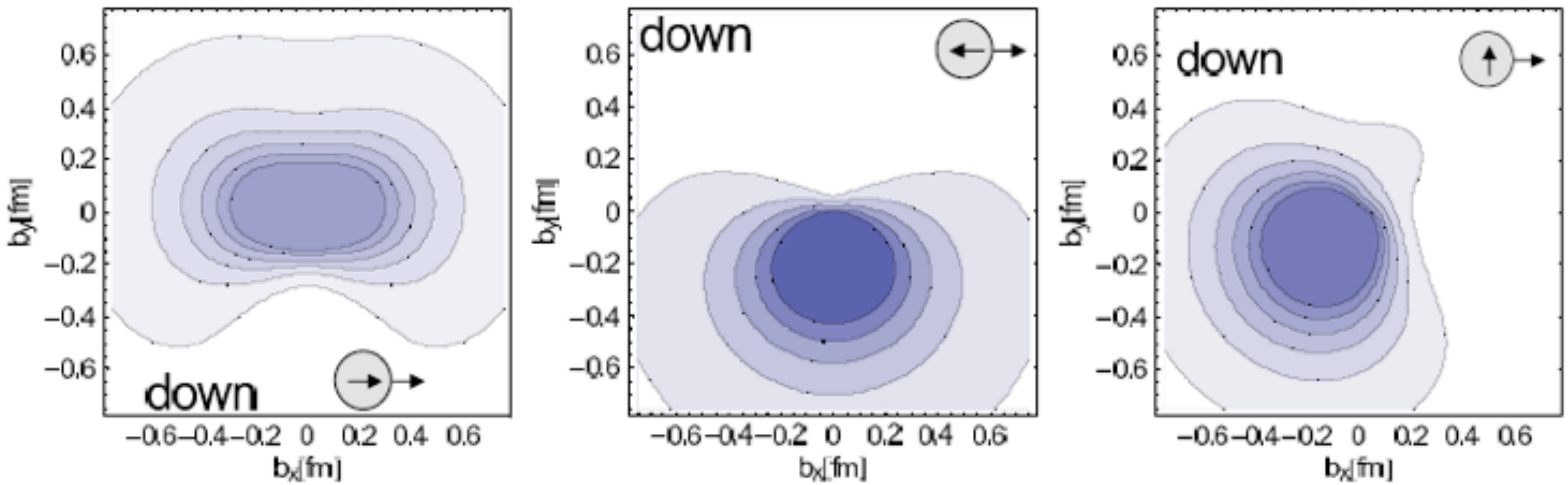
- In a model nucleon:bare nucleon + pion cloud - parameters adjusted to give negative definite F_1, pion at center causes negative central transverse charge density
- Boosting the matrix element of J^0 to the infinite momentum frame changes G_E to F_1

Rinehimer and Miller
PRC80,015201, 025206

Spin dependent densities-transverse-Lattice QCDSF, Zanotti, Schierholz...



This is not zero!



Transverse Momentum Distributions - momentum space density

In a state of fixed momentum

$\Phi_q^\Gamma(x, \mathbf{K})$ give probability of quark of given 3-momentum

$h_{1T}^\perp$ gives momentum-space spin-dependent density

measurable experimentally

hard to calculate on lattice because - gauge link

Relation or not between GPD and TMD

$$\text{GPD}:$$

$$\langle P', S'| \int \frac{dx^-}{4\pi} \bar{q}(-\frac{x^-}{2}, \mathbf{0}) \gamma^+ q(\frac{x^-}{2}, \mathbf{0}) e^{ix\bar{p}^+ x^-} |P, S\rangle_{\, x^+ = 0}$$

$$= \frac{1}{2\bar{p}^+} \bar{u}(P', S') \left(\gamma^+ H_q(\xi, t) + i \frac{\sigma^{+\nu}\Delta_\nu}{2M} E_q(x, \xi, t) \right) u(P, S)$$

$$\text{TMD}:$$

$$\Phi_q^\Gamma(x = \frac{k^+}{P^+}, \mathbf{k}) = \langle P, S| \int \frac{d\zeta^- d^2\zeta}{2(2\pi)^3} e^{ik\cdot\zeta} \bar{q}(0) \Gamma q(\zeta) |P, S\rangle_{\zeta^+ = 0}$$

GPD: nucleons have different momenta, but FT local in coordinate space if integrate over x

TMD: nucleons have same momenta, operator is local in momentum space

Both can be obtained Wigner distribution operator

$$W_q^\Gamma(\zeta^-, \boldsymbol{\zeta}, k^+, \mathbf{k})$$

$$= \frac{1}{4\pi}\int d\eta^- d^2\eta e^{ik\cdot\eta}\bar{q}(\zeta^- - \frac{\eta^-}{2}, \boldsymbol{\zeta} - \frac{\boldsymbol{\eta}}{2})\Gamma q(\zeta^- + \frac{\eta^-}{2}, \boldsymbol{\zeta} + \frac{\boldsymbol{\eta}}{2})$$

$$H_q(x,\xi,t) = \langle P', S'| \int \frac{d^2\mathbf{k}}{(2\pi)^2} W_q^{\gamma^+}(\zeta^- = 0, \zeta = 0, k^+, \mathbf{k})|P, S\rangle$$

$$\Phi_q^\Gamma(x,\mathbf{k}) = \langle P, S| \int \frac{d\zeta^-}{(2\pi)^2} W_q^\Gamma(\zeta^-, \zeta, k^+, \mathbf{k})|P, S\rangle$$

Summary

- **Form factors, GPDs, TMDs, understood from unified light-front formulation**
- **Neutron central transverse density is negative- consistent with Cloudy Bag Model**
- **Proton is not round- lattice QCD spin-dependent- density is not zero**
- **Experiment can whether or not proton is round by measuring** $h_{1T}^{\perp}$

Summary

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The Proton

Cloudy Bag Model~1980

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Cloudy bag model of the nucleon

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(Received 28 January 1981)

A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and g_A, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of α_s is smaller than that of the MIT bag model.

Many successful predictions

One feature- pion penetrates to the bag interior

What is charge density at the center of the neutron?

- **Neutron has no charge, but charge density need not vanish**
- **Is central density positive or negative?**

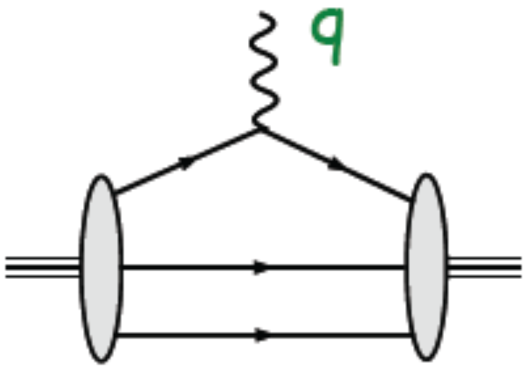
Fermi: n fluctuates to $p\pi^-$

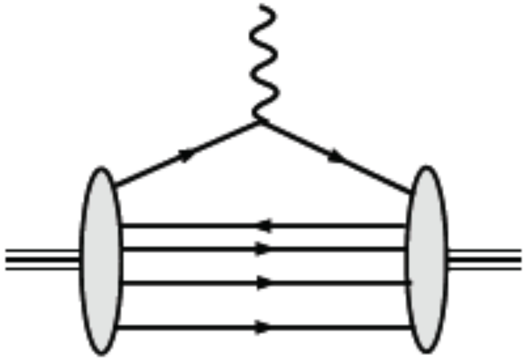
p at center, pion floats to edge

One gluon exchange favors dud

Real question- how does form factor relate to charge density?

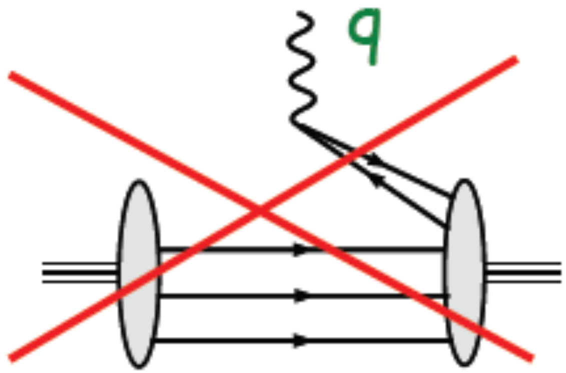
interpretation of FF as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

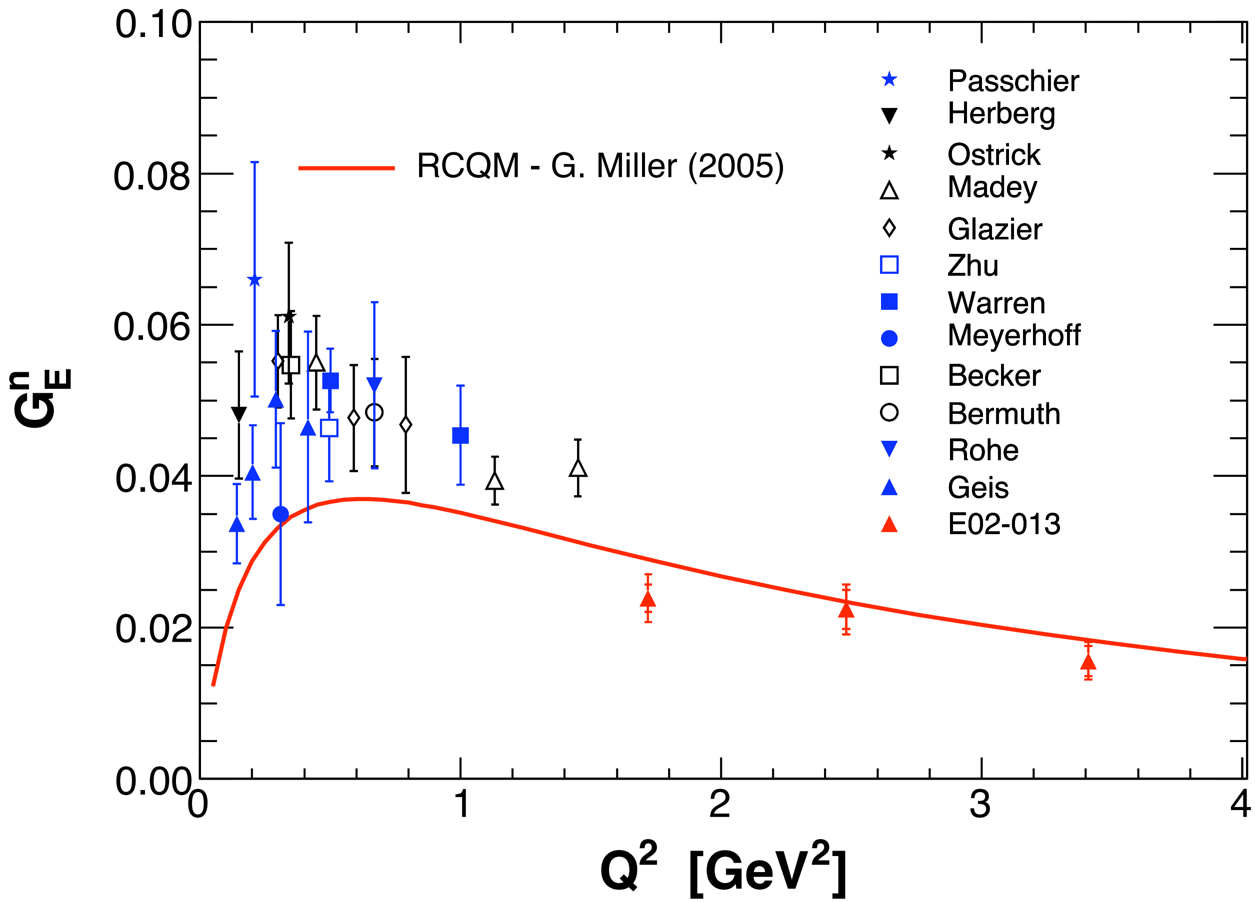
Absent in a Drell-Yan Frame

$$\boxed{q^+ = q^0 + q^3 = 0}$$

From Marc Vanderhaeghen

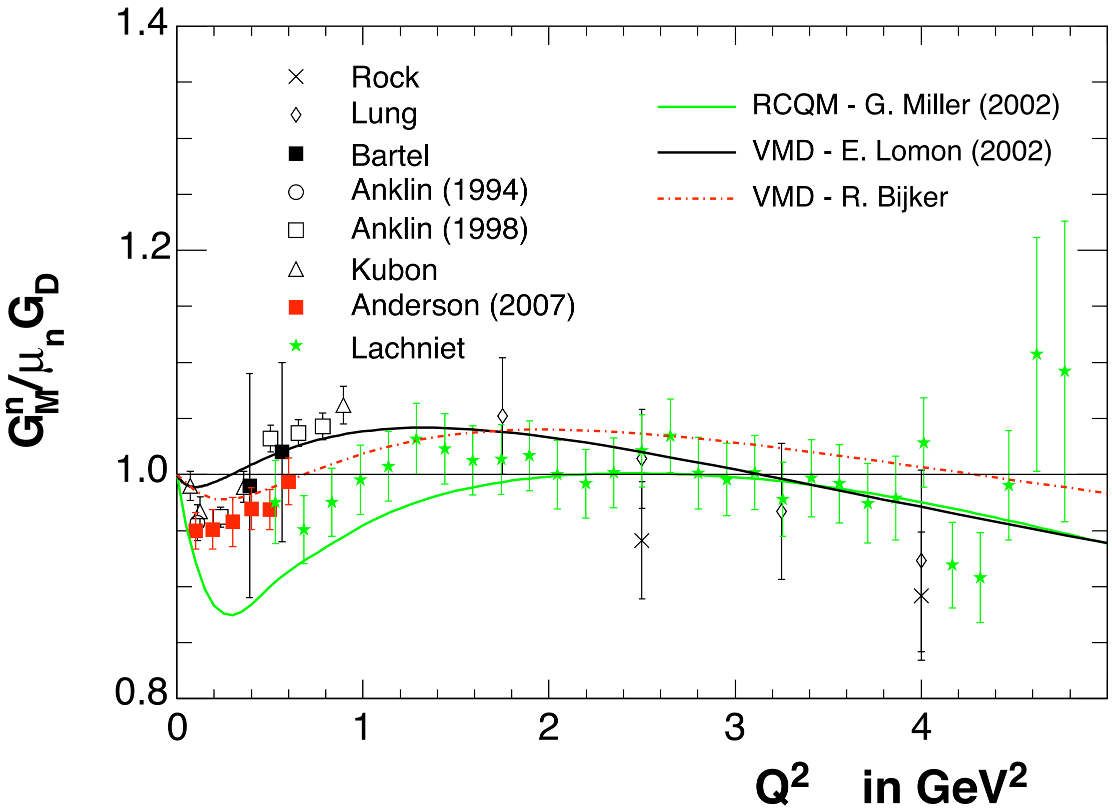
Electric Form Factor of the Neutron up to Q^2=3.4 GeV^2 using the Reaction He^3(e,e'n)pp.

S. Riordan *et al*. Aug 2010. e-Print: 1008.1738



Overview of results for G_M^n

dejager slide



High-quality data set now available up to ~4.5 GeV²

Jerry Gilfoyle

→ A systematic difference of several % between results from JLab and MAMI in Q^2-range 0.4 – 1.0 GeV^2

→ Reminder that at least two independent experiments are always needed