Pion-nucleon scattering around the delta-isobar resonance

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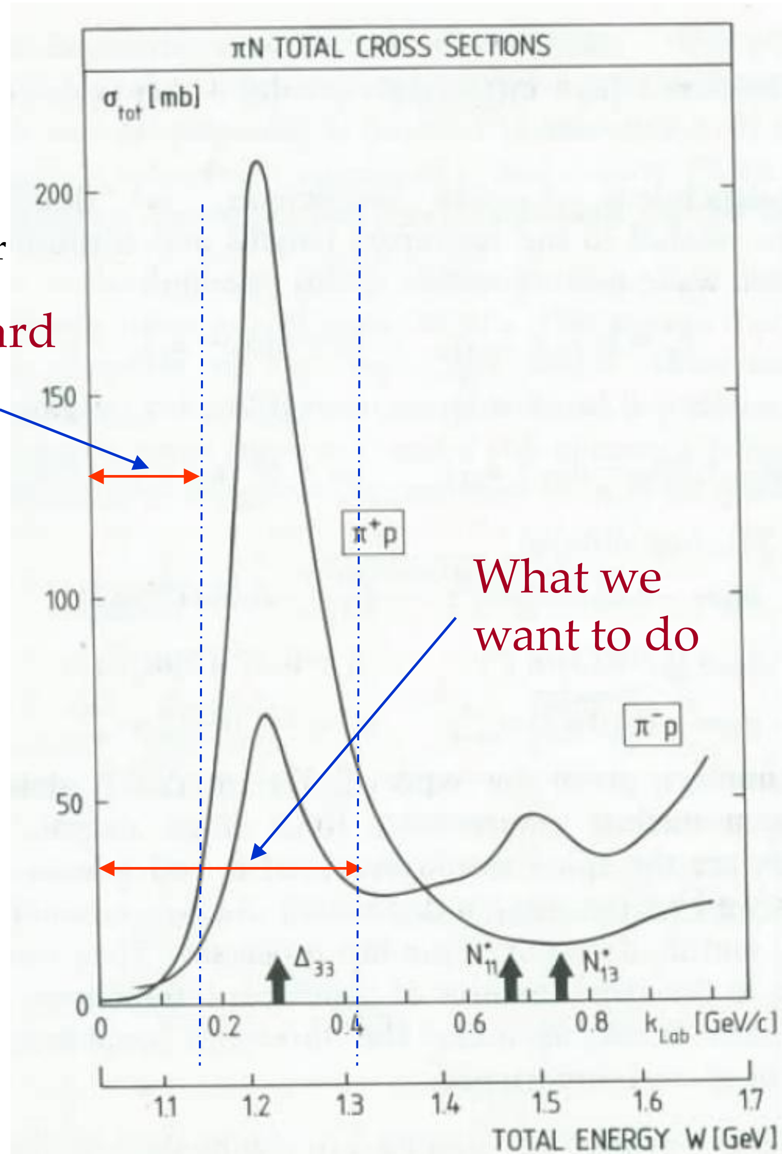
What do we really do

Fettes & Meissner 2001...

Standard ChPT



What we want to do

$$\Delta(1232)\ J^P = \frac{3}{2}^+$$

Isospin 3/2

Outline

Why delta? why pion-nucleon scattering with EFT?

Standard ChPT power counting

Power counting of resumming delta

Results, P-wave phase shifts

Summary

Cast of nuclear physics (non-Strange)

Baryons

N 940MeV

Δ 1232MeV

$N^\star$ 1440MeV (Roper)

⋮

N, Δ excitations

Mesons (mediating long-range forces)

π 140MeV

σ (5~6)00MeV

ρ 770MeV

ω 782MeV

⋮

break-down scale

chiral EFT

π N Δ $N^\star$ (1440)

Delta in nuclear forces

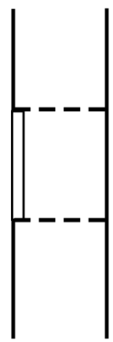
$$\delta = m_\Delta - m_N \simeq 300\mathrm{MeV} \simeq 2m_\pi \ll M_{\mathrm{QCD}} \sim 1\mathrm{GeV}$$

Delta-ful

Delta-less

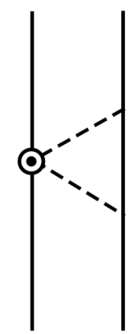
Delta as an explicit DOF

→ better convergence of EFT expansions, at the price of more DOFs



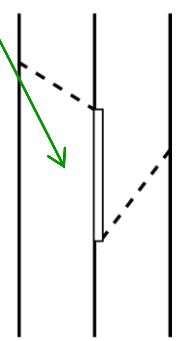
Δ → short range physics

→

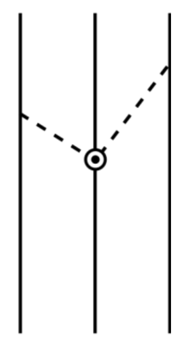


Delta becomes short-range physics

EFT expansion in powers of $m_\pi / \delta \sim 0.5$



→



Why effective field theory ?

Several non-EFT or quasi-EFT approaches describe pion-nucleon data well

- K-matrix based models
- Unitarized meson-exchange
- Unitarized chiral perturbation theory (ChPT)

  ……

But, with a proper EFT

- Model-independent, **controlled** low-energy approximation to QCD
- Provides inputs to other nuclear reactions through low-energy constants (**LECs**)
- Natural framework to take data from **lattice QCD**

Energy levels by EFT $\xrightarrow{\text{LQCD data}}$ LECs $\xrightarrow{\text{EFT in cont.}}$ nuclear reactions

A "recipe" for EFT

Tweak

- Relevant ***degrees of freedom*** at low energies
- ***Symmetries***

Tweak

- ***Power counting***

  Most non-trivial

  a scheme to weigh numerous contributions

  → order-by-order in Q/M

  Q: external momenta

  M: underlying scale

- ***Renormalization***

observables

independent of Λ

Not achieved

renormalization group (RG) invariance

Model independence

Energy

high-energy theory

Λ cutoff

low-energy EFT

Standard ChPT counting

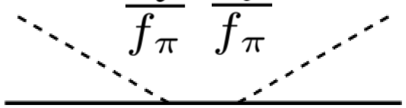
Q : generic momenta

--- πN scattering

chiral symmetry → derivative couplings

$\frac{Q}{f_\pi}$ $\frac{Q}{f_\pi}$

$\frac{Q}{f_\pi^2}$:



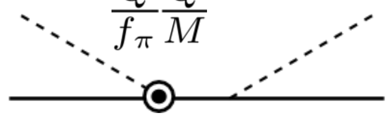
nucleon prop. $\sim 1/Q$

……

M : scale of underlying theory

$\frac{Q}{f_\pi}\frac{Q}{M}$

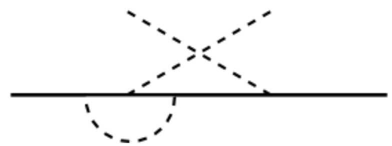
$\frac{Q}{f_\pi^2}\frac{Q}{M}$:



one more derivative

……

$\frac{Q}{f_\pi^2}\frac{Q^2}{M^2}$:



$\frac{Q}{f_\pi}\frac{Q^2}{(4\pi f_\pi)^2}$

……

pion loop suppressed by $Q^2/(4\pi f_\pi)^2$

naturalness → $M \sim 4\pi f_\pi \simeq 1.2\mathrm{GeV}$

But, perturbative series does not generate resonances → $M \sim m_\sigma < 4\pi f_\pi$

What do we really do

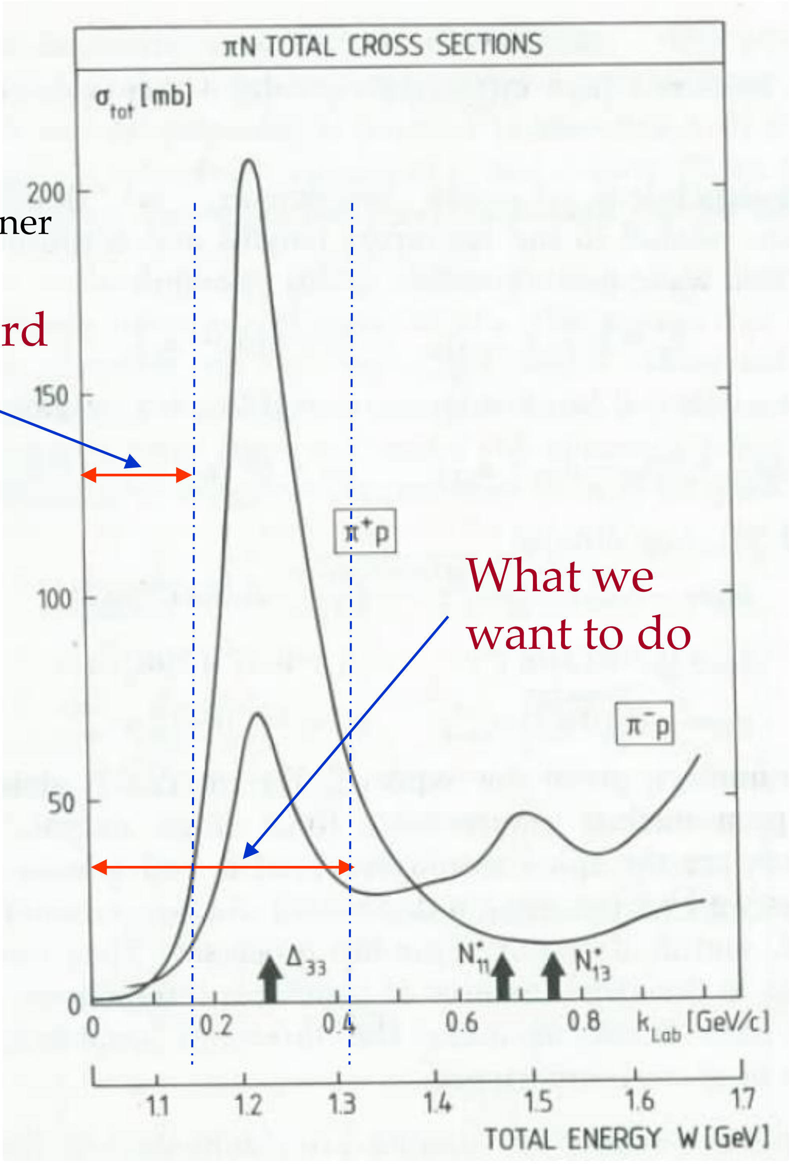
Fettes & Meissner
2001...

Standard
ChPT

$\Delta(1232)\ J^P = \frac{3}{2}^+$

Isospin 3/2

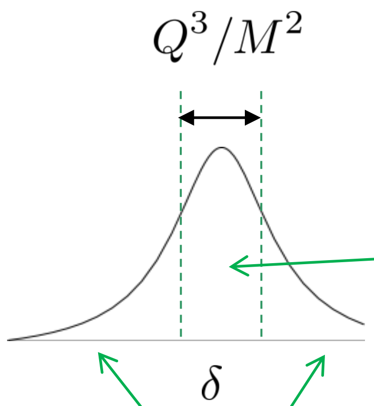
What we
want to do



Power counting ingredients

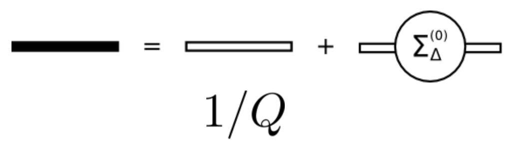
Pascalutsa & Phillips (2003)

Bertulani, Hammer, UvK(2002)



(1) resum Δ pole diags. within the resonance window

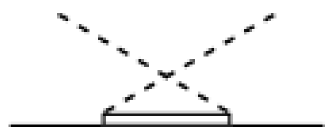
not necessary to resum



$\sim \dfrac{M^2}{Q^3}$

(2) dressed Δ propagator enhanced by $\sim M^2/Q^2$ relative to bare one

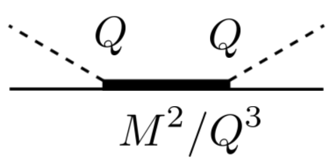
(3) u-channel not enhanced



$\sim \dfrac{Q^2}{E+\delta} \sim Q$

Background vs. Pole

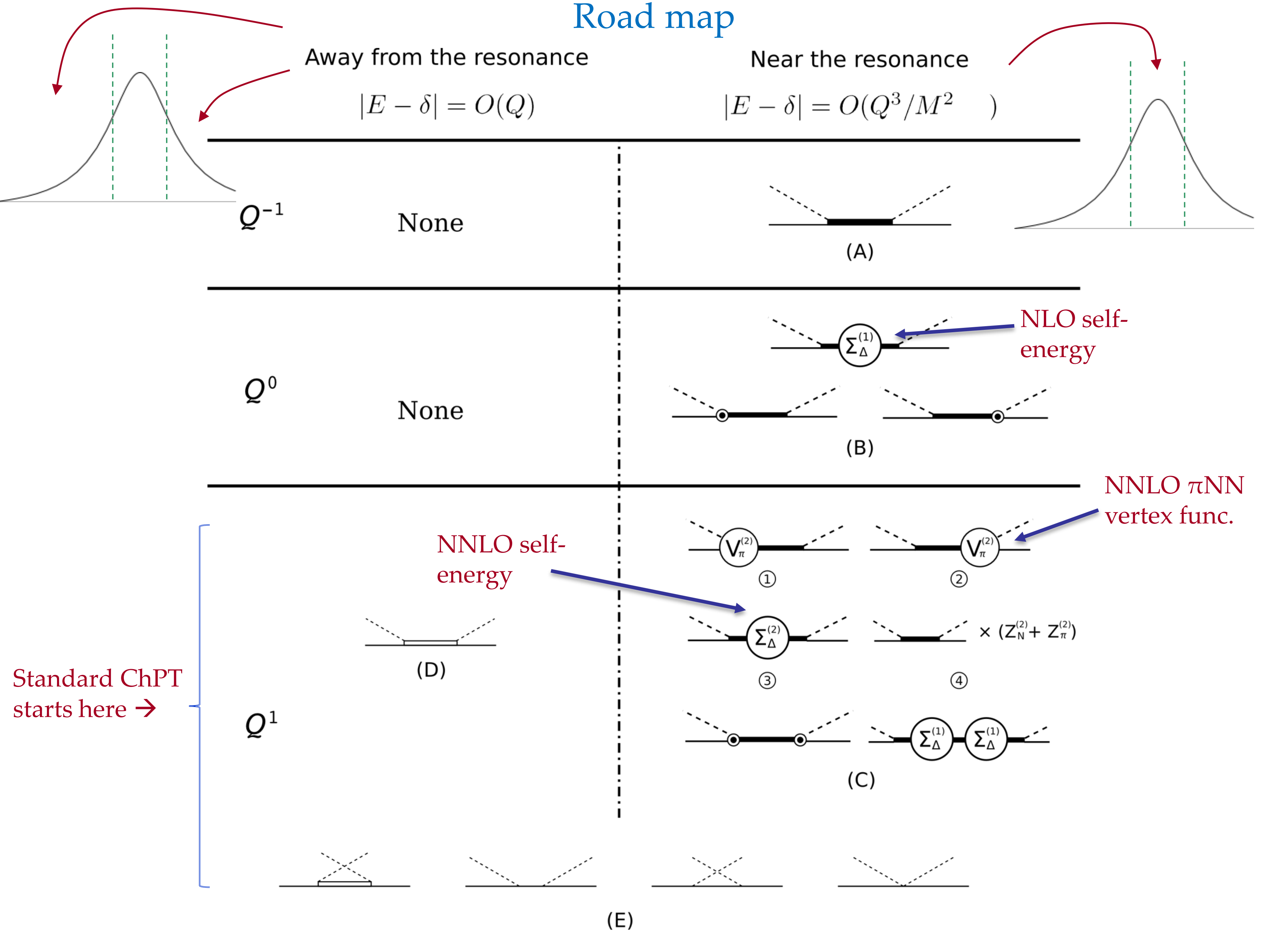
LO pi-N scattering :



$\sim Q^{-1}$ Breit-Wigner

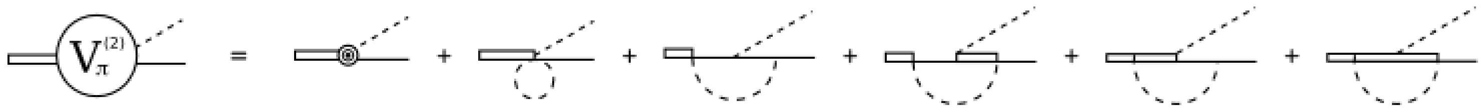
Higher orders → systematic improve over Breit-Wigner

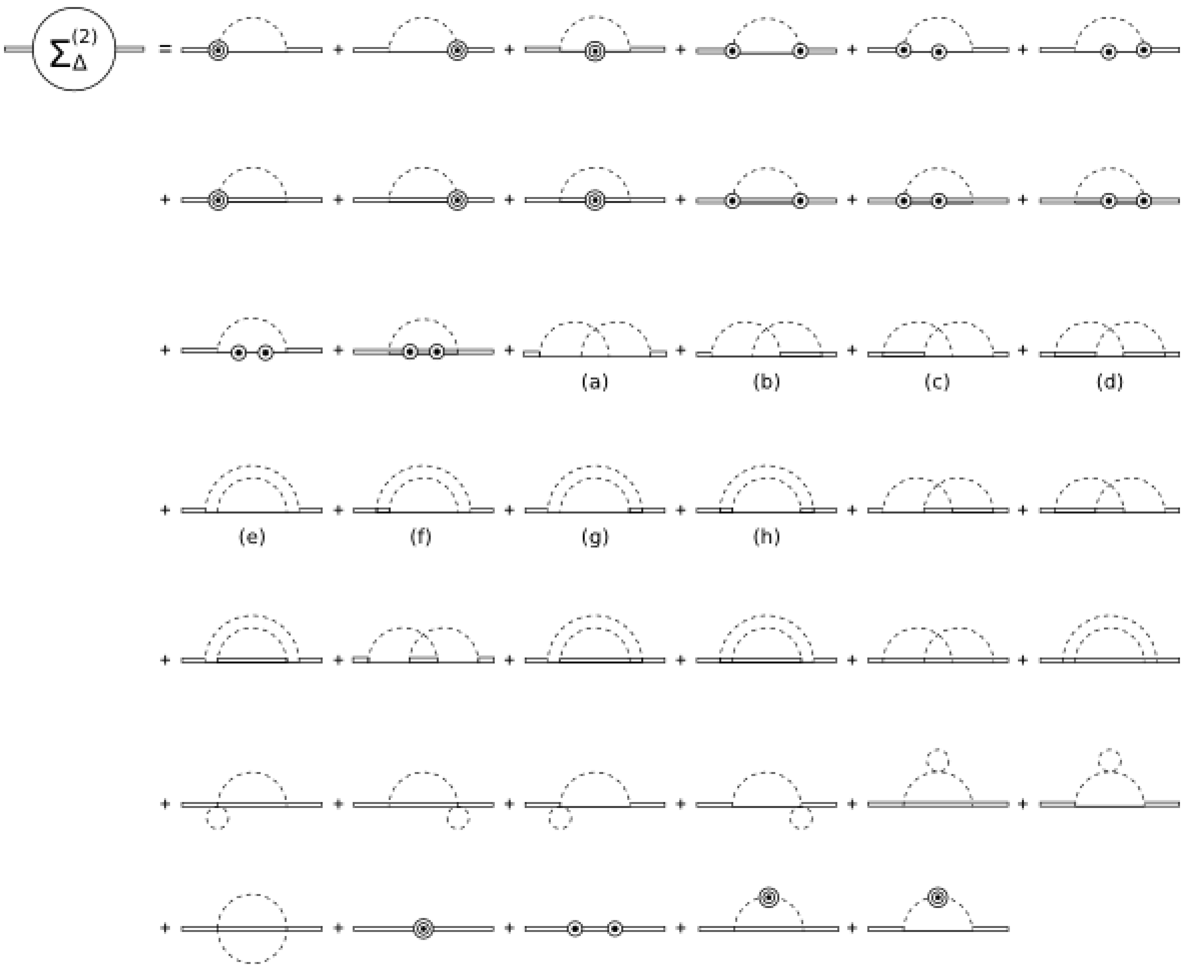
Road map



Away from the resonance
$|E-\delta| = O(Q)$
Near the resonance
$|E-\delta| = O(Q^3/M^2)$
Q^{-1}
None
(A)
Q^{0}
None
NLO self-energy
$\Sigma_\Delta^{(1)}$
(B)
Q^{1}
NNLO self-energy
NNLO πNN vertex func.
$V_\pi^{(2)}$
①
②
$\Sigma_\Delta^{(2)}$
③
$\times\ (Z_N^{(2)} + Z_\pi^{(2)})$
④
(D)
(C)
Standard ChPT starts here →
(E)

Not that simple...





Only the imaginary part needed

deal with two-loop graphs

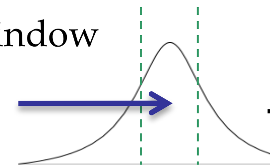
Define δ

$$S^{-1} = E - \delta + \Sigma(E) \qquad S = \text{———}$$

$$\delta - \mathrm{Re}\Sigma(\delta) \to \delta \quad \text{so that } \mathrm{Re}\Sigma(\delta) = 0$$

with a narrow resum window

$$|E-\delta| \sim Q^3/M^2$$



$$\rightarrow \Sigma(E) = \Sigma(\delta) + \overbrace{(E-\delta)\Sigma'(\delta)}^{\mathcal{O}(Q^2/M^2)\ \text{smaller}} + \cdots$$

$$\begin{aligned} S^{-1} \quad = \quad & E - \delta + \mathrm{Im}\Sigma^{(0)}(\delta) + \mathrm{Im}\Sigma^{(1)}(\delta) - Z_\Delta^{(2)}(E-\delta) \\ & + (E-\delta)\,\mathrm{Im}\Sigma^{(0)}(\delta) + \underbrace{\mathrm{Im}\Sigma^{(2)}(\delta)} + \cdots \end{aligned}$$

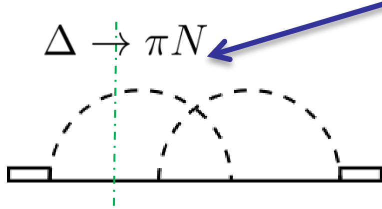
$$Z_\Delta^{(2)} = -\mathrm{Re}\Sigma^{(0)\prime}(\delta)$$

Δ field ren. const. , divergent

optical theorem

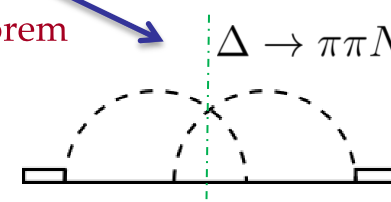
$\Delta \to \pi N$

$$\frac{Q^5}{M^4}$$



pion momenta $Q \sim \delta$

$\Delta \to \pi\pi N$

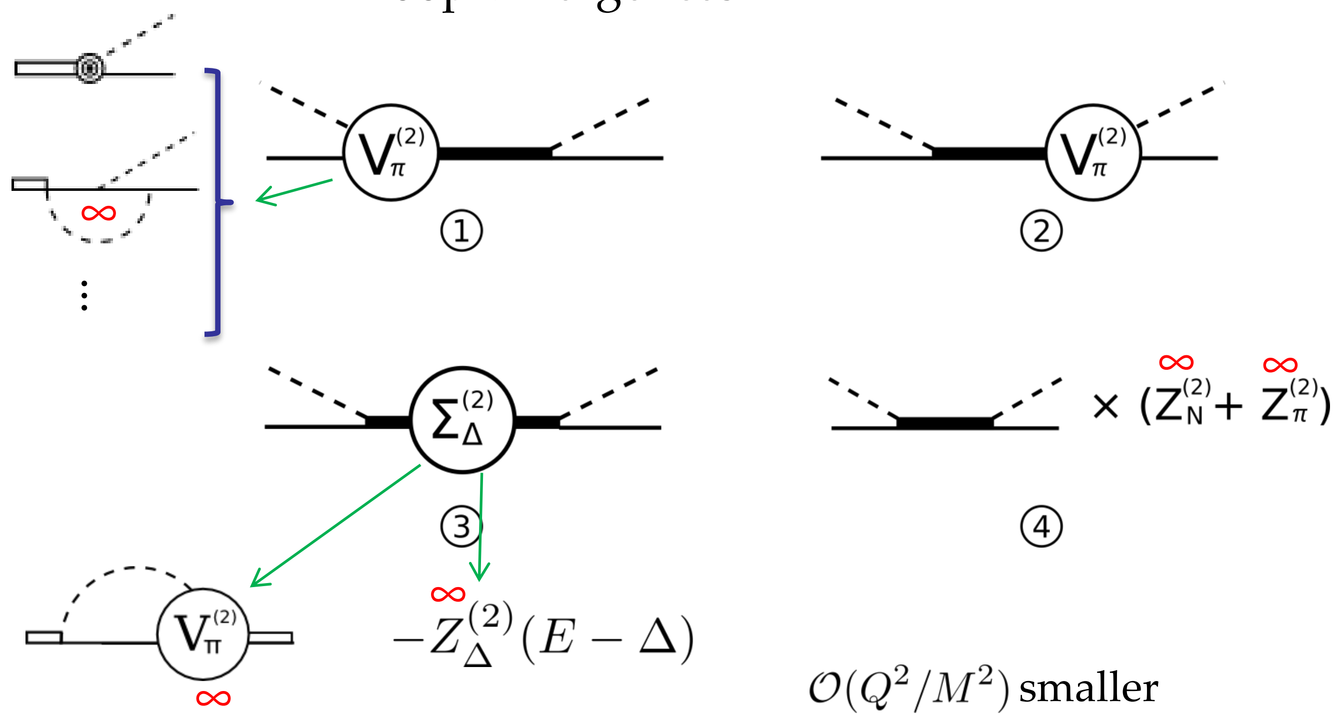


→ can be ignored

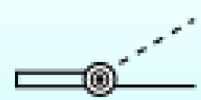
$\tilde{Q} \sim 40\mathrm{MeV} \ll \delta$

NNLO Renormalization

∞ : loop divergences



$$|E-\delta| \sim Q^3/M^2 \rightarrow V_\pi(E) = V_\pi(\delta) + \overbrace{(E-\delta)V_\pi'(\delta)}^{\mathcal{O}(Q^2/M^2)\ \text{smaller}} + \cdots$$

→All ∞ absorbed into δ, h_A, and 

Effective Lagrangian

f_π : pion decay constant

g_A : nucleon axial coupling

g_A^Δ : $\pi\Delta\Delta$ coupling

$\boldsymbol{T}(\vec{S})$: 2×4 matrices in isospin (spin) space

$D = 1 + \frac{\boldsymbol{\pi}^2}{4f_\pi^2}\,, \quad \vec{\boldsymbol{D}} = \frac{\vec{\nabla}\boldsymbol{\pi}}{2f_\pi} + \cdots\,, \quad \vec{\mathscr{D}}N = \vec{\nabla}N + \cdots$

iso-vector contraction

$$\mathcal{L}^{(0)} = 2f_\pi^2\boldsymbol{D}^2 - \frac{1}{2D}m_\pi^2\boldsymbol{\pi}^2 + N^\dagger i\mathscr{D}_0 N + g_A N^\dagger\boldsymbol{\tau}\vec{\sigma}N\cdot\cdot\vec{\boldsymbol{D}}$$
$$+\Delta^\dagger\left(i\mathscr{D}_0 - \delta\right)\Delta + g_A^\Delta\Delta^\dagger\boldsymbol{t}^{(\frac{3}{2})}\vec{S}^{(\frac{3}{2})}\Delta\cdot\cdot\vec{\boldsymbol{D}} + h_A\left(N^\dagger\boldsymbol{T}\vec{S}\Delta + H.c.\right)\cdot\cdot\vec{\boldsymbol{D}} + \cdots\,,$$

constrained by Lorentz inv.

$\pi N\Delta$ couplings

spatial-vector contraction

$$\mathcal{L}^{(1)} = \frac{1}{2m_N}\left(N^\dagger\vec{\mathscr{D}}^2N + \Delta^\dagger\vec{\mathscr{D}}^2\Delta\right) - \frac{h_A}{m_N}\left(iN^\dagger\boldsymbol{T}\vec{S}\cdot\vec{\mathscr{D}}\Delta + H.c.\right)\cdot\boldsymbol{D}_0 + \cdots\,,$$

Cohen, Friar, Miller, & van Kolck (1996)....

- Remove redundancy due to integrations by part, EOM
- Next-to-leading $\pi N\Delta$ coupling : pure relativistic correction
- No new low-energy constant (LEC)

Effective Lagrangian

- Next-to-next-to-leading $\pi N\Delta$ couplings : 1 LEC

$$\mathcal{L}^{(2)} = -\frac{\delta}{2m_N^2}\Delta^\dagger \vec{\mathscr{D}}^2\Delta + \cancel{d_1\left(N^\dagger \boldsymbol{T}\vec{S}\Delta + H.c.\right)\cdot\cdot\vec{\mathscr{D}}^2\vec{\boldsymbol{D}}}$$

removable due to pion EOM, to be corrected on arXiv

$$+ d_2\frac{m_\pi^2}{D}\left(1-\frac{\boldsymbol{\pi}^2}{4f_\pi^2}\right)\left(N^\dagger\boldsymbol{T}\vec{S}\Delta + H.c.\right)\cdot\cdot\vec{\boldsymbol{D}}$$

$$+\frac{h_A}{2m_N^2}\left[\left(N^\dagger\boldsymbol{T}\vec{S}\vec{\mathscr{D}}^2\Delta - N^\dagger\boldsymbol{T}\vec{S}\cdot\vec{\mathscr{D}}\vec{\mathscr{D}}\Delta\right) + H.c.\right]\cdot\cdot\vec{\boldsymbol{D}}$$

$$+\frac{h_A}{8m_N^2}\left[\left(\delta_{lm}N^\dagger\boldsymbol{T}\vec{S}\cdot\vec{\mathscr{D}}\Delta + 3N^\dagger\boldsymbol{T}S_l\mathscr{D}_m\Delta + 2\epsilon_{ijl}N^\dagger\boldsymbol{T}\Omega_{im}\mathscr{D}_j\Delta\right) + H.c.\right]\cdot\mathscr{D}_l\boldsymbol{D}_m + \cdots .$$

$1/m^2$ relativistic corrections

Results

- LO : recovering the Breit-Wigner formula

P_{33} partial-wave amplitude (P-wave, I = 3/2, & J=3/2)

$$T^{\mathrm{LO}}_{P_{33}} = -\frac{\gamma^{(0)}(\delta)}{E-\delta+i\gamma^{(0)}(\delta)/2}$$

$$\begin{aligned}\gamma^{(0)}(\delta) &= \mathcal{N}(k_\delta)\frac{h_A^2 k_\delta^3}{24\pi f_\pi^2} \\ &= \frac{h_A^2}{24\pi f_\pi^2}\frac{\left(\delta^2-m_\pi^2\right)^{\frac{3}{2}}}{16\left(\delta+m_N\right)^5}\left[(\delta+2m_N)^2-m_\pi^2\right]^{\frac{3}{2}}\left[(\delta+m_N)^2+m_N^2-m_\pi^2\right]\end{aligned}$$

Relativistic kin. assumed

$$\sqrt{k_\delta^2+m_\pi^2} = \delta+m_N-\sqrt{k_\delta^2+m_N^2}$$

with $\mathcal{N}(k) \equiv \dfrac{\sqrt{k^2+m_N^2}}{\sqrt{k^2+m_N^2}+\sqrt{k^2+m_\pi^2}}$

2 free parameters : h_A, δ

leading $\pi N \Delta$ coupling

- NLO : vanishes

Results

- NNLO : beyond the Breit-Wigner formula

$$T_{P_{33}}^{\mathrm{LO+NNLO}} = -\frac{\Gamma(E)}{E-\delta+i\Gamma(E)/2}\left[1+iT_B(E)\right]+T_B(E)$$

$$\Gamma(E) \equiv \frac{h_A^2 k^3}{24\pi f_\pi^2}\mathcal{N}(k)\,(1+\varkappa)^2$$ to be fitted

Δ axial coupling

$$\varkappa = \cancel{-\frac{d_1}{h_A}k_\delta^2} + \frac{d_2}{h_A}m_\pi^2 + \frac{k_\delta^2}{(4\pi f_\pi)^2}\left\{\left[\left(\frac{2}{3}g_A^2+\frac{1}{54}h_A^2+\frac{27}{128}{g_A^\Delta}^2\right)-\frac{15}{108}h_A^2\left(1+\frac{m_\pi^2}{k_\delta^2}\right)\right]\right.$$

$$\times\left(1+\frac{m_\pi^2}{k_\delta^2}\right)^{-\frac{1}{2}}\ln\left(\frac{\sqrt{1+(m_\pi/k_\delta)^2}-1}{\sqrt{1+(m_\pi/k_\delta)^2}+1}\right)$$

$$\left.+\pi\left(\frac{27}{128}{g_A^\Delta}^2-\frac{2}{3}g_A^2\right)\frac{m_\pi^3}{k_\delta^3}\left(1+\frac{m_\pi^2}{k_\delta^2}\right)^{-\frac{1}{2}}\right\}$$

NNLO $\pi N\Delta$ coupling

$$T_B(E) = \frac{k^3}{24\pi f_\pi^2}\left(4\frac{g_A^2}{E}+\frac{1}{9}\frac{h_A^2}{E+\delta}\right)$$ non-resonant “background”

Results

- NNLO : non-delta P-waves P_{11} , P_{13} , & P_{31}

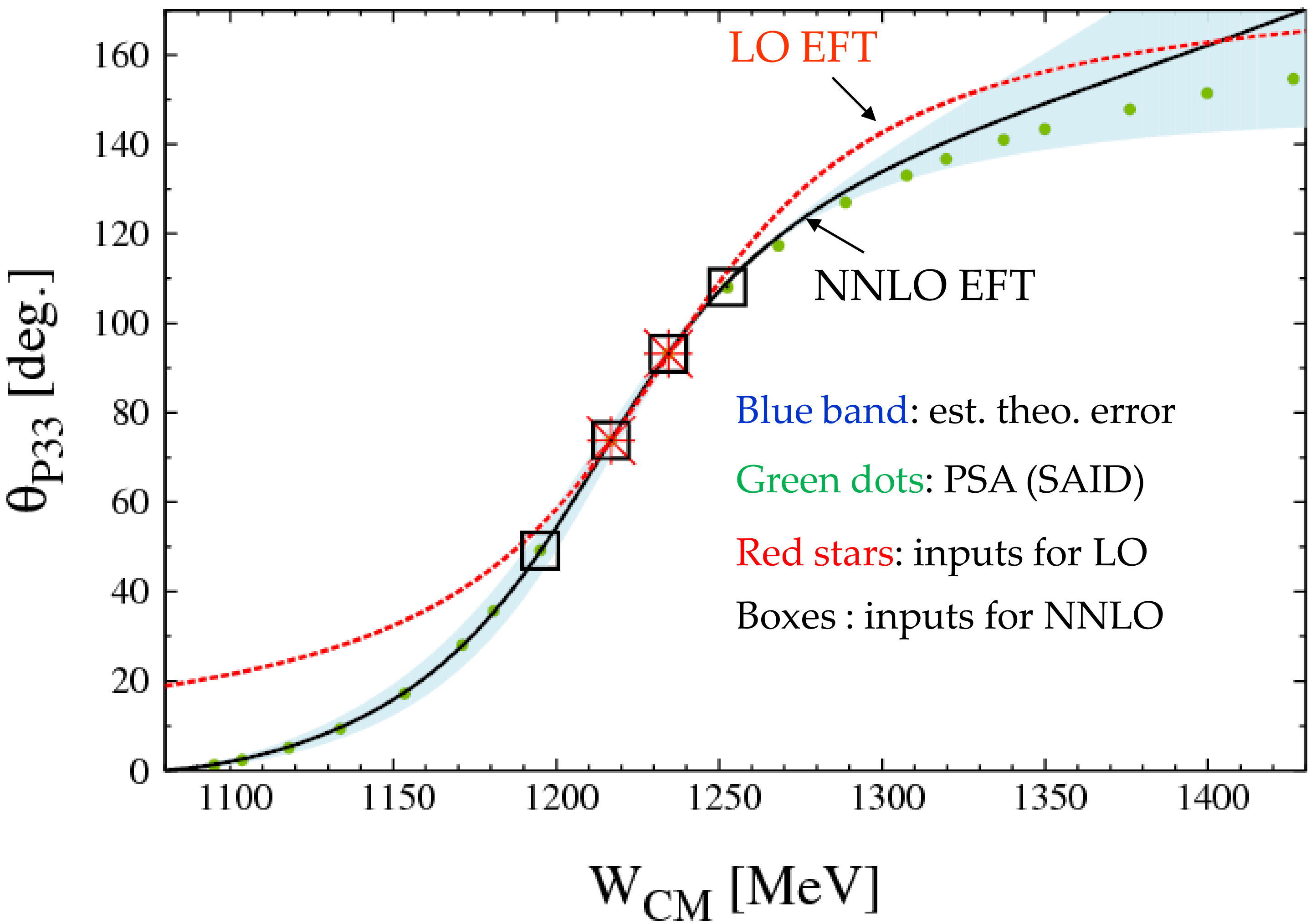
$$T^{\mathrm{NNLO}}_{P_{13}} = T^{\mathrm{NNLO}}_{P_{31}} = \frac{1}{4} T^{\mathrm{NNLO}}_{P_{11}} = \frac{(E^2 - m_\pi^2)^{\frac{3}{2}}}{12\pi f_\pi^2} \left[\frac{2}{9} \frac{h_A^2}{E+\delta} - \frac{g_A^2}{E} \right]$$

contribution of the delta

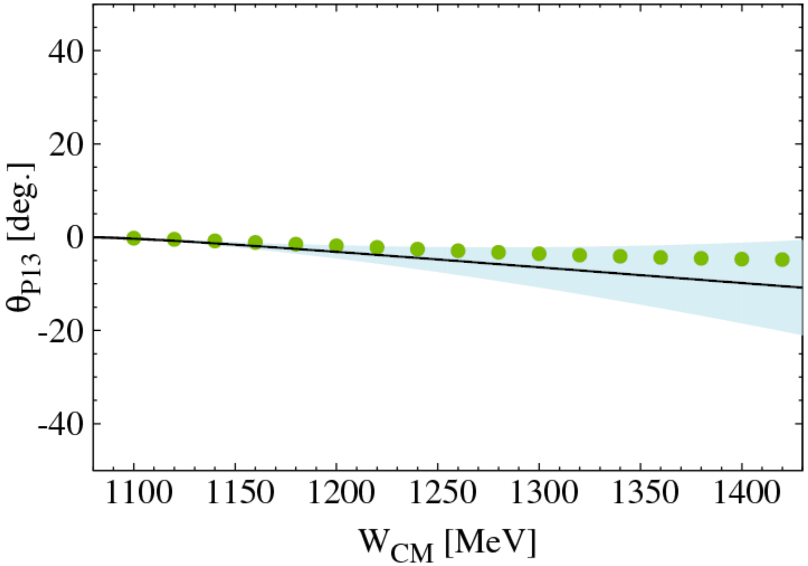
Large N_c limit: $h_A^2 = \frac{9}{2} g_A^2$

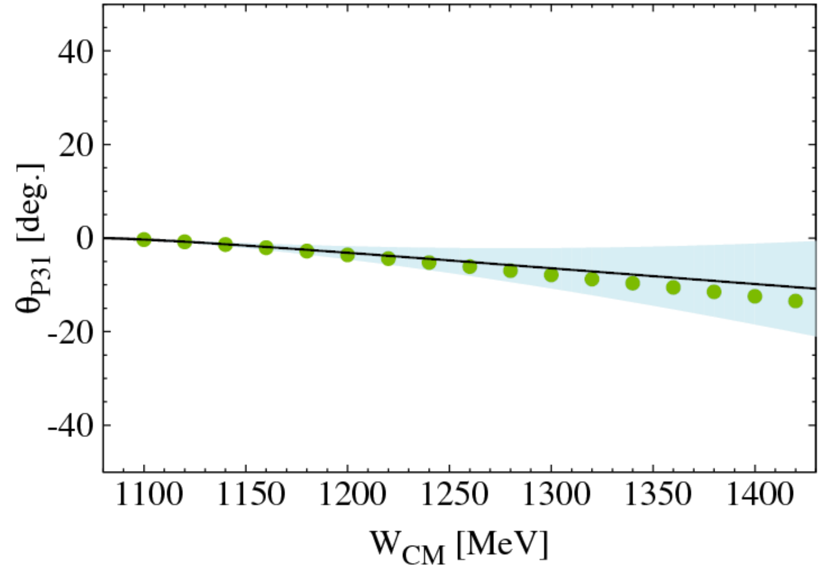
P_{33} phase shifts

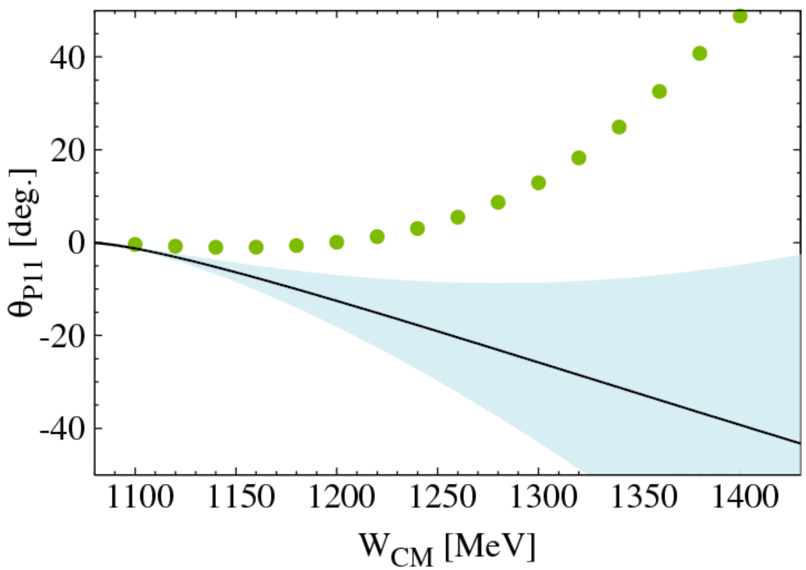
Fitted to the phase-shift analysis of SAID (George Washington group)



other P-waves







Still at O(Q)

Roper N*(1440)

Scattering volumes : threshold parameters in P-waves

$$a \equiv \lim_{k \to 0} (k^3 \cot\theta)^{-1}$$

m_π^{-3}

	EFT	PSA
a_{11}	-0.13	-0.080
a_{13}	-0.027	-0.032
a_{31}	-0.027	-0.042
a_{33}	0.20	0.21

PSA by Matsinos et al (2006)

Delta pole position

MeV

	EFT	GW PSA
Re	1211	1211
Im	-50	-49 .5

Particle Data Group (2008)

Including Roper (N*) --- Preliminary

A **pilot** study before looking deep into the Roper region

Kinematic domain : $W_{\mathrm{CM}} \sim m_\Delta$ or $E \sim \delta$

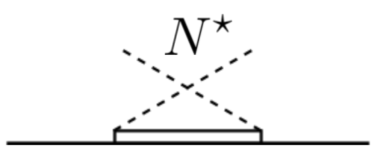
$$\delta \equiv m_\Delta - m_N \sim 300\mathrm{MeV}$$

$$\rho \equiv m_{N^\star} - m_N \sim 500\mathrm{MeV}$$

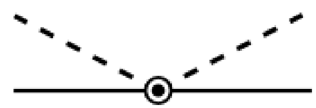
Q

$\frac{1}{E-\rho} \sim \frac{1}{Q}$ → P11

Q^2



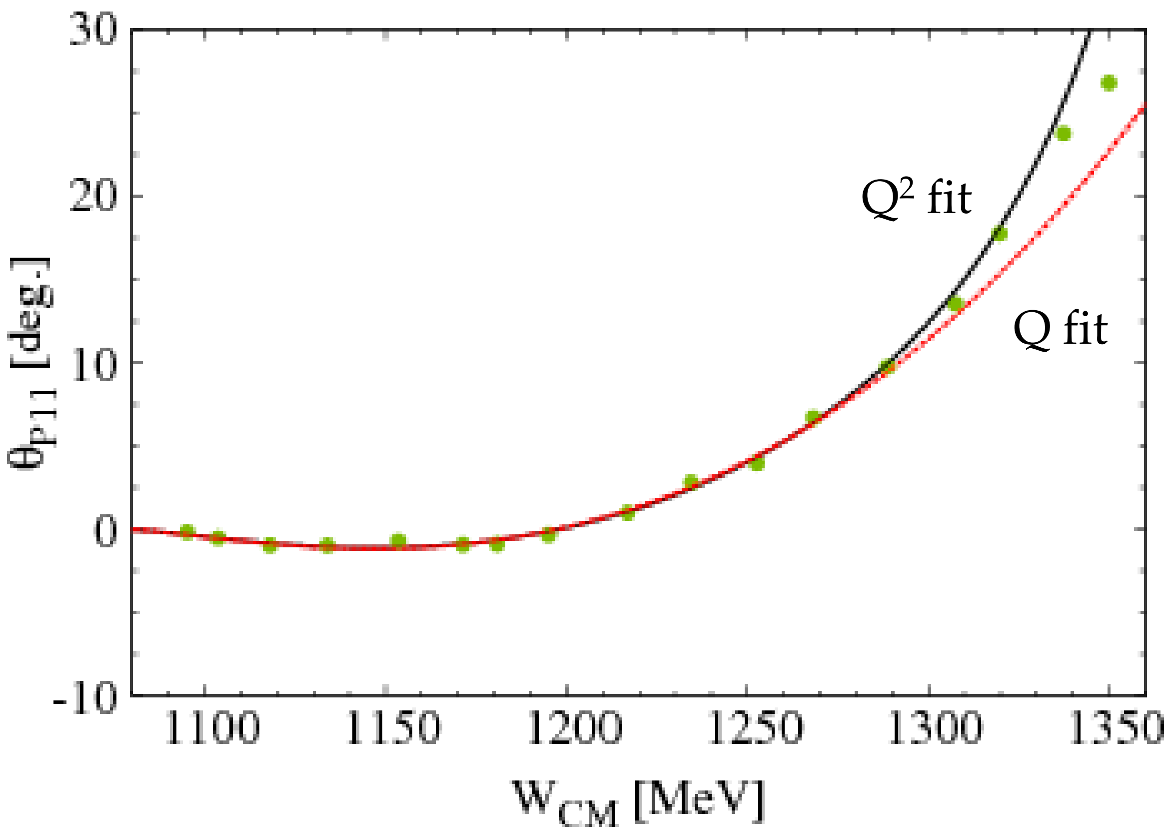
$\frac{1}{E+\rho} \sim \frac{1}{M}$



→ All S, P-waves

Let's focus on P11 → How much the Roper helps ?

P11 with Roper --- Preliminary



$$\rho \equiv m_{N^\star} - m_N$$

Q	1005 MeV
Q^2	460 MeV

Q^2 only slightly better, but with more natural LECs, e.g.,

Roper helps EFT converge --- a good fit without one-loop correction

Outlook & Summary

Model-independent framework to study pion-nucleon system

Power counting that unifies **pole** and **background** contributions

Extracting delta-related LECs → other nuclear reactions, eg. nucl-force

Energy levels by ChEFT $\xrightarrow{\text{LQCD data}}$ Delta pole position

Pushing for higher-order calculations

To the Roper : N*(1440)