Hadron electric polarizability from lattice QCD

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Jefferson Lab - November 14, 2011

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Outline

- Motivation
- Electric field on the lattice
- Neutron electric polarizability
- Pion electric polarizability & finite volume corrections
- Conclusions & outlook

Preliminaries

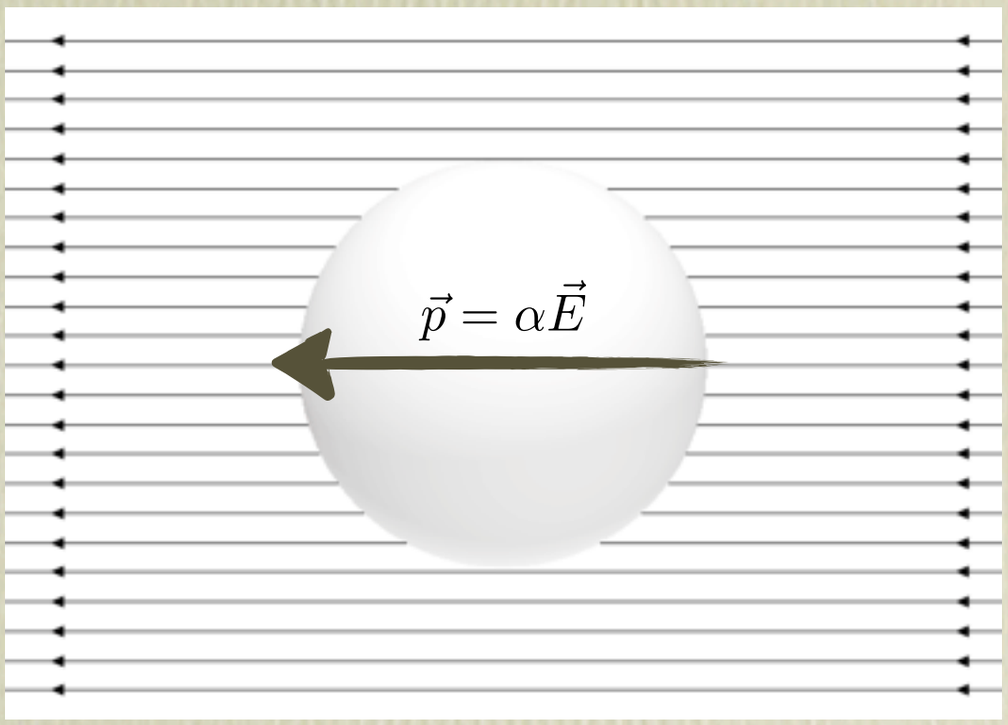
- The hadron mass changes when placed in a electromagnetic field

$$H_{\rm em} = -\boldsymbol{\mu}\cdot\boldsymbol{B} - \frac{1}{2}\alpha\boldsymbol{E}^2 - \frac{1}{2}\beta\boldsymbol{B}^2$$
$$- \frac{1}{2}\gamma_{\rm E1}\boldsymbol{\sigma}\cdot\boldsymbol{E}\times\dot{\boldsymbol{E}} - \frac{1}{2}\gamma_{\rm M1}\boldsymbol{\sigma}\cdot\boldsymbol{B}\times\dot{\boldsymbol{B}}$$
$$+ \gamma_{\rm E2}\sigma_i E_{ij} B_j - \gamma_{\rm M2}\sigma_i B_{ij} E_j$$
$$- \frac{1}{12}\alpha_{\rm E2} E_{ij}^2 - \frac{1}{12}\beta_{\rm M2} B_{ij}^2 + \ldots$$

- μ - the magnetic dipole
- α & β - the electric & magnetic polarizability
- γ - spin polarizabilities

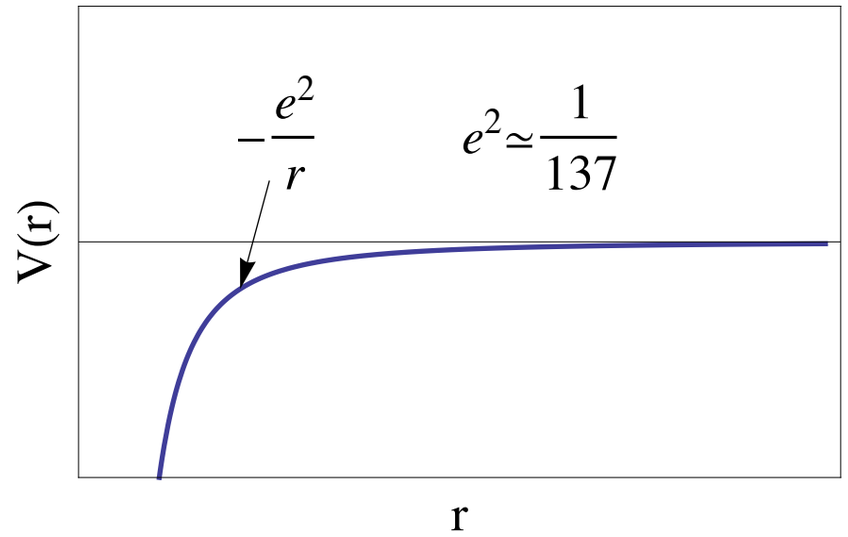
Electric polarizability

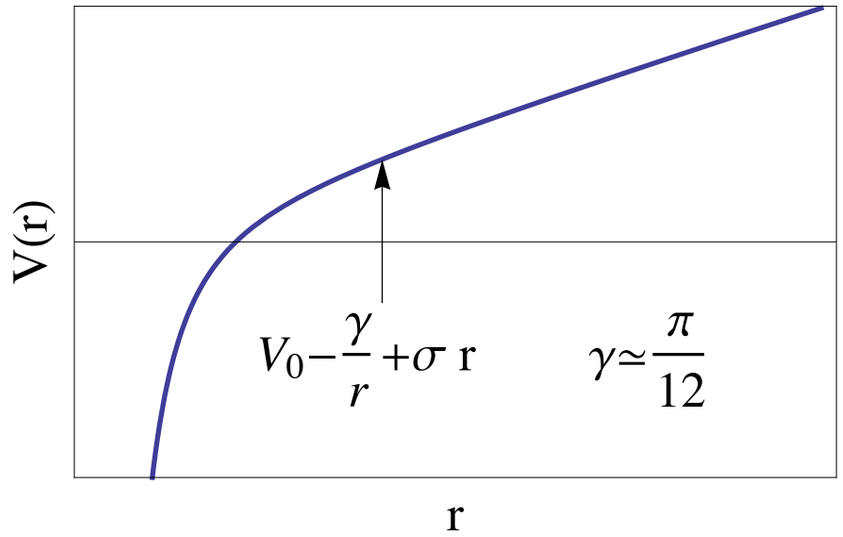
- The polarizability measures the dipole moment induced by the field
- The polarizabilities are measured in Compton scattering experiments for proton and from elastic and quasi-elastic Compton scattering on deuteron, lead, etc. for neutron



$$\Delta E = -\frac{1}{2}\alpha E^2$$

Electric polarizability

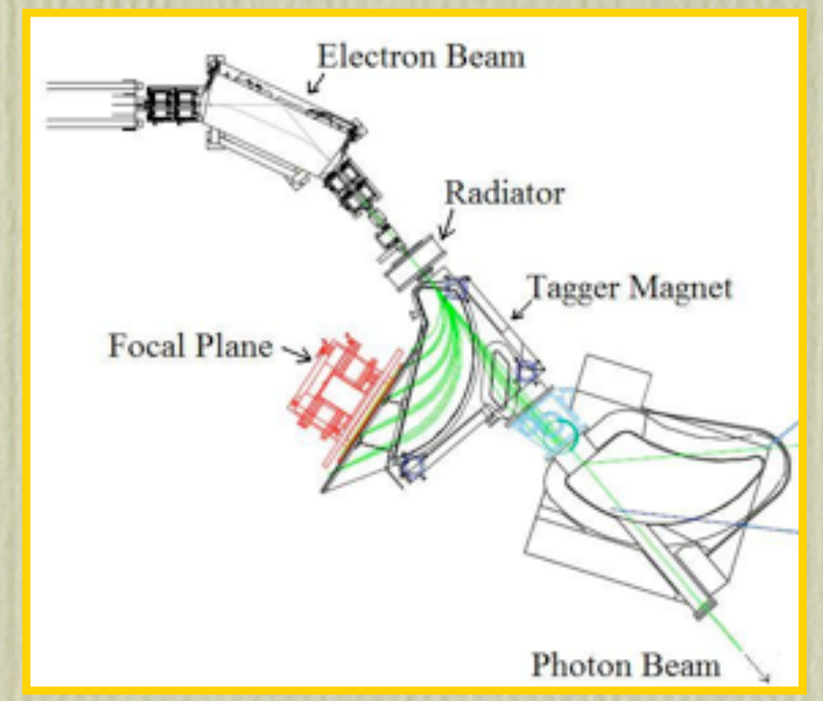


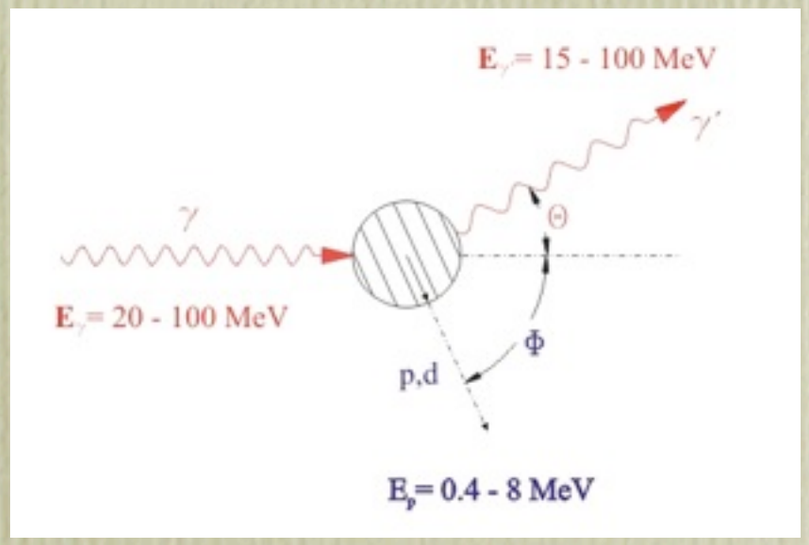


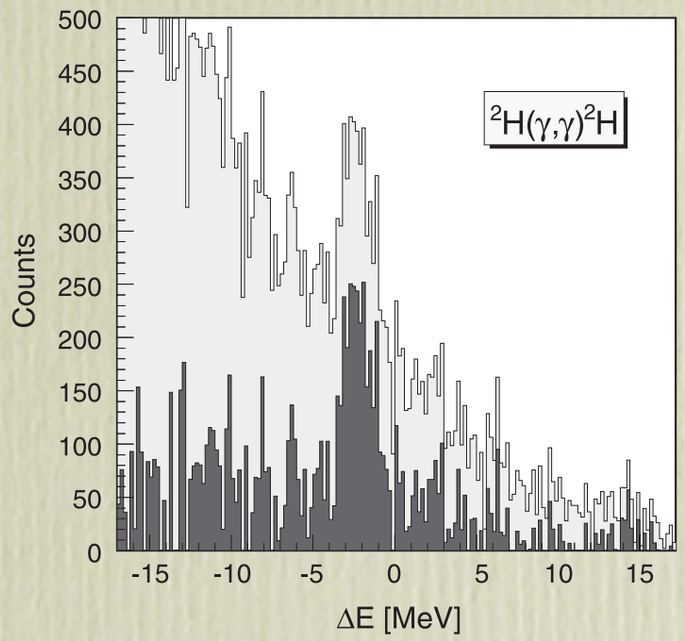
$$E = q\left(\frac{d}{R}\right)^3 \frac{1}{d^2} = \frac{qd}{R^3} \quad \Longrightarrow \quad p = qd = R^3 E \quad \Longrightarrow \quad \alpha = R^3 \sim \mathring{\mathrm{A}}^3$$

$$qE = \left(\frac{d}{R}\right)^3 \frac{\gamma}{d^2} = \frac{\gamma d}{R^3} \Longrightarrow p = qd = \frac{q^2}{\gamma} R^3 E \Longrightarrow \alpha = \frac{q^2}{\gamma} R^3 \sim 10^{-4}\,\mathrm{fm}^3$$

Compton scattering







$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi)^2}\left(\frac{\omega_1}{\omega_2}\right)\left|\alpha\omega_1\omega_2(\hat{\epsilon}_1\cdot\hat{\epsilon}_2^*) + \beta(\hat{\epsilon}_1\times\vec{k}_1)\cdot(\hat{\epsilon}_2^*\times\vec{k}_2)\right|$$

Lattice calculations

- H. Fiebig, W. Wilcox & R. Woloshyn - 1989 - staggered, quenched
- J. Christensen, W. Wilcox, F.X. Lee & L. Zhou - 2004 - clover and wilson, quenched
- M. Engelhardt - 2007 - dynamical Nf=3 staggered, 4-point functions, charged quarks
- W. Detmold, B. Tiburzi & A. Walker-Loud - 2008 - domain wall, clover valence
- A. Alexandru & F.X. Lee - 2008 - clover fermions, quenched
- W. Detmold, B. Tiburzi & A. Walker-Loud - 2009 - anisotropic clover dynamical

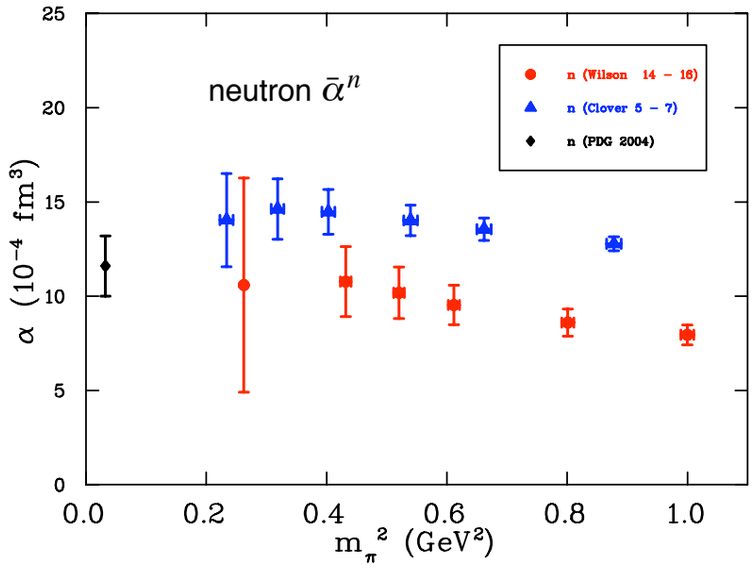
Electric field on the lattice

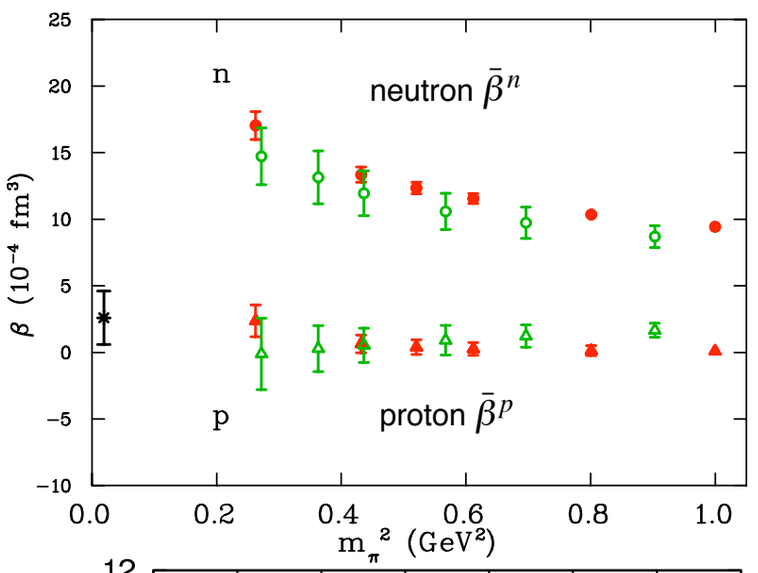
Motivation

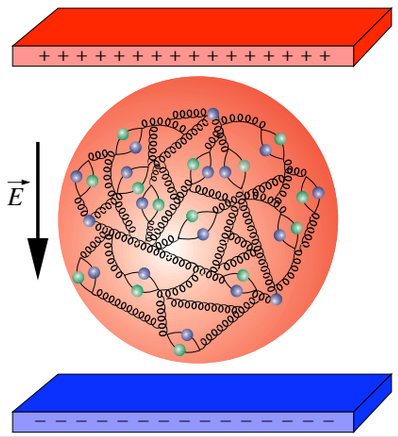
(b) Towards Polarisabilities from First Principles: Lattice QCD

Pioneering: Quenched, chiral fermions

Lee/Zhou/Wilcox/Christensen 2005-06







Ongoing:

spin-polarisabilities,

unquenching,

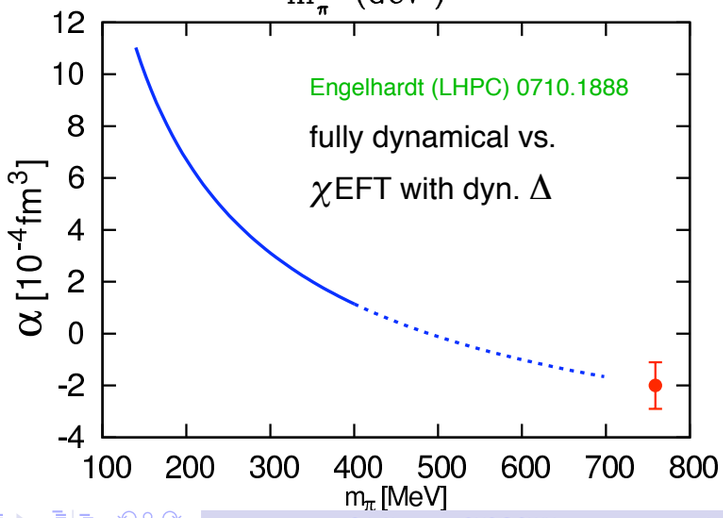
$m_\pi \searrow 200$ MeV,

larger volumes,

more statistics,. . .

Lee/. . . 2005-, LHPC 2006-,

Detmold/Tiburzi/Walker-Loud 2006-



H. W. Griesshammer, Workshop on Soft Photons and Light Nuclei, Seattle, USA, June 2008

Background field method

- Introduce a background electric field

$$D_\mu = \partial_\mu - igG_\mu - iqA_\mu$$

- The U(1) field A_μ is static
- On the lattice this amounts to changing the links

$$U_\mu \rightarrow e^{-iqaA_\mu} U_\mu$$

- The polarizability is extracted from the mass shift

Euclidean formulation

- The lattice formulation is Euclidean

$$x_4 = ix_0 \quad \text{and} \quad A_4 = -iA_0$$

- For a constant electric field in the x direction we can choose

$$A_\mu = (0, +Et, 0, 0) \quad \rightarrow \quad A_\mu = (-iEx_4, 0, 0, 0) \quad \textit{or}$$
$$A_\mu = (-Ex, 0, 0, 0) \quad \rightarrow \quad A_\mu = (0, 0, 0, +iEx_1)$$

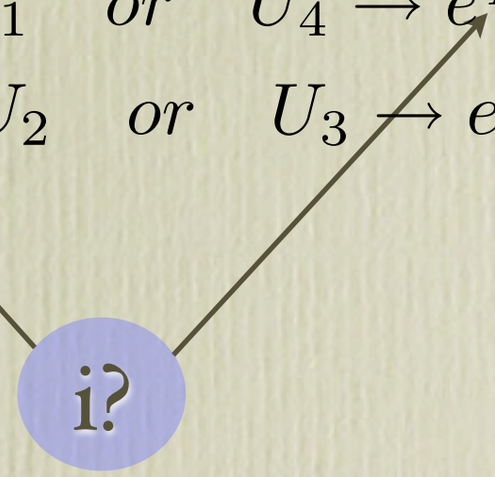
Euclidean formulation

- For a constant magnetic field in the x direction we can choose

$$A_\mu = (0, 0, Bz, 0) \quad \rightarrow \quad A_\mu = (0, Bz, 0, 0) \quad or$$
$$A_\mu = (0, 0, 0, -By) \quad \rightarrow \quad A_\mu = (0, 0, -By, 0)$$

- On the lattice we have

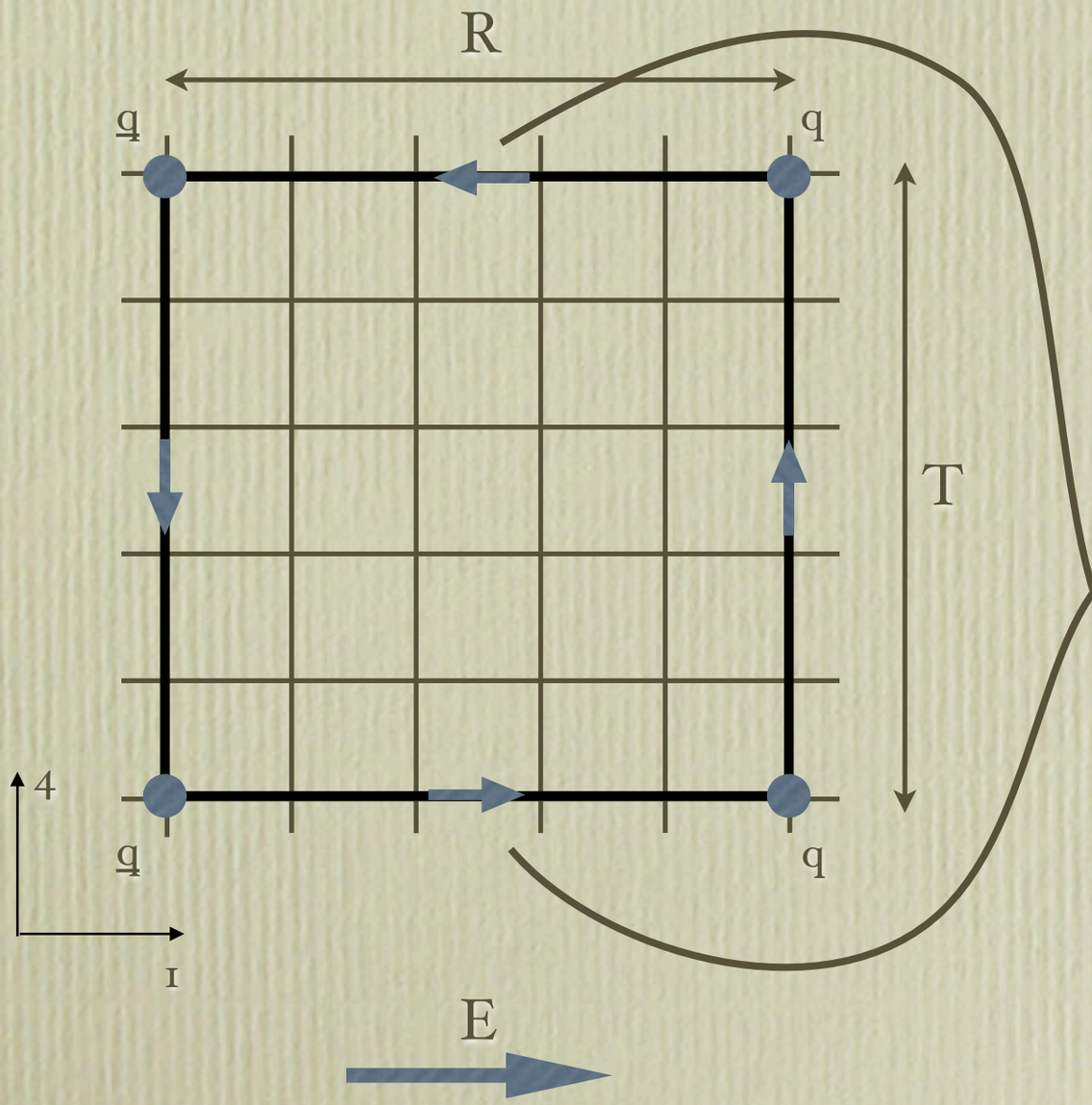
$$E_x: \quad U_1 \quad \rightarrow \quad e^{-qaEx_4} U_1 \quad or \quad U_4 \rightarrow e^{qaEx_1} U_4$$
$$B_x: \quad U_2 \quad \rightarrow \quad e^{-iqaBx_3} U_2 \quad or \quad U_3 \rightarrow e^{iqaBx_2} U_3$$



The phase factor

- The magnetic field is introduced in a similar manner in Euclidean and Minkowski formulations
- I will focus on the electric field
- I will argue that to introduce an electric field on the lattice we need to use a real factor
  - Wilson loop example
  - Charged scalar field
- I will also argue that we should use the exponential rather than linear form

Wilson loop in electric field



$$\langle W \rangle \sim e^{-V(R)T}$$

$$V(R) \to V(R) - Ep = V(R) - qER$$

$$U_1 \to e^{-qaEt} U_1 \xrightarrow{\text{R/a times}} e^{-qERt}$$

$$U_1^\dagger \to e^{qaE(t+T)} U_1^\dagger \xrightarrow{\text{R/a times}} e^{qER(t+T)}$$

$$\langle W \rangle \to \langle W \rangle \, e^{-qERt} e^{qER(t+T)} \sim e^{-V(R)T+qERT}$$

$$\Delta m = -qER$$

$$U_1 \to e^{-iqaEt} U_1 \Rightarrow \Delta m = -iqER$$

Charged scalar field

- Euclidean action: path integral of imaginary time hopping

$$\left\langle \phi_{t+1} \left| e^{-a_t \hat{H}} \right| \phi_t \right\rangle = e^{-a_t \sum_n \mathcal{L}_E(n,t)}$$

- Real time Lagrangian

$$L = \frac{1}{2} \int d^3x \left[(\partial_\mu - iqA_\mu)\phi^* (\partial^\mu + iqA^\mu)\phi - m^2 \phi^* \phi \right]$$

- Hamiltonian from Legendre transform

$$H = \int d^3x \left[\pi^* \pi + iqA^0(\pi^* \phi^* - \phi\pi) + (\vec{\nabla} - iq\vec{A})\phi^* (\vec{\nabla} + iq\vec{A})\phi + m^2 \phi^* \phi \right]$$

- Discretization (take A_0 to depend only on spatial components)

$$\hat{H} = \sum_n \left[\hat{\pi}_n^* \hat{\pi}_n + iqA_0(\hat{\pi}_n^* \hat{\phi}_n^* - \hat{\phi}_n \hat{\pi}_n) + (\tilde{\nabla}\hat{\phi}_n)^* \tilde{\nabla}\hat{\phi}_n + m^2 \hat{\phi}_n^* \hat{\phi}_n \right]$$

- Euclidean action

$$\mathcal{L}_E(n,t) = (\tilde{\nabla}\phi_t)^* \tilde{\nabla}\phi_t + m^2 \phi_t^* \phi_t + \frac{1}{2a_t^2} \left(\phi_{t+1} - e^{a_t q A_0} \phi_t \right) \left(\phi_{t+1}^* - e^{-a_t q A_0} \phi_t^* \right)$$

Linear vs exponential

- Linearize the phase factor $e^{-iqaA_\mu} \to 1 - iqaA_\mu$
  - it resembles the continuum form
  - it couples the gauge field to the conserved current
    - it removes tad-pole like terms due to quadratic terms in the gauge field
- The linear form differs from the exponential at the second order in A_μ
- This changes the value of polarizability
- The gauge symmetry in the background field is lost

$$U_1: \quad \langle W \rangle \sim e^{-V(R)T+qERT} e^{\frac{1}{2}aq^2E^2R\left[(t+T)^2+t^2\right]}$$
$$U_4: \quad \langle W \rangle \sim e^{-V(R)T+qERT} e^{\frac{1}{2}aq^2E^2T\left[(x+R)^2+x^2\right]}$$

Extracting polarizability

- We showed that to introduce an electric field on the lattice we need to use a real phase factor in the exponential form

$$U_1 \to U_1 e^{-aqEt} \Rightarrow \Delta m = -\frac{1}{2}\alpha E^2$$

- The imaginary phase factor can also be used if we remember to flip the sign

$$U_1 \to U_1 e^{-iaqEt} \Rightarrow \Delta m = +\frac{1}{2}\alpha E^2$$

- Magnetic field is introduced using an imaginary phase factor

$$U_2 \to U_2 e^{-iaqBx_3} \Rightarrow \Delta m = -\frac{1}{2}\beta B^2$$

Numerical results

Lattice parameters

- 24^4 quenched ensemble - Wilson action $\beta=6.0$
- Lattice spacing a=0.093 fm -- 300 configurations
- Clover fermions: lowest pion mass 500MeV
- Electric field ~ 10^{23} V/m - $\eta=a^2qE$ = 0.00576 -- real
- We measure the mass shift by looking at the neutron correlator ratio

$$\frac{\langle G_E(t)\rangle}{\langle G_0(t)\rangle} \sim e^{-\Delta m t}$$

Extracting the mass shift

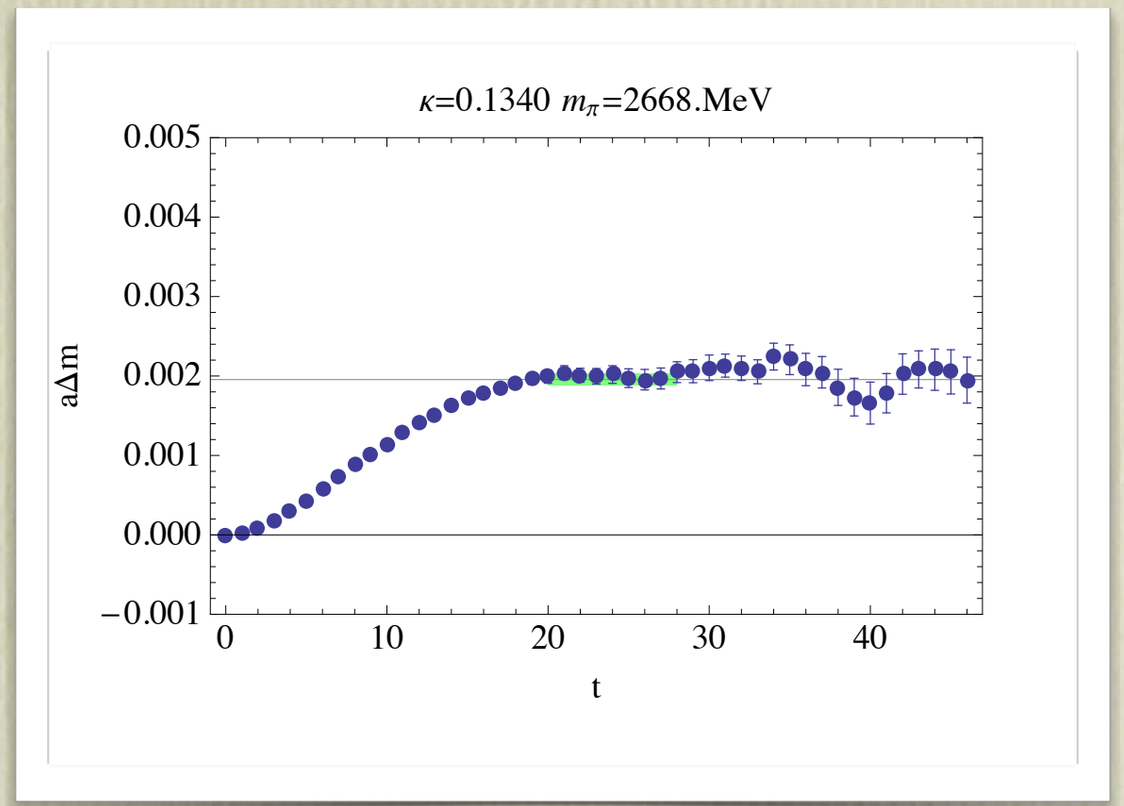
- We measure the ratio of the correlators

$$\frac{G_E(t)}{G_0(t)} \xrightarrow{t \to \infty} \frac{A' e^{-m' t}}{A e^{-mt}} \propto e^{-\delta m t}$$

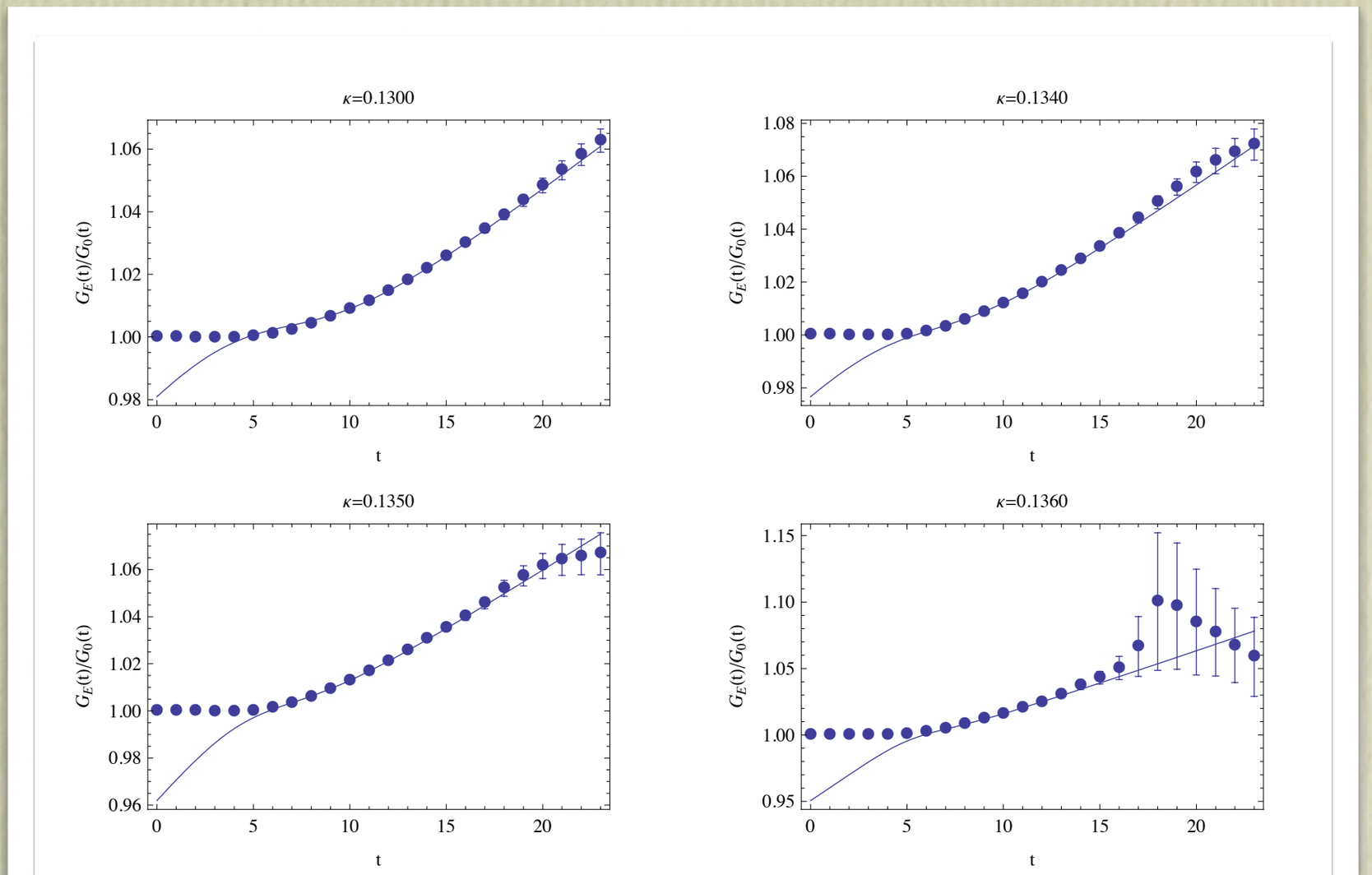
- In the large time limit this ratio becomes a simple exponential

- We fit the decaying exponential in this region and extract the mass shift from the exponent

- Look at the effective mass to determine the fitting range

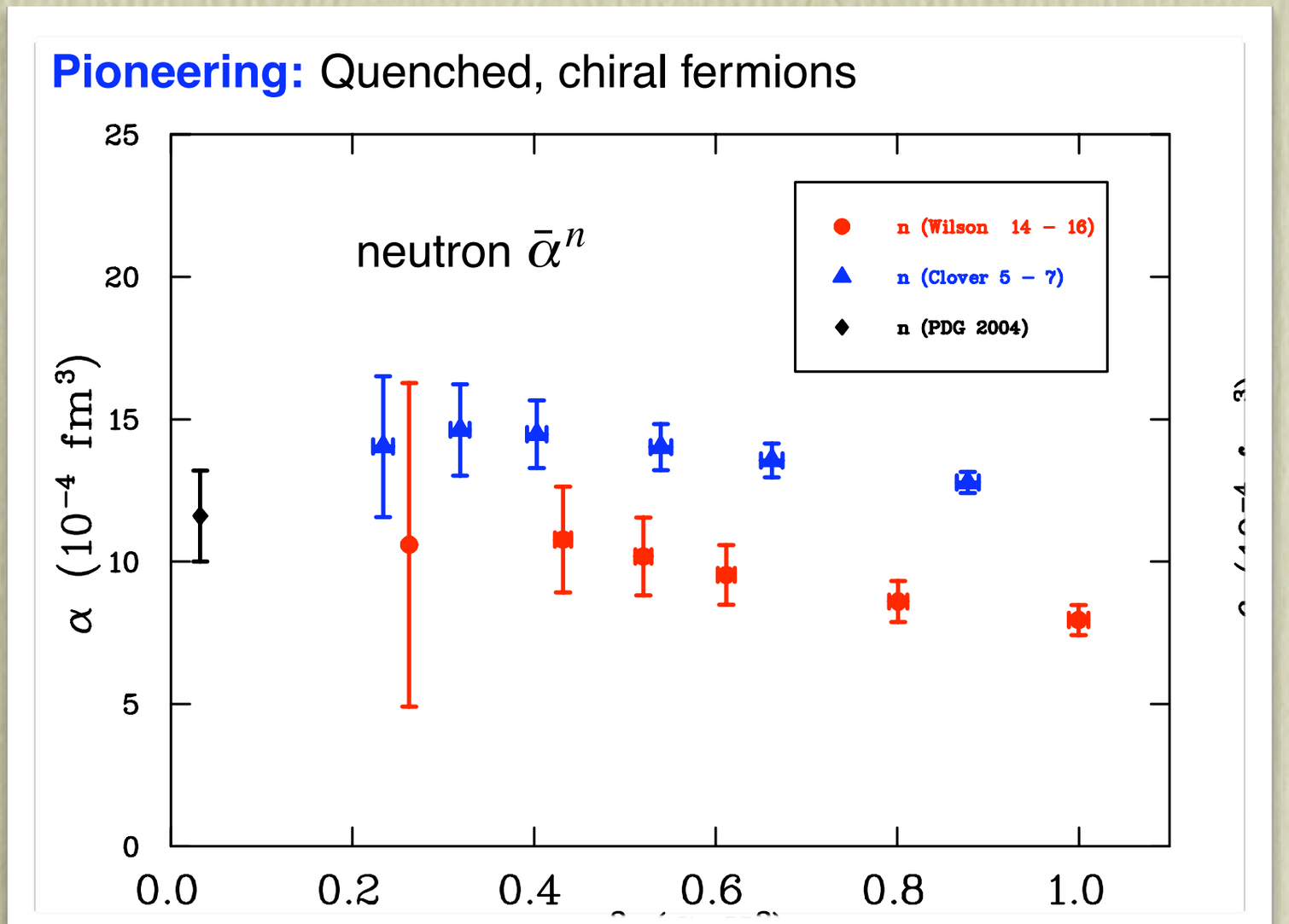


Correlator - real phase factor



The two exponentials fit the data well

Neutron polarizability

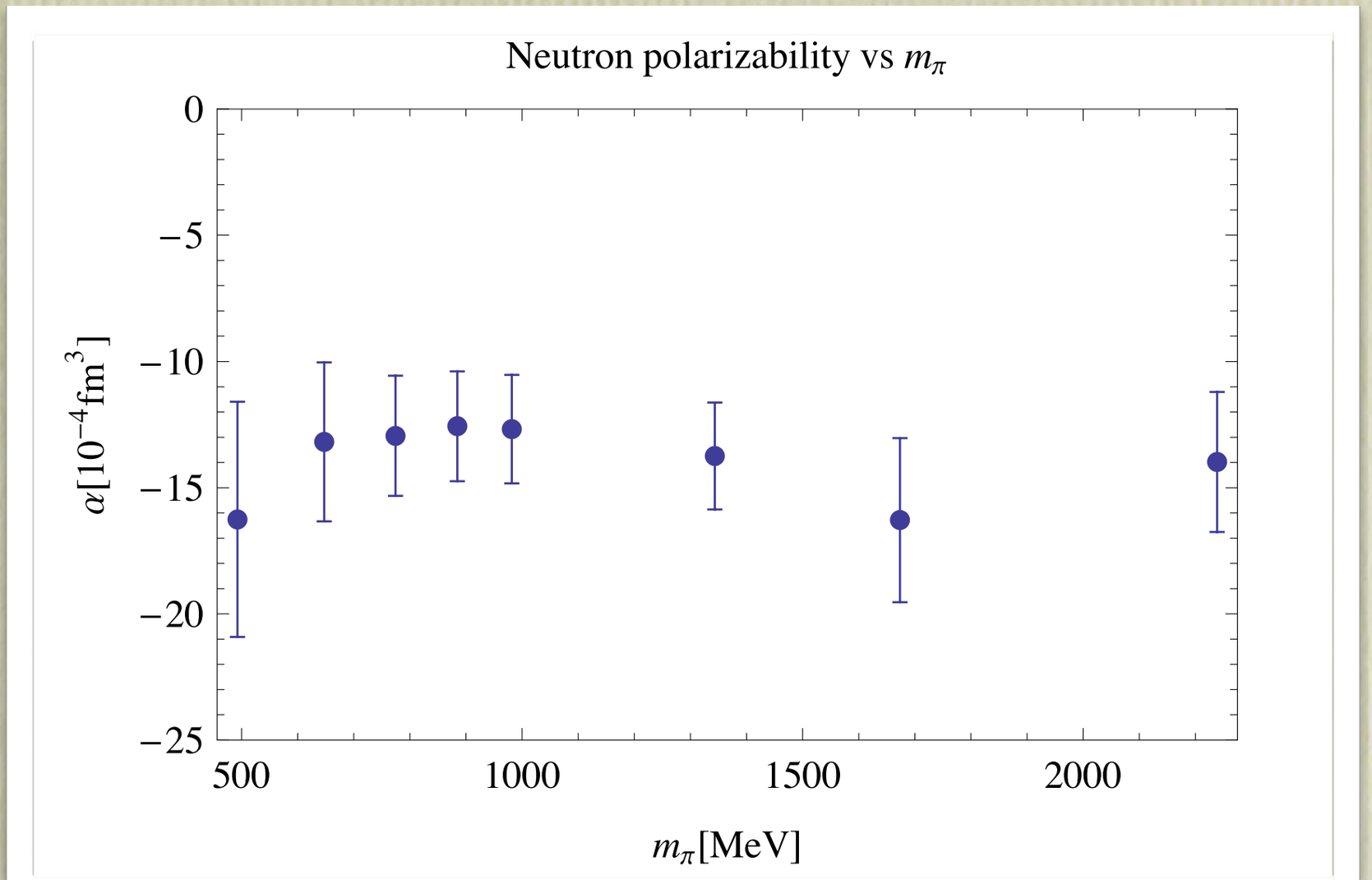


$$U_1 \to U_1 e^{-aqEt} \Rightarrow \Delta m = -\frac{1}{2}\alpha E^2$$

$$U_1 \to U_1 e^{-iaqEt} \Rightarrow \Delta m = +\frac{1}{2}\alpha E^2$$

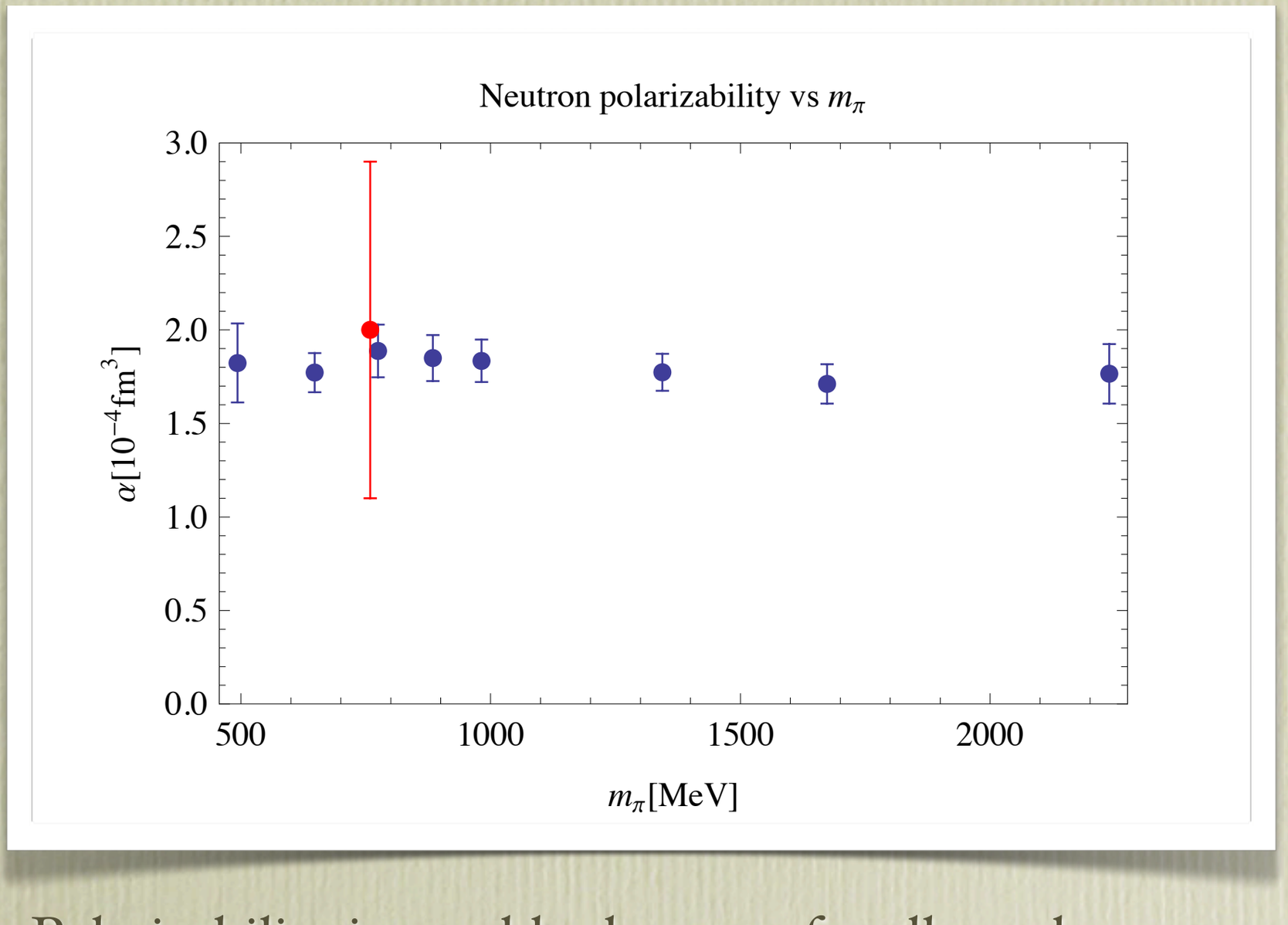
"Ancient history"

Neutron polarizability



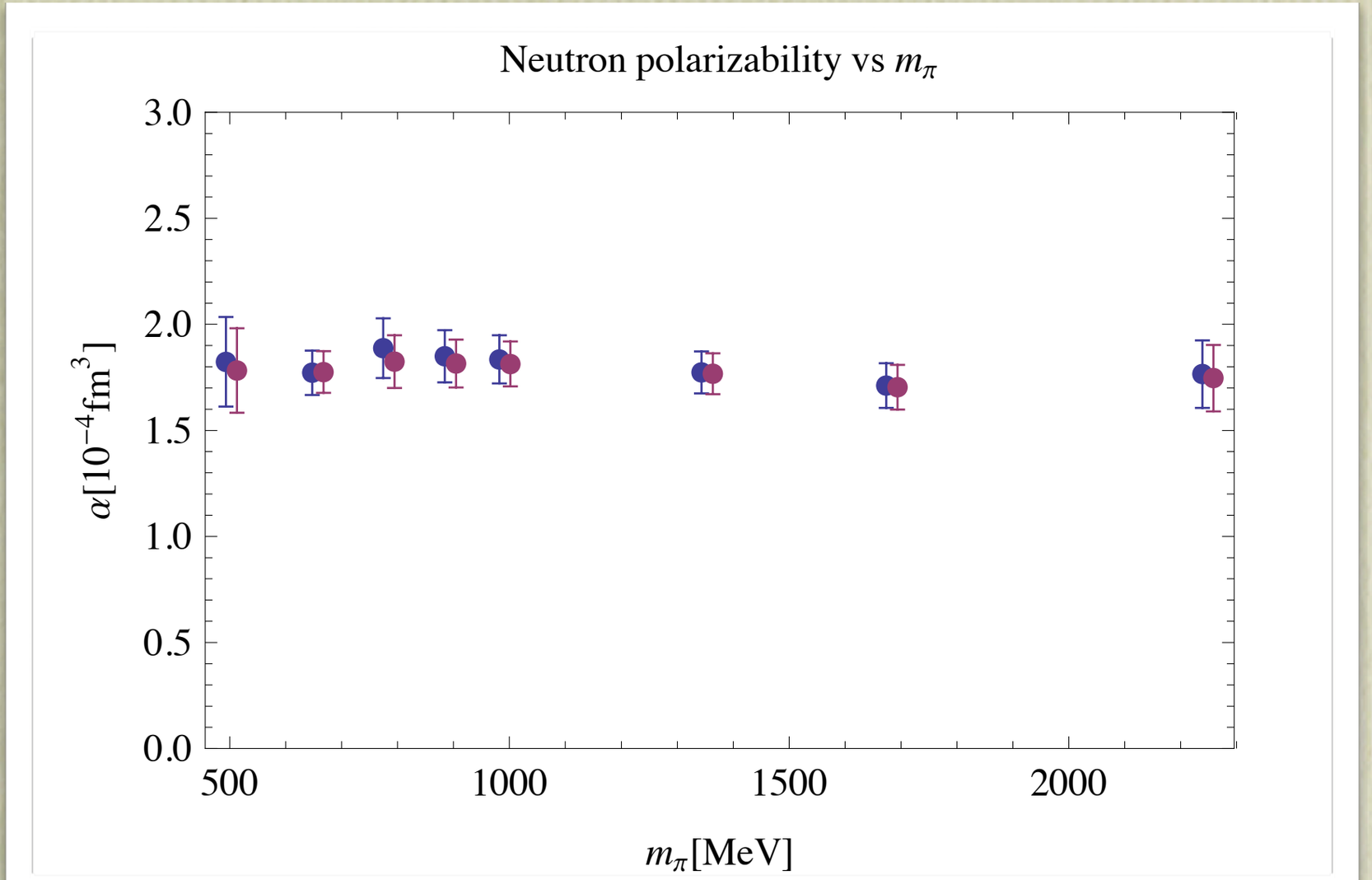
$$e^{-iqaA_\mu} \longrightarrow 1 - iqaA_\mu$$

Neutron polarizability



Polarizability is roughly the same for all quark masses
Our results compare well with previous results (Engelhardt)

Real vs imaginary



Polarizabilities measured either the real or imaginary phase are the same if you take care of the sign

Neutron electric polarizability

Neutron Polarizability Experiments

Alexandrov (Dubna – 1986)
Koester (Munich – 1986)
Schmiedmayer (Vienna/Harwell – 1986)
Koester (Munich – 1988)
Schmiedmayer (Vienna/ORNL – 1991)
Koester (Munich – 1995)
Enik (Dubna – 1997)

$\alpha_n = 12.6 \pm 1.5(\text{stat}) \pm 2.0(\text{syst})$

n scattering

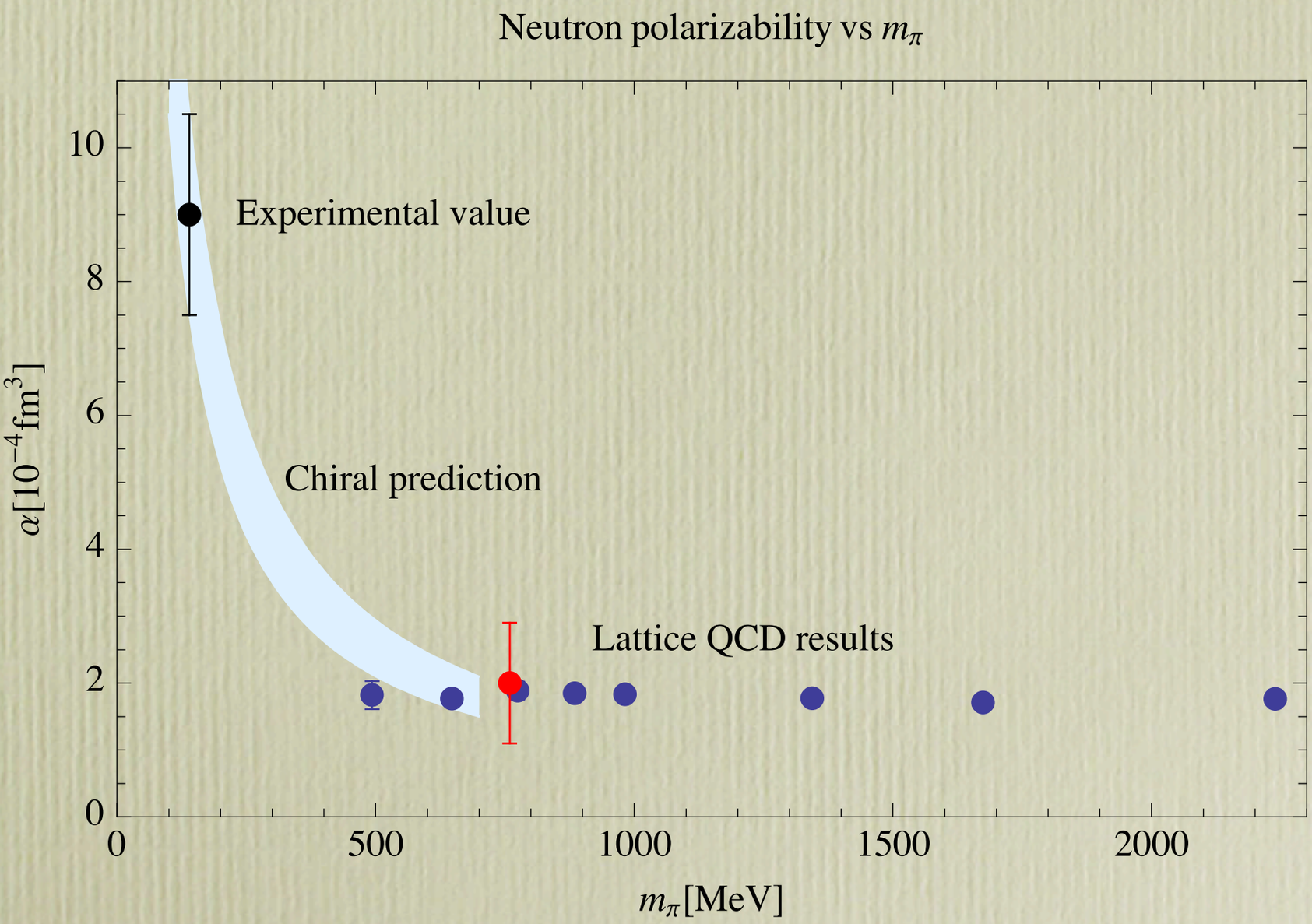
Rose (Gottingen/Mainz – 1990)
Kolb (SAL – 2000)
Kossert (Mainz – 2003)

$$\alpha_n = 12.5 \pm 1.8(\text{stat})\ ^{+1.1}_{-0.6}(\text{syst}) \pm 1.1(\text{model})$$

$$\beta_n = 2.7 \mp 1.8(\text{stat})\ ^{+0.6}_{-1.1}(\text{syst}) \mp 1.1(\text{model})$$

$D(\gamma,\gamma' n)p$

Motivation

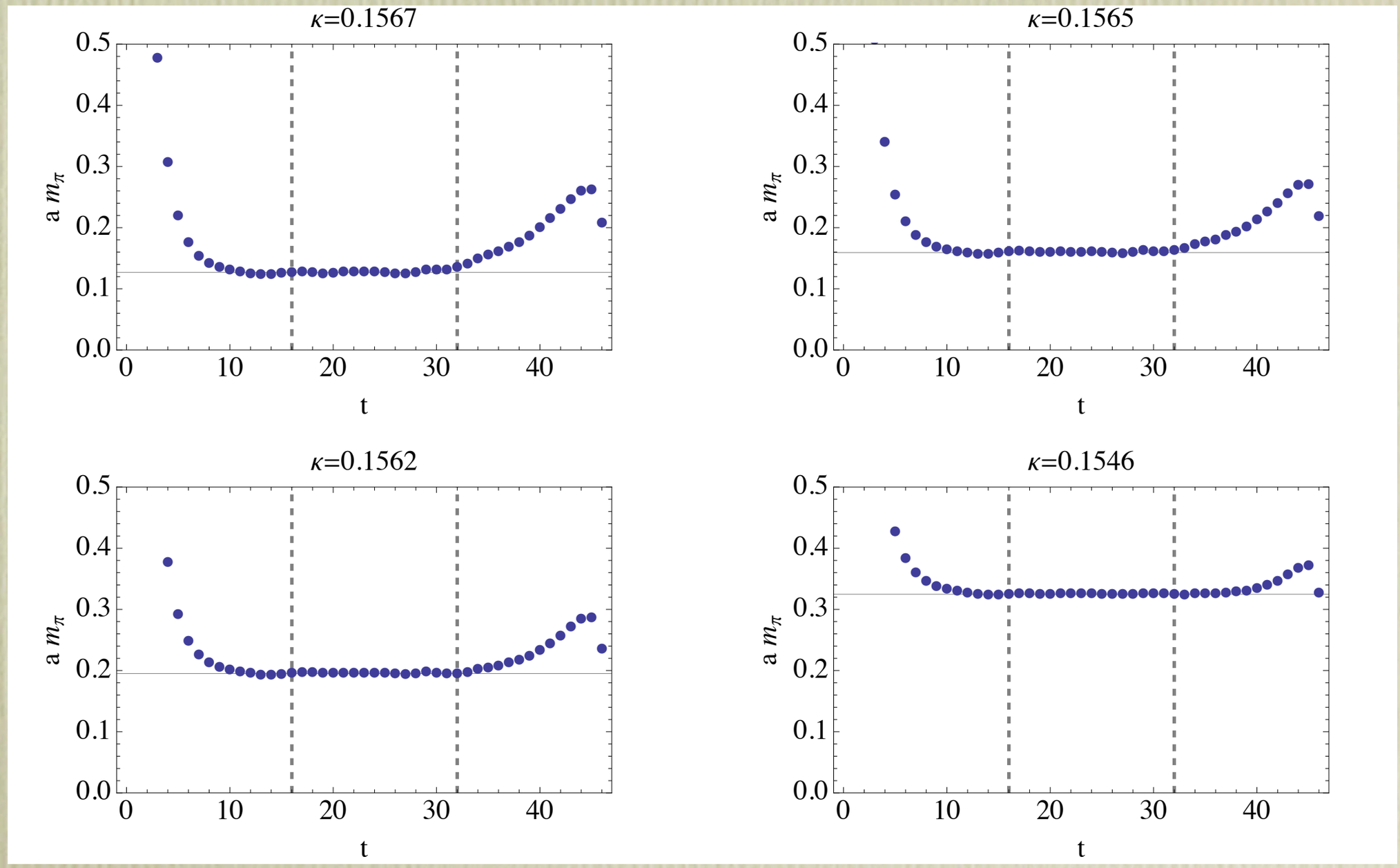


Beane et all - 2003*
Alexandru and Lee - 2008
Engelhardt - 2007*
Detmold et all - 2006

Lattice parameters

- 24^3 x 48 quenched ensemble - Wilson action β=6.0
- Lattice spacing a=0.093 fm -- 1,000 configurations
- Clover fermions: lowest pion mass 500MeV
- Wilson fermions: lowest pion mass 320MeV
- Electric field ~ 10^{23} V/m - $\eta = a^2 qE$ = 0.00576 -- imaginary

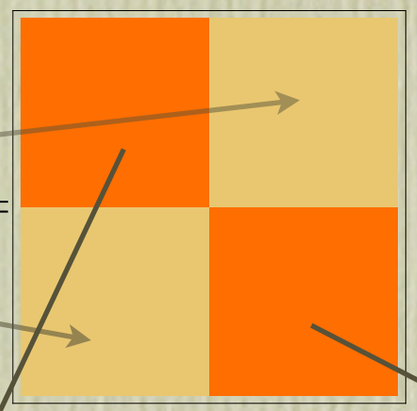
Fitting strategy



Fitting strategy

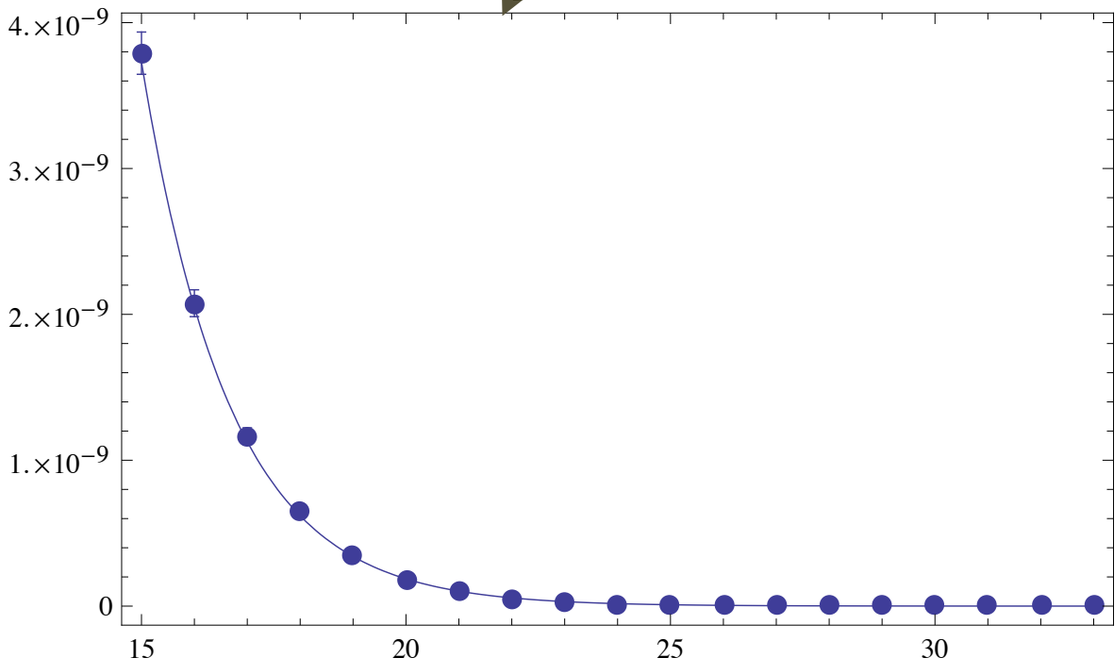
cross correlation

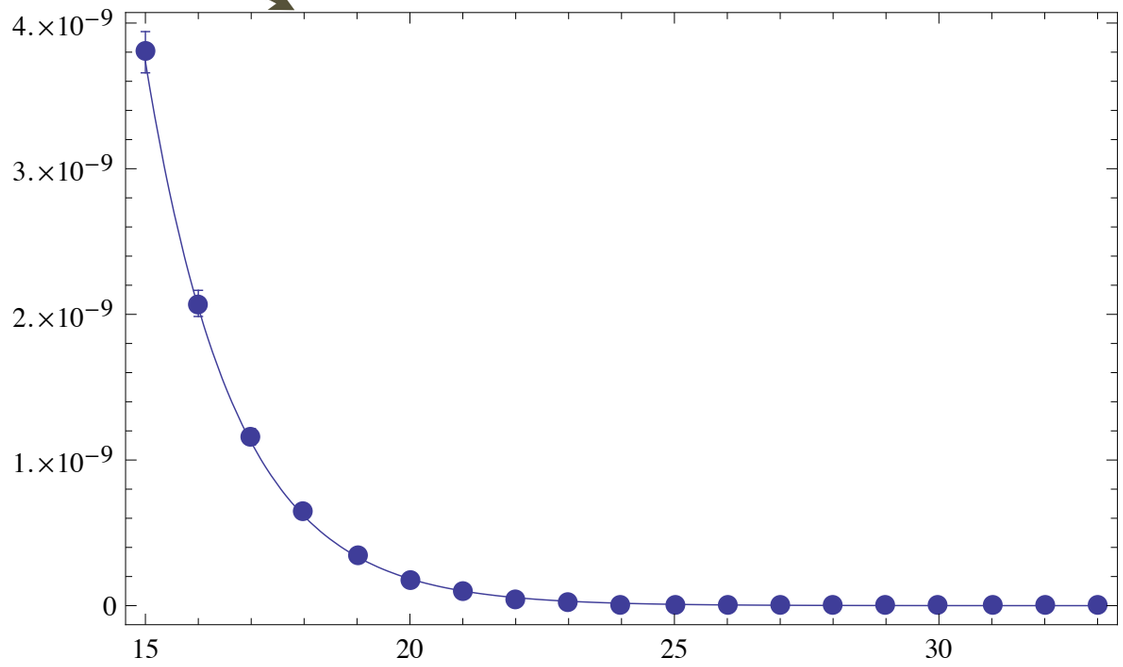
$C_{ij} =$



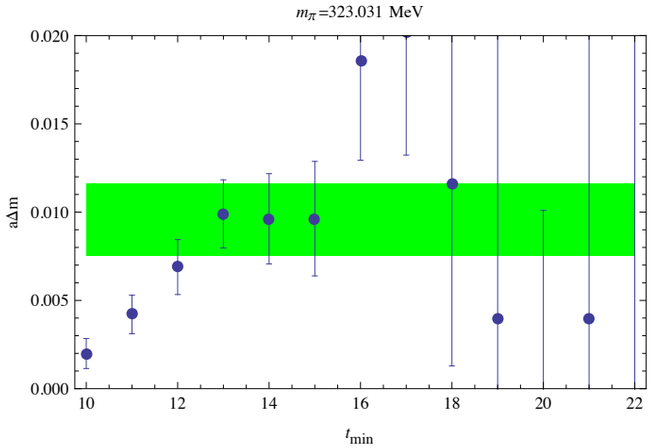
$$\chi^2 = \sum_{i,j} (f(t_i) - y_i) C_{ij}^{-1} (f(t_j) - y_j)$$

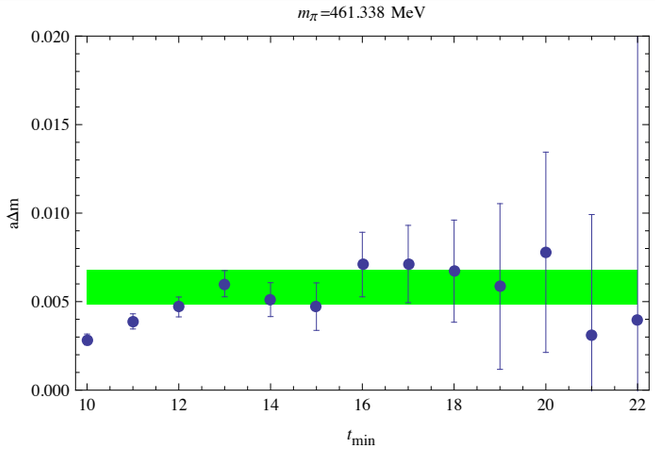
$$f(t) = Ae^{-mt} + (A + \delta A)e^{-(m+\delta m)t}$$

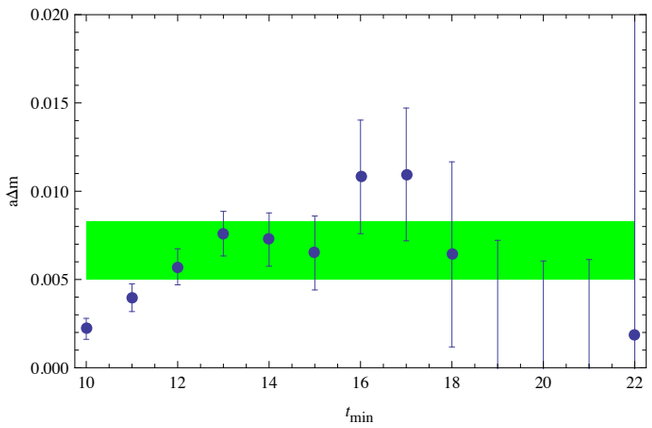


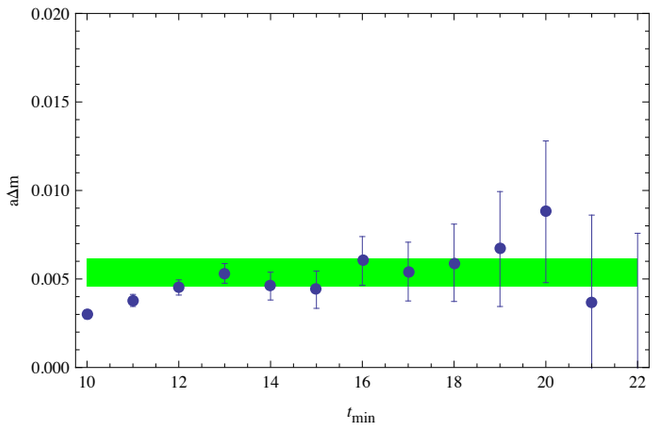


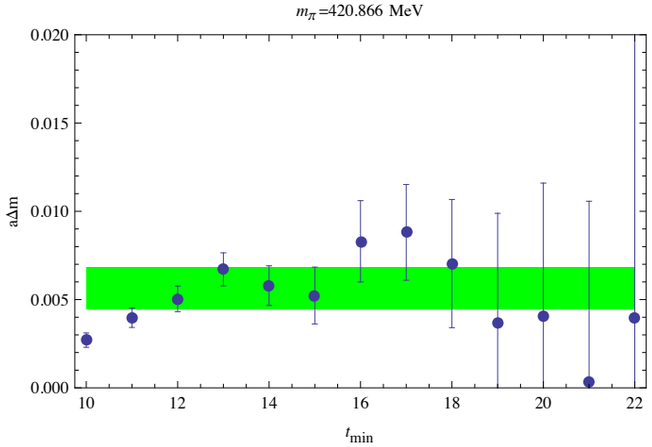
Fitting methods comparison

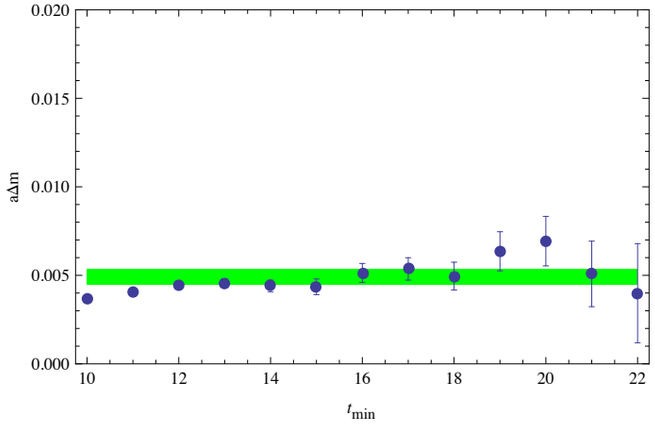




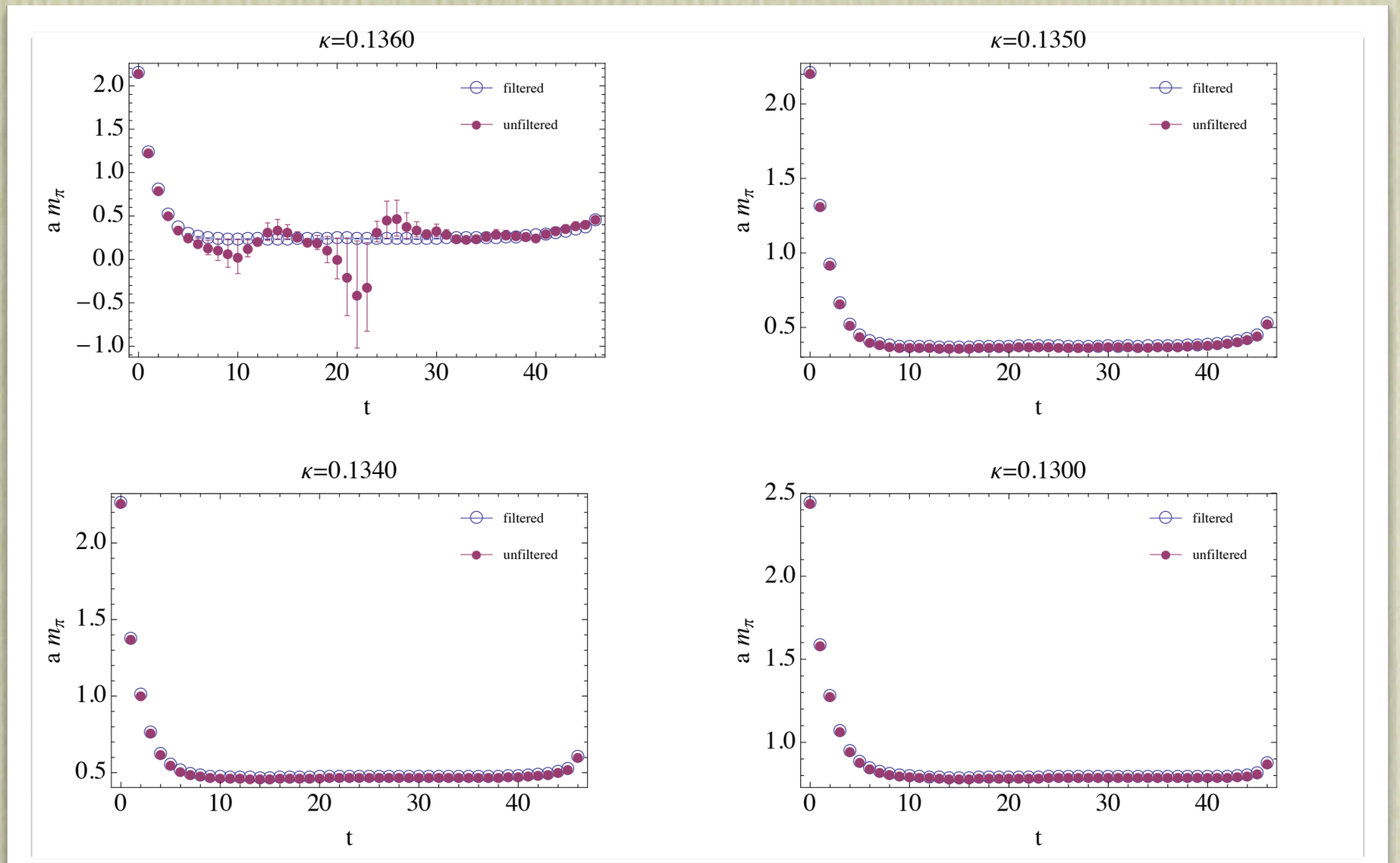




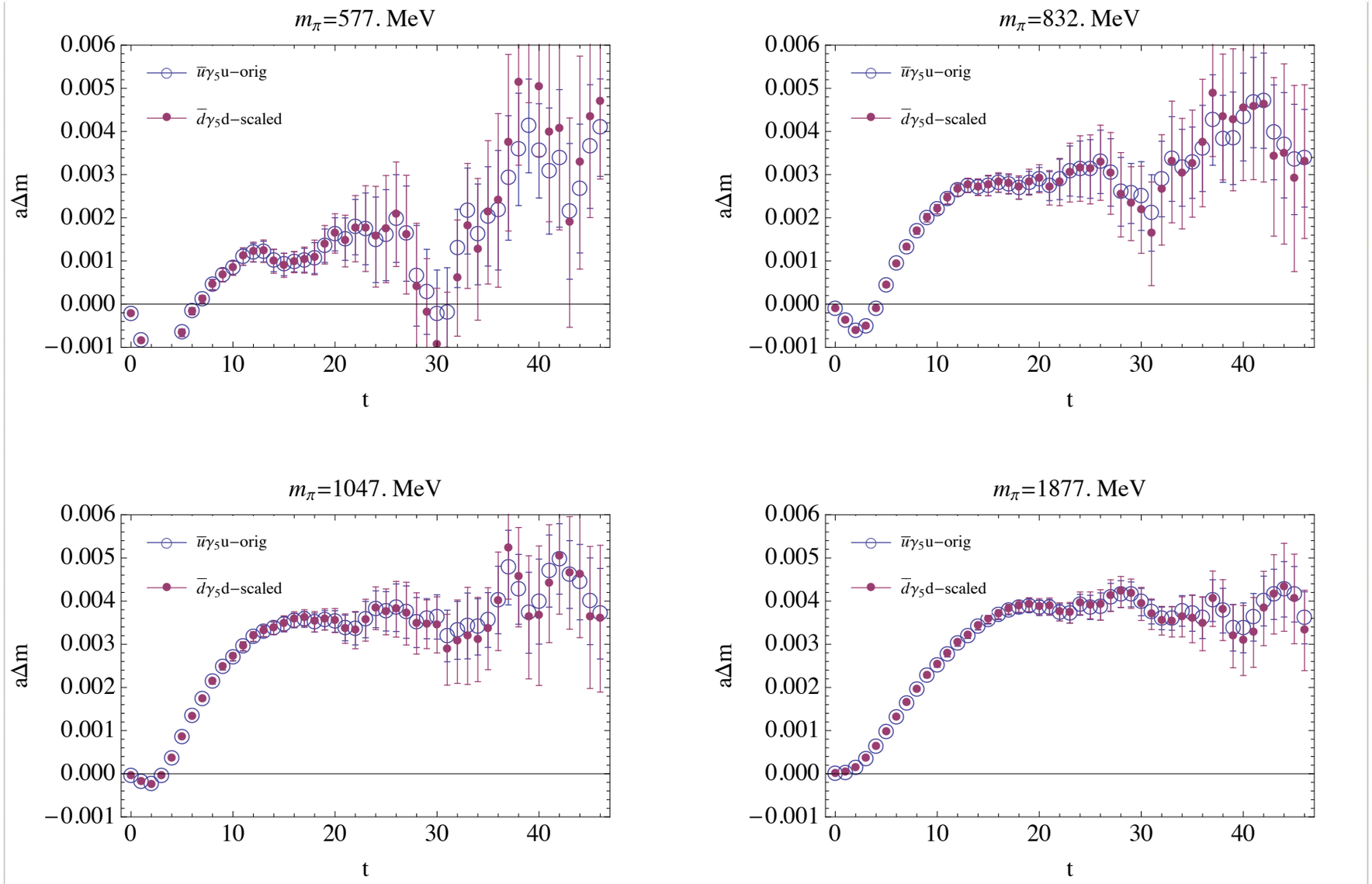




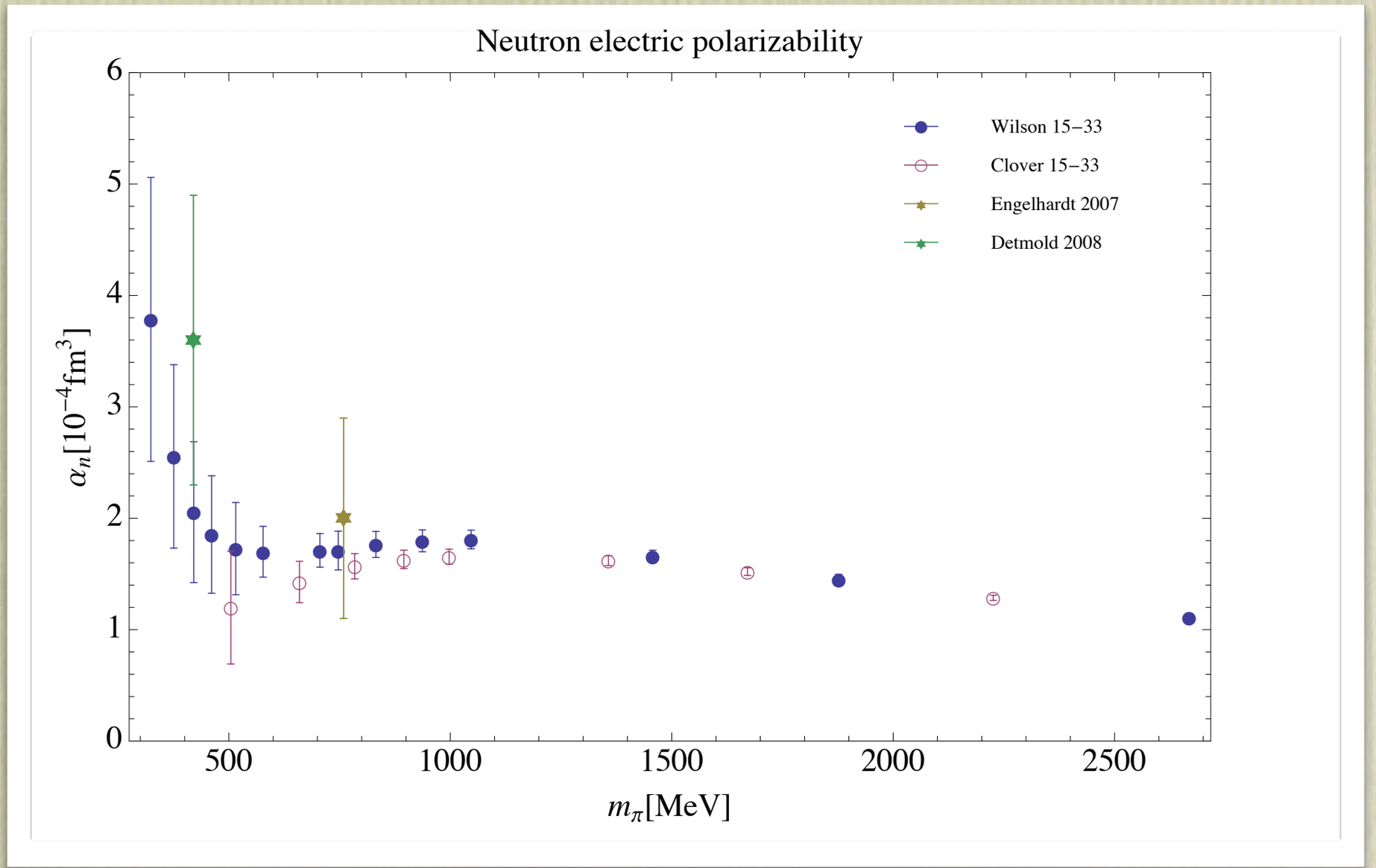
Exceptional configurations



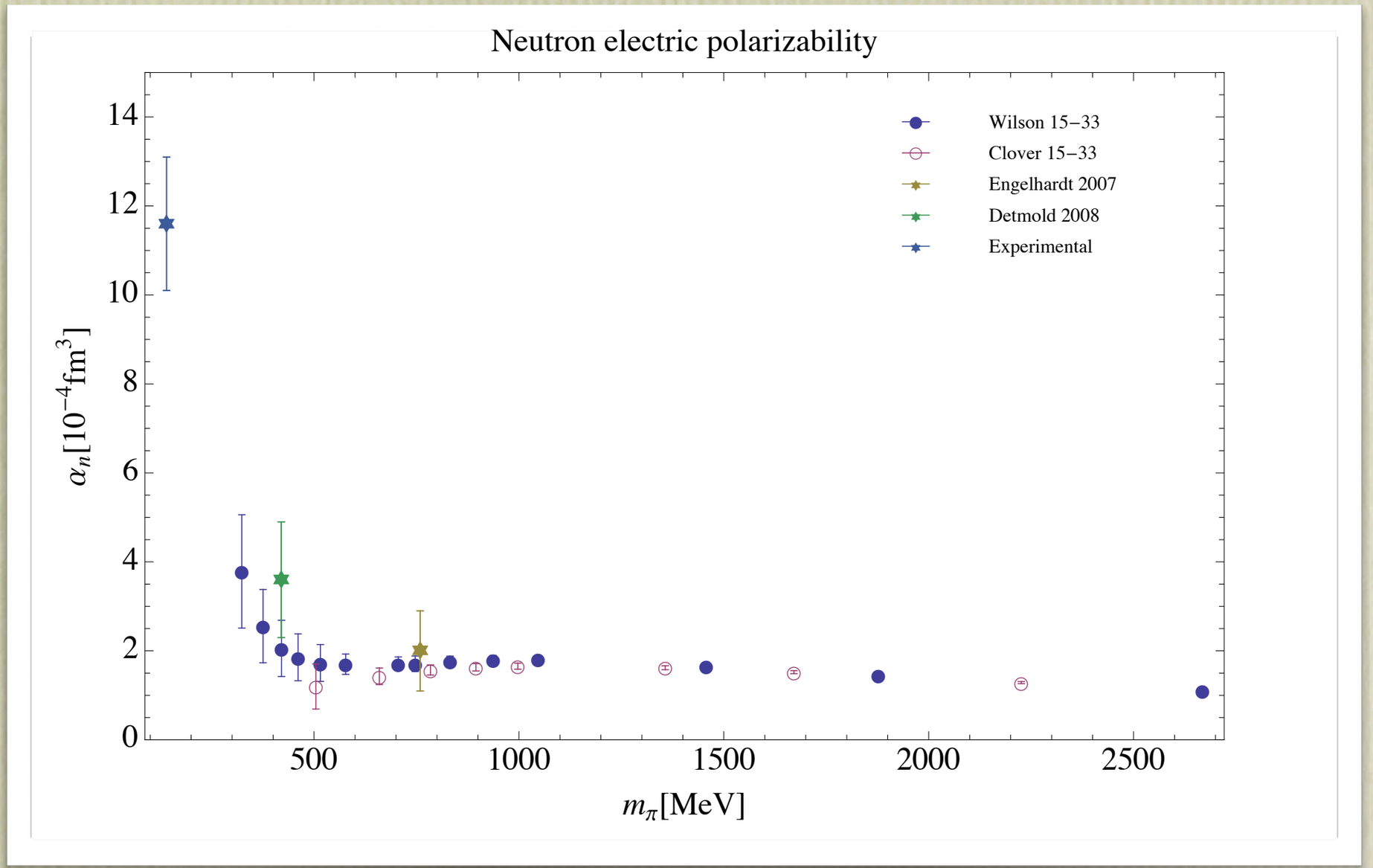
Field strength



Neutron polarizability



Neutron electric polarizability

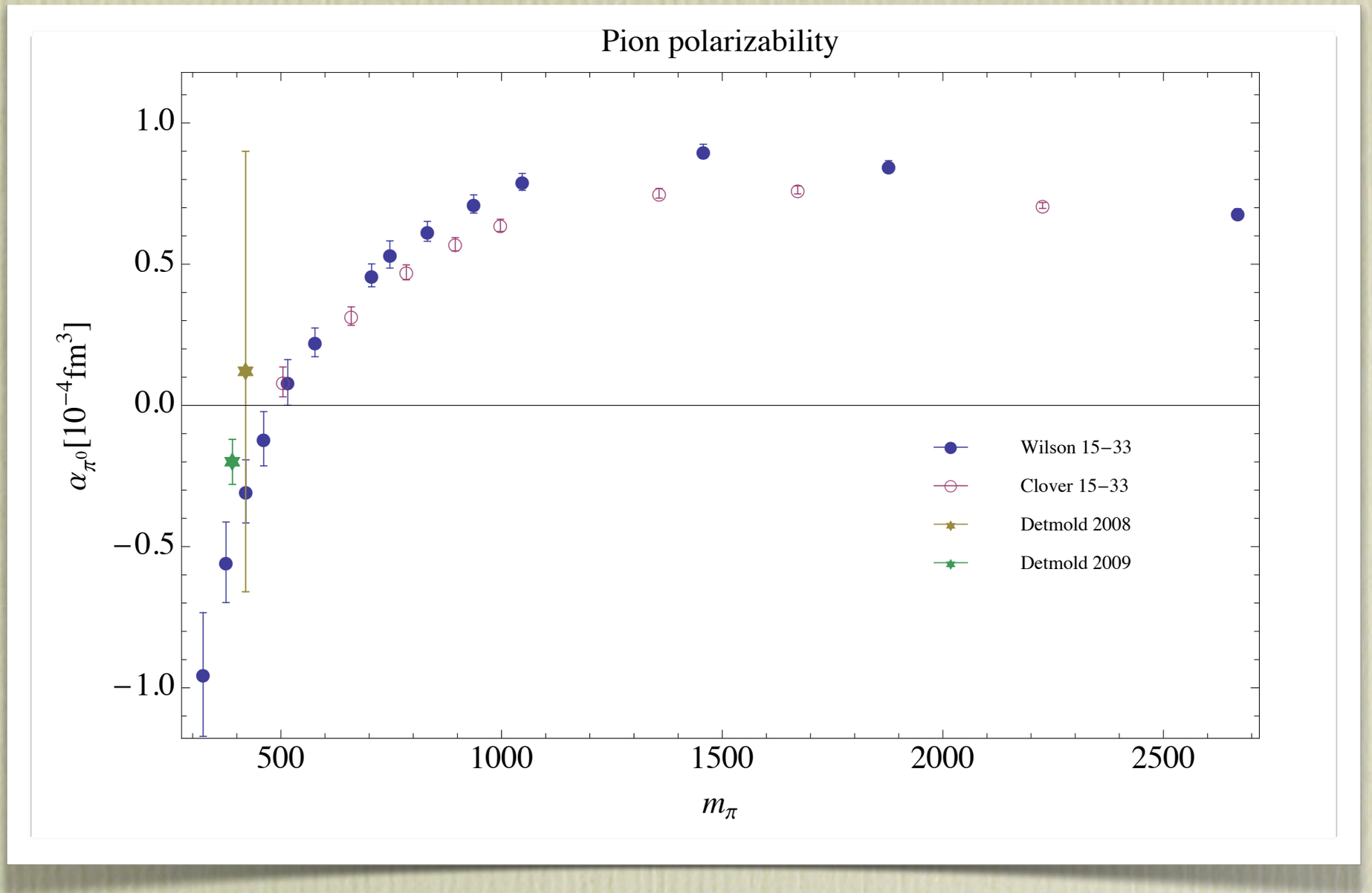


C. Amsler et al. (Particle Data Group), PL B667, 1 (2008)

Pion electric polarizability

- finite volume corrections -

Pion polarizability



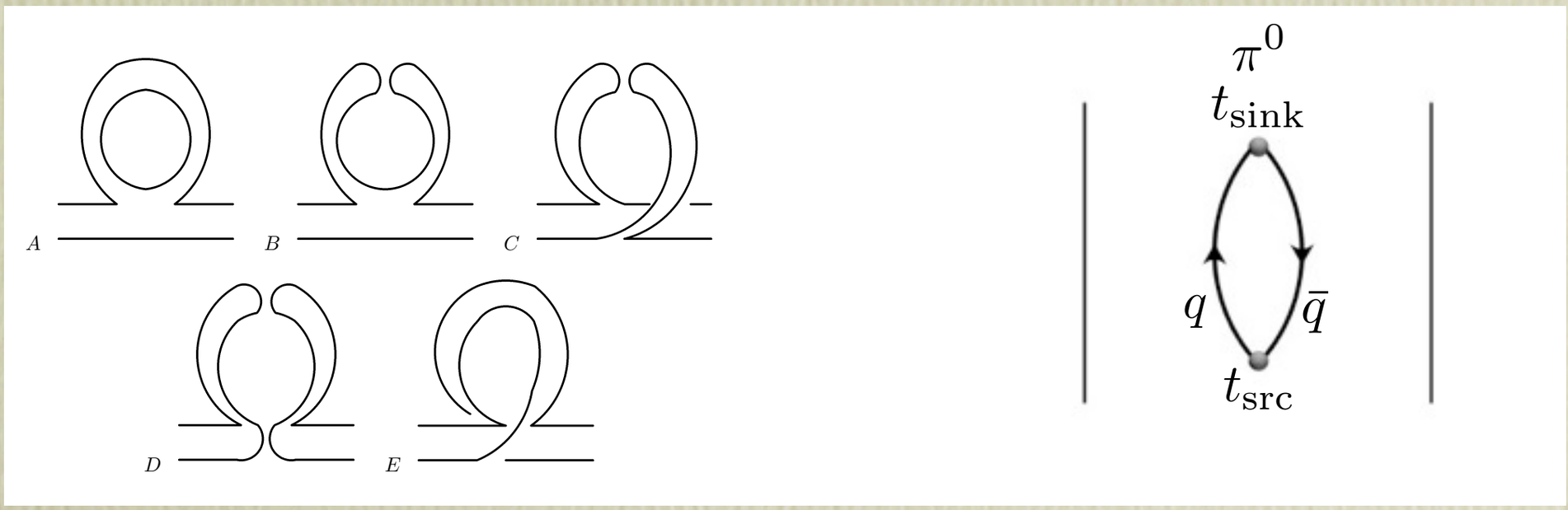
“Neutral Pion”

- The physical π^0 correlator has disconnected contributions in the presence of a background field
- The disconnected diagrams cancel only in the isospin limit -- the electric field breaks isospin symmetry
- Our calculation doesn’t include disconnected contributions
- The particle we study is more like $\bar{d}u$ when u and d have the same charge
- In this version of QCD the pions are all uncharged and χPT predicts a flat behavior (to leading order)

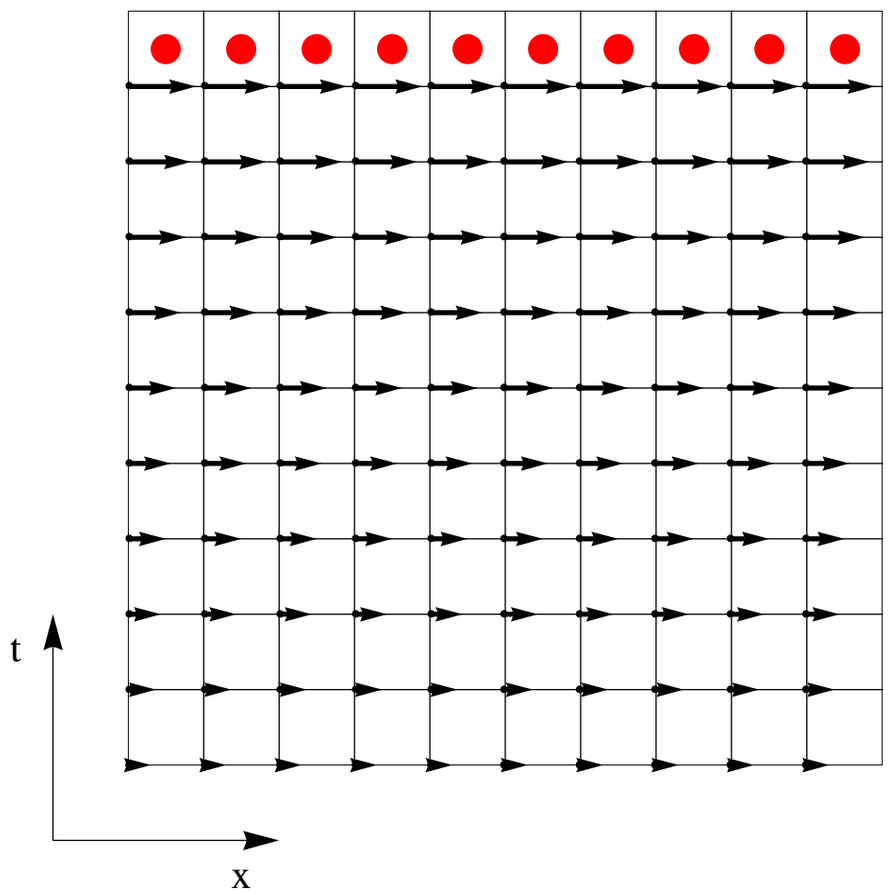
χPT expectations

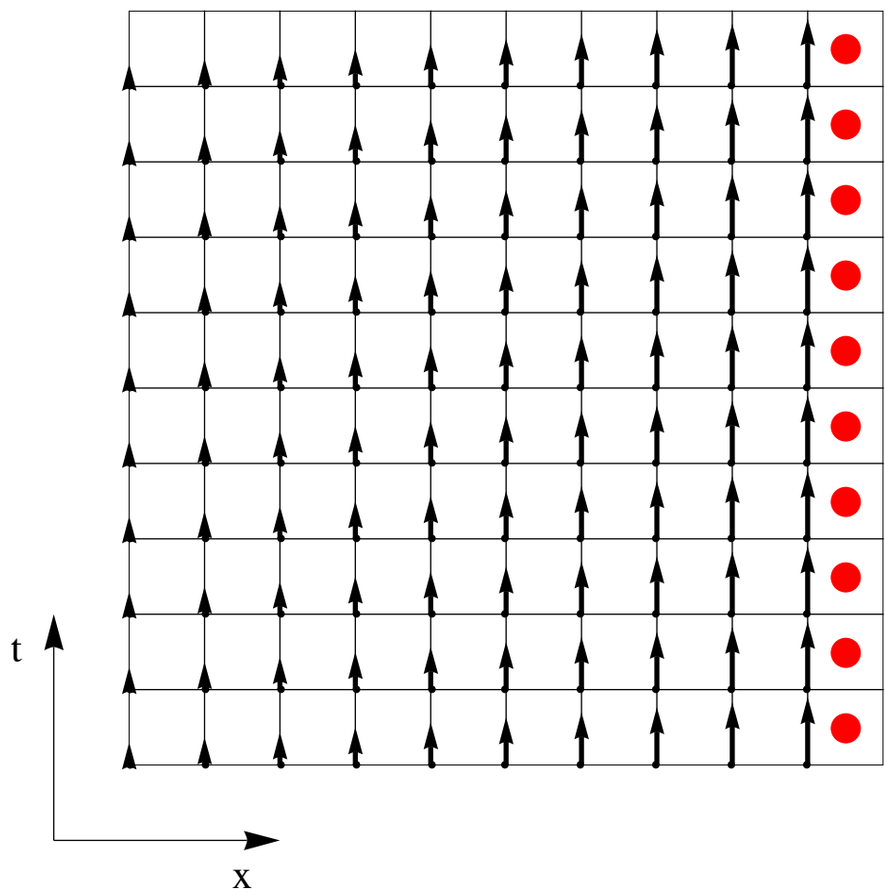
$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right) - \frac{\lambda}{2}\mathrm{Tr}\left(m_Q(\Sigma^\dagger + \Sigma)\right)$$

$$D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu \left[Q, \Sigma\right] \longrightarrow \partial_\mu \Sigma$$



Boundary conditions

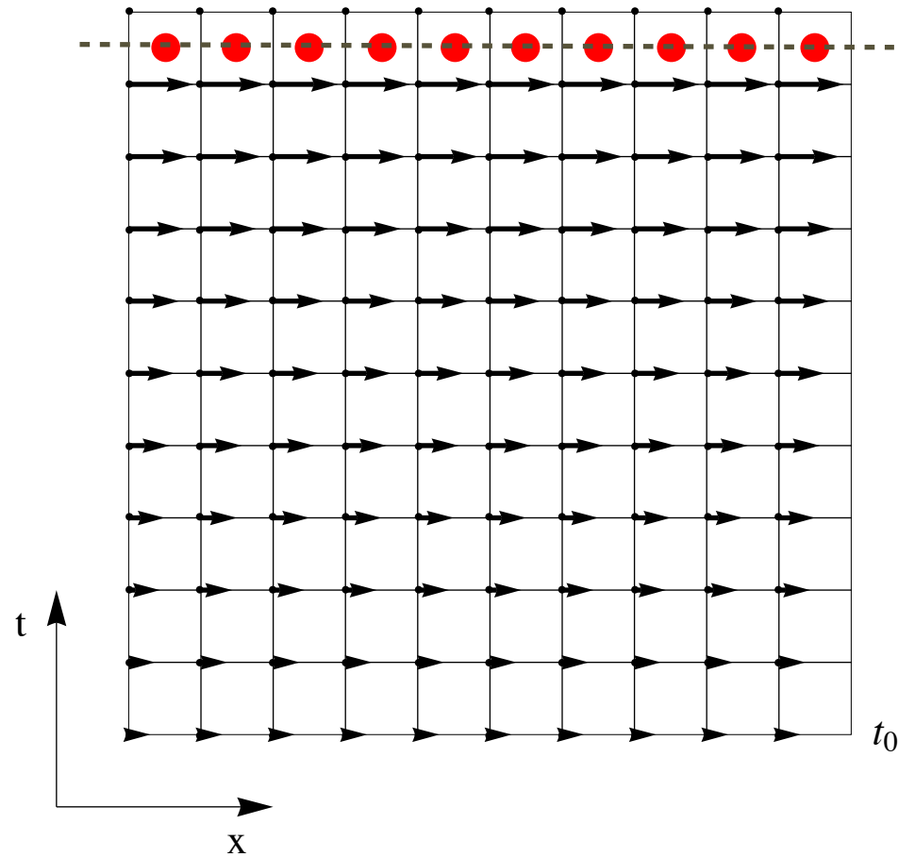


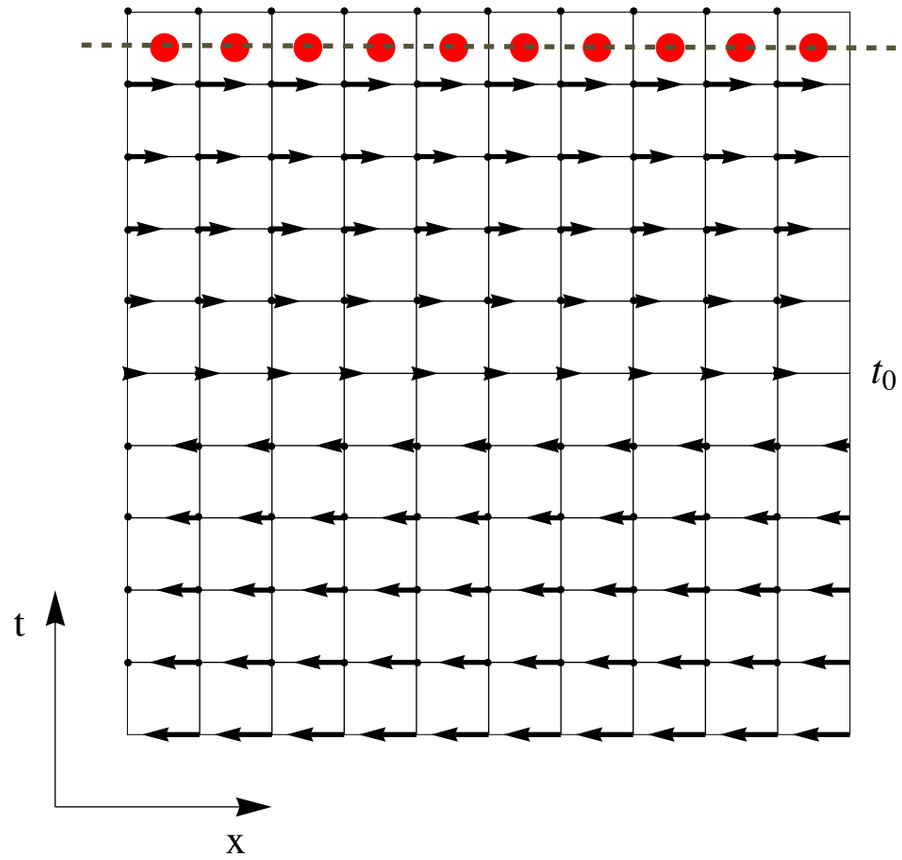


$$A_x = Et$$

$$A_t = -Ex$$

Periodic boundary conditions

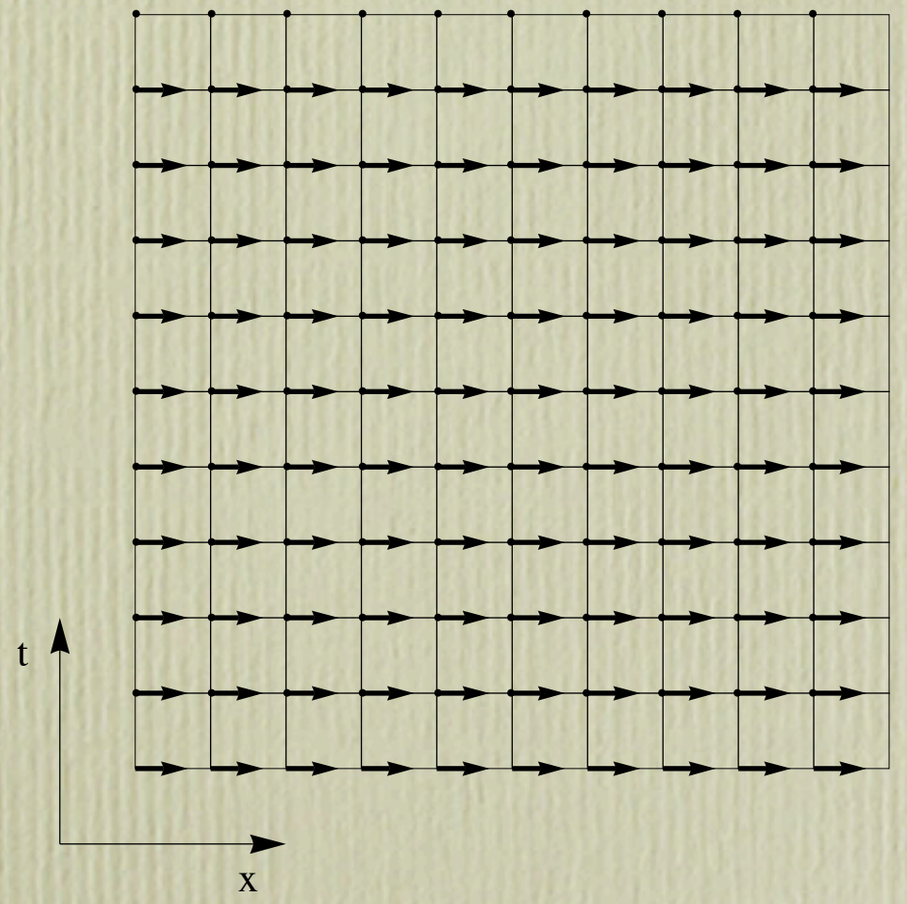


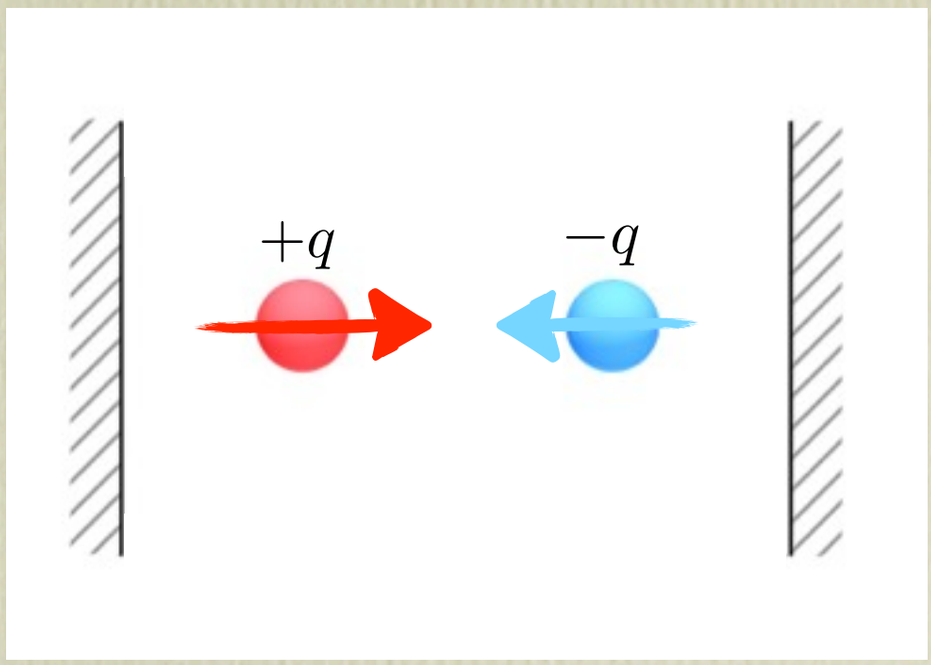


$$A_x = Et$$

$$A_x = E(t - t_0)$$

Periodic boundary conditions

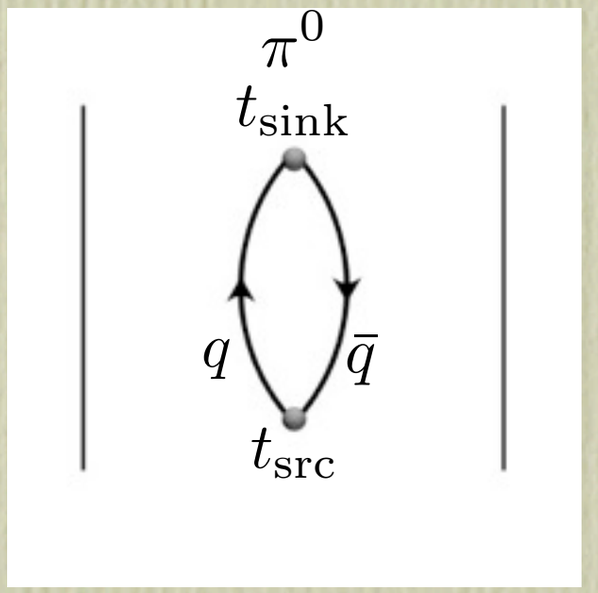


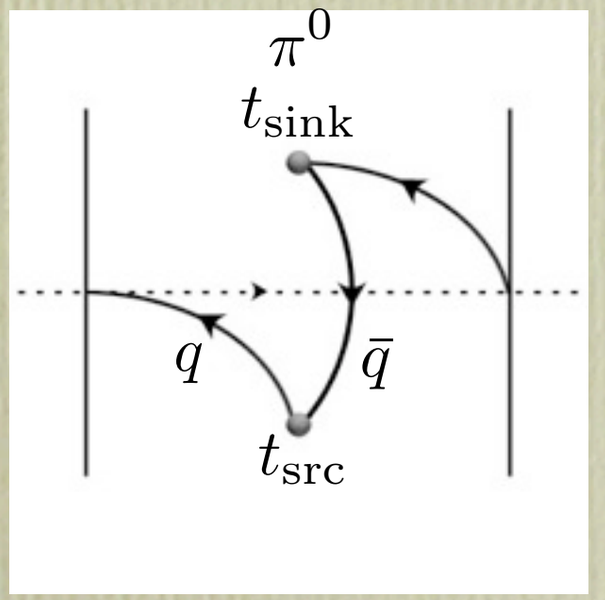


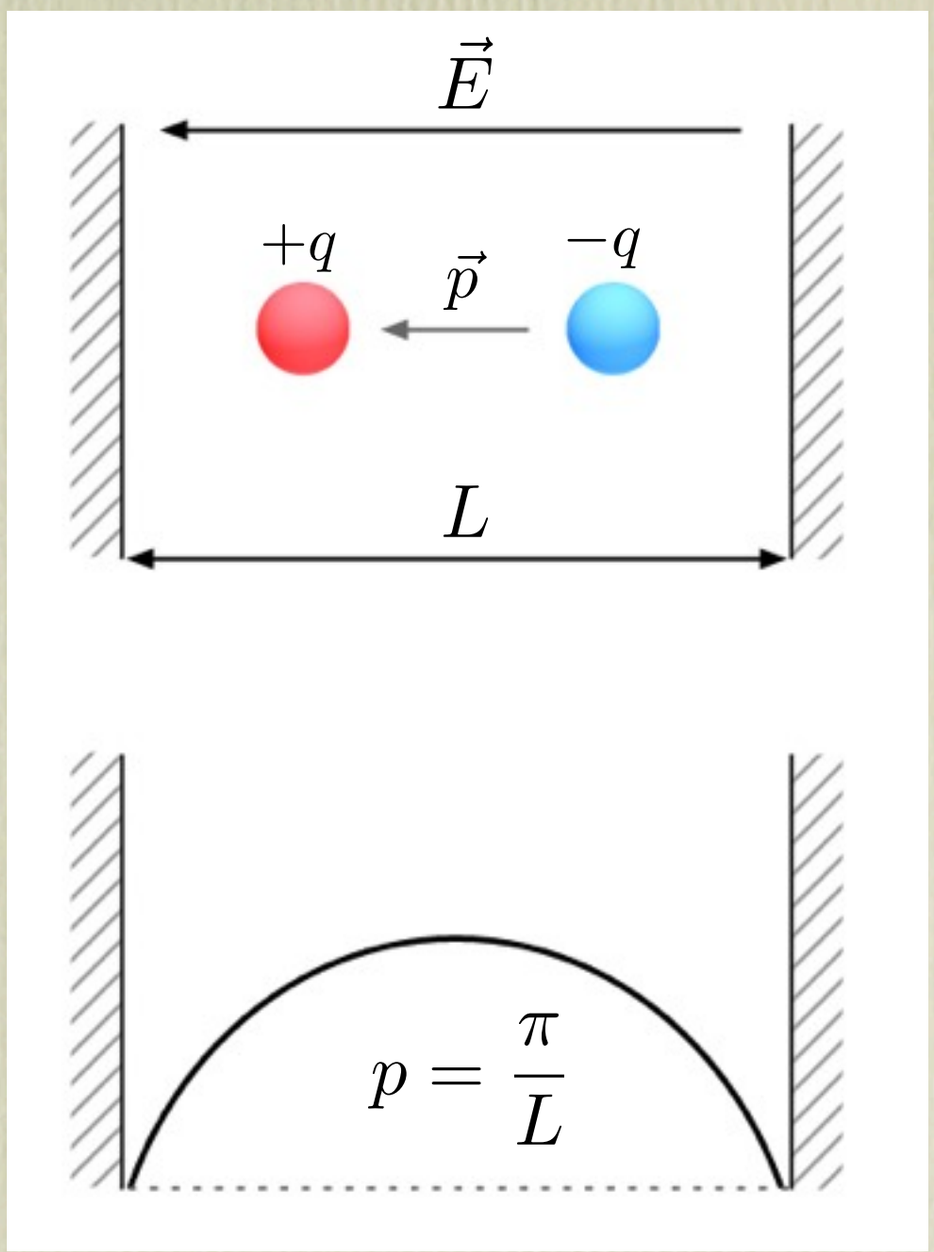
$$U_x \to e^{iaqV_0} U_x$$

$$p = qV_0 N_x$$

Dirichlet boundary conditions



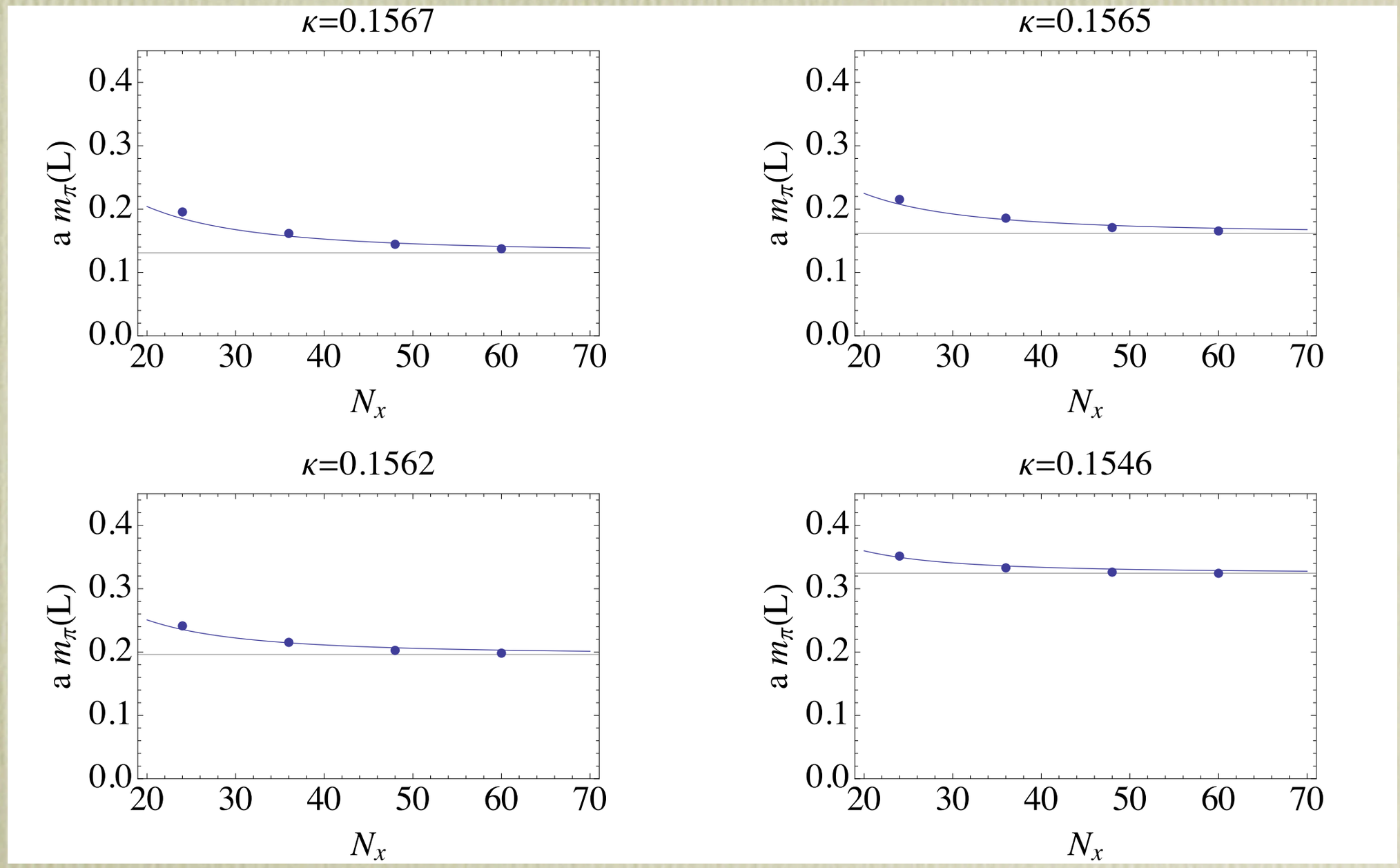




Lattice parameters - Dirichlet bc

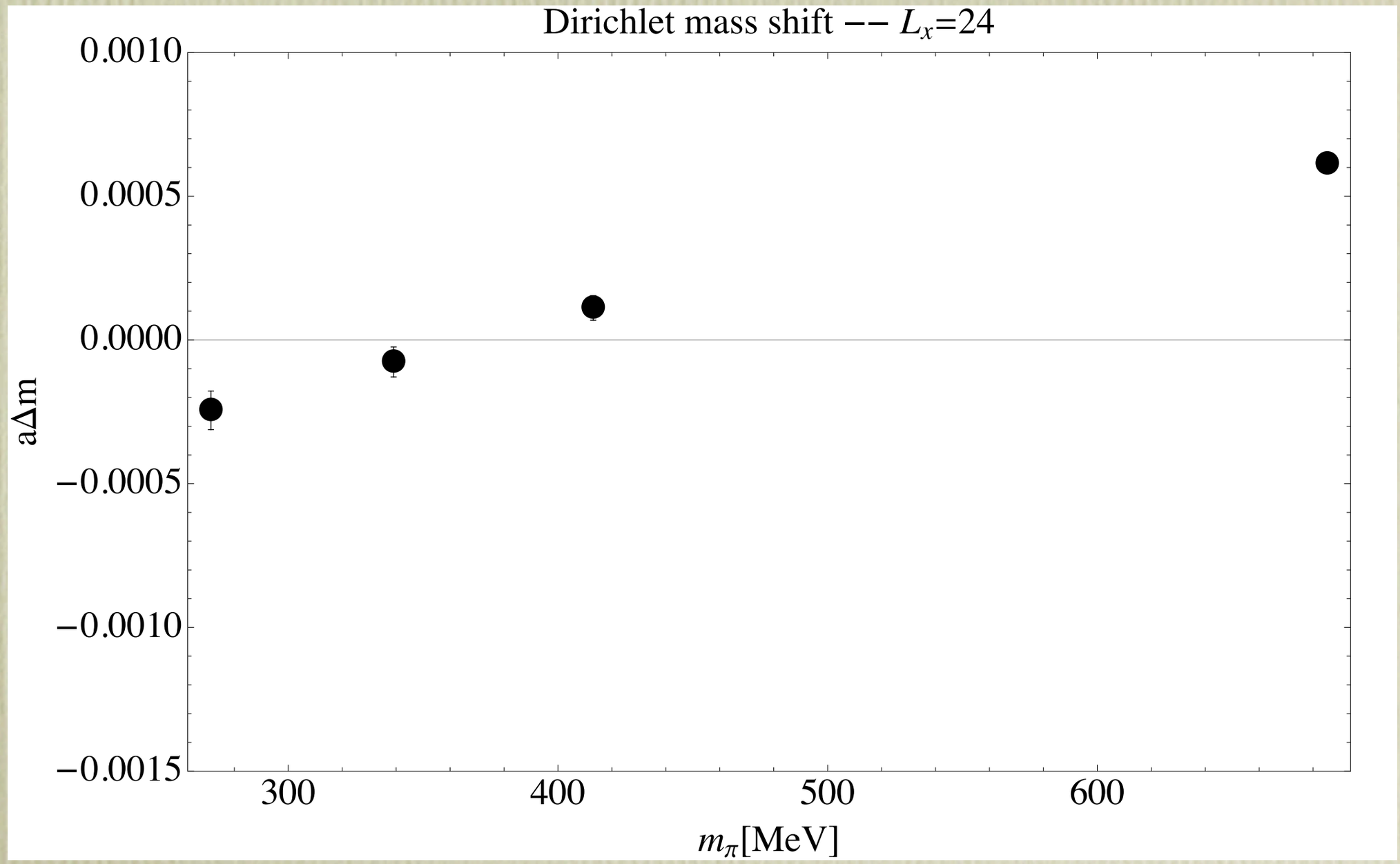
- Quenched ensemble - Wilson action β=6.0, a=0.093 fm
- Wilson fermions: 270 - 700 MeV ($m_\pi L$: 3.2-8)
- Electric field - $\eta = a^2 qE$ = 0.00576
- Four different lattice sizes: 24, 36, 48, 60 x 24^2 x 48 -- 1000 configurations

Pion ground state

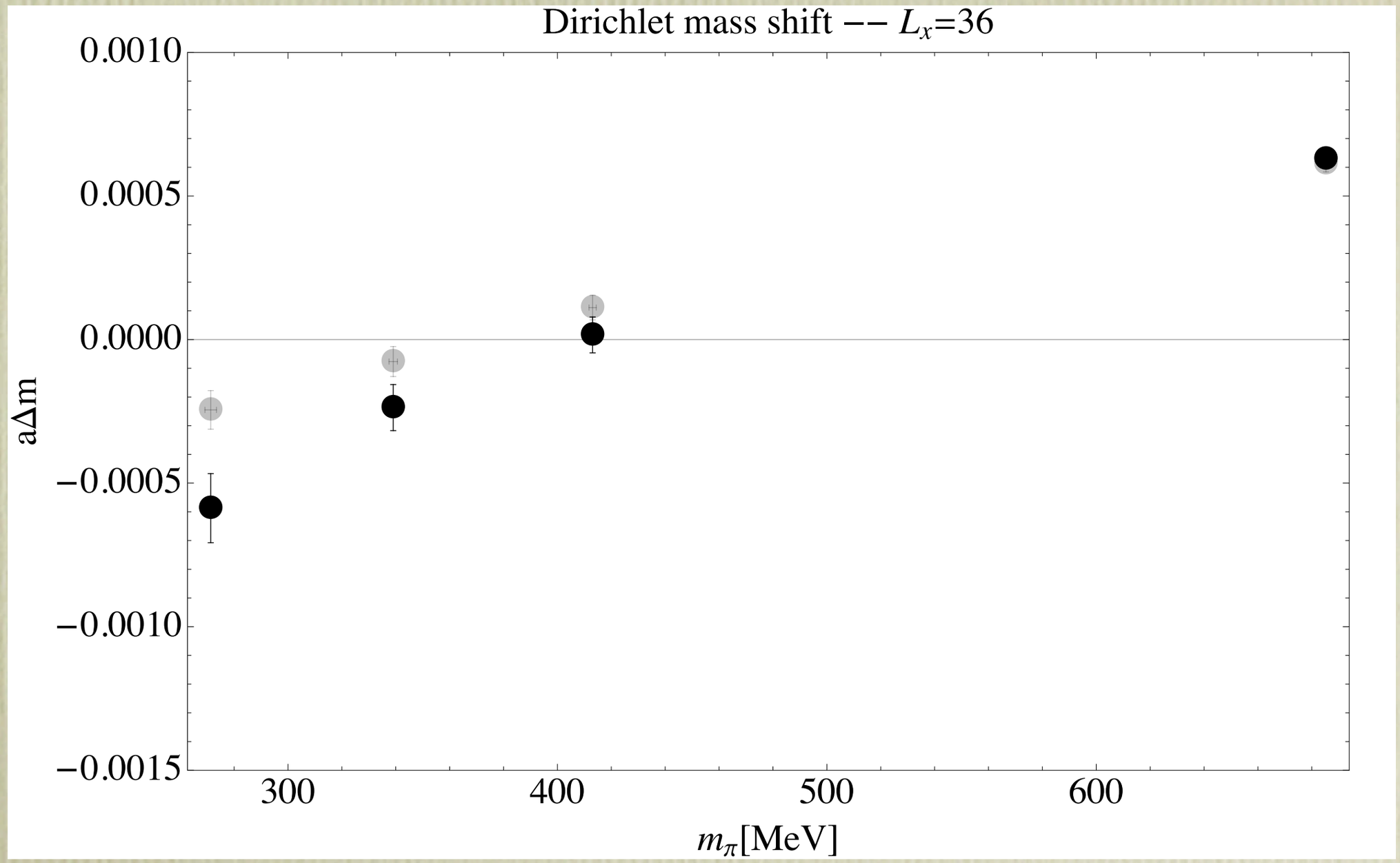


$$m_\pi(L) = \sqrt{m_\pi^2 + (\pi/L_x)^2}$$

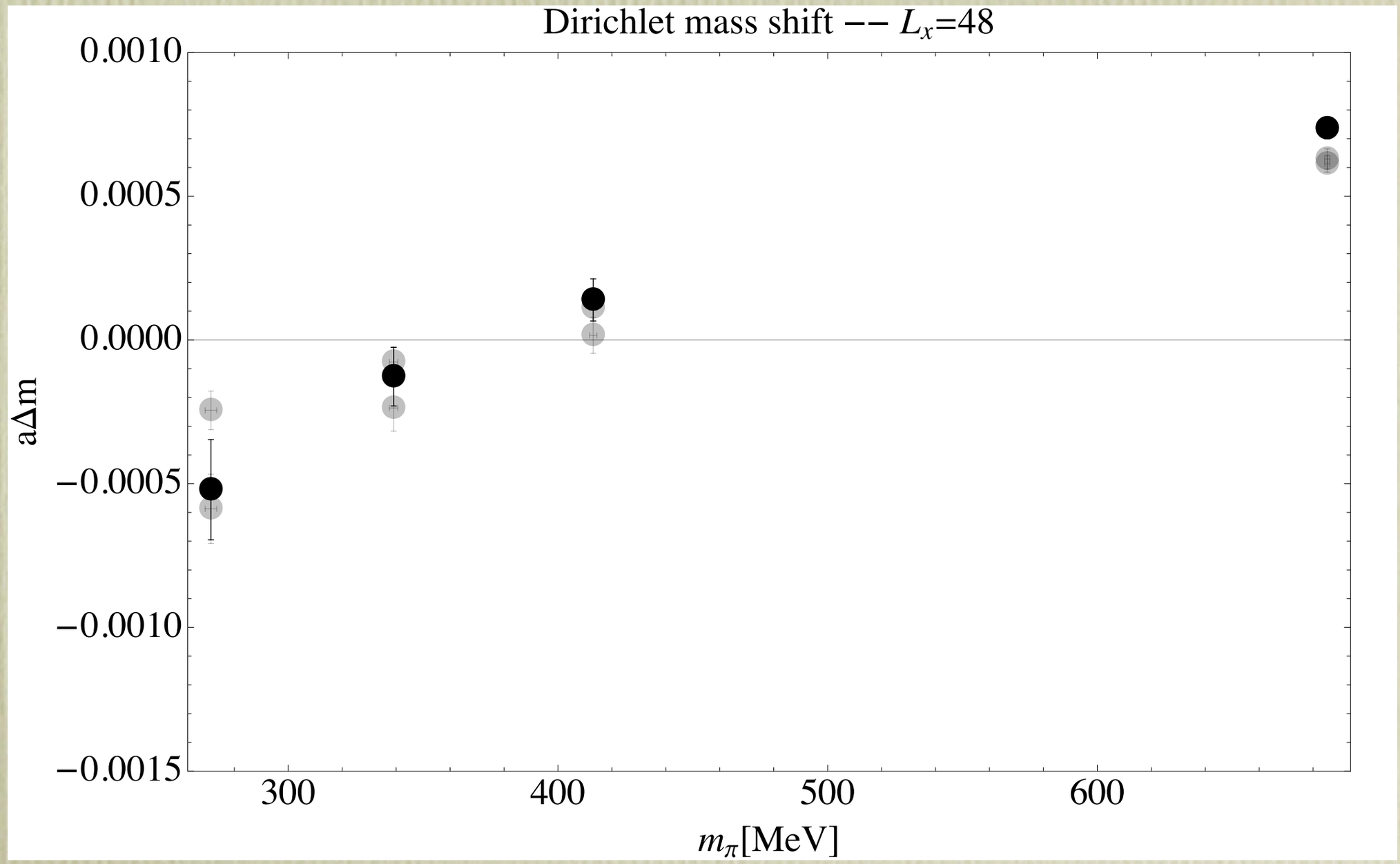
Dirichlet bc -- mass shift



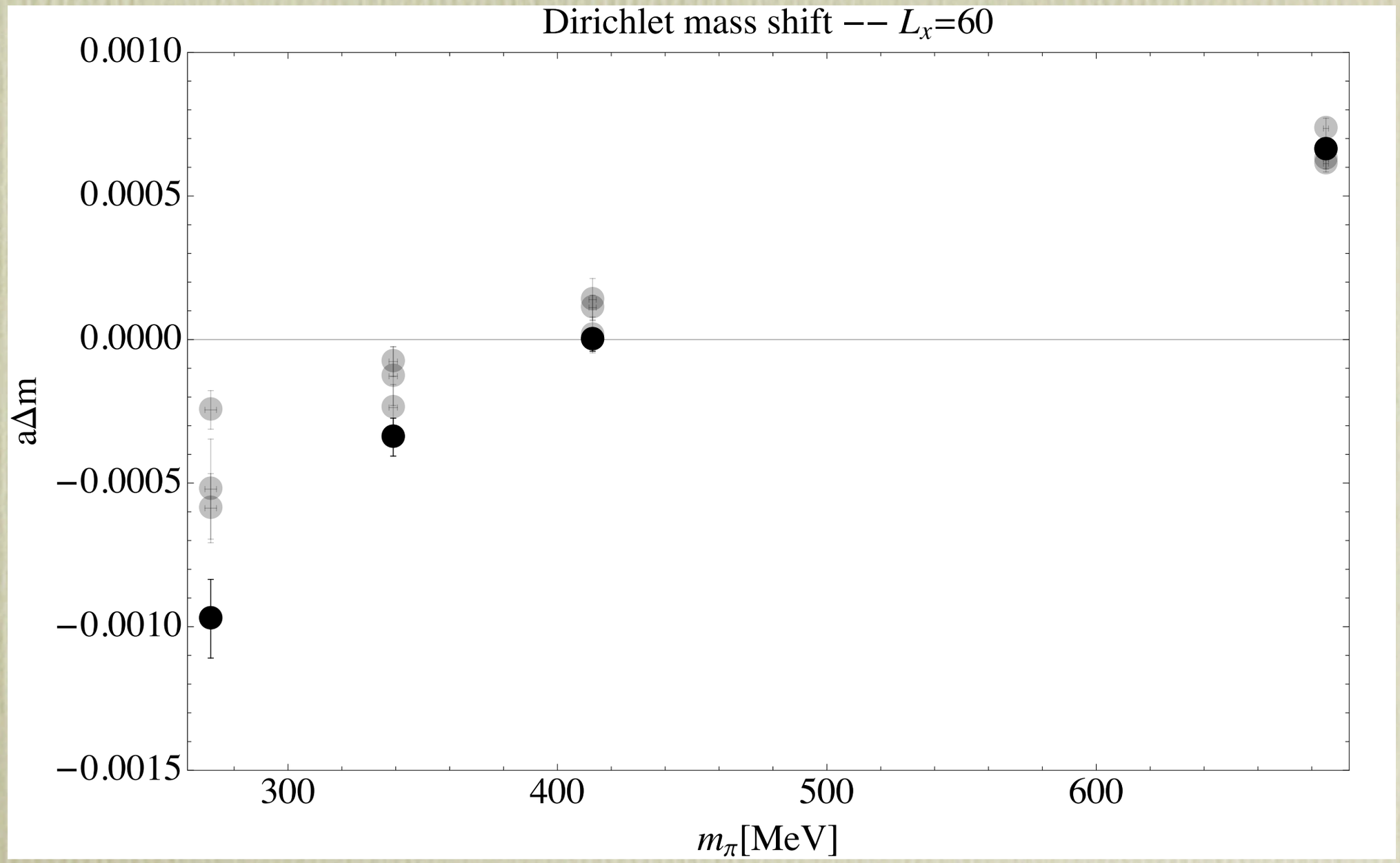
Dirichlet bc -- mass shift



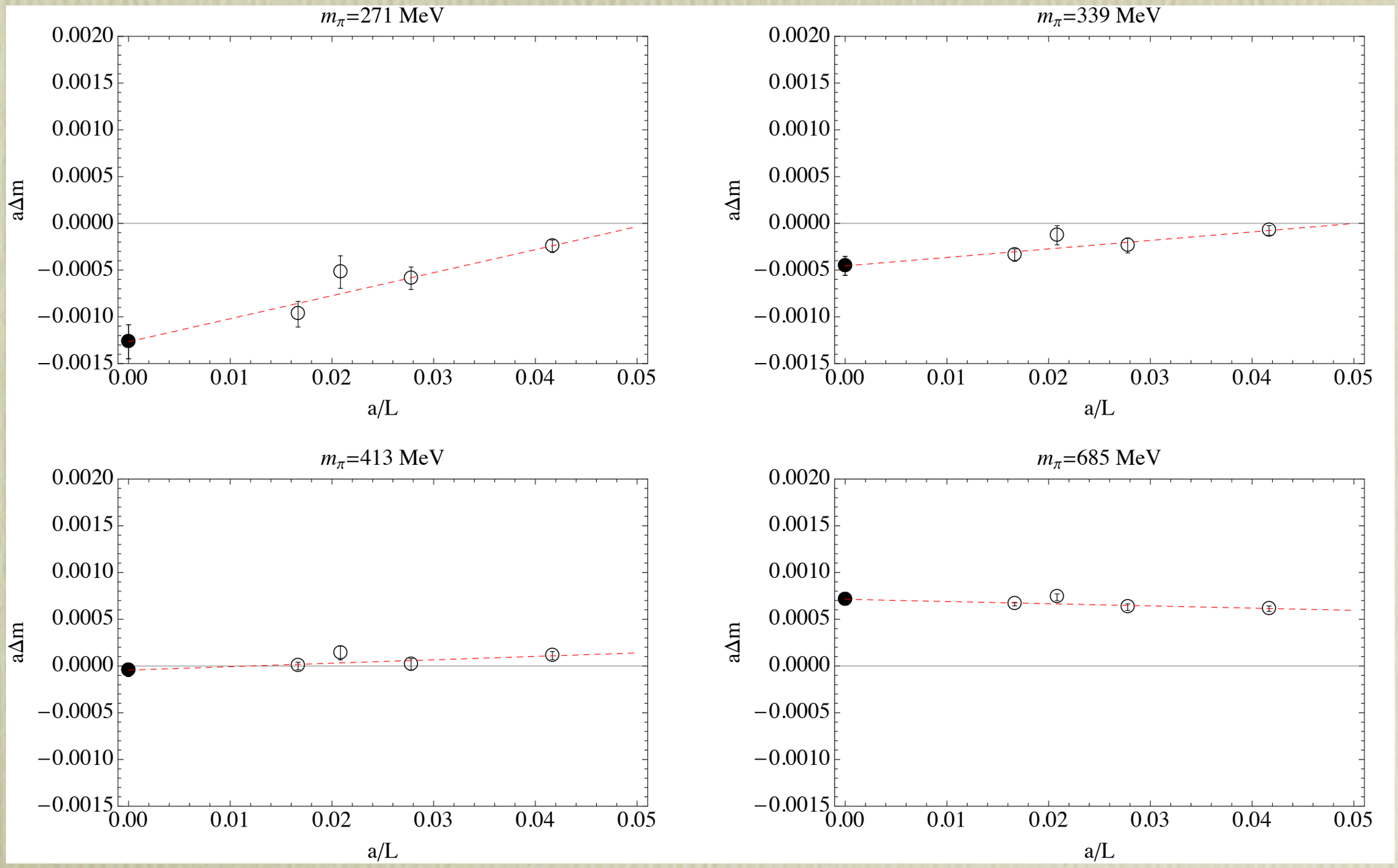
Dirichlet bc -- mass shift



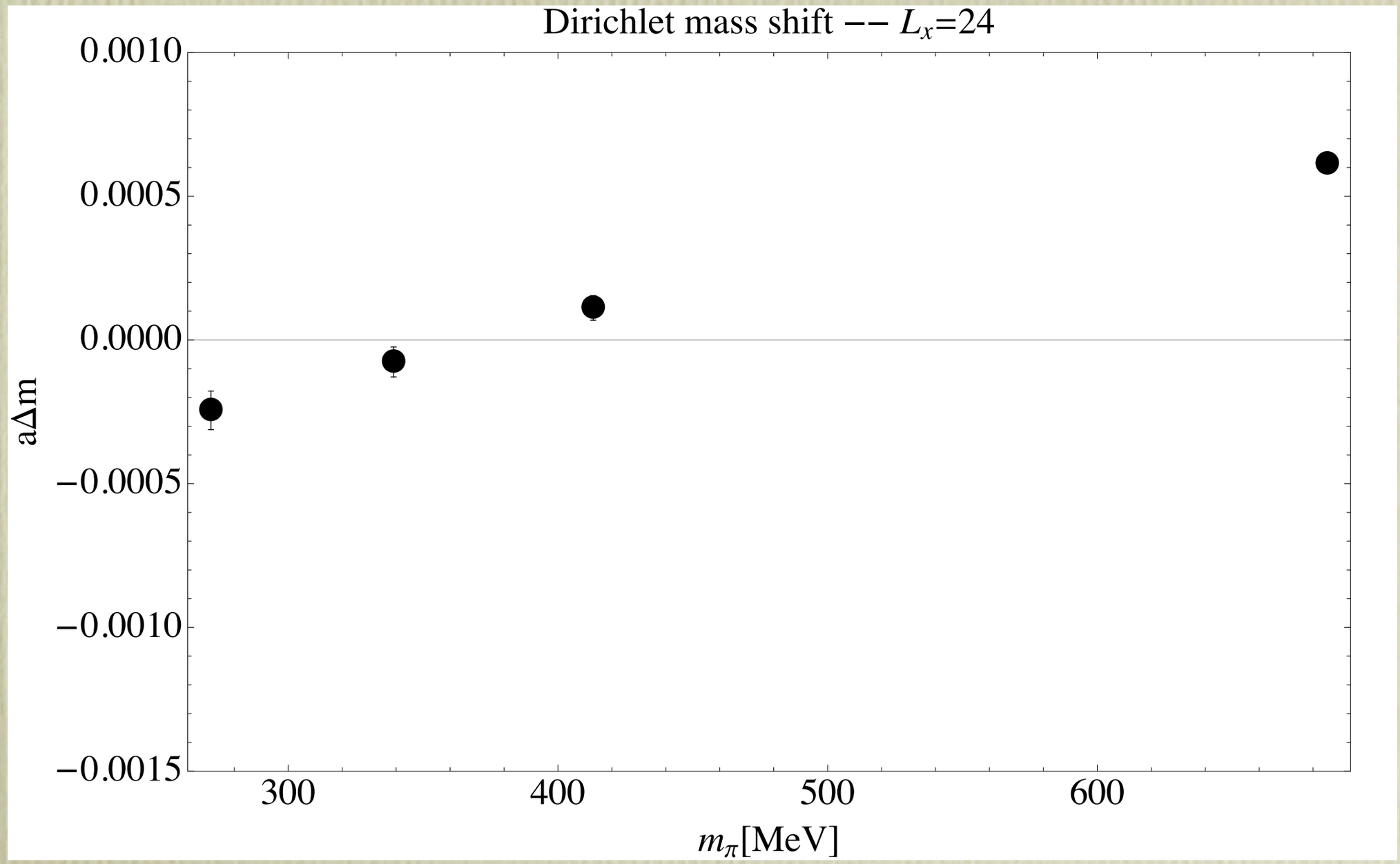
Dirichlet bc -- mass shift



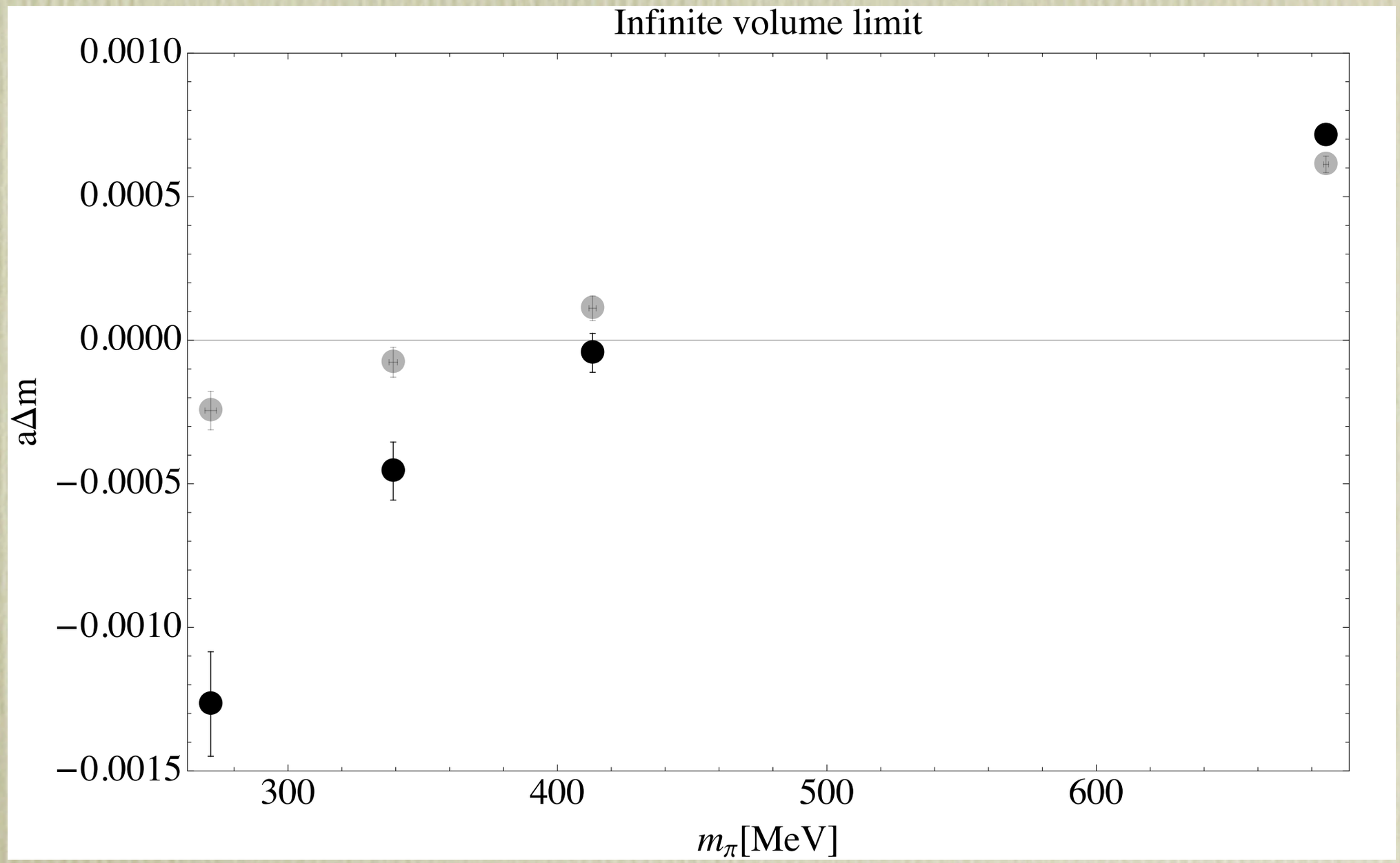
Infinite volume extrapolation



Infinite volume extrapolation



Infinite volume extrapolation



Conclusions

- We computed neutron and “pion” polarizability down to 300 MeV pion using quenched Wilson fermions
- The neutron electric polarizability is still about 1/3 of the physical value
- The “pion” polarizability is negative and it looks like it is diverging -- this seems to contradict the expectations from chiral perturbation theory
- Finite volume corrections are important (especially for smaller quark masses) but the “pion” polarizability remains negative in the large volume limit
- The results of our simulation are consistent with dynamical simulations using clover fermions -- the dynamical effects are still small compared to the error bars
- On the volumes we studied, exceptional configurations limit clover fermions to 500 MeV and Wilson to 300 MeV

Outlook

- Smaller quark masses -- overlap
- Determine finite volume corrections with periodic boundary conditions and compare the infinite volume limit to the Dirichlet case
- Include the dynamical effects of the electric field -- they should become important at smaller quark masses
  - use the electric field in the generation of configurations -- not really feasible
  - use reweighting $$\langle G_E(t) \rangle_E = \frac{\left\langle G_E(t) \frac{\det M_E}{\det M_0} \right\rangle_0}{\left\langle \frac{\det M_E}{\det M_0} \right\rangle_0}$$
- Dynamical configurations and continuum limit