Three-Dimensional nucleon structure: Jefferson Lab 12 and beyond

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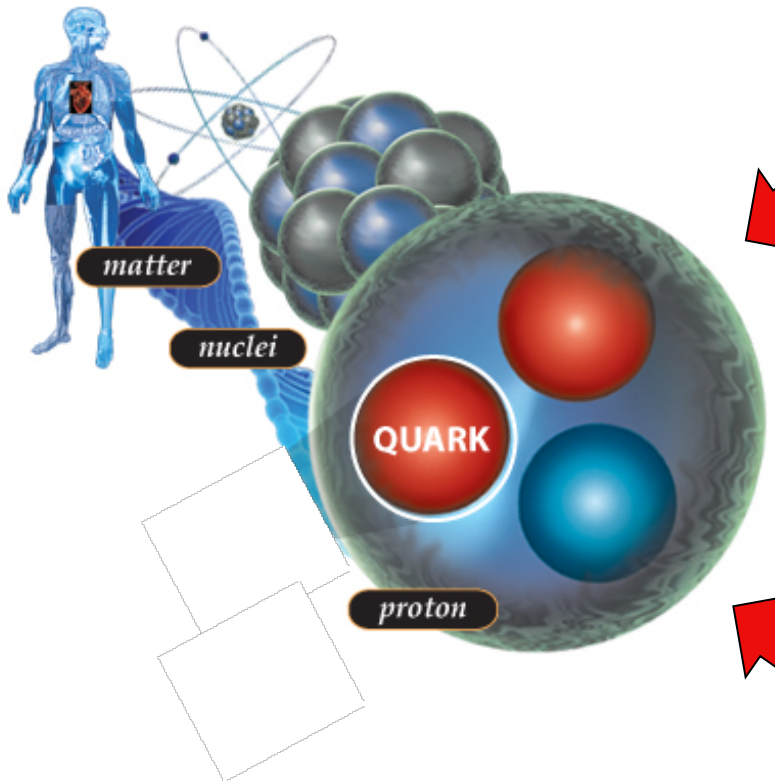
March 11, 2013

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Exploring the nucleon: a fundamental quest

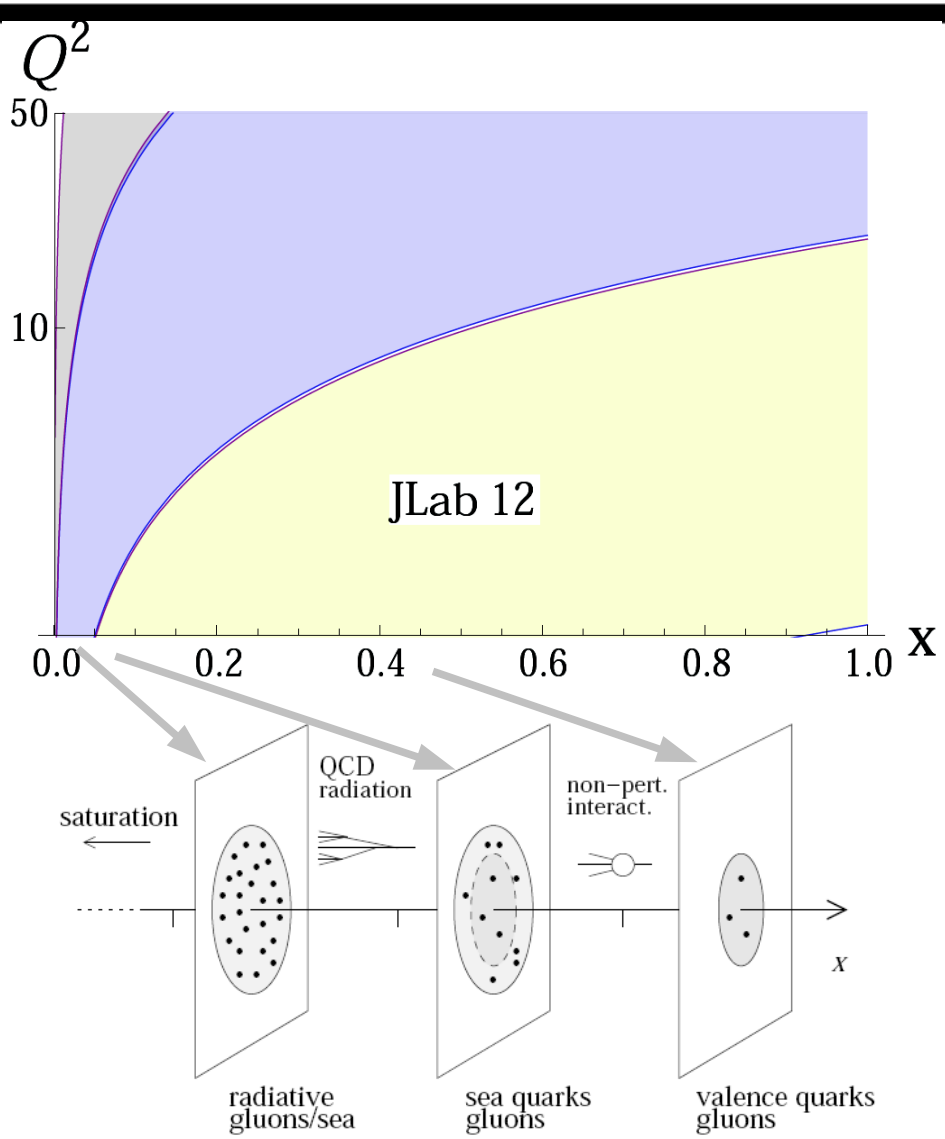


Know what we are made of !

Understand the strong force: "QCD"

Use protons as tool for discovery (e.g. LHC)

Nucleon landscape



Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How **partons move** and how they are distributed in **space** is one of the future directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions and Transverse Momentum Dependent distributions

These distributions are also referred to as **3D (three-dimensional) distributions**

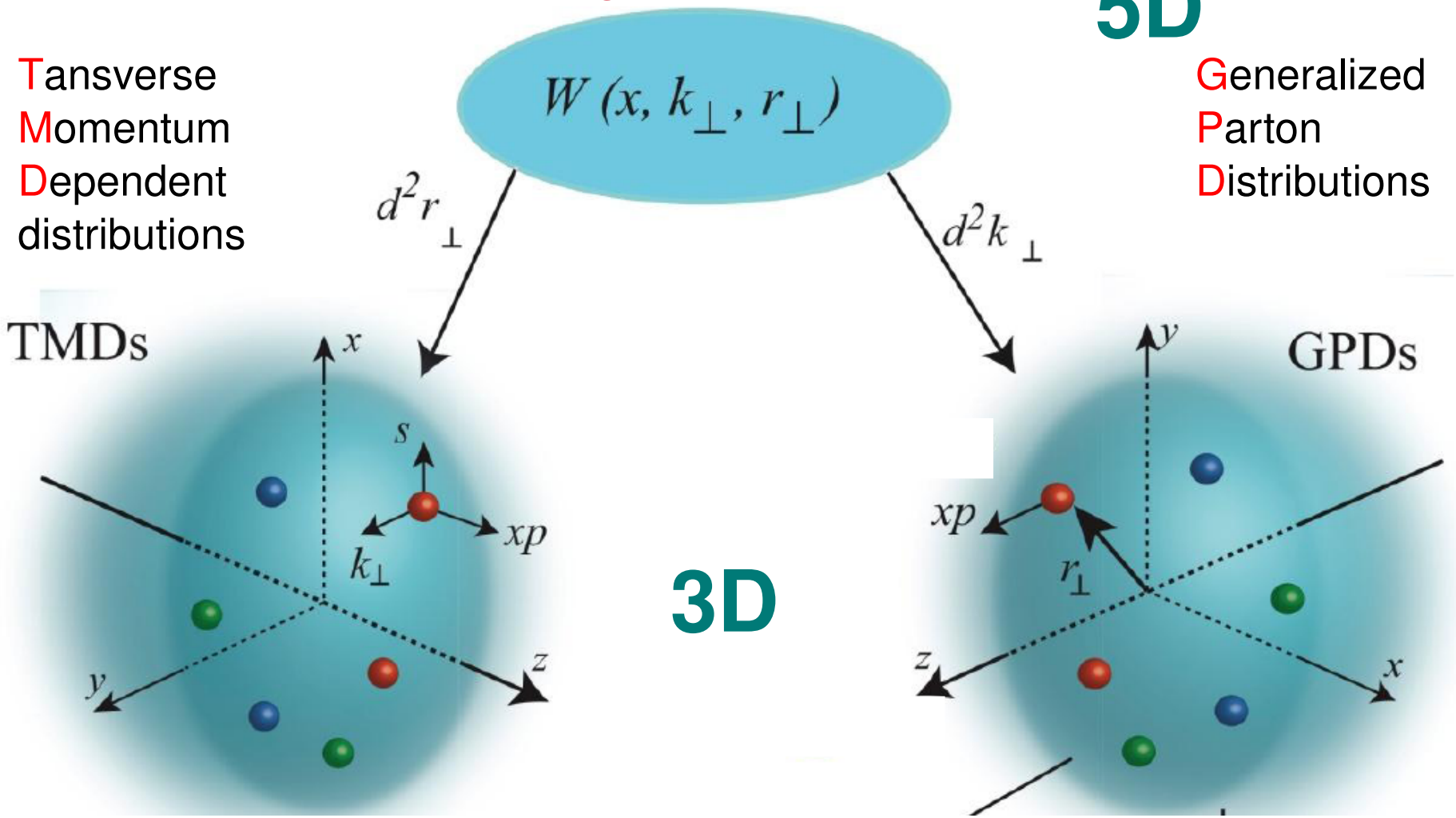
Unified View of Nucleon Structure

Wigner function

5D

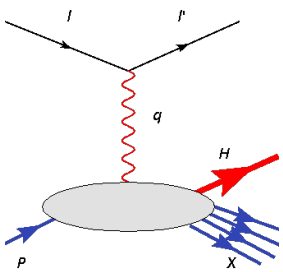
Tansverse
Momentum
Dependent
distributions

Generalized
Parton
Distributions



Unified View of Nucleon Structure

TMD

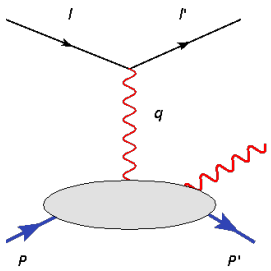


Wigner function

$$W(x, \mathbf{k}_\perp, \mathbf{b}_\perp)$$

5D

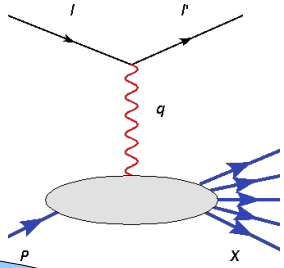
GPD

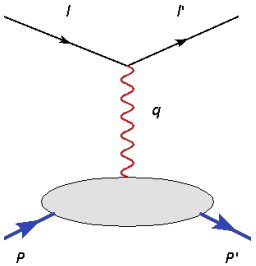


$$f(x, \mathbf{k}_\perp)$$

3D

$$H(x, \xi, t)$$





PDF

FF

$$f(x)$$

1D

$$F(Q^2)$$

Particular processes to study. Polarization is required!

JLab: 21st Century Science Questions

- What is the role of gluonic excitations in the spectroscopy of light mesons? Can these excitations elucidate the origin of quark confinement?
- Where is the missing spin in the nucleon? Is there a significant contribution from valence quark orbital angular momentum?
- Can we reveal a novel landscape of nucleon substructure through measurements of new multidimensional distribution functions?
- What is the relation between short-range N-N correlations and the partonic structure of nuclei?
- Can we discover evidence for physics beyond the standard model of particle physics?

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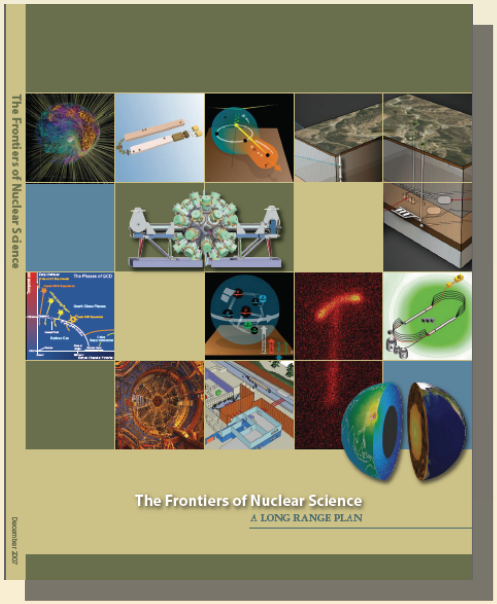
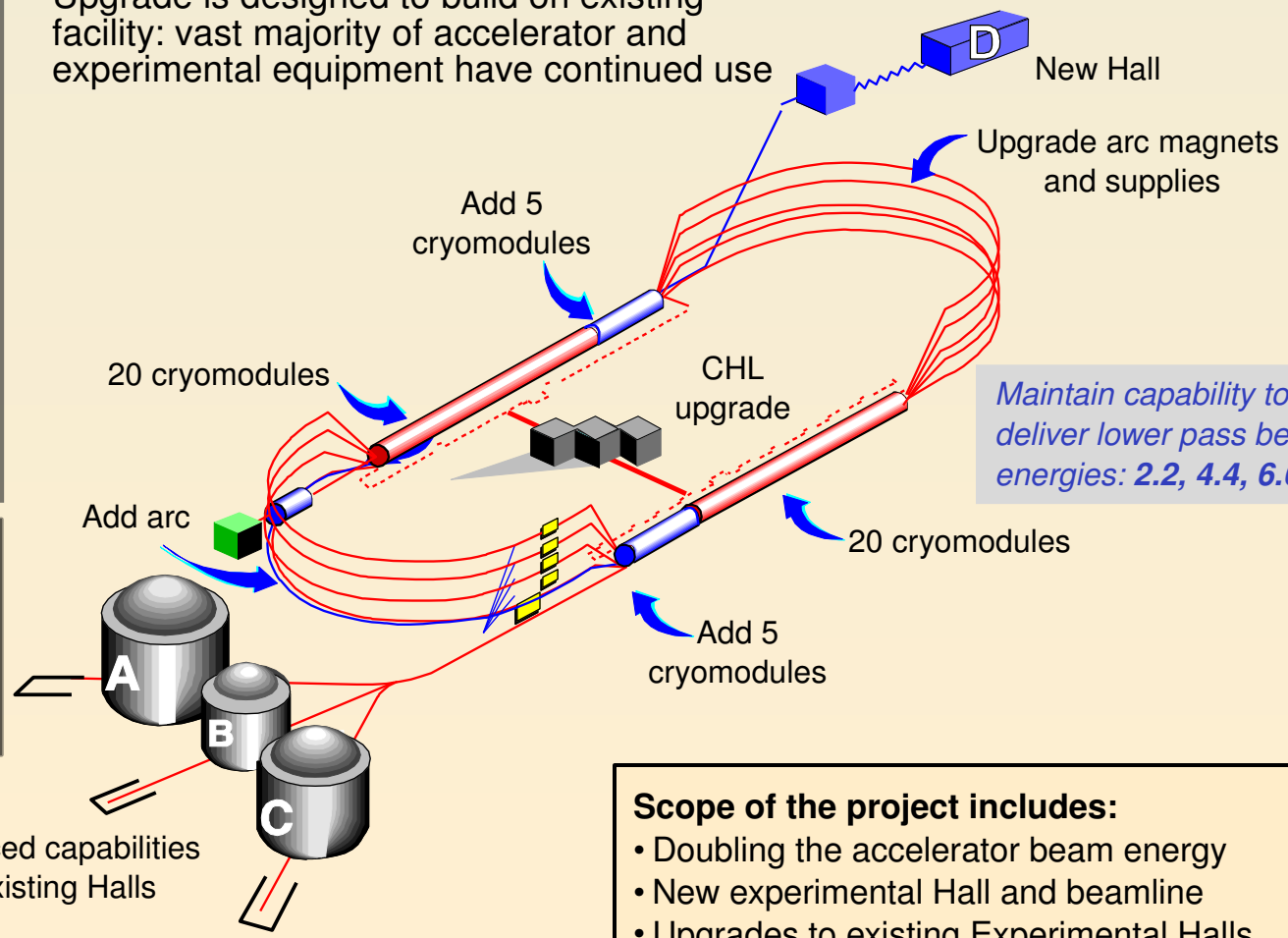
12 GeV Upgrade Project



The completion of the 12 GeV Upgrade of CEBAF was ranked the highest priority in the 2007 NSAC Long Range Plan.

Upgrade is designed to build on existing facility: vast majority of accelerator and experimental equipment have continued use



Maintain capability to deliver lower pass beam energies: ***2.2, 4.4, 6.6….***

Scope of the project includes:

- Doubling the accelerator beam energy
- New experimental Hall and beamline
- Upgrades to existing Experimental Halls

12 GeV Approved Experiments by Physics Topics

	Hall A	Hall B	Hall C	Hall D	Total
The Hadron Spectra as Probes of QCD (GluEx & heavy baryon and meson spectroscopy)		1		1	2
The Transverse Structure of the Hadrons (elastic and transition form factors)	4	3	2		9
The Longitudinal Structure of the Hadrons (Unpolarized and polarized parton distributions)	2	2	5		9
The 3D Structure of the Hadrons (GPDs and TMDs)	5	10	3		18
Hadrons and Cold Nuclear Matter	3	2	6		11
Low-Energy Tests of the Standard Model and Fundamental Symmetries	2			1	3
Total	**16**	**18**	**16**	**2**	**52**

Program Advisory Committee in the period 2006 – 2011 approved 52 experiments

12 GeV Approved Experiments by Physics Topics

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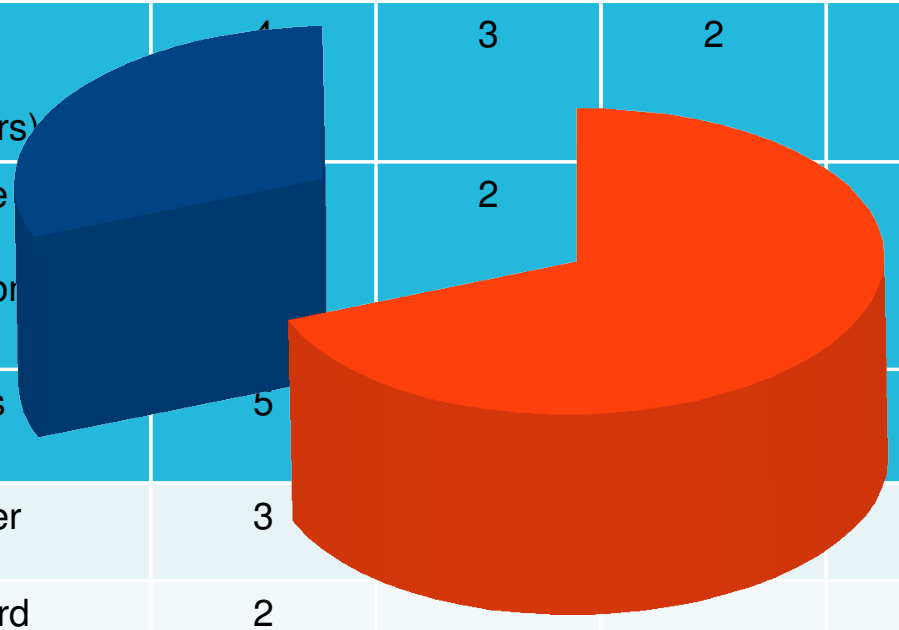
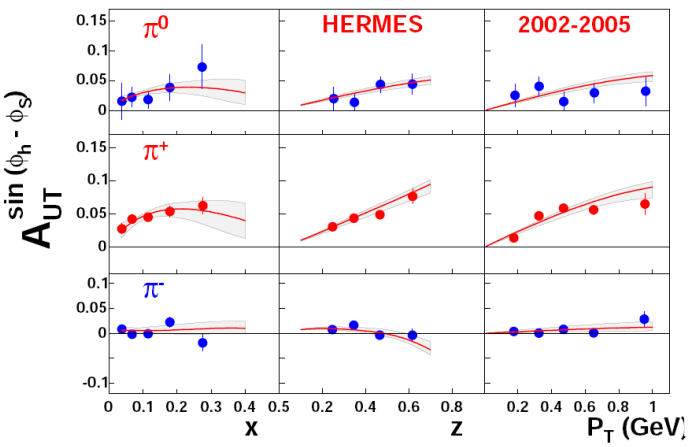


>50% is dedicated to hadron structure!

What do we measure?

Experimentally usually asymmetries are measured:

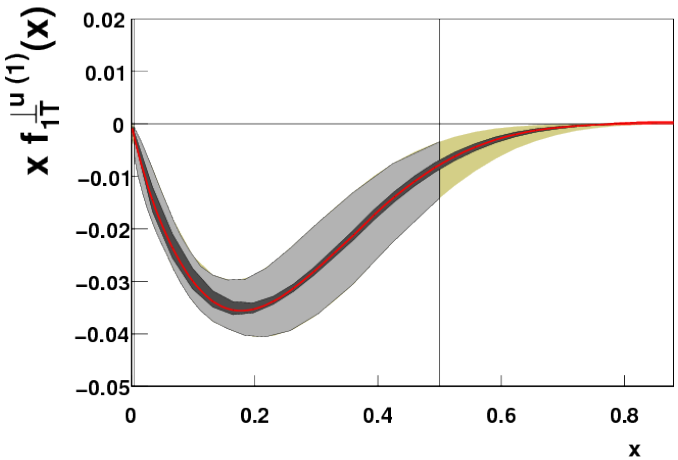
$$A \propto \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$



Observables are analysed in terms of partonic distributions

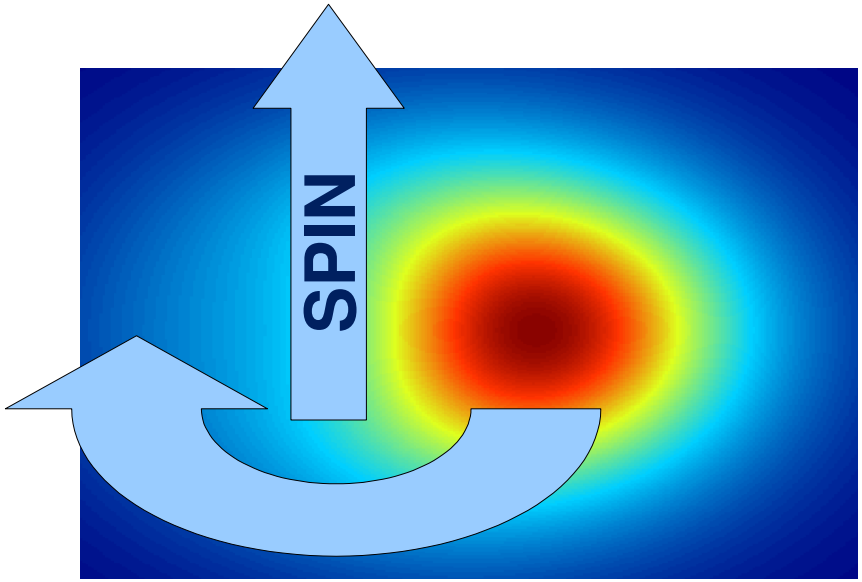
$$\sigma^{\uparrow} \propto f \otimes D$$

Results



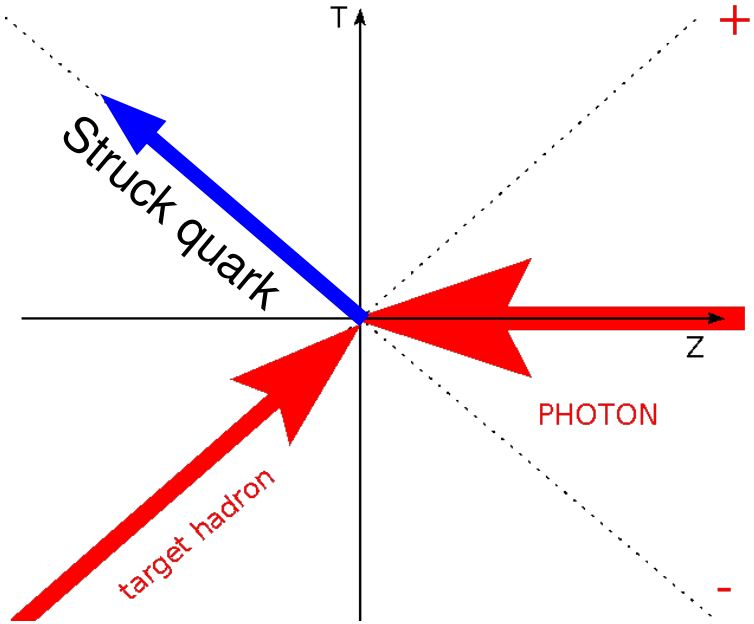
Global analysis of the data

My scientific interests

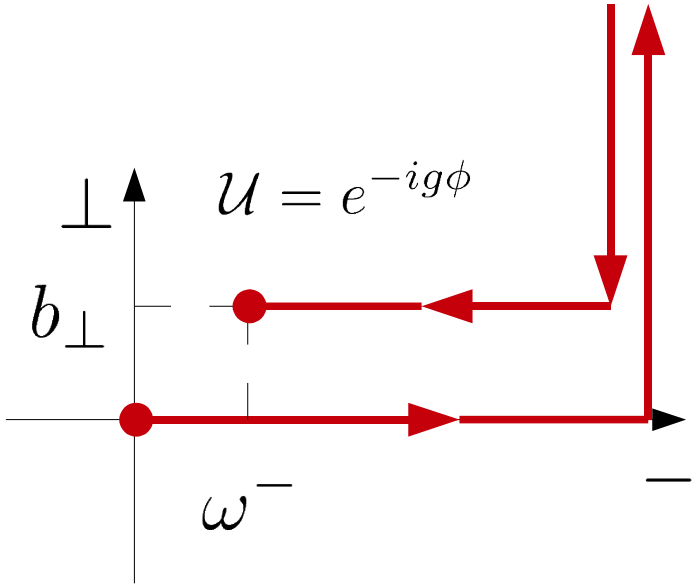


Transverse Momentum Dependent distributions

Electron Scattering in Infinite Momentum Frame:



Quark passing through the gauge field of the hadron aquires a phase called gauge link

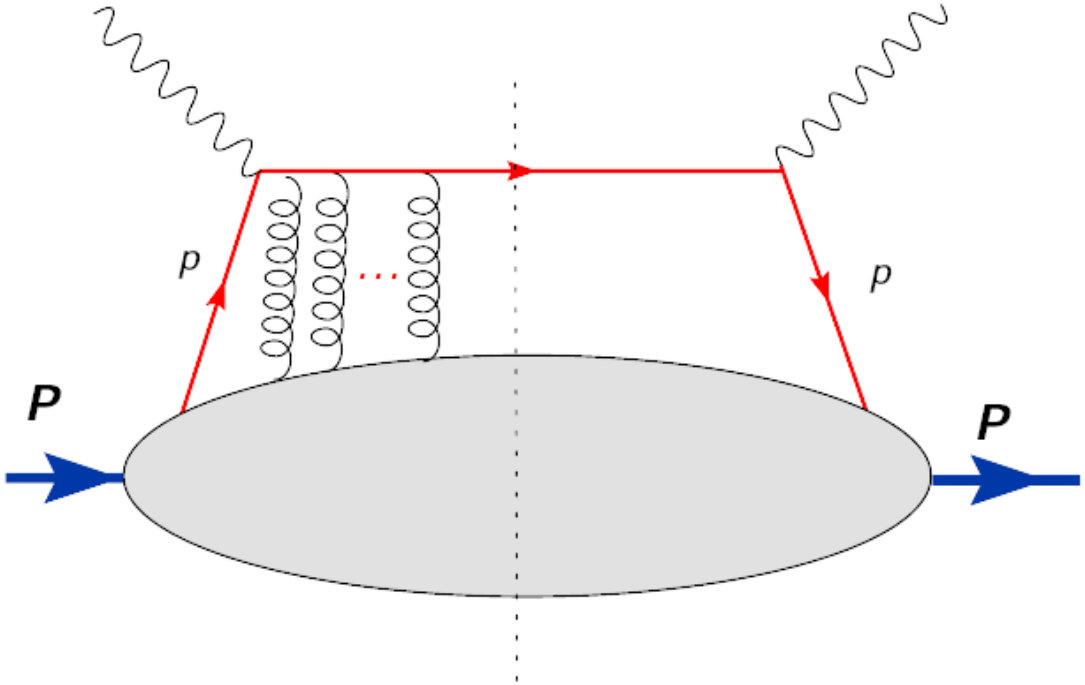


TMDs are sensitive to gauge field!

Ensures gauge invariance of the distribution

Transverse Momentum Dependent distributions

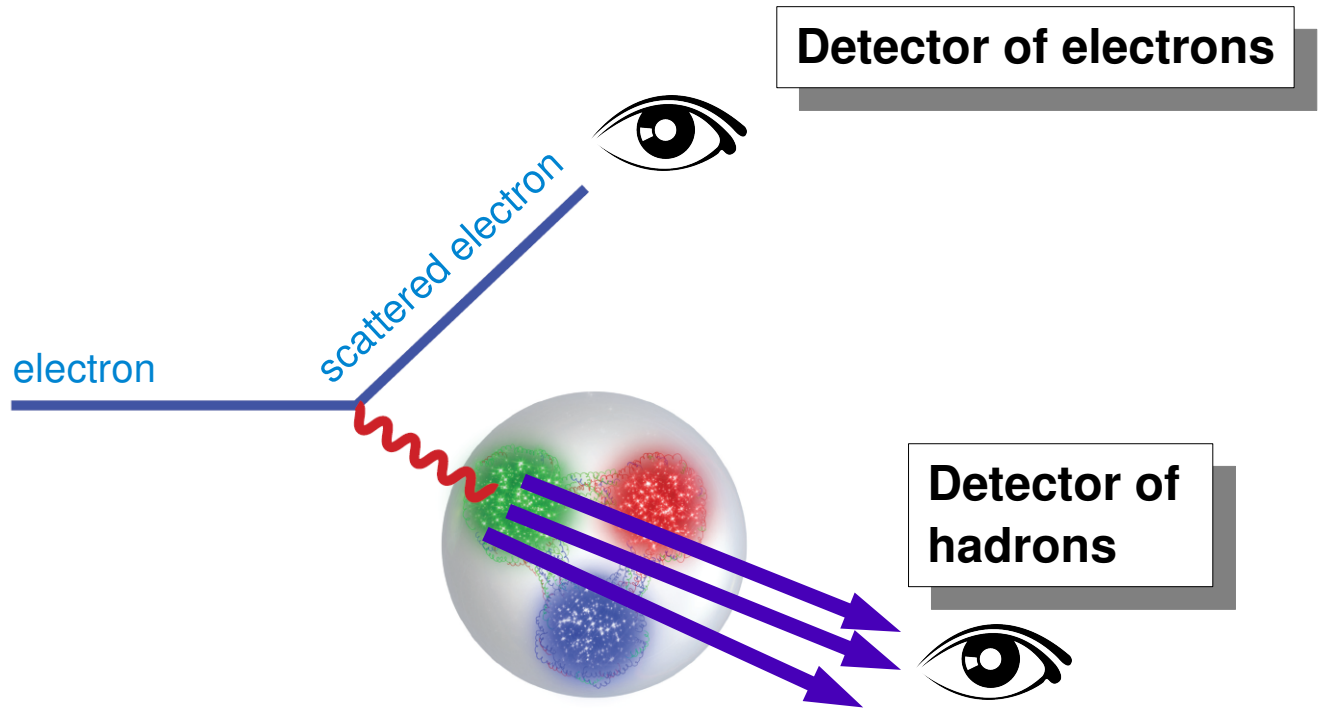
Origin of the gauge link



Transverse Momentum Dependent distributions

$$f_{ij}(x,\mathbf{k}_\perp) = \int \frac{d\omega^-}{(2\pi)} \frac{d^2b_\perp}{(2\pi)^2} e^{-ixP^+\omega^- + i\mathbf{k}_\perp \mathbf{b}_\perp} \langle P, S_P | \bar{\psi}_j(\omega) \mathcal{U}(\omega, \mathbf{0}) \psi_i(0) | P, S_P \rangle |_{\omega^+=0}$$

Process



Factorization

$$\sigma \propto \mathcal{H} \cdot f(x, k_\perp) \otimes D(z, p_\perp)$$

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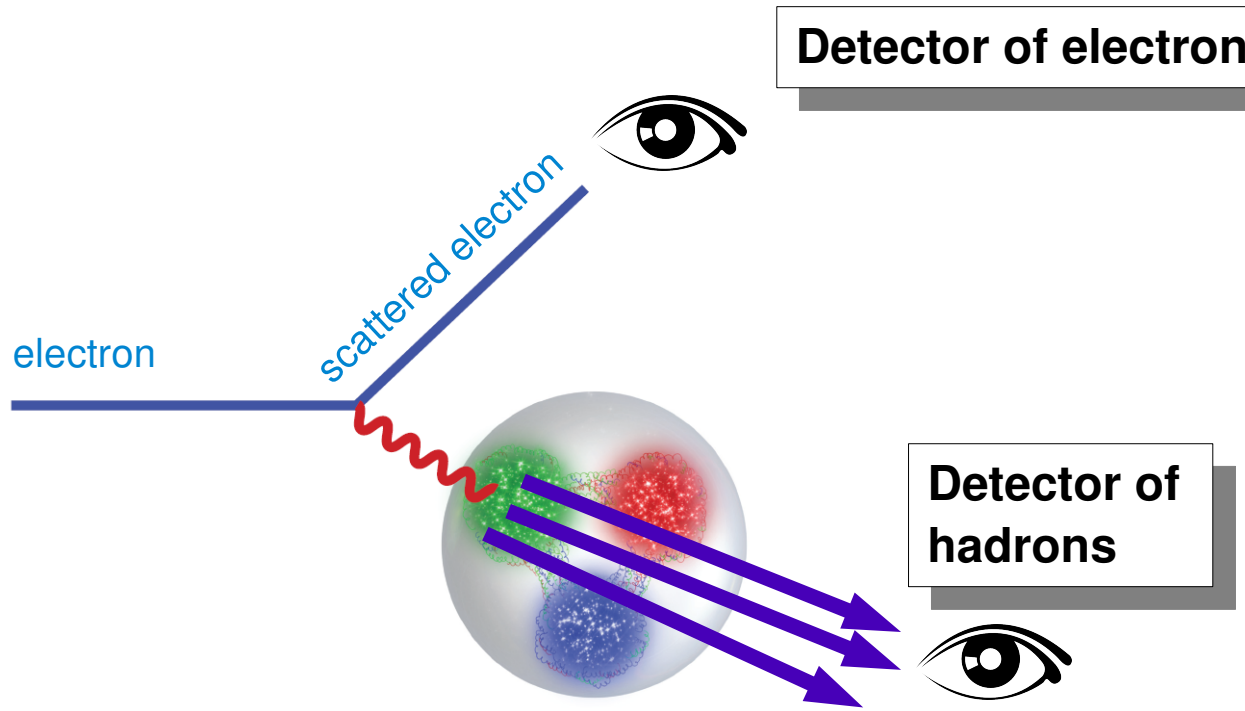

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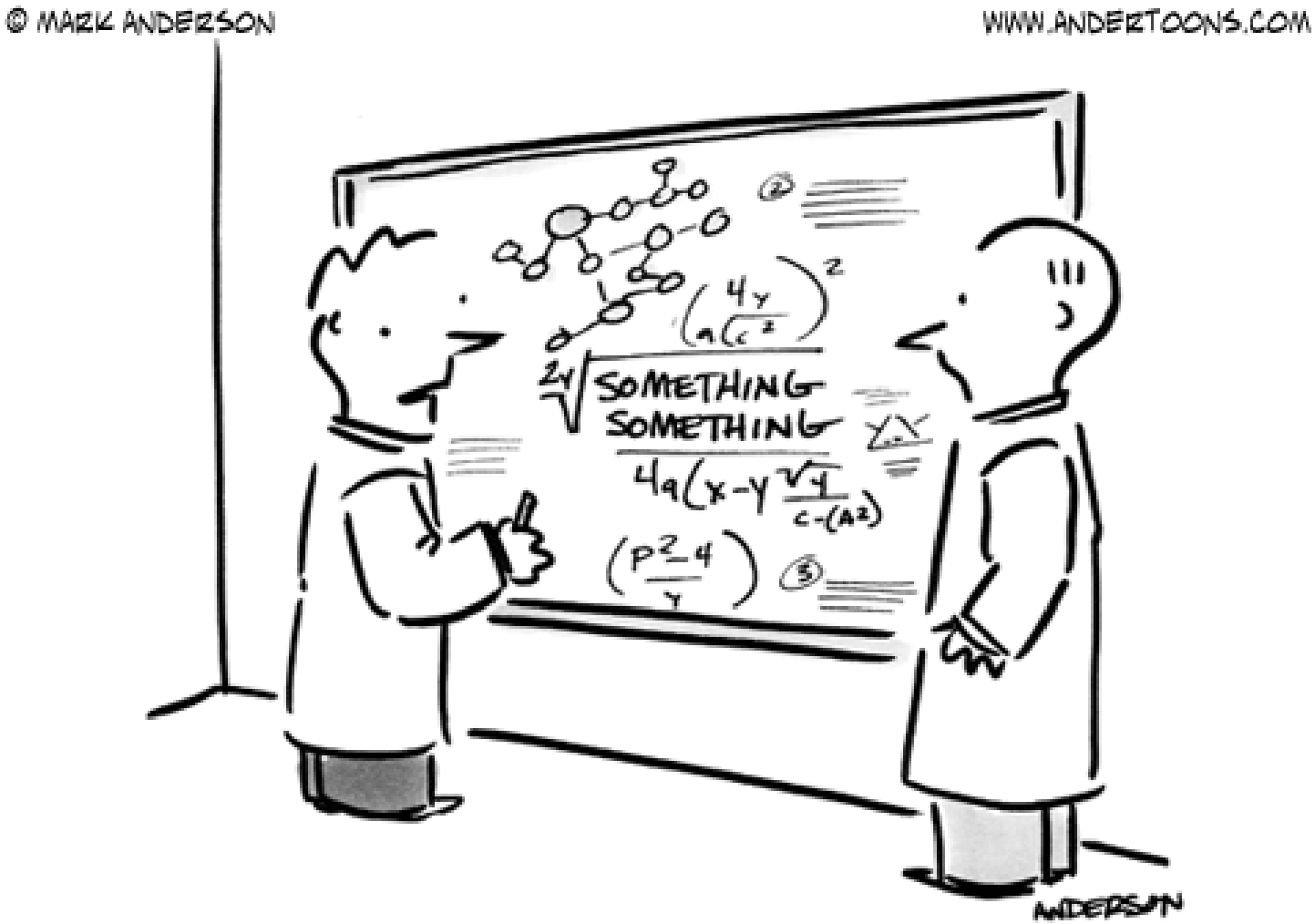
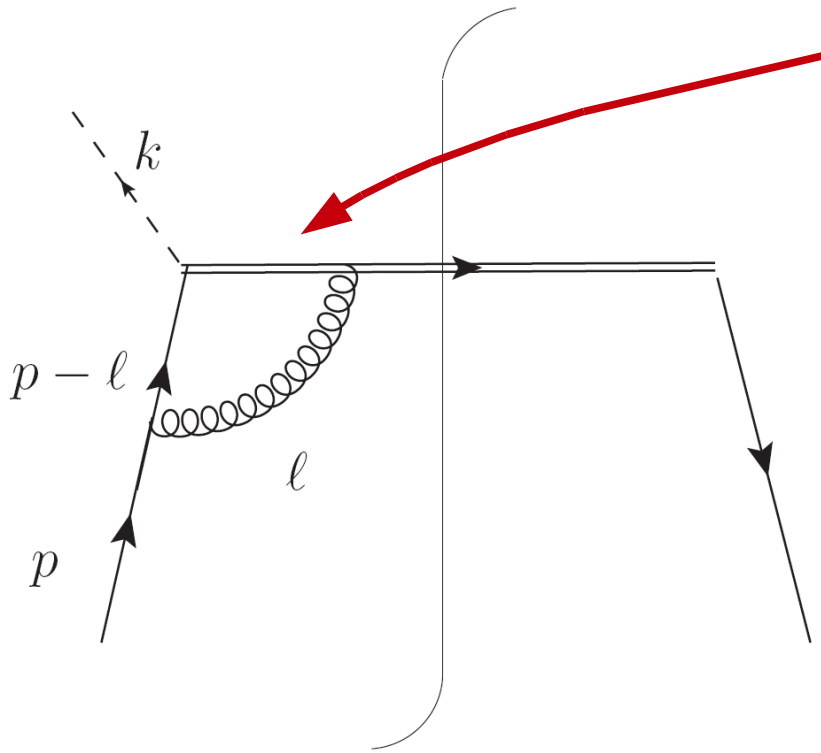
Transverse motion of partons

It seems too easy...



In fact it is not easy...



If we consider NLO corrections this diagram diverges

$$\ell = (1-\alpha)p$$

$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

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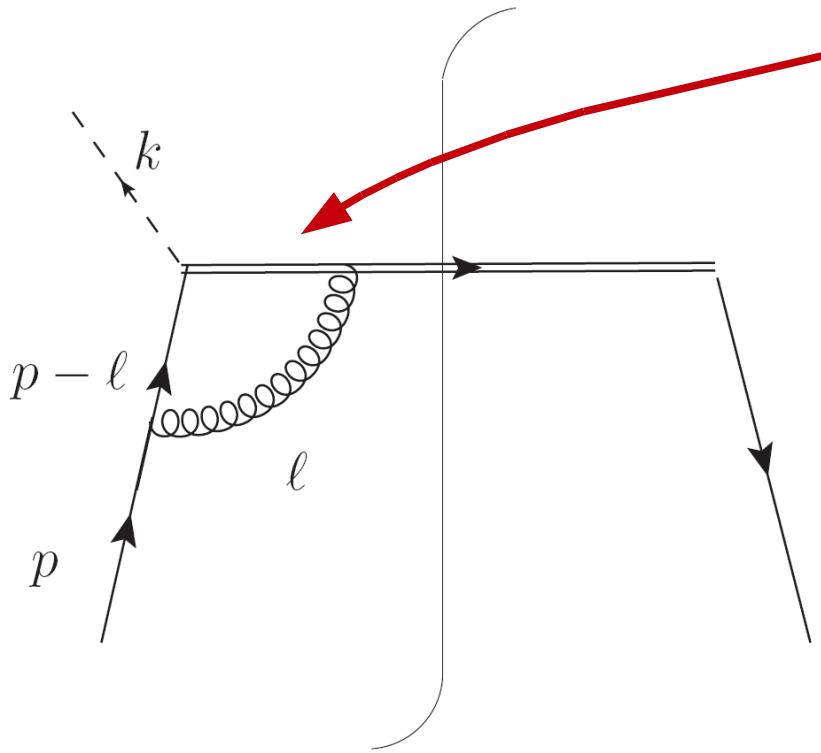

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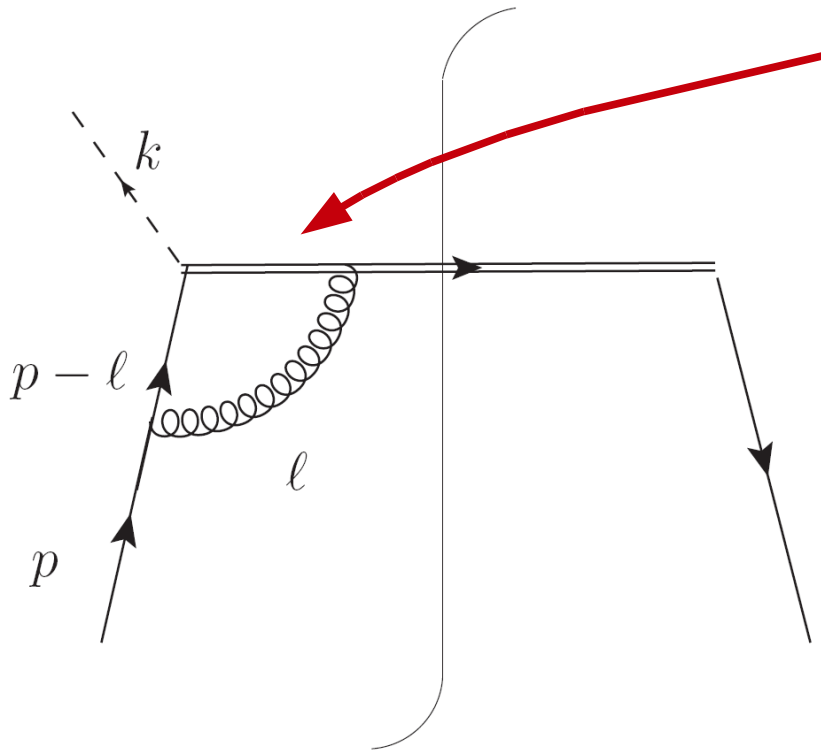

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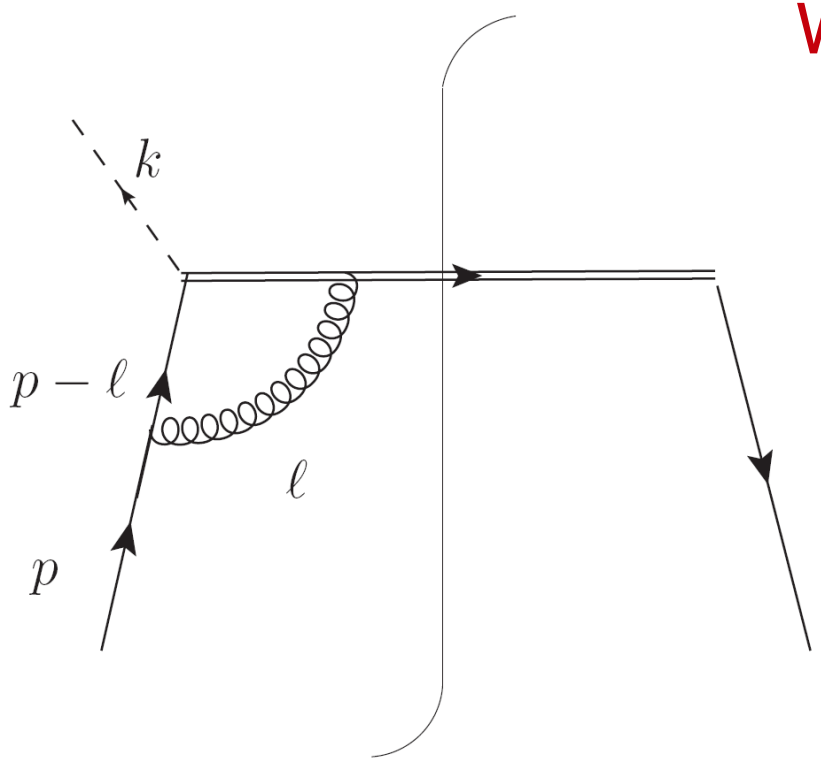
$$\propto \int_0^1 d\alpha \frac{\alpha}{1-\alpha}$$

Physics: The gluon becomes collinear to the Wilson line (struck quark) and its rapidity goes to $-\infty$

"Rapidity divergence"

In fact it is not easy...



We know how to deal with it:

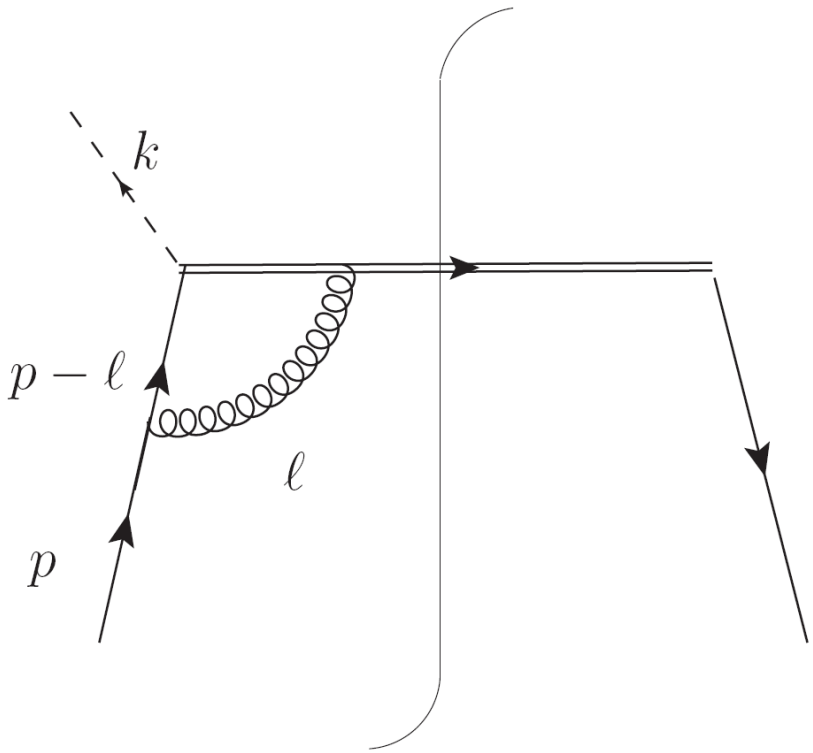
$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha) - T(1)}{(1-\alpha)}$$

"+ prescription"

$$T(\alpha = 1) - T(1) = 0$$

In fact it is not easy...



Not working for TMDs:

$$\propto \int_0^1 d\alpha \frac{\alpha}{(1-\alpha)_+} T(\alpha, k_\perp) =$$

$$= \int_0^1 d\alpha \frac{\alpha T(\alpha, k_\perp) - T(1, 0_\perp)}{(1-\alpha)}$$

"+ prescription"

$$T(\alpha = 1, k_\perp) - T(1, 0_\perp) \neq 0$$

John Collins, Acta Phys.Polon. B34 (2003) 3103

Road to TMDs

Road to TMDs

Collinear factorization, 1980's

Road to TMDs

Collinear factorization, 1980's

TMD formulation, 1990's

Road to TMDs

Collinear factorization, 1980's

TMD formulation, 1990's

First proofs of factorization and experiment, 2000's

Road to TMDs

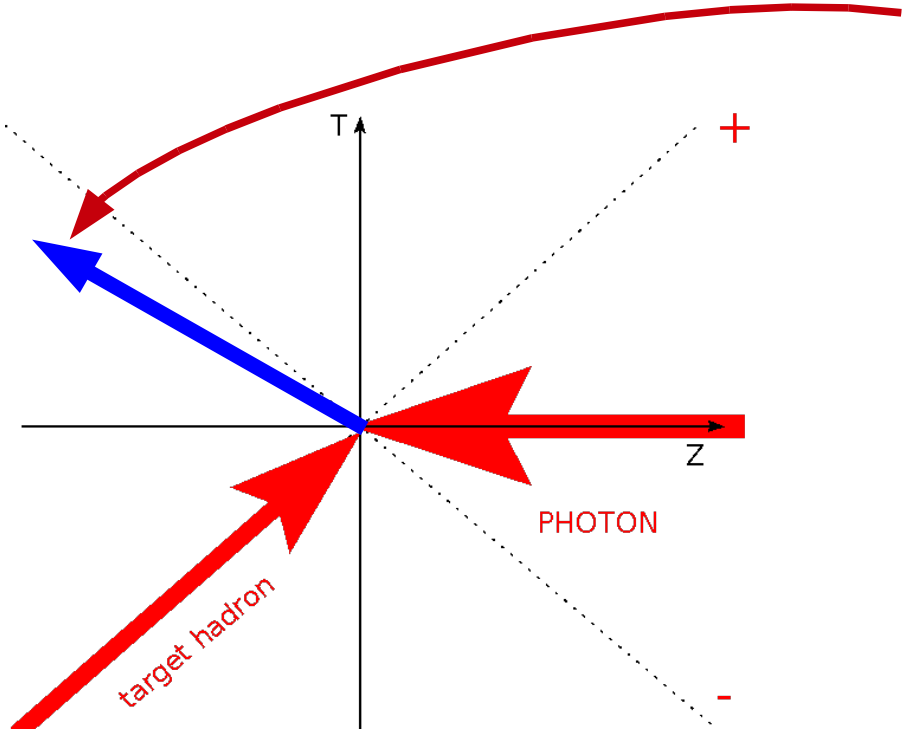
Collinear factorization, 1980's

TMD formulation, 1990's

First proofs of factorization and experiment, 2000's

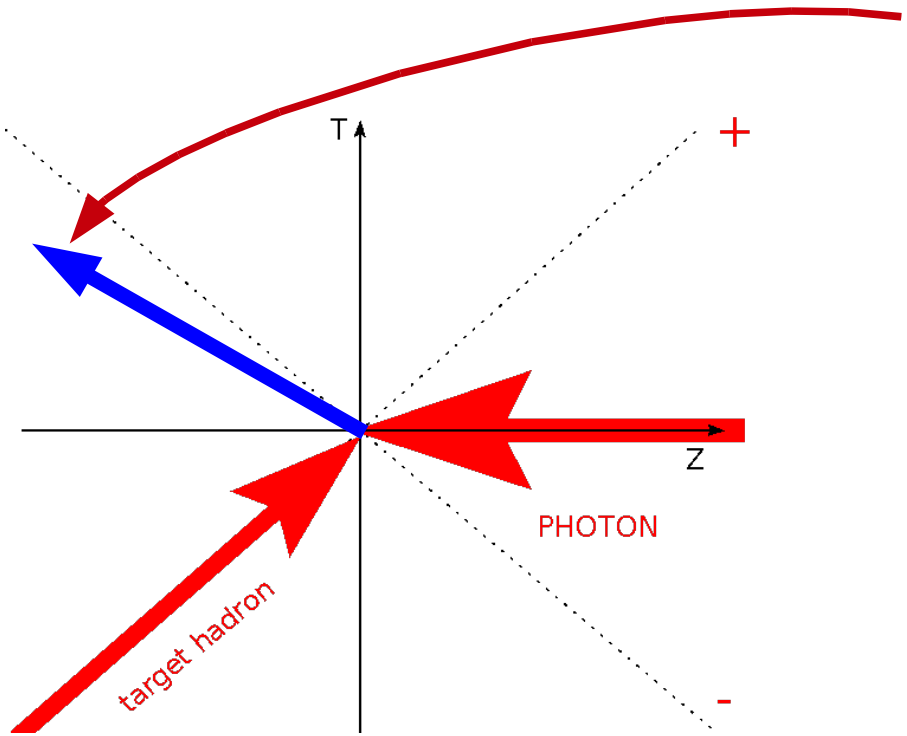
Rigorous proof of factorization and NLO, 2011-now

Definition of TMDs



Tilt slightly to regularize rapidity divergence (tilt into the space-like region as overwise gluons can be still collinear)

Definition of TMDs

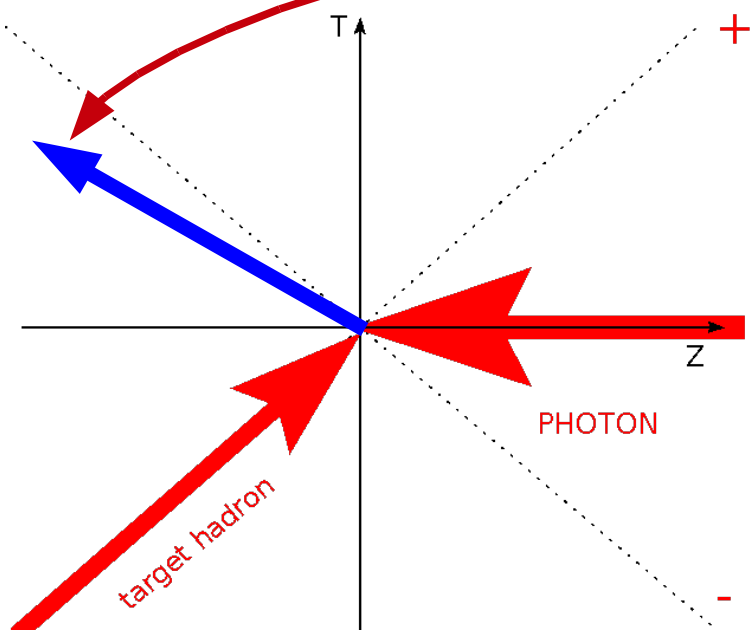


Tilt slightly to regularize rapidity divergence

$$f(x, k_\perp; \mu, \zeta)$$

Definition of TMDs

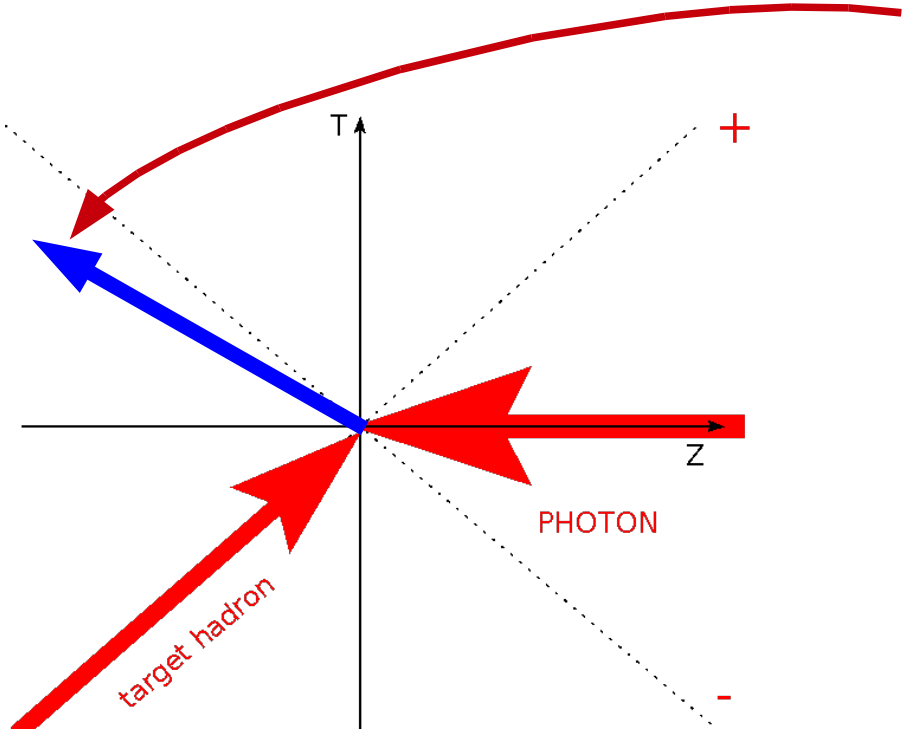
Tilt slightly to regularize rapidity divergence



$$f(x, k_\perp, \mu, \zeta)$$

Renormalization Group (RG) parameter

Definition of TMDs



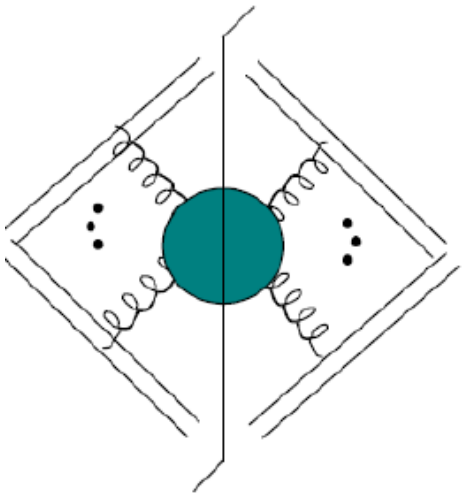
Tilt slightly to regularize rapidity divergence

Additional parameter to cancel rapidity divergence

$$f(x, k_\perp; \mu, \zeta)$$

Definition of TMDs

We also need to add something:
Soft Factor to implement subtractions



$$= S(y_A, y_B)$$

Definition of TMDs

Cross section becomes

$$\sigma \propto \mathcal{H} \cdot f(x, k_\perp; \mu, \zeta) \otimes D(z, p_\perp; \mu, \zeta)/S(+\infty, -\infty)$$

Drawback: We want to separate it into Distribution * Fragmentation !

Definition of TMDs

Cross section becomes

$$\sigma \propto \mathcal{H} \cdot \frac{f(x, k_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}} \otimes \frac{D(z, p_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}}$$

Distribution * Fragmentation !

Definition of TMDs

Cross section becomes

$$\sigma \propto \mathcal{H} \cdot \frac{f(x, k_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}} \otimes \frac{D(z, p_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}}$$

Distribution * Fragmentation !

“Famous” square root

TMD evolution

$$\frac{\partial \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_\perp, \mu)$$

John Collins, 2011

$$\frac{d\tilde{K}(b_\perp, \mu)}{d \ln \mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_\perp, \mu, \zeta)}{d \ln \mu} = \gamma_F(g(\mu), \zeta)$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Solve evolution equations:

$$\widetilde{f}_1^f(x, b_T; \mu, \zeta_F) = \sum_i (\tilde{C}_{f/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu, \zeta_F)} e^{g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{f0}}}} \hat{f}_{\rm NP}^q(x, b_T)$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

Calculate everything at NLO:

$$\tilde{C}_{j'/j}(x,\boldsymbol{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{\ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \frac{1}{2}(1-x)+\right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x,\boldsymbol{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{\ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\left(\frac{1+x^2}{1-x}\right)_+ + \frac{1}{2}(1-x)+\right.$$
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$$\delta\tilde{C}_{j'/j}(x,\boldsymbol{b}_T;\mu;\zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\rm F}}{\pi}\left\{\ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\left(\frac{2x}{1-x}\right)_+ +\right.$$
$$\left. + \delta(1-x)\left[-\ln^2\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right) + \ln\left(\frac{2e^{-\gamma_{\rm E}}}{\mu b_T}\right)\ln\left(\frac{\zeta_F}{\mu^2}\right)\right]\right\} + \mathcal{O}(\alpha_s^2)\,.$$

TMD evolution:helicity and transversity

A. Bacchetta, AP, 2013

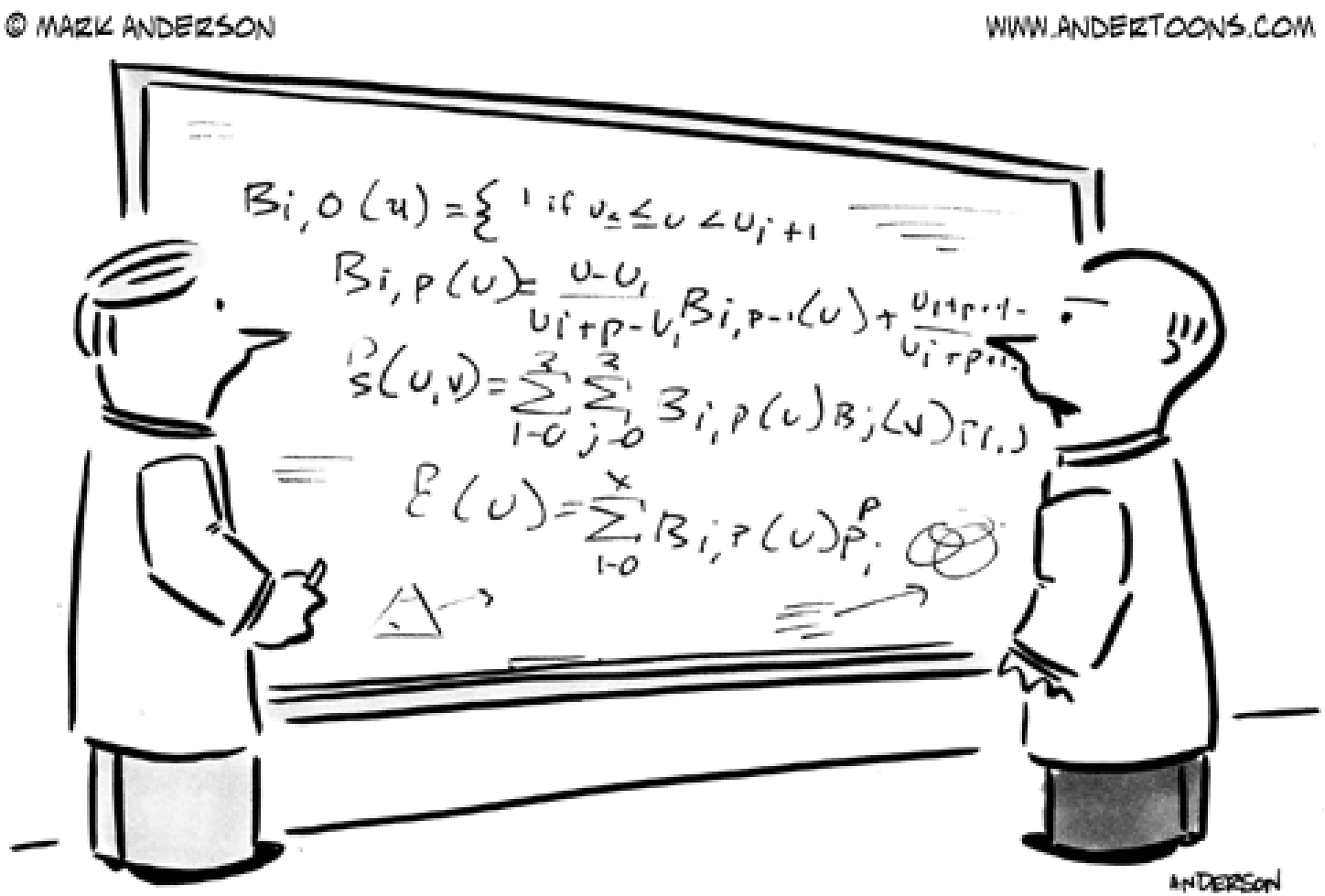
Simplify:

$$\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\Delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \delta_{j'j}\frac{\alpha_s C_{\mathrm{F}}}{2\pi}(1-x) + \mathcal{O}(\alpha_s^2)\,,$$

$$\delta\tilde{C}_{j'/j}(x, b_*; \mu_b) = \delta_{j'j}\delta(1-x) + \mathcal{O}(\alpha_s^2).$$

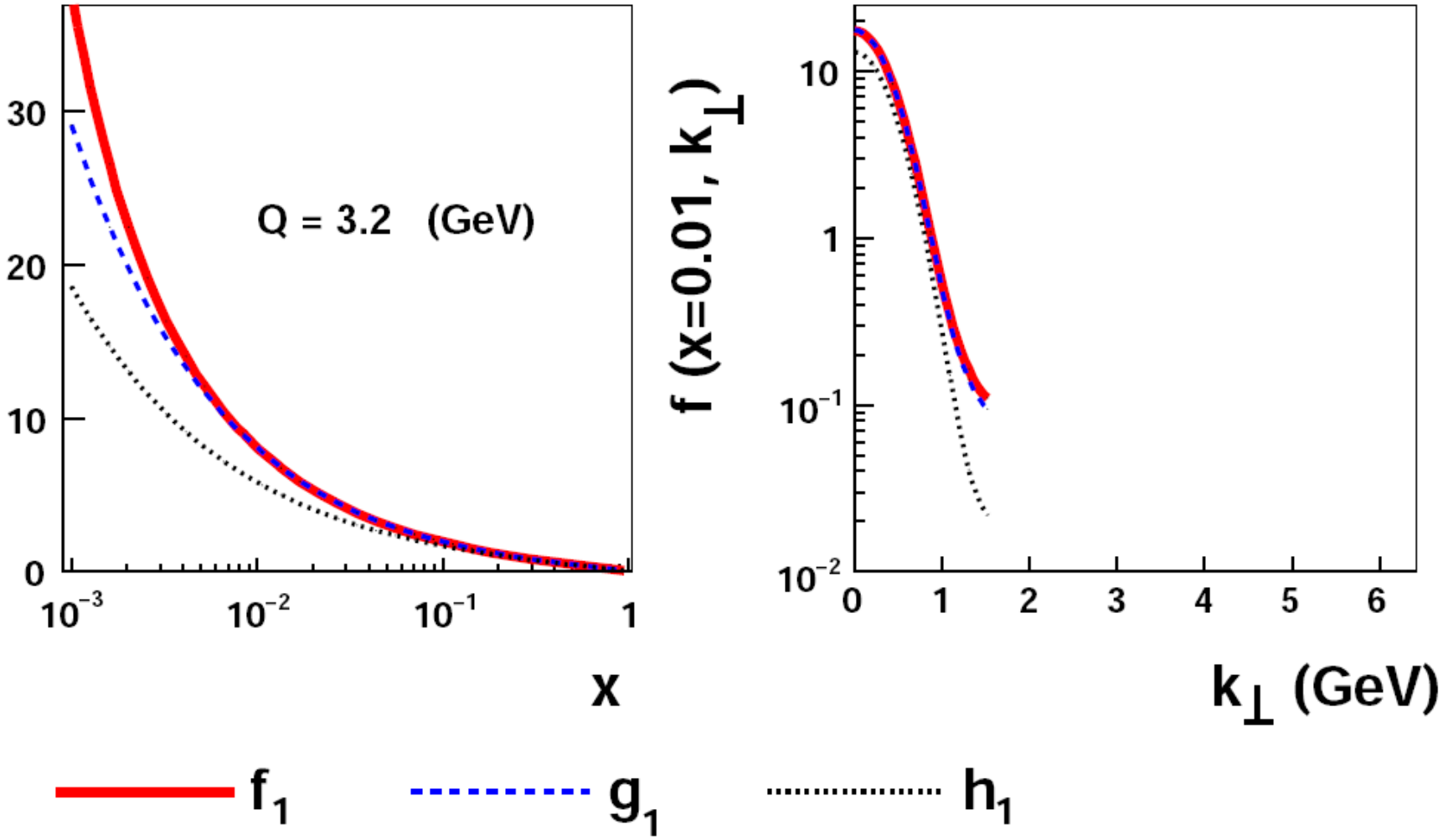
What does it mean?



"What the hell is *that* supposed to mean?!"

Phenomenology

A. Bacchetta, AP, 2013



Phenomenology

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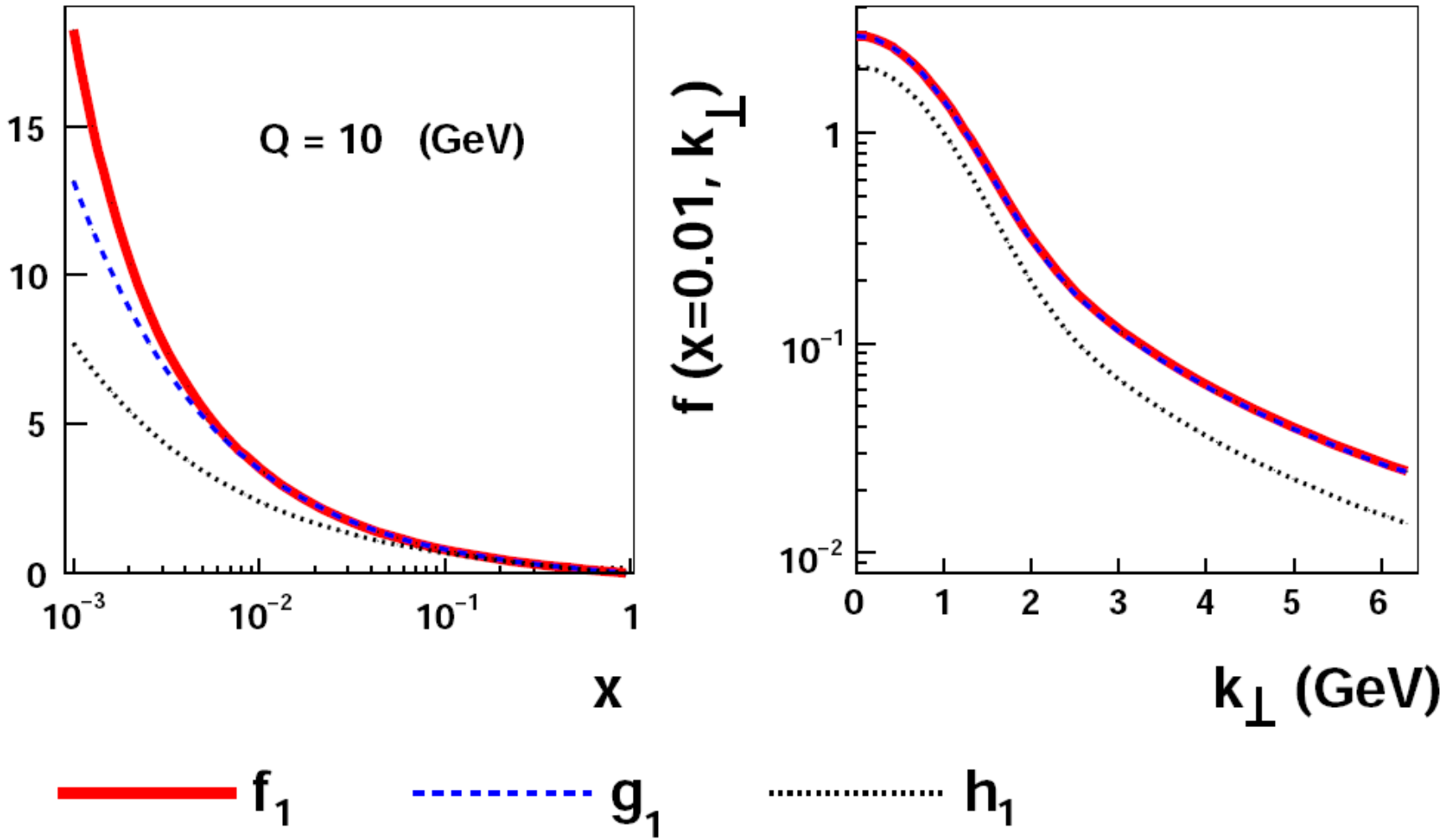

Phenomenology

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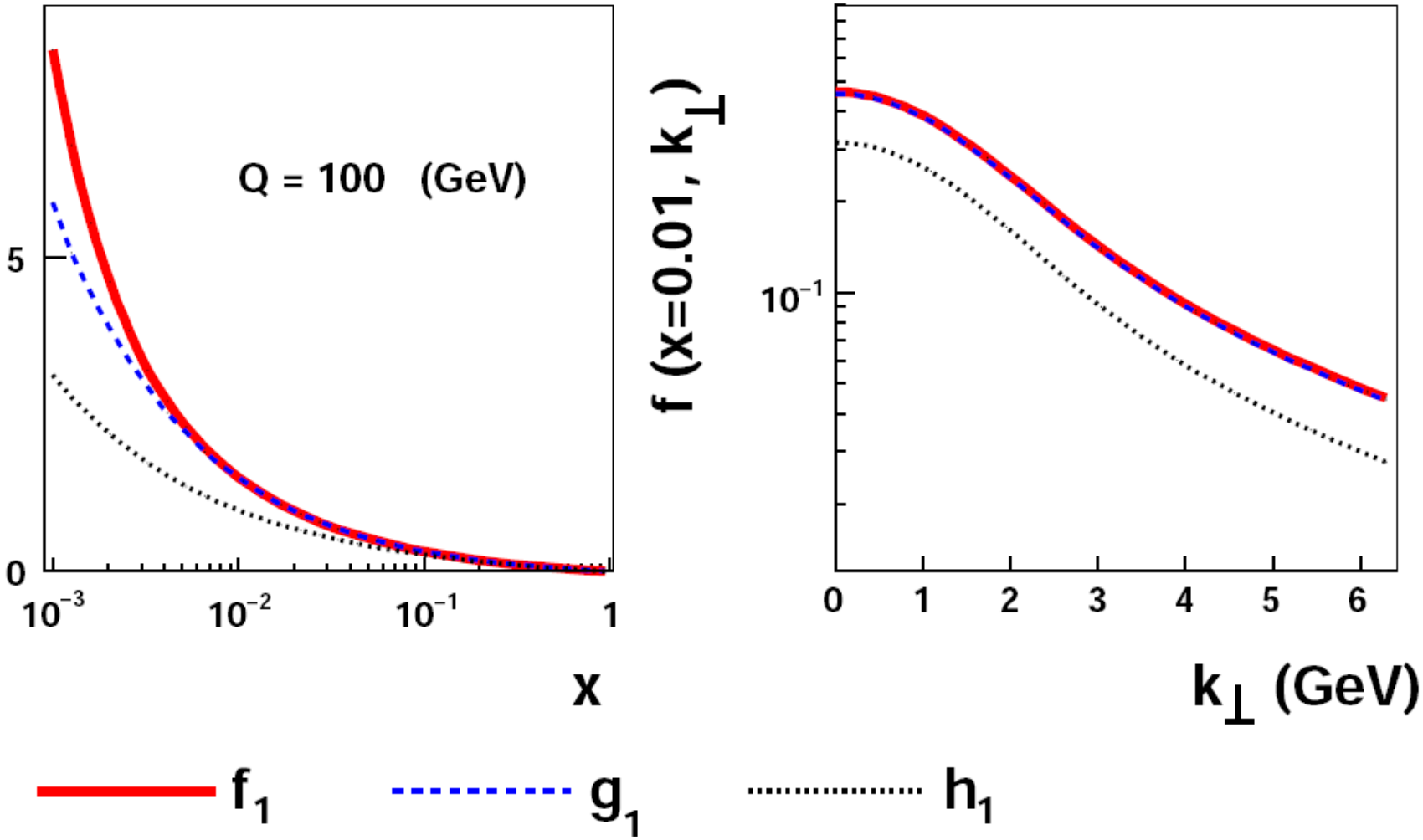

Phenomenology

It is extremely important to test our knowledge by **predicting** results of future measurements

Phenomenology

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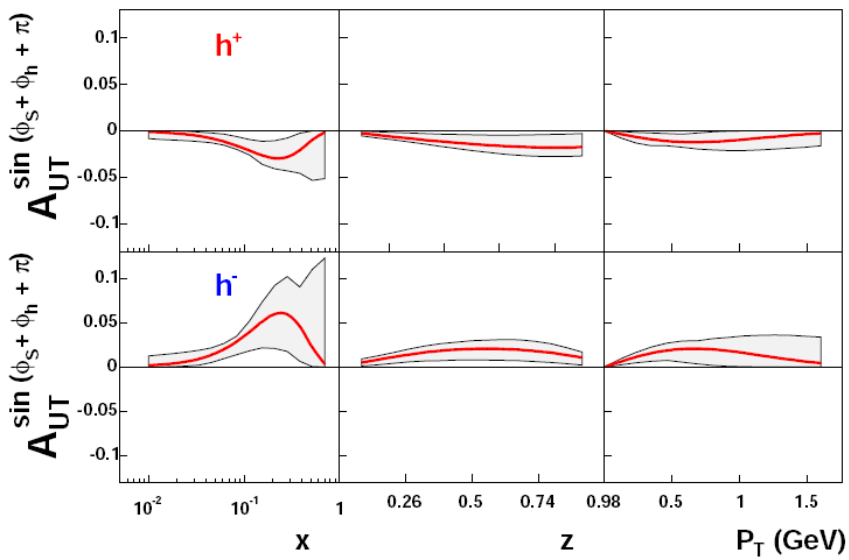
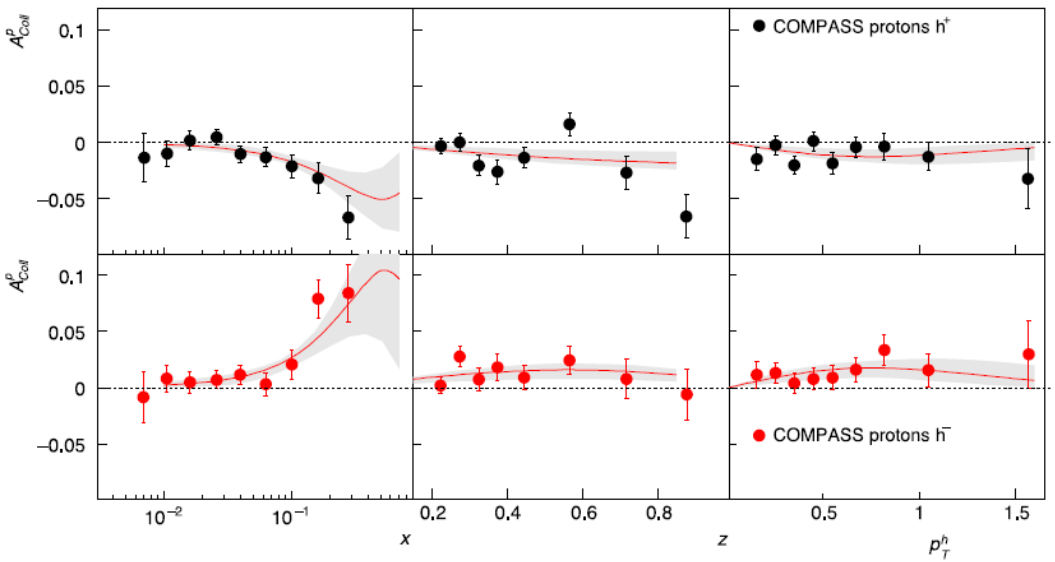


Prediction
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, **AP**, Turk
PRD75 (**2007**) 054032

Phenomenology

It is extremely important to test our knowledge by **predicting** results of future measurements

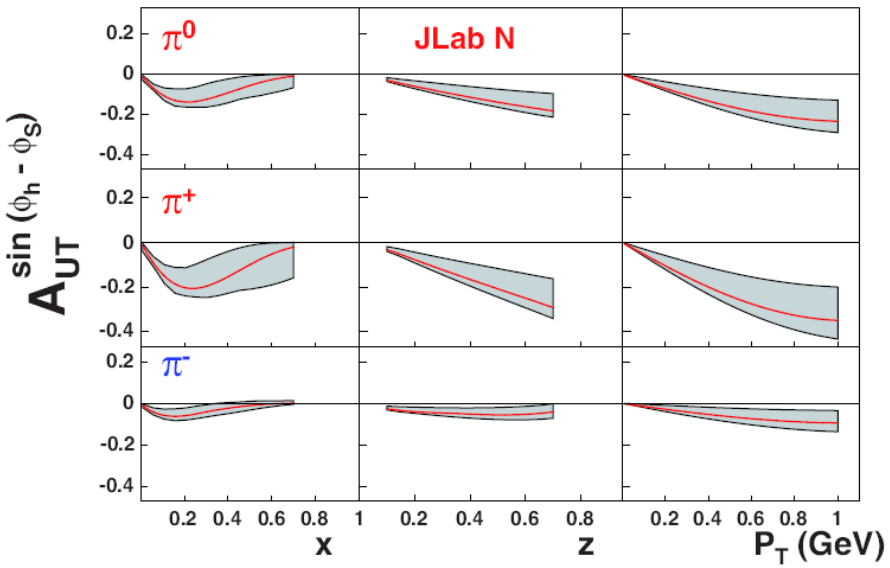


Prediction
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, **AP**, Turk
PRD75 (**2007**) 054032

Measurement
COMPASS Collaboration CERN
PLB717 (**2012**) 376-382

Phenomenology

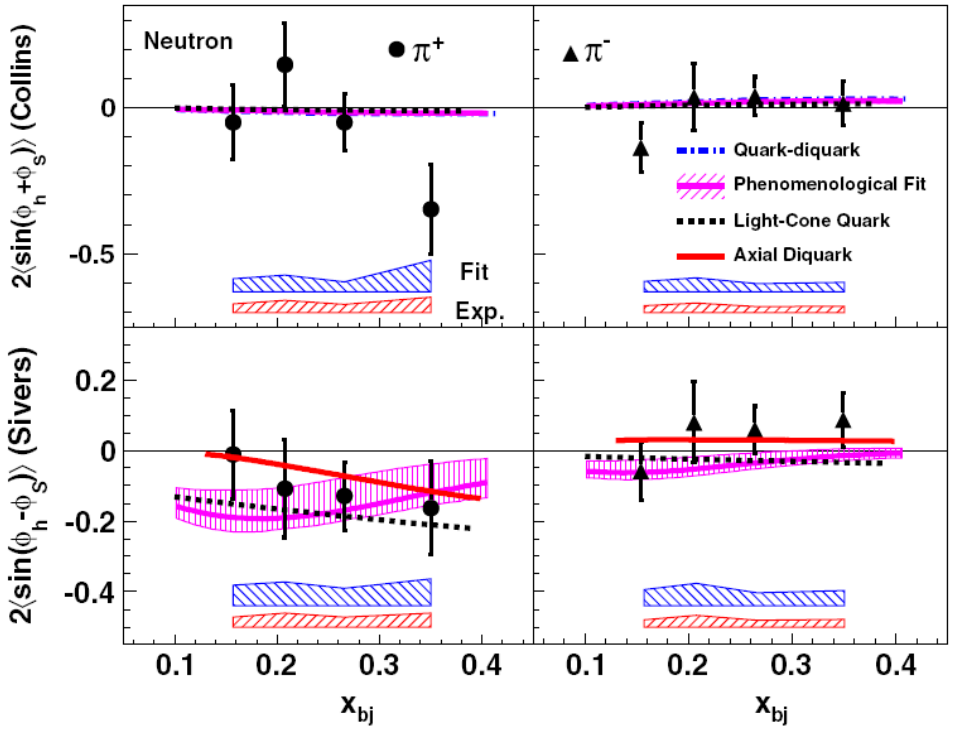
It is extremely important to test our knowledge by **predicting** results of future measurements



Prediction
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Melis, **AP**, Turk
EPJA 39 (**2009**) 89-100

Phenomenology

It is extremely important to test our knowledge by **predicting** results of future measurements



Prediction

Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Melis, **AP**, Turk

EPJA 39 (**2009**) 89-100

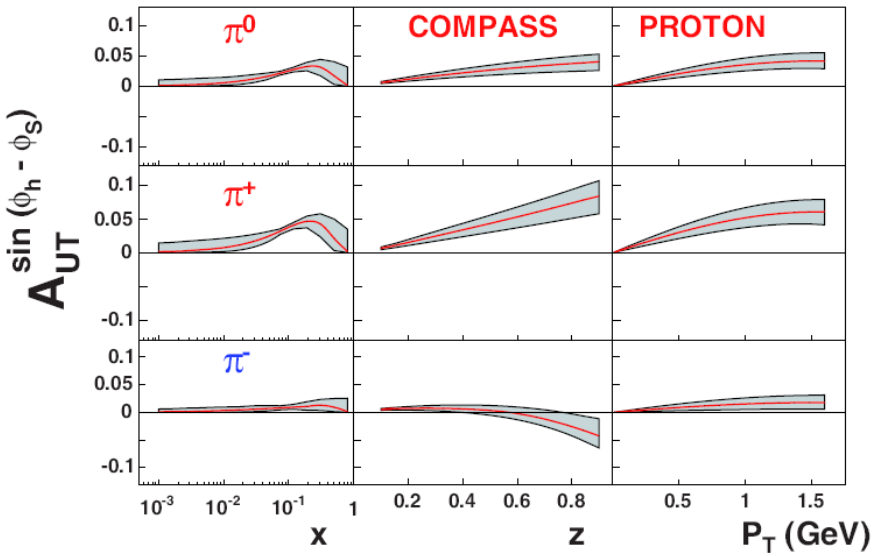
Measurement

X. Qian et all (JLab HALL A Coll)

PRL 107 (**2011**) 072003

Phenomenology

It is extremely important to test our knowledge by **predicting** results of future measurements



Prediction
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Melis, **AP**, Turk
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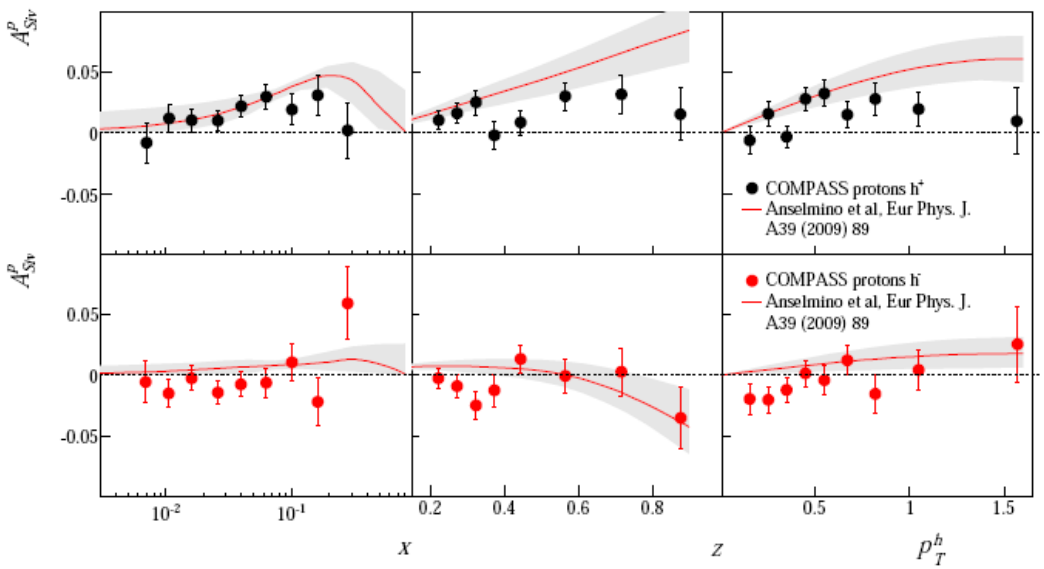
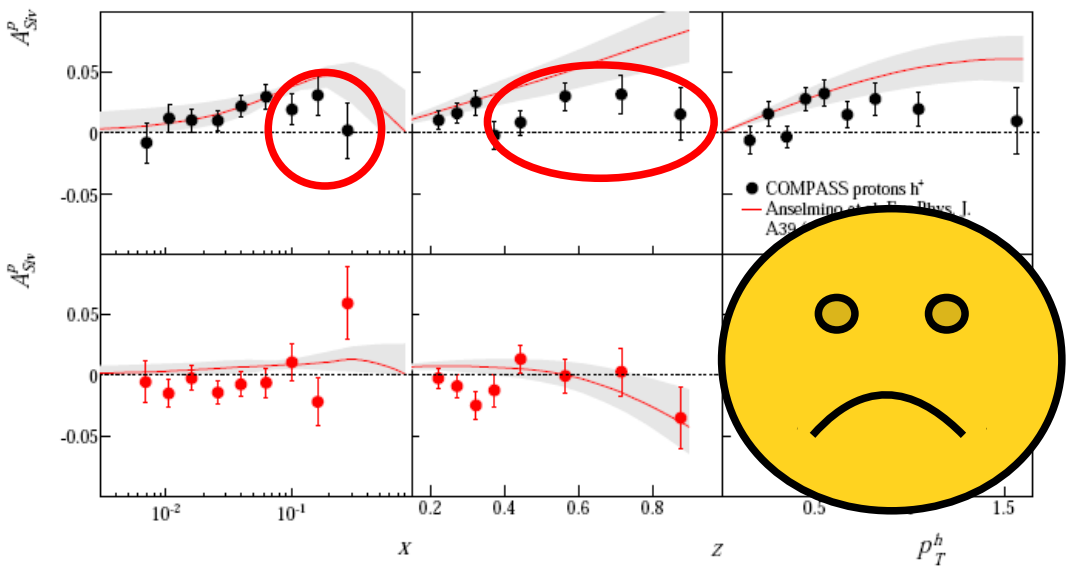


Prediction
Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Melis, **AP**, Turk
EPJA 39 (**2009**) 89-100

Measurement
Compass Collaboration
Fizika B20 (**2011**) 93-106

Phenomenology

It is extremely important to test our knowledge by **predicting** results of future measurements



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Compass Collaboration
Fizika B20 (**2011**) 93-106

Phenomenology

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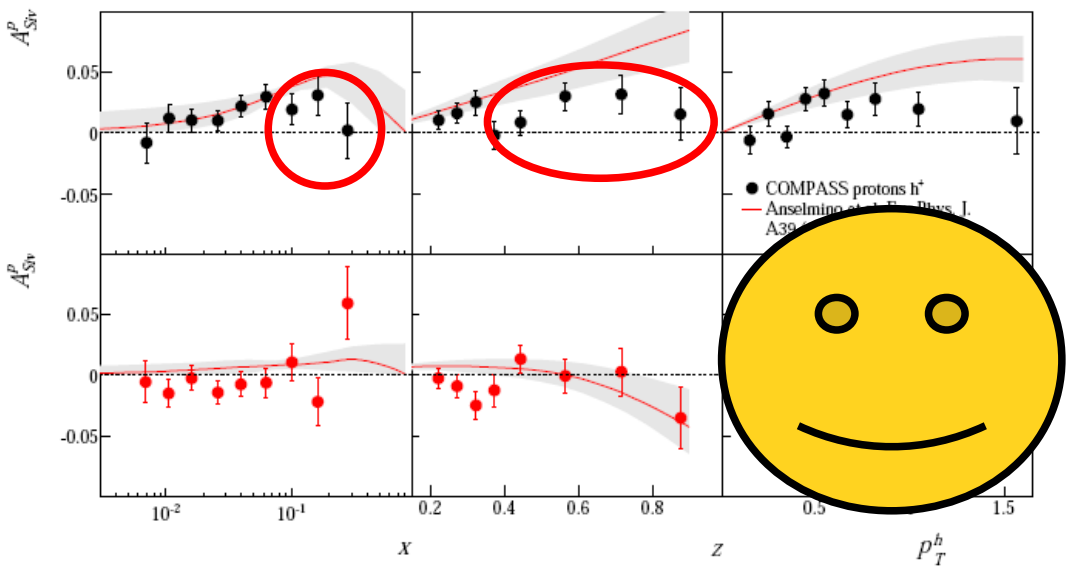

Prediction
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EPJA 39 (**2009**) 89-100

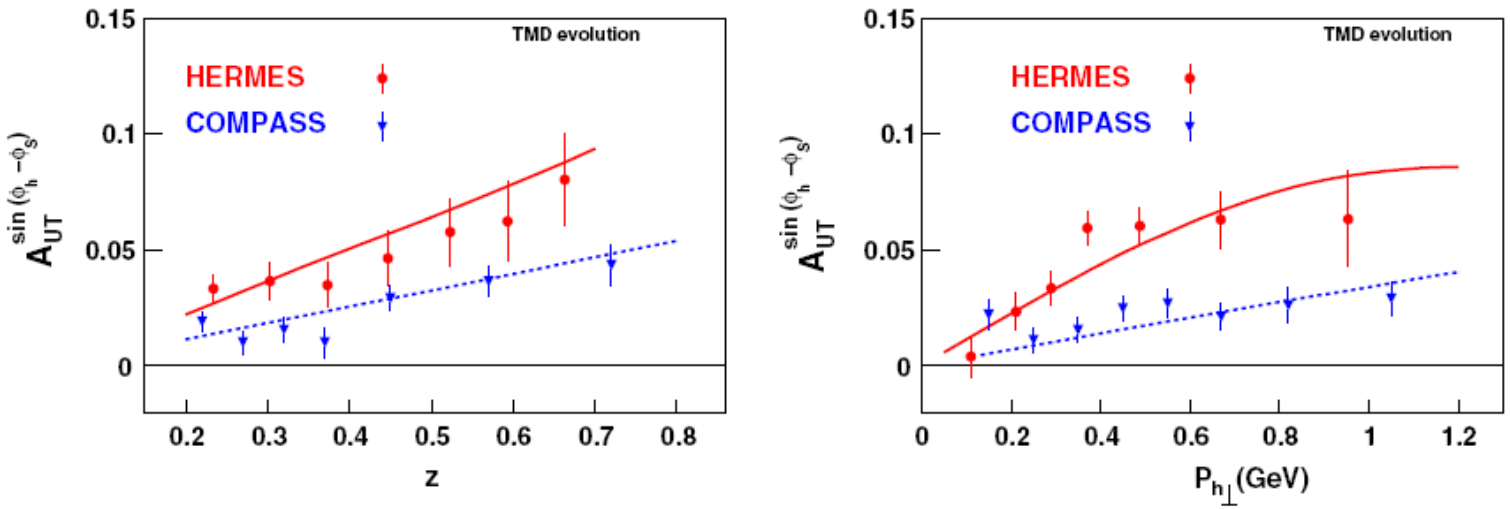
Measurement
Compass Collaboration
Fizika B20 (**2011**) 93-106

This is a chance to learn something new!

Phenomenology

It is extremely important to test our knowledge by **explaining** results of future measurements

TMD evolution taken into account:



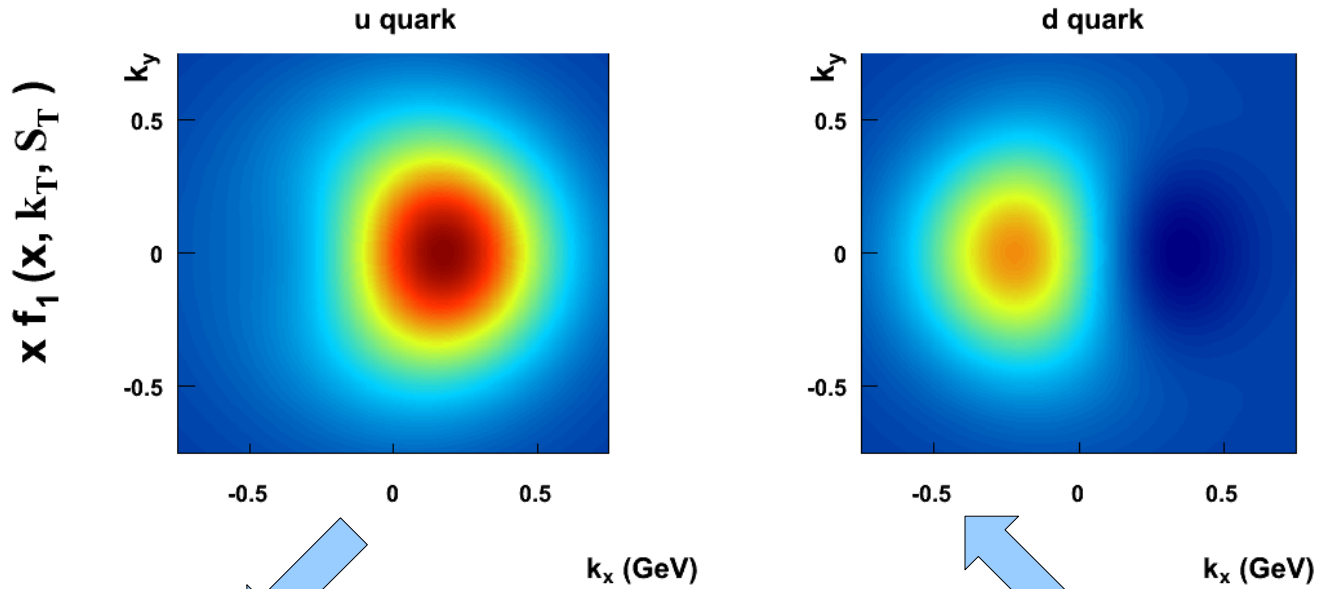
Mert Aybat, **AP**, Ted Rogers, PRL 108 (**2012**) 242003

This is a chance to learn something new!

How does phenomenology work?

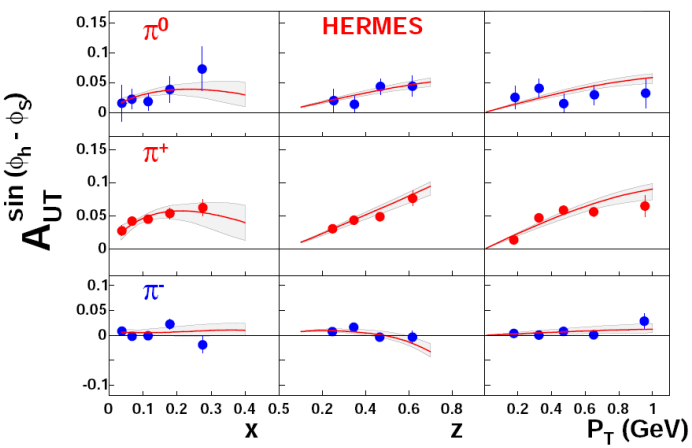
Tomographic scan of the nucleon

Theory



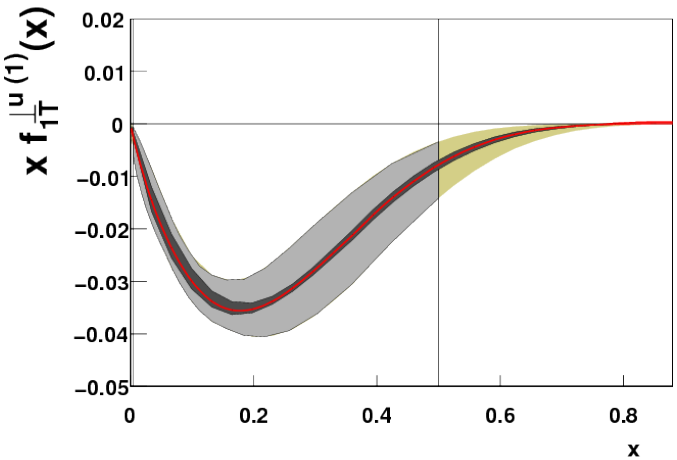
AP 2012

Measurement



Global analysis of the data

Results



Can motion of quarks and spin of the proton be correlated?

Yes, it can!

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\epsilon_T^{ij} \mathbf{k_{Ti} S_{Tj}}}{M}$$

Sivers function

Yes, it can!

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0, 1)$

Deformation in momentum space is: $x \cdot f(x^2 + y^2)$

This is called “dipole” deformation.

Yes, it can!

$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

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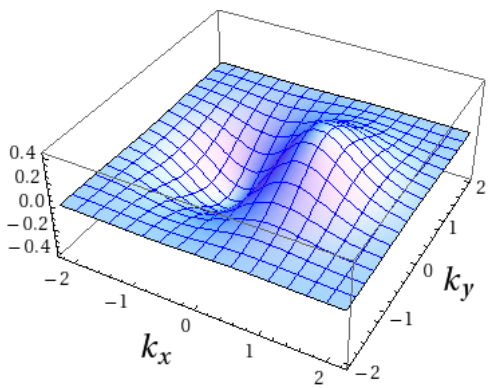

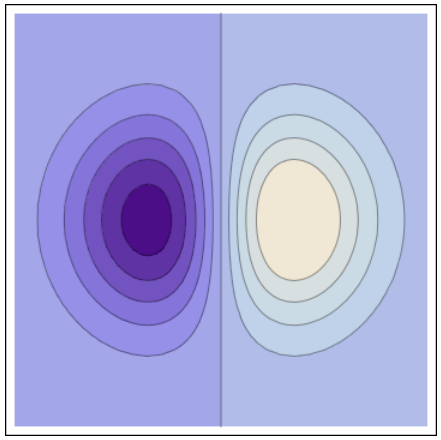

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$$f(x, \mathbf{k_T}, \mathbf{S_T}) = f_1(x, \mathbf{k_T^2}) - f_{1T}^{\perp}(x, \mathbf{k_T^2}) \frac{\mathbf{k_x}}{M}$$

Suppose the spin is along Y direction: $S_T = (0, 1)$

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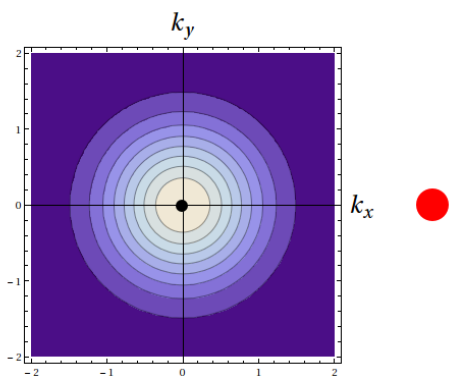
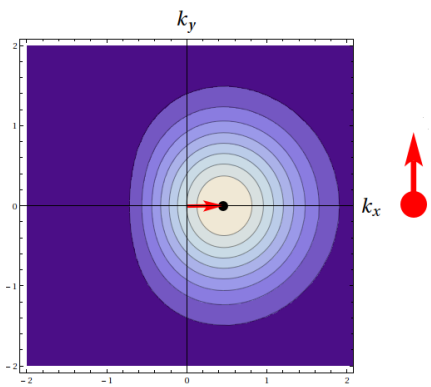
This is called “dipole” deformation.

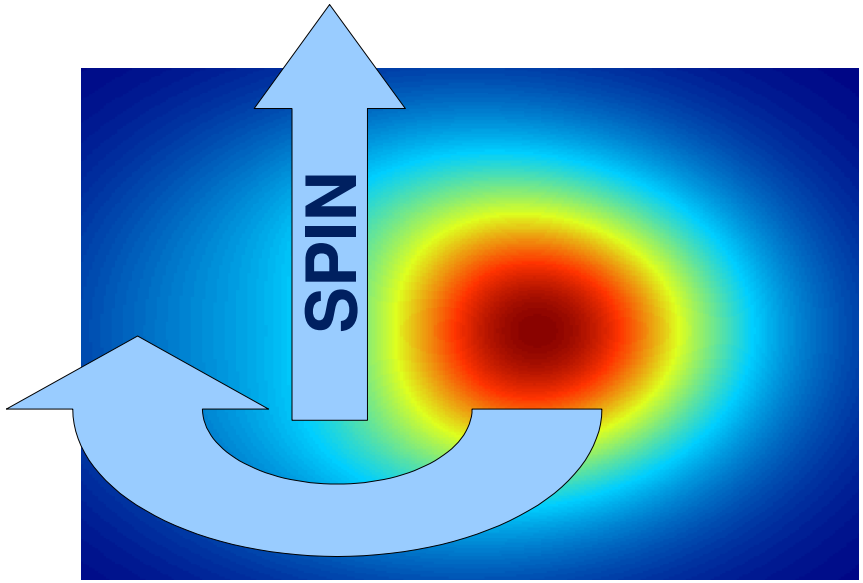
No correlation:



Correlation:



Tomographic scan of the nucleon



Internal motion of quarks is correlated with the spin of the proton!

What we will achieve at Jefferson Lab 12?

What will be achieved? Example:

Expected result for tensor charge extraction:

1 - JLab 12
2 - Anselmino et al., Nucl.Phys.Proc.Suppl. (2009)
3 - Cloet, Bentz and Thomas, Phys.Lett.B (2008)
4 - Wakamatsu, Phys.Lett.B (2007)
5 - Gockeler et al., Phys.Lett.B (2005)
6 - He and Ji, Phys. Rev. D (1995)
7 - Pasquini et al, Phys. Rev. D (2007)
8 - Gamberg and Goldstein, Phys. Rev. Lett. (2001)
9 - Hecht, Roberts and Schmidt Phys. Rev. C (2001)

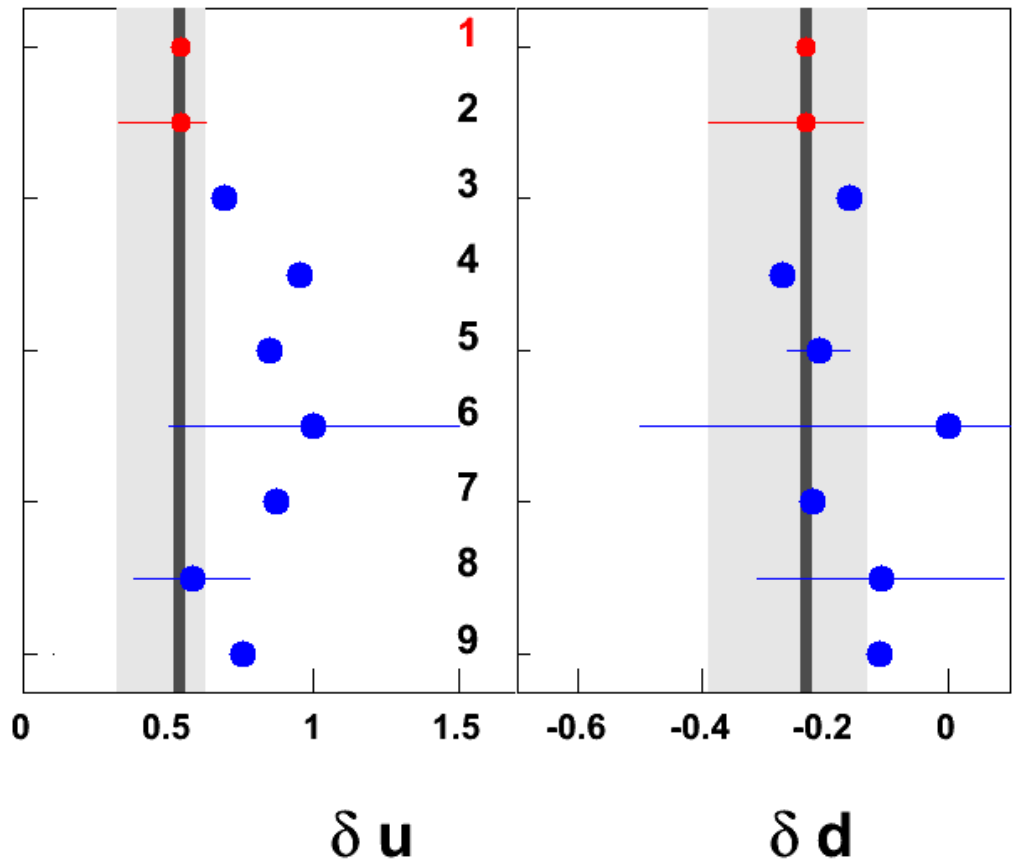
$$\delta q = \int dx \, (h^{q}(x) - h_1^{\bar{q}}(x))$$

$$\delta u = 0.54^{+0.09}_{-0.22}, \; \delta d = -0.23^{+0.09}_{-0.16}$$

JLab 12 Proton and He^3 targets

$$\delta u = 0.54^{+0.02}_{-0.02}, \; \delta d = -0.23^{+0.01}_{-0.01}$$

Statistical errors only



A. Prokudin (2012) contribution
To JLab12 white paper

New experimental methods are important

SIDIS cross section can be Fourier transformed experimetally

Boer, Gamberg, Musch,AP 2011

Complicated convolution

$$\sigma \propto \mathcal{H} \cdot \frac{f(x, k_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}} \otimes \frac{D(z, p_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}}$$

New experimental methods are important

SIDIS cross section can be Fourier transformed experimetally

Boer, Gamberg, Musch,AP 2011

Complicated convolution

$$\sigma(P_T) \propto \mathcal{H} \cdot \frac{f(x, k_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}} \otimes \frac{D(z, p_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}}$$

Experimental Fourier transform

$$\tilde{\sigma}(b_\perp) \propto \mathcal{H} \cdot \frac{\tilde{f}(x, b_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}} \cdot \frac{\tilde{D}(z, b_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}}$$

Product!

New experimental methods are important

SIDIS cross section can be Fourier transformed experimetally

$$\tilde{\sigma}(b_\perp) \propto \mathcal{H} \cdot \frac{\tilde{f}(x, b_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}} \cdot \frac{\tilde{D}(z, b_\perp; \mu, \zeta)}{\sqrt{S(+\infty, -\infty)}}$$

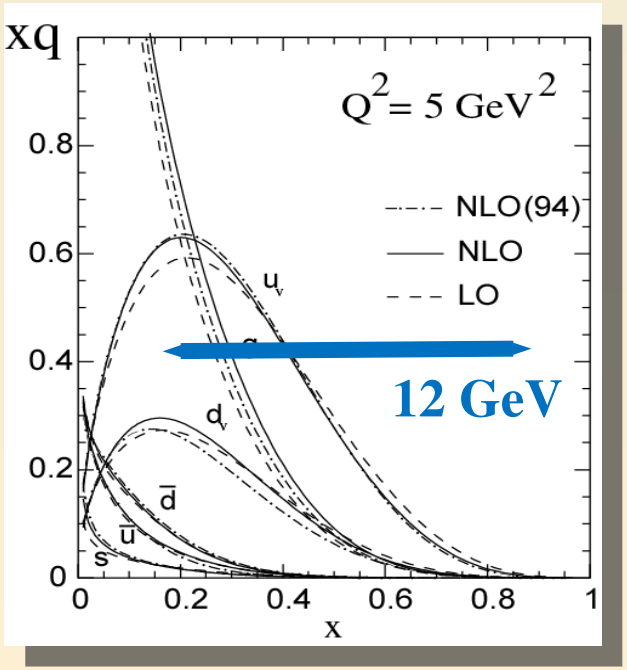
Experimental Fourier transform

Aghasyan, Avakian, Gamberg, AP, etc 2013

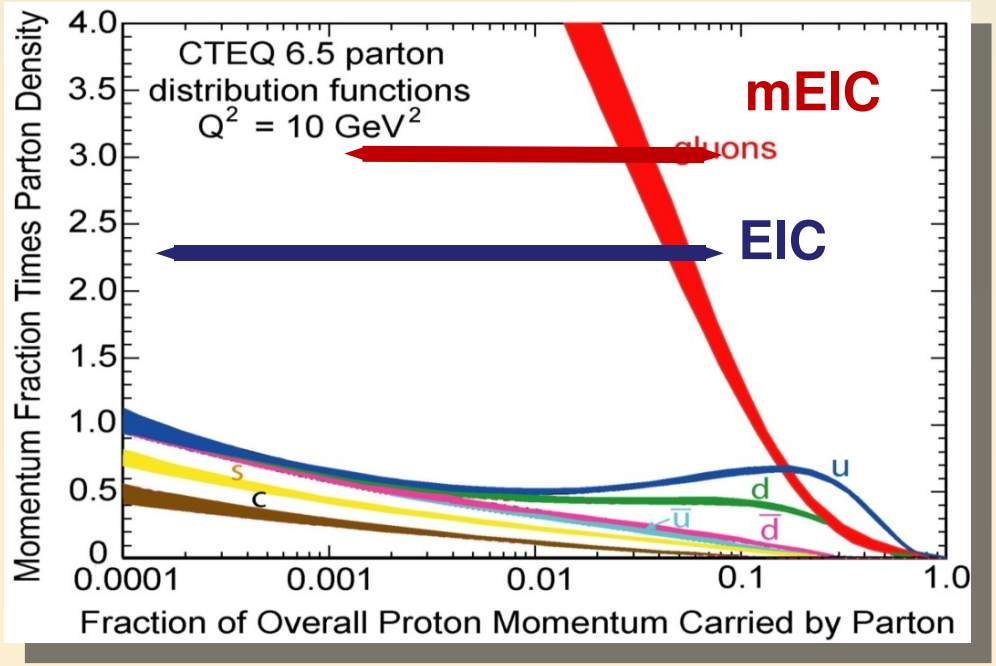
This method complements standard methods of data analysis.
It is quite general and will be useful not only for TMD community.

Into the "sea": Electron Ion Collider

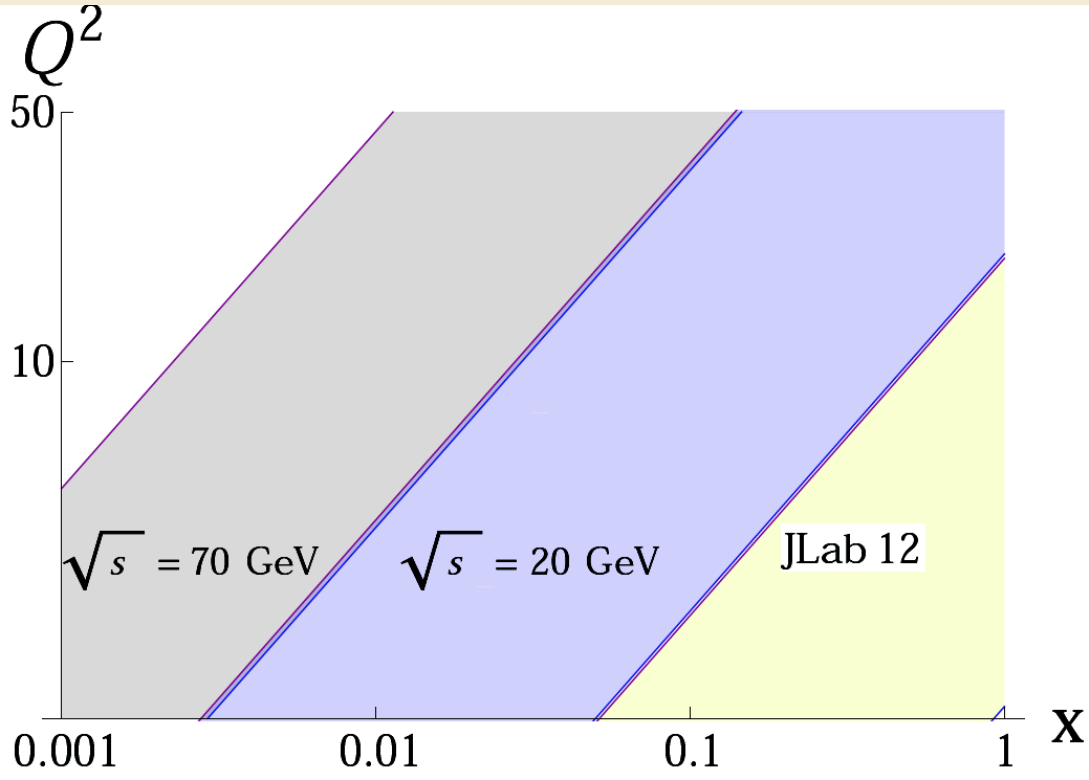
- With 12 GeV we study mostly the valence quark component



An EIC aims to study the sea quarks, gluons, and scale dependence.

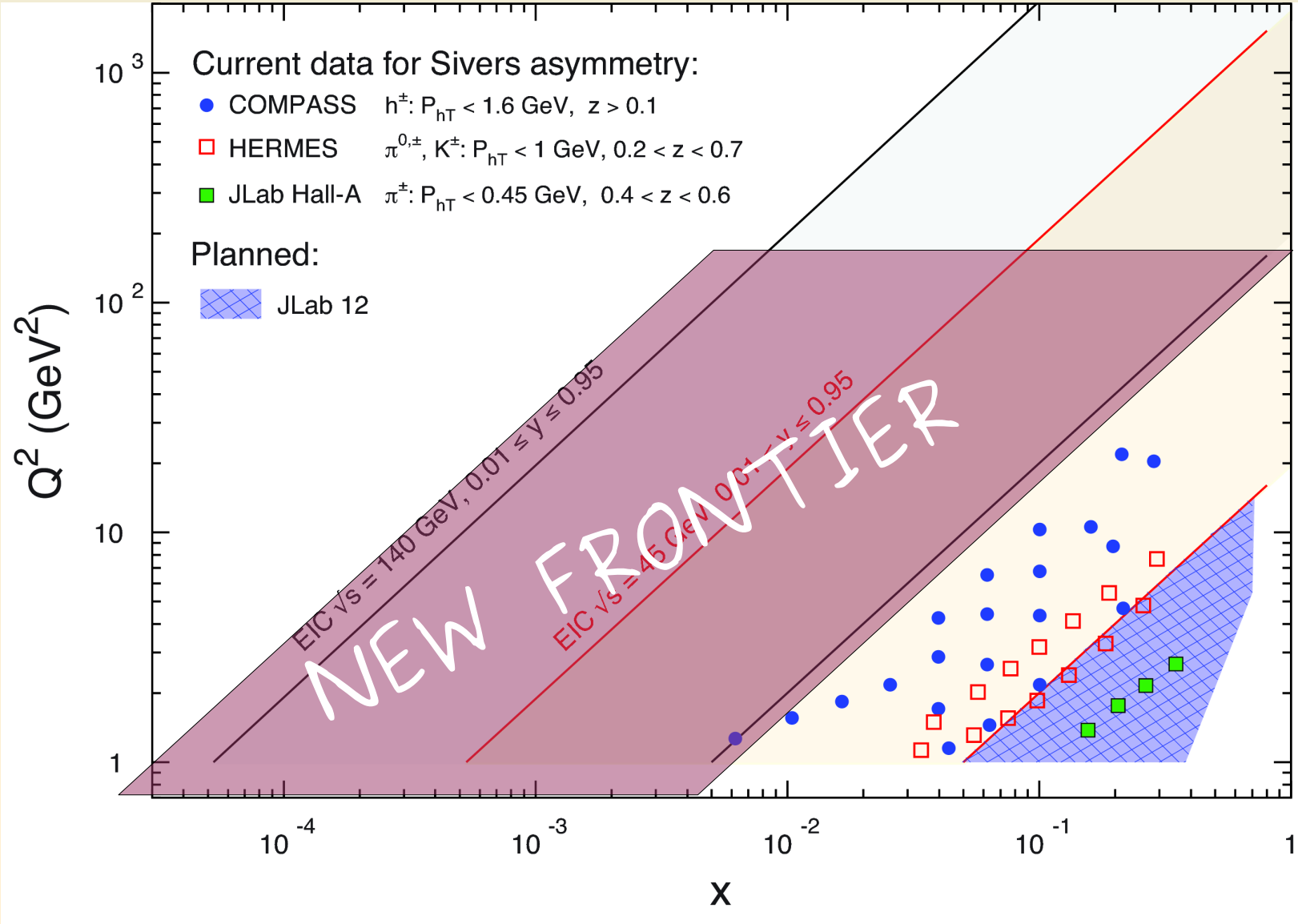


Into the "sea": Electron Ion Collider



JLab 12 and future Electron Ion Collider are complimentary

Into the "sea": Electron Ion Collider



Medium Energy EIC@JLab

Physics driven design

Spin and 3D quark/gluon structute of the hadron

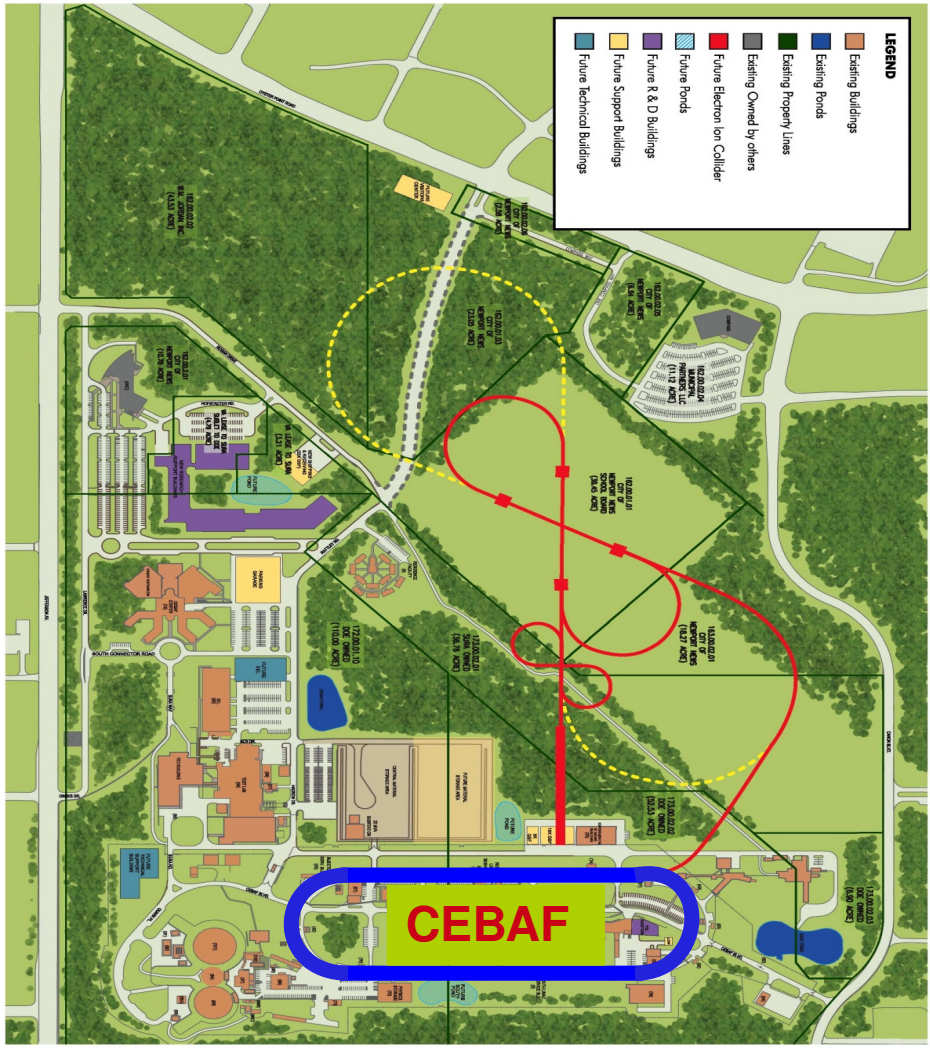
Dynamics of color fields in nuclei

Emergence of hadrons from color charge

JLab Concept

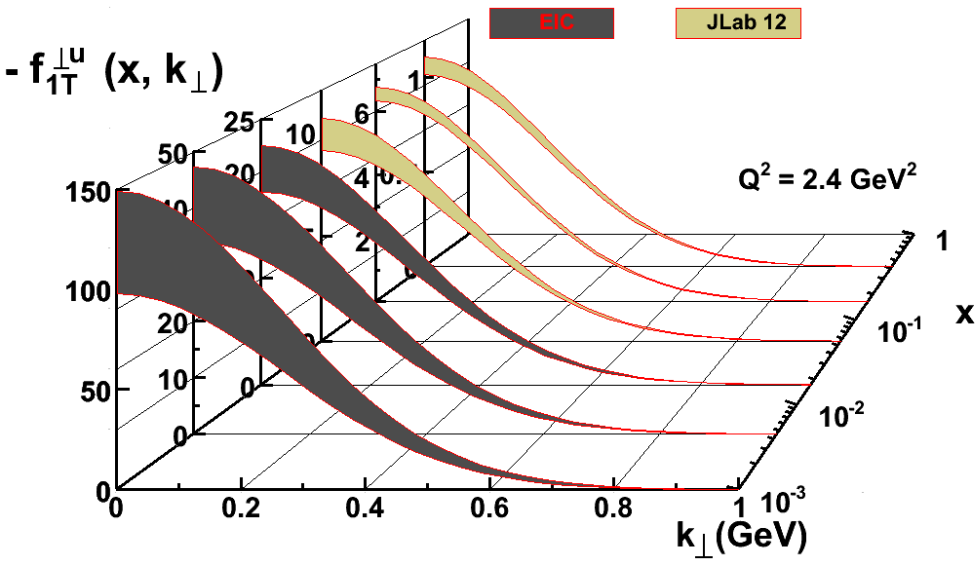
Initial configuration (MEIC):
3-12 GeV on 20-100 GeV ep/eA collider
fully-polarized, longitudinal and transverse
luminosity: few x 10^{34} e-nucleons cm^{-2} s^{-1}

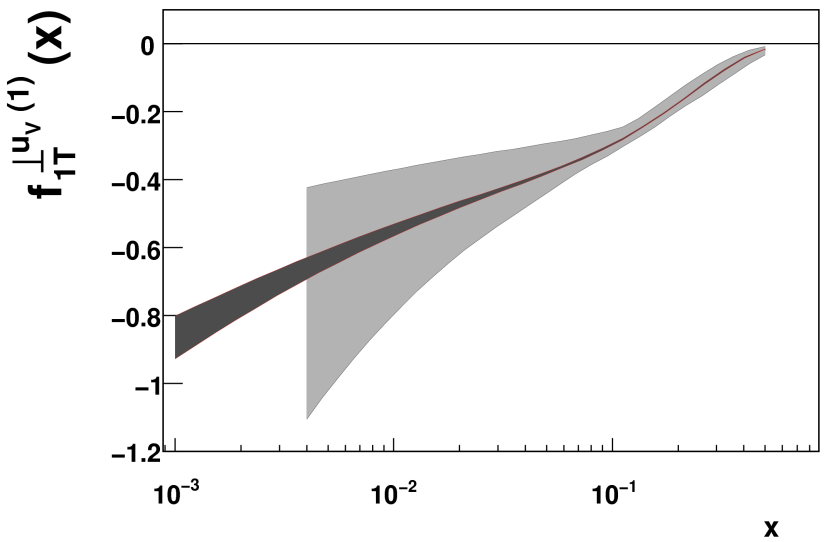
Upgradable to higher energies (250 GeV)



What will be achieved? Example:

Expected result for an individual function:

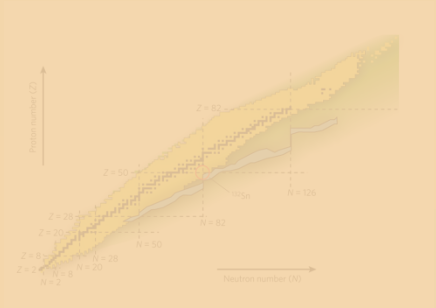


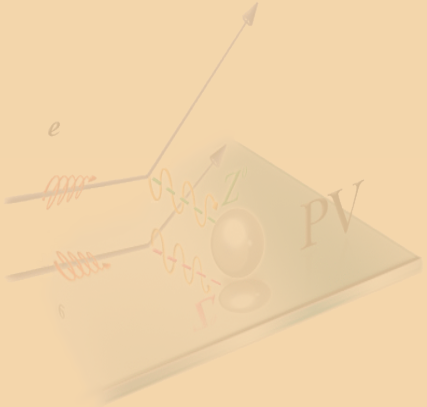


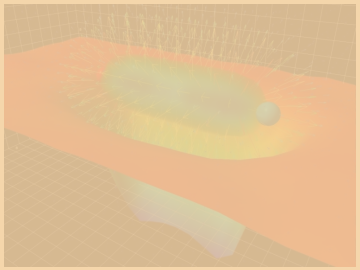
Expected accuracy of TMD profile

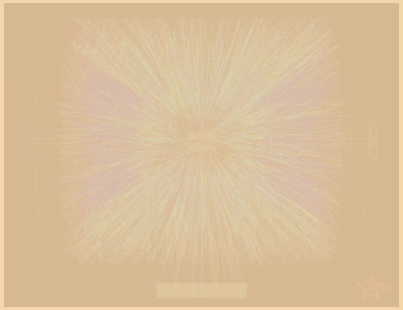
A. Prokudin (2012) contribution to EIC white paper

Conclusions









We started here...


We have become this...

Collaborations are important

PDFs

GPDs

TMDs

Experiment

Part of my collaborators is listed

LANL
Znongbo Kang
Xin Qiang

BNL
Jianwei Qiu
Elke Aeschenauer

PENN STATE
Leonard Gamberg

Jefferson Lab
Anatoly Radyushkin
Christian Weiss
Alberto Accardi
Harut Avakian
etc

UCONN
Peter Schweitzer

Stony Brook
Ted Rogers

China
Bo-Qiang Ma

NETHERLANDS
Daniel Boer

GERMANY
Werner Vogelsang
Marc Schlegel

ITALY
University of Torino
Mauro Anselmino
Stefano Melis
Elena Boglione
Vincenzo Barone
University of Cagliari
Umberto D'Alesio
Franceso Murgia
University of Pavia
Alessandro Bacchetta

Workshops


Quarks and Gluons
All the visible matter in the universe is made of quarks and gluons. They form the particles studied at Jefferson Lab.
CALIF
QCD Evolution Workshop
May 14-17, 2012 Jefferson Lab

Workshops

http://www.jlab.org/conferences/qcd2013/

QCD Evolution Workshop

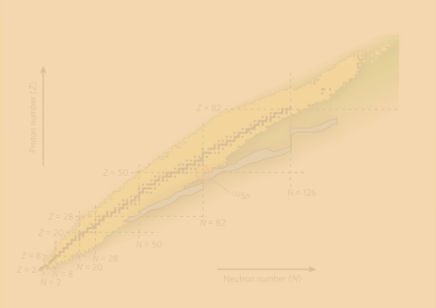
May 6-10, 2013, Jefferson Lab

Schools

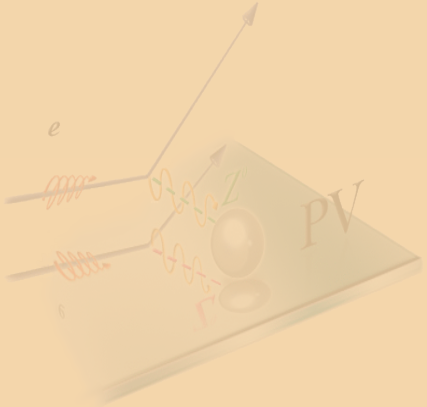

HUGS2012
27th Annual Hampton University Graduate Studies Program

Schools


Coordinator,lecturer
http://www.jlab.org/hugs/
HUGS2013
May 28-June14, 2013, Jefferson Lab

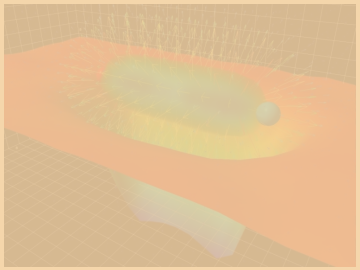

Proton number (Z)
Neutron number (N)
Z = 82
Z = 50
Z = 28
Z = 20
Z = 8
Z = 2
N = 126
N = 82
N = 50
N = 28
N = 20
N = 8
N = 2

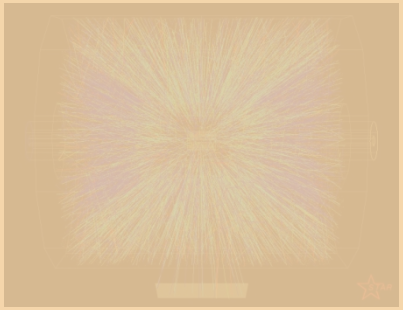

Proton Spin
Quark Spin


e
PV

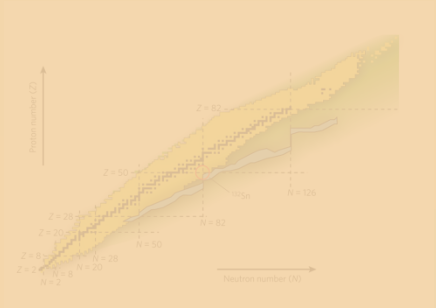

D
A
B
C

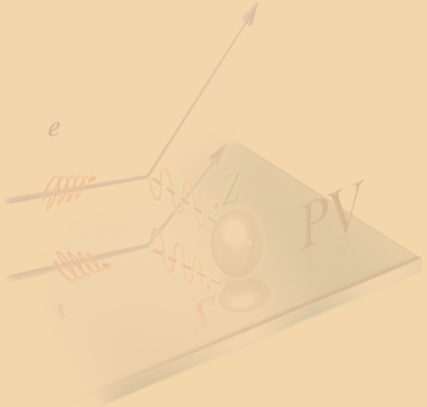
THANK YOU!

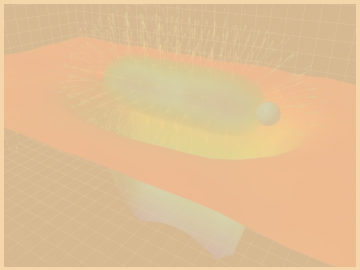


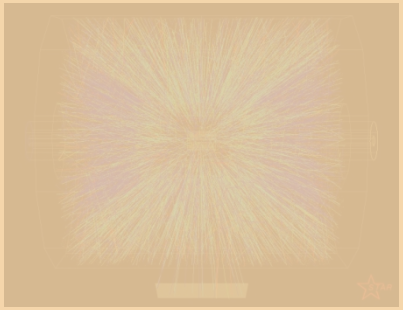


Backup slides

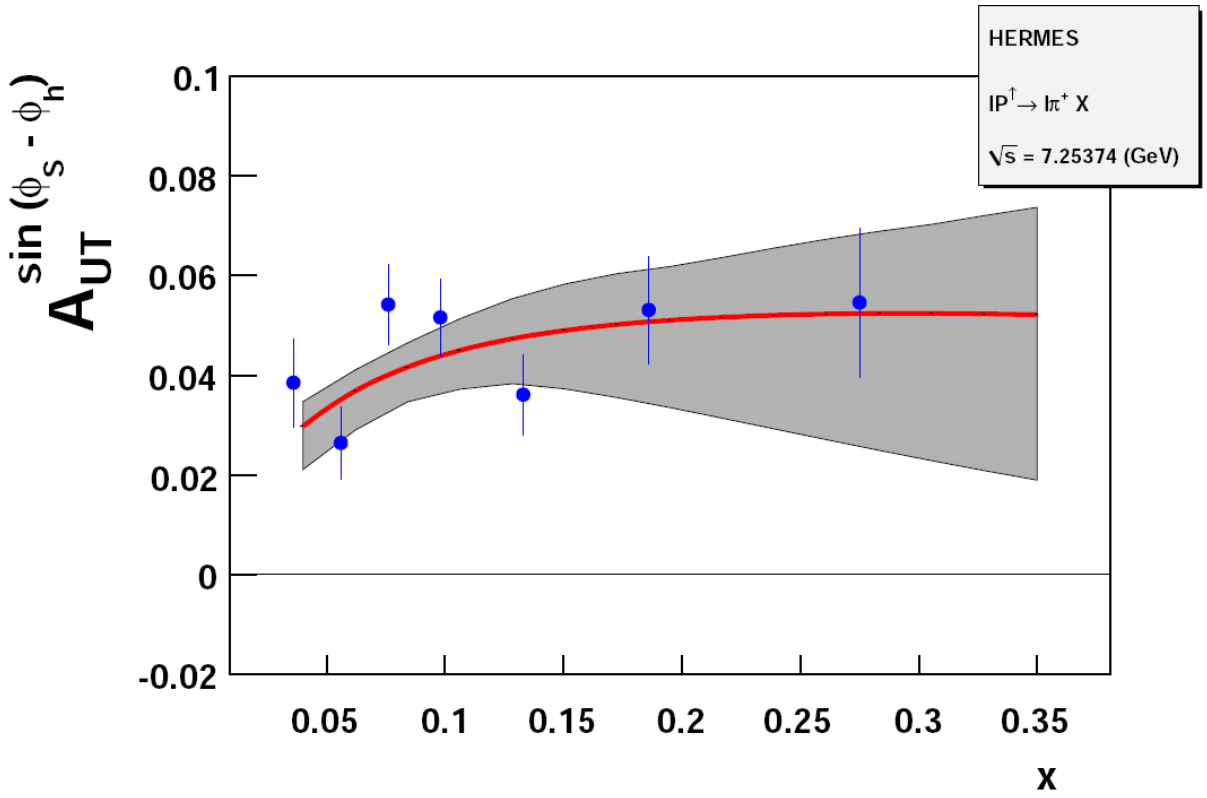








How do we estimate the uncertainty

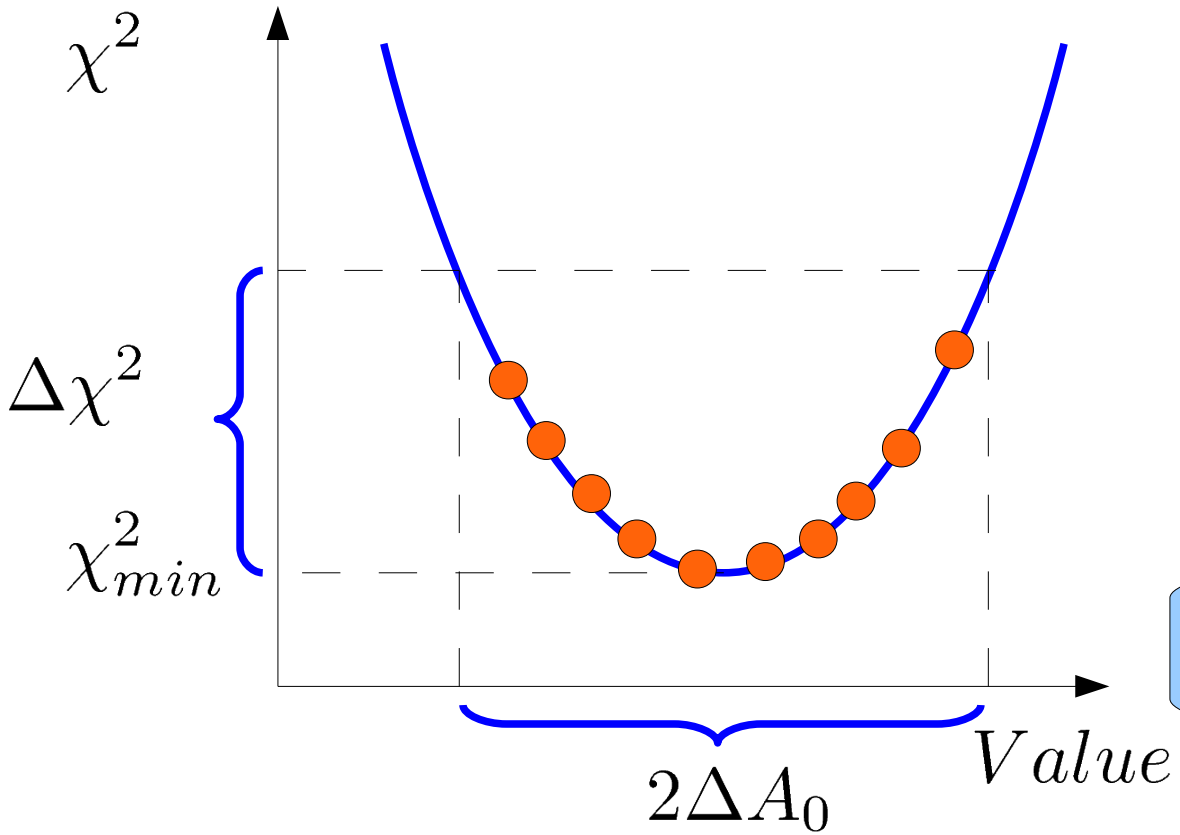


Fit the existig data

Anselmino et al 2009; AP 2012

Estimate of current uncertainty

Estimate of uncertainty is based on Monte Carlo method



Choice of $\Delta\chi^2$ is important

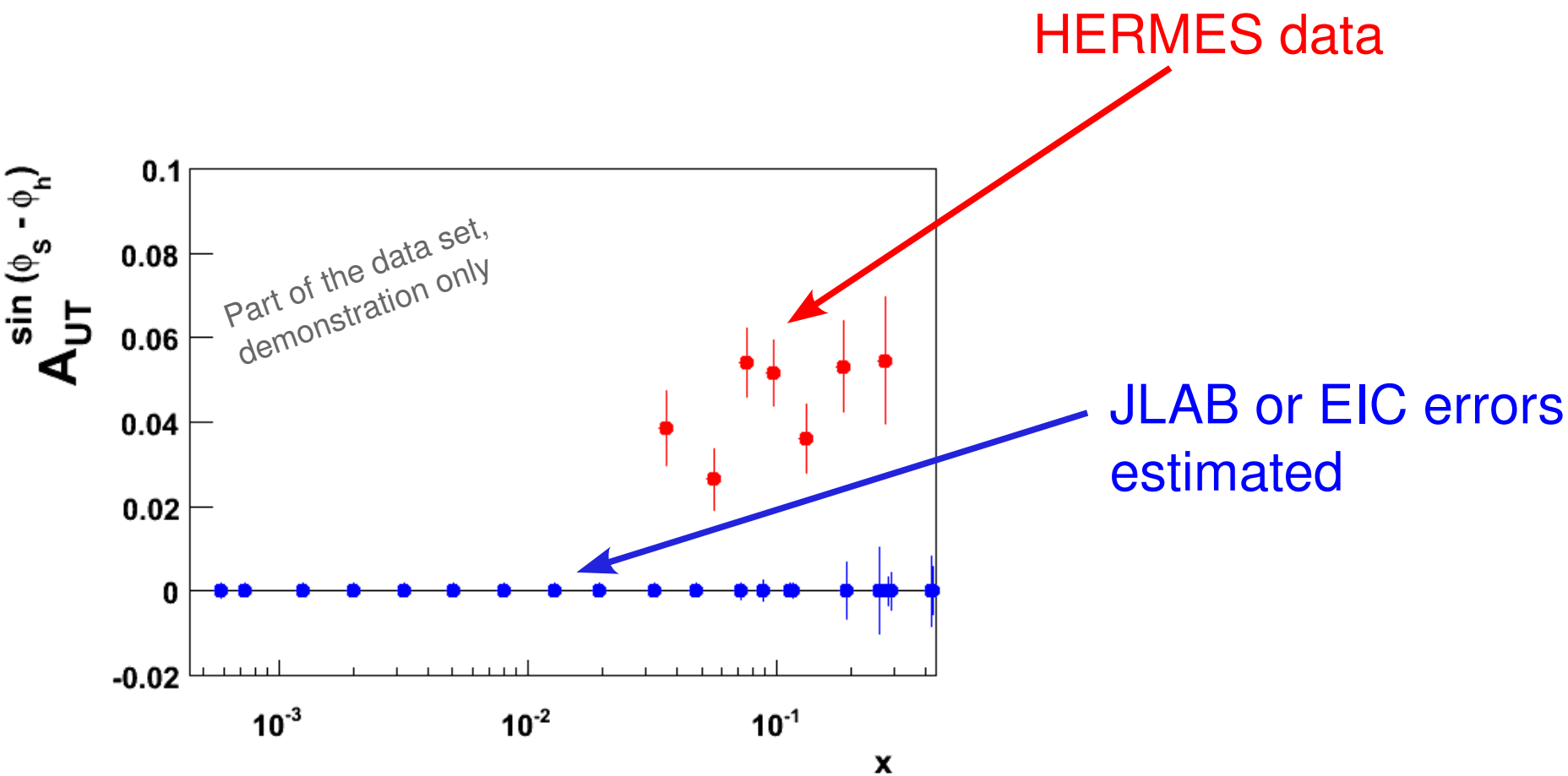
$$\Delta\chi^2 = 1$$

$$\Delta\chi^2 = 3\%\chi^2$$

$$\Delta\chi^2 = f(N_{par})$$

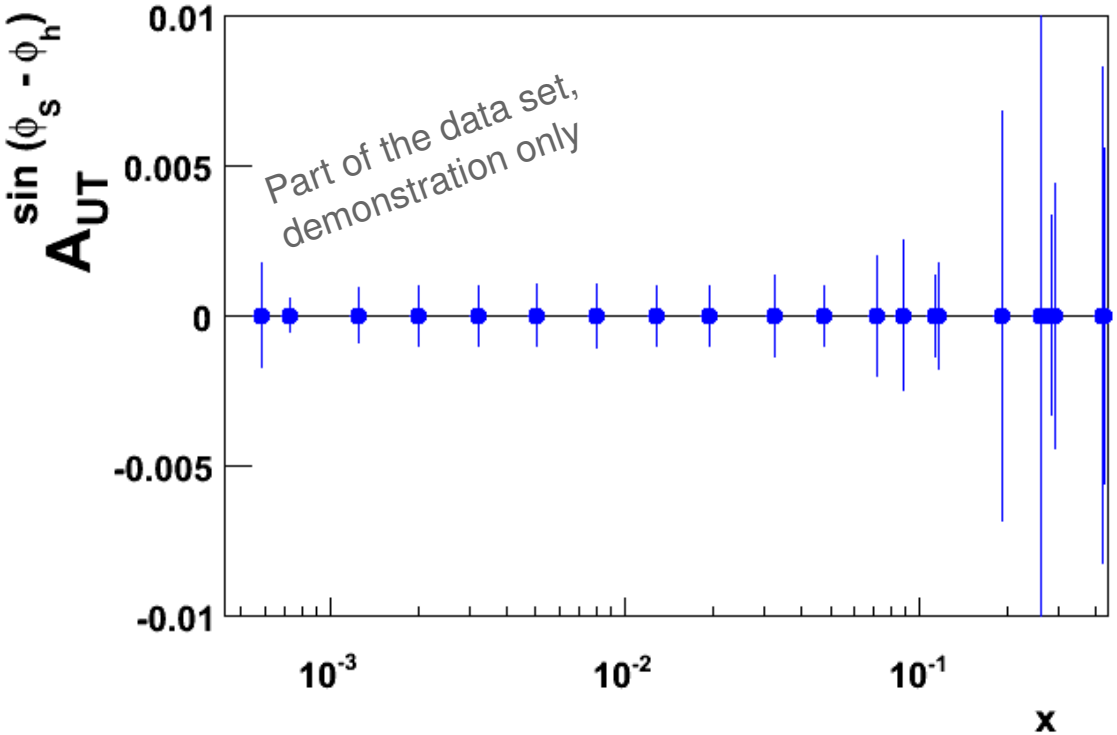
Anselmino et al 09

How do we predict uncertainty



Estimates of experimental error for JLAB or EIC at 10/fb

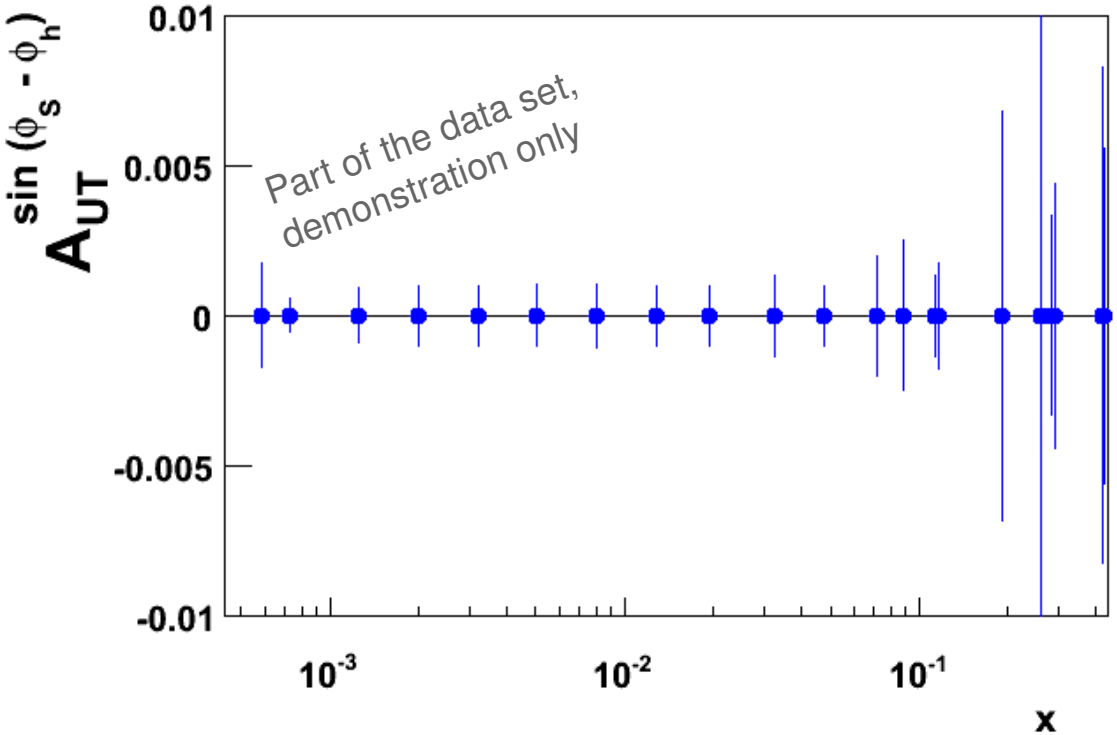
Future experimental stat error



Very small error

Estimates of experimental error for JLAB or EIC at 10/fb

Generate pseudo-data

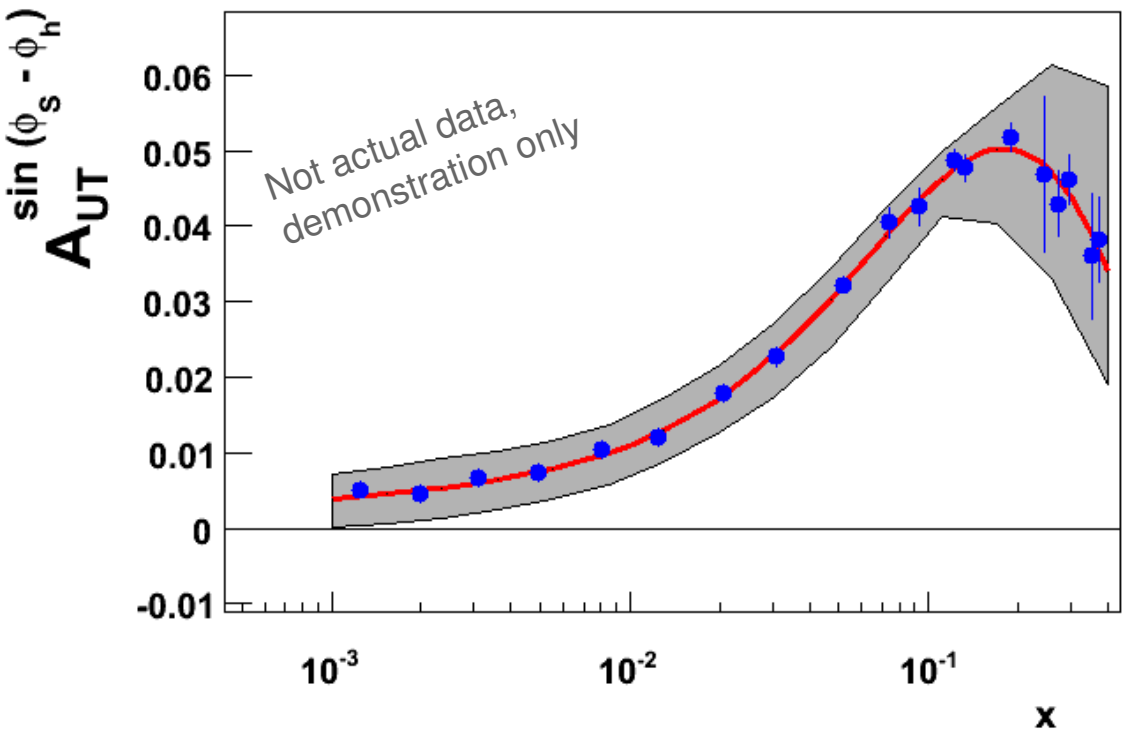


Generate pseudodata around our predictions –

We assume the same physics
as in our extractions

Estimates of experimental error for JLAB or EIC at 10/fb

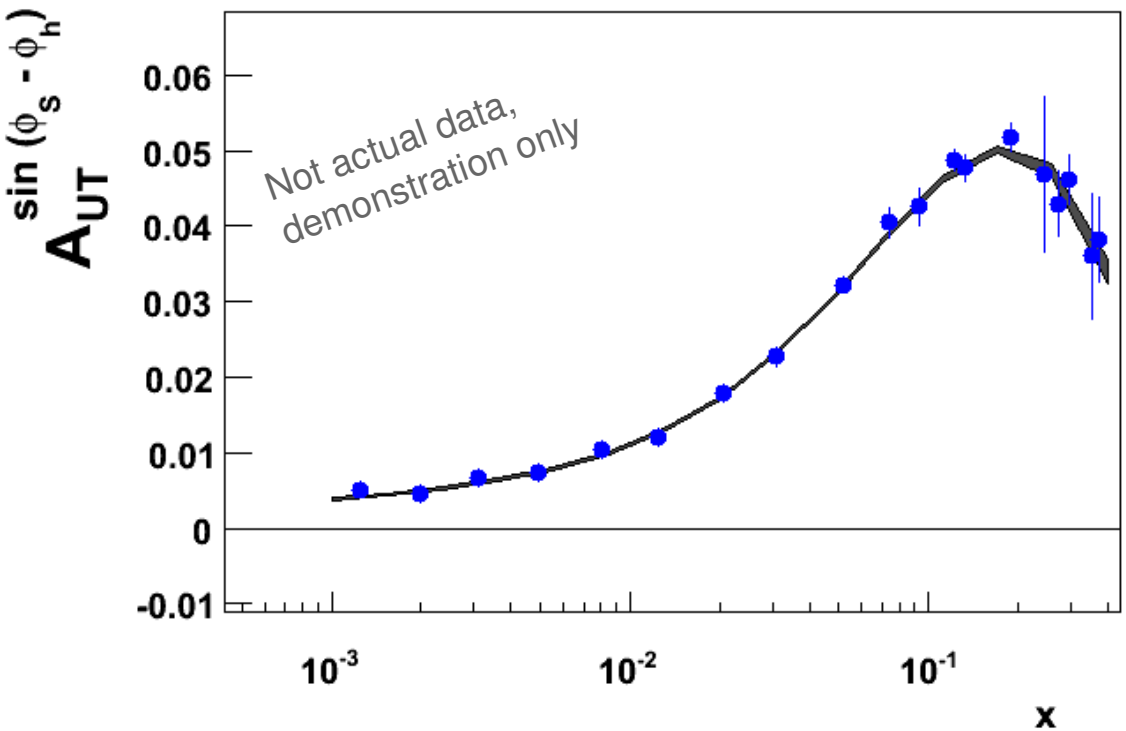
Generate pseudo-data



Generate pseudodata 1-σ around our predictions

Fit based on Anselmino et al 2009, analysis AP 2012

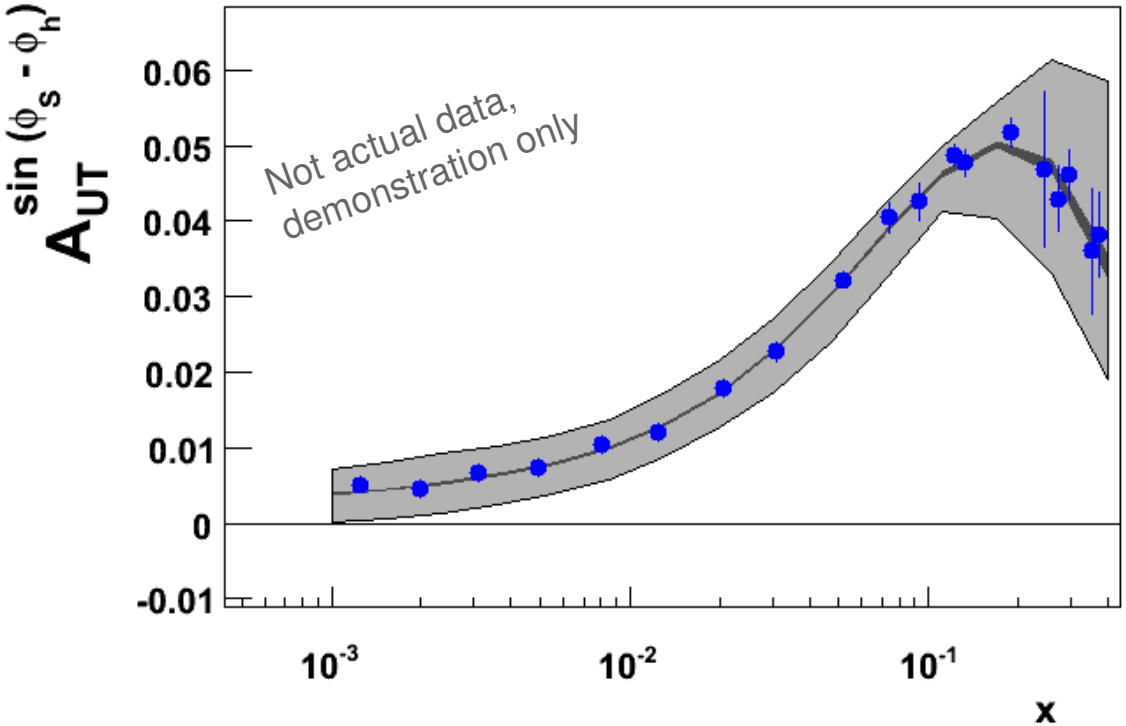
Fit the pseudo-data



Fit the pseudodata and estimate the error using the same method as in actual analysis

Fit based on Anselmino et al 2009, analysis AP 2012

Generate pseudo-data

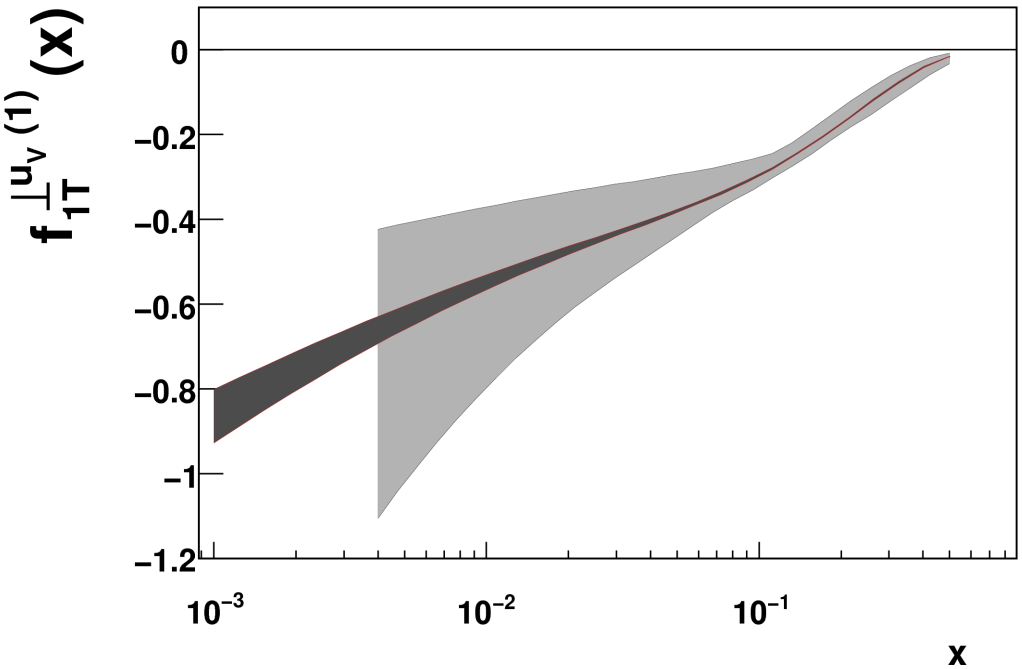


Compare the error to the existing one

Fit based on Anselmino et al 2009, analysis AP 2012

Compare functions:

Expected result for an individual function:



A. Prokudin (2012) contribution to EIC white paper