BFKL pomeron in nuclear physics

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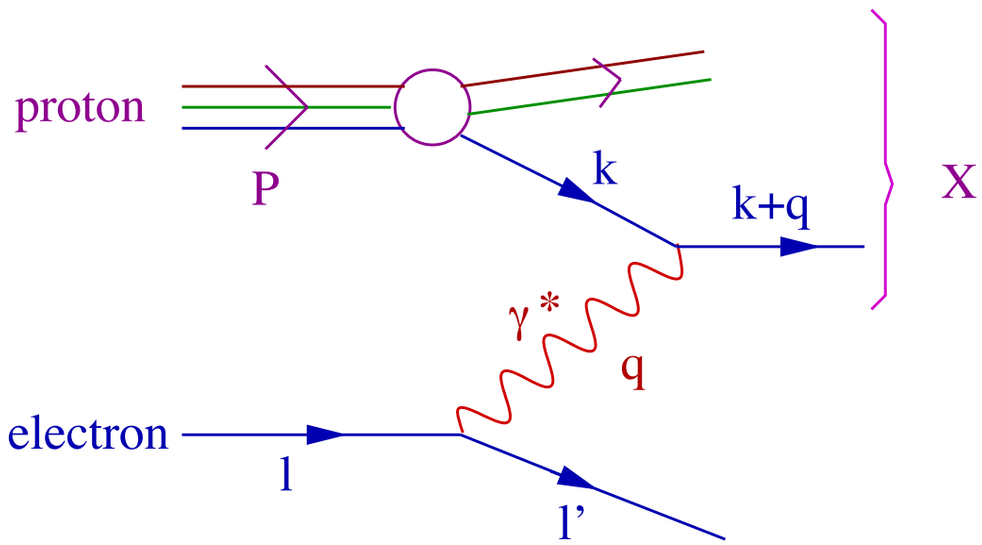
Deep inelastic scattering



Deep inelastic regime: $Q^2 = -q^2 \gg M^2$

Bjorken variable: $x = \dfrac{Q^2}{s + Q^2 - M^2}$

This kinematic variables are fixed by initial condition (l, P) and l'

They determine transverse area $1/Q$ and longitudinal momentum of the parton xP that is involved in the scattering

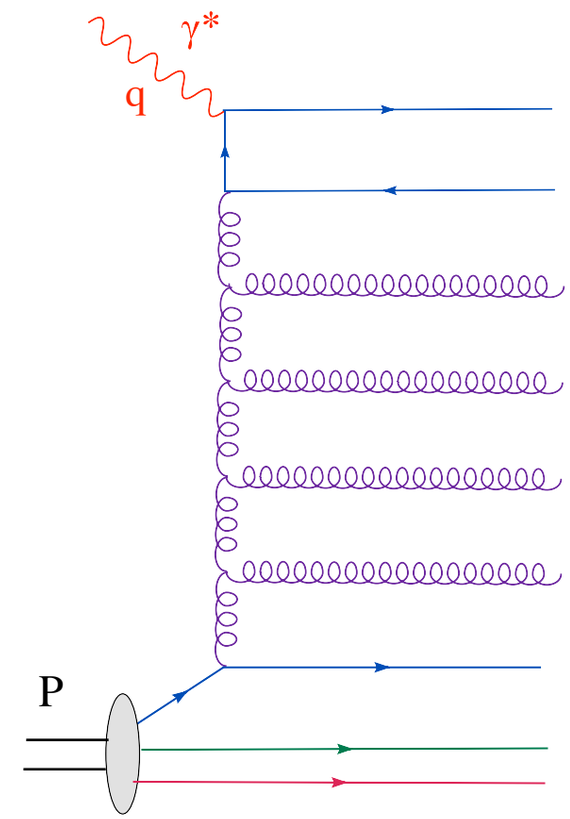
Deep inelastic scattering

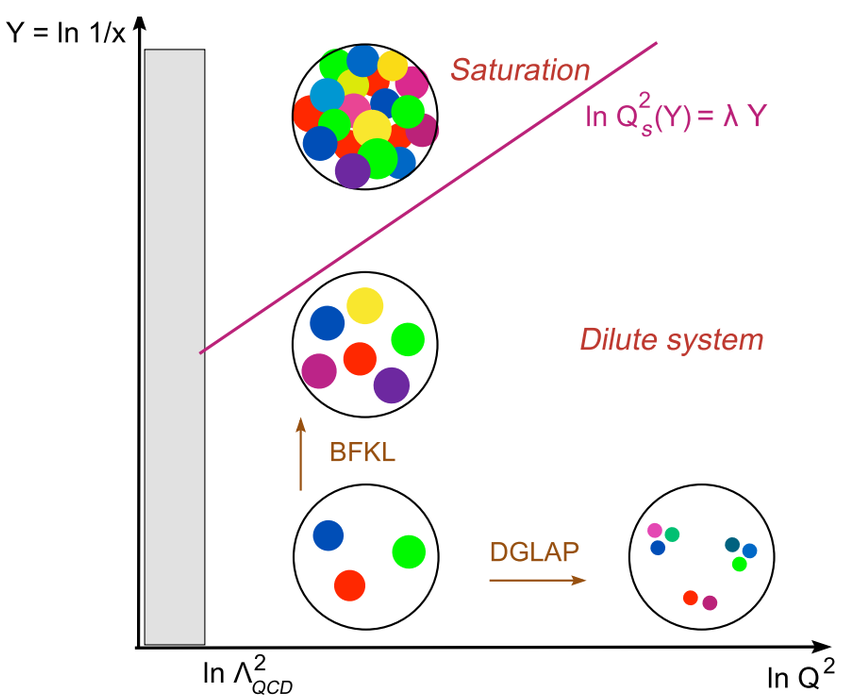
Emission of intermediate gluon is enhanced by large factor $ln(1/x)$

$$\alpha_s \ll 1, \quad \alpha_s ln(1/x) \sim 1$$

BFKL equation sums all ladder diagrams

$$\frac{\partial N(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(N(y,r_1,r_3)+N(y,r_2,r_3)-N(y,r_1,r_2)\Big)$$



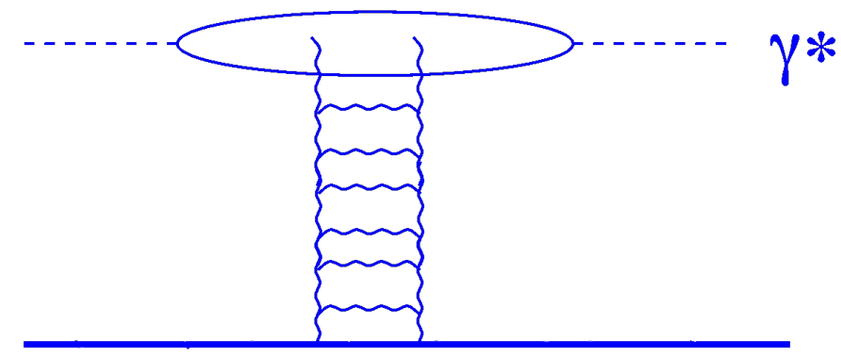


At large densities saturation is important.
One should take into account non-linear effects

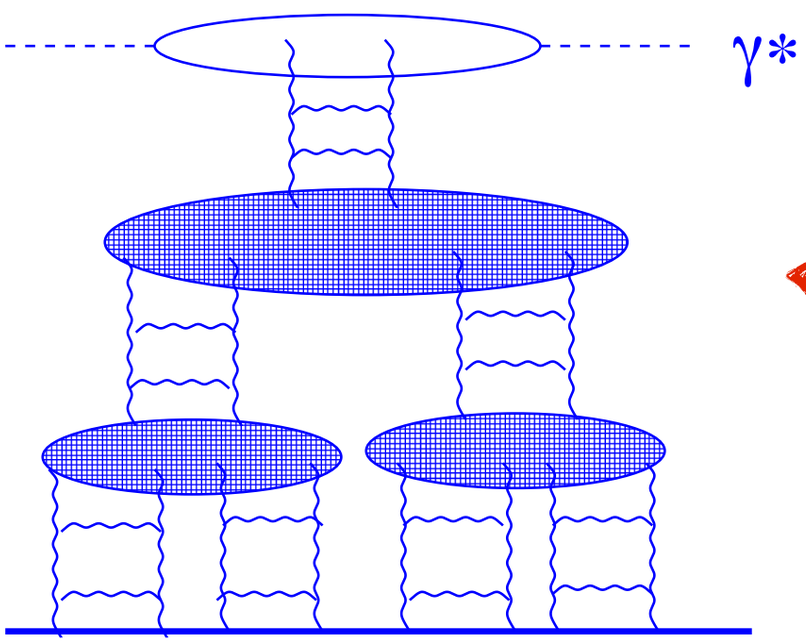
Balitsky-Kovchegov equation

$$\frac{\partial N(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(N(y,r_1,r_3)+N(y,r_2,r_3)-N(y,r_1,r_2)\Big)$$

$$\alpha_s \ll 1, \quad \alpha_s ln(1/x) \sim 1$$

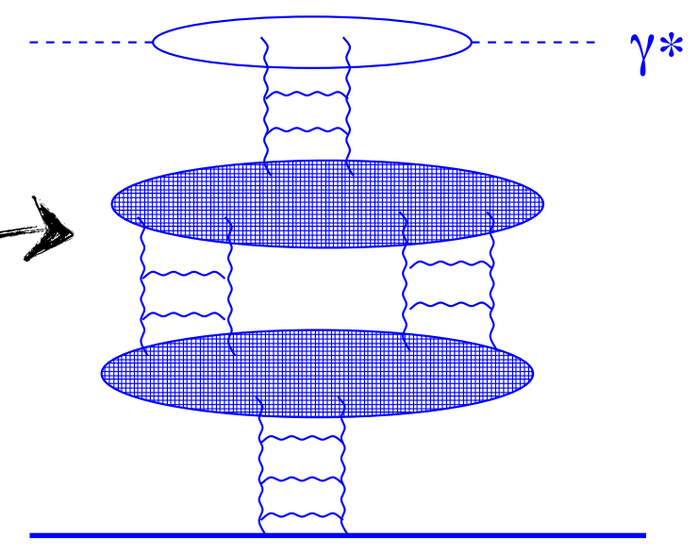


$$\frac{\partial N(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(N(y,r_1,r_3)+N(y,r_2,r_3)-N(y,r_1,r_2)-N(y,r_1,r_3)N(y,r_3,r_2)\Big)$$



$$\alpha_s \ll 1, \quad \alpha_s ln(1/x) \sim 1, \quad \alpha_s^2 A^{1/3} e^{\omega(0)y} \sim 1$$

$$\gamma \equiv \alpha_s^2 e^{\omega(0)y} \ll 1$$



Non-BK pomeron loops

$$\gamma \equiv \alpha_s^2 e^{\omega(0)y} \sim 1$$

Small-x interactions in QCD

There are some alternative approaches to describe scattering in QCD at small-x (high-energies):

1) Balitsky's Wilson-line approach

2) JIMWLK approach, based on the idea of strong gluonic fields

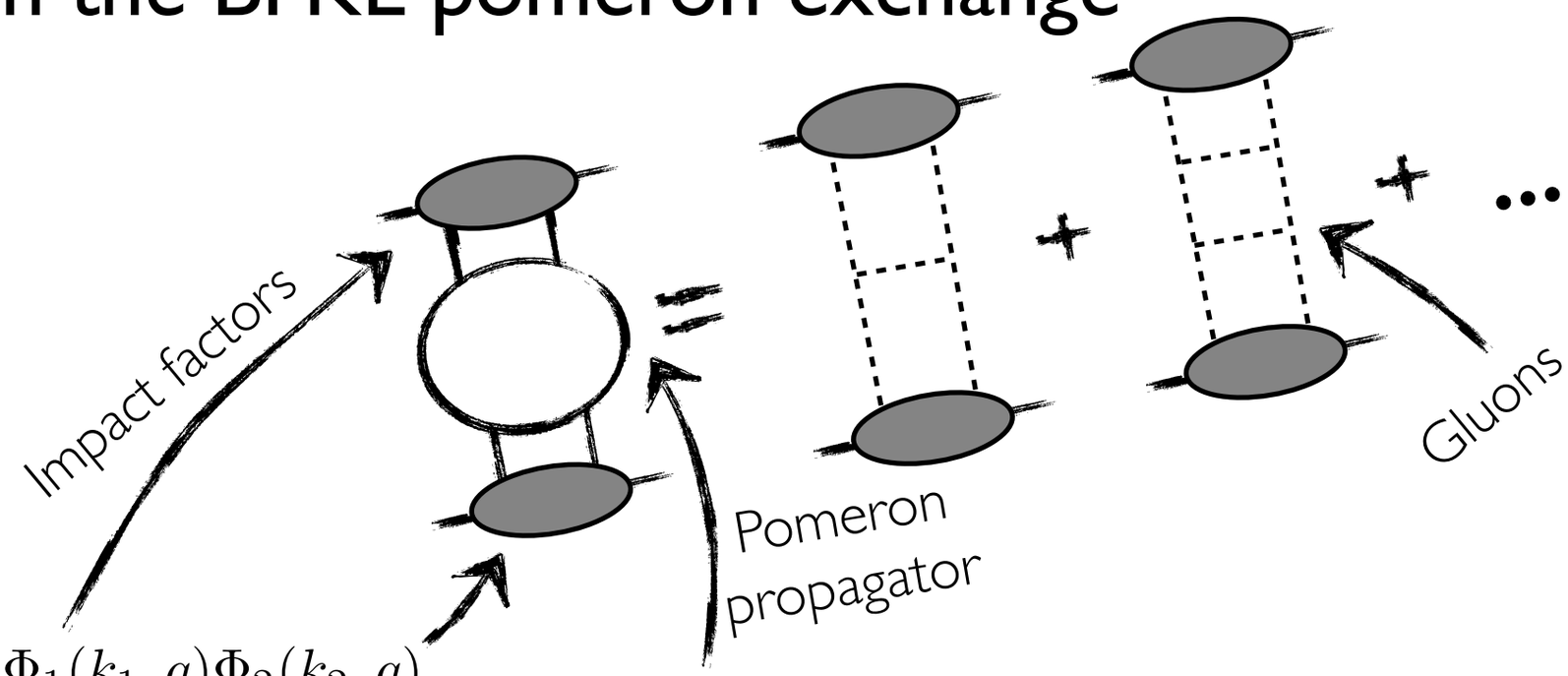
3) Colour dipole model

4) RFT-like diagram technique of interacting BFKL pomerons

The identity of the approaches is not fully clear. But fortunately all basic results, like Balitsky-Kovchegov equation, are equivalent to each other.

Small-x interactions in QCD

Equivalence of all this models imply that dynamic of interaction is based on the BFKL pomeron exchange



$$\frac{Im\mathcal{A}(s,t)}{s} = \frac{\mathcal{G}}{(2\pi)^4}\int d^2k_1 d^2k_2 \frac{\Phi_1(k_1,q)\Phi_2(k_2,q)}{k_2^2(k_1-q)^2} F(s,k_1,k_2,q)$$

One can try to formulate an effective field theory of the BFKL pomeron which is based on its general properties and does not deal directly with underlying gluonic dynamic.

Effective local pomeron field theory

A predecessor of this theory, reggeon field theory, was formulated a long time ago.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I + \mathcal{L}_E$$

$$\mathcal{L} = \phi^\dagger S \phi + \lambda \phi^\dagger \phi (\phi + \phi^\dagger) + g \rho \phi$$

Interaction with a nucleon

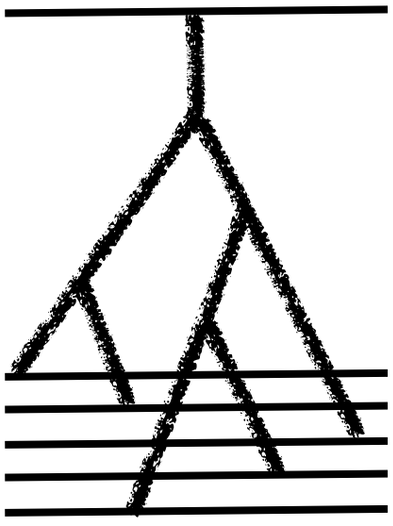
$$\rho = AT(b)\delta(y)$$

$$S = \frac{1}{2}(\overrightarrow{\partial}_y - \overleftarrow{\partial}_y) + \alpha' \nabla_b^2 + \epsilon$$

The theory uses Lagrangian approach. One can use standard Feynman diagram calculations.

Classical equations of motion.

$$\frac{\delta S}{\delta \Phi(y,b)} = 0, \quad \frac{\delta S}{\delta \Phi^\dagger(y,b)} = 0$$



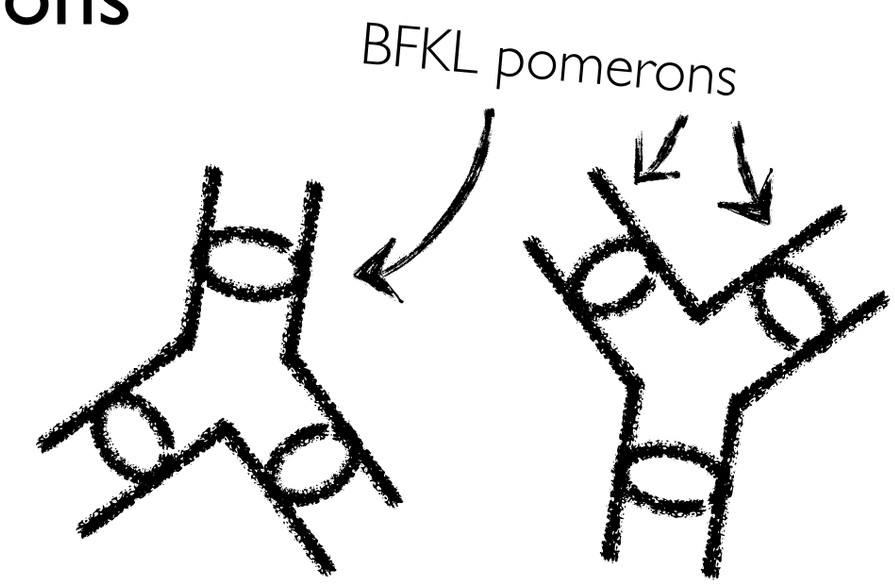
$$\frac{\partial \Phi^\dagger}{\partial y} - \alpha' \nabla_b^2 \Phi^\dagger - \epsilon \Phi^\dagger - \lambda \Phi^{\dagger 2} = g\rho(y,b)$$

$$\Phi^\dagger(y,b) = \frac{gAT(b)e^{\epsilon y}}{1 + \lambda g AT(b) \frac{1}{\epsilon}(e^{\epsilon y} - 1)}$$

A. Schwimmer, Nucl. Phys. B 94 (1975) 445

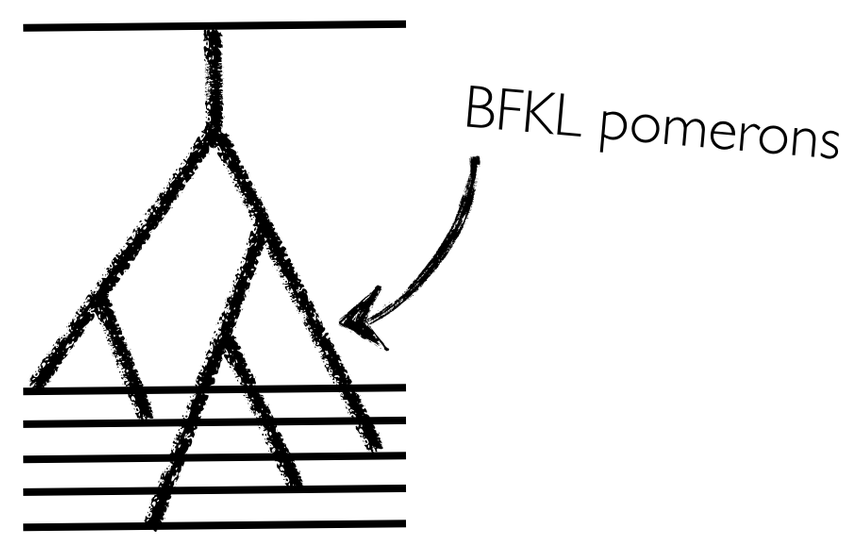
Effective BFKL pomeron field theory

Strong interaction at high energies is mediated by the exchange of BFKL pomerons



In the limit $N_c \to \infty$ they split and fuse by triple pomeron vertex

For <u>*hadron-nucleus*</u> scattering the relevant tree (fan) diagrams are summed by the Balitsky-Kovchegov (BK) evolution equation



Effective BFKL pomeron field theory

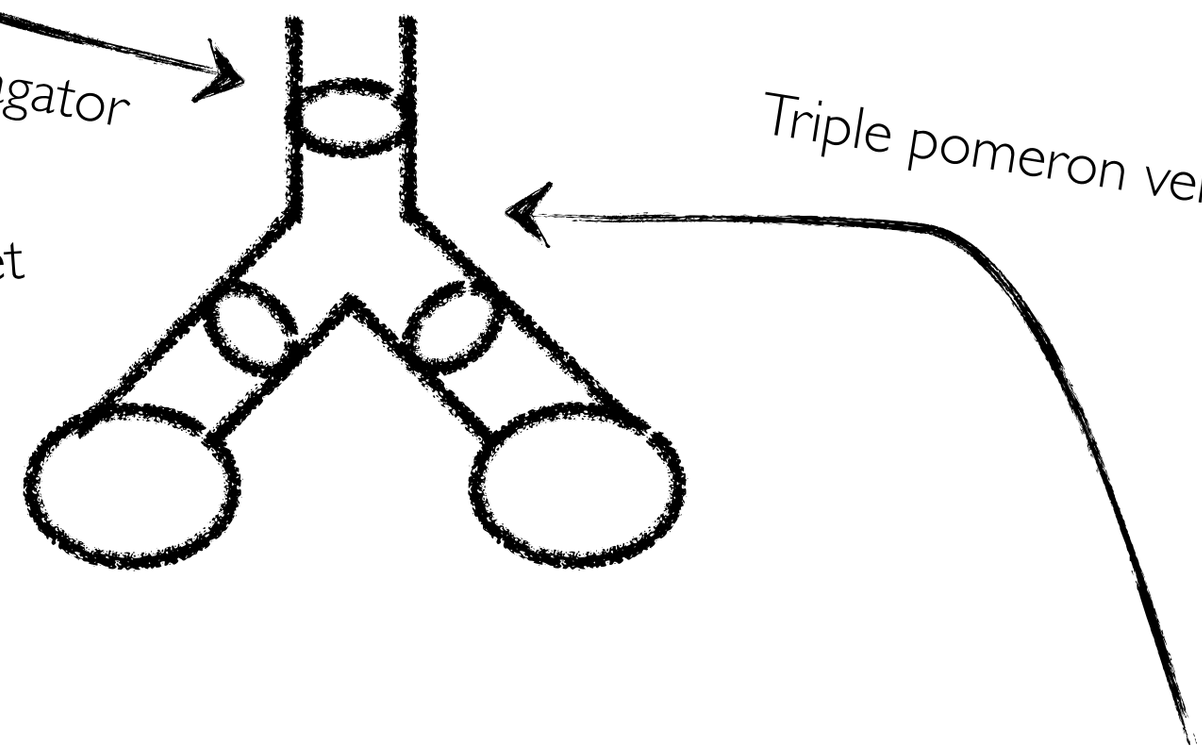
We use effective non-local field theory with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I + \mathcal{L}_E$

$$\mathcal{L}_0 = \int d^2r_1 d^2r_2 \Phi^\dagger \nabla_1^2 \nabla_2^2 \Big(\frac{\partial}{\partial y} + H_{BFKL} \Big) \Phi$$

BFKL pomeron propagator

Triple pomeron vertex

interaction with the nuclear target



$$\mathcal{L}_E = - \int d^2r_1 d^2r_2 \Phi^\dagger J$$

$$\mathcal{L}_I = \frac{2\alpha_s^2 N_c}{\pi} \int \frac{d^2r_1 d^2r_2 d^2r_3}{r_{12}^2 r_{23}^2 r_{13}^2} \Phi^\dagger(y, r_1, r_2) \Phi^\dagger(y, r_2, r_3) K_{31} \Phi(y, r_3, r_1) + (\Phi \leftrightarrow \Phi^\dagger)$$

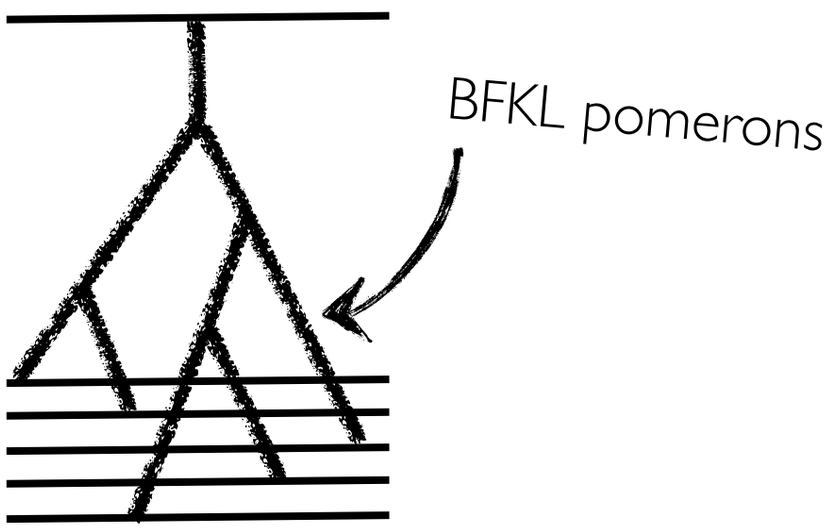
M. Braun, Phys. Lett. B 632 (2006) 297

Balitsky-Kovchegov equation

Can be obtained it this approach as a solution of classical equations of motion

$$\frac{\delta S}{\delta \Phi(y, r_1, r_2)} = 0, \quad \frac{\delta S}{\delta \Phi^\dagger(y, r_1, r_2)} = 0$$

$$\frac{\partial \Phi(y, r_1, r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi} \int d^2 r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2} \big(\Phi(y, r_1, r_3) + \Phi(y, r_2, r_2) - \Phi(y, r_1, r_2) - \Phi(y, r_1, r_3)\Phi(y, r_2, r_3)\big)$$



BK approximation can be justified if $\gamma = \alpha_s^2 \exp \omega(0) y$ is small

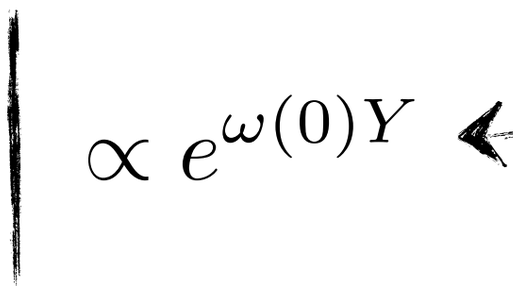
rapidity

triple pomeron coupling

pomeron intercept

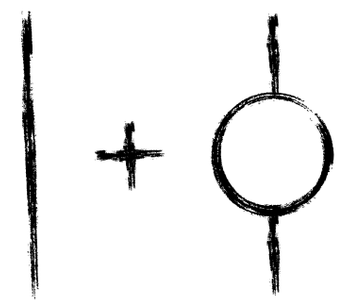
BFKL pomeron loops

$$G(Y, r_1, r_2; Y', r_1', r_2') = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu d^2 r_0 e^{\omega(\nu,n)(Y-Y')} f(n,\nu) E^*_{\nu n}(r_{1'0}, r_{2'0}) E_{\nu n}(r_{10}, r_{20})$$



$\propto e^{\omega(0)Y}$

Naive estimation shows:



$\propto e^{\omega(0)Y}(1 + \alpha_s^2 e^{\omega(0)Y})$

$\gamma = \alpha_s^2 \exp \omega(0) y$ plays a role of parameter in expansion in the number of loops

For supercritical BFKL pomeron $\omega(0) > 0$. Parameter γ grows with rapidity

1) loop contribution becomes not small
2) one can not apply the perturbative approach

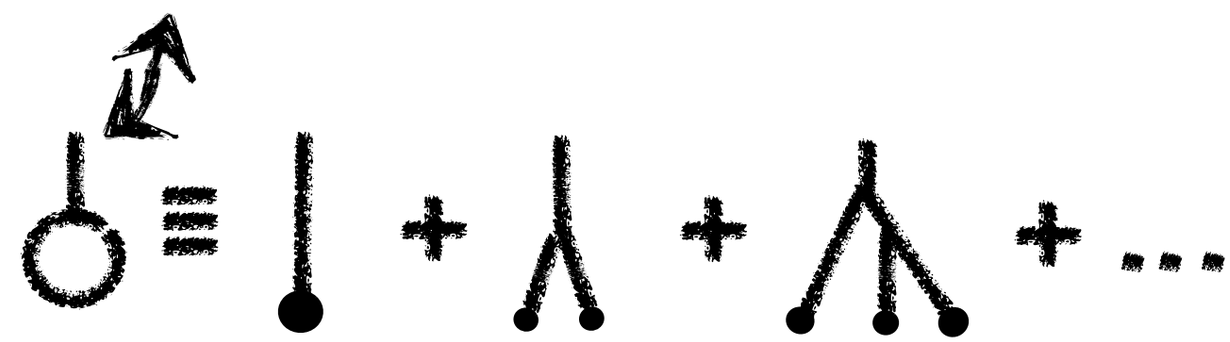
BFKL pomeron in the external field of the nucleus

Calculations of loops may become easier if one starts with the perturbative approach inside the nucleus from the start

$$\mathcal{L} = \Phi^\dagger\left(\frac{\partial}{\partial y} + H\right)\Phi + \lambda\Phi^\dagger\Phi(\Phi + \Phi^\dagger) + g\rho\Phi$$

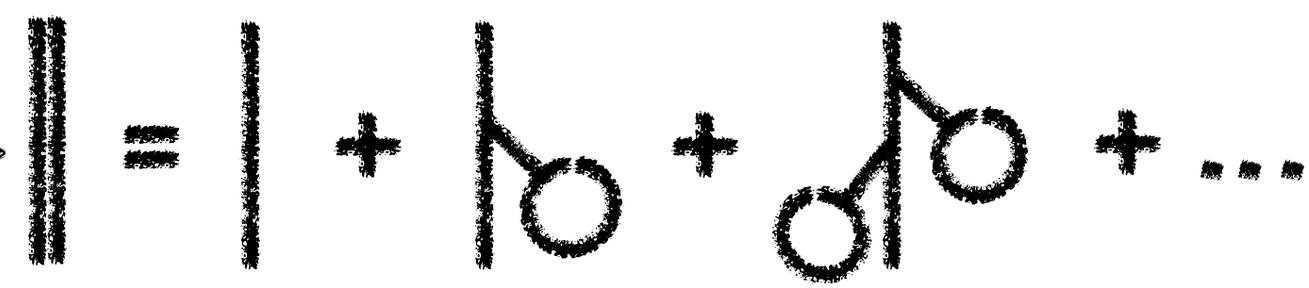
we make a shift in the quantum field

$$\Phi^\dagger(y,b) = \Phi_1^\dagger(y,b) + \Phi_0^\dagger(y,b)$$



The model in the nuclear background field:

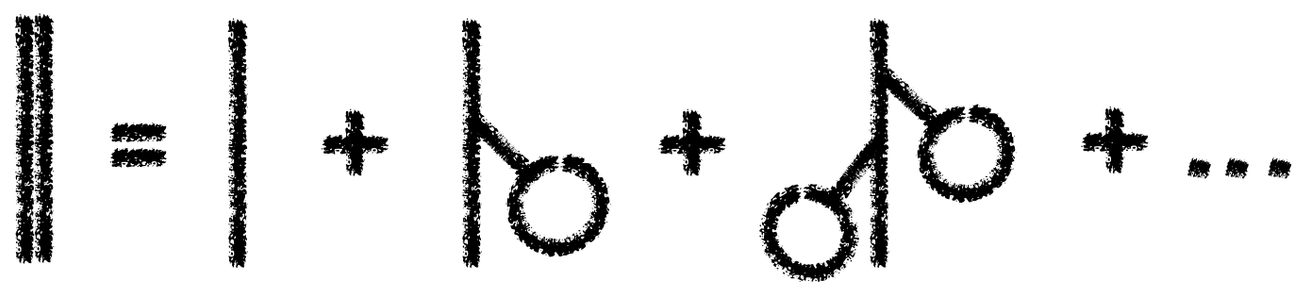
$$\mathcal{L} = \Phi_1^\dagger(S + 2\lambda\Phi_0)\Phi + \lambda\Phi_0\Phi^2 + \lambda\Phi_1^\dagger\Phi(\Phi + \Phi_1^\dagger)$$



M.A. Braun, A.N. Tarasov, Eur. Phys. J. C 58 (2008) 383

BFKL pomeron in the external field of the nucleus

The nuclear field transforms the supercritical pomeron into a subcritical one with the intercept smaller than unity.



For the BFKL pomeron we use effective non-local field theory

propagator in the external field

$$\frac{\partial P(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(P(y,r_1,r_3)+P(y,r_2,r_3)-P(y,r_1,r_2)$$

$$-\Phi(y,r_1,r_3)P(y,r_2,r_3)-\Phi(y,r_2,r_3)P(y,r_1,r_3)\Big)$$

with boundary condition

M.A. Braun, A.N. Tarasov, Nucl. Phys. B 851 (2011) 533

$$P(y=y',r_1,r_2;y',r_1',r_2') = \nabla_1^{-2}\nabla_2^{-2}\delta^2(r_1-r_1')\delta^2(r_2-r_2')$$

BFKL pomeron

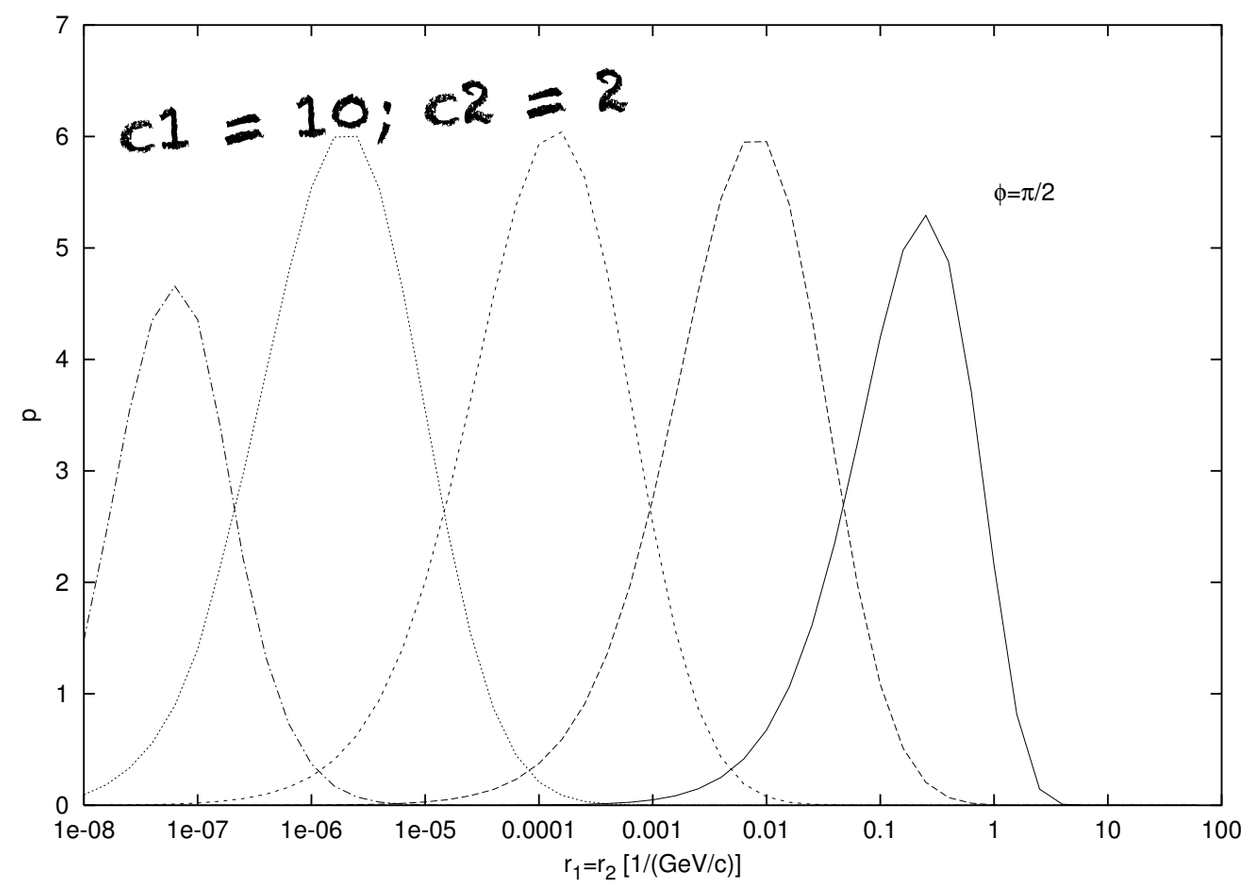
$$P(y, r_1, r_2) = \int d^2 r_1' d^2 r_2' P(y, r_1, r_2; y', r_1', r_2') \nabla_1^2 \nabla_2^2 \psi(r_1', r_2')$$

With the chosen set of initial conditions, the convoluted BFKL pomeron propagator vanishes at large rapidity distances

$$P(y = 0, r_1, r_2) = 1 - e^{-c_1 r_{12}^2 e^{-b^2/c_2}}$$

initial condition for evolution of the convoluted propagator

Calculation is possible only numerically



BFKL pomeron in (2+1) dimensional QCD

Propagator in the external field:

$$\frac{\partial P(y,r_1,r_2)}{\partial y} = \frac{\bar{\alpha}}{2\pi}\int d^2r_3 \frac{r_{12}^2}{r_{13}^2 r_{23}^2}\Big(P(y,r_1,r_3)+P(y,r_2,r_3)-P(y,r_1,r_2)$$

$$-\Phi(y,r_1,r_3)P(y,r_2,r_3)-\Phi(y,r_2,r_3)P(y,r_1,r_3)\Big)$$

$$2\alpha_s N_c \theta(r_{2,1}^{max}-r_3)\theta(r_3-r_{2,1}^{min})$$

the kernel in (2+1)-dimensional QCD

We expect to find analytical solution in (2+1) dimensional QCD

J. Bartels, V.S. Fadin, L.N. Lipatov, Nucl. Phys. B 698 (2004) 255

M.A. Braun, A.N. Tarasov, Nucl. Phys. B 863 (2012) 495

BFKL pomeron in (2+1) dimensional QCD

In terms of S-matrix $S_{r_2 r_1}(y) = 1 - \Phi_{r_2 r_1}(y)$

BK equation

$$\frac{\partial S_{r_2 r_1}}{\partial y} = \int_{r_1}^{r_2} dr_0 \Big(S_{r_2 r_0} S_{r_0 r_1} - S_{r_2 r_1} \Big)$$

$$\Psi_{r_2 r_1}(y) = e^{r_{21} y} S_{r_2 r_1}$$

$$\frac{\partial \Psi(y)}{\partial y} = \Psi^2(y)$$

$$\Psi(y) = \Psi(0)[1 - y\Psi(0)]^{-1}$$

Pomeron propagator

$$\frac{\partial P_{r_2 r_1}}{\partial y} = \int_{r_1}^{r_2} dr_0 \Big(S_{r_2 r_0} P_{r_0 r_1} + P_{r_2 r_0} S_{r_0 r_1} - P_{r_2 r_1} \Big)$$

$$Q_{r_2 r_1}(y) = e^{r_{21} y} P_{r_2 r_1}(y)$$

$$\frac{\partial Q(y)}{\partial y} = \Big\{ Q(y), \Psi(y) \Big\}$$

$$Q(y, y') = \Psi^{-1}(y') \Psi(y) Q(y', y') \Psi(y) \Psi^{-1}(y')$$

$$\equiv T(y, y') Q(y' y') T(y, y')$$

Solution was found in matrix notation

Initial condition

We shall first study the Green function which satisfies

$$G_{r_2 r_1 | r_2' r_1'}(y', y') = \delta(r_2 - r_2')\delta(r_1 - r_1')$$

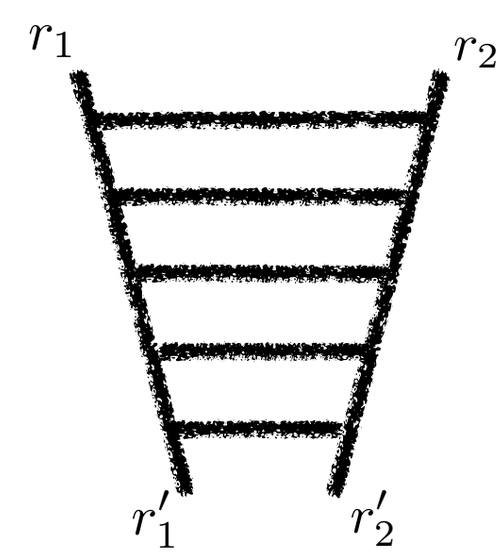
$$Q(y, y') = T(y, y')Q(y', y')T(y, y')$$

The solution factorizes

$$G_{r_2 r_1 | r_2' r_1'}(y, y') = e^{y' r_{21}' - y r_{21}} T_{r_2 r_2'}(y, y') T_{r_1' r_1}(y, y')$$

The expression is different from zero only in the interval:

$$g_{r_2 r_1 | r_2' r_1'}(y, y') \neq 0 \quad \text{only if} \quad r_1 < r_1' < r_2' < r_2$$



Initial condition

The initial condition for the BFKL pomeron propagator:

$$g_{r_2 r_1 | r_2' r_1'}(y', y') = \nabla_1^{-2} \nabla_2^{-2} \delta(r_2 - r_2') \delta(r_1 - r_1')$$

In the one-dimensional space the initial condition can be presented as:

$$g_{r_2 r_1 | r_2' r_1'}(y', y') = (r_2 - r_2') \theta(r_2 - r_2')(r_1' - r_1) \theta(r_1' - r_1)$$

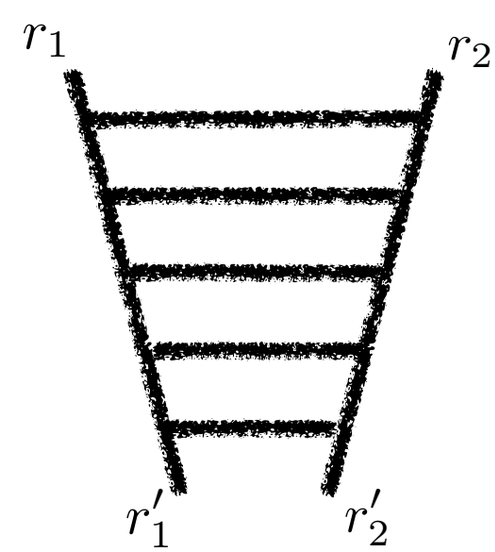
$$Q(y, y') = T(y, y') Q(y', y') T(y, y')$$

The solution factorizes

$$g_{r_2 r_1 | r_2' r_1'}(y, y') = e^{-r_{21}(y - y')} U_{r_{22'}}(y, y') U_{r_{1'1}}(y, y')$$

$$U_r(y, y') = \int_0^r dr' (r - r') T_{r'}(y, y') e^{-y' r'}$$

$$g_{r_2 r_1 | r_2' r_1'}(y, y') \neq 0 \quad \text{only if} \quad r_1 < r_1' < r_2' < r_2$$



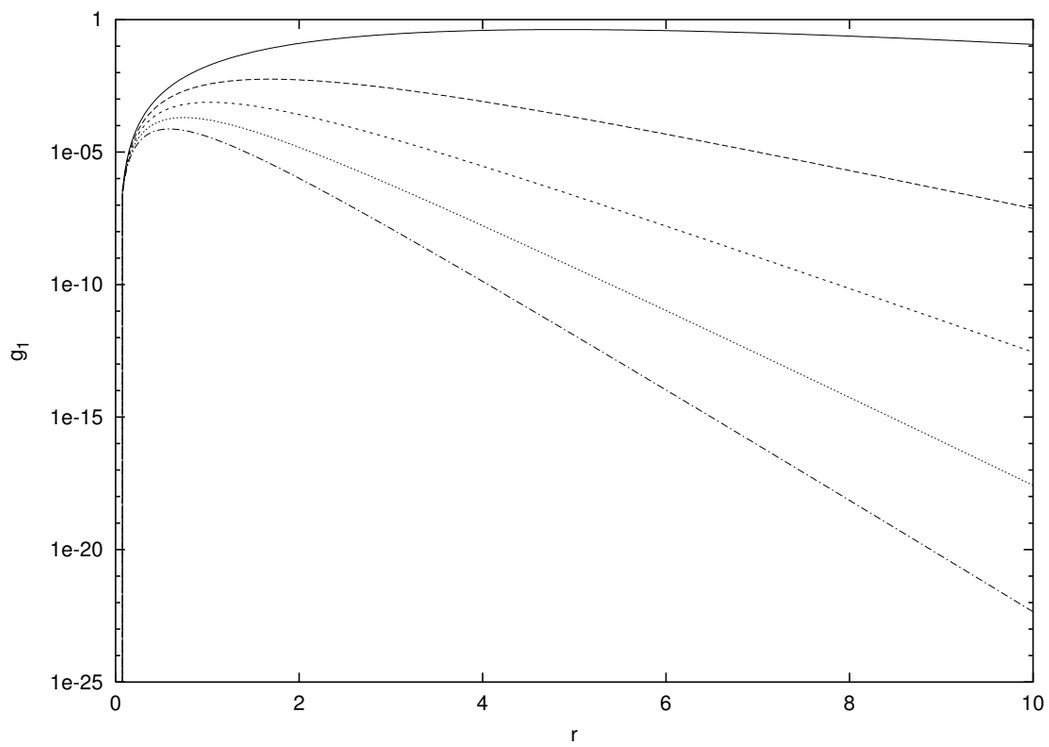
Numerical results

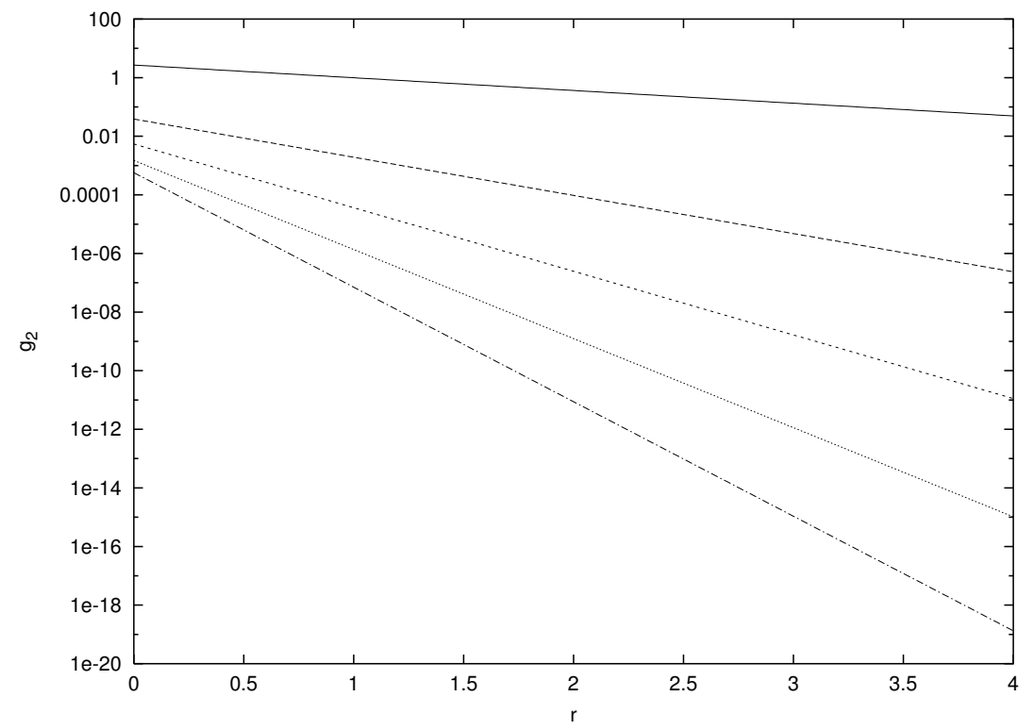
In our calculations we have taken:

$$S_r(0) = e^{-r^\gamma}$$

$$g_1(y, r \equiv r_{21}) = e^{-yr} \int_0^r dr_1 U_{r_1}(y,0) \int_0^{r-r_1} dr_2 U_{r_2}(y,0)$$

$$g_2(y, r \equiv r'_{21}) = e^{-yr} \int_0^{r_m - r} dr_1 U_{r_1}(y,0) e^{-yr_1} \int_0^{r_m - r - r_1} dr_2 U_{r_2}(y,0) e^{-yr_2}$$

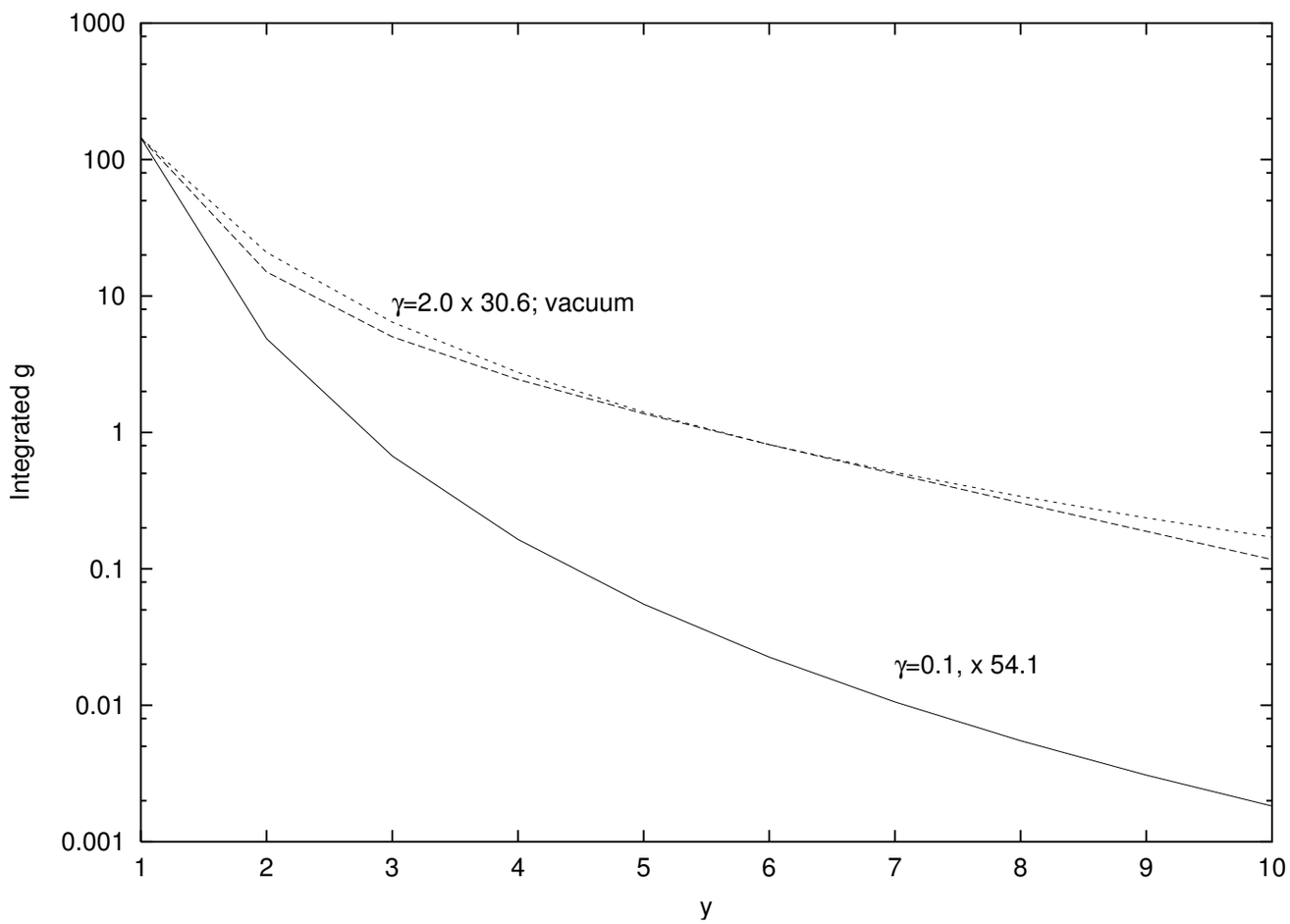




Numerical results for $\gamma = 0.1$ and $y = 1, 3, 5, 7, 9$

Numerical results

The influence of the field and the role of value of γ



$\gamma < 1$ corresponds to vanishing $S_r(y)$

For $\gamma > 1$ S-matrix tends to unity

1) With $\gamma < 1$ the propagator the propagator in the nuclear field goes to zero much faster than in the vacuum

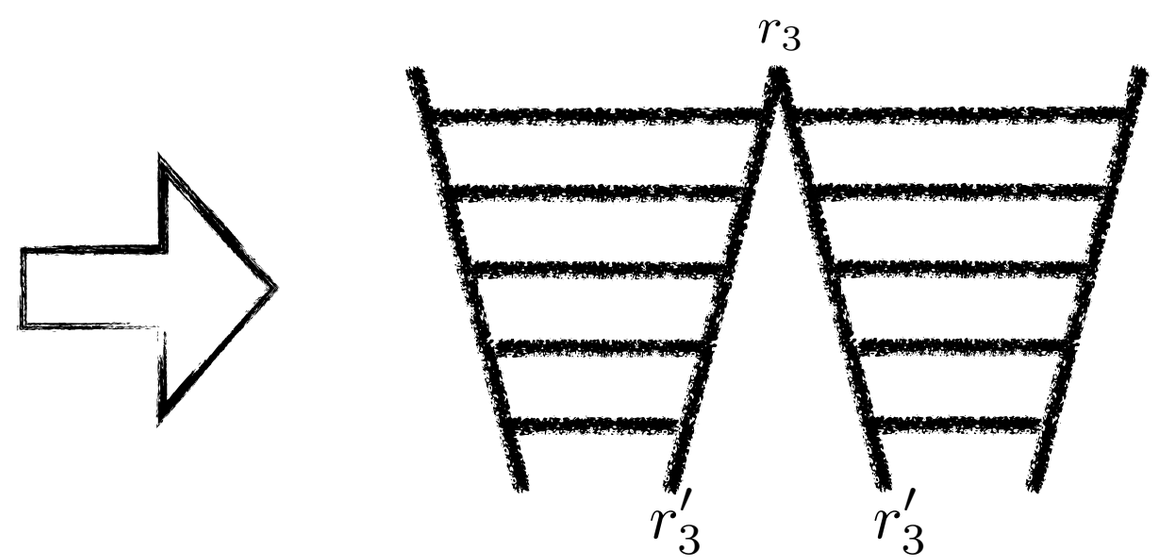
2) With $\gamma > 1$ we do not observe any influence of the field

BFKL pomeron loops in (2+1) dimensional QCD

The pomeron self-mass:

$$\Sigma(y, r_2, r_1|y', r_2', r_1')$$
$$= -(8\pi\alpha_s^2 N_c)^2 \int_{min\{r_2,r_1\}}^{\max\{r_2,r_1\}} dr_3 \int_{min\{r_2',r_1'\}}^{\max\{r_2',r_1'\}} dr_3' g(y, r_2, r_3|y', r_2', r_3') g(y, r_3, r_1|y', r_3', r_1')$$

It is trivial to see that this expression is zero due to the properties of the pomeron propagator



Pomerons cannot form loops in the nuclear field.

Conclussions

1) We have studied numerically the BFKL pomeron propagator in the external field created by the solution of the BK equation in the nuclear matter

2) We have found that for more or less arbitrary set of initial conditions the convoluted propagator vanishes at large rapidities. This gives reasons to believe that the propagator itself vanishes at large rapidities in the nuclear background

3) In 2+1 dimensional QCD, BFKL pomerons in the nuclear field cannot form loops in the nuclear field. This implies that the BK equation gives the complete solution to the quantum field theory of interacting pomerons