Exploring Hadron Structure with Transverse Momentum Dependent Factorization

Ted C. Rogers

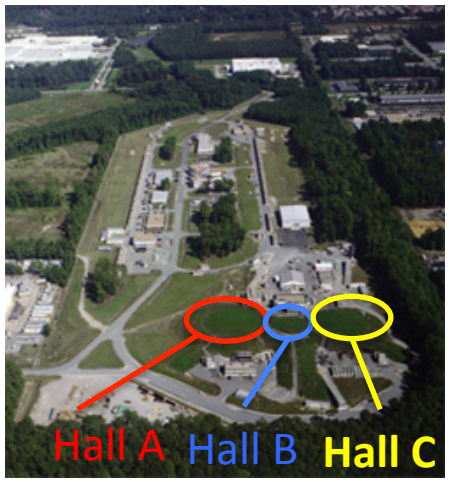
C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

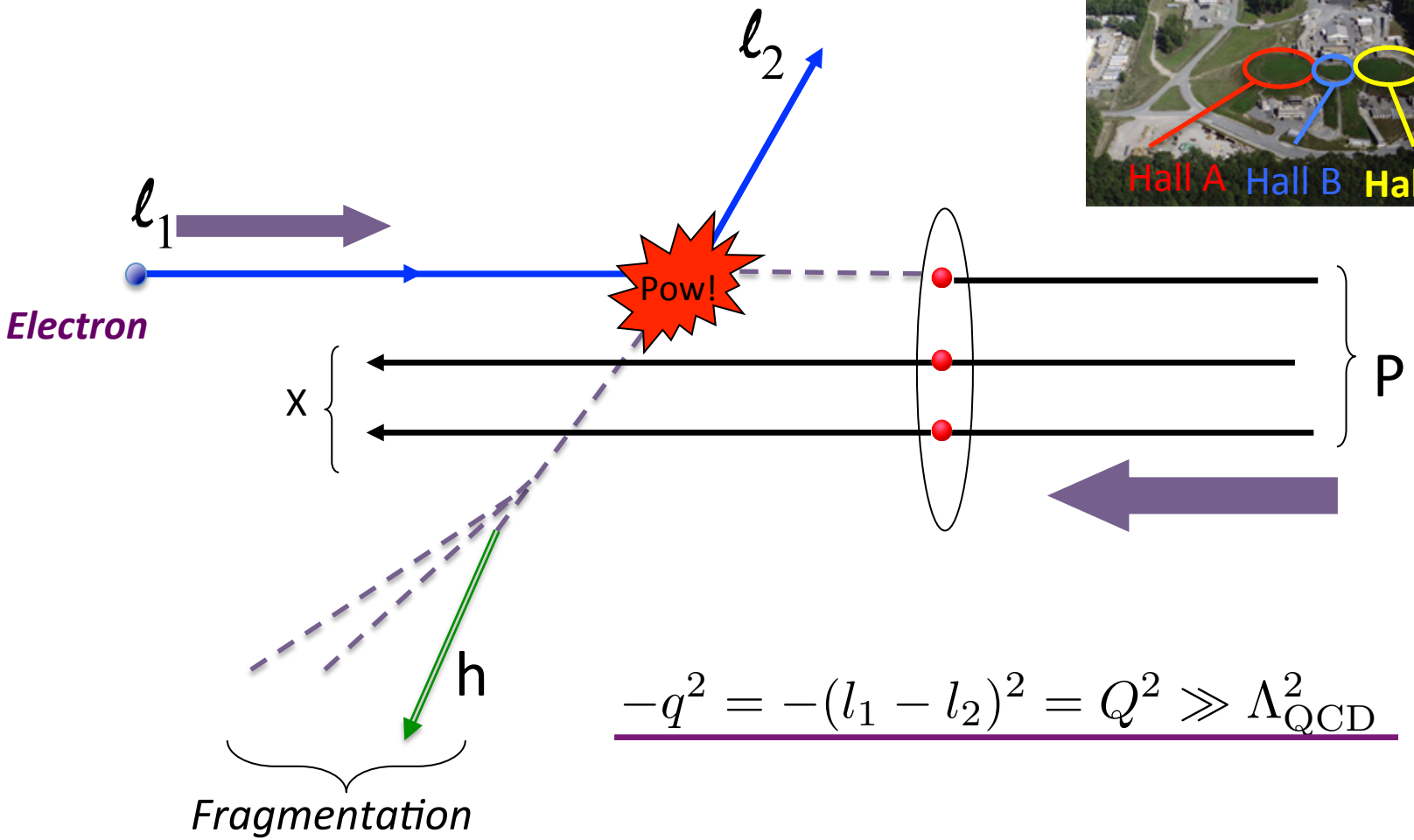
- Perturbative QCD and Collinear Factorization.
- Transverse Momentum Dependent (TMD) Factorization.
- TMD project: Implementing TMD-factorization.

Jefferson National Laboratory – February 6, 2013

Example:

- Semi-Inclusive Deep Inelastic Scattering (SIDIS):



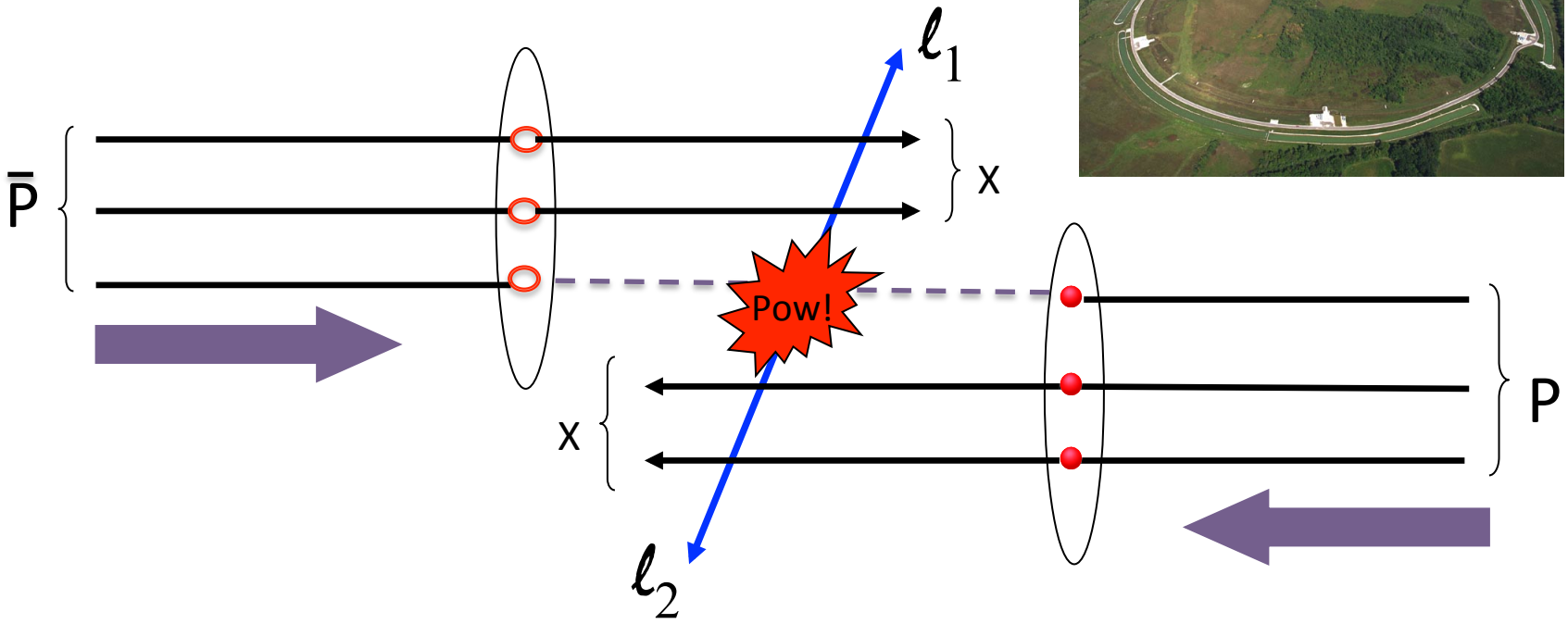


$$-q^2 = -(l_1 - l_2)^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$$

Example 2:

Tevatron

- Drell-Yan:

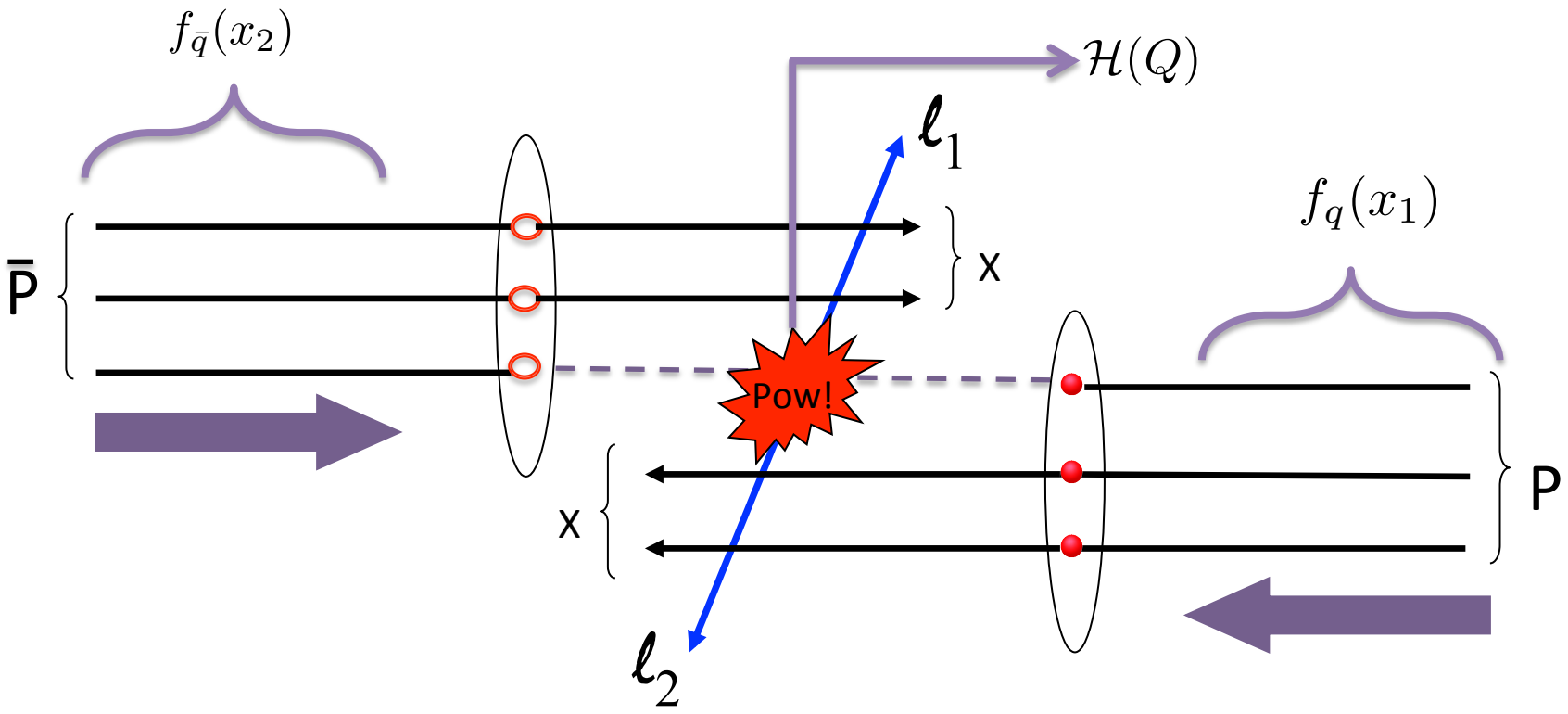


$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$$

$\sim 1/Q$

Small Scales

Example 2:



$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\rm QCD}$$

$\sim 1/Q$

Small Scales

Collinear (Standard) Case

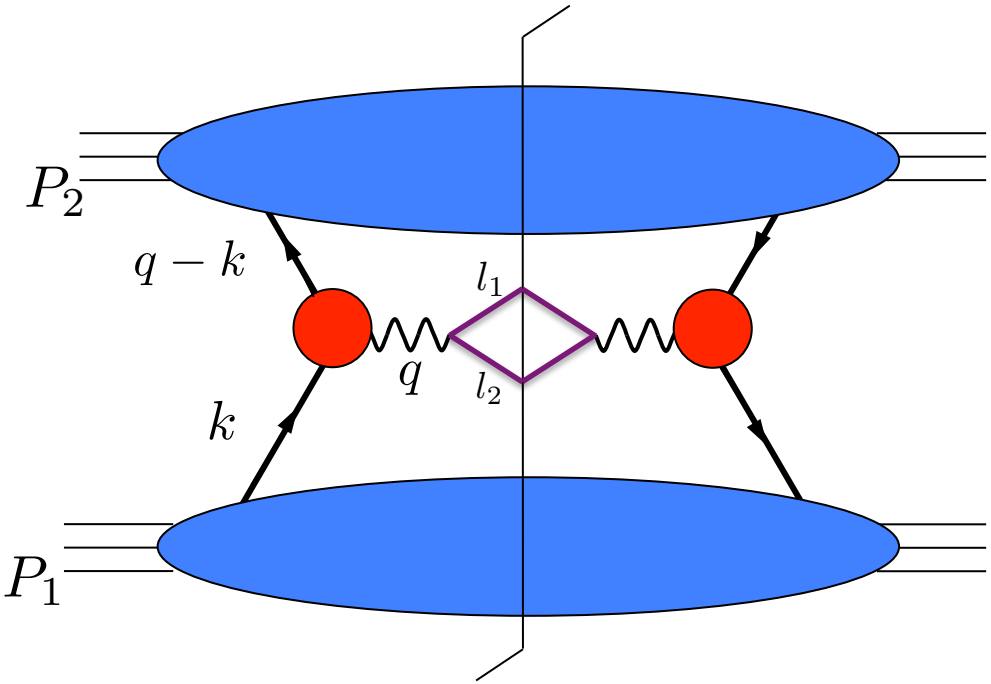
- Parton Model Picture

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)}_{\text{Elementary collision}} \otimes \underbrace{f_{q/P}(x_1)}_{\text{Number densities}} \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}_{\text{Number densities}}$$

"Parton Distribution Functions" (PDFs)

Example 2:

- Drell-Yan:



Collinear case

Get: $\int d\mathbf{q}_T \frac{d\sigma}{d\mathbf{q}_T \cdots}$

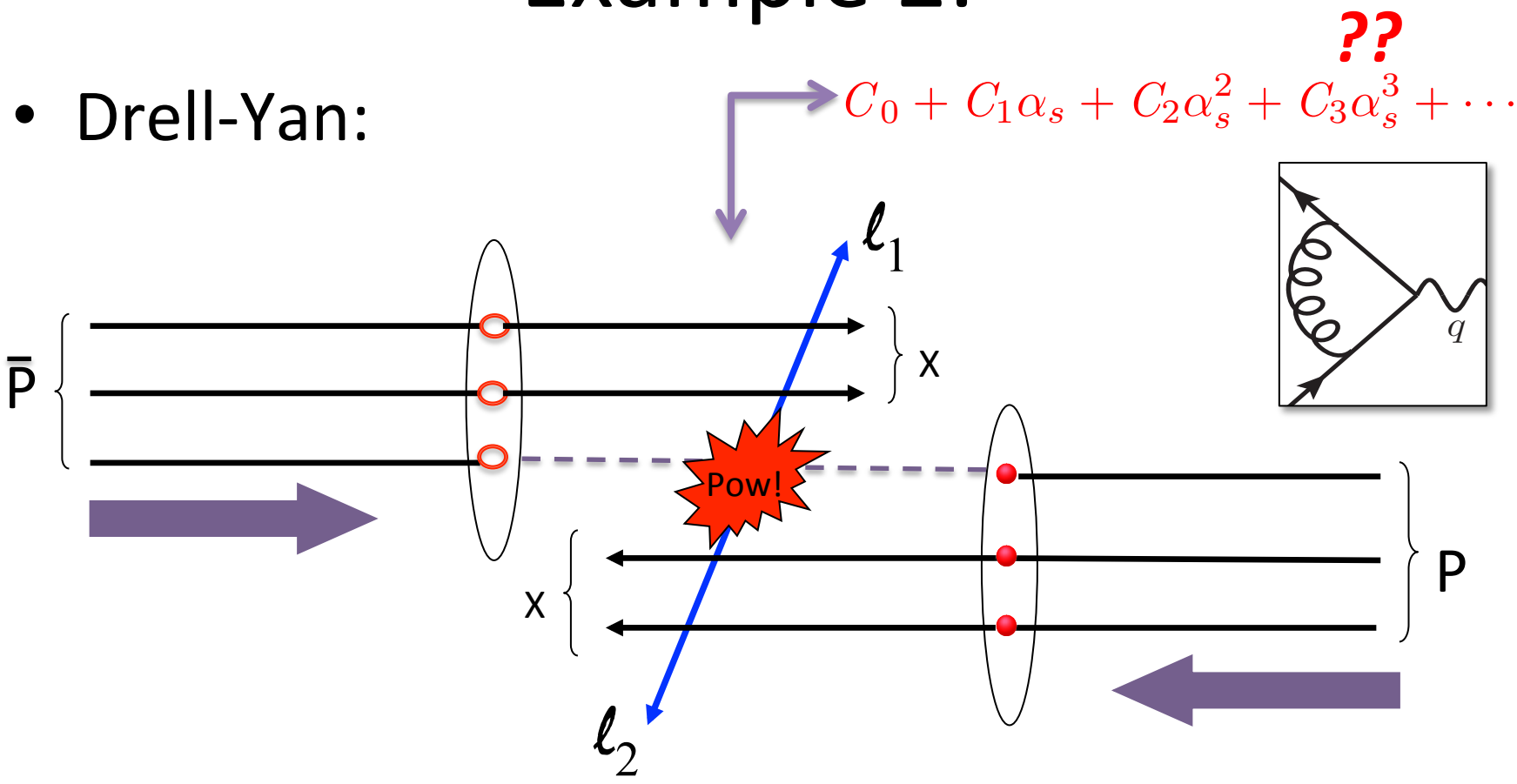
(Or large q_T)

$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$$

$$k_1 \equiv k$$
$$k_2 \equiv q - k$$
$$\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{q}_T$$

Example 2:

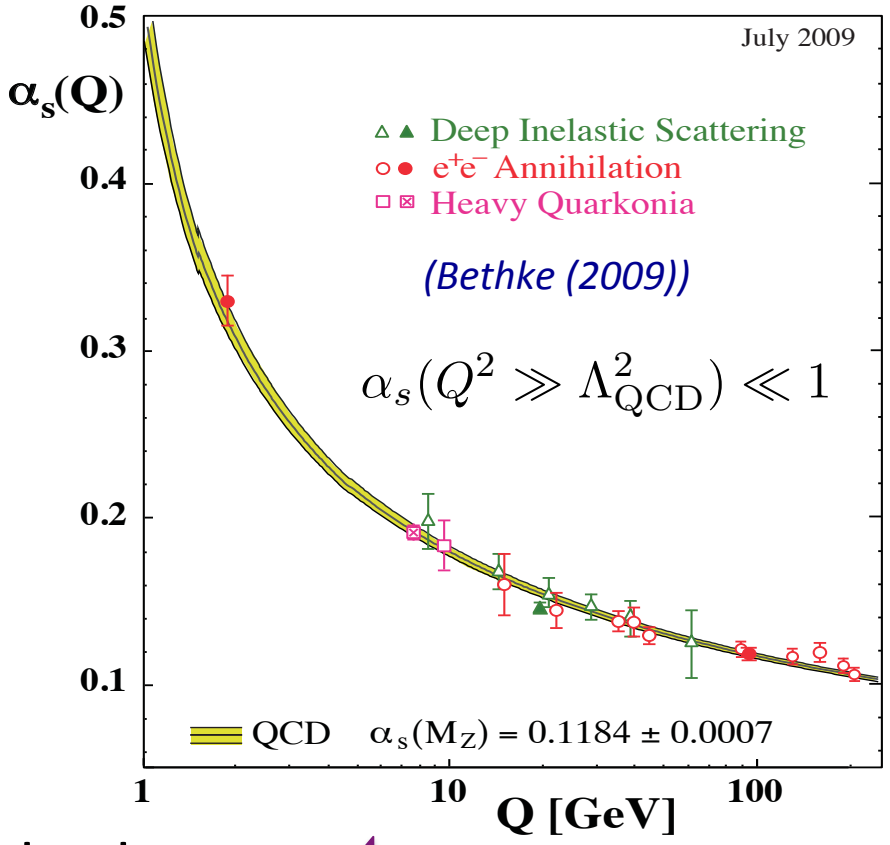
- Drell-Yan:



α_s = QCD coupling strength

QCD Factorization

- Short Distances; Asymptotic Freedom
  - Perturbation Theory



- Large Distance Scales:
  - Hadron Structure
    Confinement

- Physical processes involve both.
  - Need to be separated: *QCD Factorization*

$d \sim 1/Q$

Collinear (Standard) Case

- Parton Model

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)}_{\text{Elementary collision}} \otimes \underbrace{f_{q/P}(x_1)}_{\text{Hadron Structure: large distance scales}} \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}_{\text{Hadron Structure: large distance scales}}$$

Short distance scales

$$\sim 1/Q$$

Collinear (Standard) Case

- Perturbative QCD factorization theorem:

$$\sigma \sim \int \underbrace{\mathcal{H}(Q, \mu/Q, \alpha_s(\mu))}_{\text{Small Coupling: Perturbation Theory}} \otimes f_{q/P}(x_1;\mu) \otimes f_{\bar{q}/\bar{P}}(x_2;\mu)$$

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

$$\text{Error} \sim \Lambda_{\text{QCD}}/Q$$

Red : Perturbatively Calculable

Blue : Non-Perturbative Input Needed

Collinear (Standard) Case

- Perturbative QCD factorization theorem:

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Small Coupling: Perturbation Theory

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

Defined in terms of elementary fields

$$\text{Error} \sim \Lambda_{\text{QCD}}/Q$$

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Collinear (Standard) Case

- Perturbative QCD factorization theorem:

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$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

Auxiliary parameter: Arbitrary

$$\text{Error} \sim \Lambda_{\text{QCD}}/Q$$

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Evolution (Standard) Case

- Perturbative QCD factorization theorem:

$$\sigma \sim \int \mathcal{H}(Q, \mu/Q, \alpha_s(\mu)) \otimes f_{q/P}(x_1;\mu) \otimes f_{\bar{q}/\bar{P}}(x_2;\mu)$$

Auxiliary parameter: Arbitrary

- DGLAP evolution
  (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- *Factorization* + *Evolution*: Universal PDFs

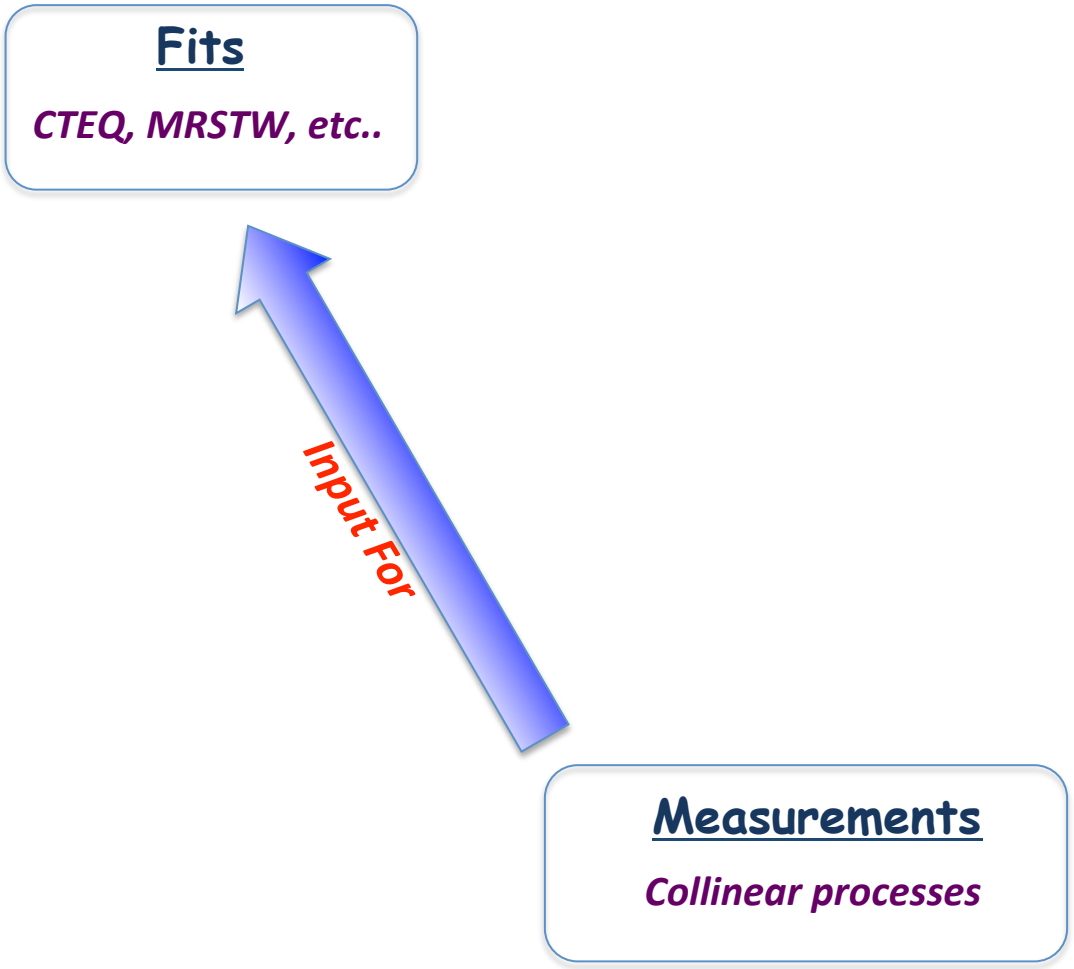
"Portable"

Implementing Collinear Factorization

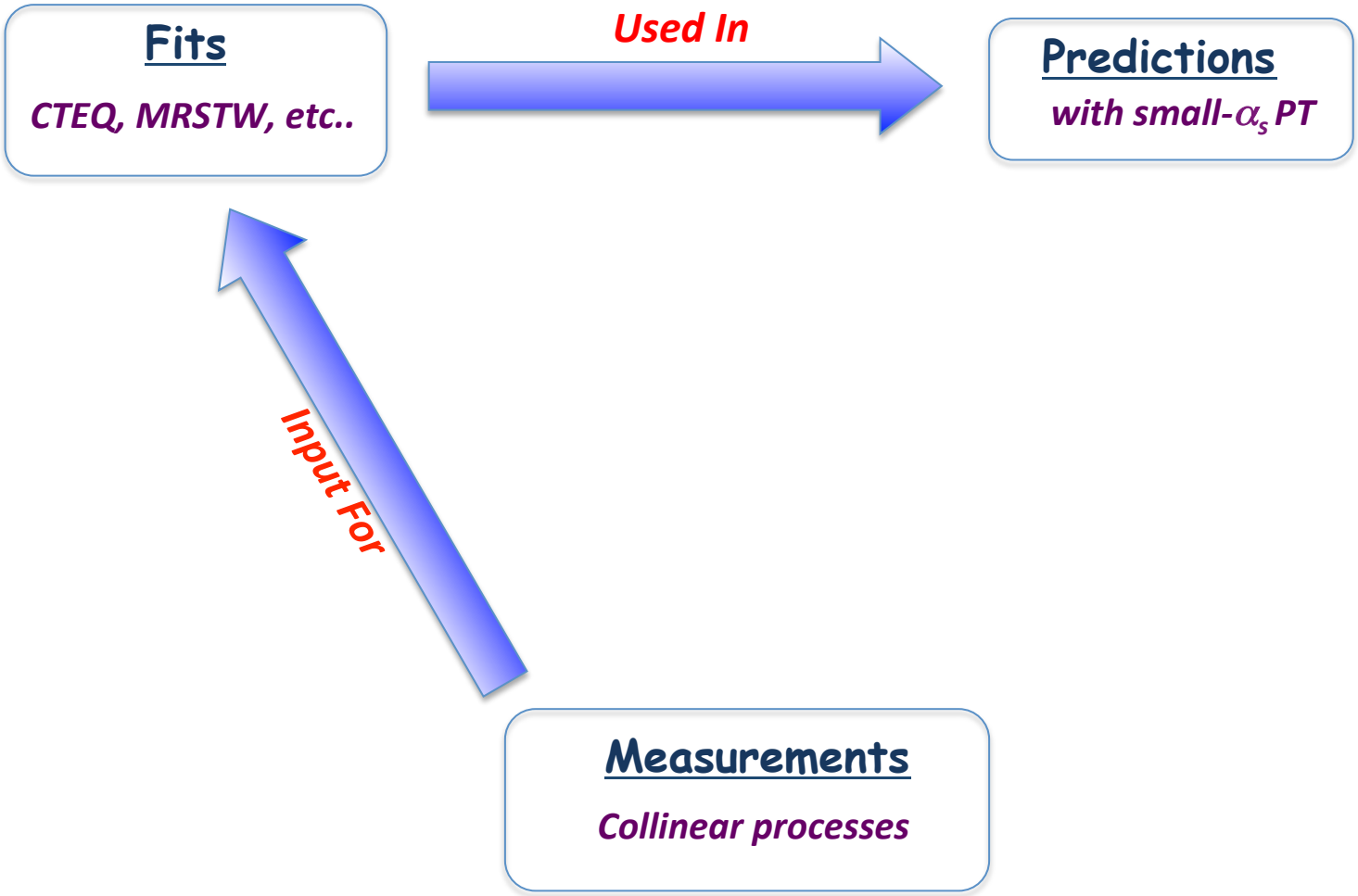
Measurements

Collinear processes

Implementing Collinear Factorization



Implementing Collinear Factorization



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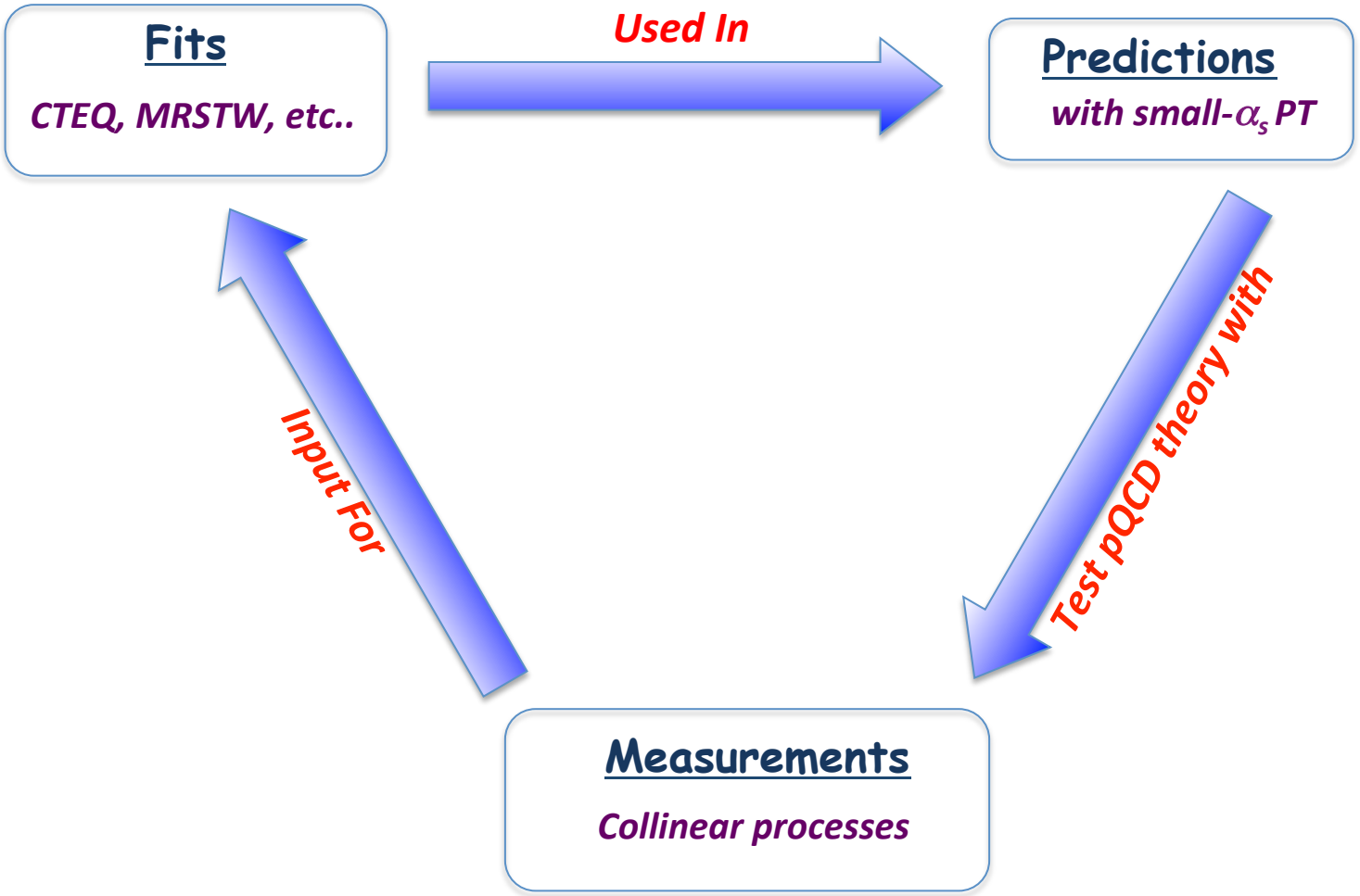

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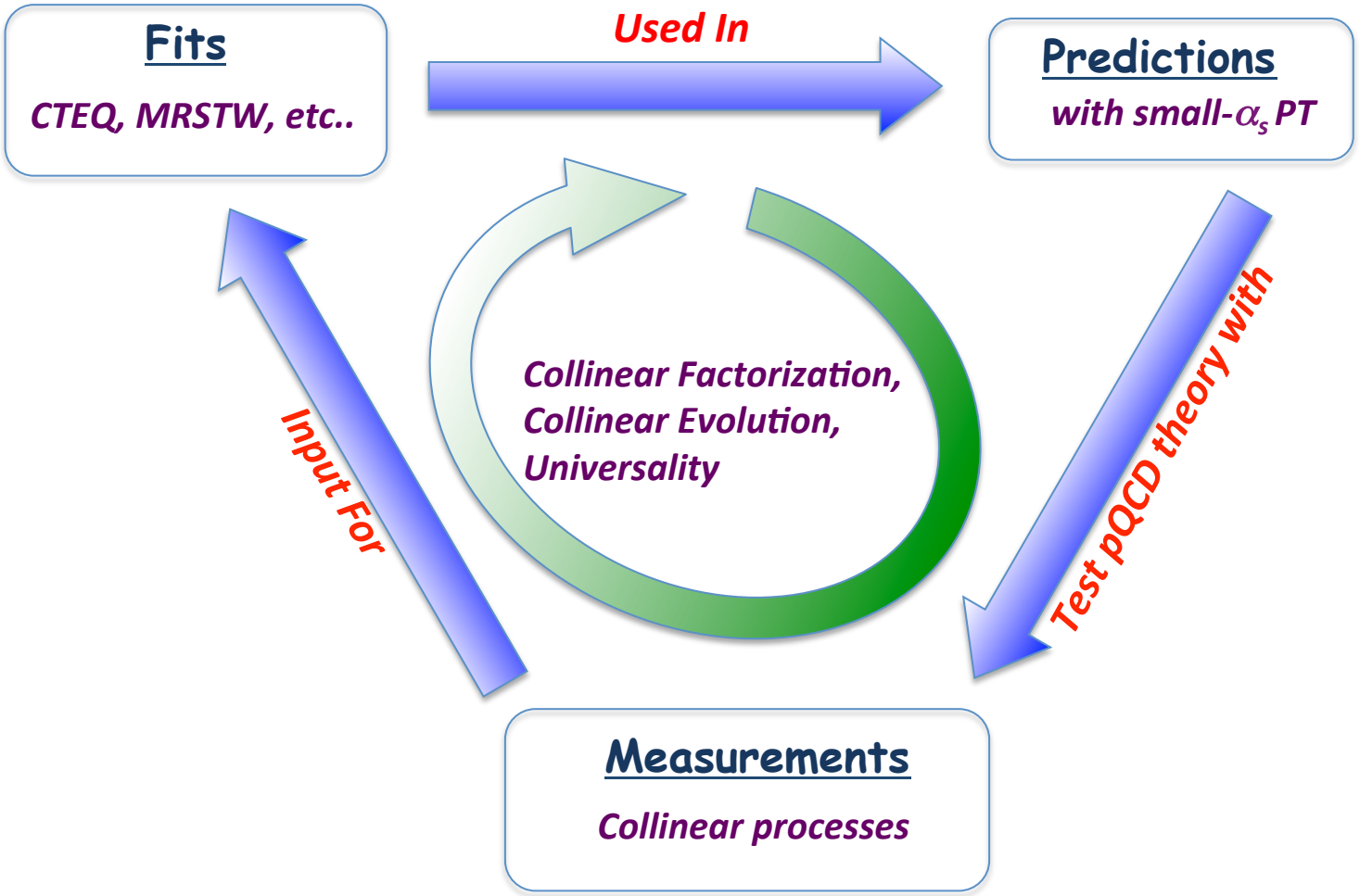

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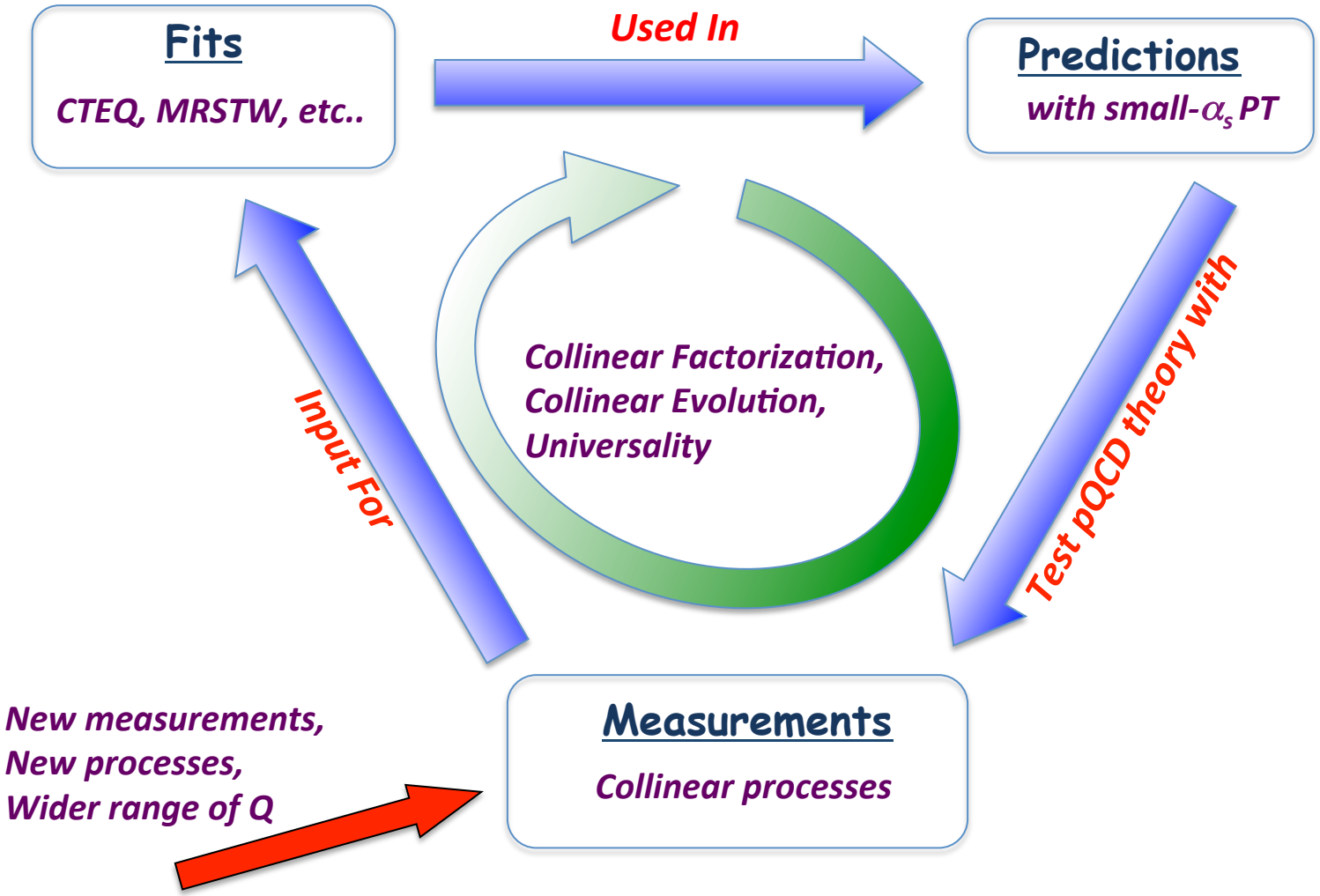

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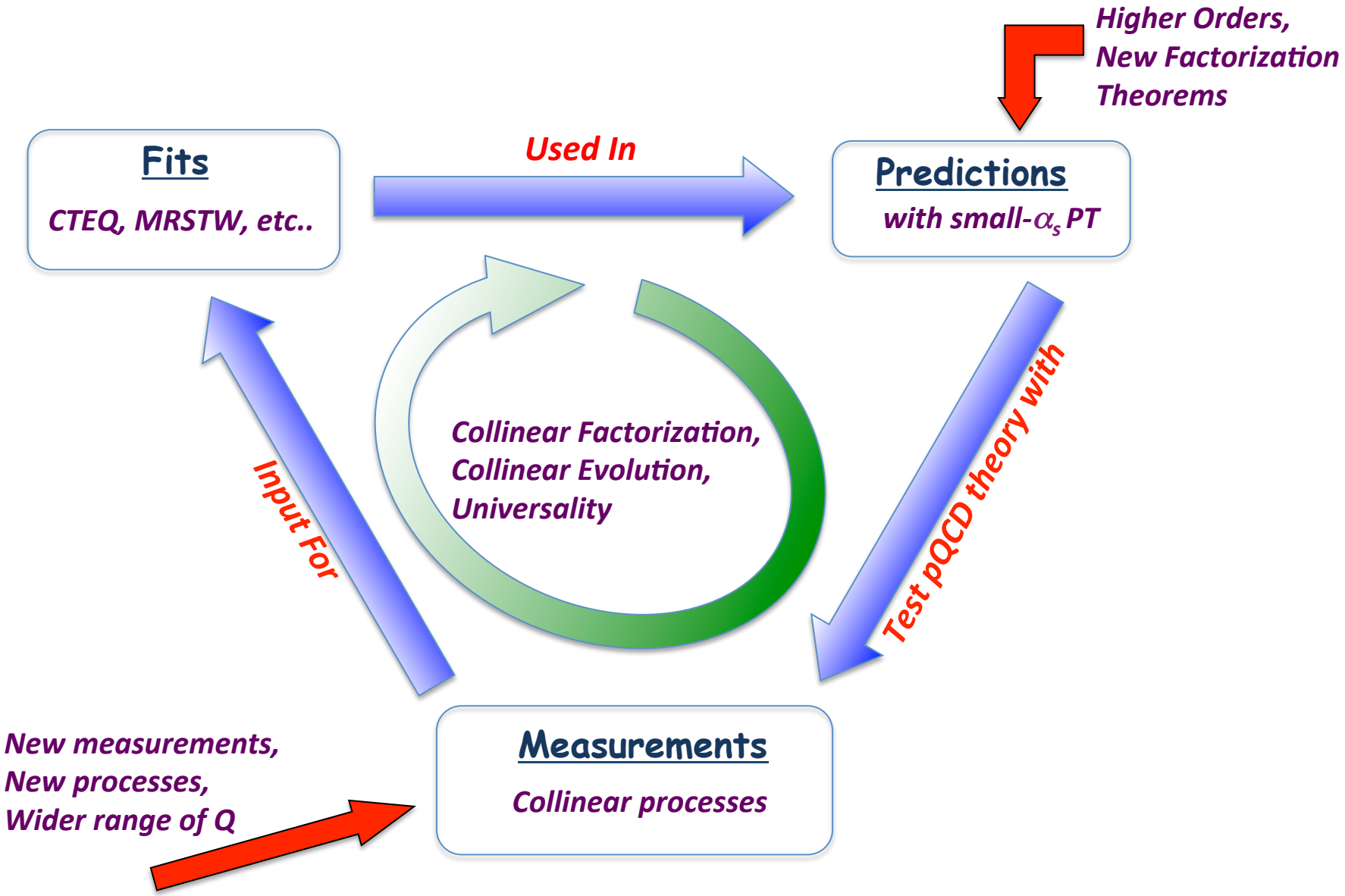
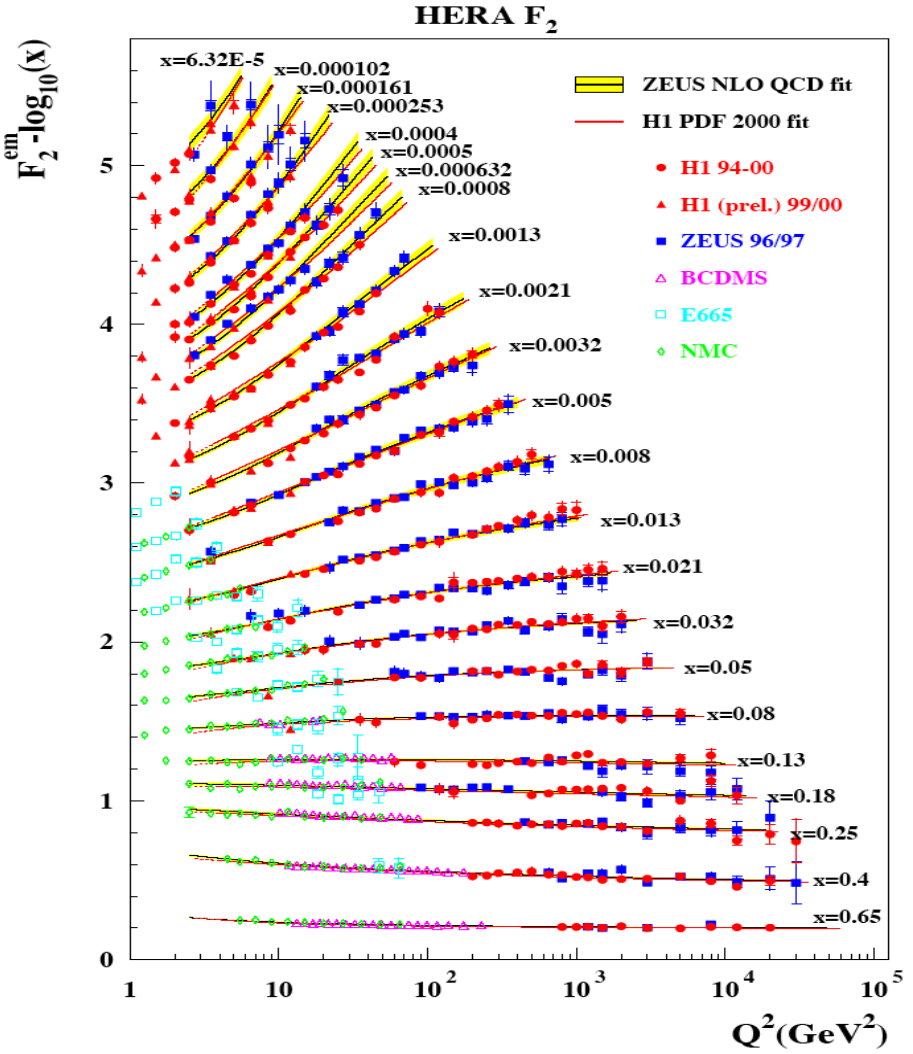


Phenomenology

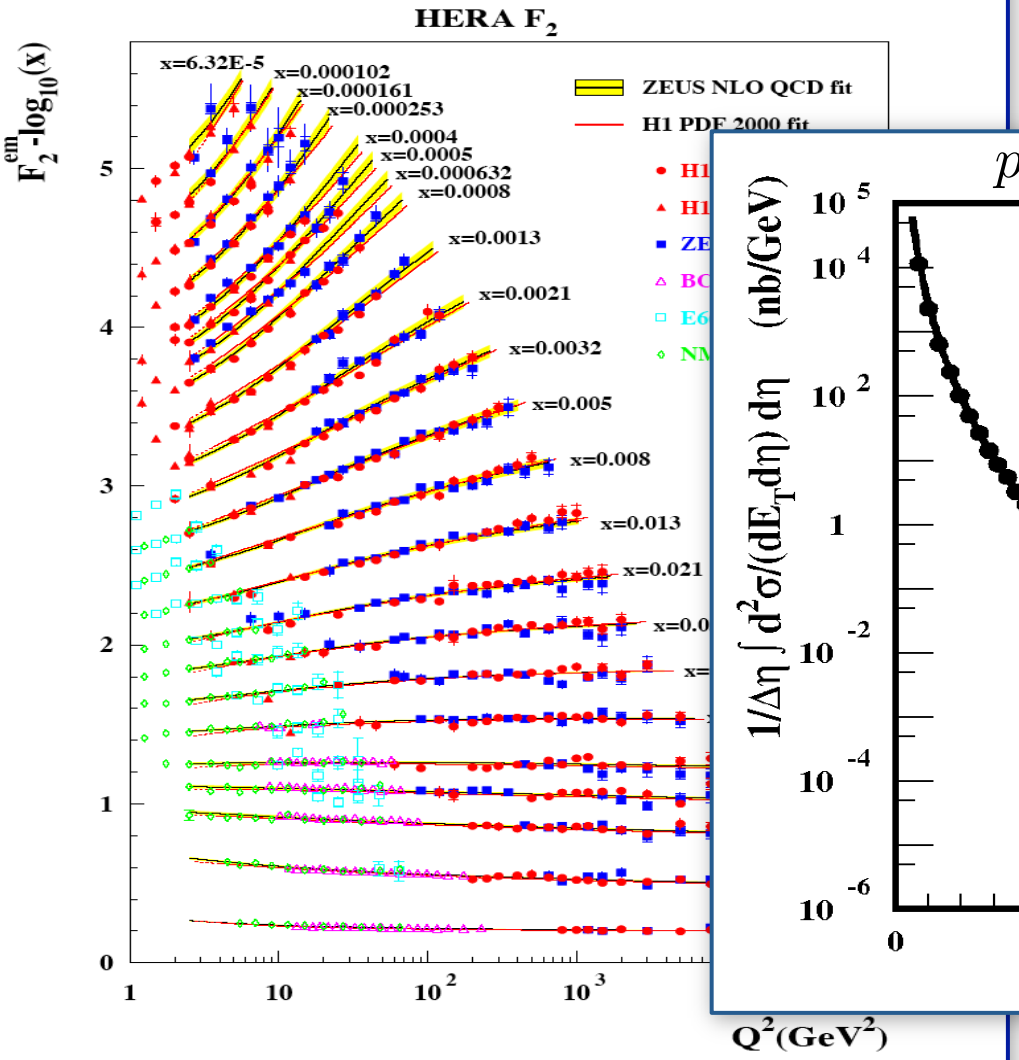
Deep Inelastic Scattering



Fitting

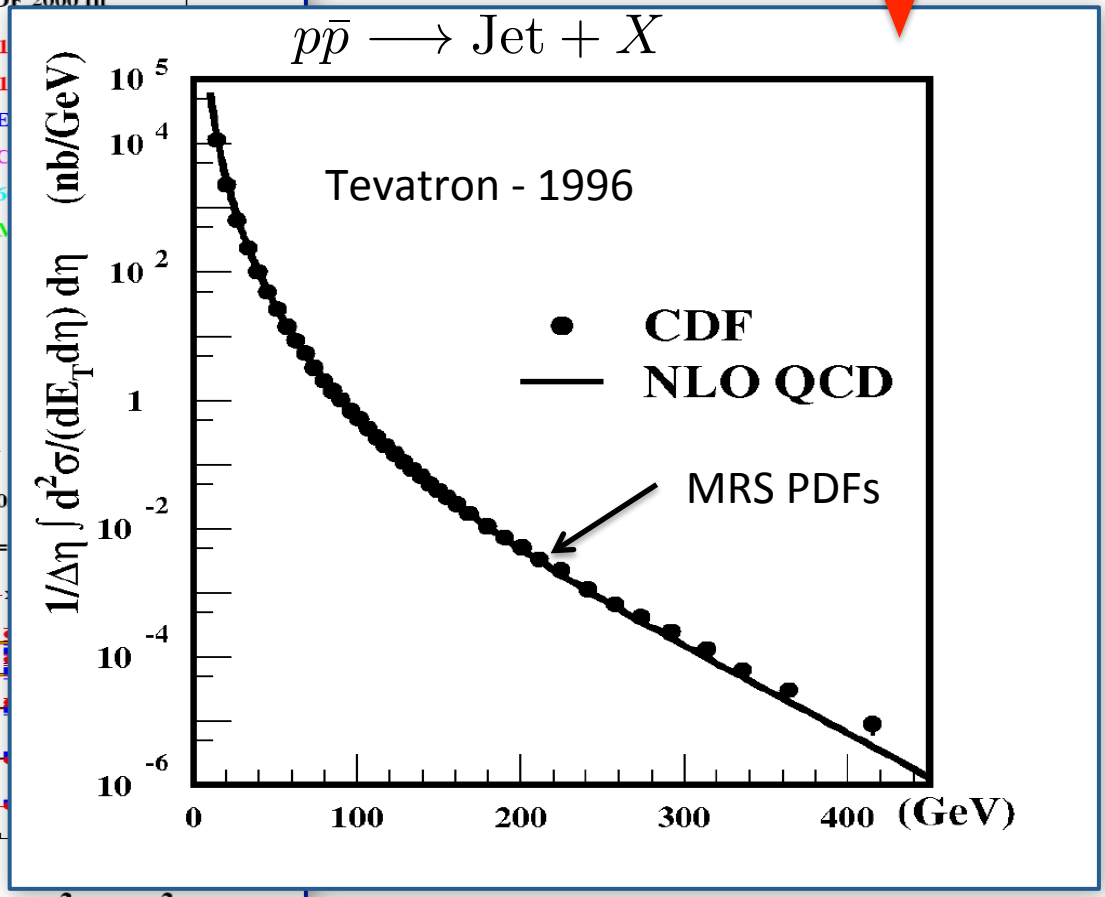
Phenomenology

Deep Inelastic Scattering



Fitting

Predicting



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Collinear (Standard) Case

- Perturbative QCD factorization theorem:

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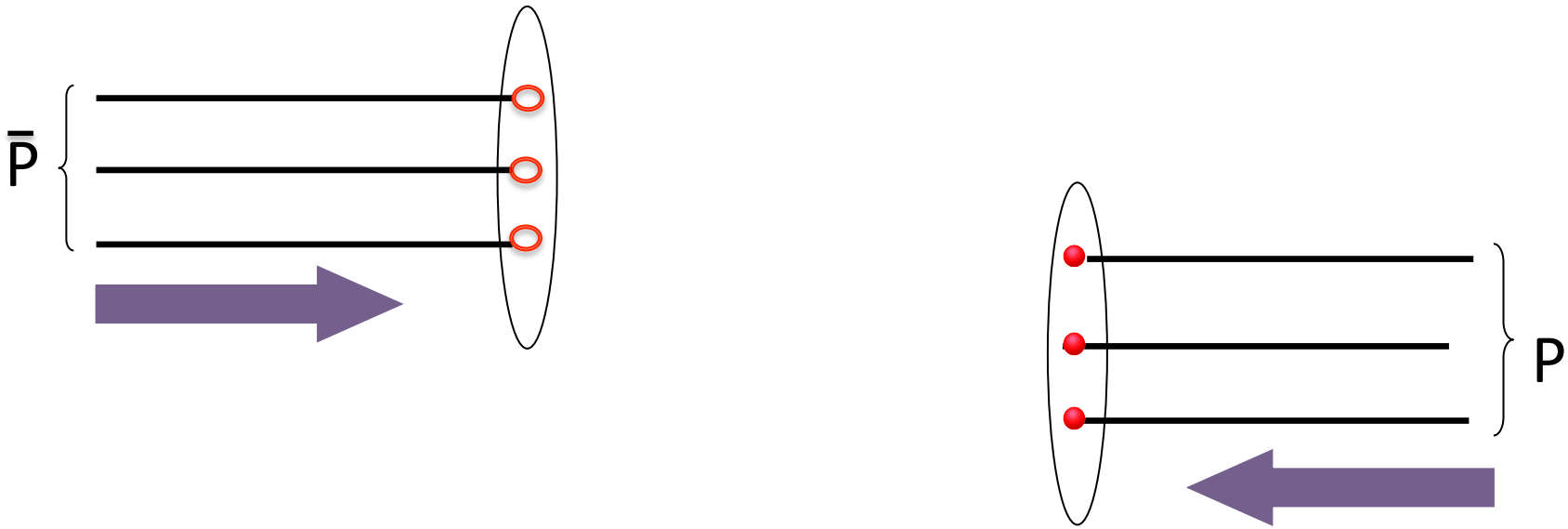

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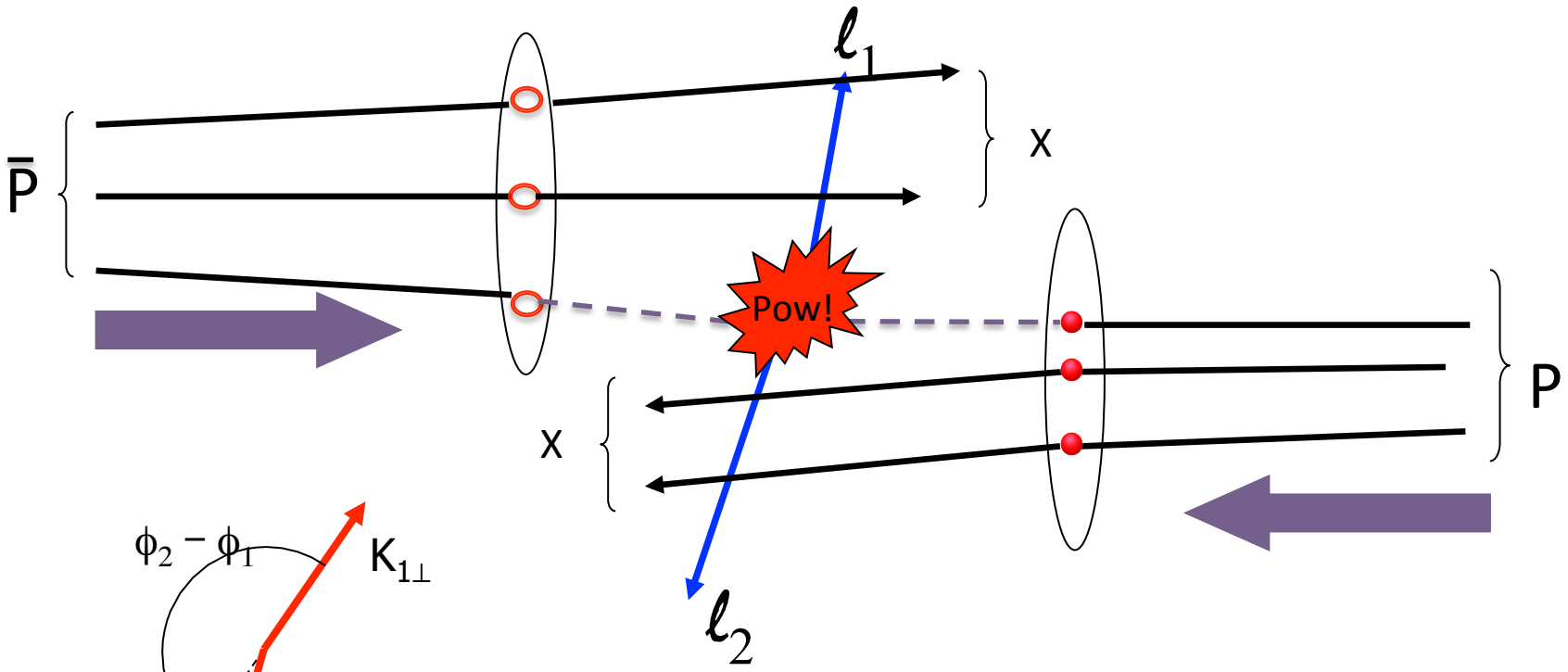

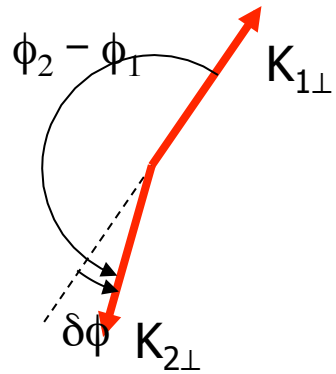

$$\mathbf{q}_T = \mathbf{l}_{1T} + \mathbf{l}_{2T}$$

Example 2:

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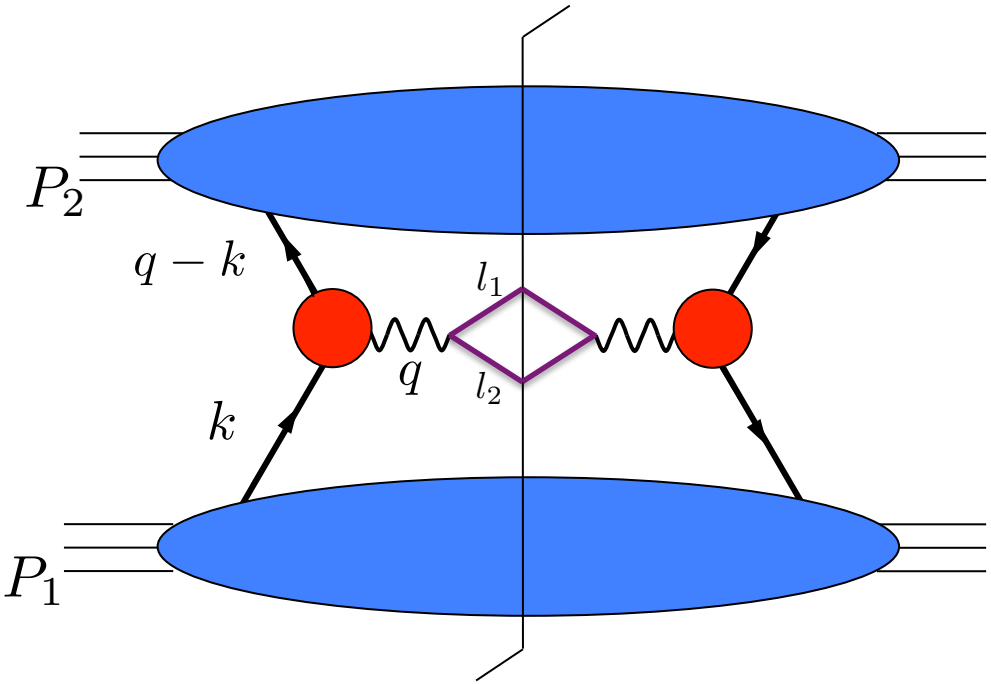

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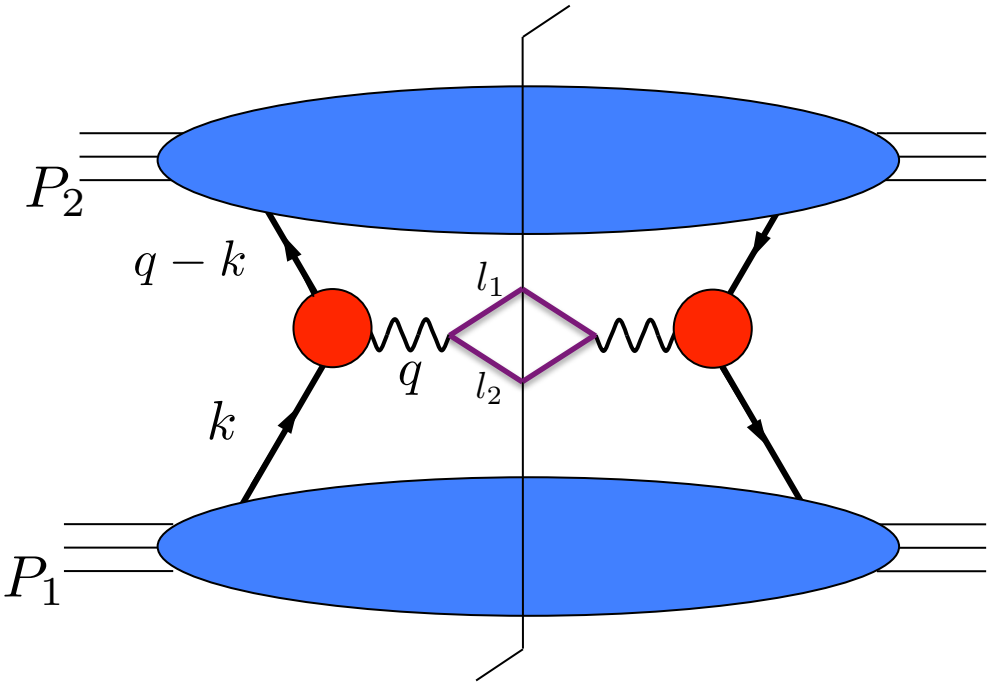

TMD *case*

Get: $\dfrac{d\sigma}{d\mathbf{q}_T \cdots}$

For all q_T.

$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\mathrm{QCD}}$$

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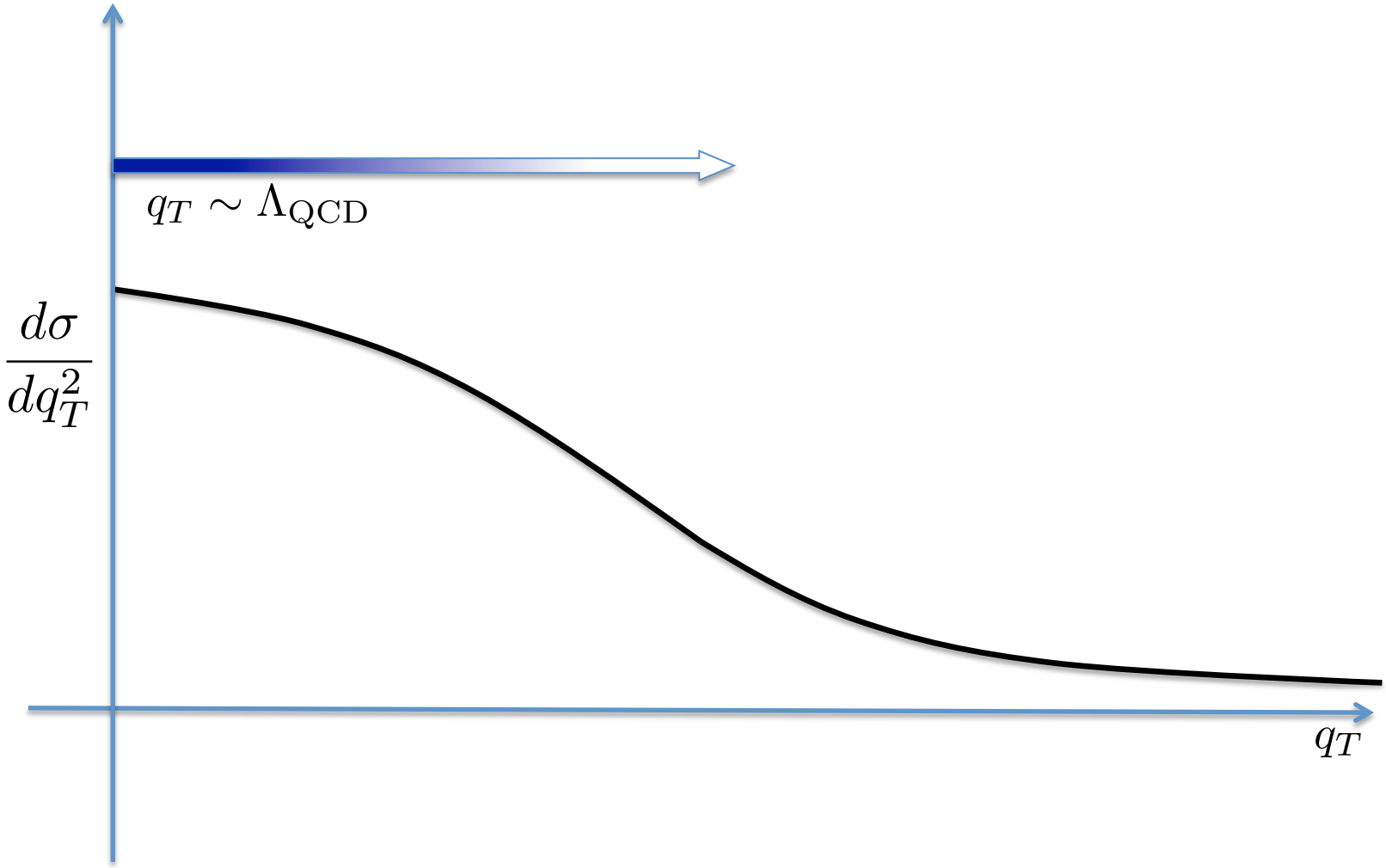
Transverse Momentum Distributions



TMD-Factorization

- Collinear factorization theorem relies on *collinear* approximations.

TMD-Factorization

- Collinear factorization theorem relies on *collinear* approximations.

- Accounting for intrinsic transverse momentum requires *new factorization theorems*.

Collinear (Standard) Case

- Parton Model

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)}_{\text{Elementary collision}} \otimes \underbrace{f_{q/P}(x_1)}_{\text{Hadron Structure: large distance scales}} \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}_{\text{Hadron Structure: large distance scales}}$$

Elementary collision Short distance scales

Hadron Structure: large distance scales

- Perturbative QCD factorization theorem

$$\sigma \sim \int \underbrace{\mathcal{H}(Q, \mu/Q, \alpha_s(\mu))}_{\text{Small Coupling: Perturbation Theory}} \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/\bar{P}}(x_2; \mu)$$

Small Coupling: Perturbation Theory

Auxiliary parameter: Arbitrary Defined in terms of QFT operators

TMD-Factorization

- Parton Model

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)}_{\text{Elementary collision}} \otimes \underbrace{F_{q/P}(x_1, \mathbf{k}_{1T})}_{\text{Number densities}} \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}_{\text{Number densities}}$$

Parton Model

Short distance scales

$\sim 1/Q$

Number densities

"Transverse Momentum Dependent Parton Distribution Functions" (TMD PDFs)

TMD-Factorization

- Parton Model

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Parton Model

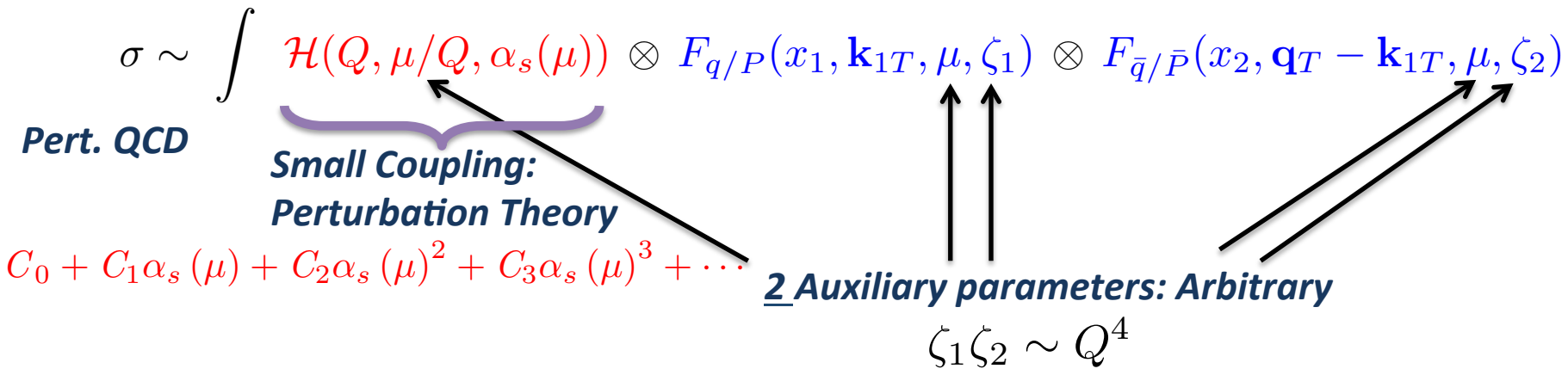
Short distance scales $\sim 1/Q$

"Transverse Momentum Dependent Parton Distribution Functions" (TMD PDFs)

- Past Approaches:
  - Non-perturbative descriptions:
    - Model
    - Fitting
  - Transverse Momentum Resummation

TMD-Factorization: QCD

- Unified Formalism



(Collins, Soper, Sterman (CSS) formalism (1982,1983))

(Collins Extension: (2011), Chapts. 10,13,14)

TMD-Evolution

- Recall Collinear / DGLAP:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

TMD-Evolution

- Recall Collinear / DGLAP:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- TMD Case:

$$\frac{\partial \ln \tilde{F}(x,b_T;\mu,\zeta)}{\partial \ln\sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln \tilde{F}(x,b_T;\mu,\zeta)}{d\ln\mu} = \gamma_F(g(\mu);\zeta/\mu^2)$$

((CSS) formalism (1982,1983))

(Collins Extension: (2011), Chapts. 10,13,14)

TMD PDF Definitions

- Defined in terms of elementary field operators.

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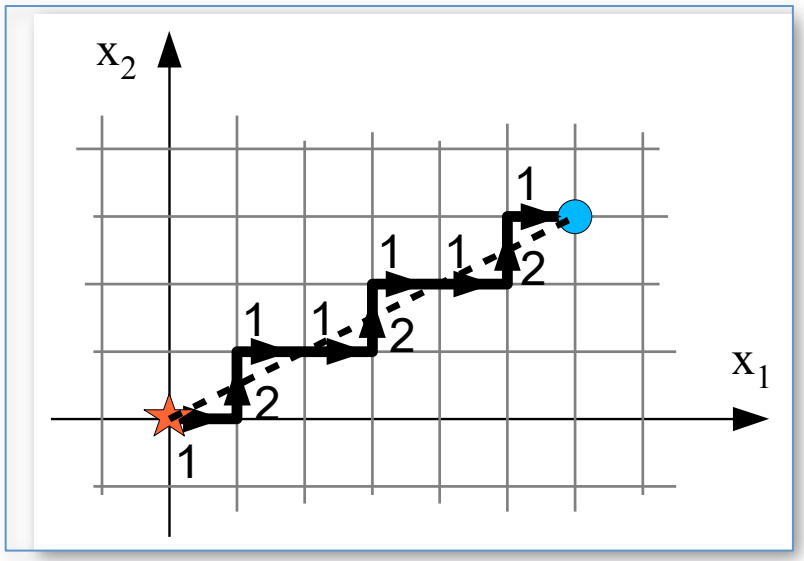
- Needed to address questions of hadronic structure.
  - Lattice QCD.



(M. Engelhardt et al., (2012))
(B. Musch et al., (2011))

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  - Models and Effective Theories.
- Universality / Modified Universality.
- Constrained by factorization derivation.
- TMD evolution is independent of method of regulation.

(Collins, TCR (2012))

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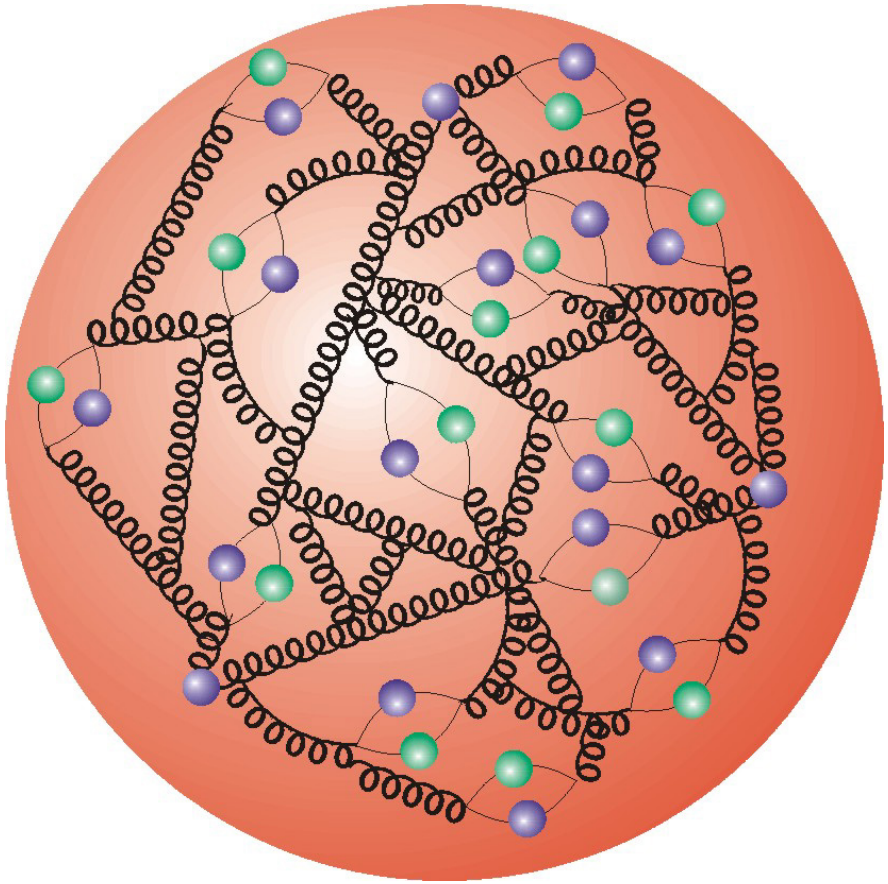
TMD vs. Collinear

- TMDs: Rich source of information about hadron structure.
  - *TMD Zoo*

TMD vs. Collinear



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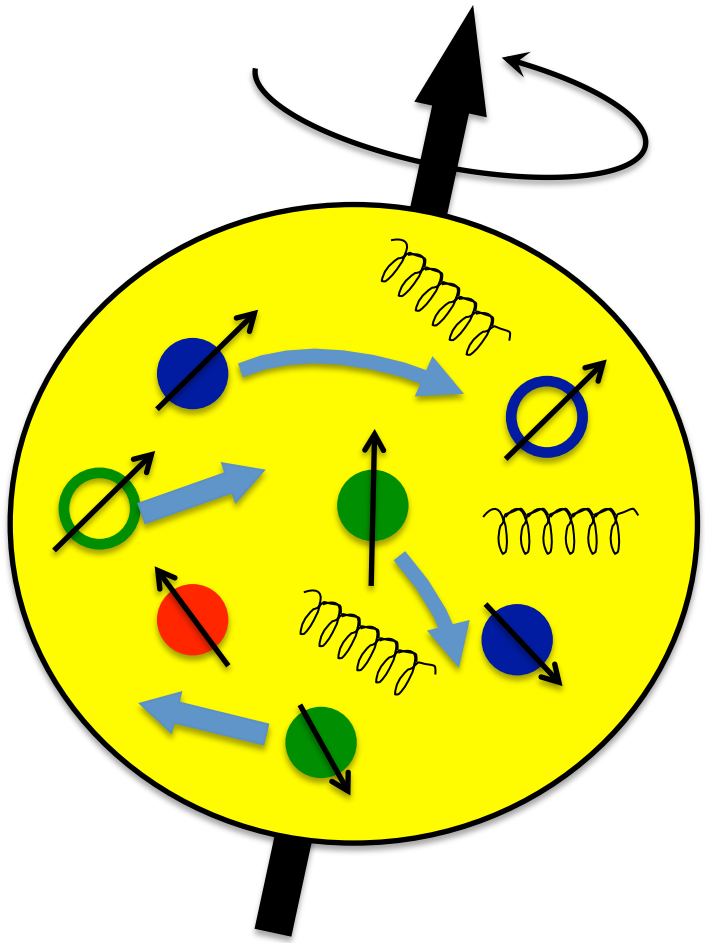


TMD vs. Collinear

- TMDs: Rich source of information about hadron structure.
  - *TMD Zoo*

- More complicated fitting:

$$f_{f/P}(x) \longrightarrow F_{f/P}(x, \mathbf{k}_T)$$

Need non-perturbative descriptions

TMD vs. Collinear

- TMDs: Rich source of information about hadron structure.
  - *TMD Zoo*

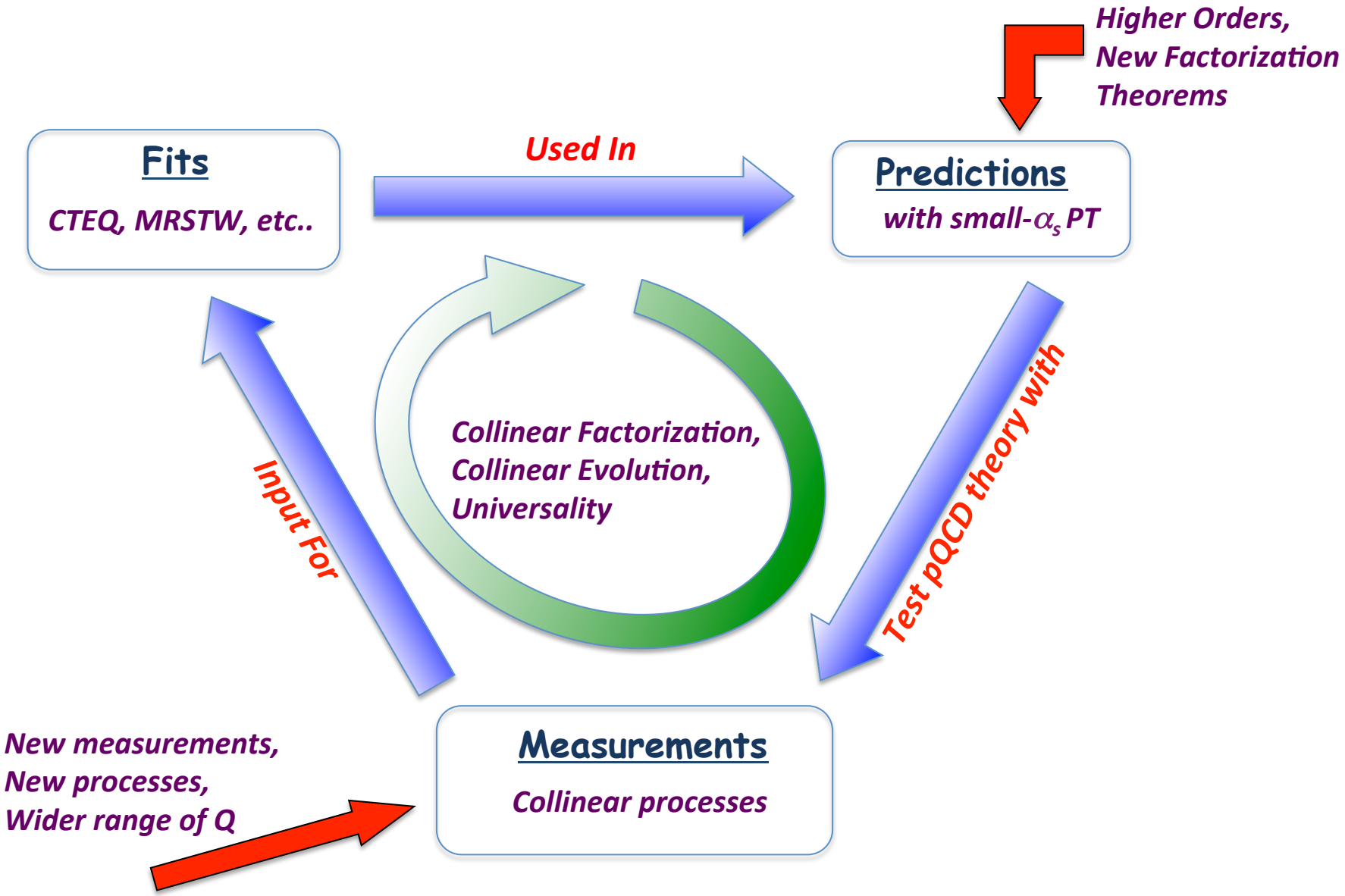
- More complicated fitting:

$$f_{f/P}(x) \longrightarrow F_{f/P}(x, \mathbf{k}_T)$$

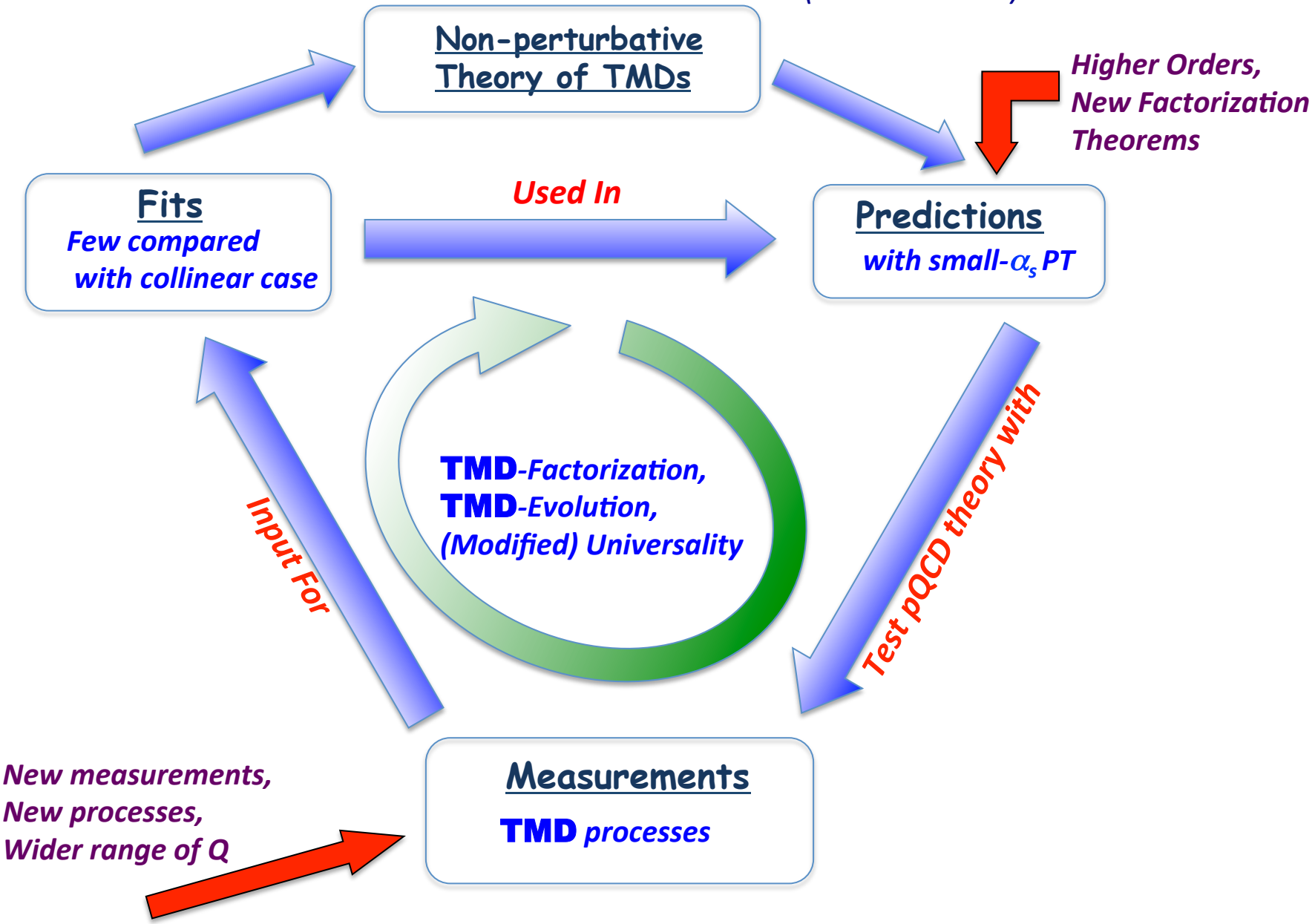
Need non-perturbative descriptions

- Cases of non-universality / TMD-factorization breaking.

Implementing Collinear Factorization



Implementing TMD-Factorization *(Current Work)*



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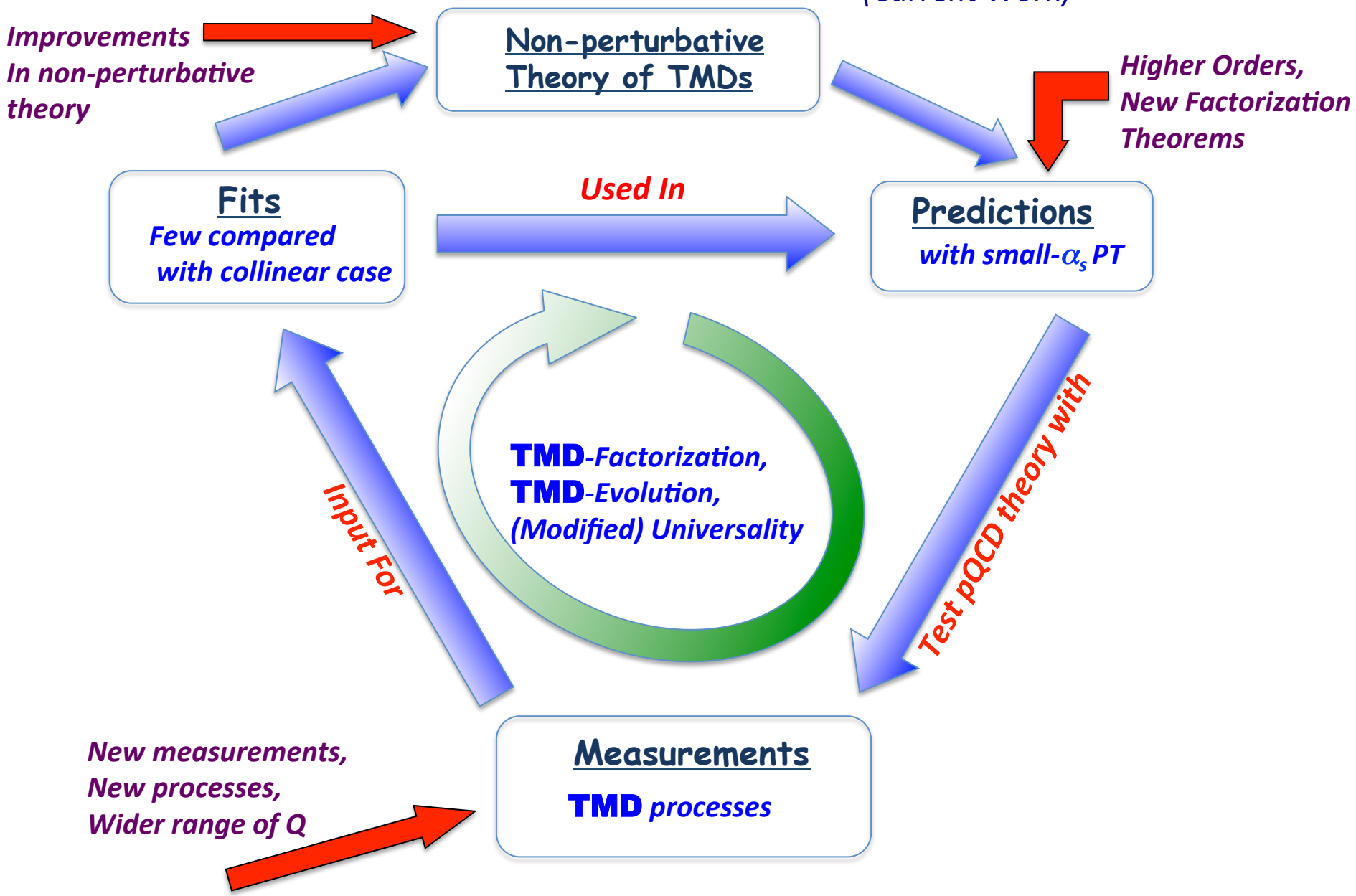

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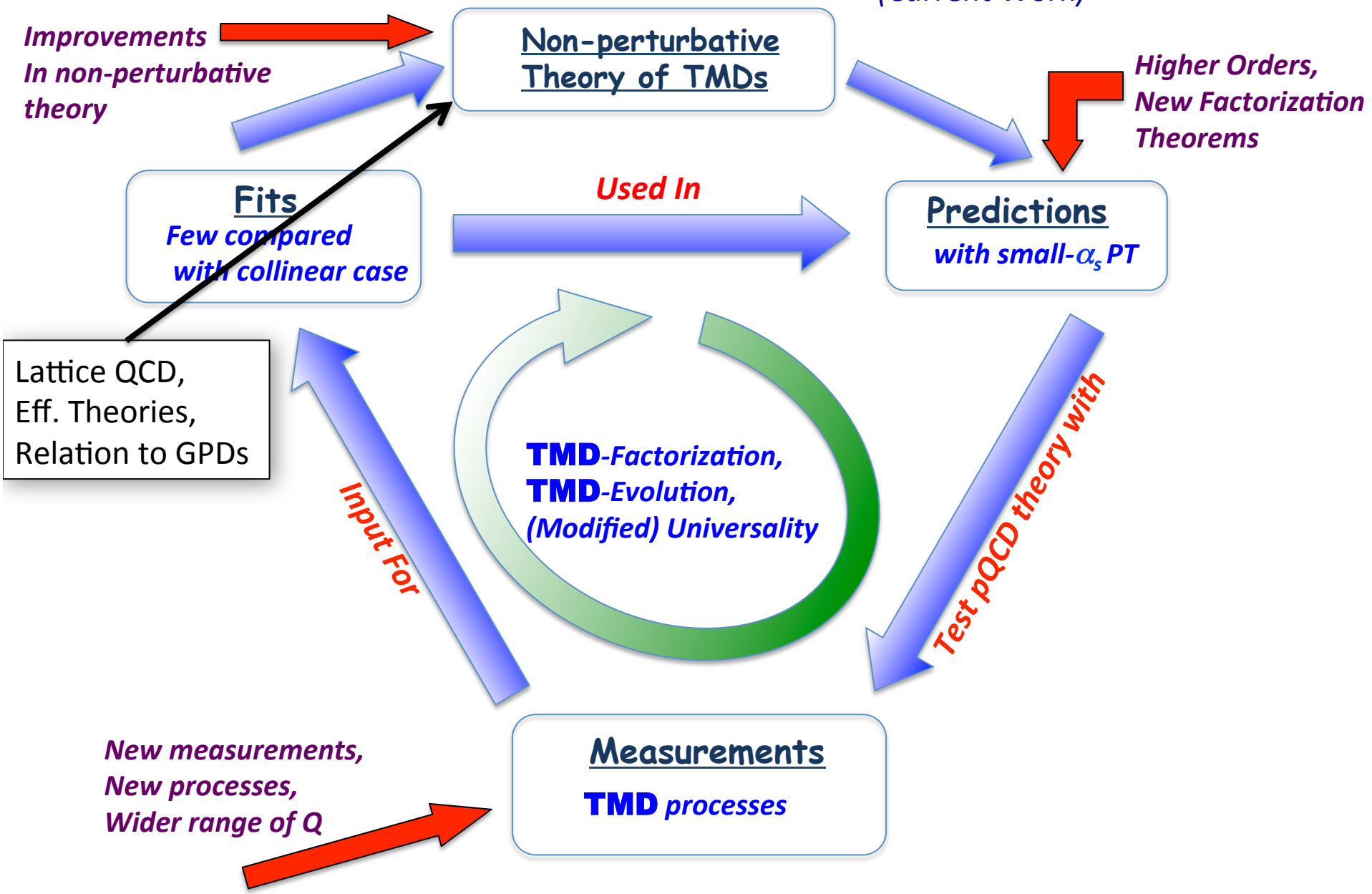


Implementing TMD-Factorization *(Current Work)*

Improvements In non-perturbative theory

Non-perturbative Theory of TMDs

Higher Orders, New Factorization Theorems

Fits
Few compared with collinear case

Used In

Predictions
with small-α_s PT

Lattice QCD,
Eff. Theories,
Relation to GPDs

Jlab@12GeV
EIC
LHC
Belle
…

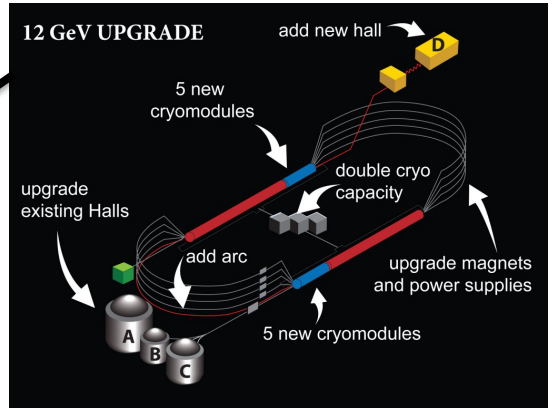
Input For

TMD-*Factorization,*
TMD-*Evolution,*
(Modified) Universality

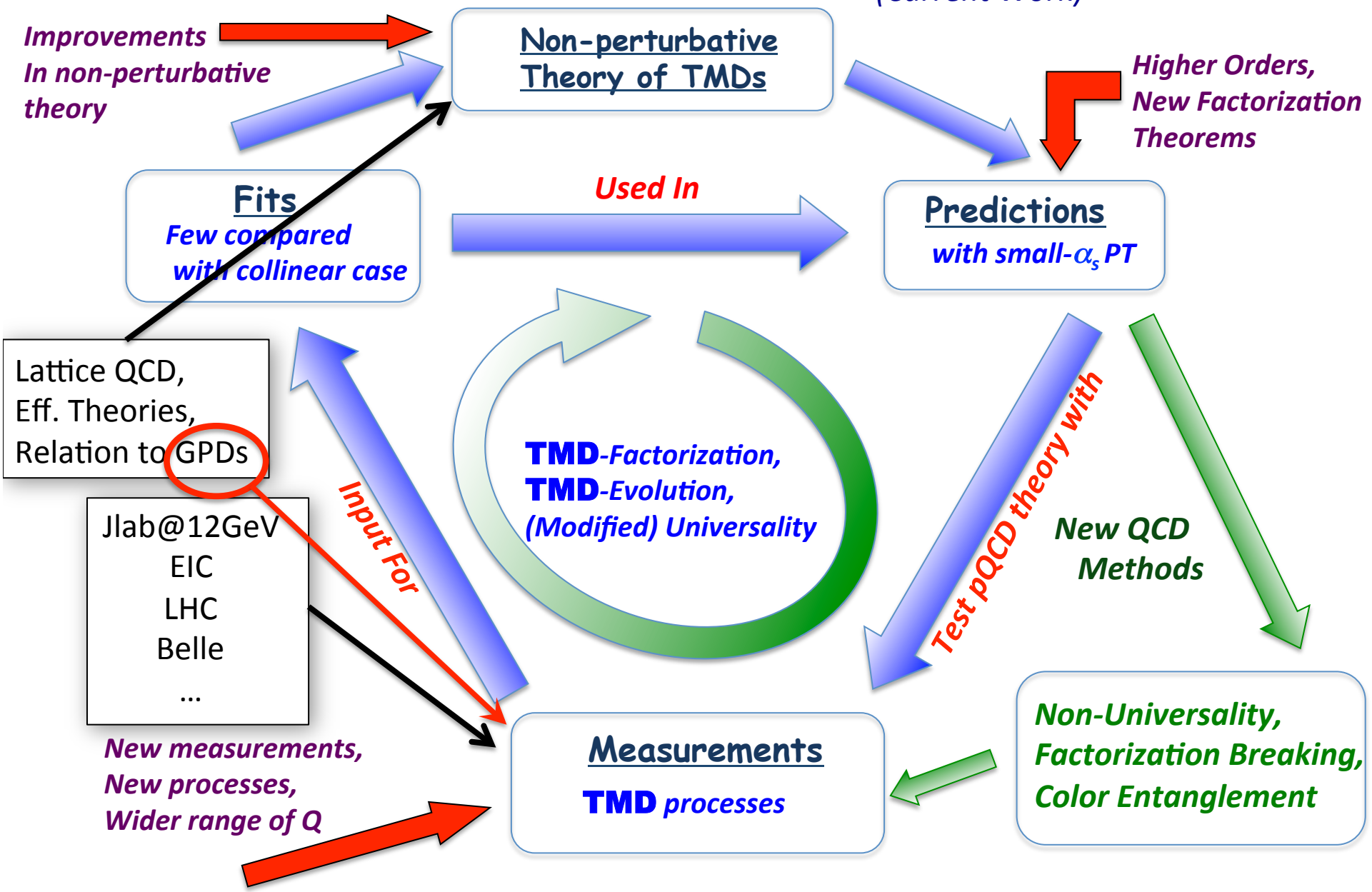
Test pQCD theory with

New measurements, New processes, Wider range of Q

Measurements
TMD *processes*



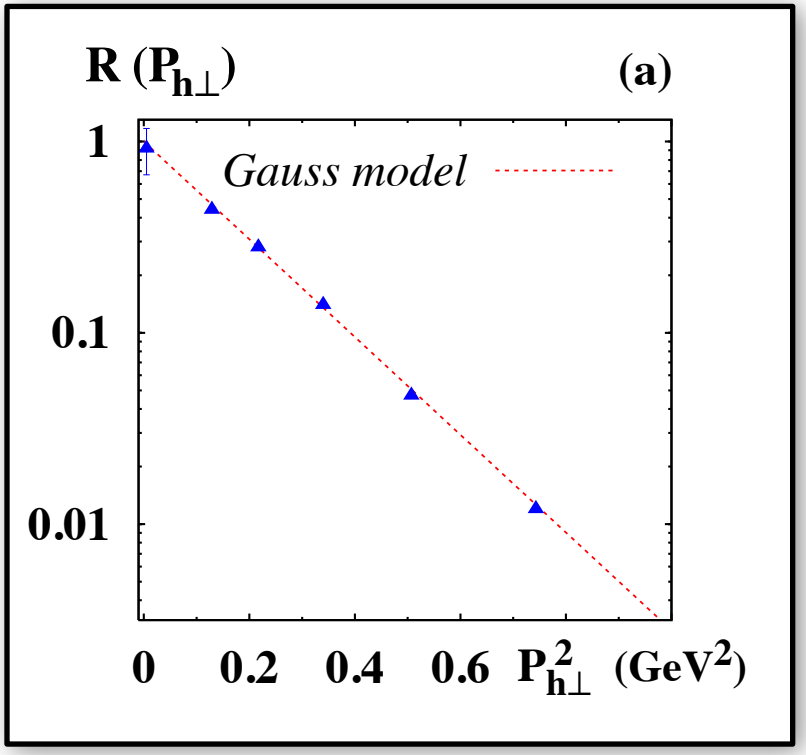
Implementing TMD-Factorization *(Current Work)*



Specific Fits: Two Approaches

Fixed Scale

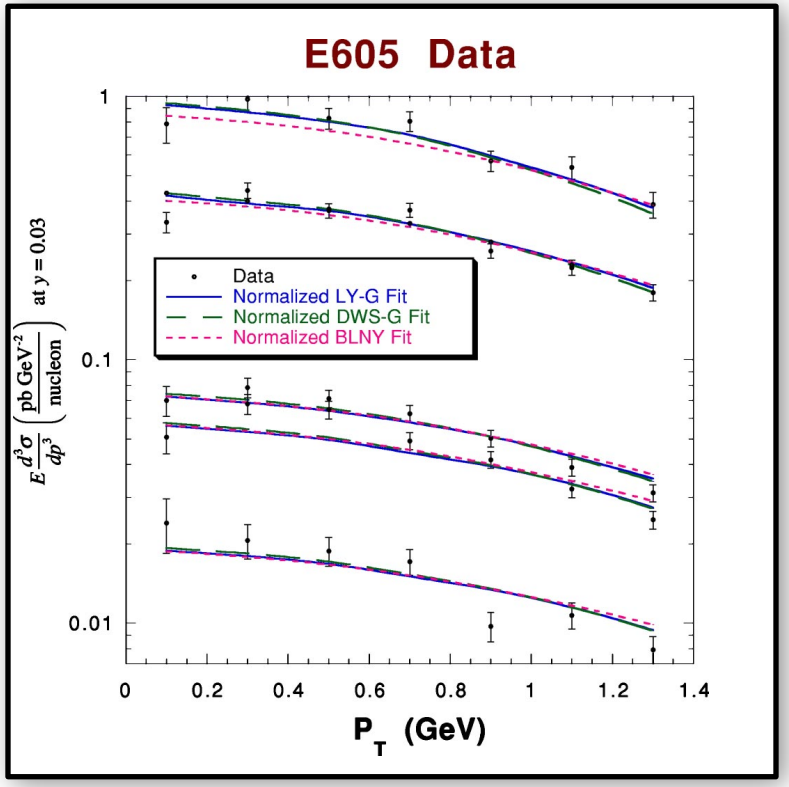
Single-Inclusive DIS



(Schweitzer, Teckentrup, Metz (2010))

Resummation in CSS

Drell-Yan



(Landry, et al. (2003))

Unpolarized TMD PDFs

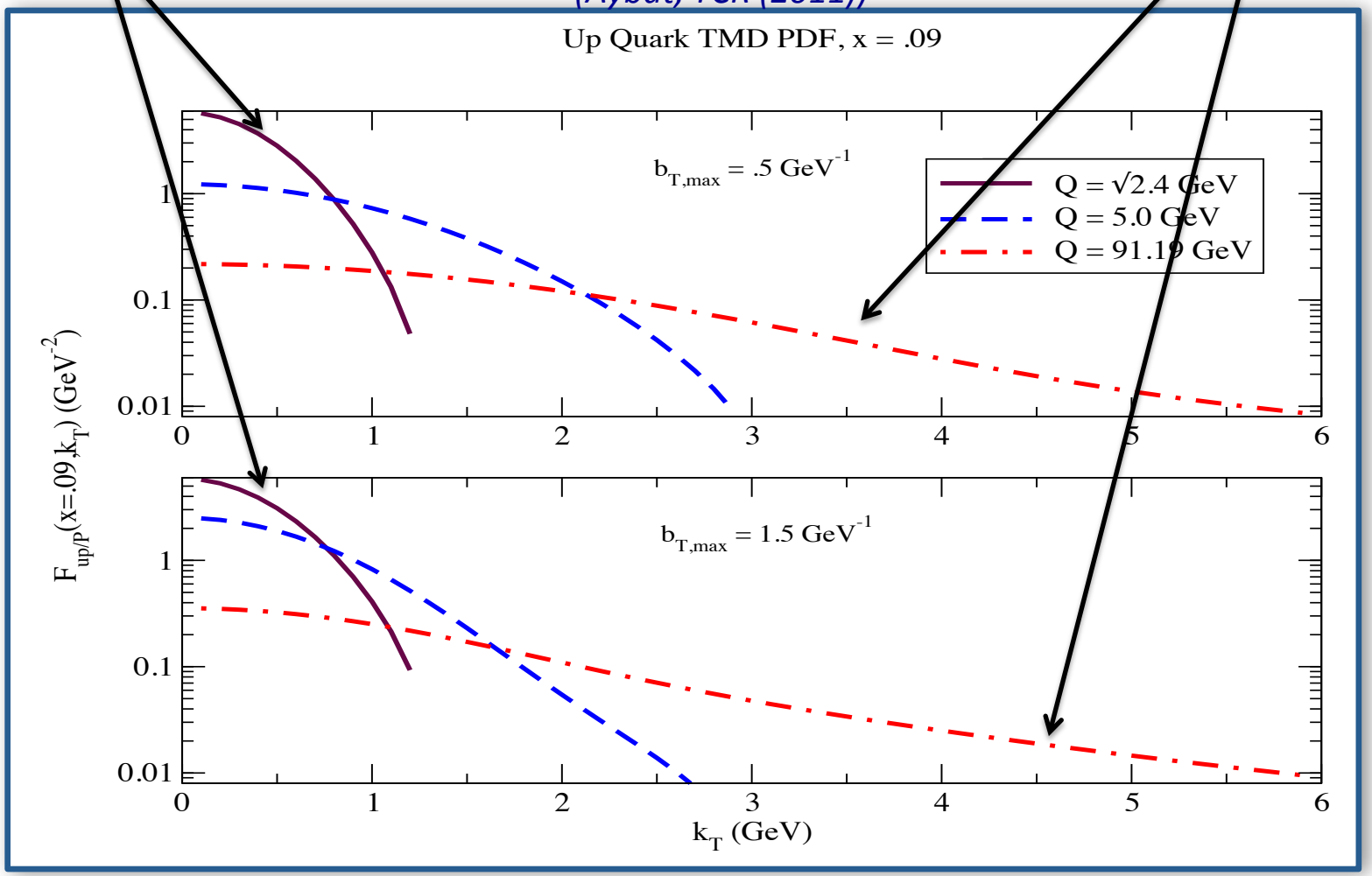
Fixed Scale

(Schweitzer, Teckentrup, Metz (2010))

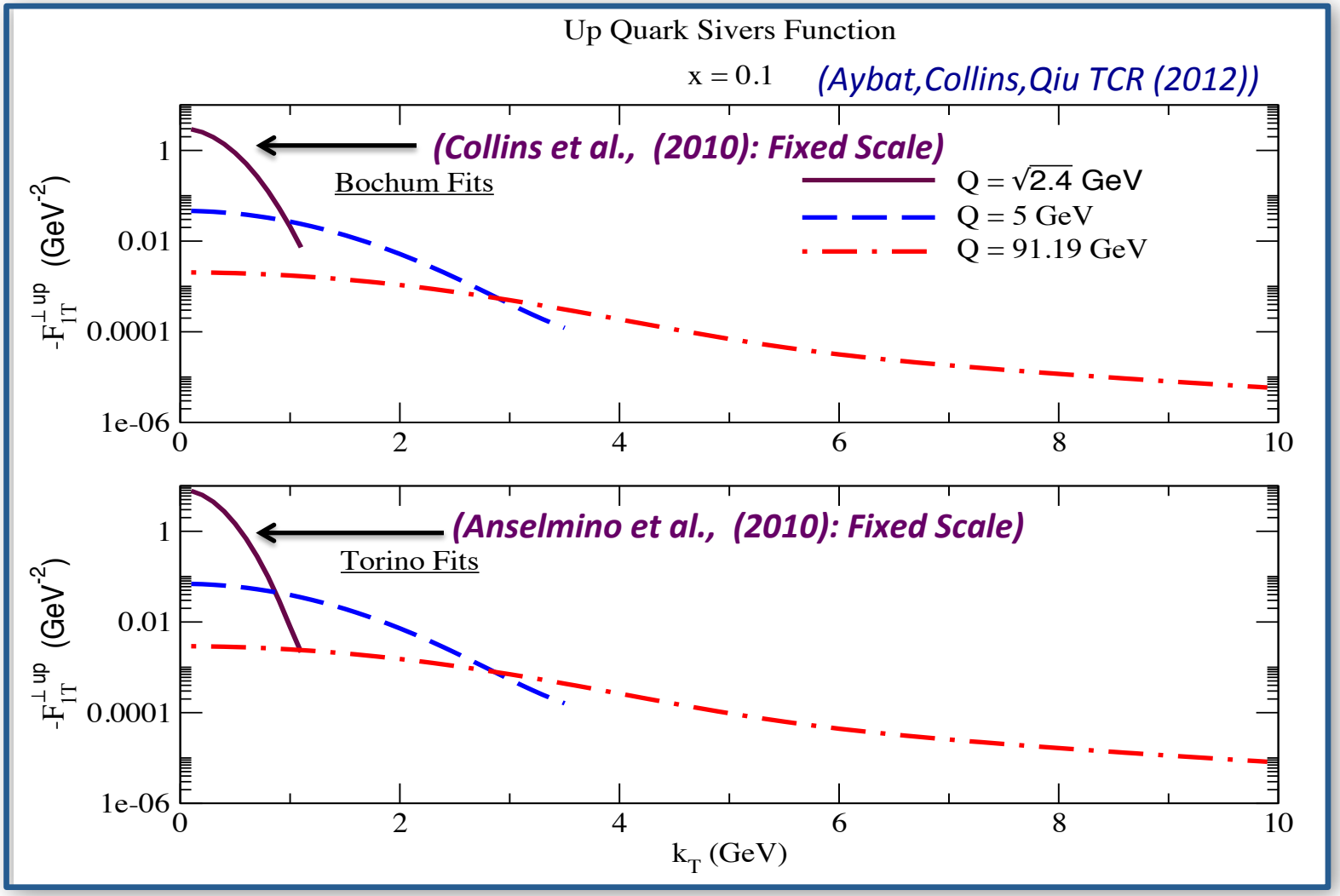
Resummation in CSS

(Landry, et al. (2003))

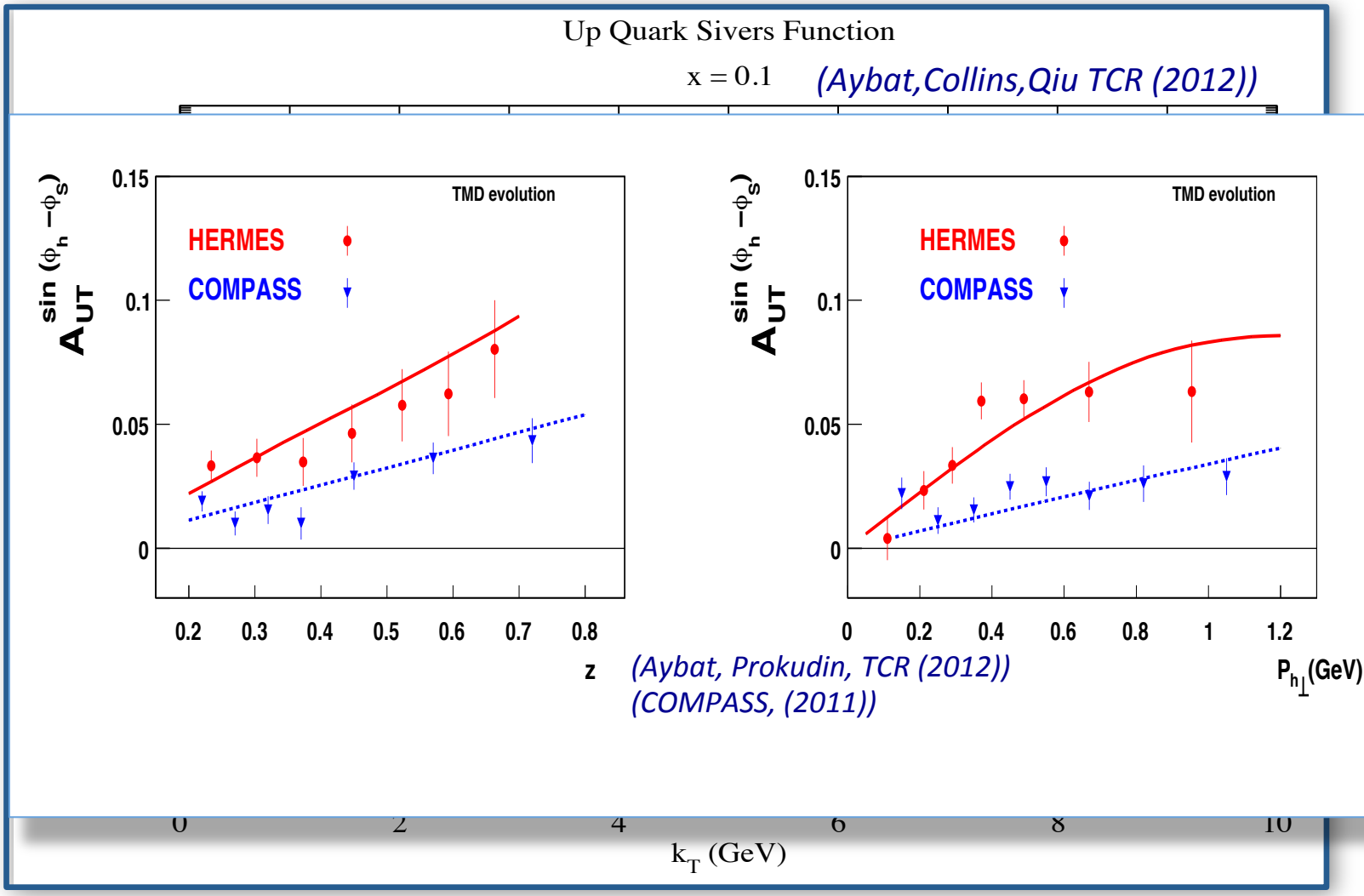
(Aybat, TCR (2011))



Sivers Function



Sivers Function



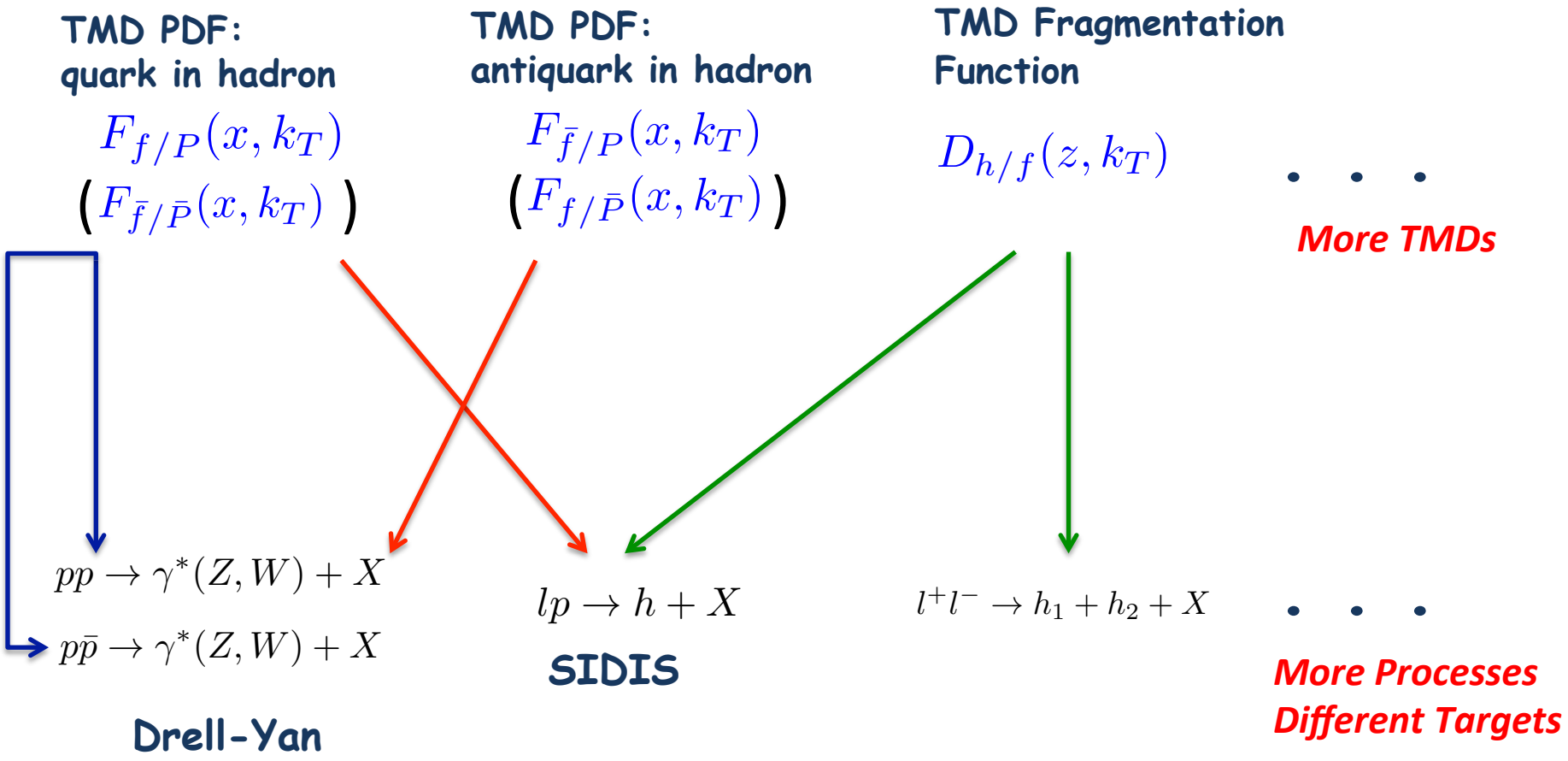
(Aybat,Collins,Qiu TCR (2012))

(Aybat, Prokudin, TCR (2012))
(COMPASS, (2011))

New fits: *To do*

- Incorporate data from *all* types of processes.
  - SIDIS, DY, e^+e^-, different targets….
- Incorporate *all* types of observables.
  - Unpolarized cross sections, spin asymmetries…

Constraining Non-Perturbative Parts



Unpolarized Proton vs. Anti-proton

TMD PDF: quark in hadron

$F_{f/P}(x, k_T)$

$(F_{\bar{f}/\bar{P}}(x, k_T))$

TMD PDF: antiquark in hadron

$F_{\bar{f}/P}(x, k_T)$

$(F_{f/\bar{P}}(x, k_T))$

TMD Fragmentation Function

$D_{h/f}(z, k_T)$

. . .

More TMDs

$pp \to \gamma^*(Z, W) + X$

$p\bar{p} \to \gamma^*(Z, W) + X$

Drell-Yan

$lp \to h + X$

SIDIS

$l^+l^- \to h_1 + h_2 + X$

. . .

More Processes
Different Targets

Unpolarized Proton vs. Anti-proton

TMD PDF: quark in hadron

$F_{f/P}(x, k_T)$

$(F_{\bar{f}/\bar{P}}(x, k_T))$

TMD PDF: antiquark in hadron

$F_{\bar{f}/P}(x, k_T)$

$(F_{f/\bar{P}}(x,$

TMD Fragmentation Function

$D_{h/f}(z, k_T)$

$pp \to \gamma^*(Z, W) + X$

$p\bar{p} \to \gamma^*(Z, W) + X$

Drell-Yan

$lp \to h +$

SIDIS

Experiment	Reference	Reaction	$\sqrt{S}$(GeV)	δN_{exp}
R209	[14]	$p+p \to \mu^+\mu^- + X$	62	10%
E605	[15]	$p+Cu \to \mu^+\mu^- + X$	38.8	15%
E288	[16]	$p+Cu \to \mu^+\mu^- + X$	27.4	25%
CDF-Z (Run-0)	[17]	$p+p \to Z+X$	1800	–
DØ-Z (Run-1)	[18]	$p+\overline{p} \to Z+X$	1800	4.3%
CDF-Z (Run-1)	[19]	$p+\overline{p} \to Z+X$	1800	3.9%

Ex: (Landry, et al. (2003))

Different Targets

Unpolarized Proton vs. Anti-proton

Current Project with C. Aidala and B. Field

(In Preparation)

- Focus only on pp and p-pbar Drell-Yan.
- Focus on small q_T.
- Use all available data.
- Vary non-perturbative functional forms.
- Test correlation of x and b_T.
- Extract any difference between valence and sea.

(See,e.g., Schweitzer, Strikman, Weiss (2012))

New fits: *To do*

- Incorporate data from *all* types of processes.
  - SIDIS, DY, e^+e^-, different targets….
- Incorporate *all* types of observables.
  - Unpolarized cross sections, spin asymmetries…
- Test/Compare non-perturbative functional forms
- Treatment of large transverse momentum
- Higher orders.
  - Much already exists, some translating to TMD formalism needed
- Test non-universality
  - Sivers sign
  - Factorization breaking

TMD-Factorization Breaking

- Need tests of TMD-factorization.

- Polarizations in final state:
  - Lambda Hyperon: Transverse Polarization
    - Seen in hadron-hadron collision
    - Not seen in SIDIS, e^+e^-

Factorization Breaking

(TCR, Mulders (2010))
(TCR, in preparation)

Summary

- Theoretical advances.
- Implementations needed.
- Progress being made.
- Next:
  - Include Higher Orders
  - Improve treatment of large q_T region
  - Unpolarized SIDIS data.

Please see updates:

https://projects.hepforge.org/tmd/

Exploring Hadron Structure with Transverse Momentum Dependent Factorization

Ted C. Rogers

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

- ~~Perturbative QCD and Collinear Factorization.~~ ✓
- ~~Transverse Momentum Dependent (TMD) Factorization.~~ ✓
- ~~TMD project: Implementing TMD factorization.~~ ✓

Jefferson National Laboratory – February 6, 2013

Exploring Hadron Structure with Transverse Momentum Dependent Factorization

Ted C. Rogers

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook

Thank You!

Jefferson National Laboratory – February 6, 2013

Solutions

- After evolution and $\mu = \sqrt{\zeta} = Q$

Example Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}$$

$$\mu_*(b_T) = C_1/b_*$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

Collinear PDFs

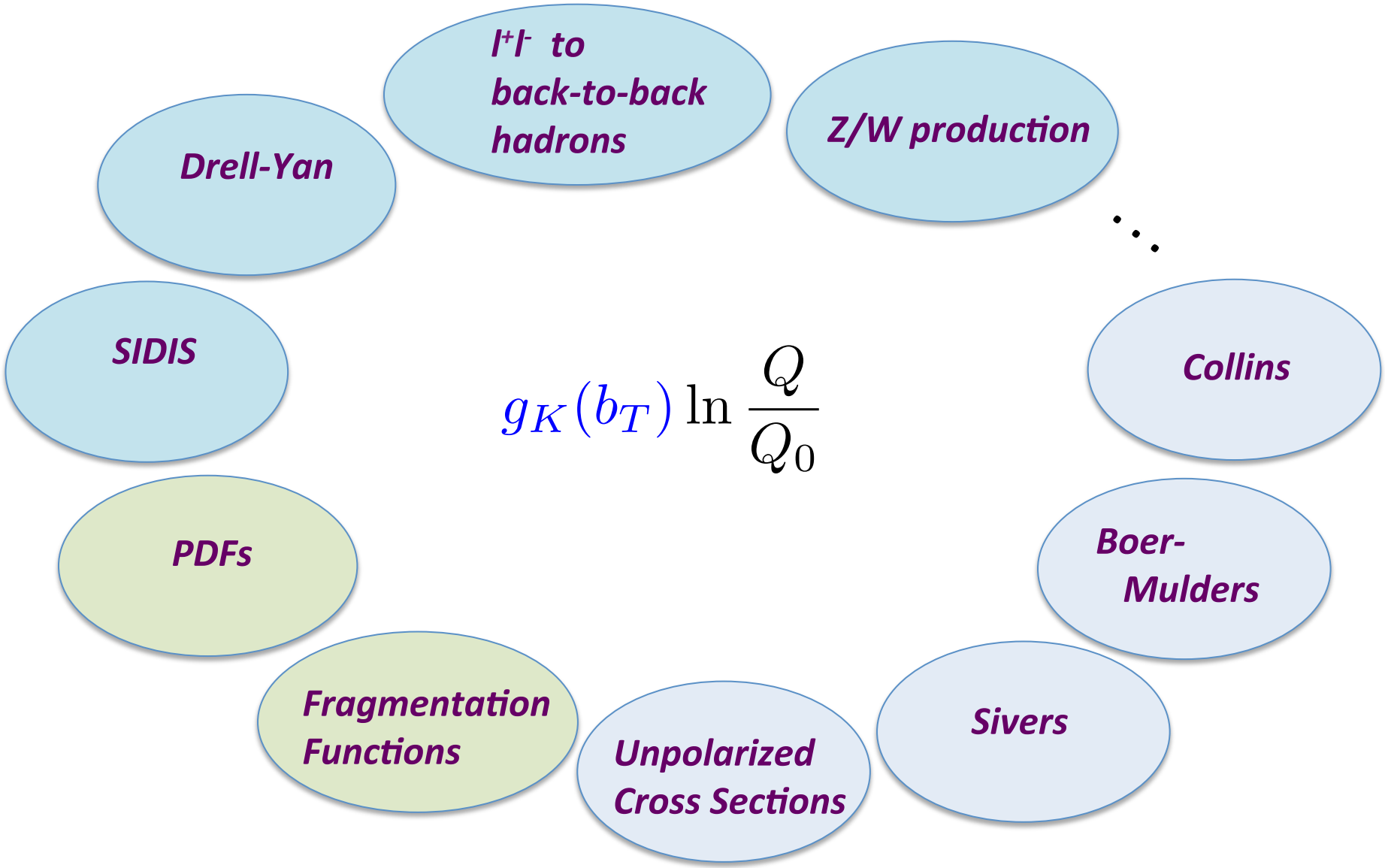
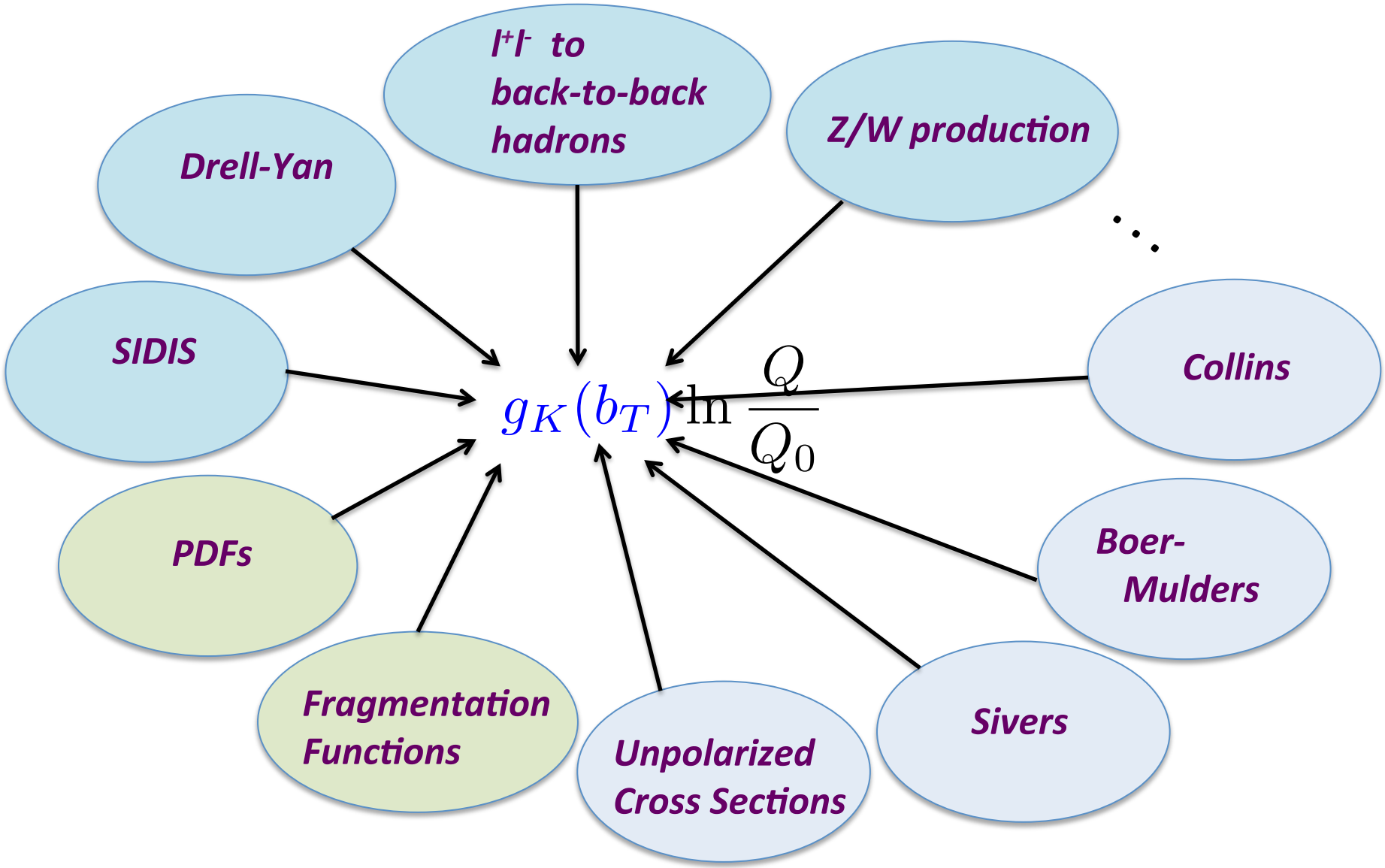
$$\mathrm{A}\left\{ \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times \right.$$

$$\mathrm{B}\left\{ \times \exp\left\{ \ln\frac{Q}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\} \times \right.$$

$$\mathrm{C}\left\{ \times \exp\left\{ g_{f/P}(x, b_T) + g_K(b_T) \ln\frac{Q}{Q_0} \right\} \right.$$

Nonperturbative large b_T behavior

Constraining Non-Perturbative Parts



Constraining Non-Perturbative Parts