Effective Field Theories in Nuclear and Hadron Physics

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Outline

- Introduction
- Proton Compton Scattering
- Pion production in $NN \to NN\pi$
- Few-Body Calculations in Stochastic Variational Method

Effective Field Theories

Effective Field Theory:

- low-energy theory of some "fundamental" theory
- external momenta much smaller than some high-energy scale: $Q \ll M_{\text{hep}}$
- the S-matrix calculated in an EFT is an expansion in the powers of $\chi = Q/M_{\text{hep}}$
- the degrees of freedom (DOFs) $\neq$ those of the underlying theory
- fundamental symmetries constrain the dynamics of the EFTs
- renormalizable order by order
- a finite number of parameters (LECs) arises at each order; their values are found by matching with the fundamental theory or from experiment
- counting rules tell what order is to be assigned to a particular graph

Effective Field Theories

Chiral EFT

- based on the spontaneously broken (and approximate) chiral symmetry of QCD
- theory of low-energy interactions of baryons and pions (pseudo-Goldstone bosons of QCD)
- the underlying scale is set by the mass of the lightest non-Goldstone boson σ, $m_\sigma \sim 600$ MeV
- the nucleon mass, $M \simeq 940$ MeV, is not a light scale compared with m_σ
  - treat baryons with heavy-particle formalism — heavy-baryon ChEFT (non-relativistic reduction) (Jenkins, Manohar, 1991)
  - ...or no reduction: on-mass-shell regularization (Gegelia, Japaridze, Wang, 1999, Fuchs, Gegelia, Japaridze, Scherer, 2003)
    a covariant calculation + an appropriate renormalization

- proton Compton scattering in covariant baryon ChEFT

Compton Scattering

Compton Scattering:

- allows to study the structure and the e.m. properties of the nucleon
- the nucleon polarizabilities:

$$T(\omega,\omega') = \frac{-e^2 Z^2}{4\pi M} + \alpha(\vec{\varepsilon}'^{\,*}\cdot\vec{\varepsilon})\omega\omega' + \beta(\vec{\varepsilon}'^{\,*}\times\vec{k}\,')\cdot(\vec{\varepsilon}\times\vec{k}\,) + \mathcal{O}(\omega^4)$$

  experimental values (proton, Griesshammer et al. (2012)):
  $\alpha_E^{(p)} = 10.5 \pm 0.9,\ \beta_M^{(p)} = 2.7 \pm 0.9$ (units: 10^{-4} fm^{-3})

- test ChEFT: just a few loops and no LECs contribute to polarizabilities at the leading order: (Bernard, Kaiser, Meißner, 1992)

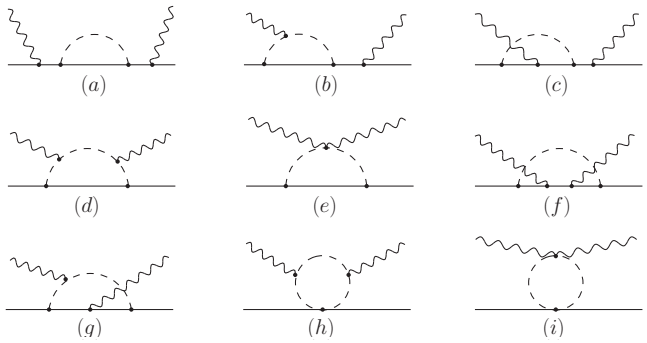
$$\alpha_E^{HBLO} = \frac{5}{24}\frac{e^2 g_A^2}{(4\pi)^2 f_\pi^2 m_\pi} = 12.5,\ \beta_M^{HBLO} = \frac{1}{48}\frac{e^2 g_A^2}{(4\pi)^2 f_\pi^2 m_\pi} = 1.3.$$

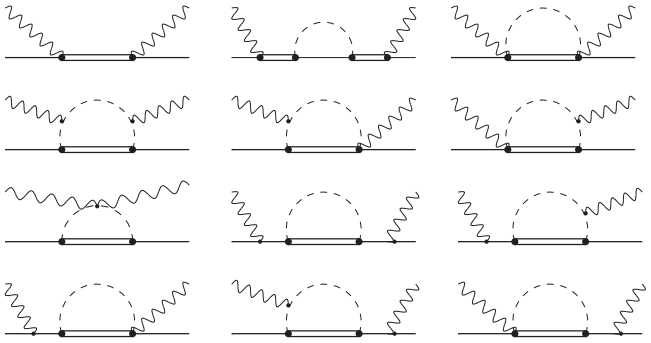
$$\alpha_E^{BLO} = \frac{5}{24}\frac{e^2 g_A^2}{(4\pi)^2 f_\pi^2 m_\pi} + \cdots = 6.3,\ \beta_M^{BLO} = \frac{1}{48}\frac{e^2 g_A^2}{(4\pi)^2 f_\pi^2 m_\pi} + \cdots = -1.8.$$

- the Δ isobar gives a big contribution to $\beta \simeq 7$ (Pascalutsa, Phillips (2003)) — a large counterterm at next-to-leading order in the HBChEFT calculation;
- Δ contribution can be more naturally accomodated in covariant BChEFT (V.L., Pascalutsa (2009,2010))

Covariant vs. Heavy Baryon ChEFT

- large $1/M$ corrections to α_E and β_M
- other sizable effects of HB expansion in Compton scattering?
- adjust polarizabilities to expt. values (V.L., McGovern, Pascalutsa, Phillips (2012))





counterterm contributions to α_E and β_M

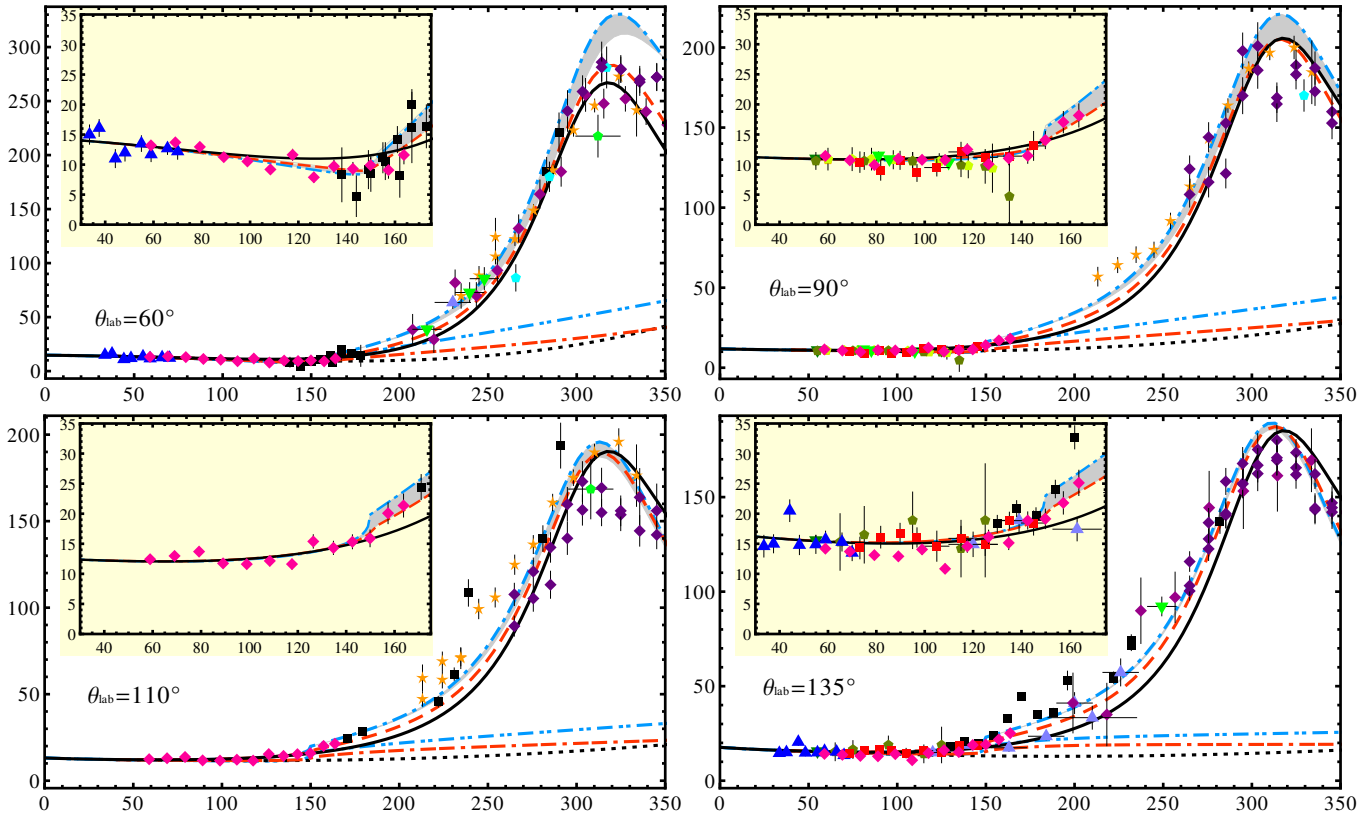
	$\delta\alpha_E$ (10^{-4} fm^3)	$\delta\beta_{M1}$ (10^{-4} fm^3)
Tree	10.5	2.7
$+\Delta$	10.6	-4.4
$+\pi$, HB, Δ	-9.8	-7.1
$+\pi$, Δ	-0.8	-1.2

- appears to remove most of the effects of HB expansion

Covariant vs. Heavy Baryon ChEFT

- adjust polarizabilities to expt. values

$d\sigma/d\theta_{\rm lab}$ [nb/sr], unpolarised, vs. lab energy [MeV]



- $\longrightarrow$ HBChEFT and covariant BChEFT give consistent results at one-loop level (difference is of higher order in the expansion);
- important in connection with experiments ongoing at MAMI and HIGS
- results for spin polarizabilities and polarized observables (in preparation, V.L., McGovern, Pascalutsa (2013))

- Pion production in *NN* collisions

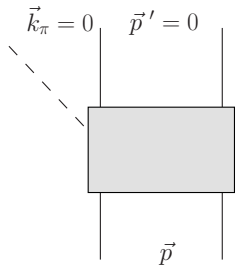
Motivation/Results

π production in NN collisions

- test of ChEFT — the lowest-lying hadron inelasticity of NN
- π production at threshold (s-wave) — key to absorptive corrections to πd scattering
  V.L. et al. (2005, 2006); Baru et al. (2007)
  $\Longrightarrow$ needed for an extraction of πN scattering lengths Baru et al. (2011)
- necessary for studying CSB and isospin violation
  Kolck, Miller, Niskanen (2000); Gårdestig et al.(2004); Filin et al.(2009); Miller, Bolton(2009)
  experiment: $A_{fb}(pn \to d\pi^0)$ — Opper et al.(2003); $dd \to \alpha\pi^0$ — Stephenson et al.(2003)
- immediate links to many other reactions ($\gamma d \to \pi NN$, $\pi d \to \gamma NN$, weak reactions, e.g. $pp \to de^+\nu$, NNN forces)
  Gårdestig, Phillips (2006); Nakamura (2008); Park et al. (2003); Epelbaum (2002); Nogga et al. (2006); Gazit et al. (2007); Baru et al. (2009)

Modified Power Counting

$$NN \to NN\pi$$



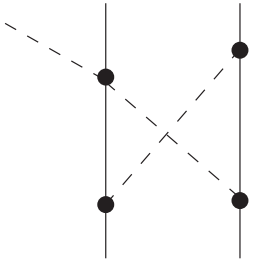
Large momentum transfer already at threshold: $|\vec{p}_{\rm thr}| = \sqrt{m_\pi(M + m_\pi/4)}$

— modification of power counting scheme is needed
Cohen, Friar, Miller, van Kolck (1996); Hanhart, van Kolck, Miller (2000), Hanhart, Kaiser (2002)

The expansion parameter in this case is

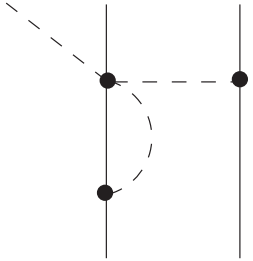
$$|\vec{p}_{\rm thr}|/M \sim \sqrt{m_\pi/M} = \sqrt{\chi}$$

— $\Delta(1232)$ should be explicitly included: $(M_\Delta - M)/M \sim |\vec{p}_{\rm thr}|$



$$\sim p^3/(m_\pi M^2)$$

$$\underbrace{\chi^2}_{\rm N^2LO} \to \underbrace{\sqrt{\chi}}_{\rm NLO}$$

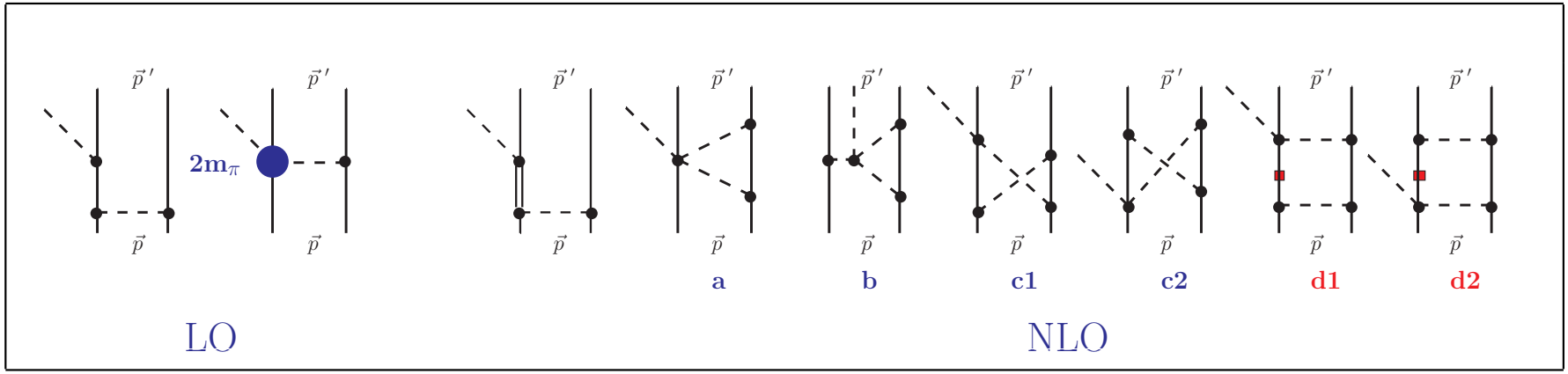


$$\sim m_\pi^2/M^2$$

$$\underbrace{\chi^2}_{\rm N^2LO} \to \underbrace{(\sqrt{\chi})^4}_{\rm N^4LO}$$

- s-wave pion production in $NN \to NN\pi$

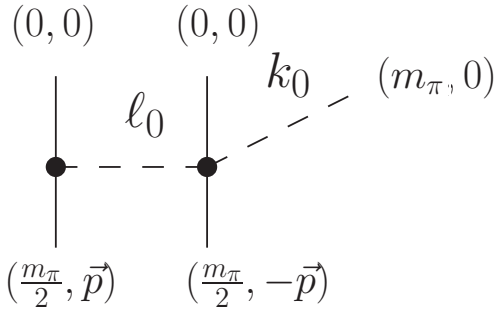
The Irreducible Diagrams up to NLO – Complete Set



NLO contribution

$$A_{d\pi^+}^{\mathrm{a+b+c+d1(irr)+d2(irr)}} = \frac{g_A^3|\vec{q}|}{256 f_\pi^5}(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\frac{\vec{q}}{2}\left(-2+3+0-\frac{1}{4}-\frac{3}{4}\right) = 0$$

Leading order pion rescattering vertex goes on shell



$$\propto\ (\ell_0+k_0) = \tfrac{3}{2}m_\pi \longrightarrow 2\ m_\pi$$

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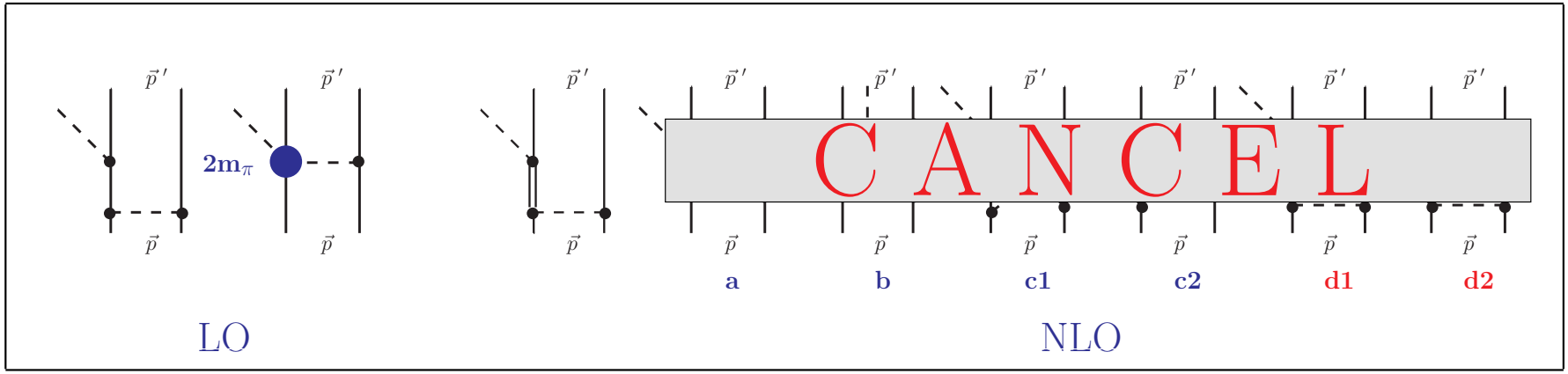

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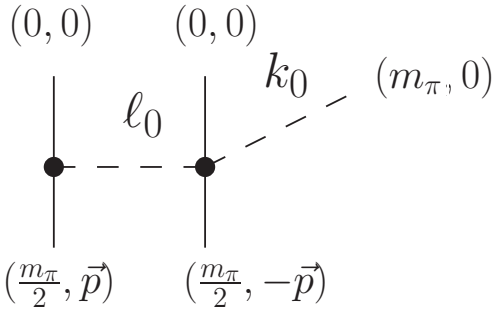
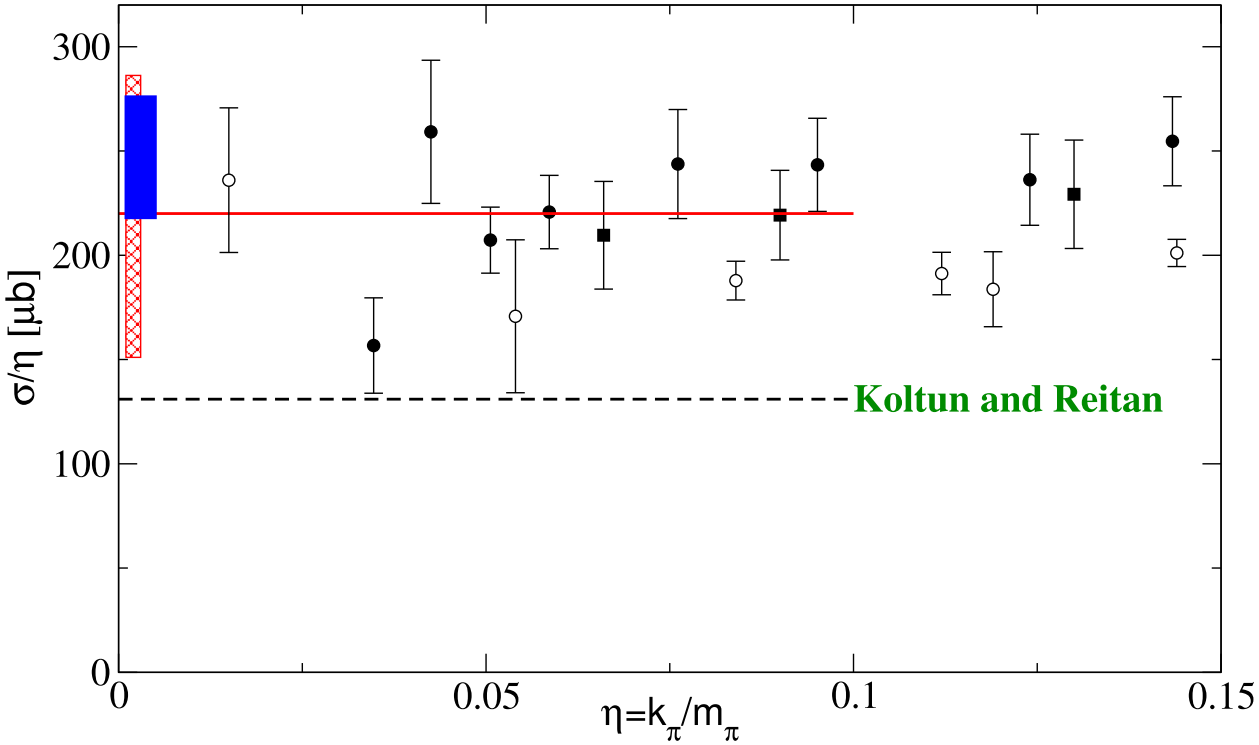
Leading order pion rescattering vertex goes on shell



$$\propto\ (\ell_0+k_0) = \tfrac{3}{2}m_\pi \longrightarrow 2\ m_\pi$$

enhancement factor $4/3$ in the amplitude

Results for $pp \to d\pi^+$



V.L. et al. (2005)

Theoretical uncertainty is

$\mathcal{O}(\frac{m_\pi}{M_N}) \sim 30\%$

(conservative estimate)—hatched bar

Experimental Data:
IUCF (1996) (filled circles),
TRIUMF (1991) (open circles),
COSY (1998) (squares),
PSI (1998) (pionic deuterium—blue bar).

- these findings allow for a calculation of dispersive and absorptive corrections to πd scattering length (V.L. et al. (2006); Baru et al. (2007))

  $\Longrightarrow$ serves for an extraction of πN scattering lengths (Baru et al. (2011))

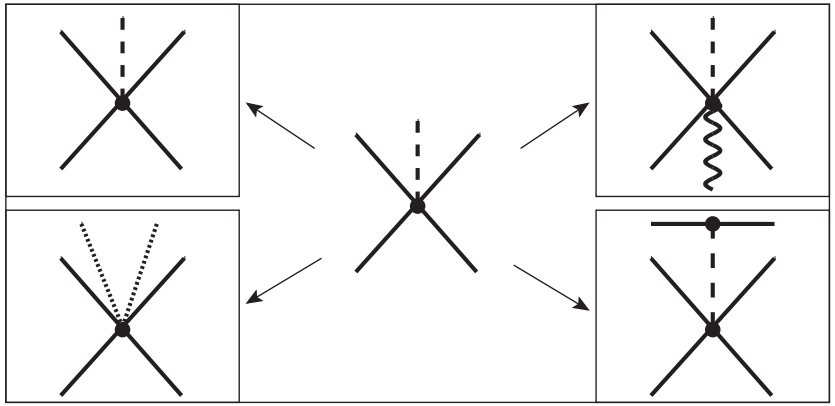
- an NNLO $NN \to NN\pi$ s-wave calculation is called for (Baru et al. (2012))
  - reduce the uncertainty in $pp \to d\pi^+$
  - necessary for a description of $pp \to pp\pi^0$

- p-wave pion production in $NN \to NN\pi$

$NN \to NN\pi$ and other reactions: $4N\pi$ LEC

$NN \to NN\pi$

$pp \to de^+\nu$
^3H β-decay



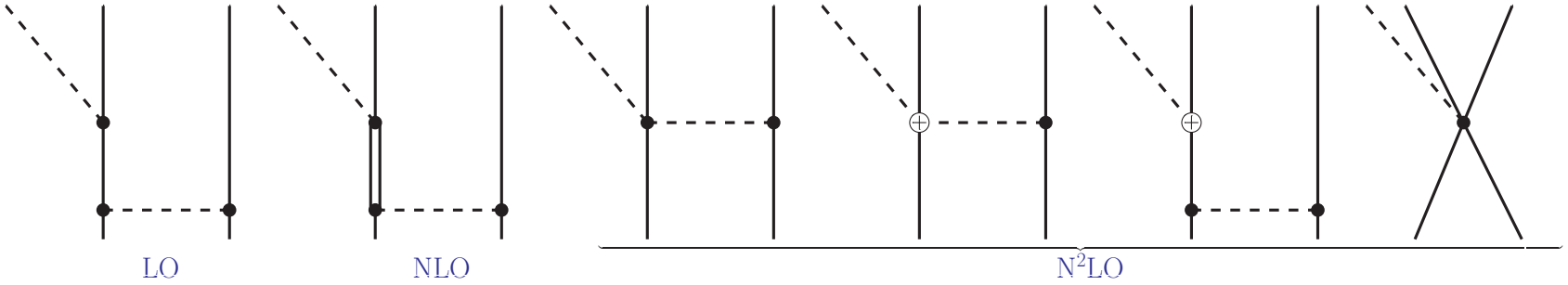
$\gamma d \to \pi NN$
$\pi d \to NN$

$pd \to pd$
NNN forces

- a single $4N\pi$ contact term with a LEC d enters all the reactions
  - momentum transfer $\sim \sqrt{m_\pi M}$: $NN \to NN\pi$
  - momentum transfer $\sim m_\pi$: all others
- we want to describe them all with one LEC d (with reasonable accuracy at NNLO)
  a very nontrivial test of ChEFT
- Gazit et al (2000): works for $3N$ binding energies
- does it work for different channels in $NN \to NN\pi$?

p-wave $NN \to NN\pi$: production mechanism

Baru et al (2009)



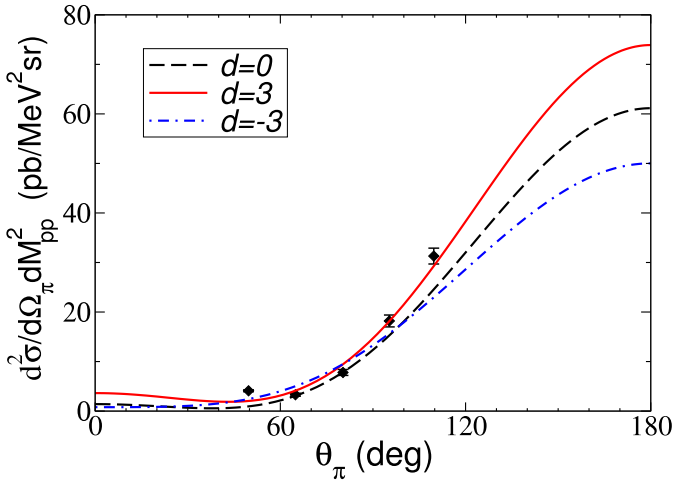
- we fit d to experimental data — cross sections and asymmetries
  - $A_y = \dfrac{d\sigma_{\uparrow\uparrow} - d\sigma_{\uparrow\downarrow}}{d\sigma_{\uparrow\uparrow} + d\sigma_{\uparrow\downarrow}}$ — target analysing power
  - $\Longrightarrow$ $^1S_0 \to {}^3S_1 p$ in $pp \to pn\pi^+$, $pp \to d\pi^+$
  - $\Longrightarrow$ $^3S_1 \to {}^1S_0 p$ in $pn \to pp\pi^-$
- $d = d(\Lambda)$ depends on the regularization/NN interaction used
- d absorbs the short-range part of the production operator

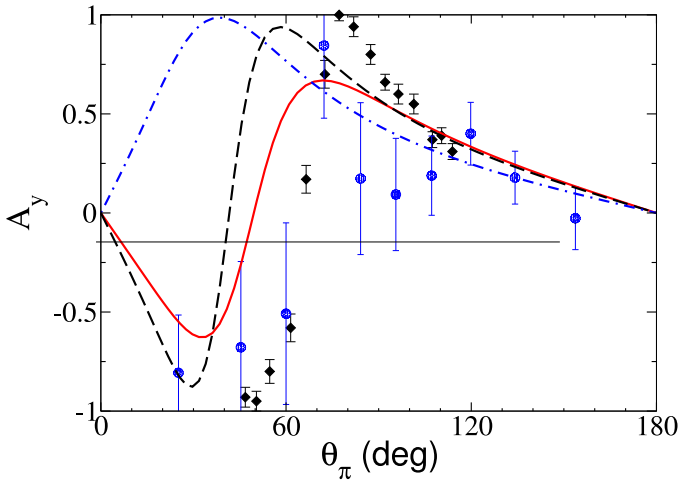
$$A_{\text{s.r.}} \propto \frac{(\vec{p} - \vec{p}\,')^2}{(\vec{p} - \vec{p}\,')^2 + m_\pi^2} = const + \mathcal{O}(N^4LO)$$

Results: $pp \to d\pi^+$, $np \to pp\pi^-$

$np \to pp\pi^-$

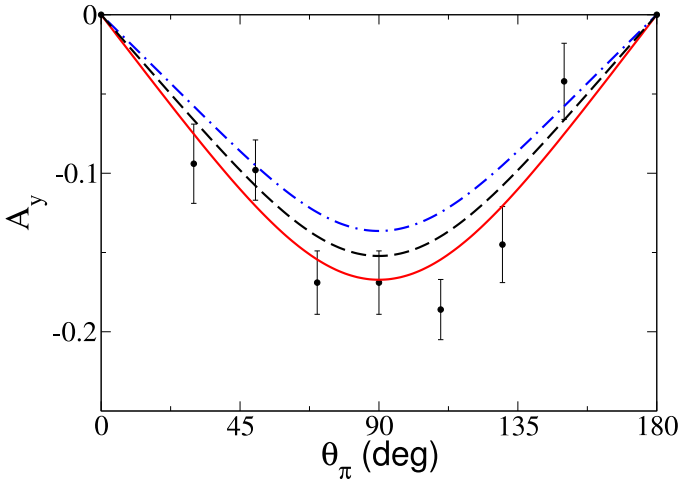
$\eta = 0.6$
energy cut-off:
$E_{pp} \leq 1.5$ MeV
data: TRIUMF, PSI

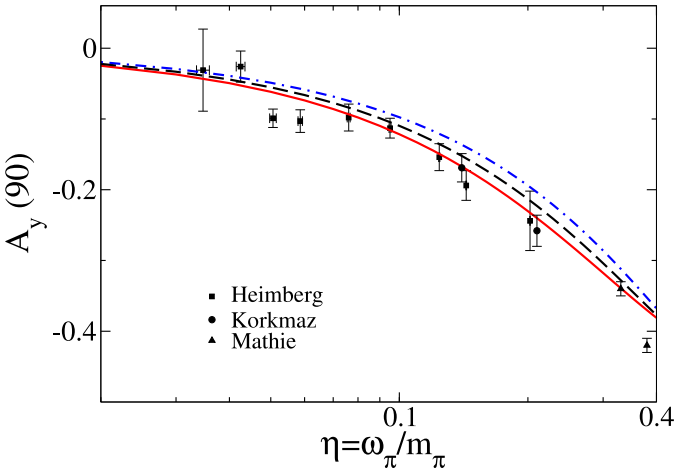




$pp \to d\pi^+$

$\eta = 0.14$
data: TRIUMF, IUCF





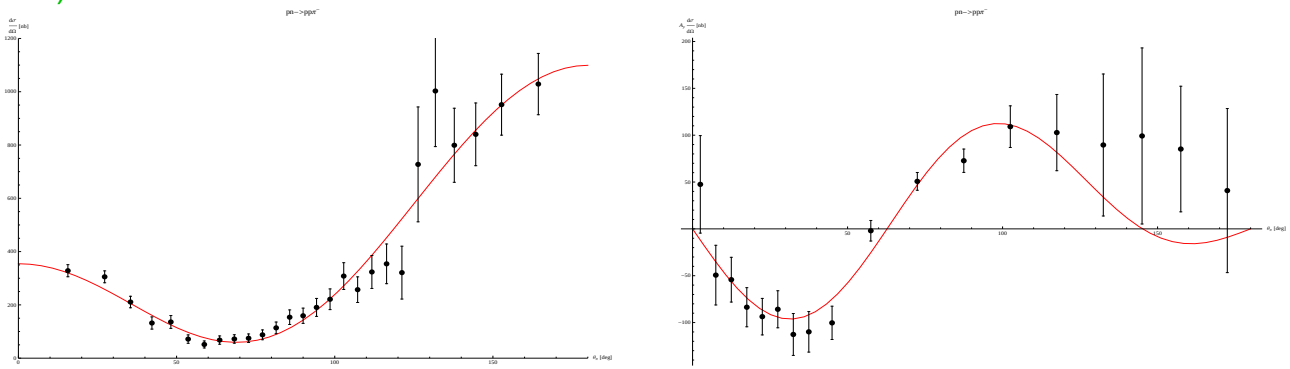
- Positive $d \sim 3$ is preferred in both reactions
- New data on $np \to pp\pi^-$ at low η (expt: ANKE@COSY (2011))

New Data: $pp \to pp\pi^0$, $np \to pp\pi^-$

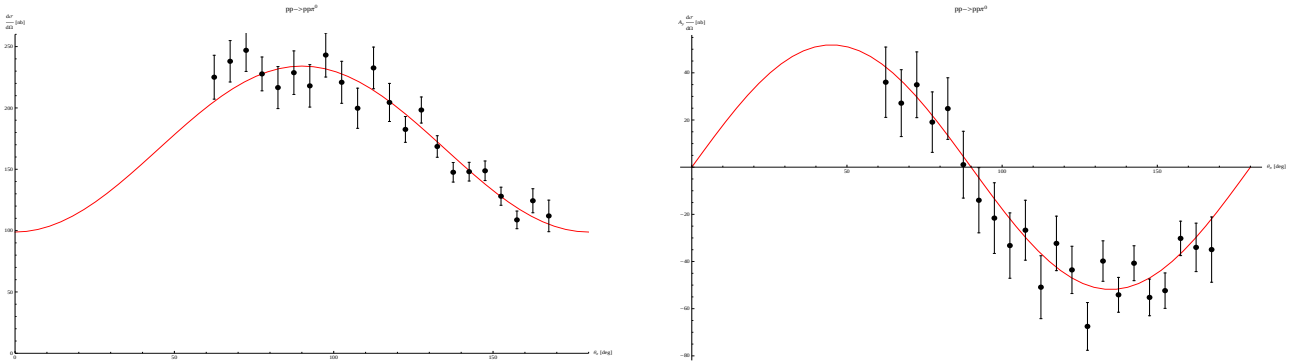
ANKE@COSY (2011)

$np \to pp\pi^-$

$\eta = 0.6$
energy cut-off:
$E_{pp} \leq 1.5$ MeV



$pp \to pp\pi^0$

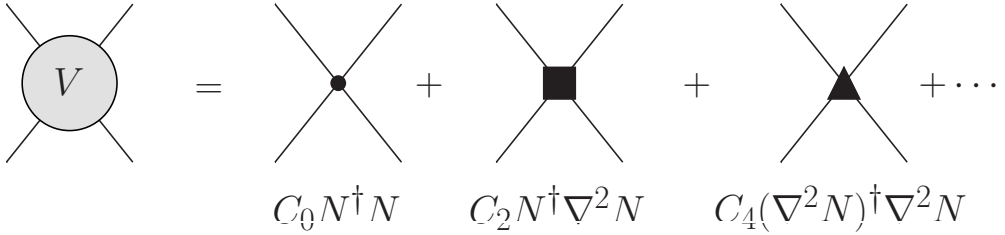


- Complete amplitude analysis: *s*-, *p*-, *d*-wave π production amplitudes
- ChEFT/hybrid helps justify fixing the phases of *d*-waves
- *d*-wave is significant!
- ChEFT calculation taking into account *d*-waves is in progress

- few-body variational calculations in pionless EFT

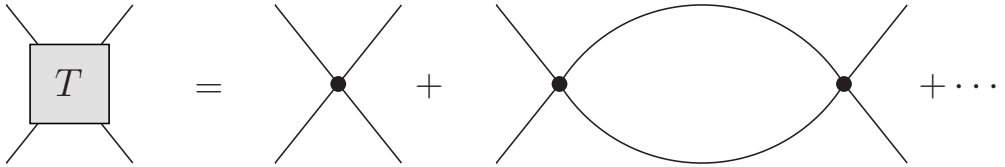
Pionless Nuclear EFT

- high momentum scale $\simeq m_\pi$: $Q \lesssim m_\pi$, $E \lesssim 20$ MeV for NN
- contact interactions (with derivatives) $\Longrightarrow$ delta-functions



Weinberg (1990), Kaplan, Savage, Wise (1998), Kong, Ravndal (1999), ...
Beane, Bertulani, Cohen, Hammer, Higa, Gelman, van Kolck, Phillips, Rupak, ...
reviews — Bedaque, van Kolck (2002), Epelbaum (2006)

- loops divergent (couple to arbitrary high momenta)



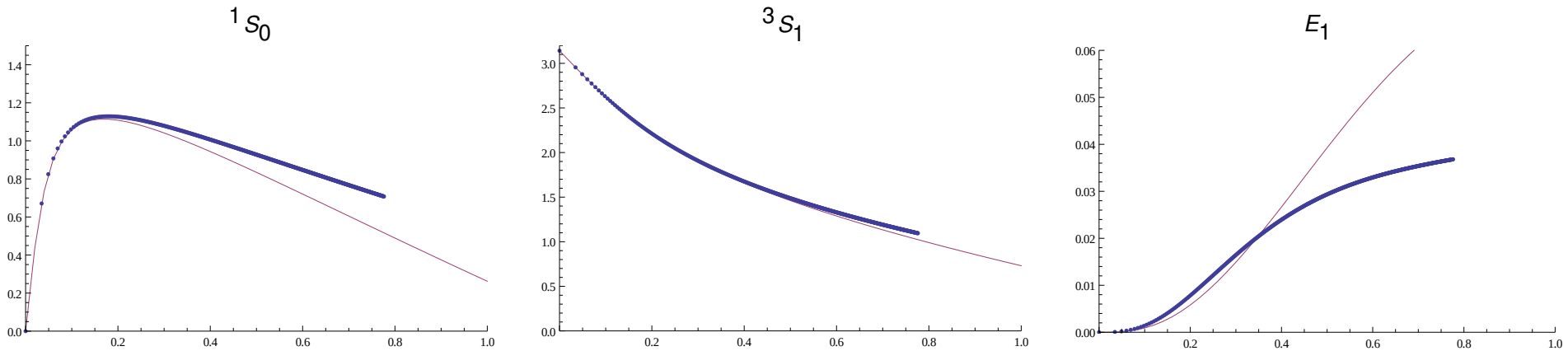
- need to regularize and renormalize

$\Longrightarrow$ use a (Gaussian) formfactor:

Two Nucleons

$$V(r) = \left(C_0 + C_0^t \vec{\tau}_1 \cdot \vec{\tau}_2\right) \exp\left(-\frac{r^2}{2\sigma^2}\right) + \left(C_1 + C_2^t \vec{\tau}_1 \cdot \vec{\tau}_2\right) r^2 \exp\left(-\frac{r^2}{2\sigma^2}\right) + \cdots$$

solve the Lippman-Schwinger equation; fit the parameters to the observables:



- works well up to $T_{lab} \sim 20$MeV;
- deuteron bound state is at the right position too;
- we want few-body bound states: ^{3}H, ^{3}He, ^{4}He, ...
  $\Longrightarrow$ we need three-body forces (one contact term at leading order), and we are going to use the
  Stochastic Variational Method (SVM) (Varga, Suzuki, 1998).

Stochastic Variational Method

Hamiltonian

$$H = T + V = \sum_{i=1}^{N+1} \frac{p_i^2}{2m_i} + \sum_{i<j}^{N+1} V_{ij}(\mathbf{r}_i - \mathbf{r_j}) + \sum_{i<j<k}^{N+1} V_{ijk}(\mathbf{r}_i - \mathbf{r}_j, \mathbf{r}_k - \mathbf{r}_j)$$

Trial function

$$|\Psi\rangle = \sum_{\alpha} \left|\psi^{\alpha}_{JJ_zLS}\right\rangle ,$$

Basis functions for the system of N nucleons:

$$\left|\psi^{\alpha}_{JJ_zLS}\right\rangle = \sum_{M,S_z} C^{JJ_z}_{LMSS_z} \left|f_{KLM}\right\rangle \left|\chi^{\alpha}_{SS_z}\right\rangle ,$$

$$|f_{KLM}\rangle = |f_{KLM}(\mathbf{x}, \mathrm{A}, u)\rangle = v^{2K} \mathbb{Y}_{LM}(\mathbf{v}) \exp\left(-\frac{1}{2}\mathrm{A}^{ij}\mathbf{x}_i^{\mathsf{T}}\mathbf{x}_j\right) ,$$

- $\mathbf{x}_i$ is the i-th Jacobi coordinate,
- $\alpha = (\mathrm{A}, u)$, where A is an $(N-1) \times (N-1)$ matrix of parameters, and u is an $(N-1)$-component vector; $\mathbf{v} = u^i \mathbf{x}_i^{\mathsf{T}}$
- A and u are parameters varied randomly and picked such that the lowest (generalized) eigenvalue of H is minimized $\Longrightarrow$ ground state

Stochastic Variational Method

Features of SVM:

- \+ (relatively) easy to implement and modify
- \+ easily scalable
- +/- best for ground states — excited states need extra care
- \- not exact (although reasonably accuracy is expected)
- \- computationally rather expensive (but still reasonable, e.g, $\sim$ 20 hours for ^{4}He)

Ongoing:

- ^{3}H, ^{3}He, ^{4}He wave functions:

  charge radii, magnetic moments;

  use the w.f. to calculate low energy reactions (e.g., Compton scattering on ^{3}He)
- study Efimov physics (it is easy to change the potential and see what happens)
- heavier nuclei (starting from ^{5,6}He... going to $A = 8$ should be easy (hopefully!) but probably a supercomputer will be needed)

Outlook

- Compton scattering
  - deuteron Compton scattering around π photoproduction energies and beyond

  $\Longrightarrow$ three-body πNN dynamics around threshold

  $\Longrightarrow$ neutron polarizabilities
- $NN \to NN\pi$
  - get a consistent description of p-wave and d-wave π production

  $\Longrightarrow$ connection to NNN forces
- few-body variational calculations
  - incorporate scattering states

  $\Longrightarrow$ few-body reactions
  - use a more “realistic“ potential, e.g., that from ChEFTs

  $\Longrightarrow$ higher energy