coupled $\pi K/\eta K$ scattering from QCD

Jozef Dudek

OLD DOMINION UNIVERSITY

Jefferson Lab

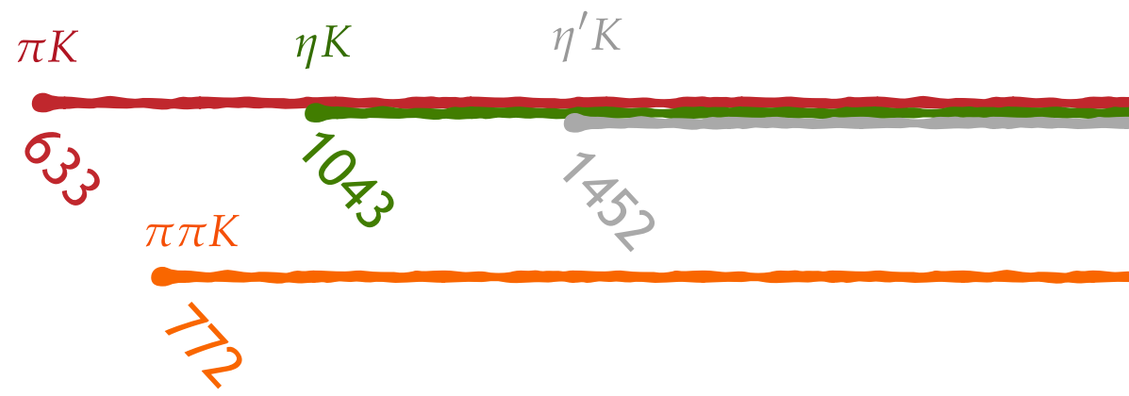
with Dave Wilson,
Robert Edwards
& Christopher Thomas

pdg kaon resonances

NATURAL PARITY

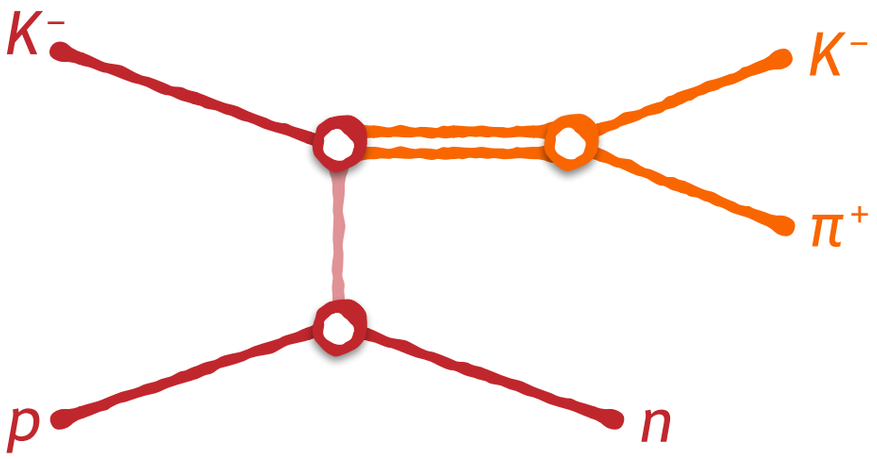
	κ	
0^+	$K_0^\star(1430)$	$\Gamma = 270(80)$ MeV b.r.$(\pi K) = 93(10)\,\%$
	$K_0^\star(1950)$	$\Gamma = 200(90)$ MeV b.r.$(\pi K) = 52(14)\,\%$
1^-	$K^\star(892)$	$\Gamma = 50(3)$ MeV b.r.$(\pi K) = 100\,\%$
	$K^\star(1410)$	$\Gamma = 232(21)$ MeV b.r.$(\pi K) = 7(2)\,\%$ b.r.$(\pi K^\star) > 40\,\%$
2^+	$K_2^\star(1430)$	$\Gamma = 100(5)$ MeV b.r.$(\pi K) = 50(1)\,\%$
	$K_2^\star(1980)$	$\Gamma = 373(70)$ MeV $\to \pi\pi K$?

ℓ	S	P	D	F	$\ldots$
J^P	0^+	1^-	2^+	3^-	$\ldots$

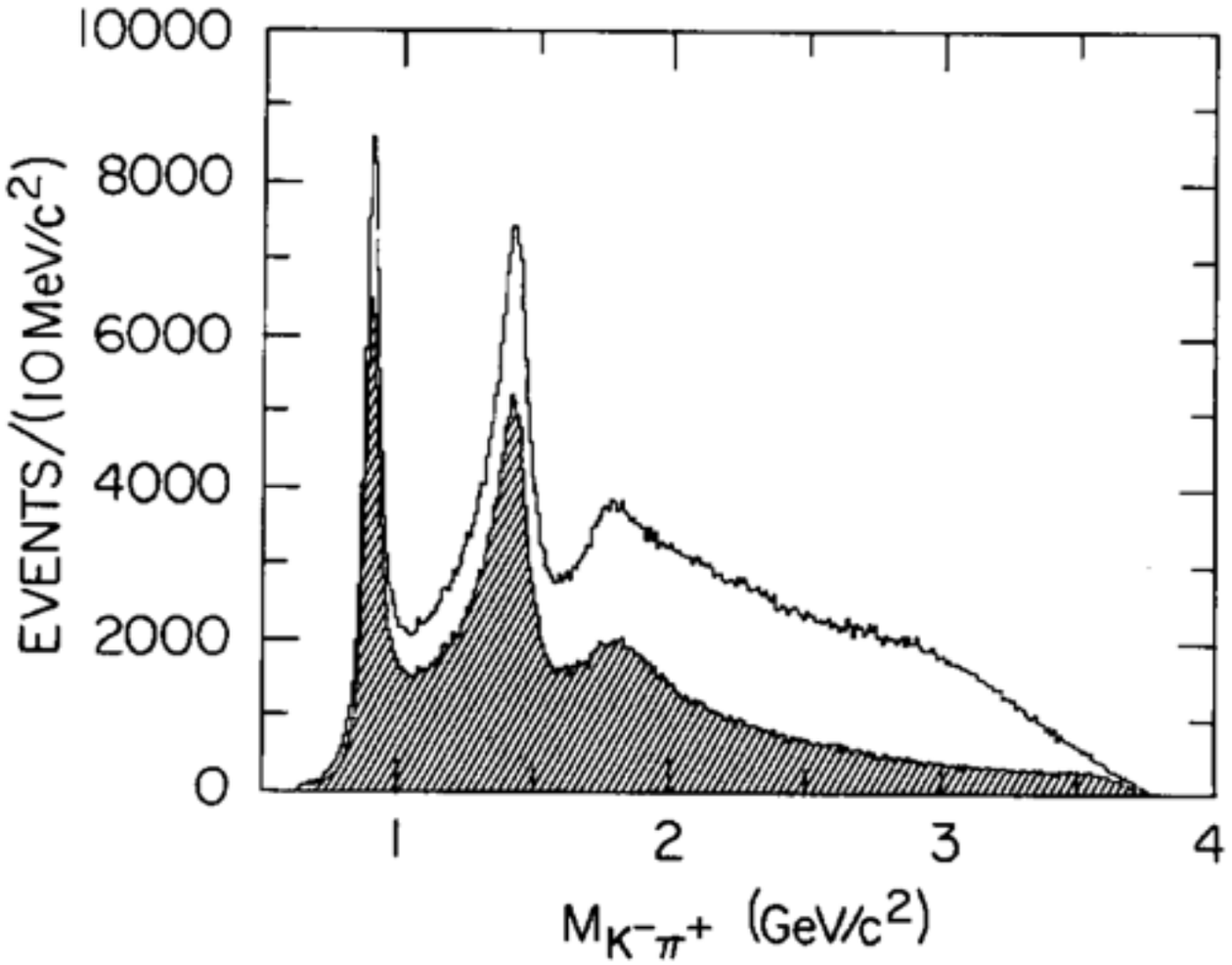


kaon beams

$K^- p \to K^- \pi^+ n$ $\quad E_K$ ~ 11 GeV



πK EVENTS

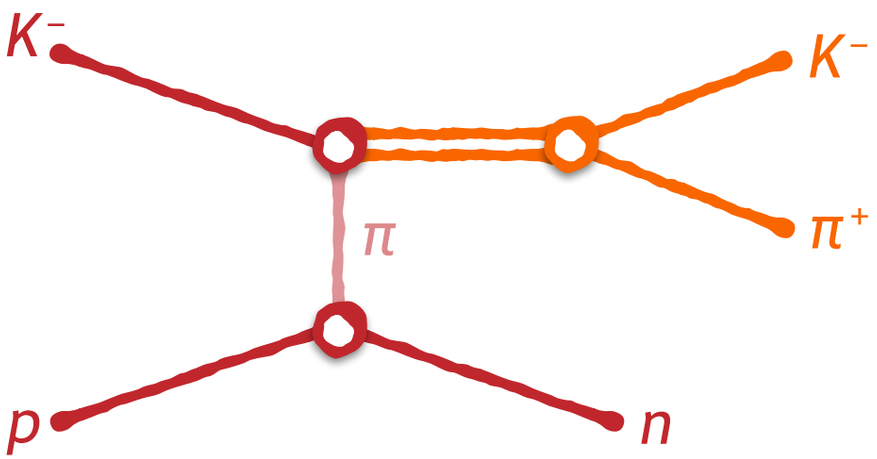


LASS, NPB296 493 (1988)

- LASS magnet currently bending tracks in Hall-D

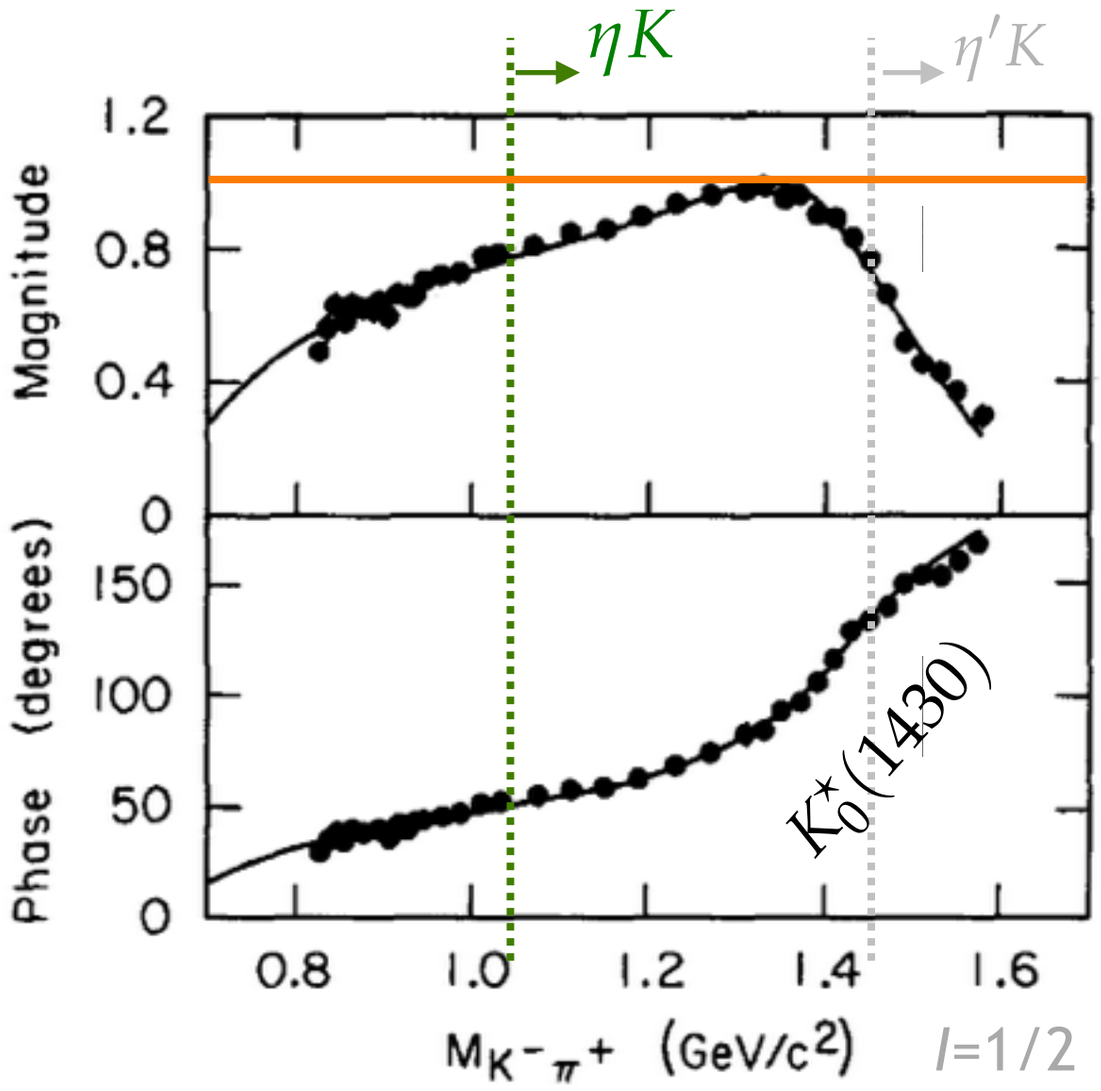
kaon beams - partial wave analysis

$K^- p \to K^- \pi^+ n$



- Chew-Low extrapolation to *t*-channel pion pole

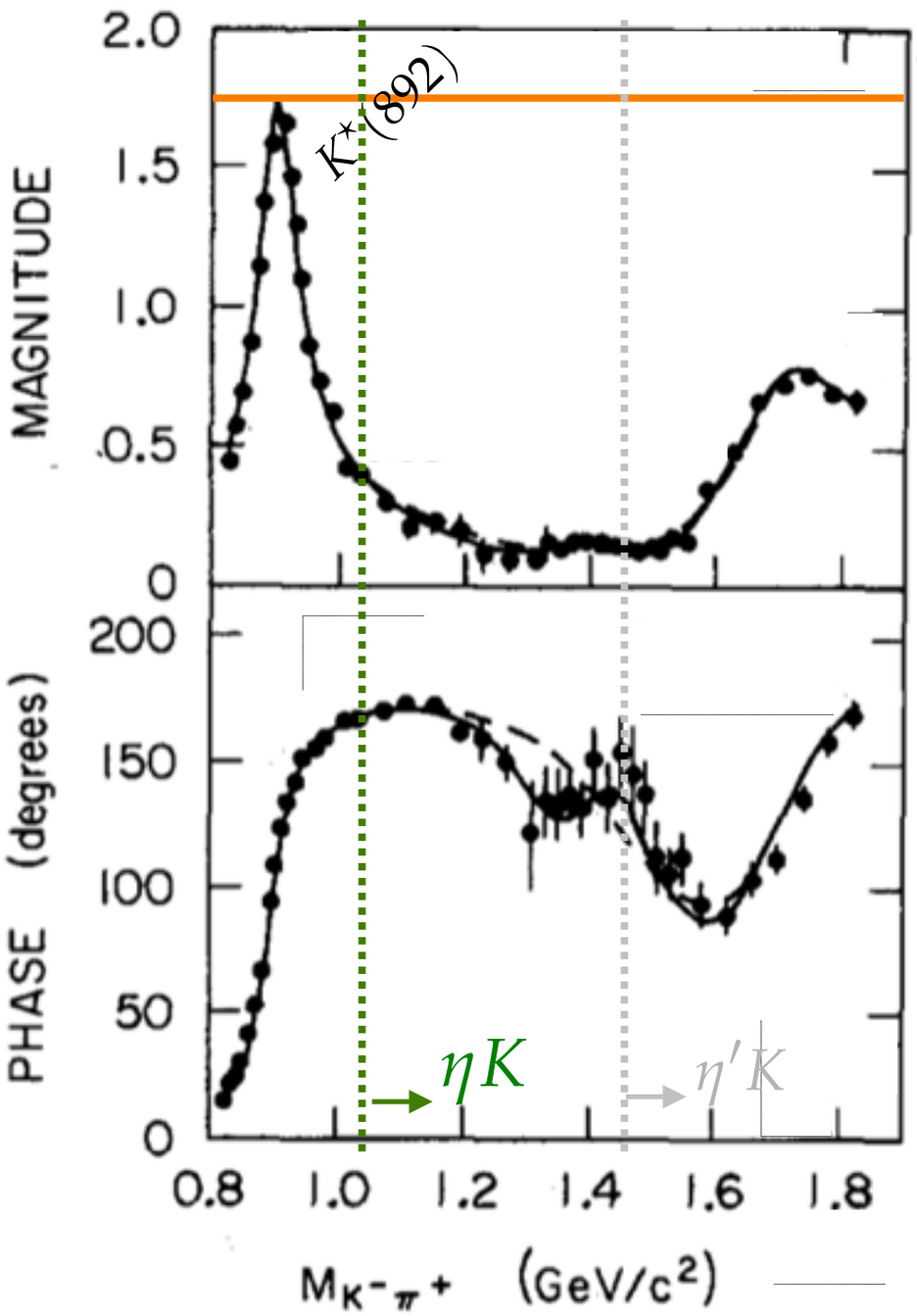
S-WAVE $\pi K \to \pi K$ AMPLITUDE



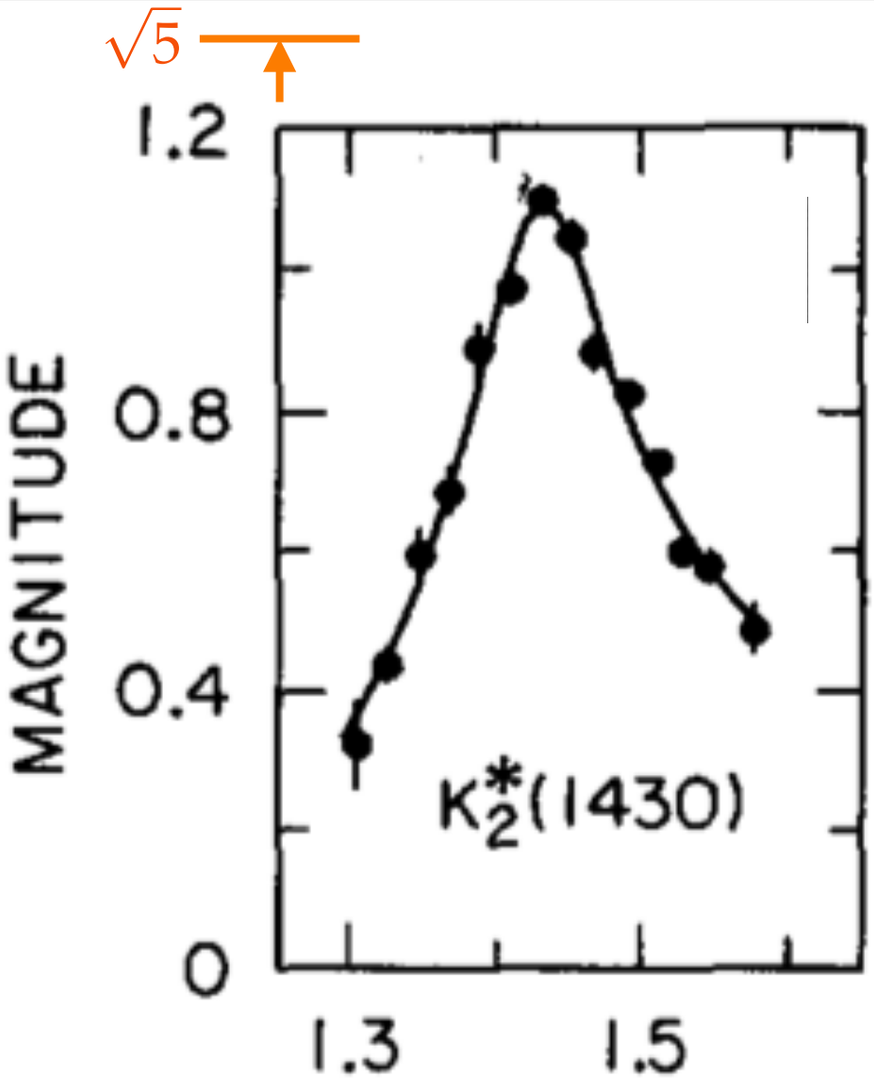
LASS, NPB296 493 (1988)

kaon beams - partial wave analysis

P-WAVE $\pi K \to \pi K$ AMPLITUDE



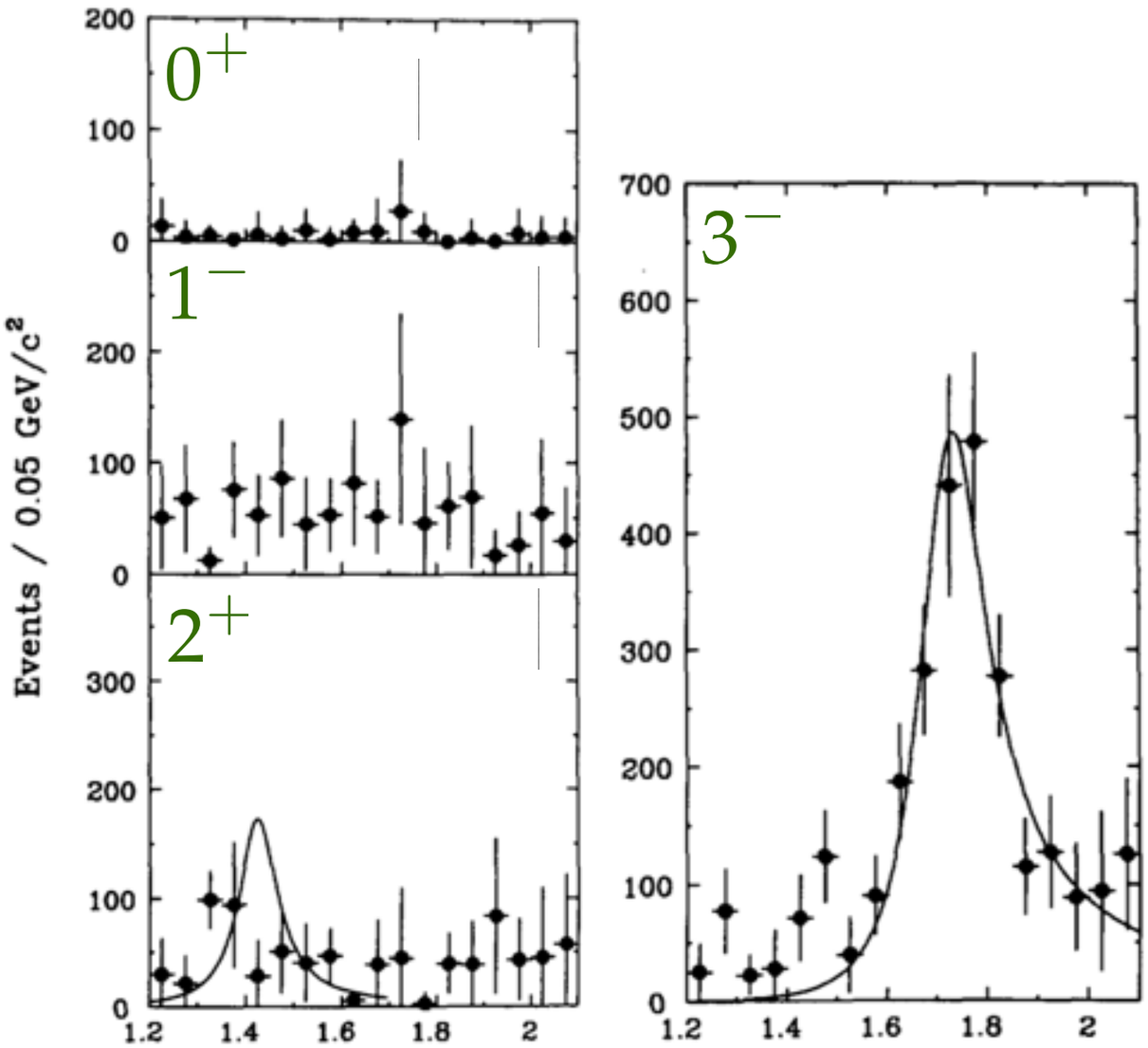
D-WAVE $\pi K \to \pi K$ AMPLITUDE



kaon beams

$K^- p \to K^- \underbrace{\pi^+\pi^-\pi^0}_{\eta}\, p$

$(\pi)K \to \eta K$ AMPLITUDES

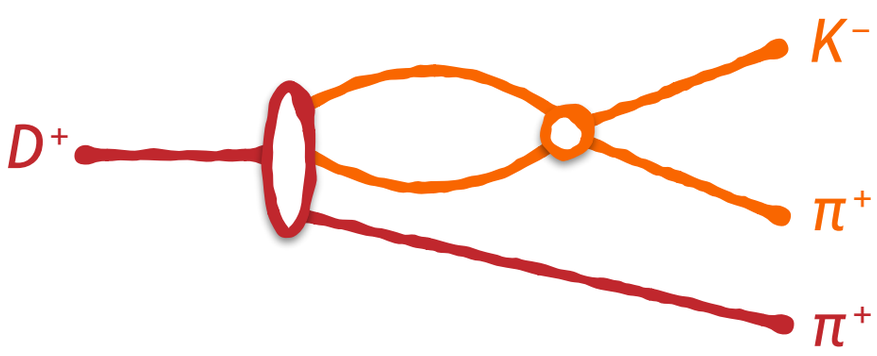


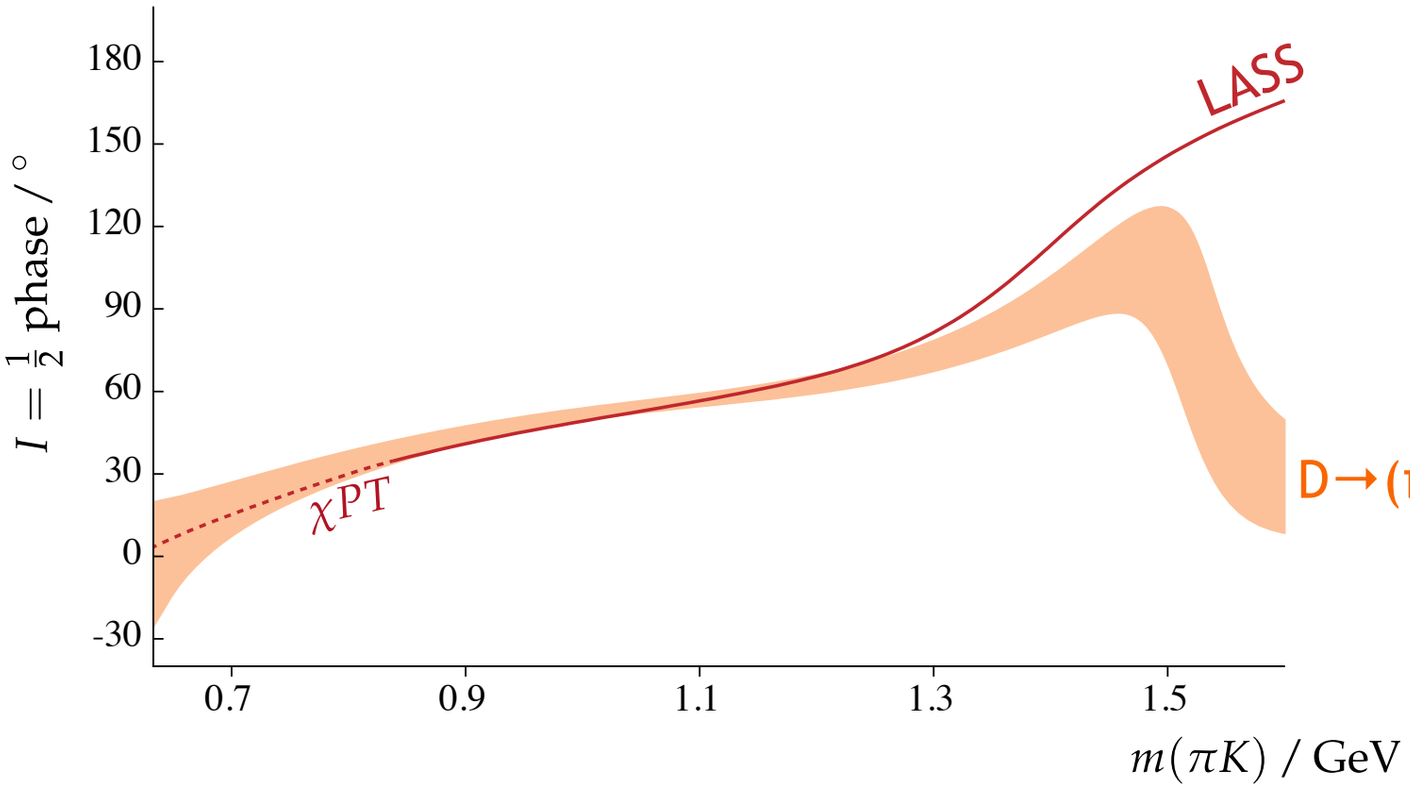
- no significant coupling to ηK for $J<3$

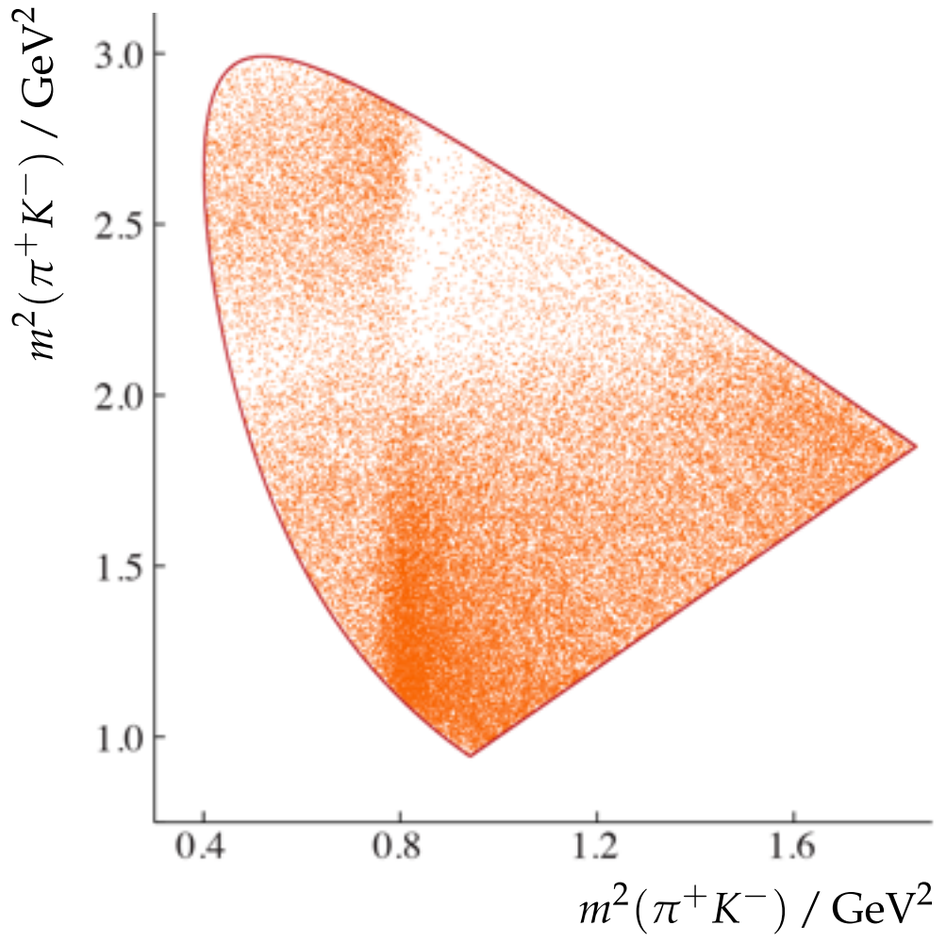
LASS, PLB201 169 (1988)

D-meson decays

$D^+ \to K^- \pi^+ \pi^+$







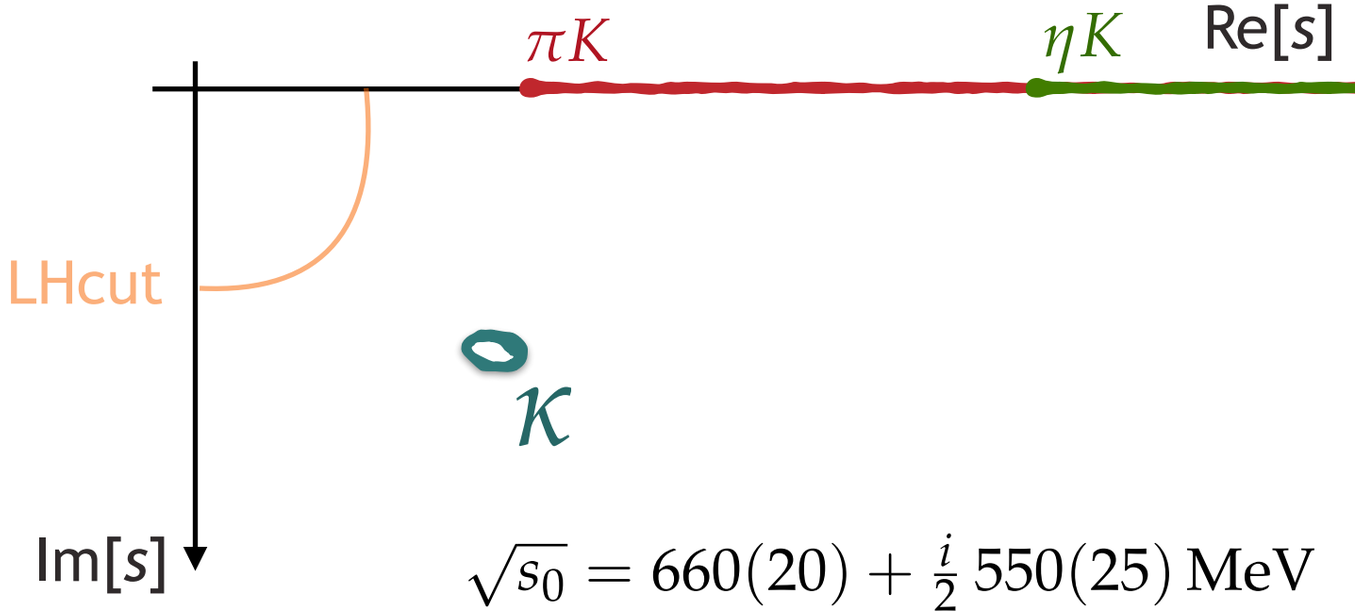
FOCUS, PLB653 1 (2007)
INCL. M.R.PENNINGTON

- determining the phase at lower energies

Roy-Steiner & the κ (kappa)

- scattering data: $\pi K \to \pi K$, $\pi\pi \to K\bar{K}$
  + analyticity, unitarity, crossing-symmetry

- dispersive representation



$$\sqrt{s_0} = 660(20) + \frac{i}{2}\,550(25)\,\mathrm{MeV}$$

Descotes-Genon & Moussallam, EPJC48 553 (2006)

- resonance pole, but deep in the complex plane

extracting the excited state spectrum

- how to get at excited QCD eigenstates ?
  - optimal operator for state $|\mathfrak{n}\rangle$: $\Omega^\dagger_{\mathfrak{n}} \sim \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$

    for a basis of meson operators $\{\mathcal{O}_i\}$

  - can be obtained (in a variational sense) from the **matrix of correlators**

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) | 0 \rangle$$

  - by solving a generalized eigenvalue problem

$$C(t) v^{(\mathfrak{n})} = C(t_0) v^{(\mathfrak{n})}\, \lambda_{\mathfrak{n}}(t)$$

eigenvalues

$$\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$$

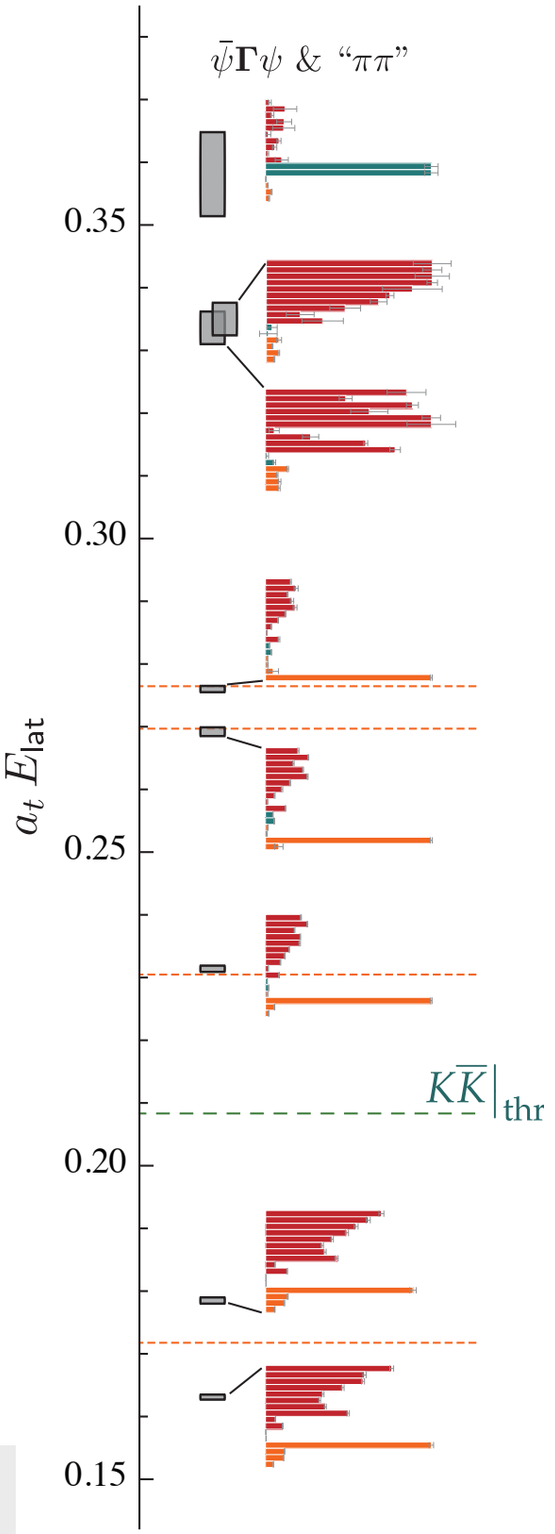
extracting the excited state spectrum

[100]

- e.g. low-energy isospin-1, $J^P=1^-$ channel *(P-wave ππ, $K\bar{K}$ …)*
  - a large basis can be constructed

$$\bar{\psi}\Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} \psi$$ "local" q$\bar{\text{q}}$-like

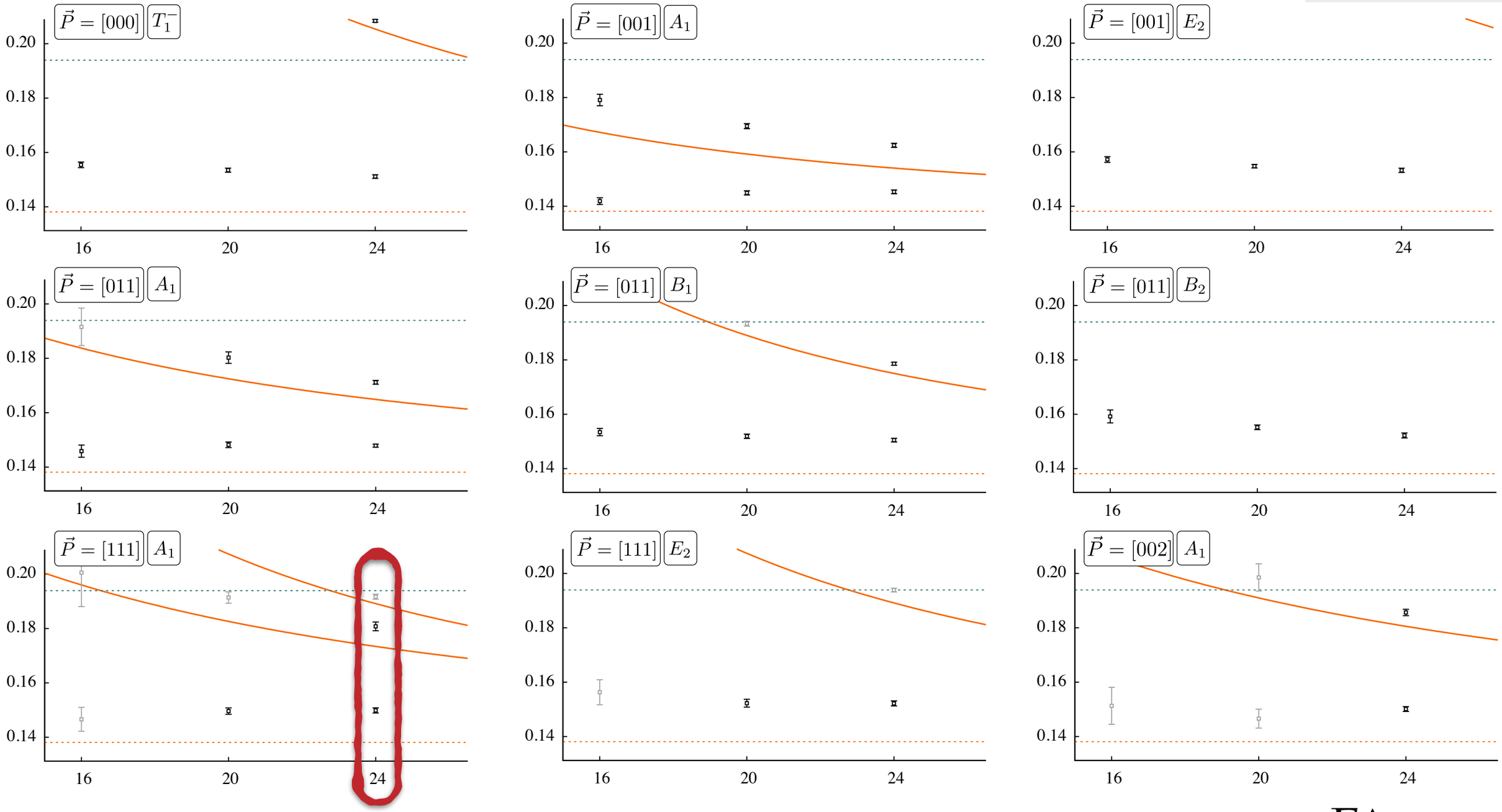
& $\pi_{\vec{p}_1}\pi_{\vec{p}_2}$ where $\pi_{\vec{p}} \sim \sum_i v_i \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \left[\bar{\psi}\Gamma_i\psi\right]_{\vec{x}}$



$m_\pi \sim 391\,\text{MeV}$

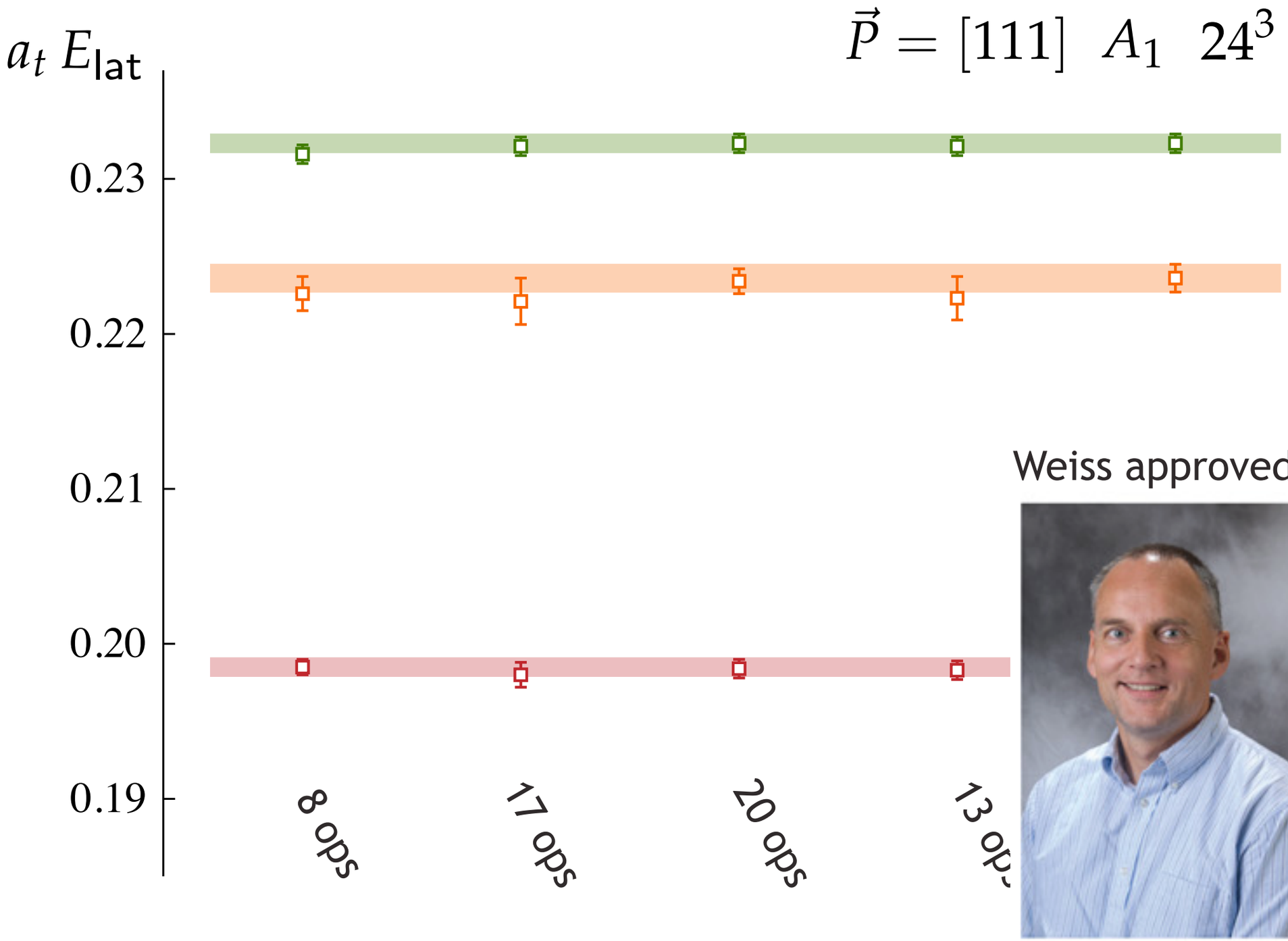
finite-volume spectra

$m_\pi \sim 391\,\mathrm{MeV}$



finite-volume spectrum

- varying the $\bar{\psi}\Gamma\psi$ content of the operator basis



3× $\pi\pi$ ops

$m_\pi \sim 391\,\text{MeV}$

Weiss approved ?

Lüscher & friends

- elastic scattering in a finite cubic volume

$$\det\left[\cot\delta_\ell(E)\,\delta_{\ell\ell'} - \mathcal{M}^{\vec{P},\Lambda}_{\ell,\ell'}(pL)\right] = 0$$

elastic scattering amplitude

known ‘kinematic’ functions

$\vec{P} = [000]$

Λ	ℓ
A_1	$0, 4 \ldots$
T_1	$1, 3, 4 \ldots$
E	$2, 4 \ldots$
T_2	$2, 3, 4 \ldots$
A_2	$3 \ldots$

$\vec{P} = [100]$

Λ	ℓ
A_1	$1, 3 \ldots$
E_2	$1, 3^2 \ldots$
B_1	$3 \ldots$
B_2	$3 \ldots$

- but at low energies, higher partial waves suppressed

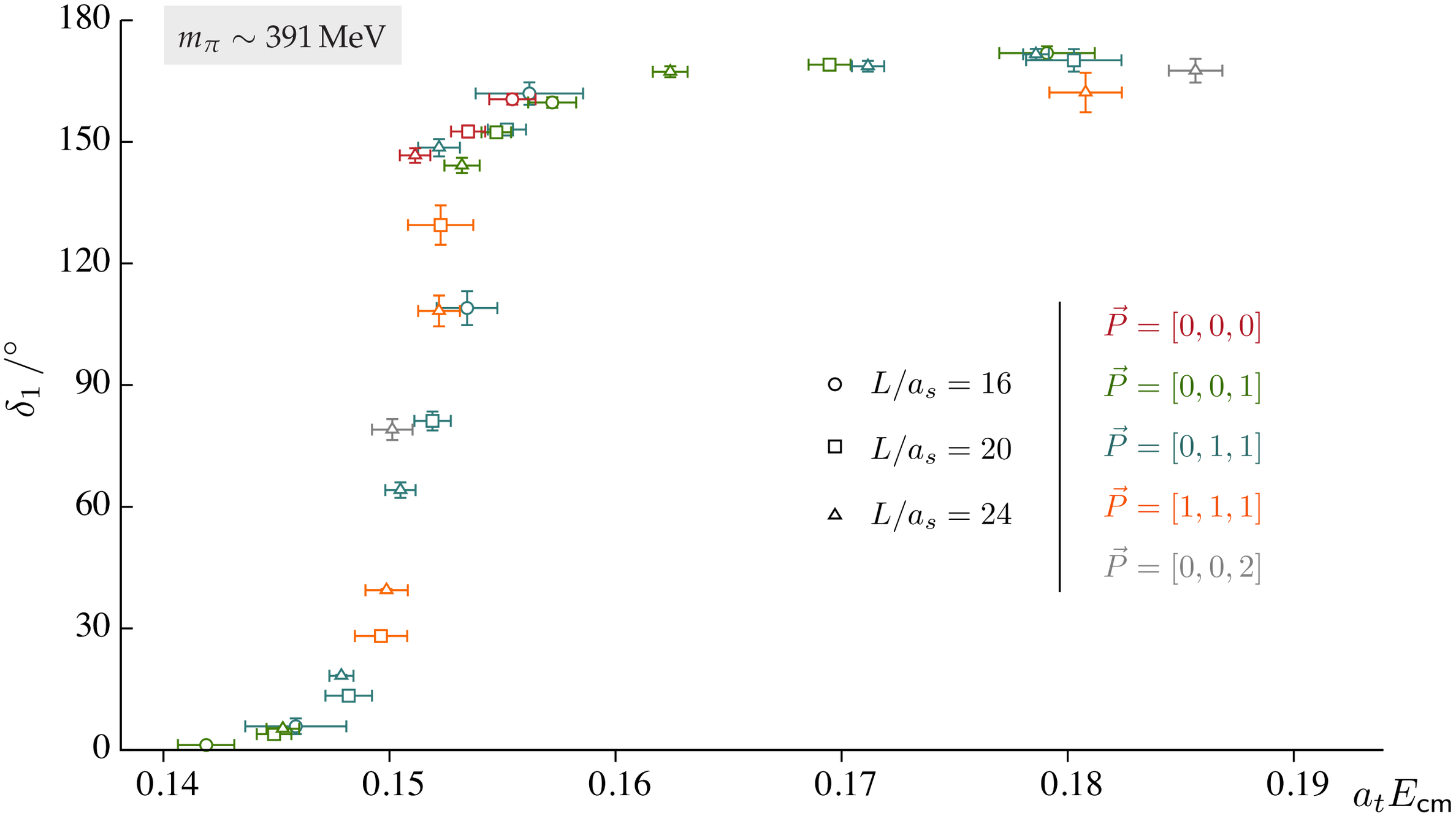
$$\delta_\ell(p \to 0) \to a\, p^{2\ell+1}$$

so if $\delta_3 \ll \delta_1$

$$\cot\delta_1(E_n) = \mathcal{M}^{\vec{P},\Lambda}_{1,1}\left(p(E_n)L\right)$$

- a value of δ_1 at each finite-volume energy, E_n

$\pi\pi$ P-wave phase-shift

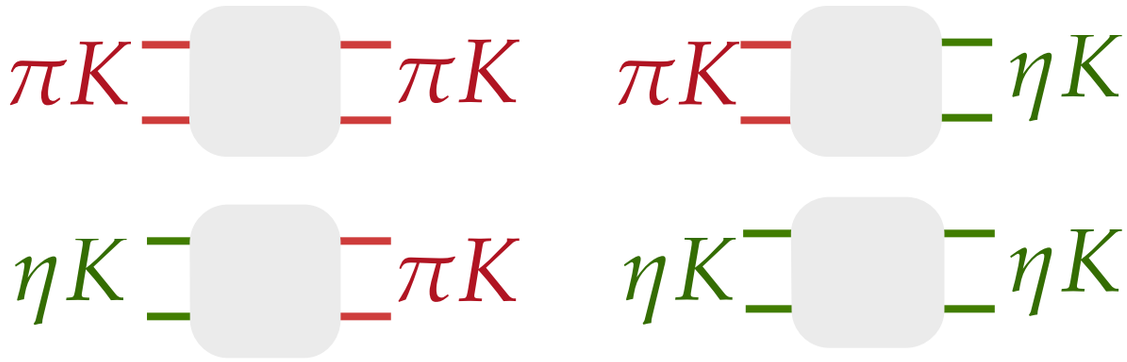


(other lattice groups have computed this in less detail)

PRD87 034505 (2013)
& erratum to appear

$\pi K/\eta K$ scattering & kaon resonances

- an example of coupled-channel scattering



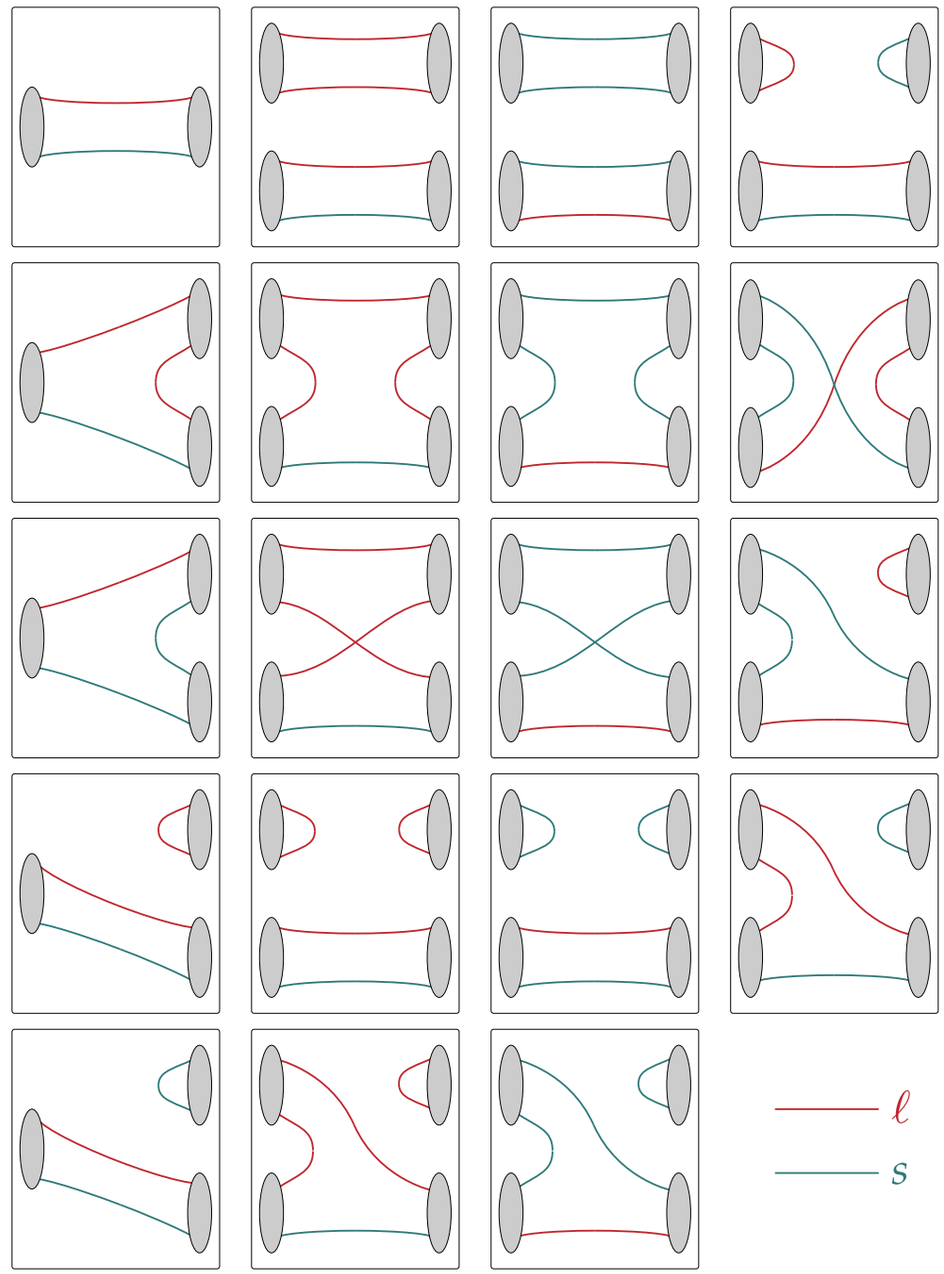
- compute finite-volume spectrum

$$\bar{u}\Gamma s$$

$$\pi_{\vec{p}_1} K_{\vec{p}_2}$$

$$\eta_{\vec{p}_1} K_{\vec{p}_2}$$

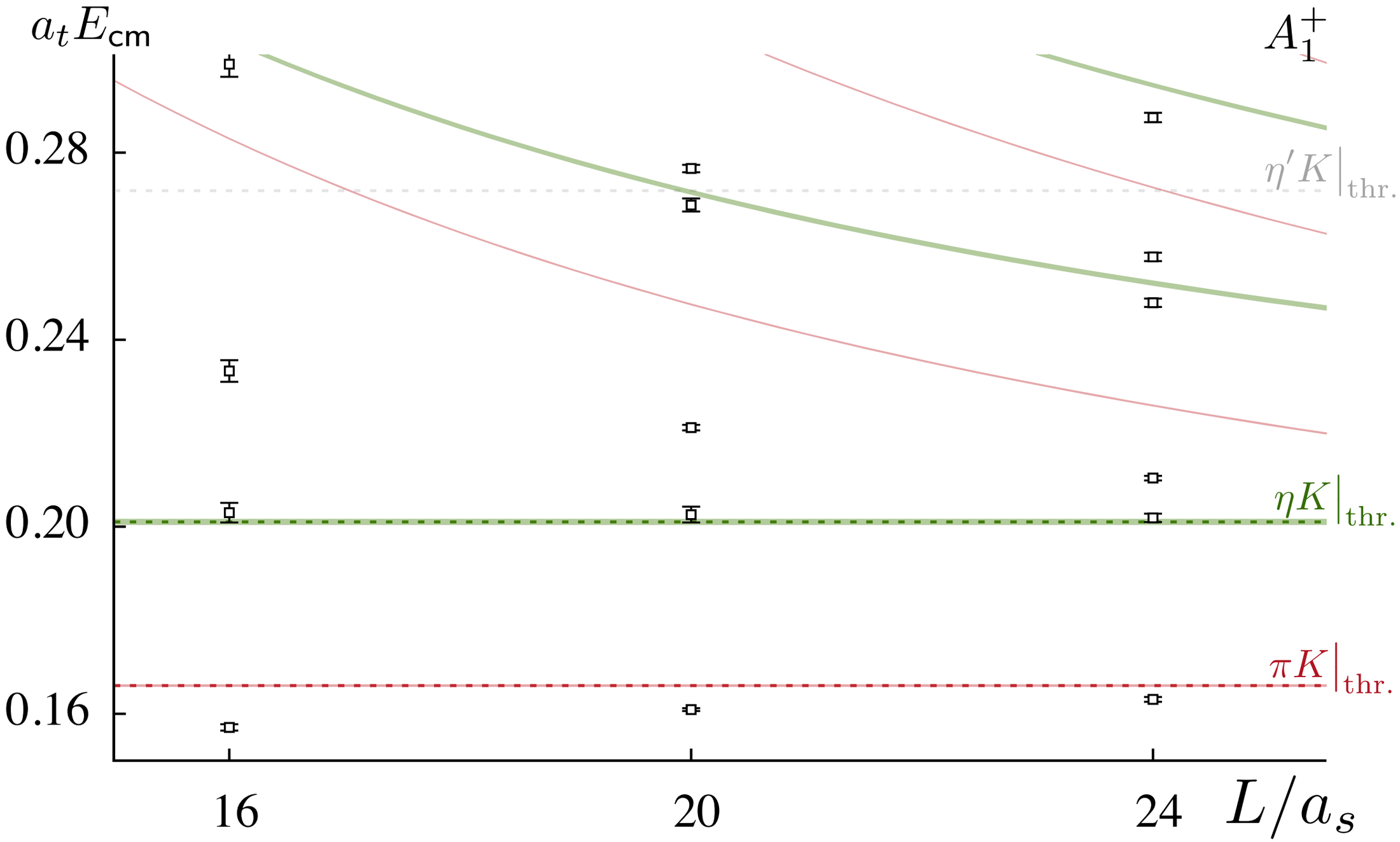
WICK CONTRACTIONS



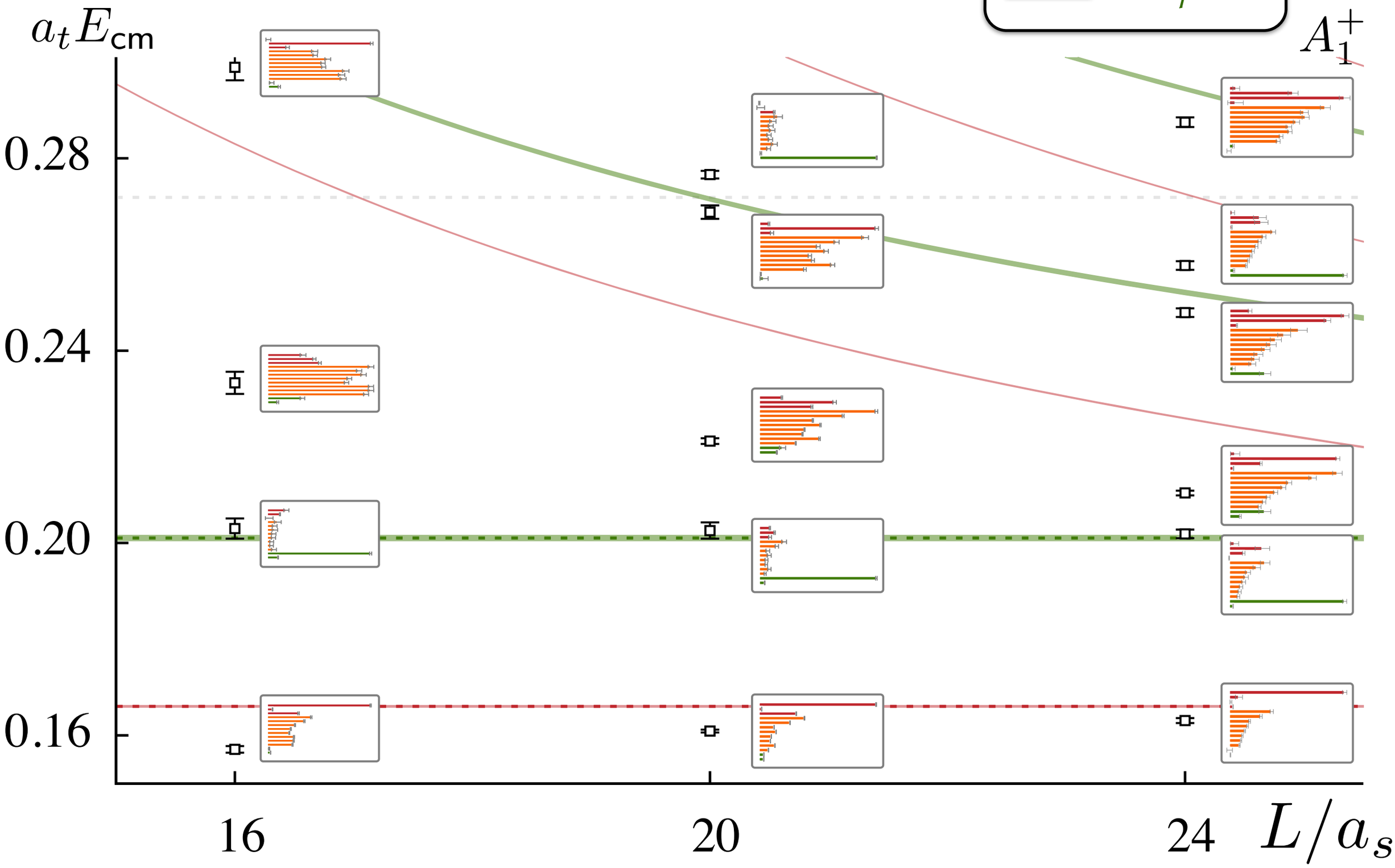
$\pi K/\eta K$ scattering & kaon resonances

- rest frame “S-wave” spectrum

$m_\pi \sim 391\,\mathrm{MeV}$



$\pi K/\eta K$ scattering



coupled-channel scattering

- finite-volume formalism recently derived (multiple methods)

He, JHEP 0507 011
Hansen, PRD86 016007
Briceno, PRD88 094507
Guo, PRD88 014051

$$\det\left[\left(\left[t^{(\ell)}(E)\right]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right)\delta_{\ell\ell'} - \delta_{ij}\,\rho_i(E)\,\mathcal{M}^{\vec{P},\Lambda}_{\ell,\ell'}(p_i L)\right] = 0$$

scattering t-matrix

phase space

$$\rho_i(E) = \frac{2p_i}{E}$$

known 'kinematic' functions

partial-wave unitarity: $\mathrm{Im}\, t^{-1}_{ij}(E) = -\delta_{ij}\,\rho_i(E)\Theta(E - E_i^{\mathrm{thr.}})$

- so this is one condition per energy level

- we want to determine $\mathbf{t}(E) = \begin{bmatrix} t_{\pi K,\pi K}(E) & t_{\pi K,\eta K}(E) \\ t_{\pi K,\eta K}(E) & t_{\eta K,\eta K}(E) \end{bmatrix}$

3 real unknowns at each energy

parameterizing coupled-channels

$s = E^2$

- coupled channel scattering - partial-wave *t*-matrix $t_{ij}^{[\ell]}(s)$

unitarity condition $\mathrm{Im}\,[t^{-1}(s)]_{ij} = -\delta_{ij}\,\rho_i(s)\,\Theta(s - s_i)$

(only an imag part above threshold)

– can ensure multichannel unitarity using a *K*-matrix form:

$$\left[t^{-1}(s)\right]_{ij} = \left[K^{-1}(s)\right]_{ij} - i\,\delta_{ij}\,\rho_i(s) \qquad K_{ij}(s) \text{ is real for real } s$$

or a slight variation:

$$\left[t^{-1}(s)\right]_{ij} = \left[K^{-1}(s)\right]_{ij} + \delta_{ij}\,I_i(s) \quad \text{where} \quad \mathrm{Im}\,I_i(s) = -\rho_i(s)\,\Theta(s - s_i)$$

(there's a particularly useful choice for this called the "Chew-Mandelstam" function)

parameterizing coupled-channels

- *K*-matrix form $\left[t^{-1}(s)\right]_{ij} = \left[K^{-1}(s)\right]_{ij} + \delta_{ij}\, I_i(s)$

  - parameterization freedom lives in the *K*-matrix

e.g. $K_{ij}(s) = \sum_p \frac{g_i^{[p]} g_j^{[p]}}{m_p^2 - s} + \sum_n \gamma_{ij}^{[n]} s^n$ “poles plus polynomial”

  - start with simple one-pole + constant

$$\mathbf{K}(s) = \frac{1}{m^2 - s} \begin{bmatrix} g_{\pi K}^2 & g_{\pi K} g_{\eta K} \\ g_{\pi K} g_{\eta K} & g_{\eta K}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

6 variable parameters

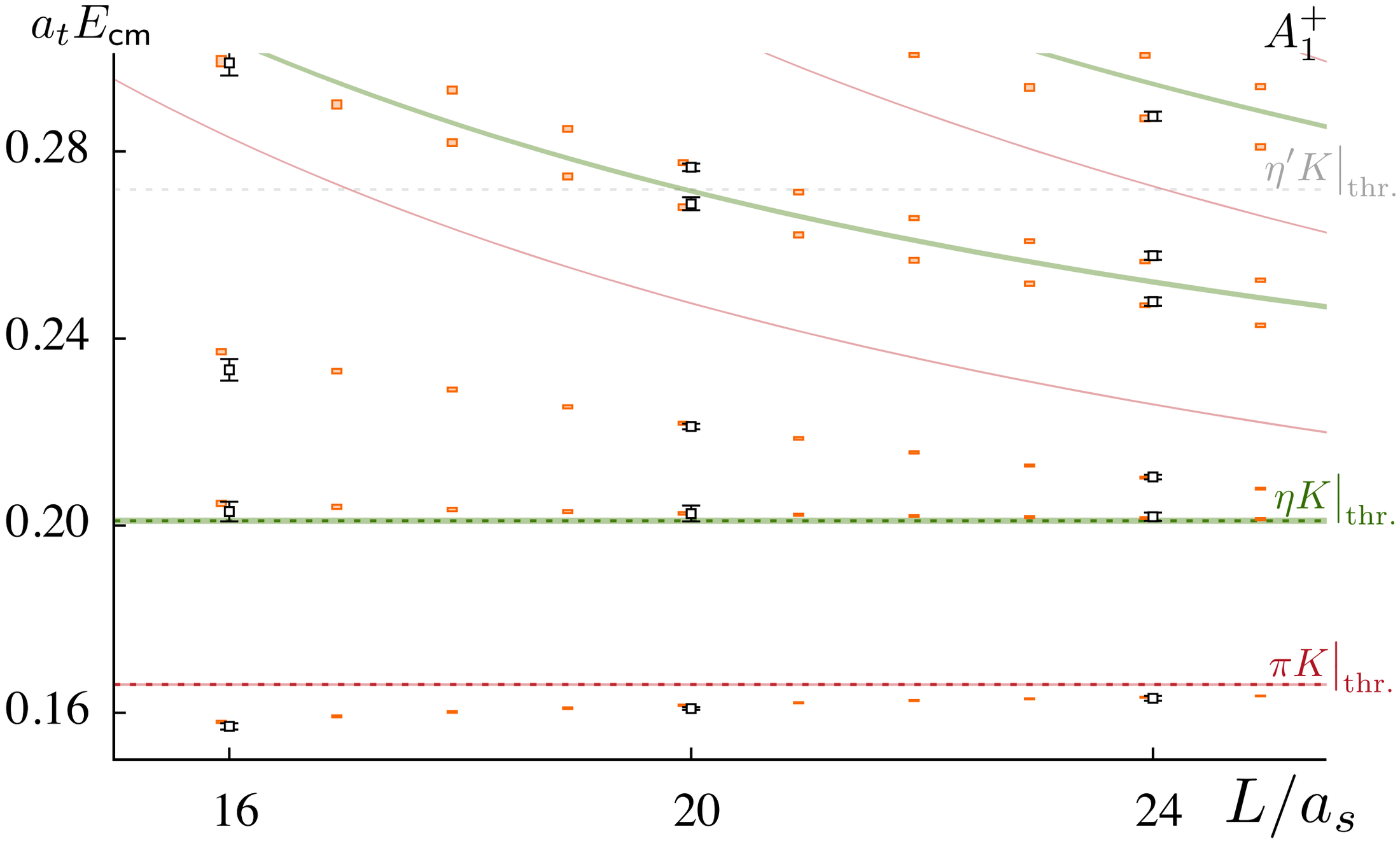
m, $g_{\pi K}$, $g_{\eta K}$,
$\gamma_{\pi K,\pi K}$, $\gamma_{\pi K,\eta K}$, $\gamma_{\eta K,\eta K}$

$\pi K/\eta K$ scattering

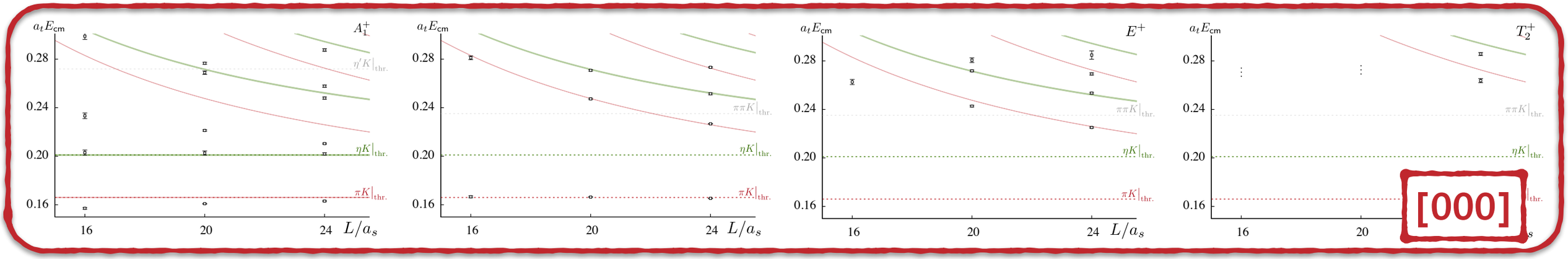
model description

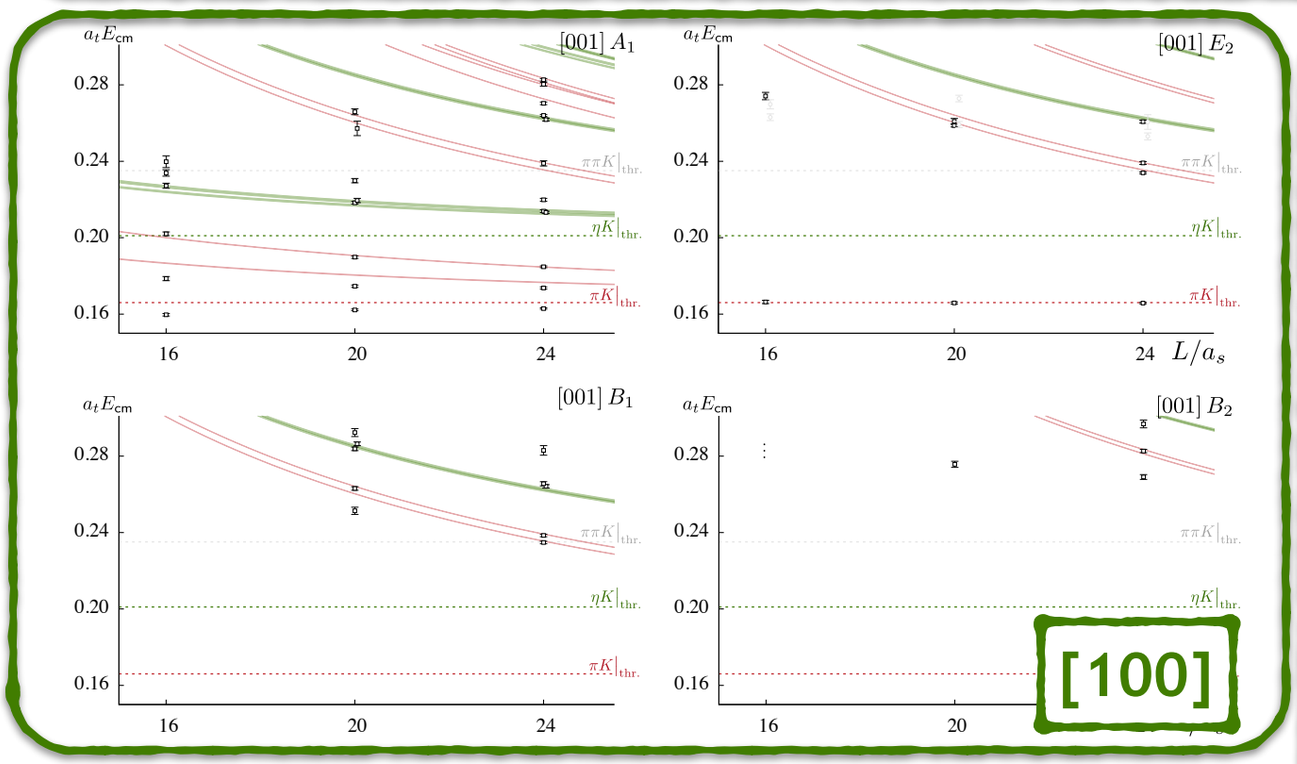
$$\chi^2/N_{\text{dof}} = \frac{6.40}{15-6} = 0.71$$

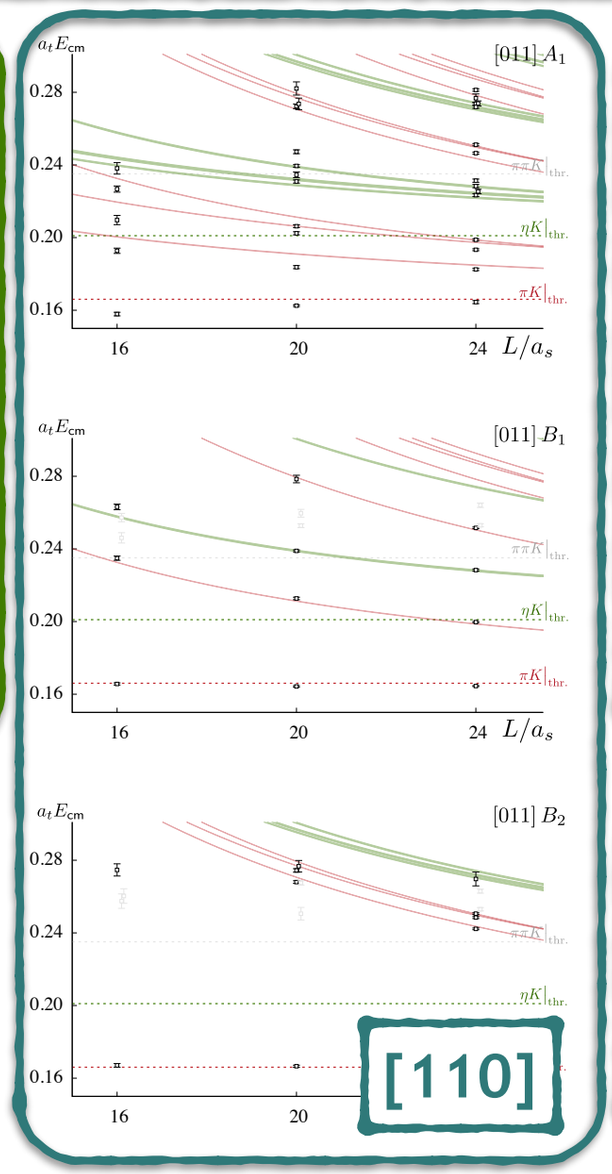
$m_\pi \sim 391\,\text{MeV}$

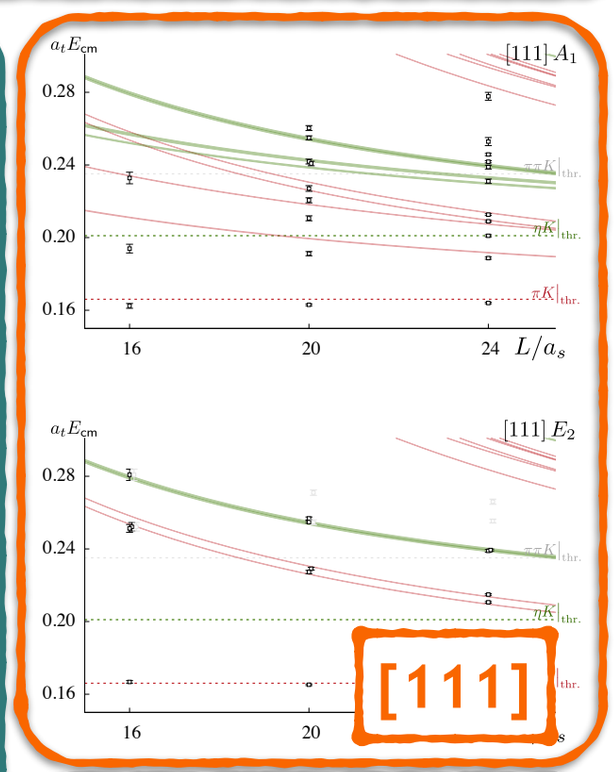


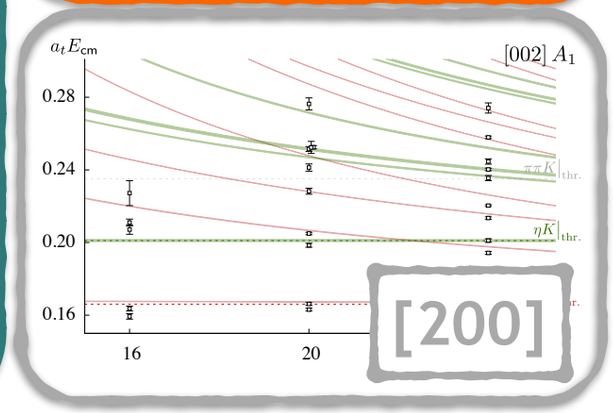
spectra in moving frames



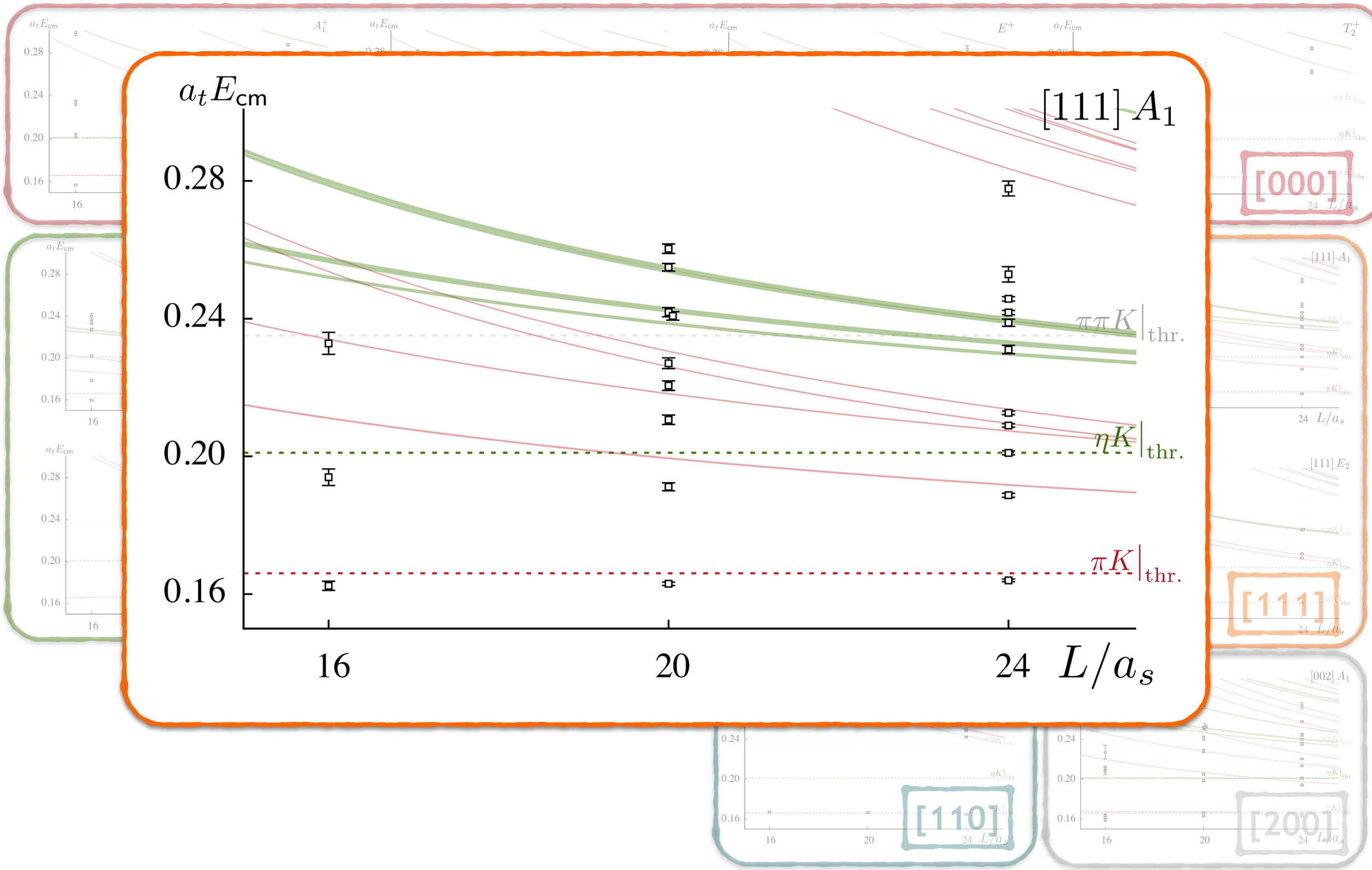




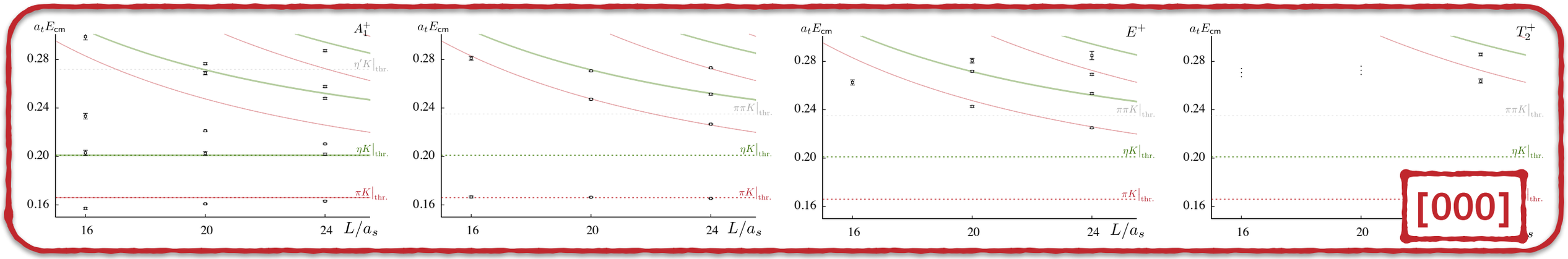


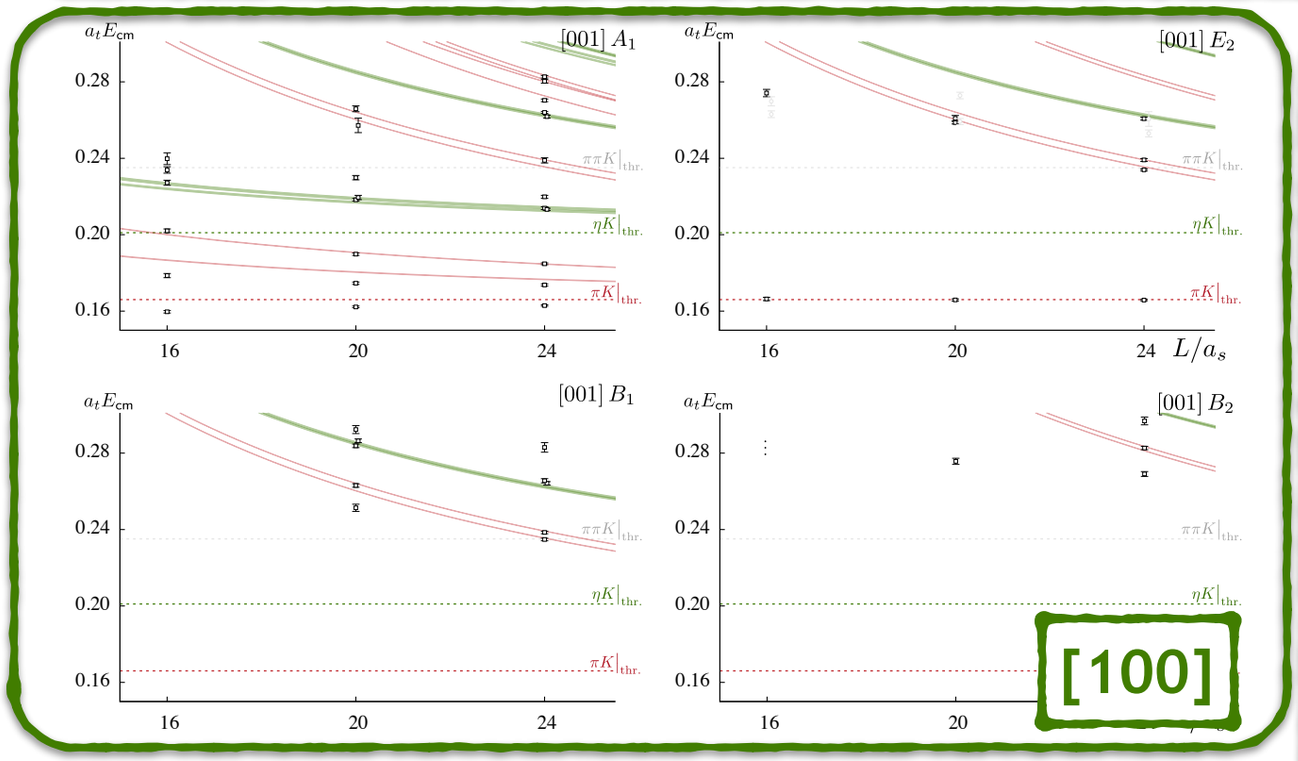


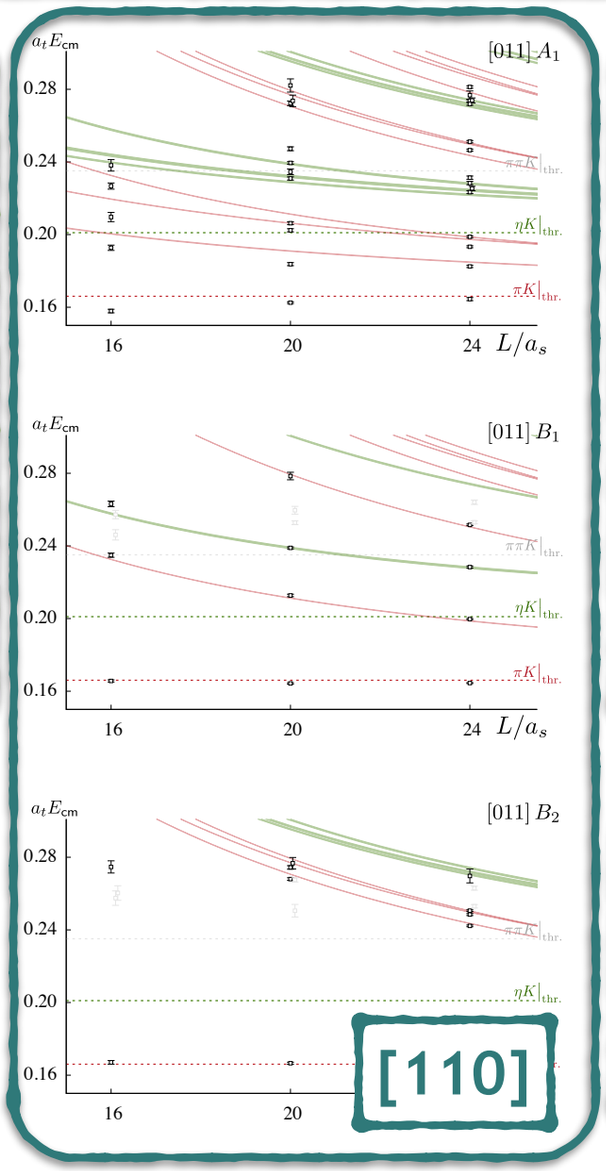
spectra in moving frames

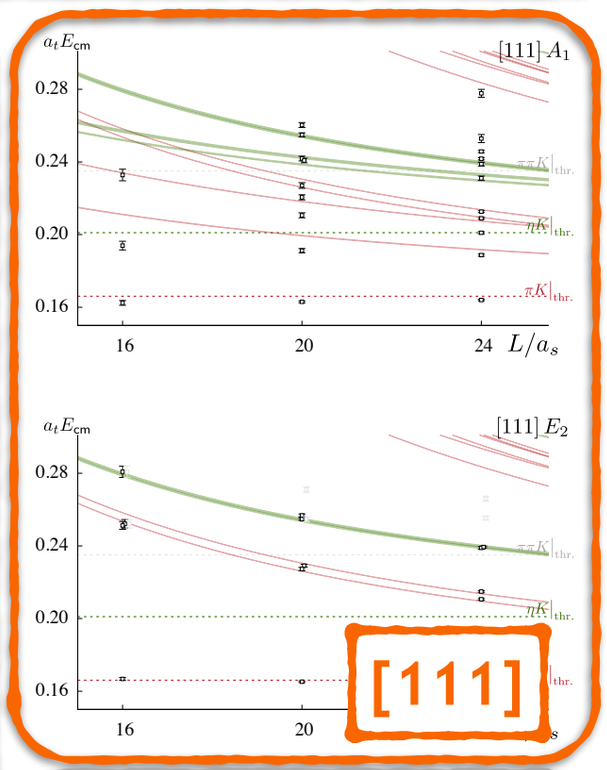


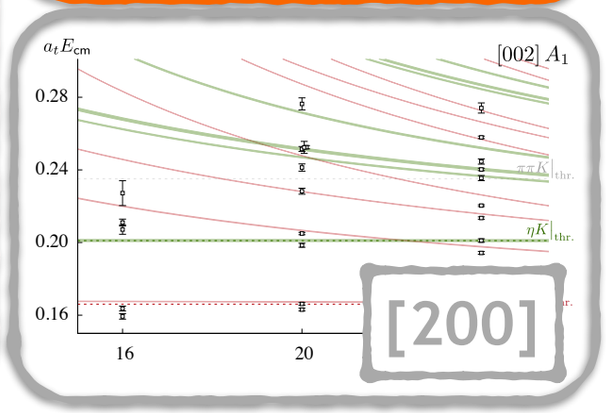
spectra in moving frames











- more information, but need to constrain multiple ℓ simultaneously

$$\chi^2/N_{\rm dof} = \frac{49.1}{61-6} = 0.89$$

$\pi K/\eta K$ scattering amplitudes

PHASE-SHIFTS & (IN)ELASTICITY

$$S_{\pi K\to\pi K} = \eta\, e^{2i\delta^{\pi K}}$$
$$S_{\eta K\to\eta K} = \eta\, e^{2i\delta^{\eta K}}$$
$$S_{\pi K\to\eta K} = i\sqrt{1-\eta^2}\, e^{i(\delta^{\pi K}+\delta^{\eta K})}$$

η=1 ⇒ decoupled scattering

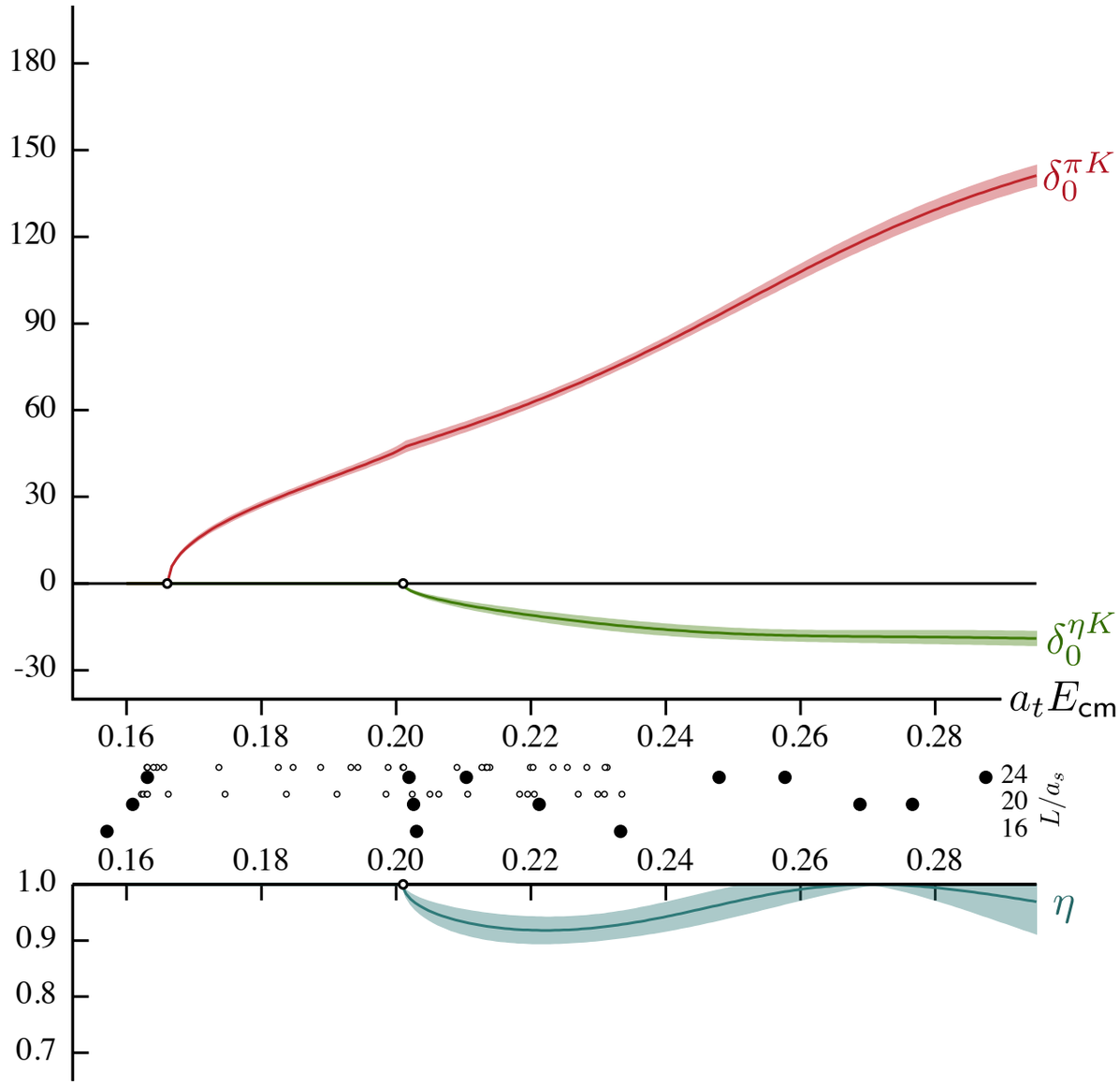
FINITE-VOLUME FIT PARAMS

$$\mathbf{K}(s) = \frac{1}{m^2-s}\begin{bmatrix} g^2_{\pi K} & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g^2_{\eta K}\end{bmatrix} + \begin{bmatrix}\gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K}\end{bmatrix}$$

$$\begin{aligned} m &= (0.2458 \pm 0.0014 \pm 0.0004)\cdot a_t^{-1} \\ g_{\pi K} &= (0.156 \pm 0.004 \pm 0.001)\cdot a_t^{-1} \\ g_{\eta K} &= (0.027 \pm 0.008 \pm 0.008)\cdot a_t^{-1} \\ \gamma_{\pi K,\pi K} &= 0.082 \pm 0.046 \pm 0.022 \\ \gamma_{\pi K,\eta K} &= 0.33 \pm 0.13 \pm 0.06 \\ \gamma_{\eta K,\eta K} &= -0.41 \pm 0.05 \pm 0.07 \end{aligned} \qquad \begin{bmatrix} 1 & 0.5 & -0.3 & 0.0 & 0.1 & -0.1 \\ & 1 & -0.4 & -0.7 & 0.5 & -0.1 \\ & & 1 & 0.3 & -0.6 & 0.3 \\ & & & 1 & 0.1 & -0.1 \\ & & & & 1 & -0.3 \\ & & & & & 1 \end{bmatrix}.$$

$\pi K/\eta K$ scattering amplitudes

S-WAVE $\pi K/\eta K$ SCATTERING



PHASE-SHIFTS & (IN)ELASTICITY

$$S_{\pi K\to\pi K} = \eta\, e^{2i\delta^{\pi K}}$$
$$S_{\eta K\to\eta K} = \eta\, e^{2i\delta^{\eta K}}$$
$$S_{\pi K\to\eta K} = i\sqrt{1-\eta^2}\, e^{i(\delta^{\pi K}+\delta^{\eta K})}$$

η

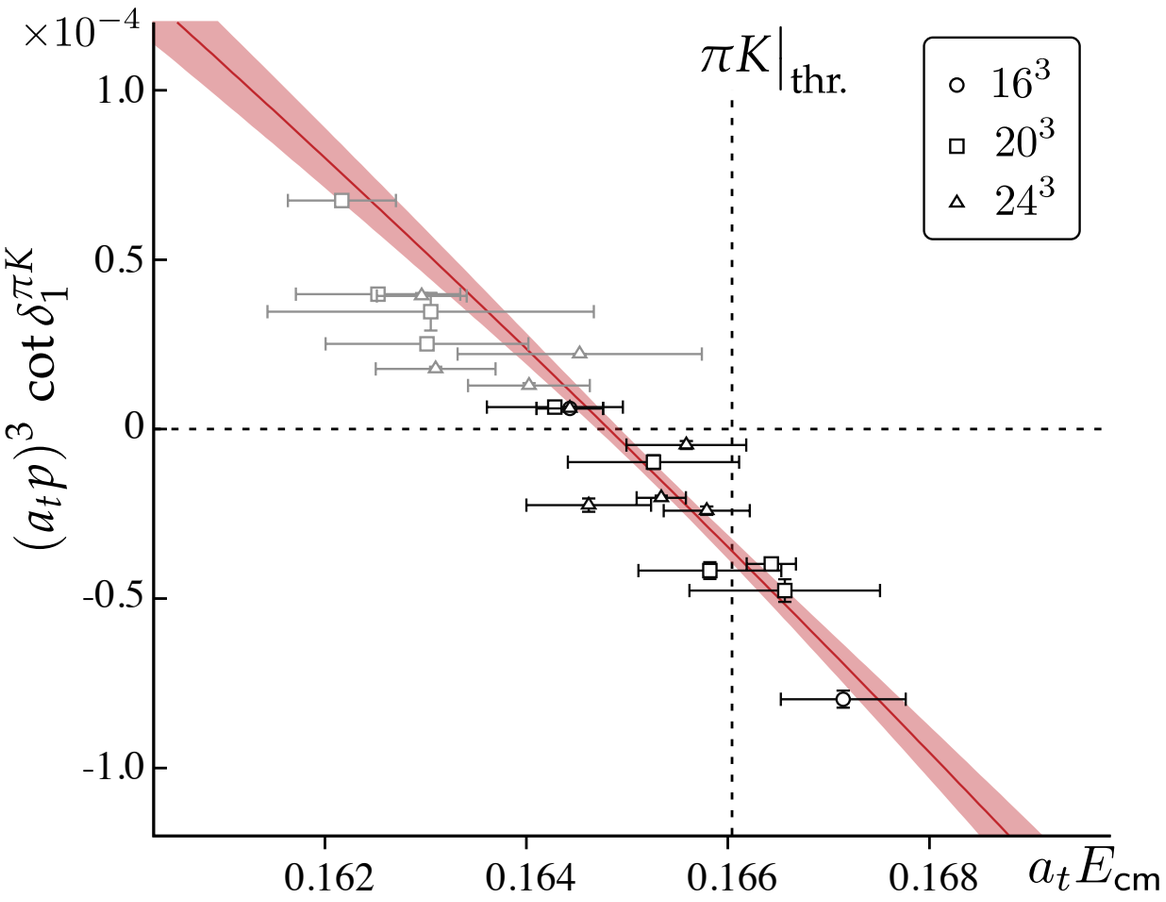
FINITE-VOLUME FIT PARAMS

$$\mathbf{K}(s) = \frac{1}{m^2-s}\begin{bmatrix} g_{\pi K}^2 & g_{\pi K}\, g_{\eta K} \\ g_{\pi K}\, g_{\eta K} & g_{\eta K}^2 \end{bmatrix} + \begin{bmatrix} \gamma_{\pi K,\pi K} & \gamma_{\pi K,\eta K} \\ \gamma_{\pi K,\eta K} & \gamma_{\eta K,\eta K} \end{bmatrix}$$

$$\begin{aligned} m &= (0.2458 \pm 0.0014 \pm 0.0004)\cdot a_t^{-1} \\ g_{\pi K} &= (0.156 \pm 0.004 \pm 0.001)\cdot a_t^{-1} \\ g_{\eta K} &= (0.027 \pm 0.008 \pm 0.008)\cdot a_t^{-1} \\ \gamma_{\pi K,\pi K} &= 0.082 \pm 0.046 \pm 0.022 \\ \gamma_{\pi K,\eta K} &= 0.33 \pm 0.13 \pm 0.06 \\ \gamma_{\eta K,\eta K} &= -0.41 \pm 0.05 \pm 0.07 \end{aligned} \qquad \begin{bmatrix} 1 & 0.5 & -0.3 & 0.0 & 0.1 & -0.1 \\ & 1 & -0.4 & -0.7 & 0.5 & -0.1 \\ & & 1 & 0.3 & -0.6 & 0.3 \\ & & & 1 & 0.1 & -0.1 \\ & & & & 1 & -0.3 \\ & & & & & 1 \end{bmatrix}.$$

$\pi K/\eta K$ scattering amplitudes

P-WAVE πK SCATTERING



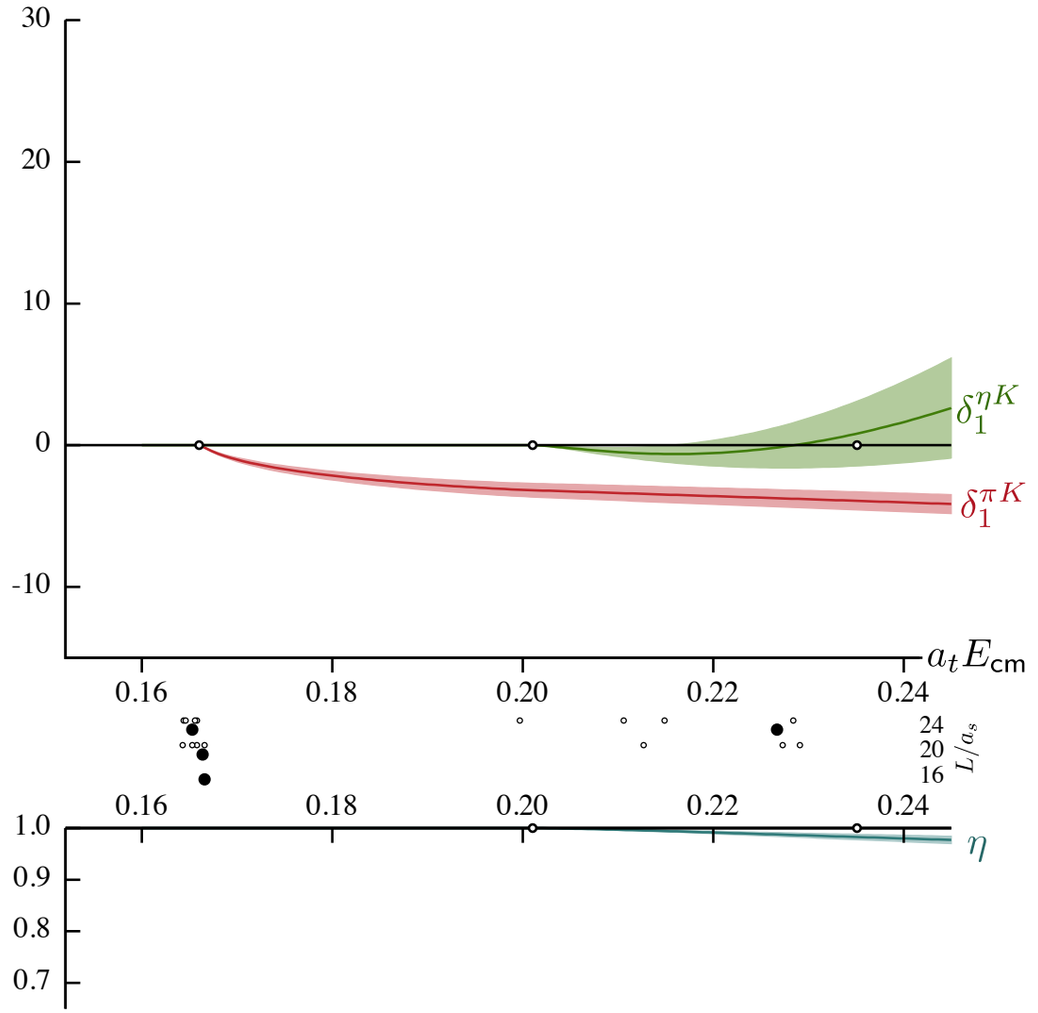
$$a_t m(K^\star) = 0.16482(15)$$

$$g = 5.93(30)$$

vector bound-state

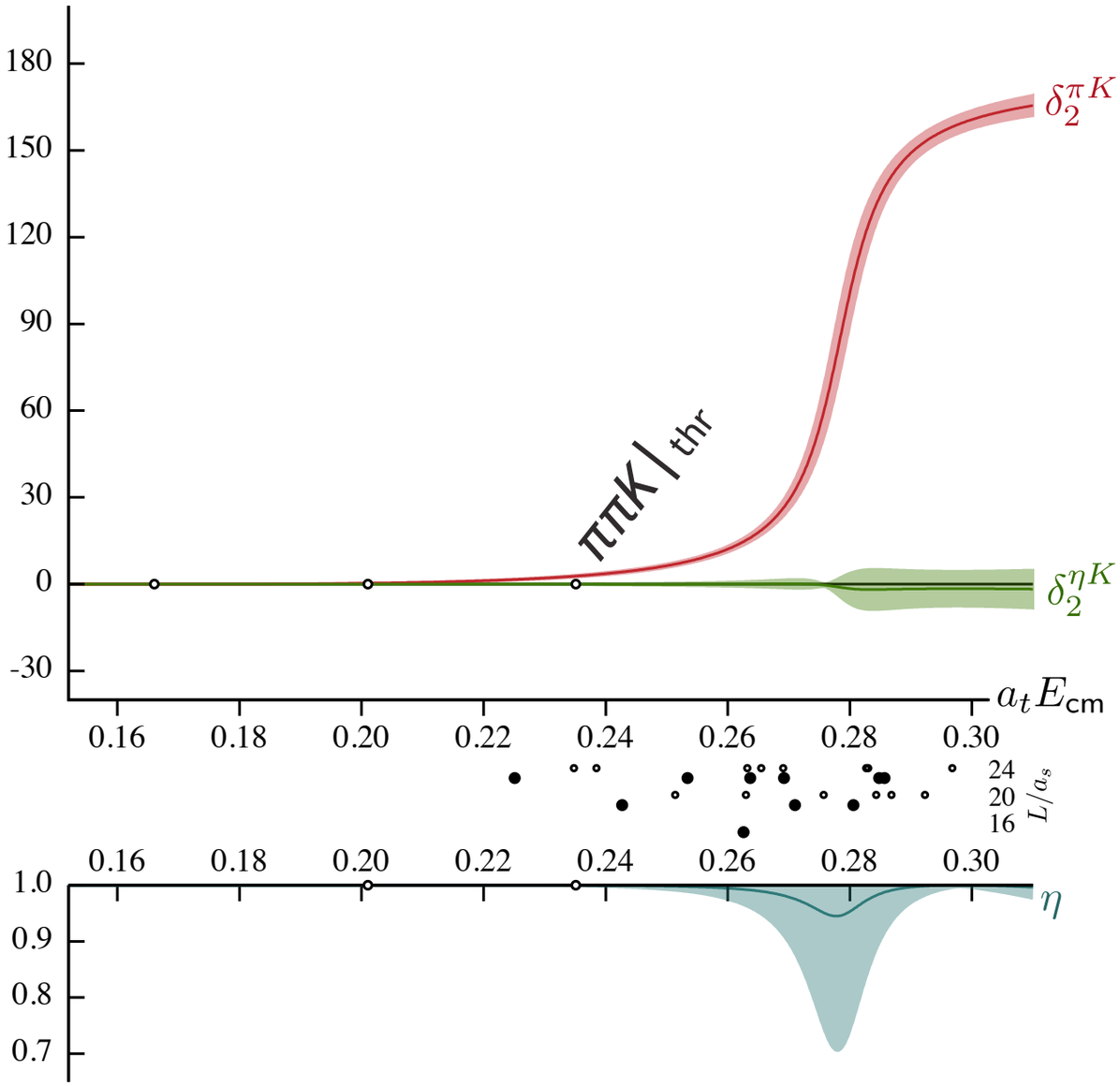
$$a_t(m_\pi + m_K) = 0.16604$$

P-WAVE $\pi K / \eta K$ SCATTERING



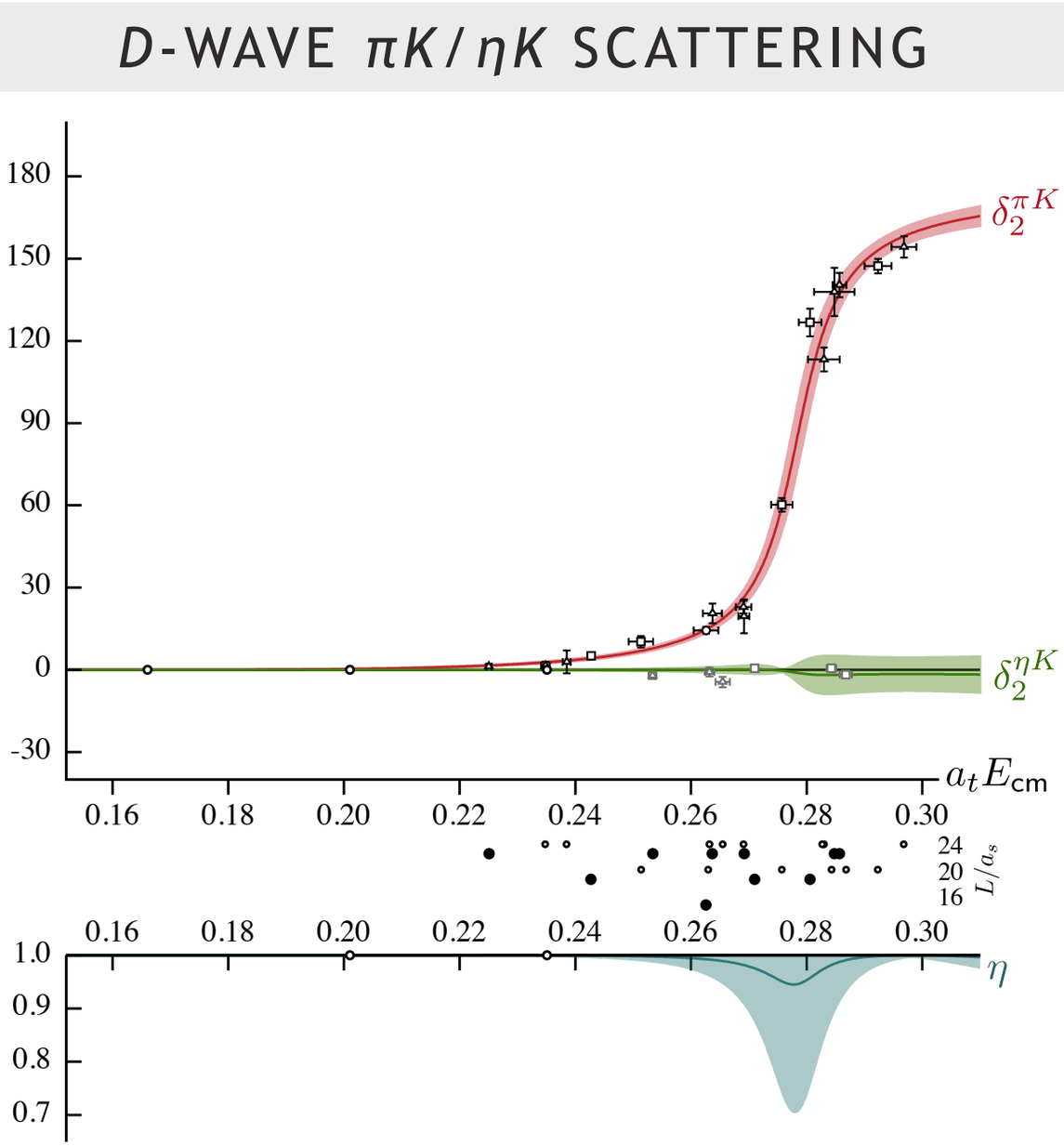
$\pi K/\eta K$ scattering amplitudes

D-WAVE $\pi K/\eta K$ SCATTERING



$\pi K/\eta K$ scattering amplitudes

- phase-shift points assuming complete decoupling of $\pi K/\eta K$

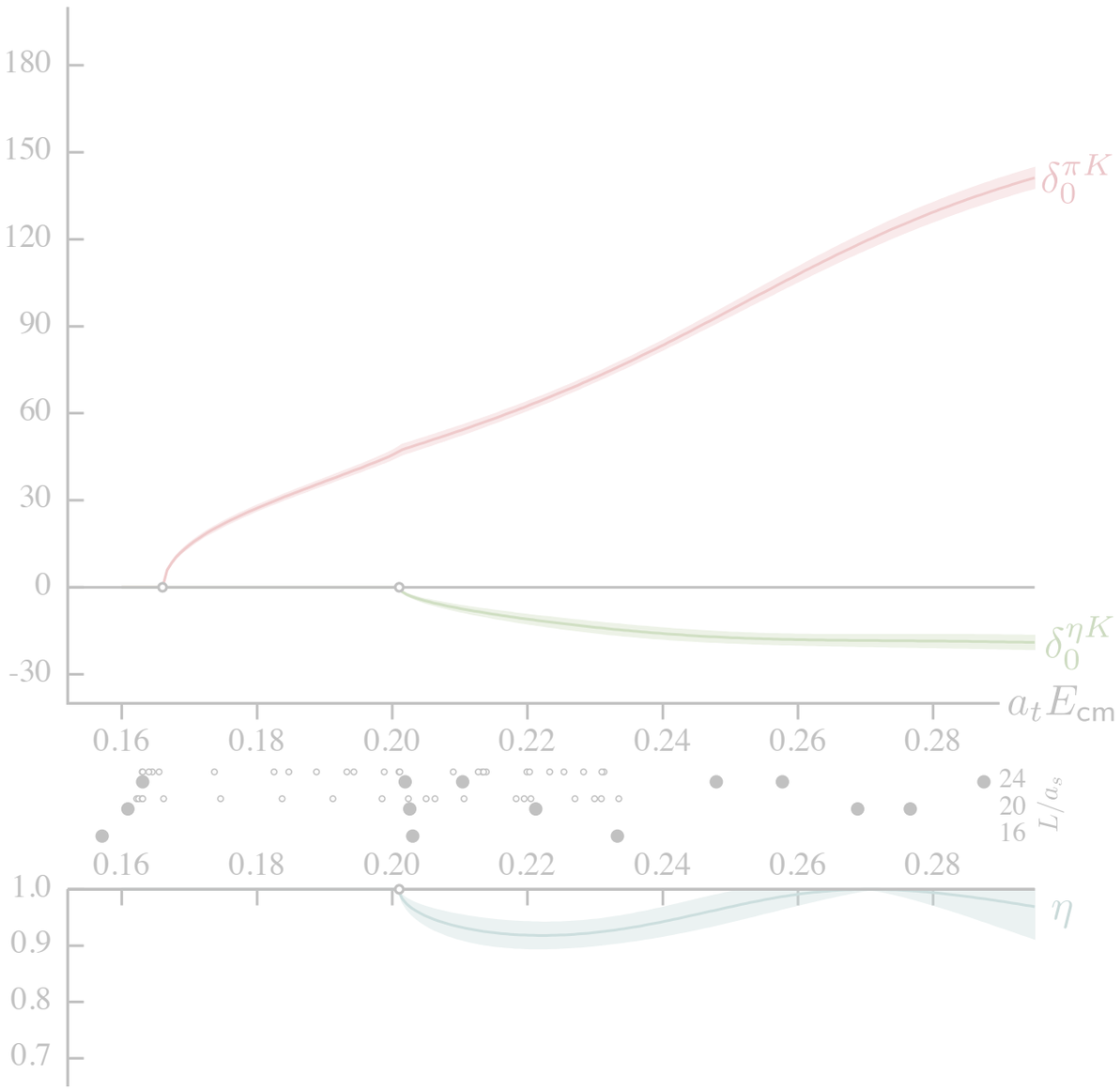


– decoupled from $\pi\pi K$?

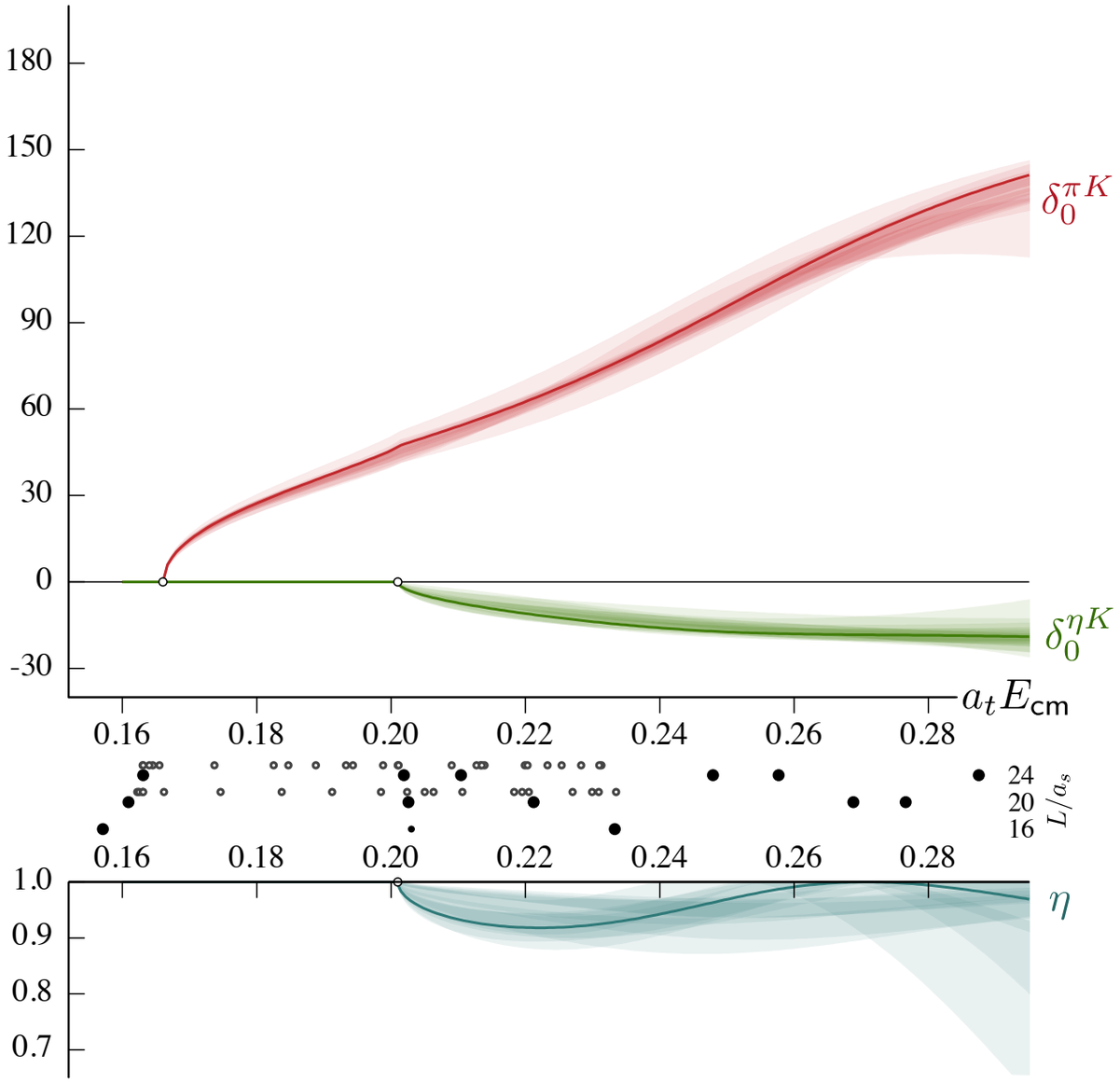
$\pi K/\eta K$ scattering amplitudes

- are the result parameterization dependent ?
  - try a range of parameterizations …

many variations of $K_{ij}^{-1}(s) = \sum_{n=0}^{N_{ij}} c_{ij}^{[n]}\, s^n$



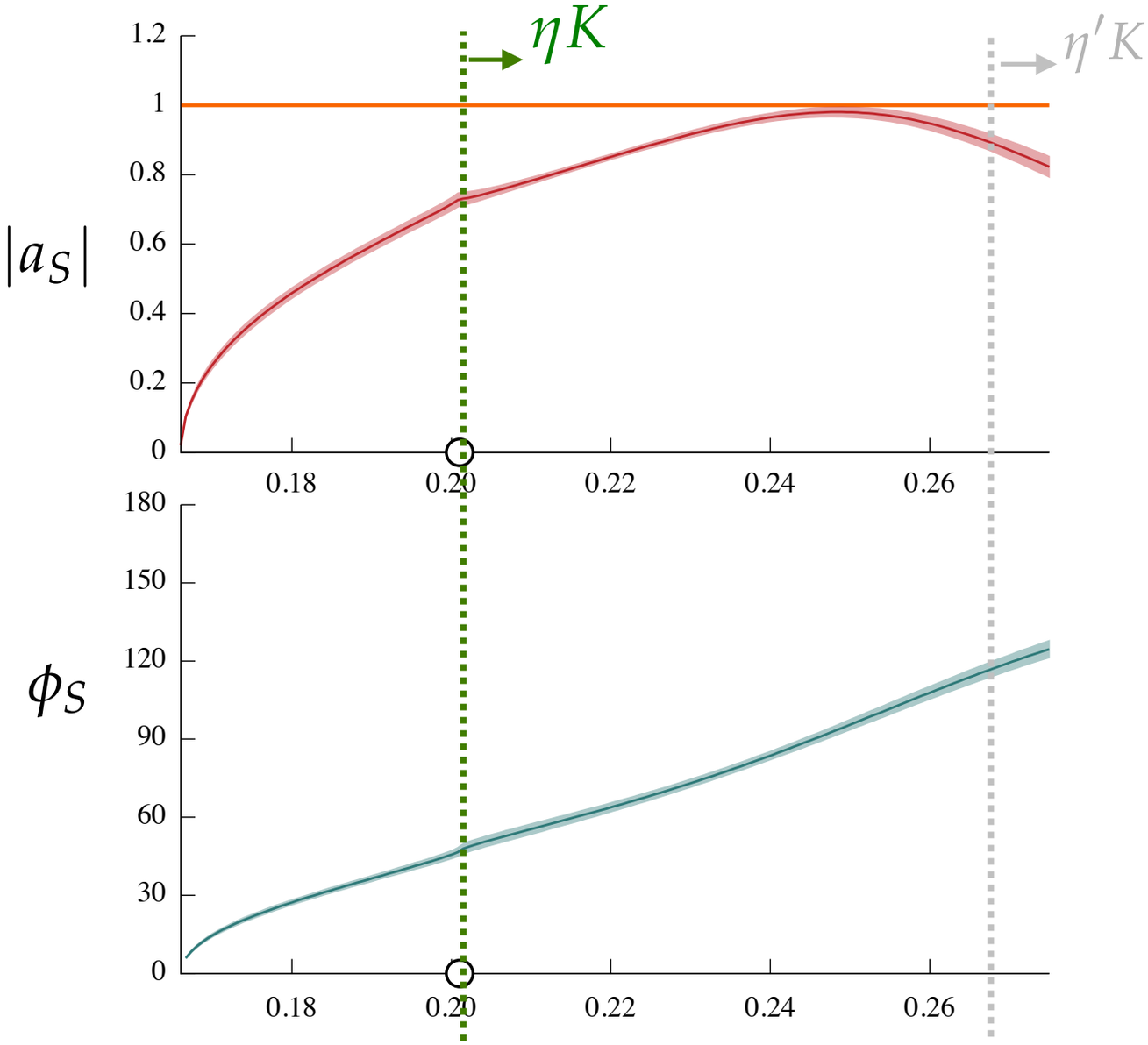
S-WAVE $\pi K/\eta K$ SCATTERING



- gross features are robust

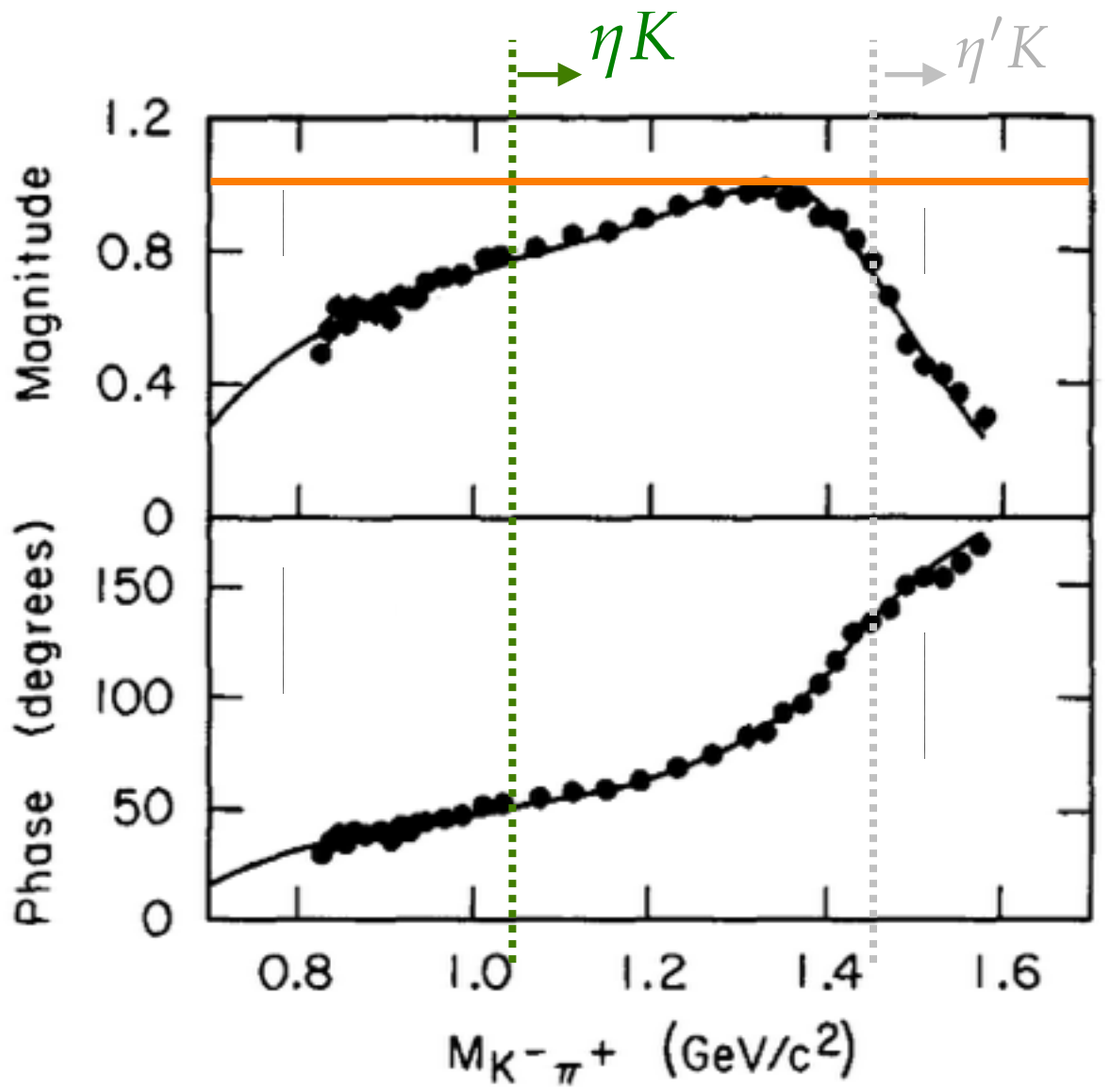
$\pi K/\eta K$ scattering amplitudes

S-WAVE $\pi K \to \pi K$ AMPLITUDE



$m_\pi \sim 391\,\mathrm{MeV}$

LASS S-WAVE



LASS, NPB296 493

analytic structure of elastic scattering

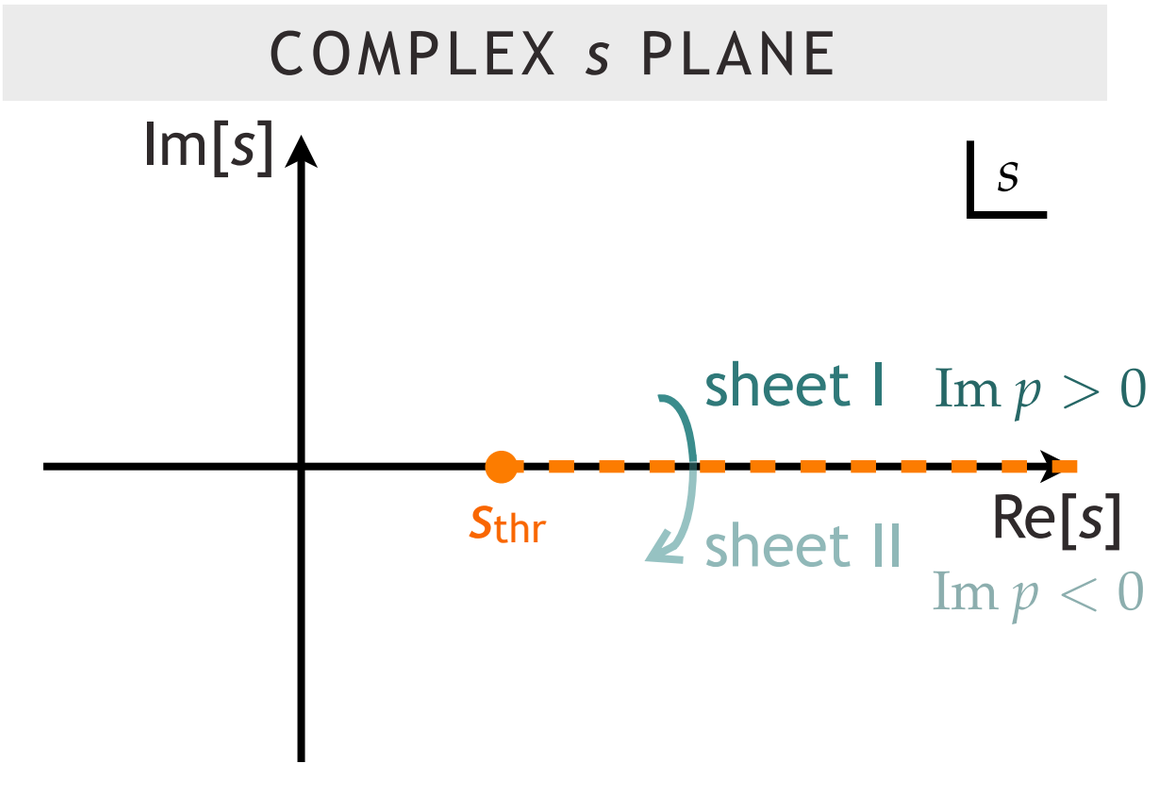
- elastic scattering amplitude has a two-sheet structure

e.g. equal mass scattering

$$\mathrm{Im}\, t(s) = \rho(s)\, |t(s)|^2$$

$$\rho(s) = \tfrac{1}{2}\sqrt{s - 4m^2}$$

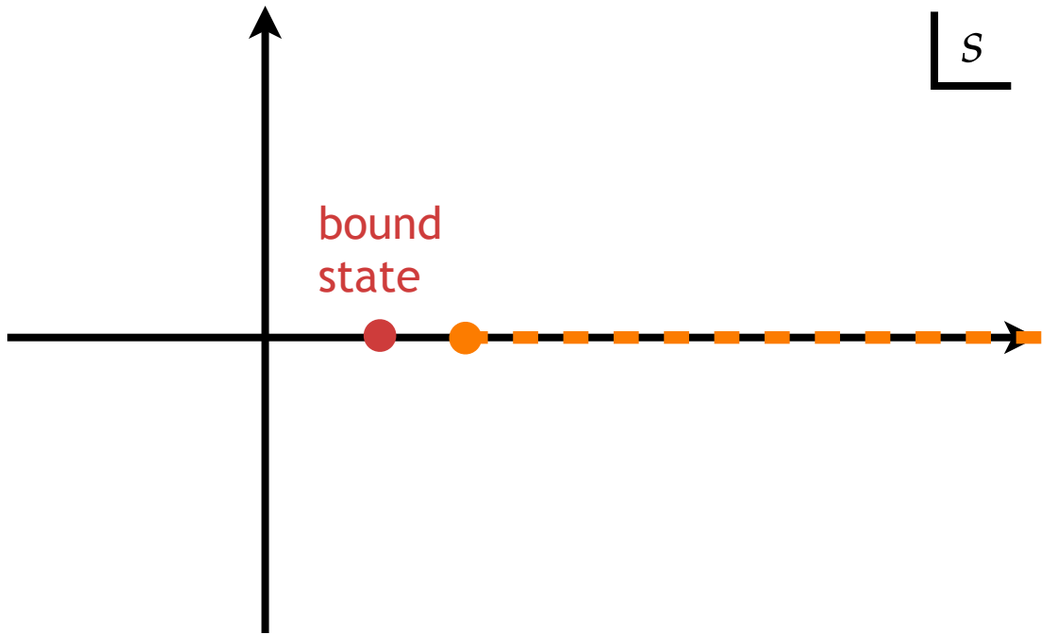
branch cut for $s > s_{\mathrm{thr}}$



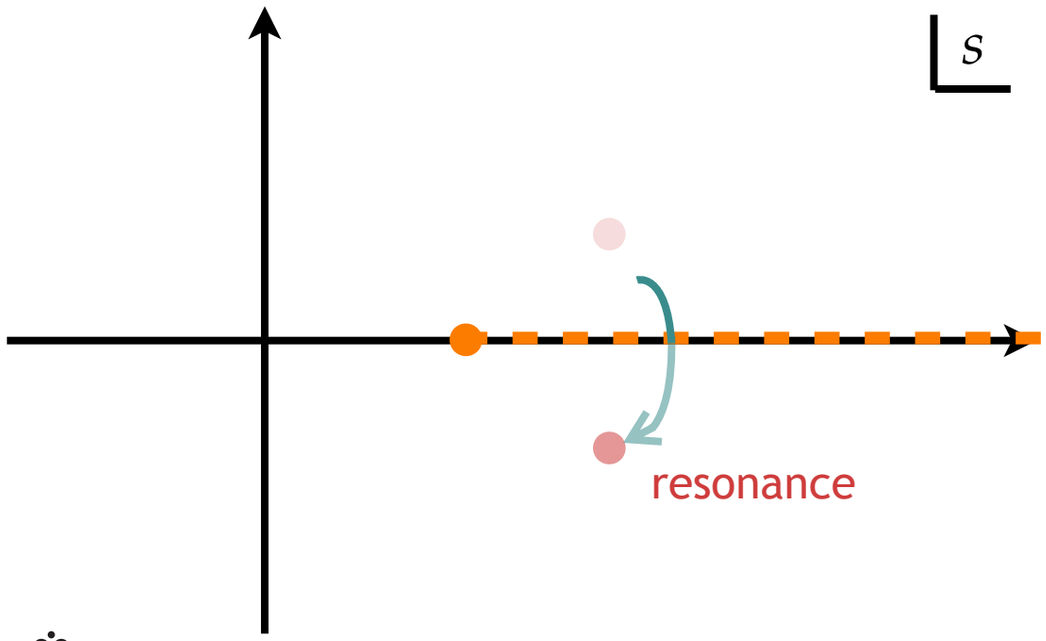
elastic scattering

- amplitude may have pole singularities

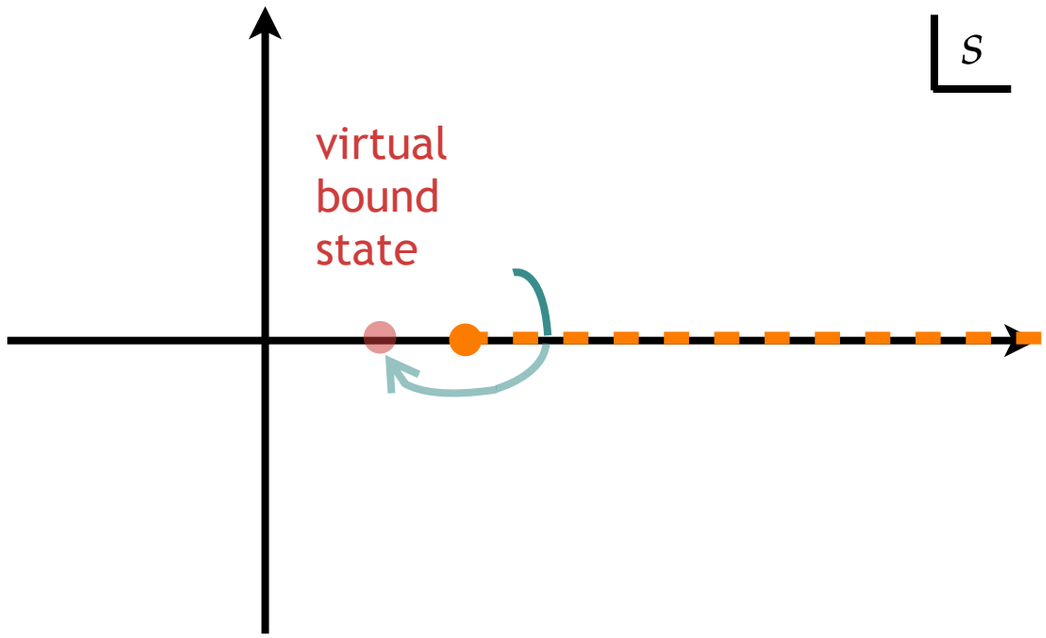
BOUND-STATE POLE



RESONANCE POLE(S)



VIRTUAL BOUND-STATE POLE

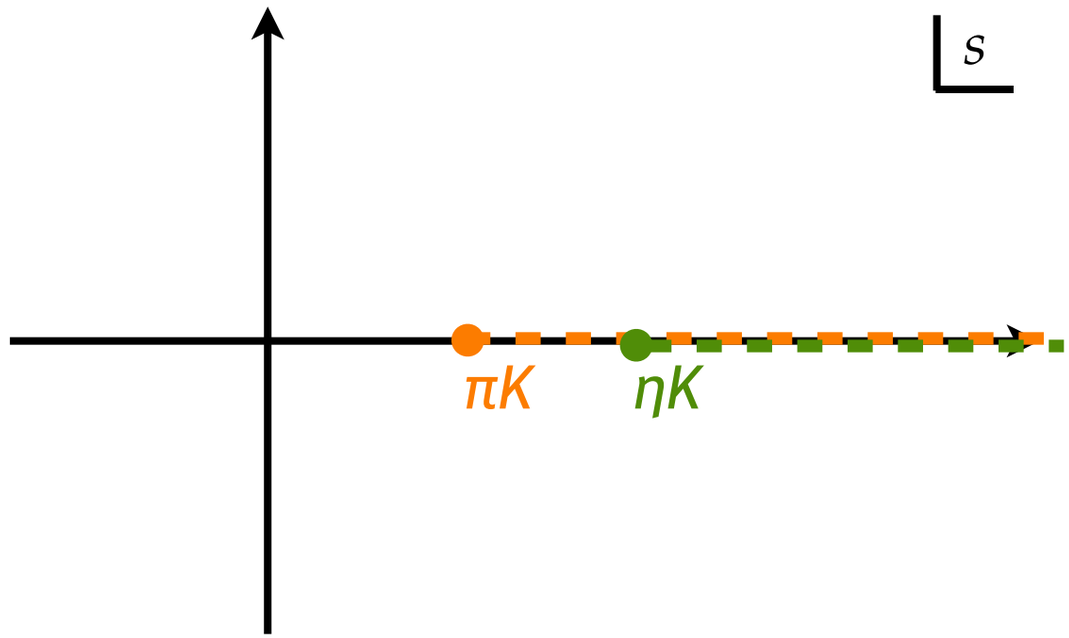


coupled-channel scattering

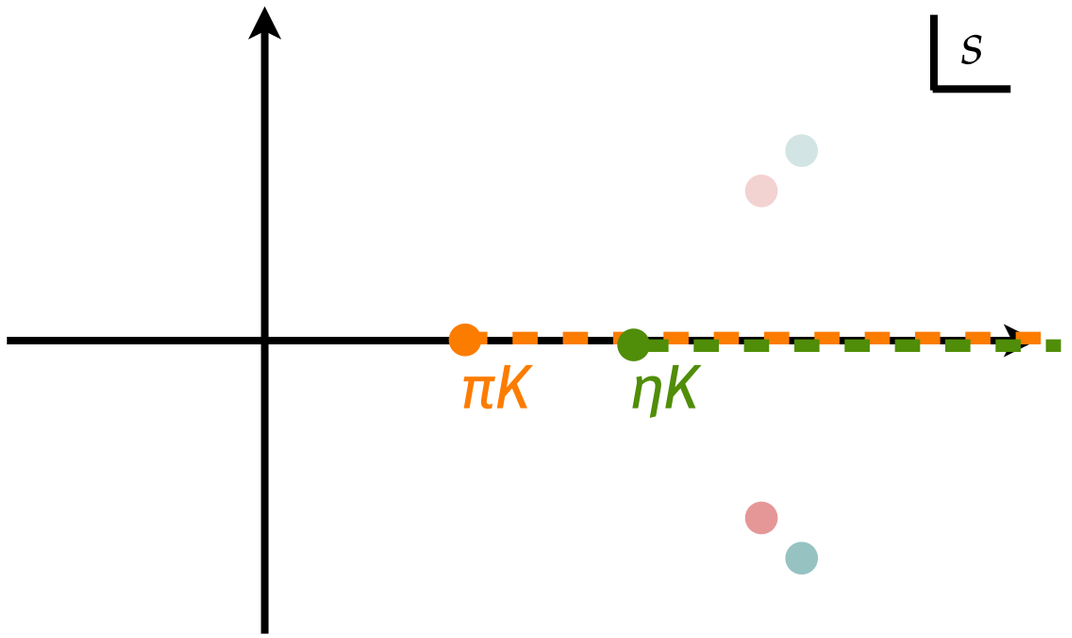
- get $2^{n_{\text{chan}}}$ sheeted structure

sheet	$\text{Im}\, p_{\pi K}$	$\text{Im}\, p_{\eta K}$
I	$+$	$+$
II	$-$	$+$
III	$-$	$-$
IV	$+$	$-$

COMPLEX *s* PLANE



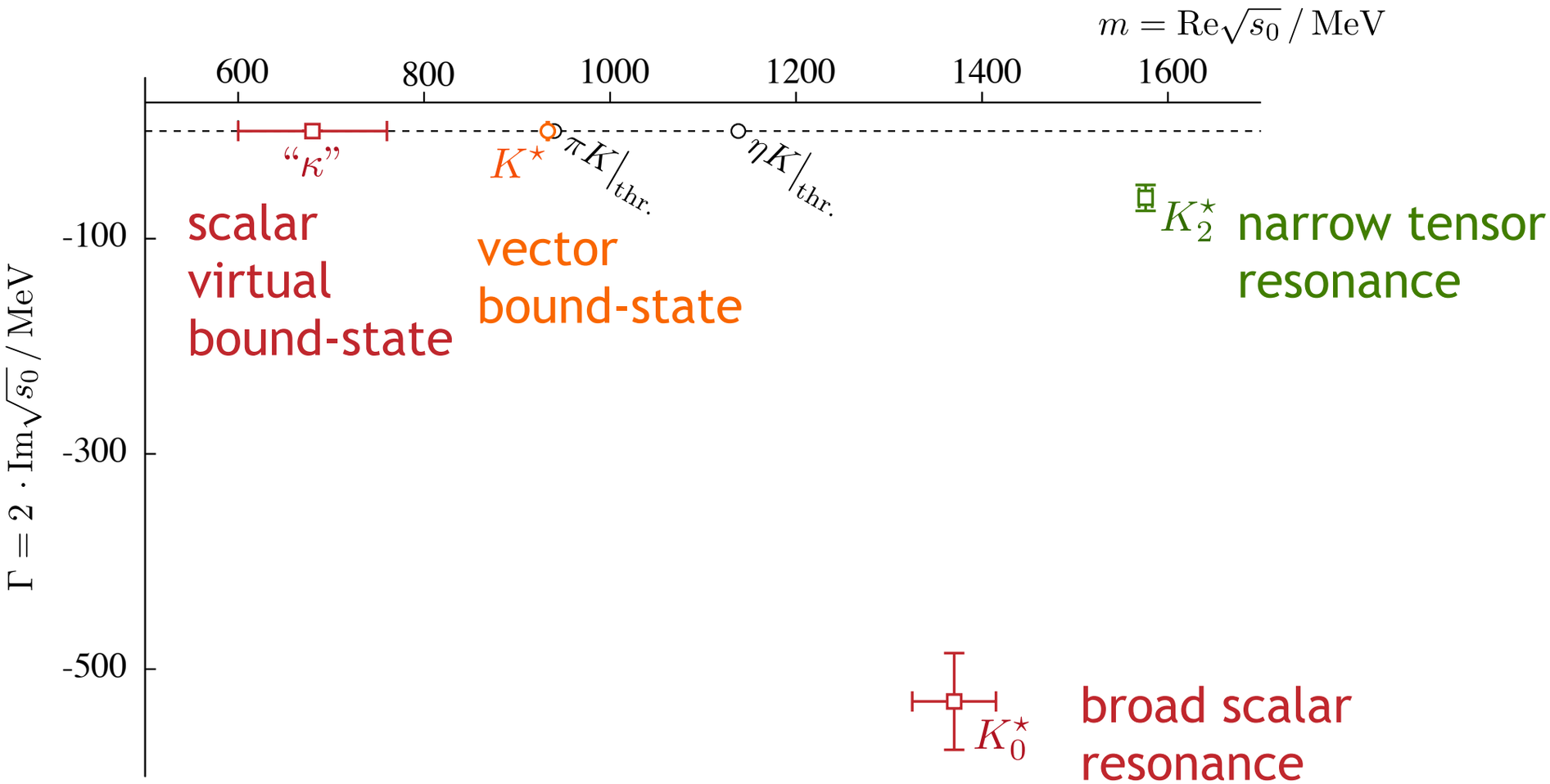
COUPLED-CHANNEL RESONANCE



‘simple’ resonances usually have poles on sheets II & III

$\pi K/\eta K$ scattering amplitudes

- pole content of parameterized *t*-matrices



scalar virtual bound-state

vector bound-state

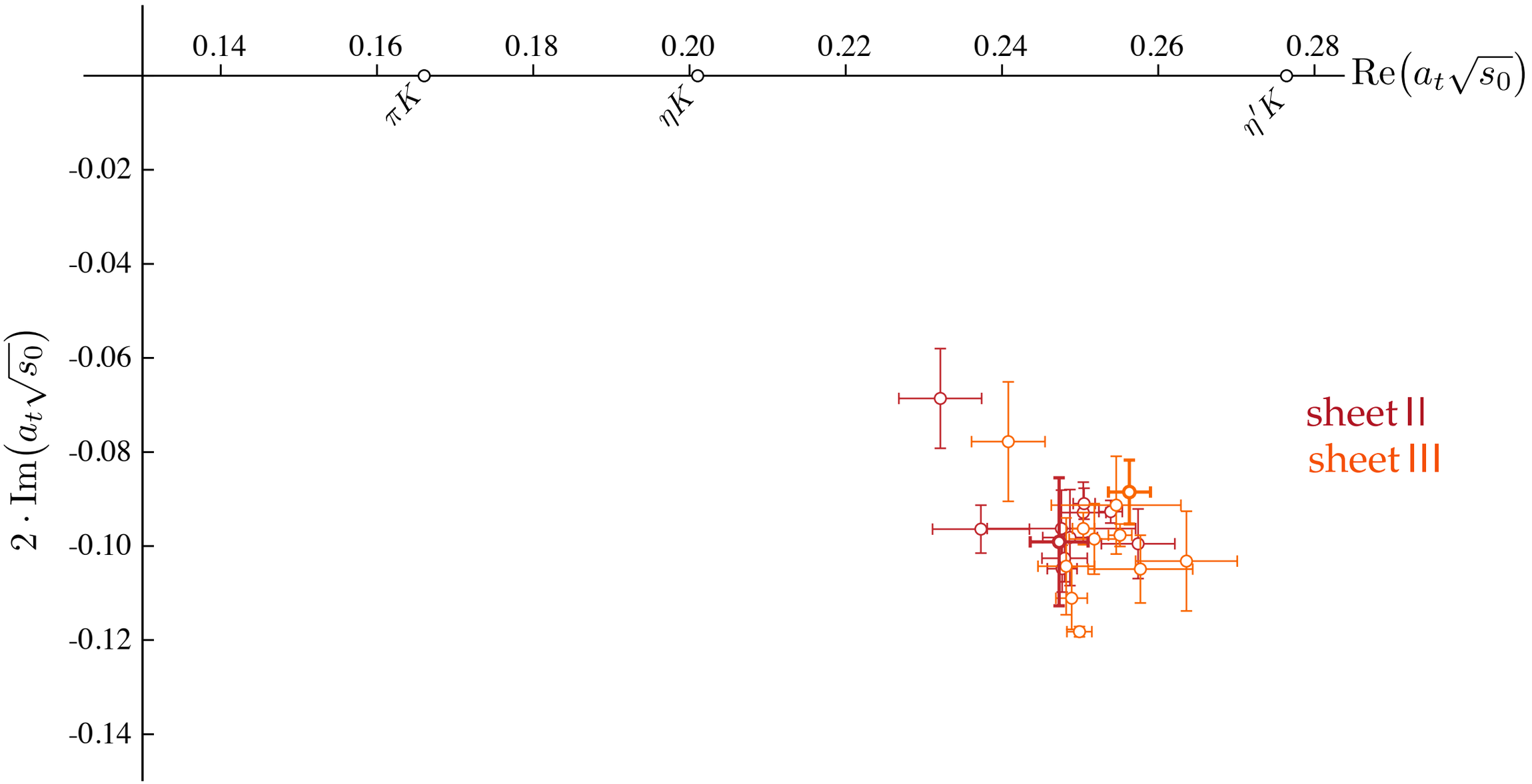
narrow tensor resonance

broad scalar resonance

$m_\pi \sim 391\,\mathrm{MeV}$

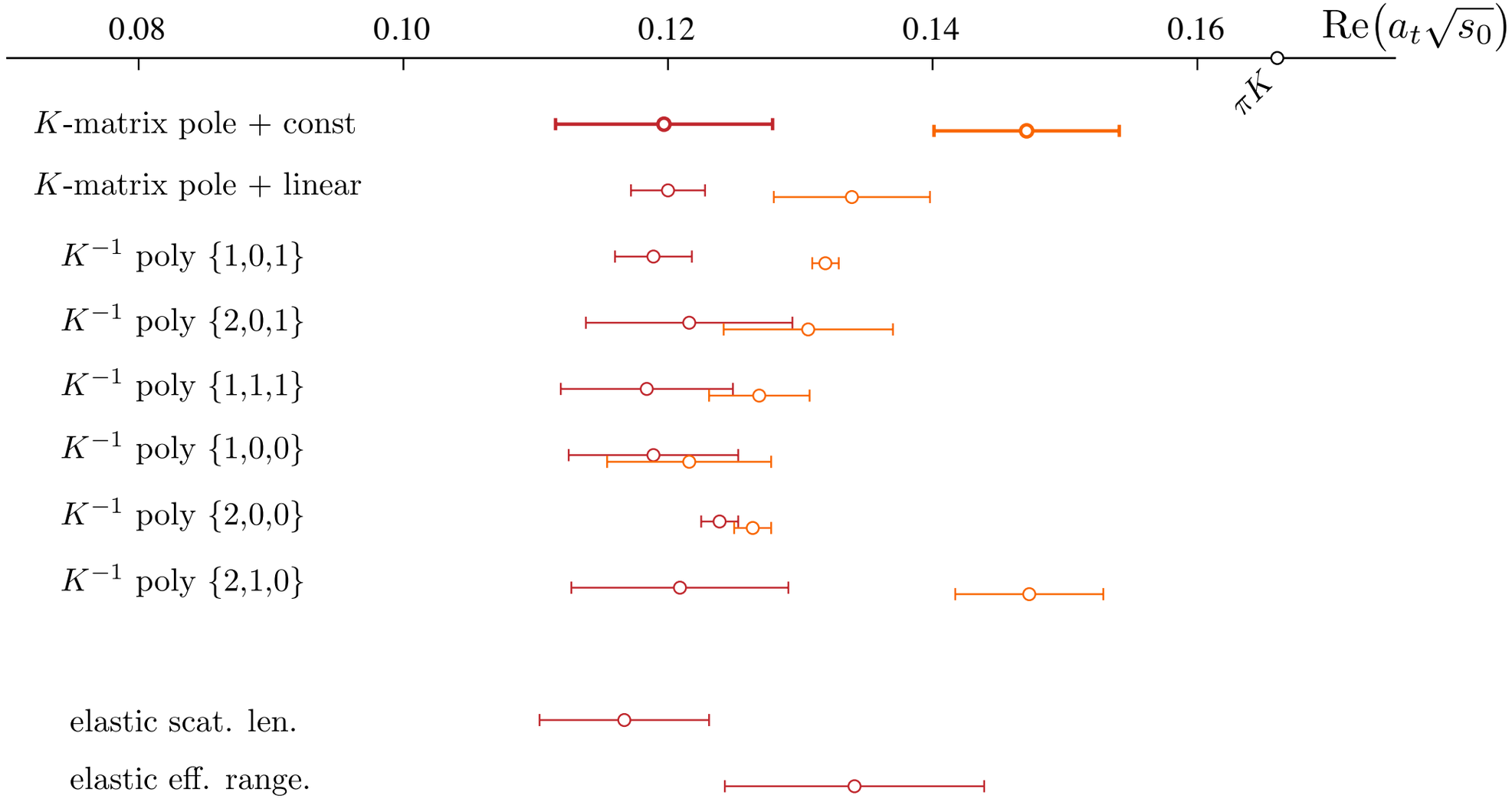
πK/ηK S-wave resonance

- *t*-matrix pole position under variation of parameterization



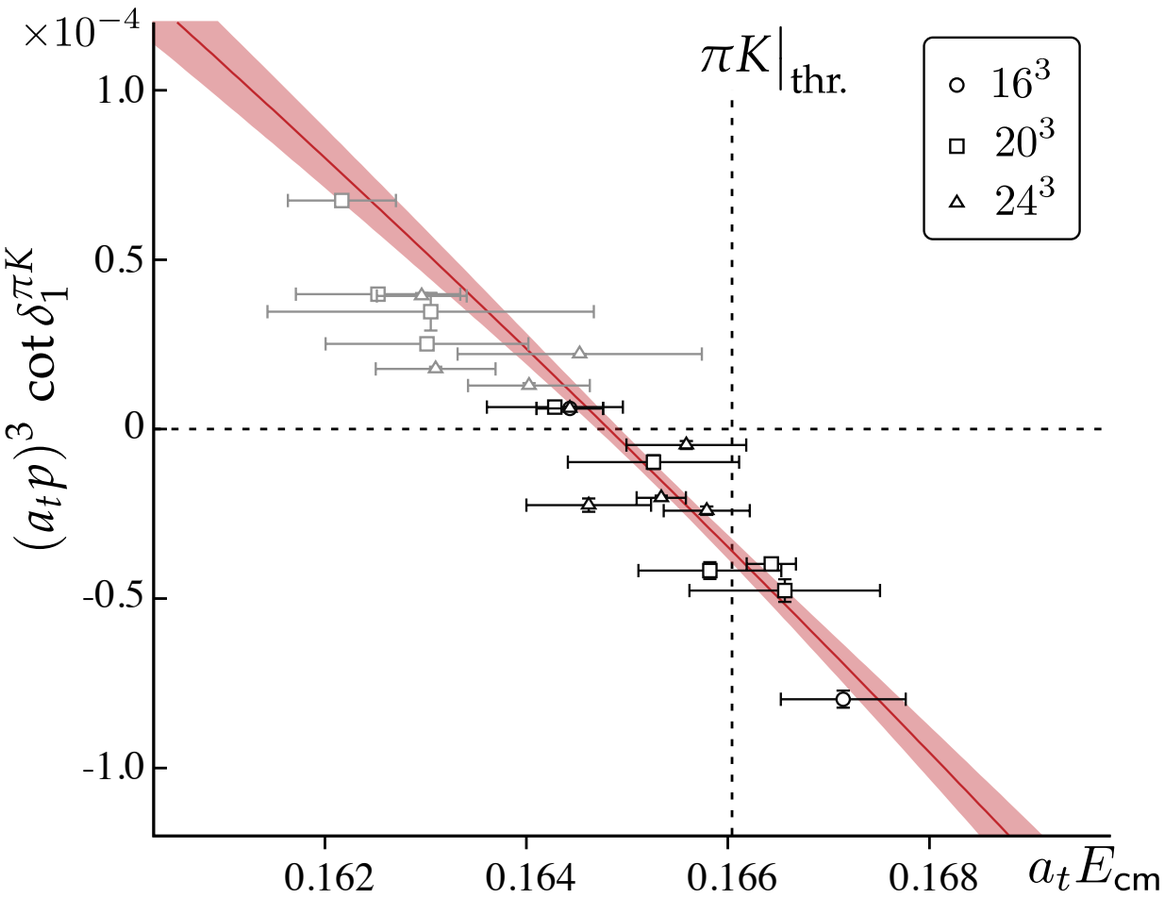
πK/ηK S-wave virtual bound state

- *t*-matrix pole position under variation of parameterization



$\pi K/\eta K$ scattering amplitudes

P-WAVE πK SCATTERING



$a_t m(K^\star) = 0.16482(15)$

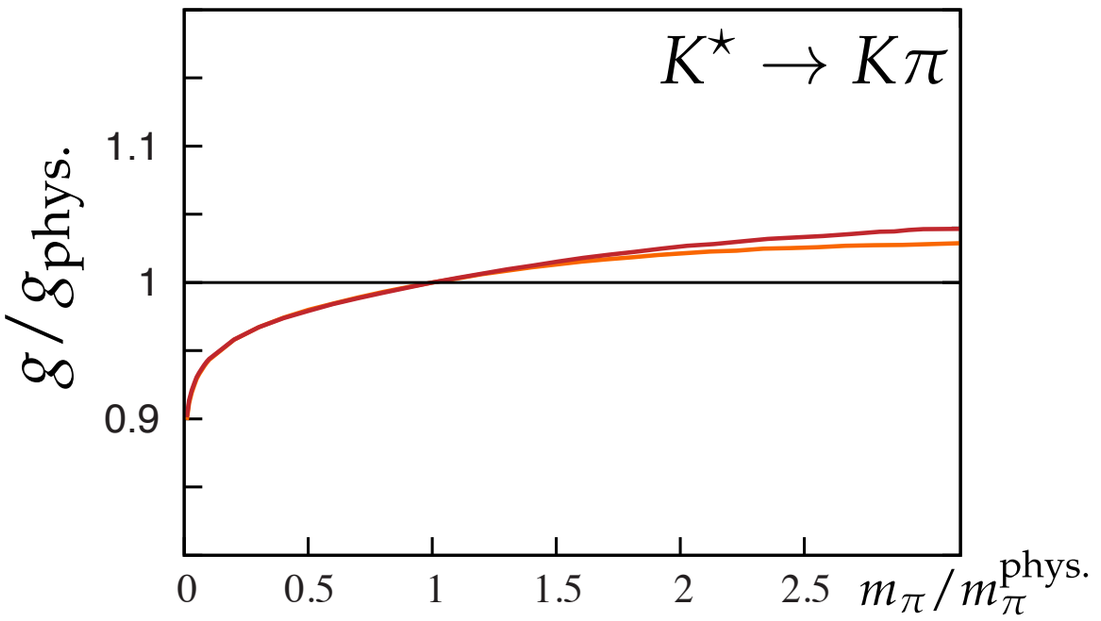
$g = 5.93(30)$

vector bound-state

$a_t(m_\pi + m_K) = 0.16604$

$g_{\text{phys.}} = 5.5(2)$ *PDG*

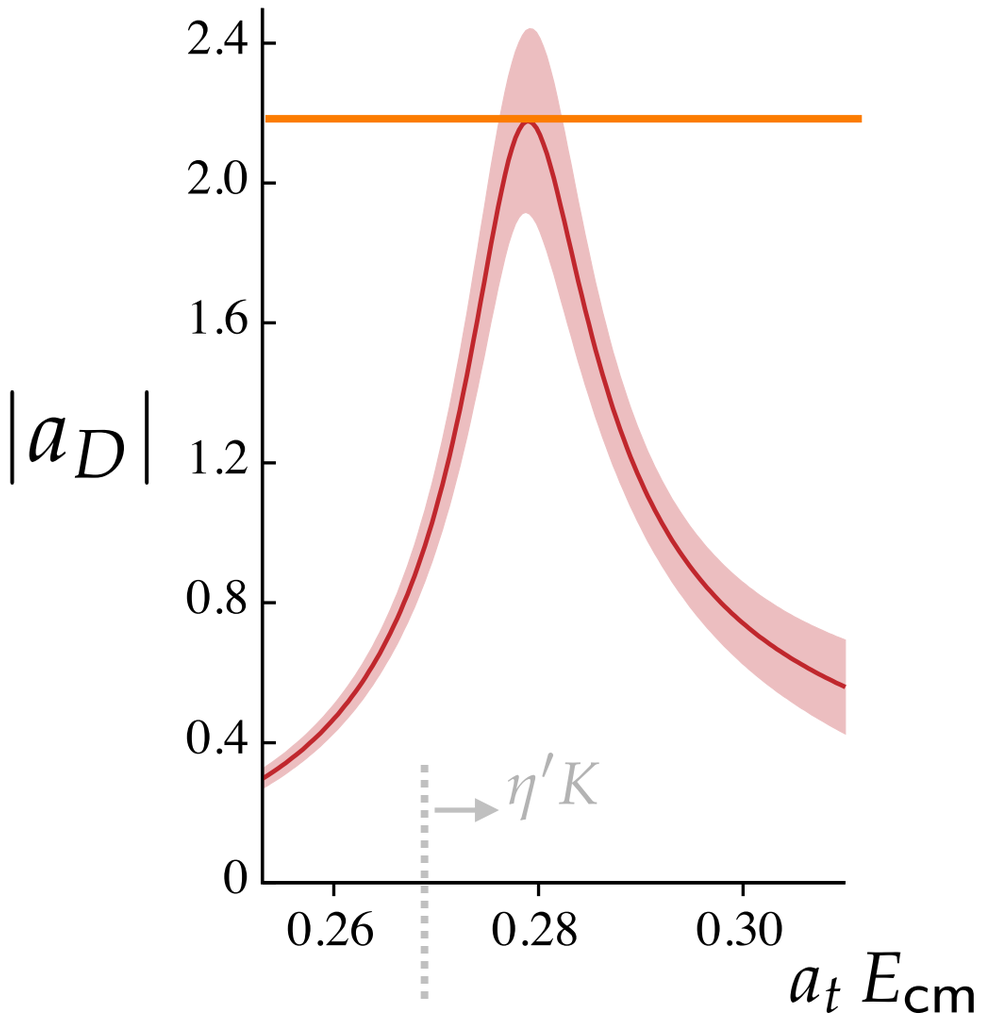
UNITARIZED χPT



NEBREDA & PELAEZ PRD81 054035 (2010)

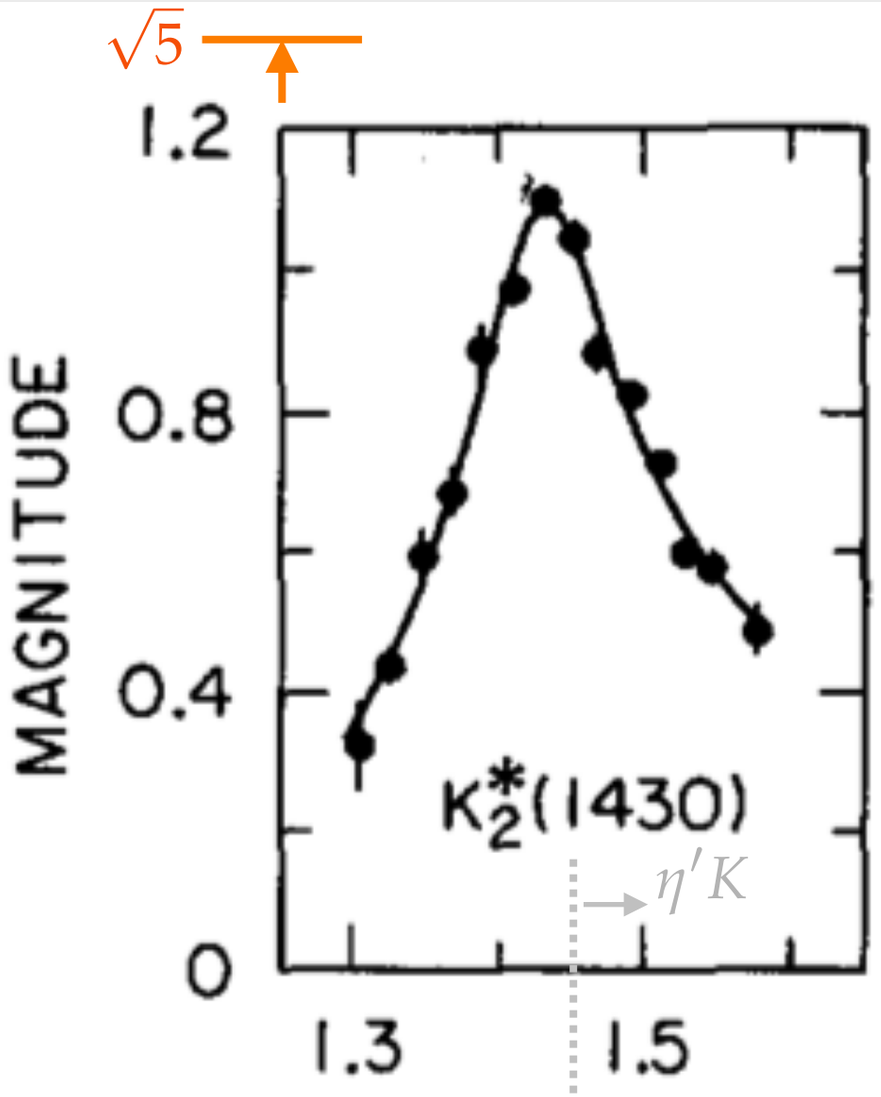
$\pi K/\eta K$ scattering amplitudes

D-WAVE $\pi K \to \pi K$ AMPLITUDE



$m_\pi \sim 391\,\text{MeV}$

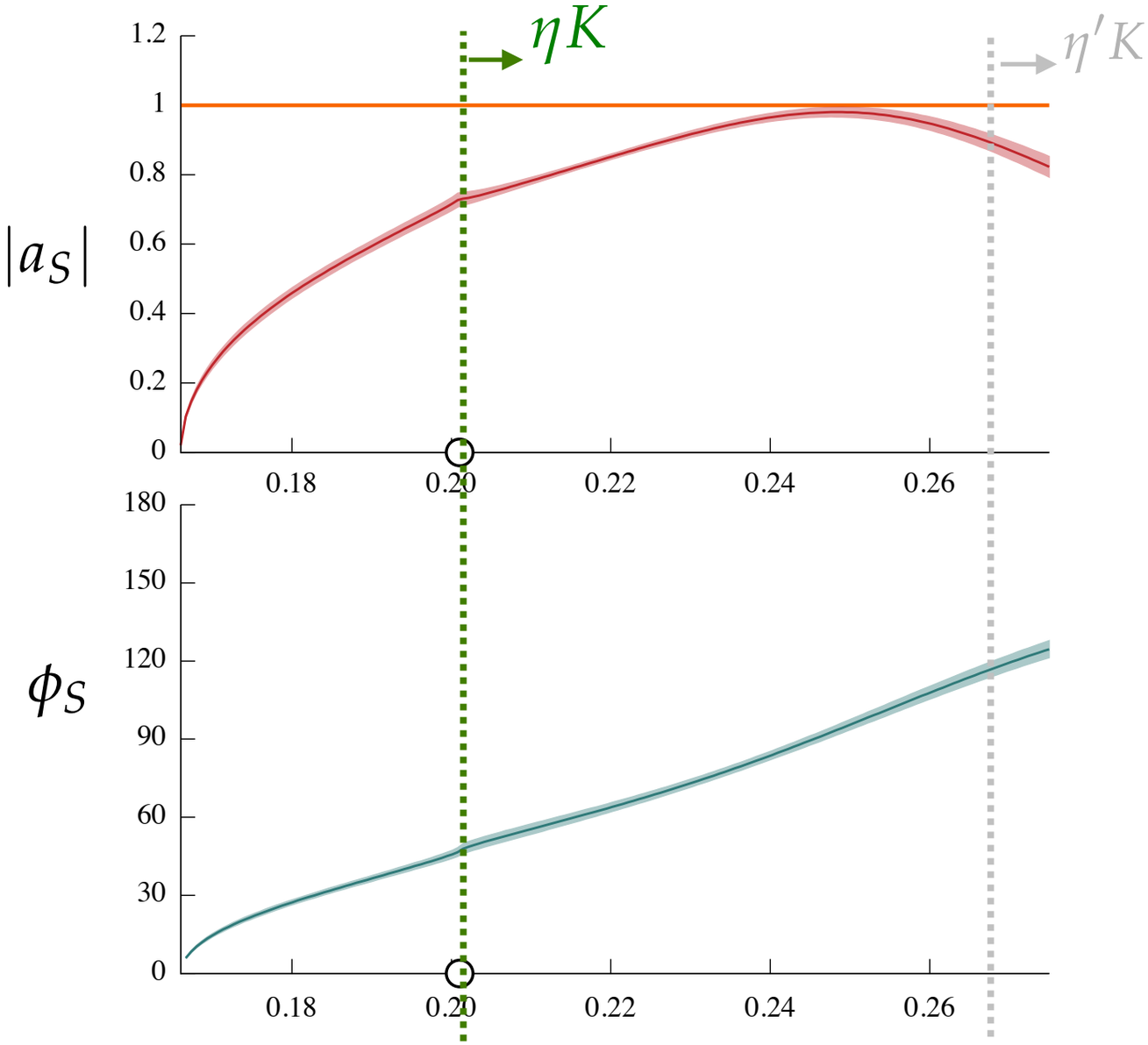
LASS D-WAVE



LASS, NPB296 493

$\pi K/\eta K$ scattering amplitudes

S-WAVE $\pi K \to \pi K$ AMPLITUDE



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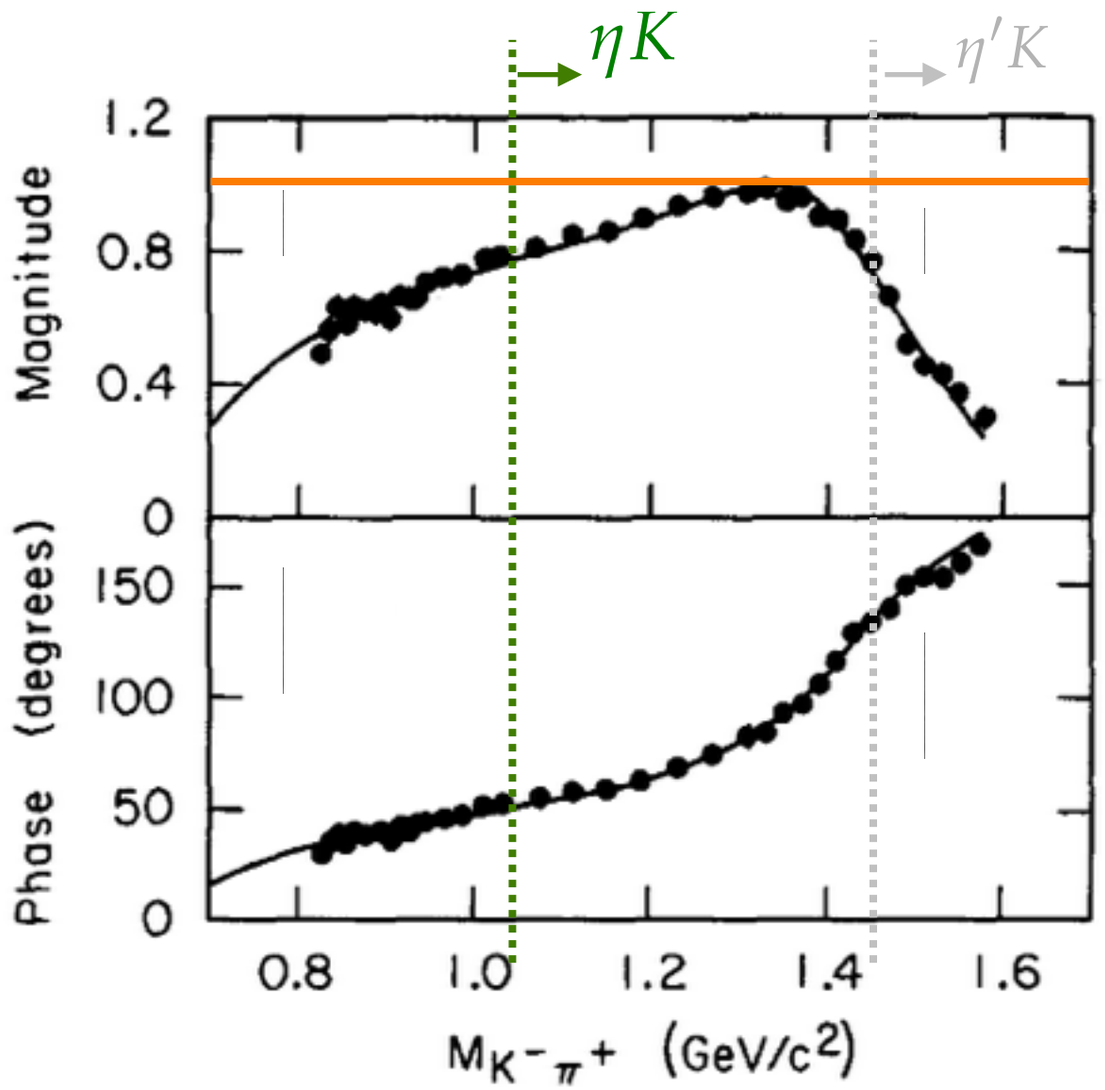
LASS S-WAVE



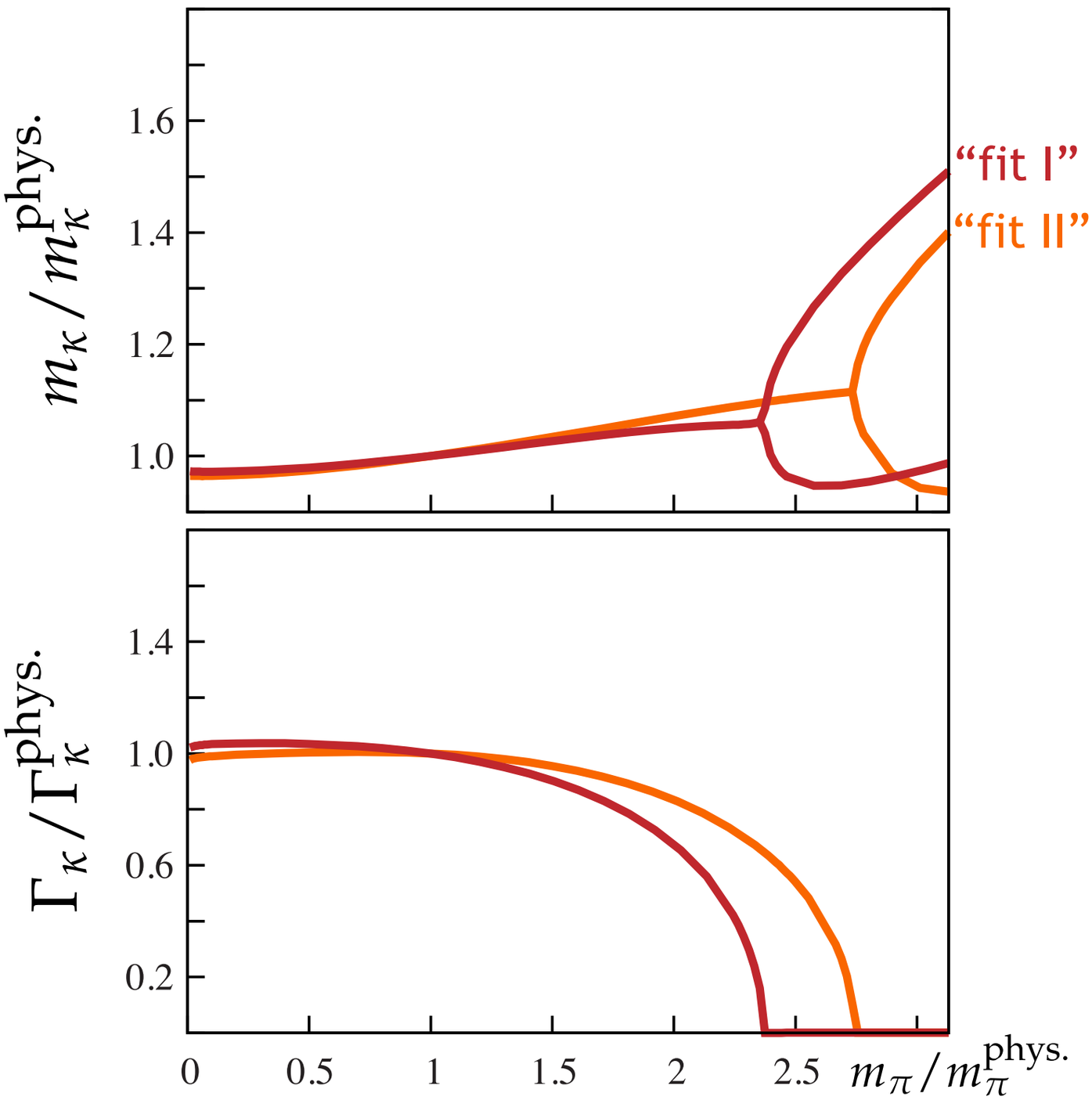
LASS, NPB296 493

κ (kappa) pole with changing quark mass

- unitarized $SU(3)_F$ chiral perturbation theory

Nebreda & Pelaez
PRD81 054035 (2010)

$$\sqrt{s_0} = m + \tfrac{i}{2}\Gamma$$



- resonance poles become virtual bound states somewhere near $m_\pi \sim 2.5\ m_\pi^{phys}$

Descotes-Genon

$$\sqrt{s_0} = 660(20) + \tfrac{i}{2}\,550(25)\,\text{MeV}$$

future directions ?

- near future
  - more two-body meson systems $\pi\eta,\ \overline{K}K$ $\pi\pi,\ \overline{K}K,\ \eta\eta$ $a_0(980)$ $\sigma,\ f_0(980)\ldots$
  - and also charmed systems $D\pi$ DK $D\overline{D}$ *WITH EUROPEAN COLLABORATORS*
  - 'quasi' two-body systems $\pi\rho$ $\pi\omega$ (ρ,ω stable at heavier quark masses)
  - external currents $\gamma\pi \to \pi\pi$ (e.g. form-factor of unstable ρ) *RAUL, CHRISTIAN, DAVE*
- needs a little more development
  - meson-baryon scattering $\pi N,\ \eta N\ldots$ $N^\star, \Delta^\star$
  - three meson systems (no complete formalism yet)

arXiv:1406.4158 [hep-ph]

original arXiv submission results were contaminated by a bug, now fixed

corrected results on the arXiv & soon to appear in PRL

PHYSICAL REVIEW LETTERS

Resonances in Coupled πK-ηK Scattering from Quantum Chromodynamics

Jozef J. Dudek,[1,2,*] Robert G. Edwards,[1] Christopher E. Thomas,[3] and David J. Wilson[2]

(for the Hadron Spectrum Collaboration)

[1]*Theory Center, Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[2]*Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA*
[3]*DAMTP, University of Cambridge, Cambridge CB3 0WA, United Kingdom*

(Received 17 June 2014; revised manuscript received 15 July 2014)

Using a first-principles calculation within quantum chromodynamics, we are able to determine a pattern of strangeness = 1 resonances that appear as complex singularities within coupled πK-ηK scattering amplitudes. We make use of numerical computation in the lattice discretized approach to the quantum field theory with light quark masses corresponding to $m_\pi \sim 400$ MeV and at a single lattice spacing. The energy dependence of scattering amplitudes is extracted through their relationship to the discrete spectrum in a finite volume, which we map out in unprecedented detail.

DOI: PACS numbers: 14.40.Df, 12.38.Gc, 13.25.Es

a longer paper with details will appear on arXiv soon …