Strong interactions for the precision frontier in particle and nuclear physics

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Colloquium

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Outline

Introduction: why the strong interactions are difficult

Strong and weak interactions

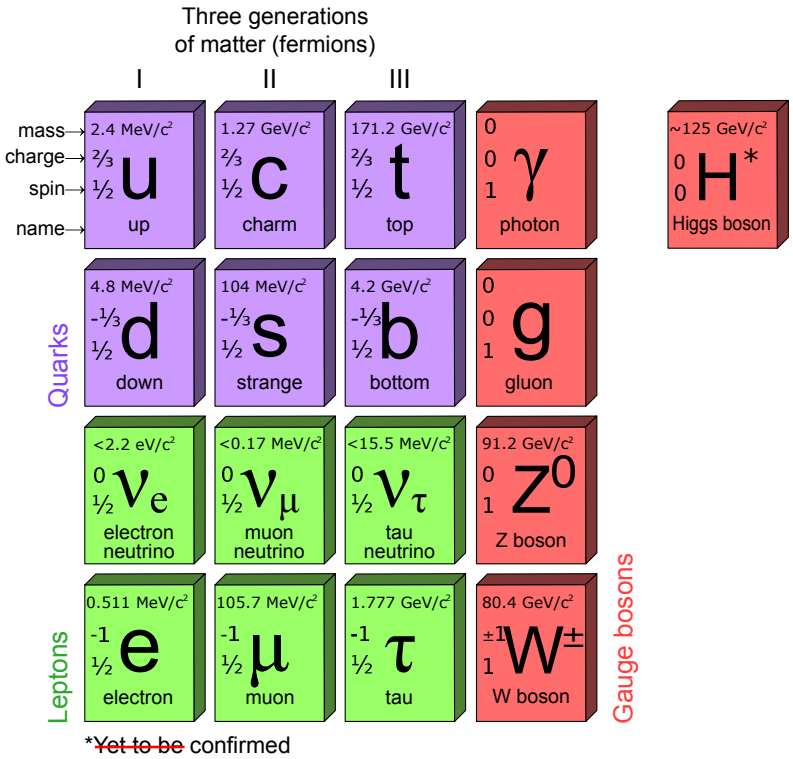
- Spectroscopy, CP violation, and Dalitz plots

Strong interactions and electromagnetism

- The anomalous magnetic moment of the muon

Summary / Outlook

The Standard Model of particle physics



- matter particle:
  quarks and leptons
- force carriers: gauge bosons
- how did we find these?
  how to find something beyond?

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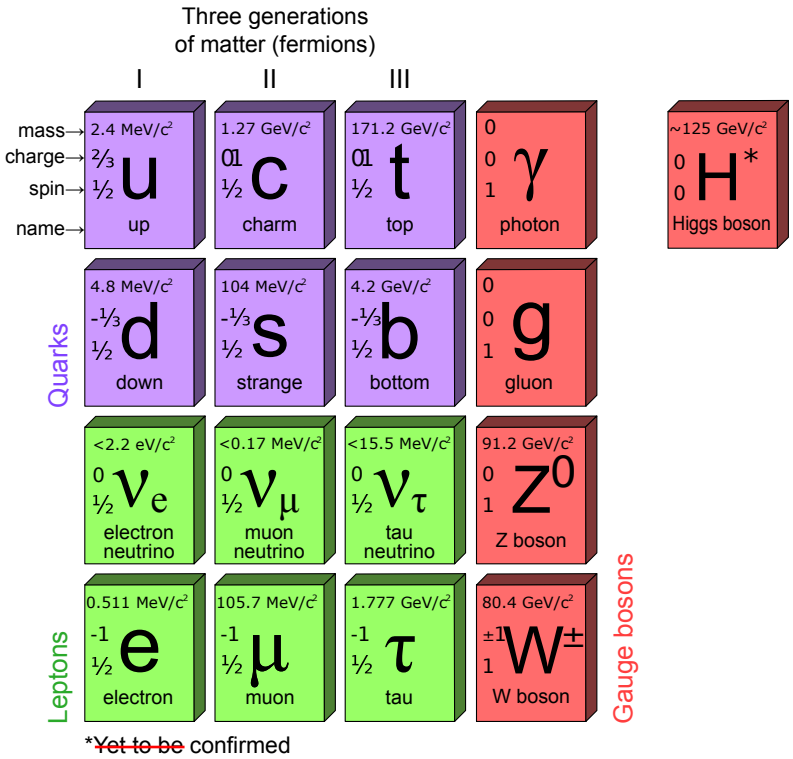

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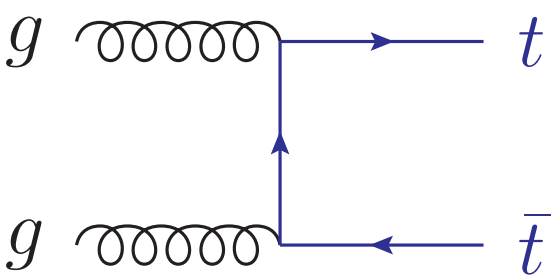

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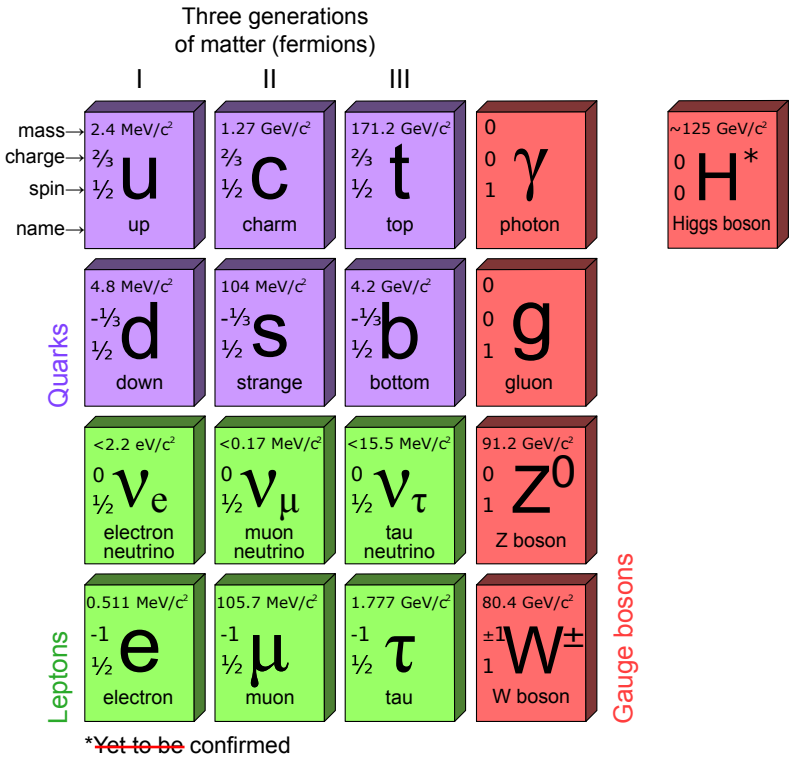

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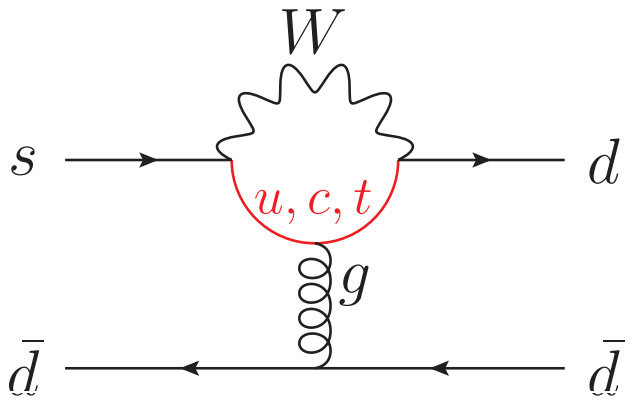
- discoveries in precision experiments: effects of virtual particles

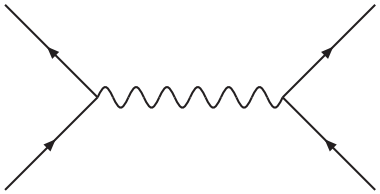


Why the strong interactions are difficult

Perturbation theory and coupling constants

- expand amplitudes in powers of coupling constants α

$$\text{scattering amplitude} \;\propto\; \alpha(\ldots)$$



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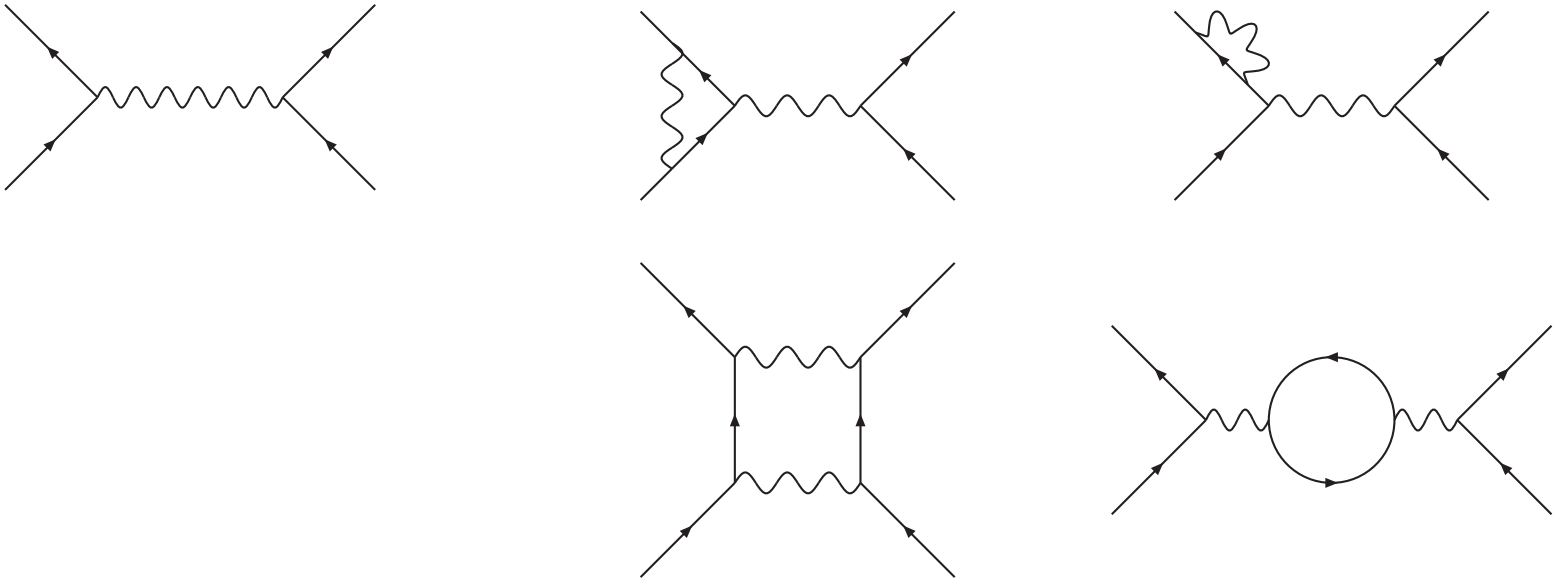
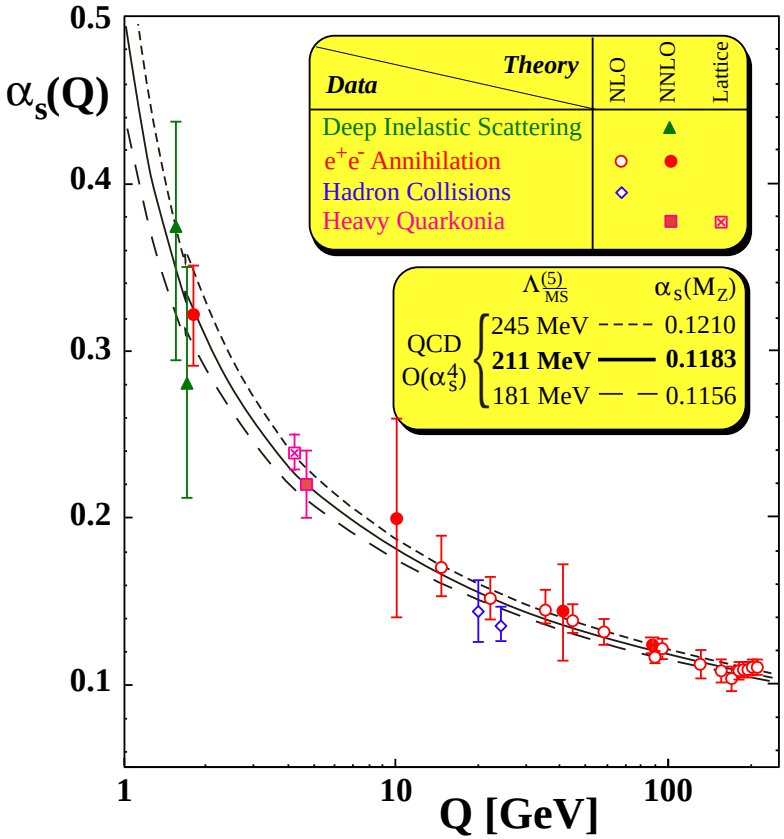
$$\text{scattering amplitude} \;\propto\; \alpha(\ldots) \;+\; \alpha^2(\ldots)$$



- works well for electromagnetic and weak interactions: $\alpha \approx 10^{-2}$

Why the strong interactions are difficult

The running coupling constant in Quantum Chromodynamics



- asymptotic freedom at high energies ("weak QCD")
- confinement at low energies ("strong QCD"):

  no quarks + gluons, only (colour-neutral) hadrons

  baryons (rgb) + mesons ($r\bar{r}$)

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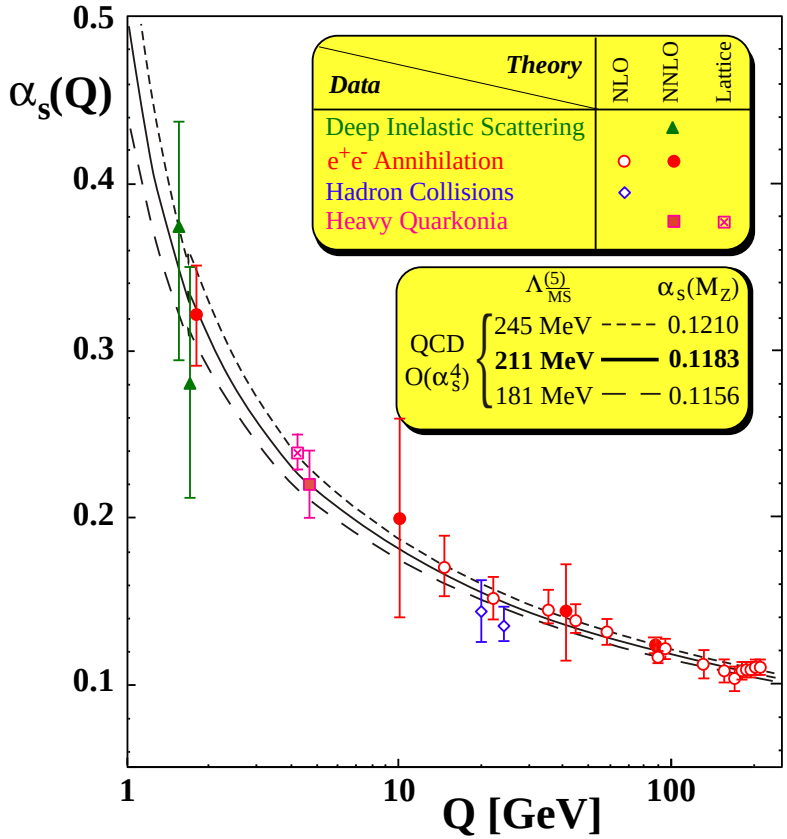

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Perturbative QCD not applicable for

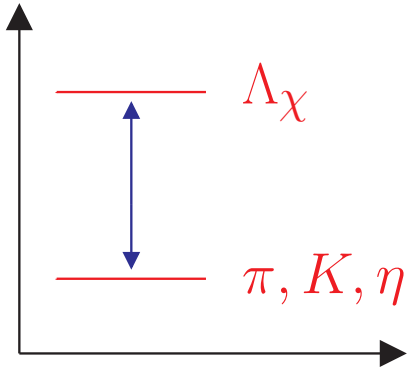
- hadron spectroscopy
- hadron scattering, decays
- nuclear physics

Why the strong interactions are difficult

Alternative methods

Effective field theories

$\longrightarrow$ symmetries, scale separation



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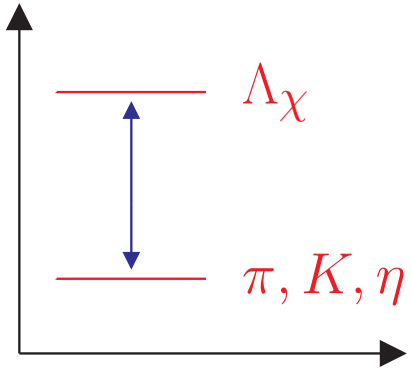
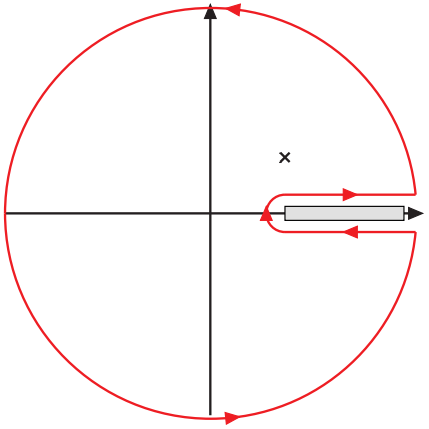


Dispersion relations

$\longrightarrow$ analyticity, unitarity, crossing symmetry



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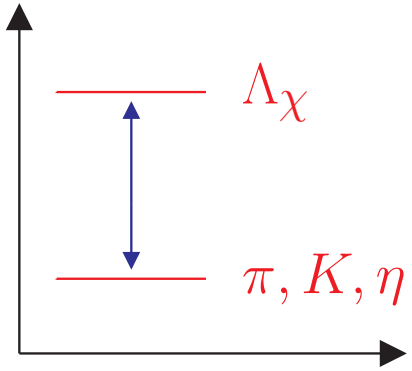

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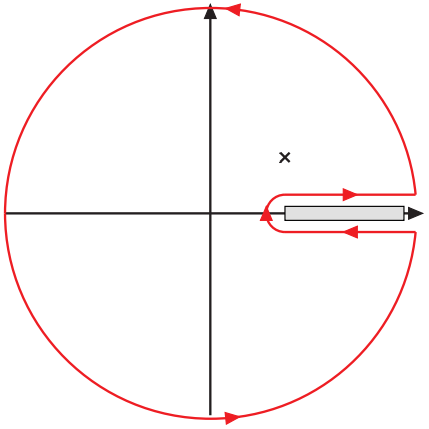
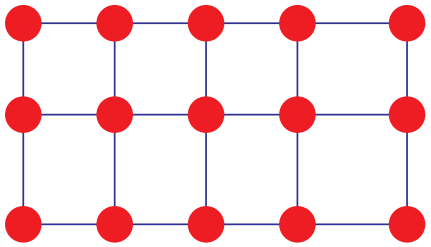


Lattice QCD

$\longrightarrow$ solve discretized version of QCD numerically



Part I:

Strong and weak interactions:

Spectroscopy, CP violation,

and Dalitz plots

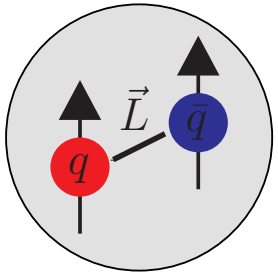
Modern spectroscopy

- mesons in the quark model: $q\bar{q}$ states

  $\longrightarrow$ certain quantum numbers J^{PC} not allowed:

  0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, ... "exotics"

- lattice QCD: 1^{-+} state $\approx$ 1.3 GeV above the ρ



Dudek et al. 2013

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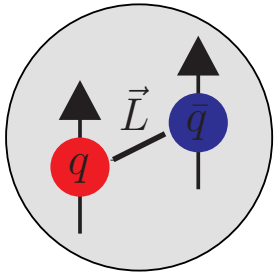

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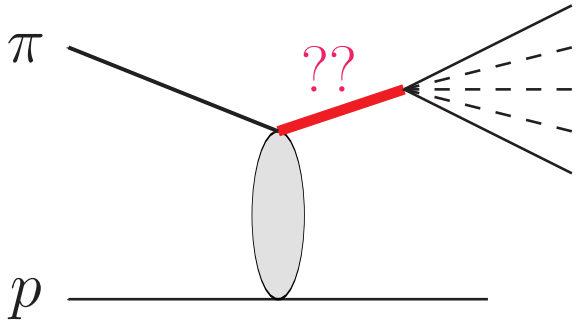
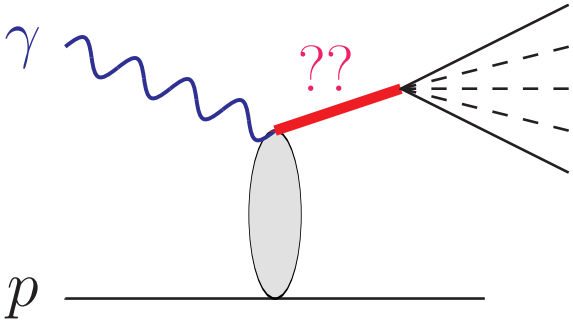
- experiments:



COMPASS@CERN



CLAS, GlueX@JLab

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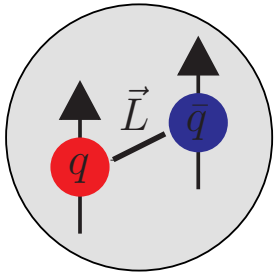

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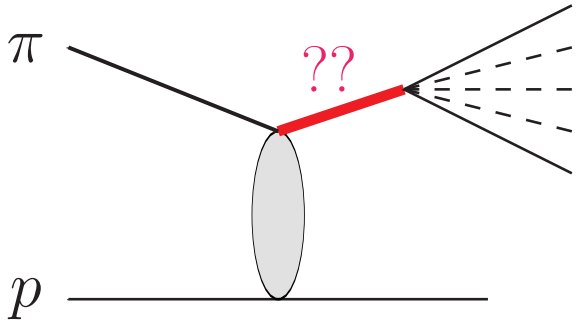

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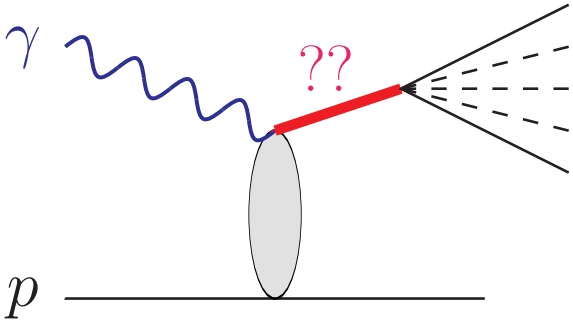

CLAS, GlueX@JLab

Things to control using dispersion relations:

- (exotic) phases interfere with known reference phases
- effects of three-/many-body final-state interactions?
- resonances $\hat{=}$ poles in the complex scattering-energy plane

CP violation in weak interactions

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- in the Standard Model: embedded in the weak interactions, Cabibbo–Kobayashi–Maskawa matrix

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CP-violation given by a complex phase in V

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  CP-violation given by a complex phase in V
- Standard-Model CP violation too small

  $\longrightarrow$ search for new/unconventional sources of CP violation
  - ▷ charm D / beauty B decays
  - ▷ electric dipole moments (proton/neutron/deuteron. . .)

CP violation in weak interactions

CP violation in partial widths $\Gamma(P \to f) \neq \Gamma(\bar{P} \to \bar{f})$

- at least two interfering decay amplitudes
- different weak (CKM) phases
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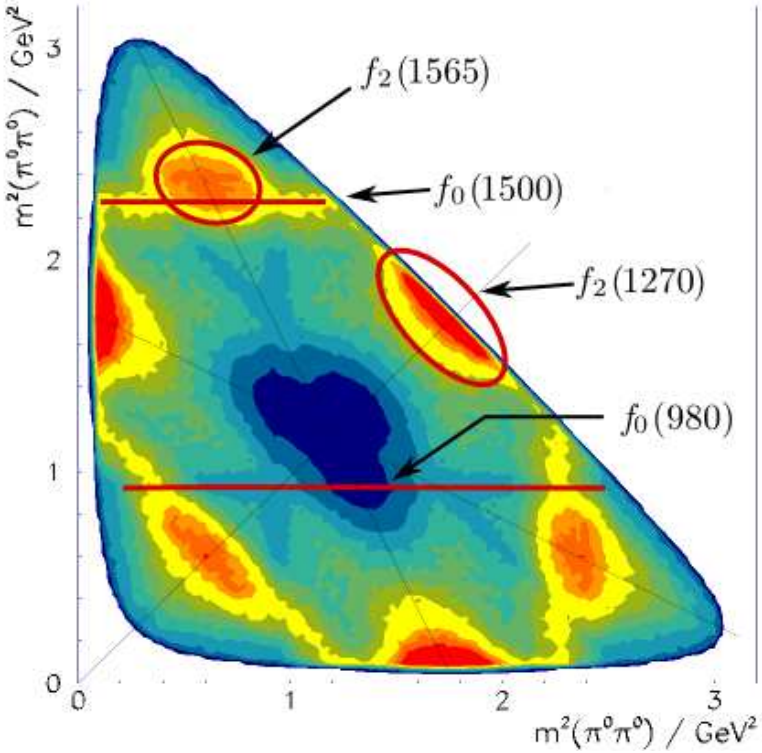
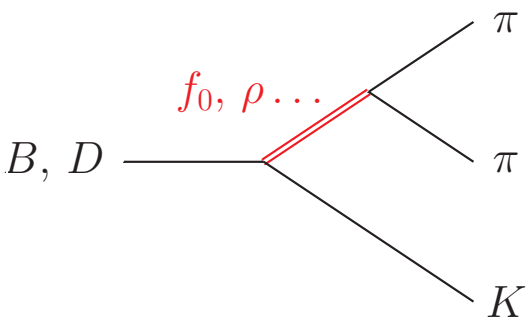
three-body decays: $D \to 3\pi,\ \pi\pi K$

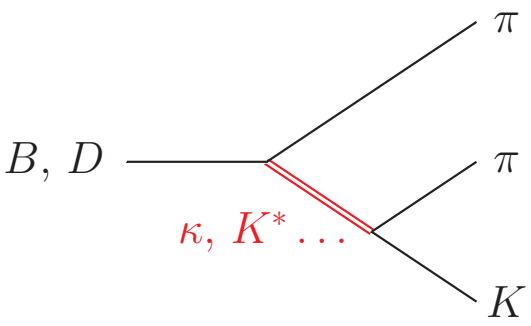
- Dalitz plot $\hat{=}$ density distribution in two kinematical variables
- resonances $\longrightarrow$ rapid phase variation enhances CP-violation in parts of the decay region

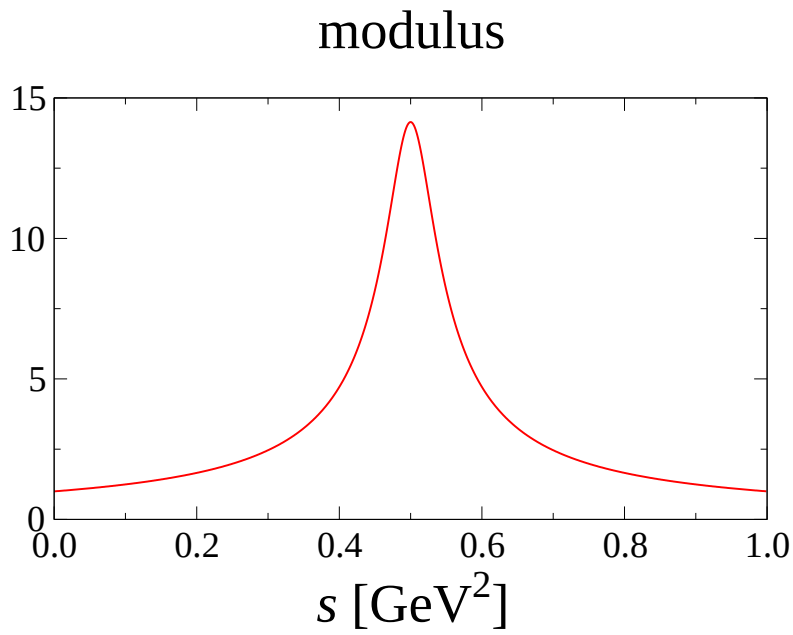


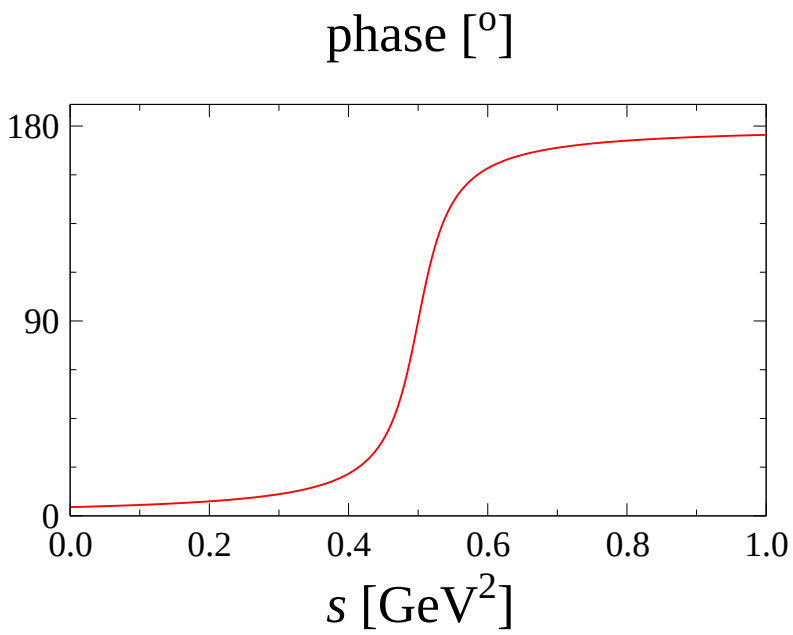
Amplitude analyses in Dalitz plots

The traditional picture: isobar model / Breit–Wigner resonances









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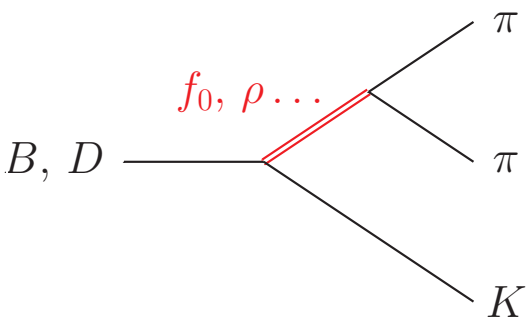
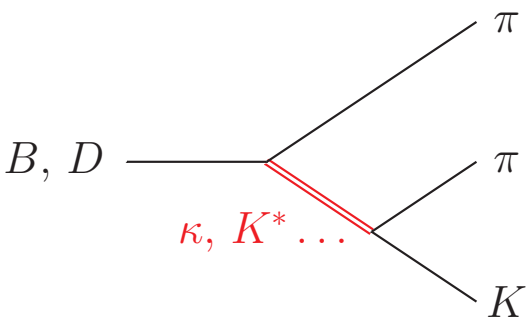
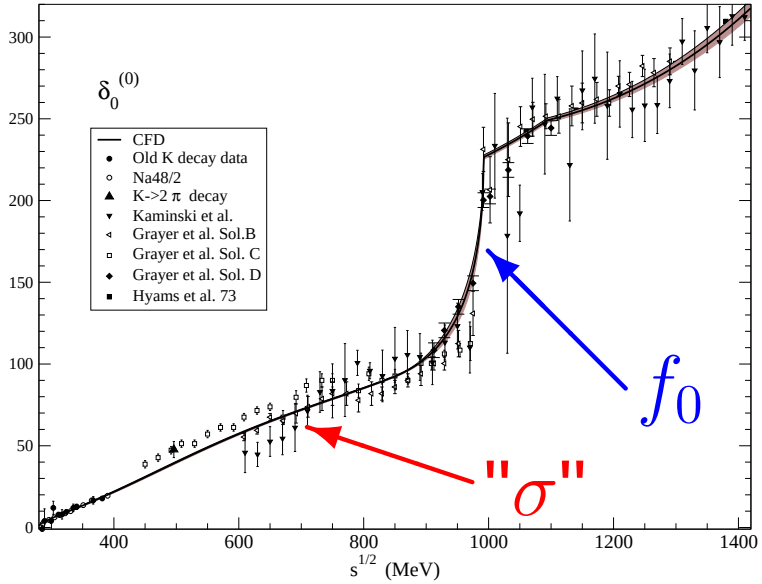




. . . so what's not to like?

- some resonances don't look like Breit–Wigners at all!

  $\longrightarrow$ use exact scattering phase shifts instead



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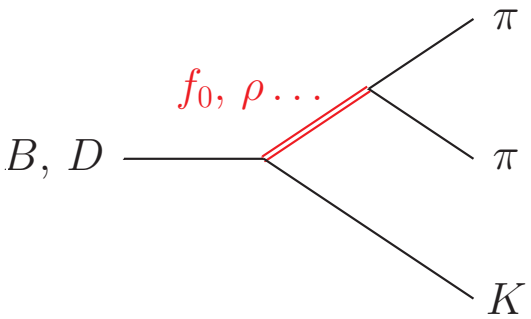

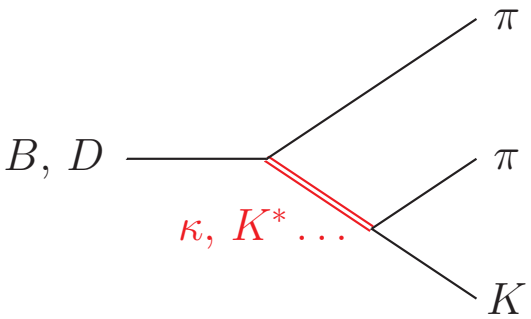

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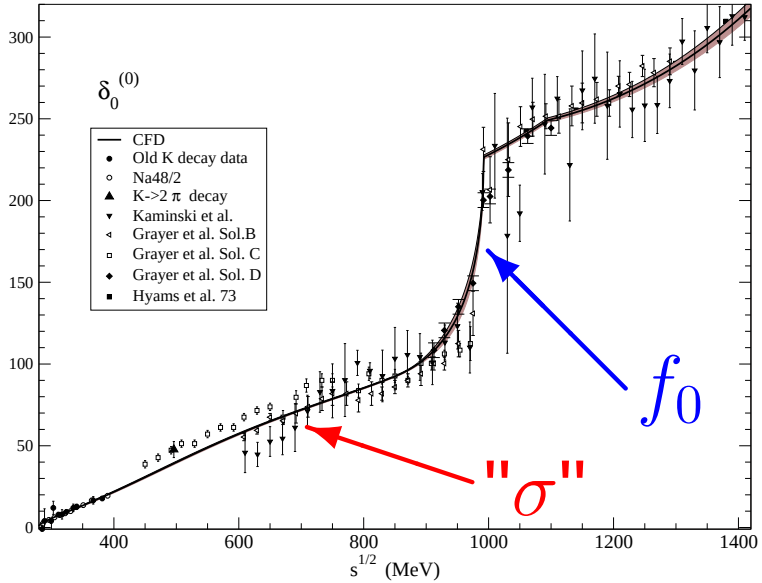
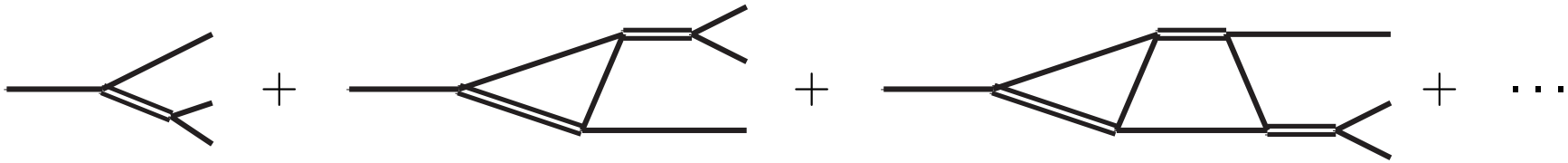
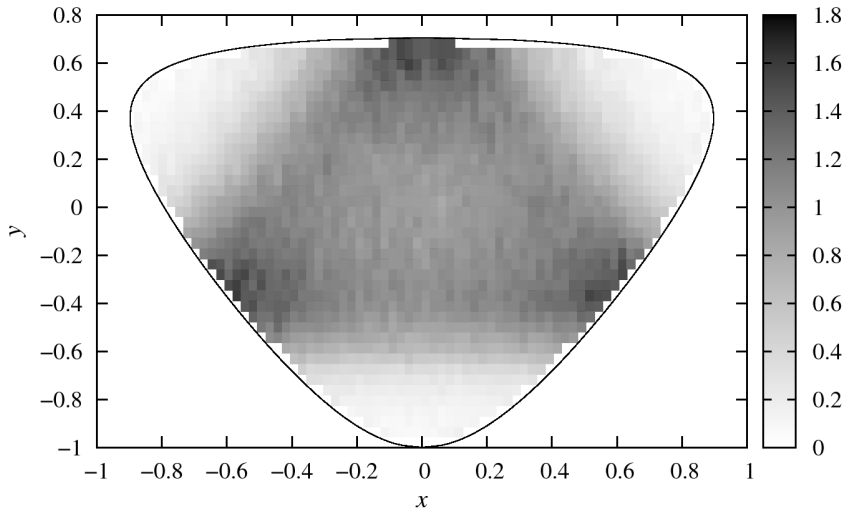
- 3-particle rescattering





A simple Dalitz plot: $\phi \to 3\pi$



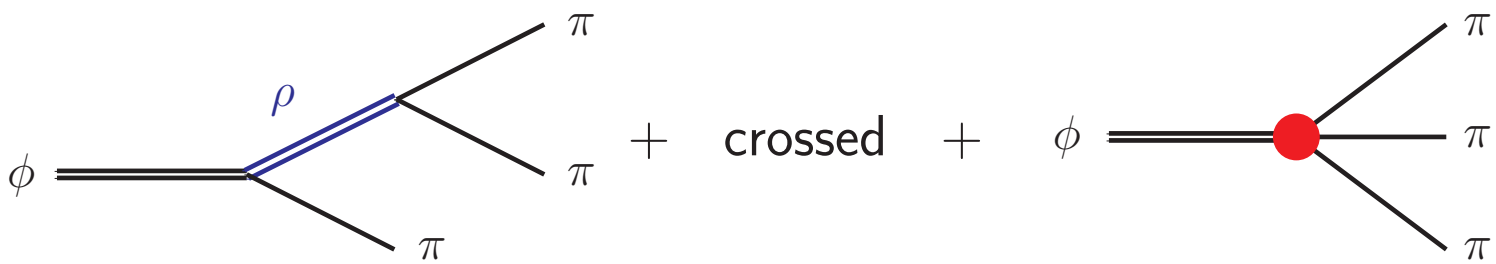
- 2×10^6 events in 1834 bins

  KLOE 2003

- analyzed in terms of:

  sum of 3 Breit–Wigners ($\rho^{\pm}$, ρ^0)

  \+ constant background term



Problem:

$\longrightarrow$ unitarity fixes Im/Re parts

$\longrightarrow$ adding a contact term destroys this relation

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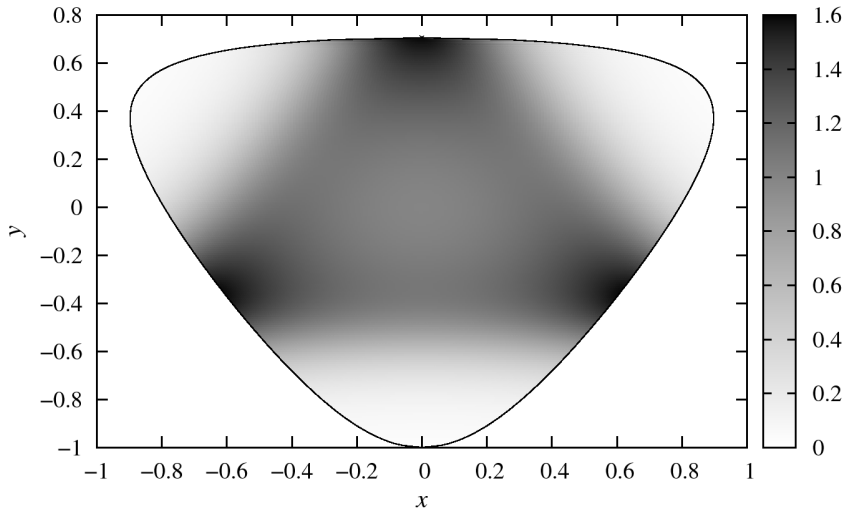

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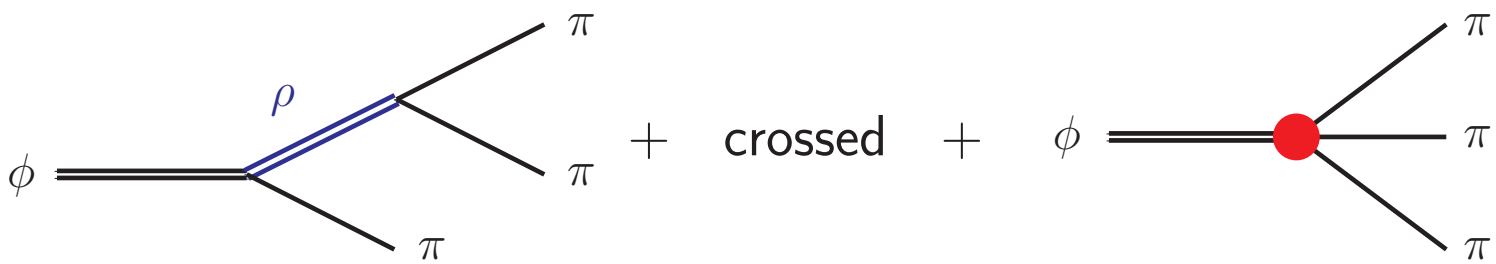

Problem:

$\longrightarrow$ unitarity fixes Im/Re parts

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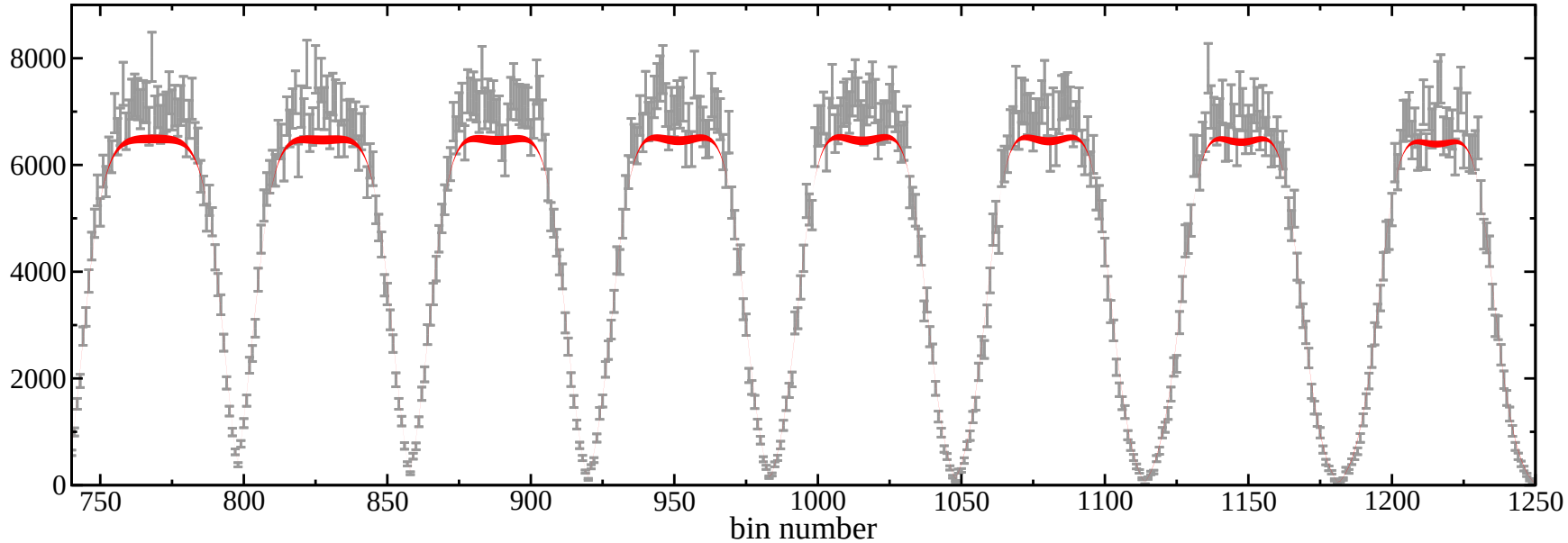
$\longrightarrow$ reconcile data with dispersion relations? Niecknig, BK, Schneider 2012

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



χ^2/ndof	$1.7 \ldots 2.1$

$\longrightarrow$ pairwise interaction only (with correct $\pi\pi$ scattering phase)

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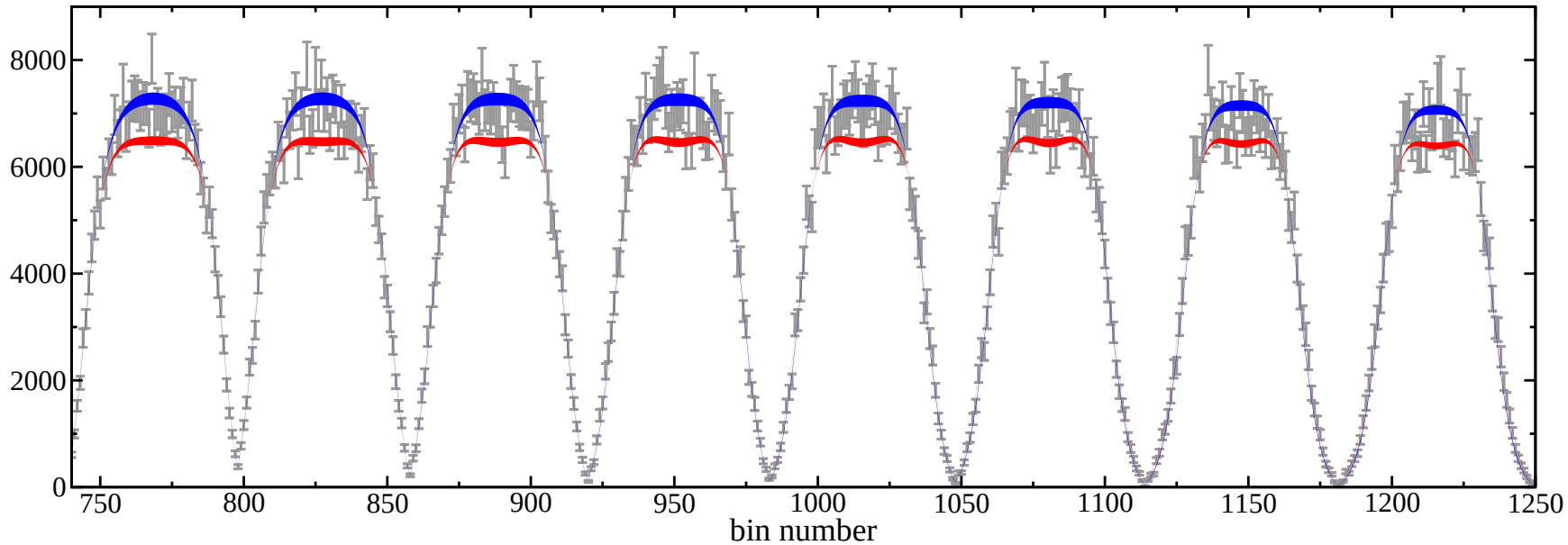

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$\longrightarrow$ full 3-particle rescattering, only overall normalization adjustable

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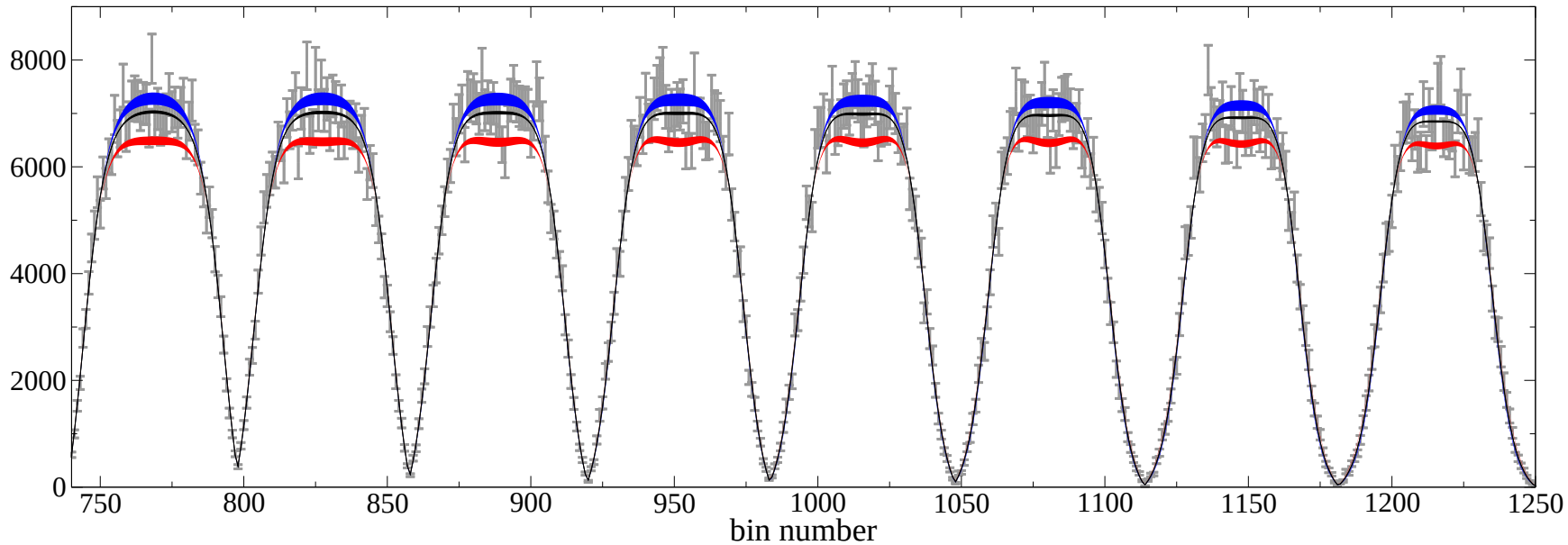

χ^2/ndof	$1.7\ldots2.1$	$1.2\ldots1.5$	1.0

$\longrightarrow$ full 3-particle rescattering, 2 adjustable parameters
(additional "subtraction constant" to suppress inelastic effects)

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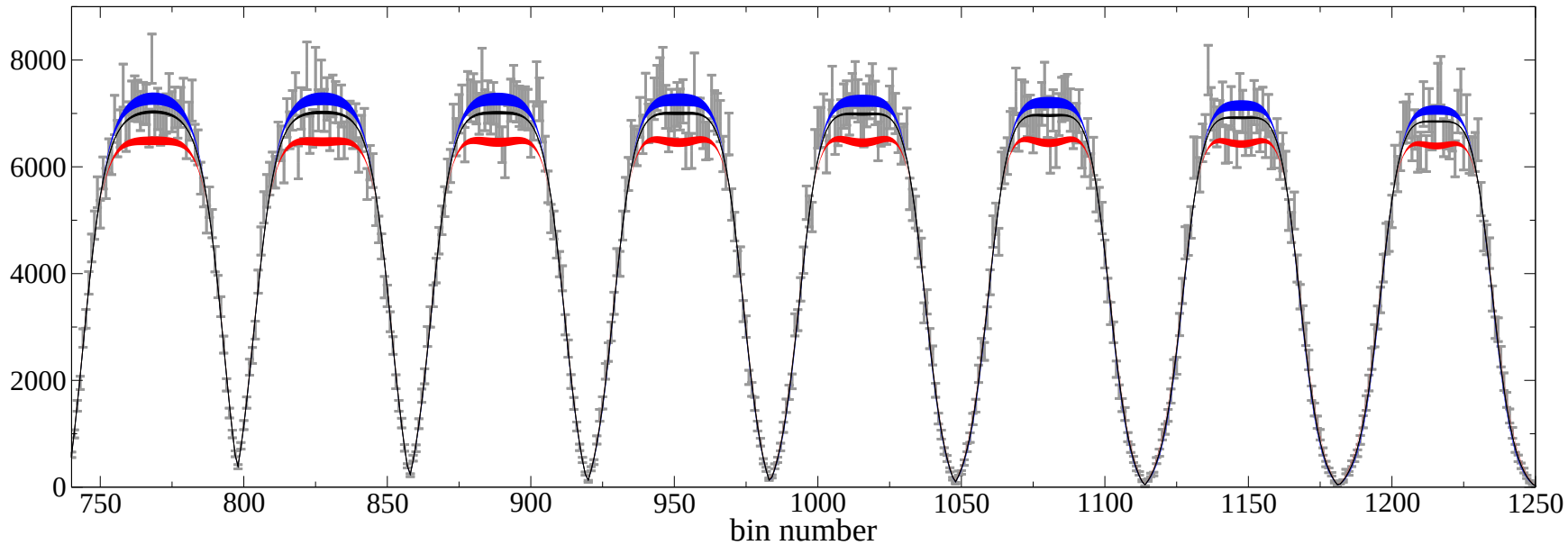

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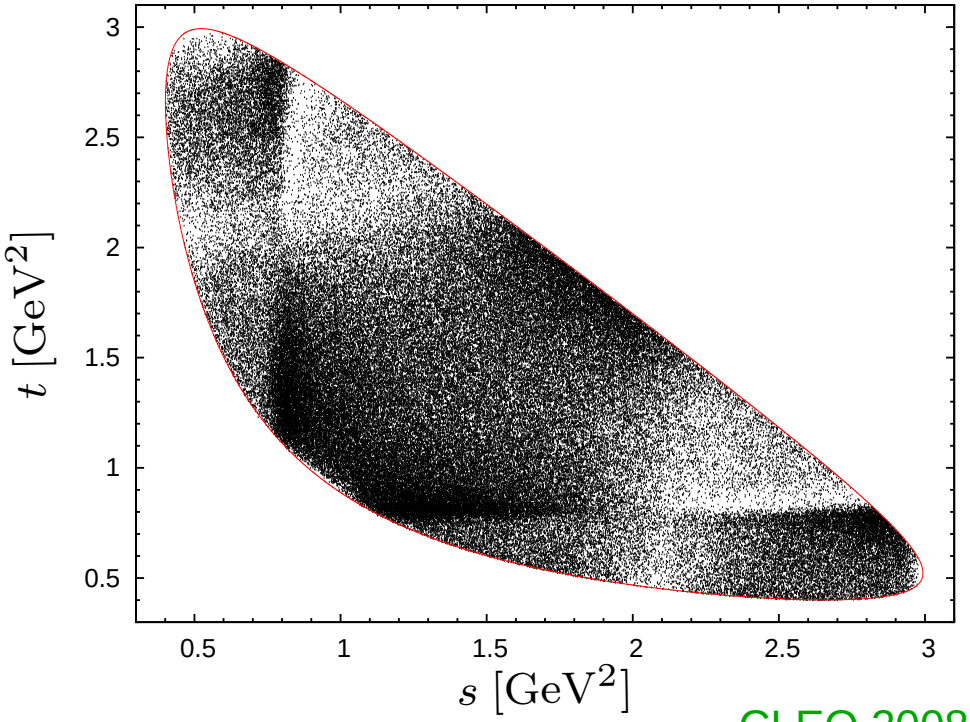
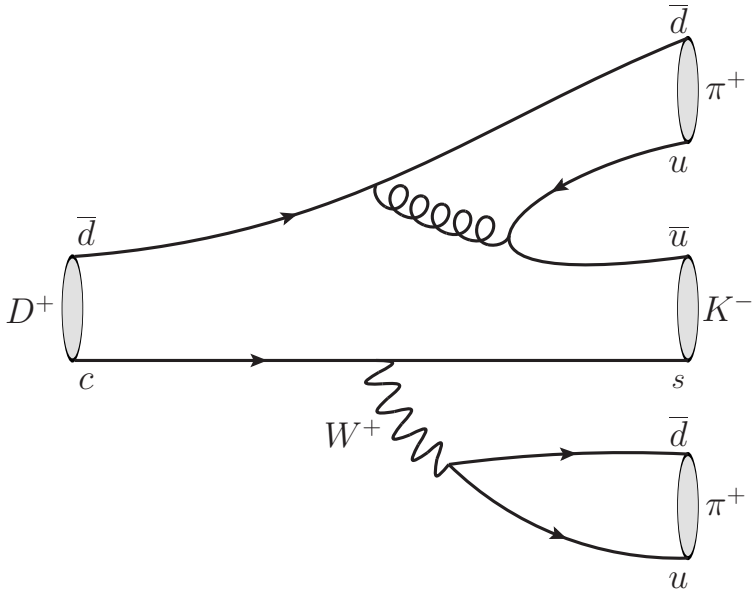
- perfect fit respecting analyticity and unitarity possible
- contact term emulates neglected rescattering effects
- no need for "background" — inseparable from "resonance"

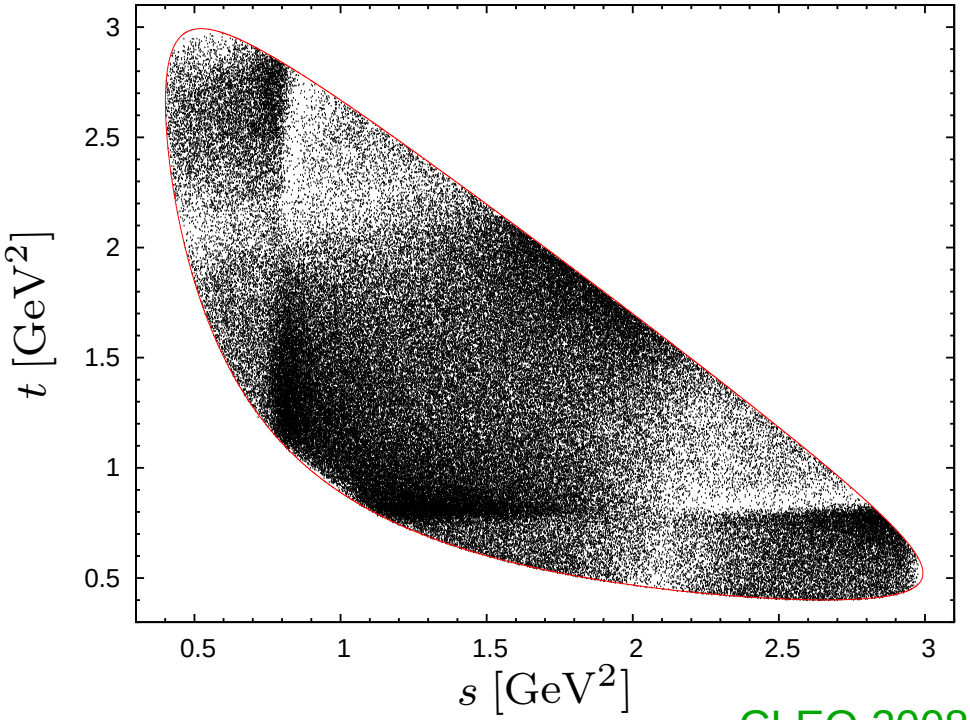
Heavier decays: $D^+ \to \pi^+\pi^+K^-$



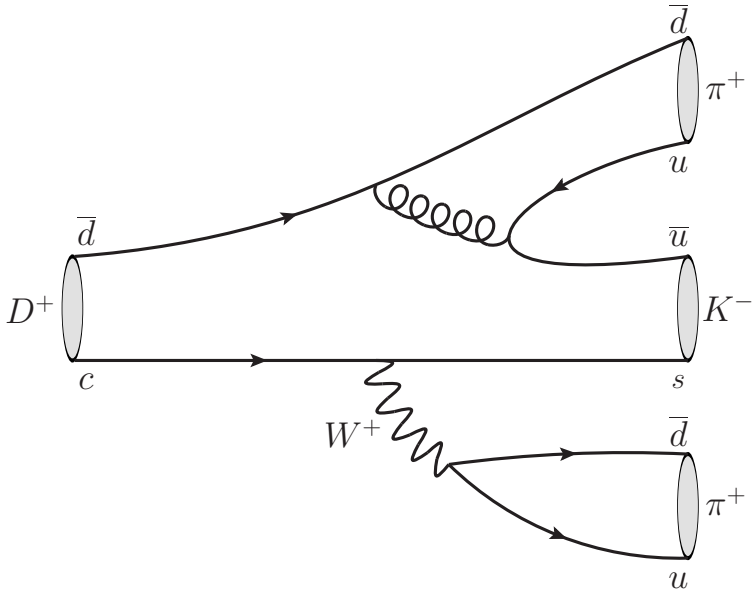
CLEO 2008



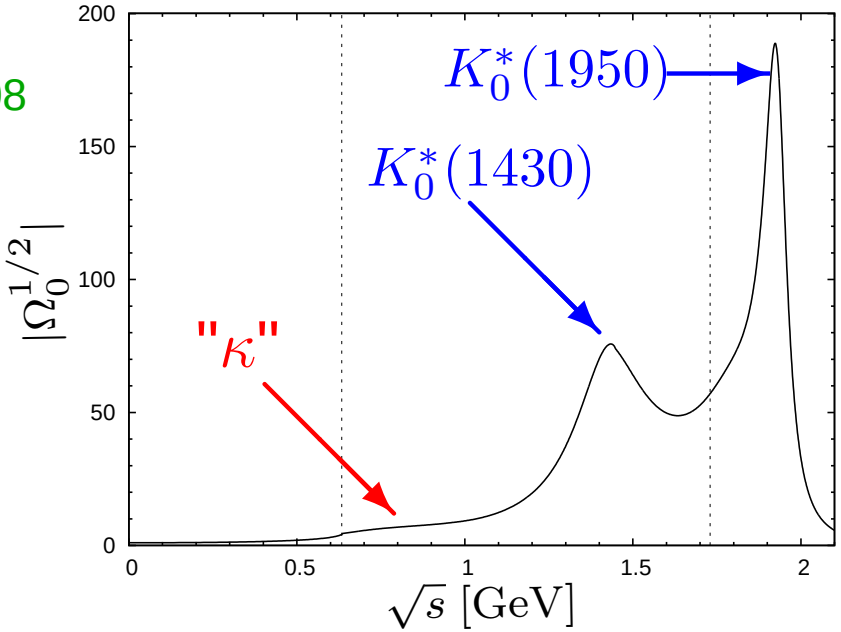
Heavier decays: $D^+ \to \pi^+\pi^+K^-$



CLEO 2008



pion–pion	$S^2_{\pi\pi}$	$P^1_{\pi\pi}$
pion–kaon	$S^{1/2}_{\pi K}$	$P^{1/2}_{\pi K}$
	$S^{3/2}_{\pi K}$	$P^{3/2}_{\pi K}$



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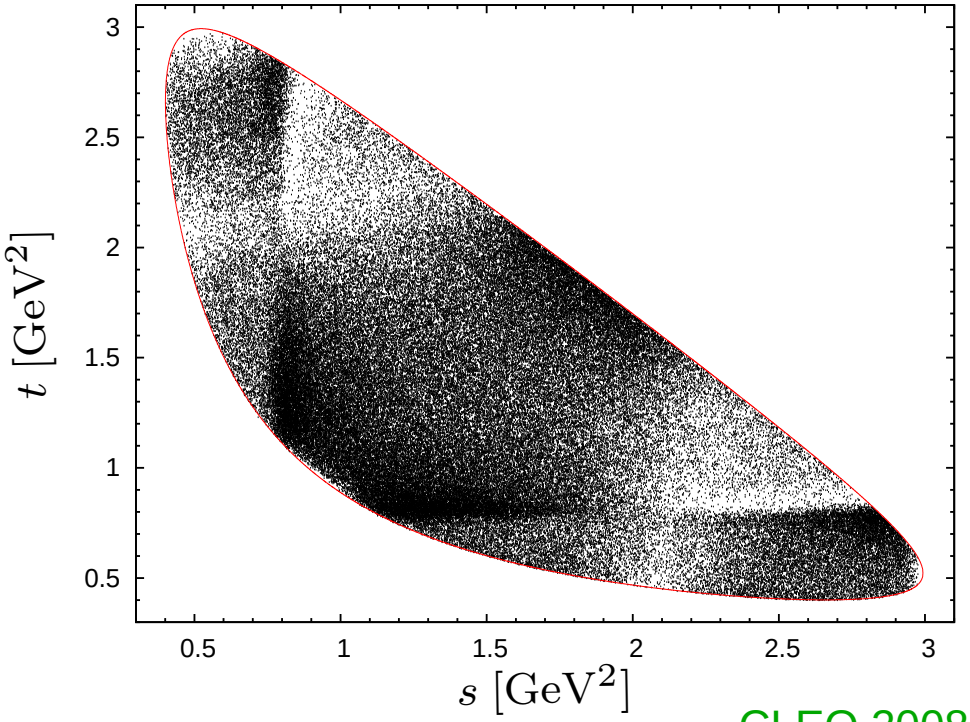

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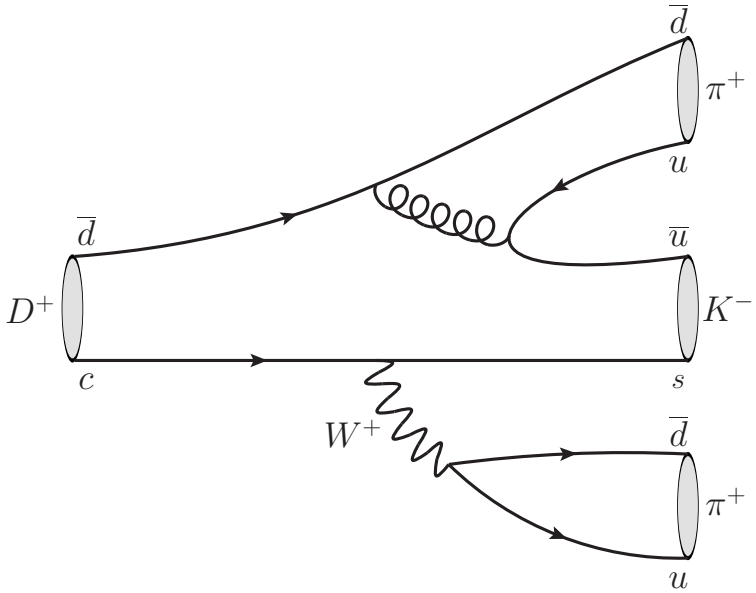

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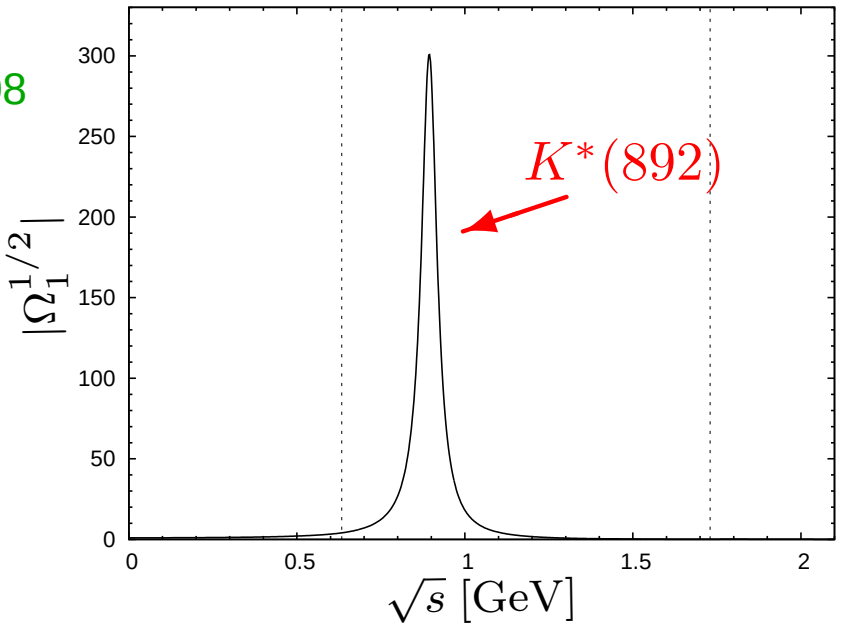

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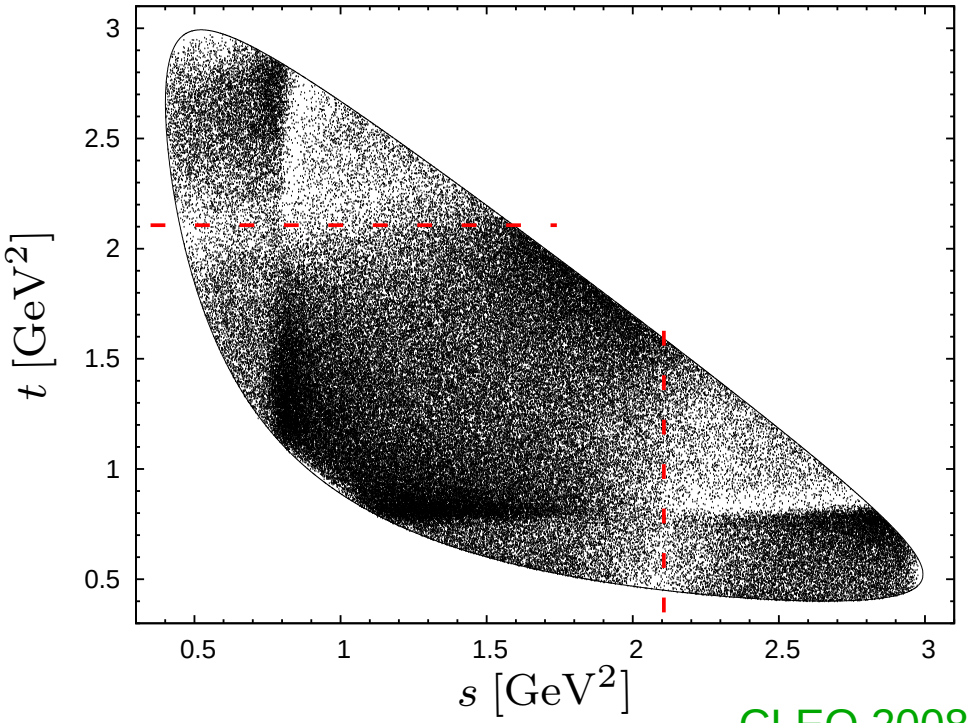

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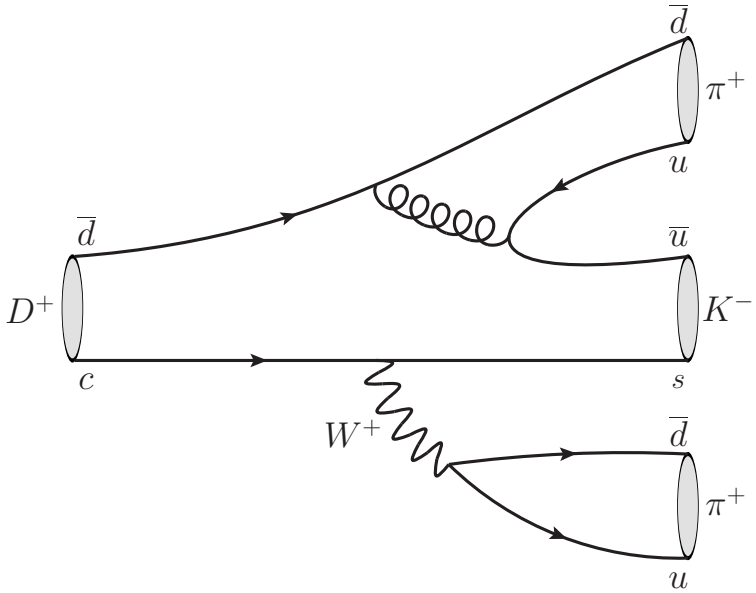

Fit in the elastic region:

- "improved isobar model" $\chi^2/\mathrm{ndof} \approx 1.4$
- full three-body rescattering $\chi^2/\mathrm{ndof} \approx 1.1$

$\longrightarrow$ visible improvement similar to $\phi \to 3\pi$ Niecknig, BK *in progress*

Part II:

Strong interactions and electromagnetism:

The anomalous magnetic moment of the muon

The anomalous magnetic moment of the muon

- gyromagnetic ratio: magnetic moment $\leftrightarrow$ spin

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

- Dirac theory for spin-1/2 fermions: $g_\mu = 2$
  rad. corr.: $g_\mu = 2(1 + a_\mu)$, a_μ "anomalous magnetic moment"
- one of the most precisely measured quantities in particle physics

$$a_\mu = (116\,592\,089 \pm 63) \times 10^{-11}$$

BNL E821 2006

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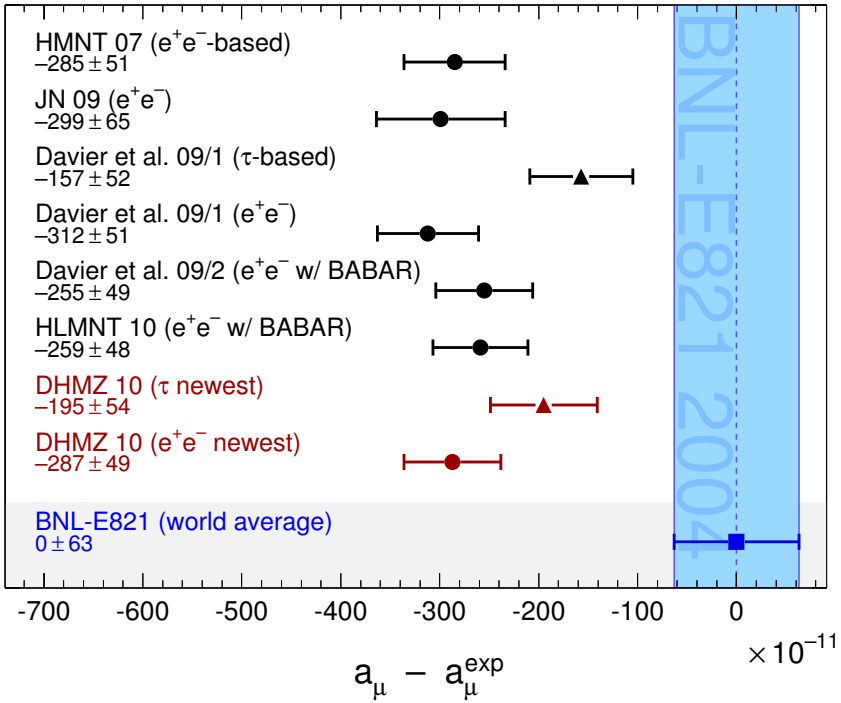

- … and one with a significant (??) deviation from the Standard Model:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (27.6 \pm 8.6) \times 10^{-11}$$

Davier et al. 2011

- new $g - 2$ experiment at Fermilab: reduce experimental error by factor 4

The Standard Model prediction for a_μ

	a_μ $[10^{-11}]$	Δa_μ $[10^{-11}]$
experiment	116 592 089.	63.
QED $\mathcal{O}(\alpha)$	116 140 973.21	0.03
QED $\mathcal{O}(\alpha^2)$	413 217.63	0.01
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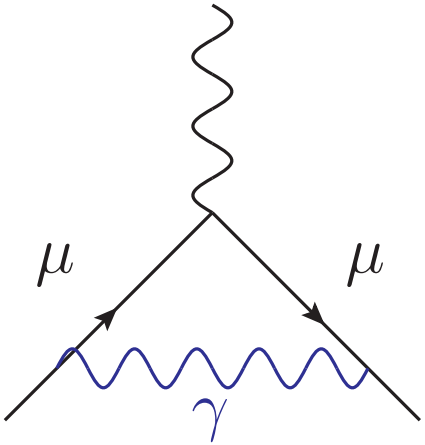

Schwinger 1948

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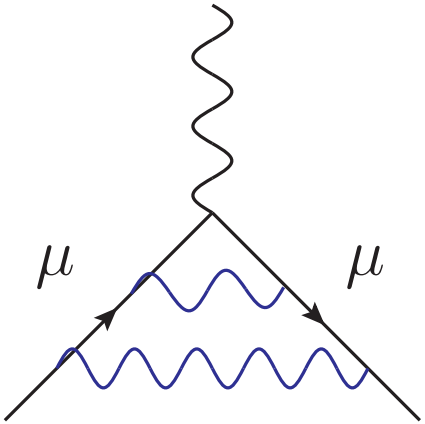

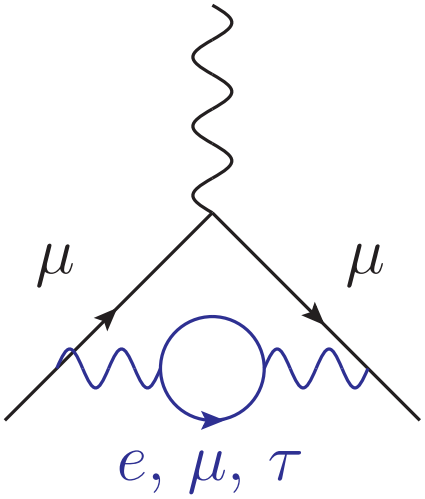

Petermann 1957

Sommerfield 1957

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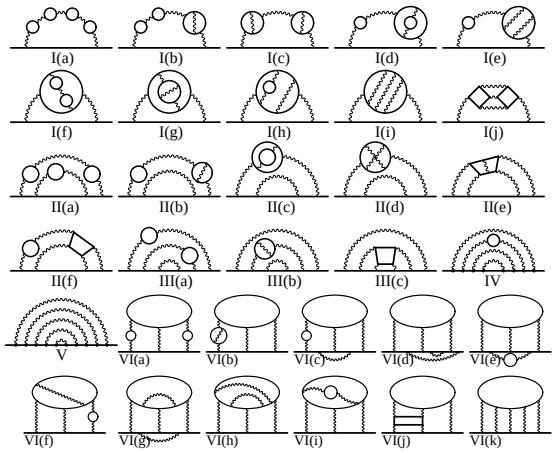

Kinoshita et al. 2012

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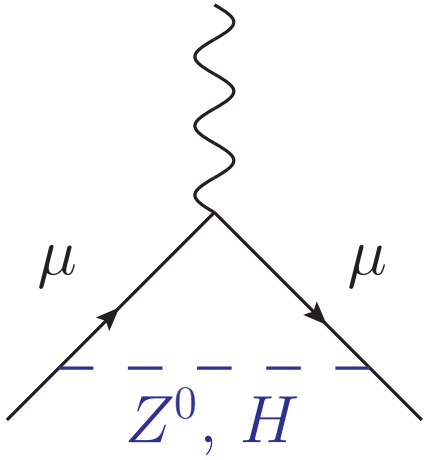

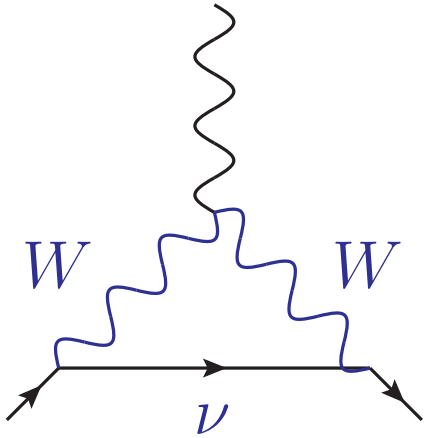

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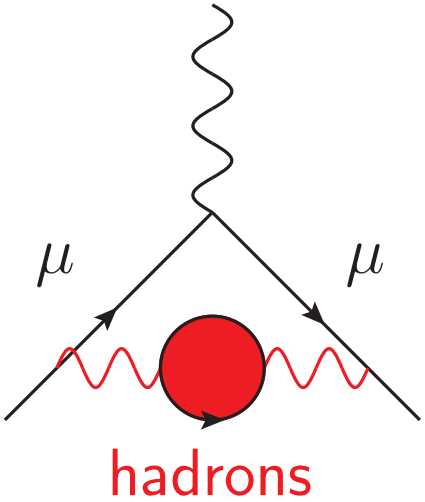

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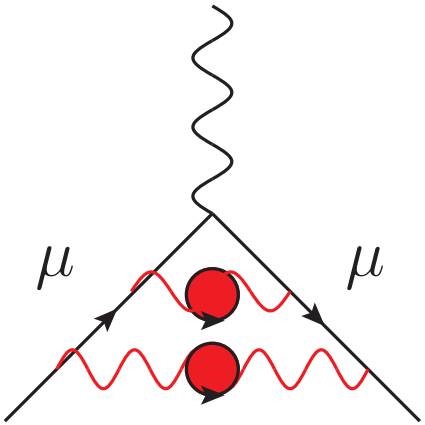

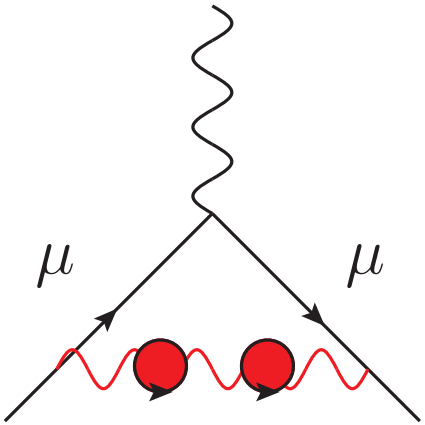

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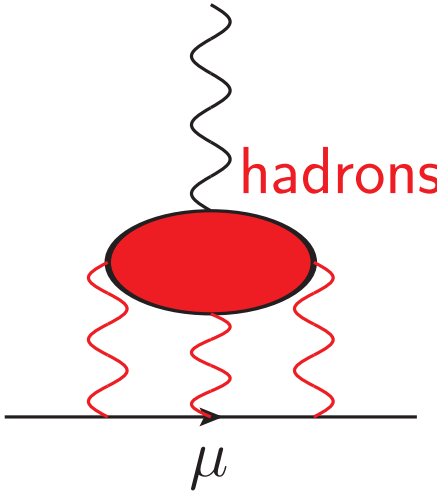

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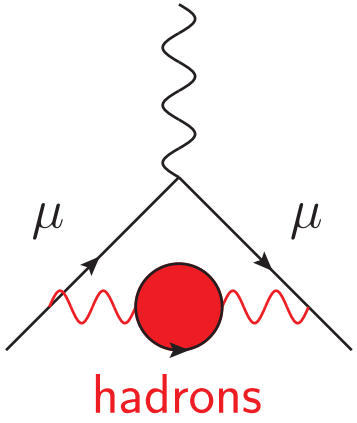
$\longrightarrow$ hadronic part dominates the uncertainty by far!

Jegerlehner, Nyffeler 2009
Davier et al. 2011
and references therein

Hadronic vacuum polarization

- how to control hadronic vacuum polarization?
- characteristic scale set by muon mass
  $\longrightarrow$ this is not a perturbative QCD problem!
- dispersion relations to the rescue:
  use the optical theorem!



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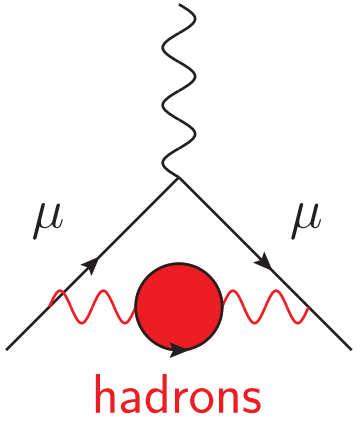
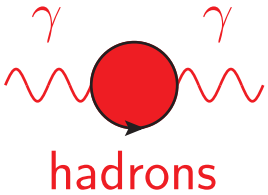
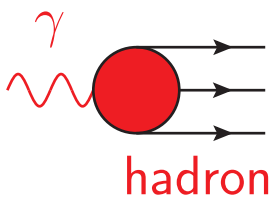


Im 

$\Leftrightarrow$ $\Big|$ 

$\Big|^2$ $\propto \sigma_{\mathrm{tot}}(e^+e^- \to \mathrm{hadrons})$

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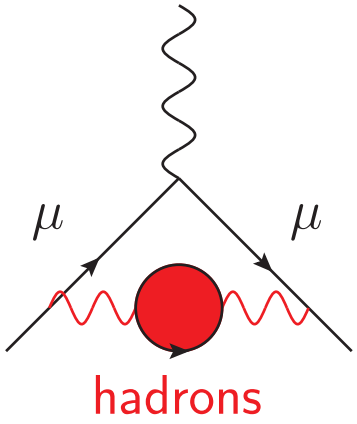

$$a_\mu^{\text{had VP}} \propto \int_{4M_\pi^2}^{\infty} K(s)\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$$

- $K(s)$: kinematical function, for large s: $K(s) \propto 1/s$,
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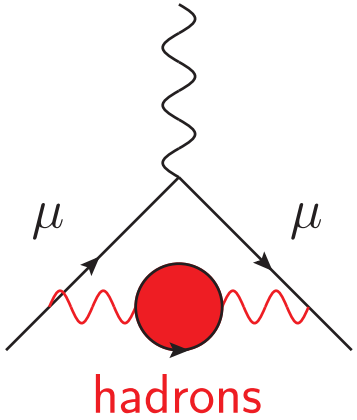

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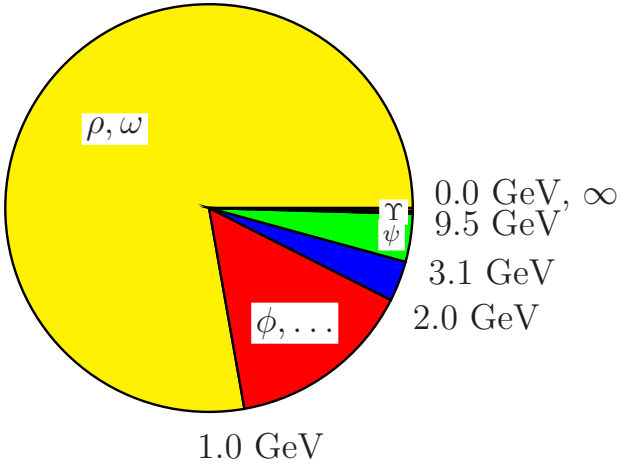
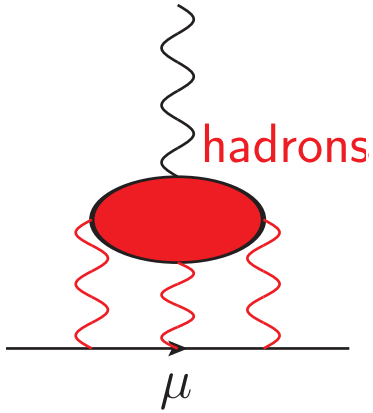
- more than 75% of $a_\mu^{\text{had VP}}$ given by
  energies $s \leq 1\,\text{GeV}^2$ Jegerlehner, Nyffeler 2009
- dominated by $e^+e^- \to \pi^+\pi^-$
  $\longrightarrow$ pion electromagnetic form factor
- well constrained by data KLOE, BABAR...



Hadronic light-by-light scattering

- hadronic light-by-light soon to dominate Standard Model uncertainty



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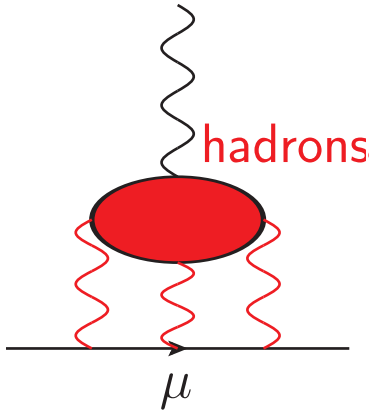
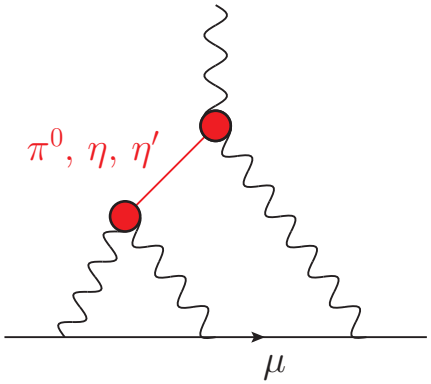
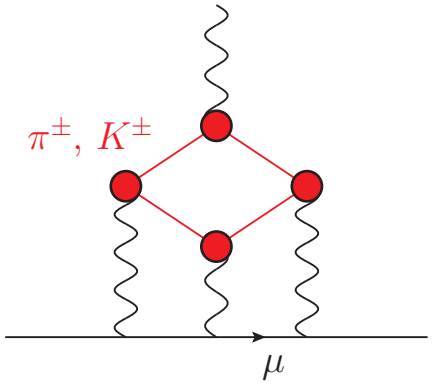


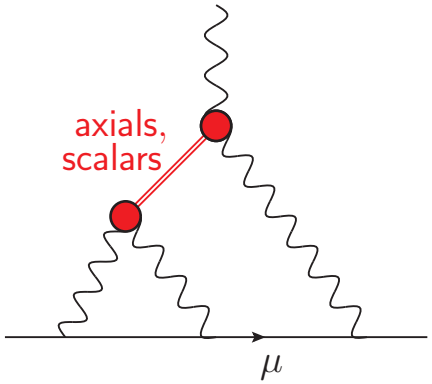
- different contributions estimated (in 10^{-11}):



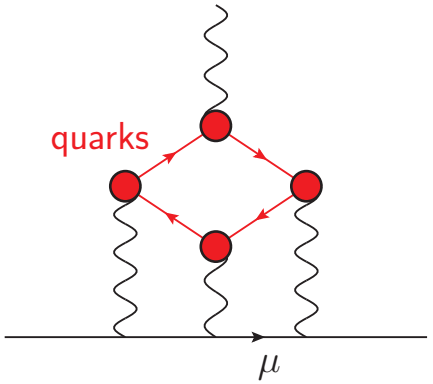
99±16



–19±13



15±7



21±3

$\longrightarrow$ hadronic modelling at its worst. . .

Jegerlehner, Nyffeler 2009

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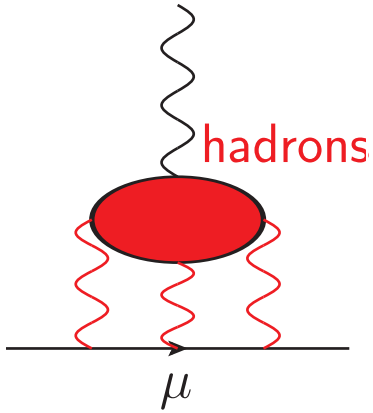
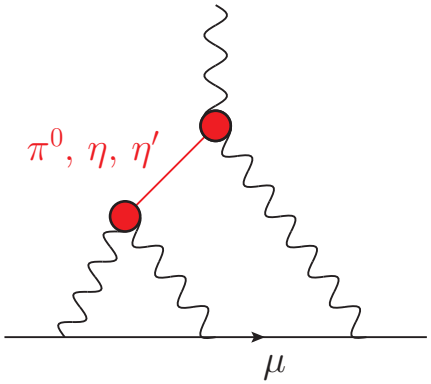
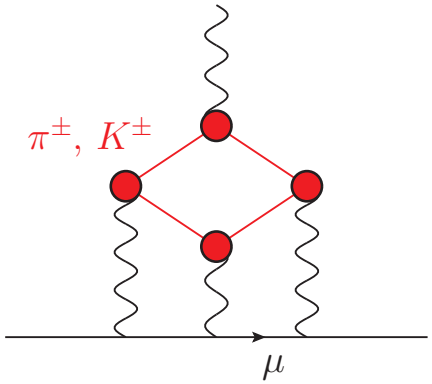


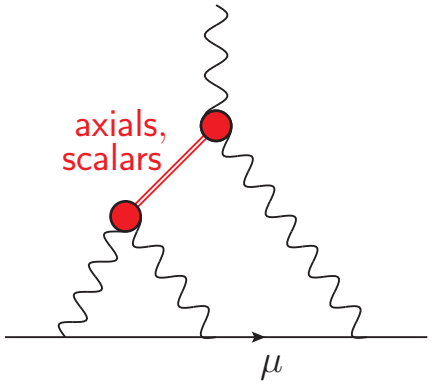
- different contributions estimated (in 10^{-11}):



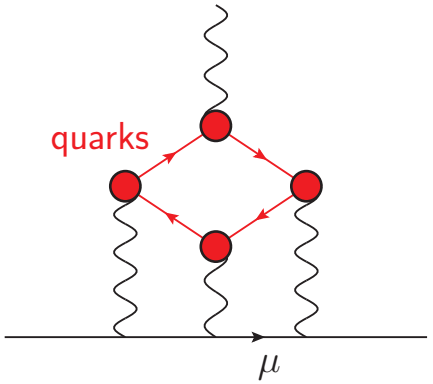
99±16



–19±13



15±7



21±3

$\longrightarrow$ hadronic modelling at its worst. . . Jegerlehner, Nyffeler 2009

- largest (and unambiguous!) individual contribution: π^0 pole term
  $\longrightarrow$ depends on $\pi^0 \to \gamma^*\gamma^*$ form factor (doubly virtual in general!)

On the road to a dispersive analysis of $\pi^0 \to \gamma^*\gamma^*$

The Wess–Zumino–Witten (chiral) anomaly

- $\pi^0 \to \gamma\gamma$ fixed by the chiral anomaly: $F_{\pi^0\gamma\gamma} = \dfrac{e^2}{4\pi^2 F_\pi}$

  F_π: pion decay constant $\longrightarrow$ measured at % level PrimEx 2011

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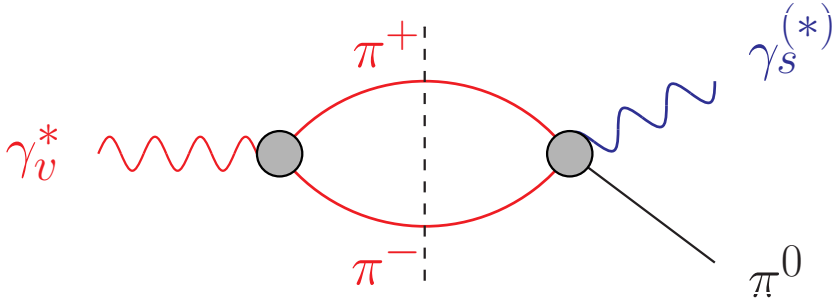

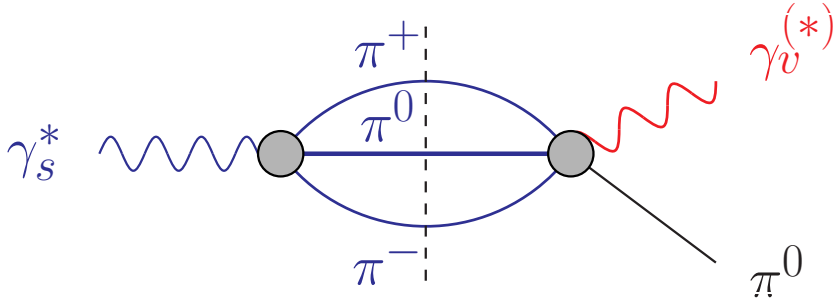

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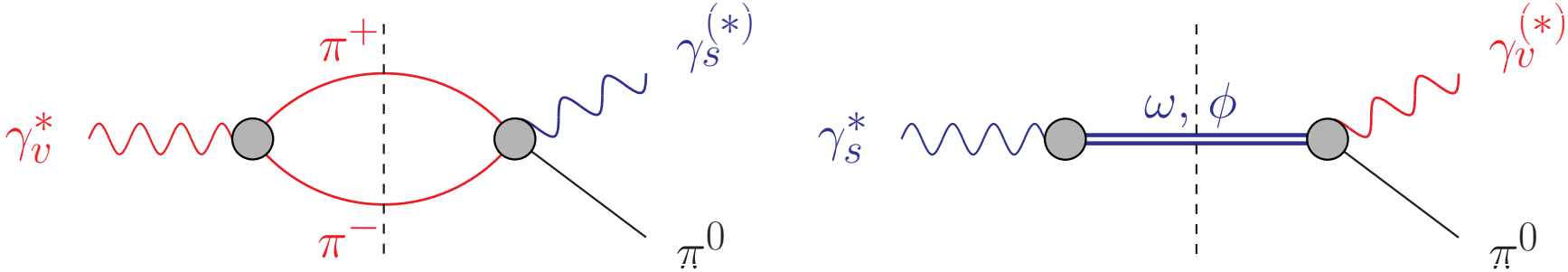

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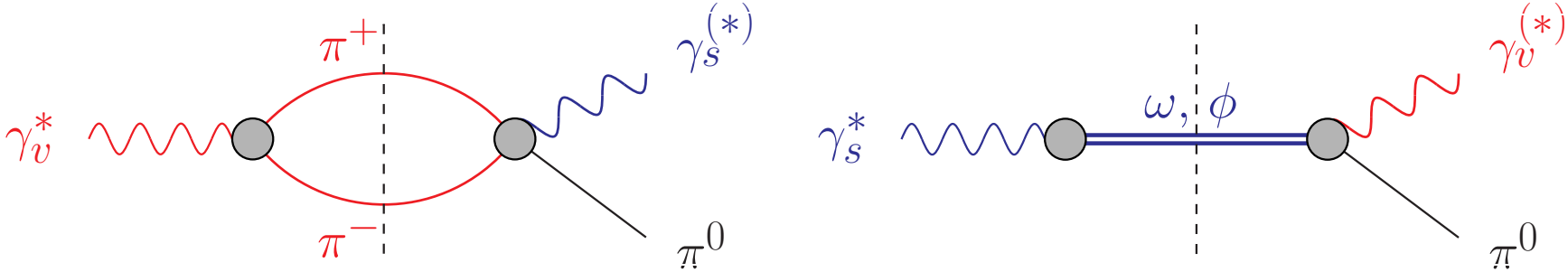

- important ingredients:
  - ▷ anomalous process $\gamma\pi \to \pi\pi$
  - ▷ vector-meson radiative decays $\omega/\phi \to \pi^0\gamma$
    transition form factors $\omega/\phi \to \pi^0\ell^+\ell^-$

  $\longrightarrow$ analyze these in turn, using dispersion relations

The anomalous process $\gamma\pi \to \pi\pi$

- $\gamma\pi \to \pi\pi$ at zero energy: $F_{3\pi} = \dfrac{e}{4\pi^2 F_\pi^3} = (9.78 \pm 0.05)$ GeV^{-3}

  how well can we test this low-energy theorem?

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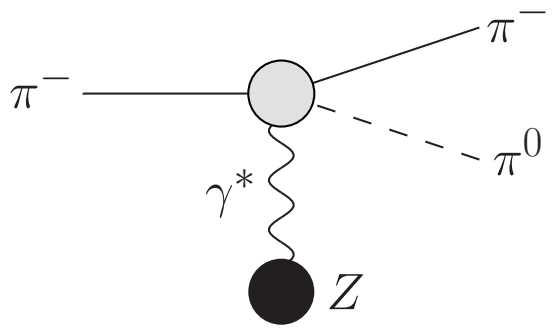
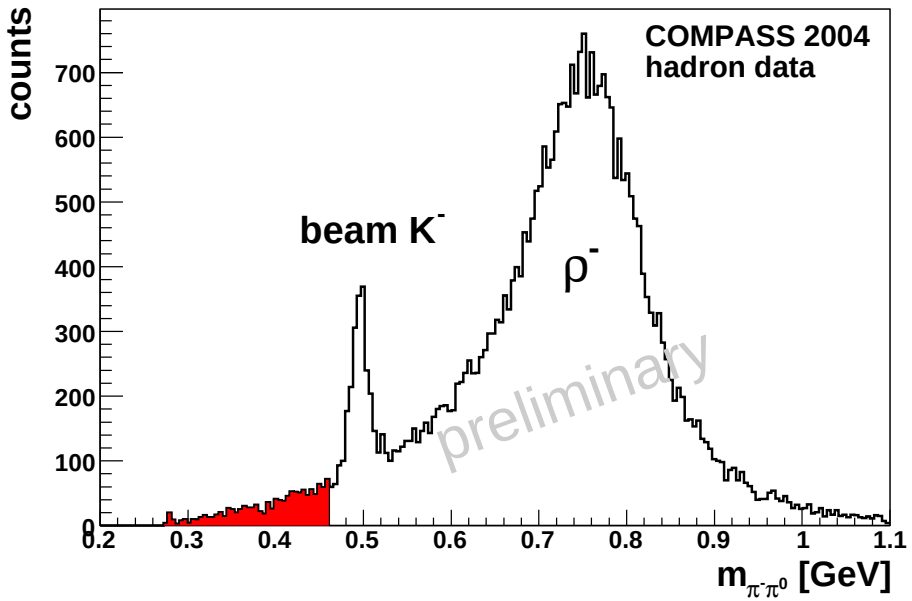
Primakoff reaction:





COMPASS, work in progress

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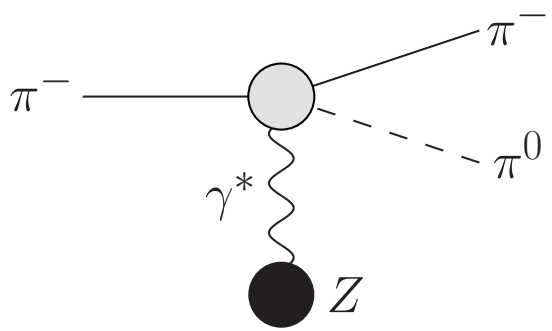
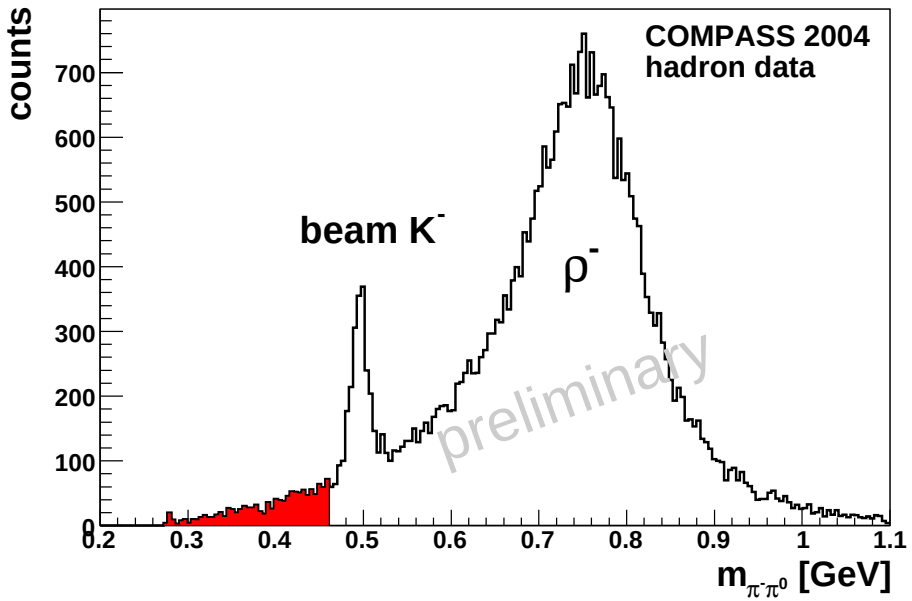
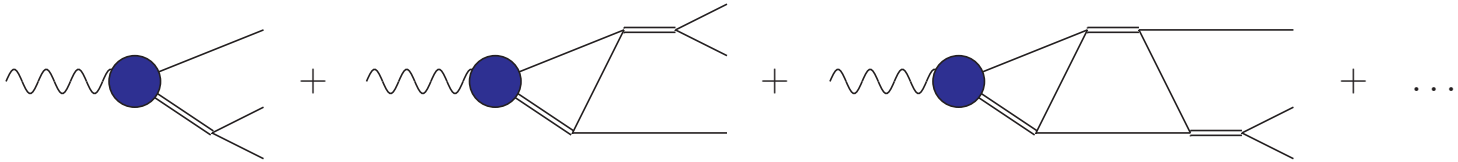




COMPASS, work in progress

- same quantum numbers as $\phi \to 3\pi$:



Truong 2002, Hoferichter, BK, Sakkas 2012

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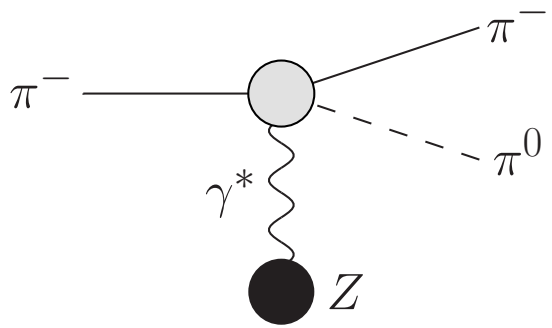
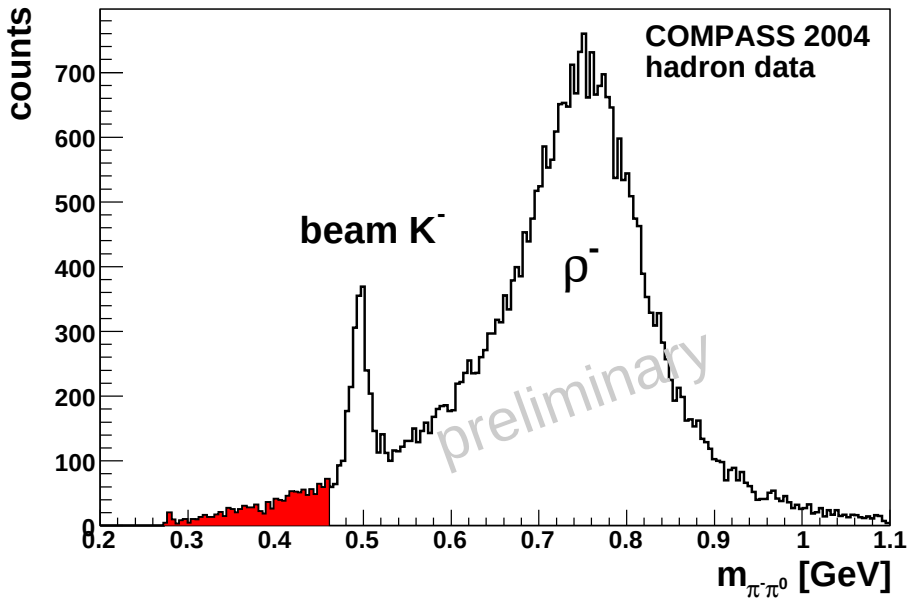




COMPASS, work in progress

- same quantum numbers as $\phi \to 3\pi$:

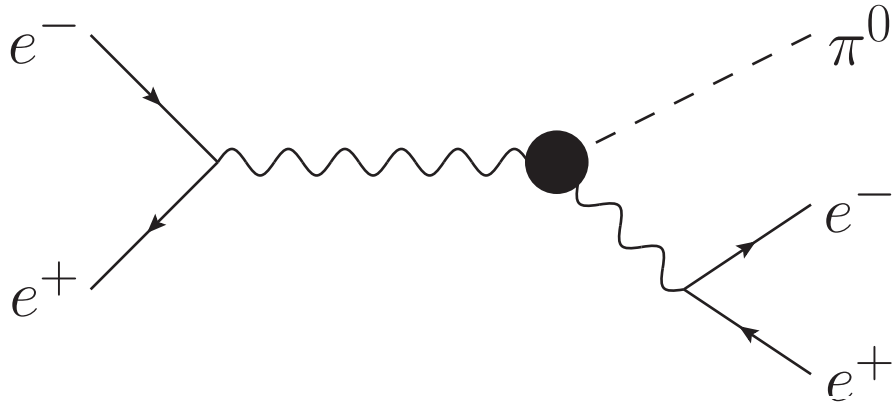
  $\longrightarrow$ extract $F_{3\pi}$ from the complete spectrum

  model-independently, using $\pi\pi$ phase shift!

Hoferichter, BK, Sakkas 2012

Transition form factors $\omega(\phi) \to \pi^0 \ell^+ \ell^-$

- $\pi^0 \to \gamma^* \gamma^*$ form factor linked to $\omega(\phi) \to \pi^0 \gamma^*$ transition:



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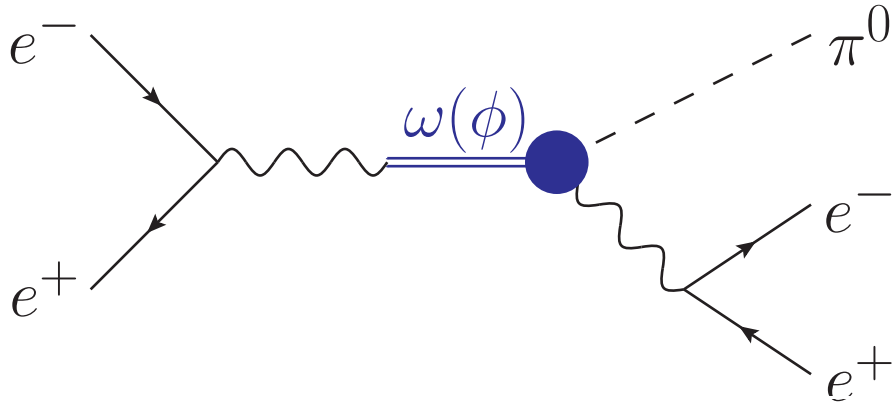

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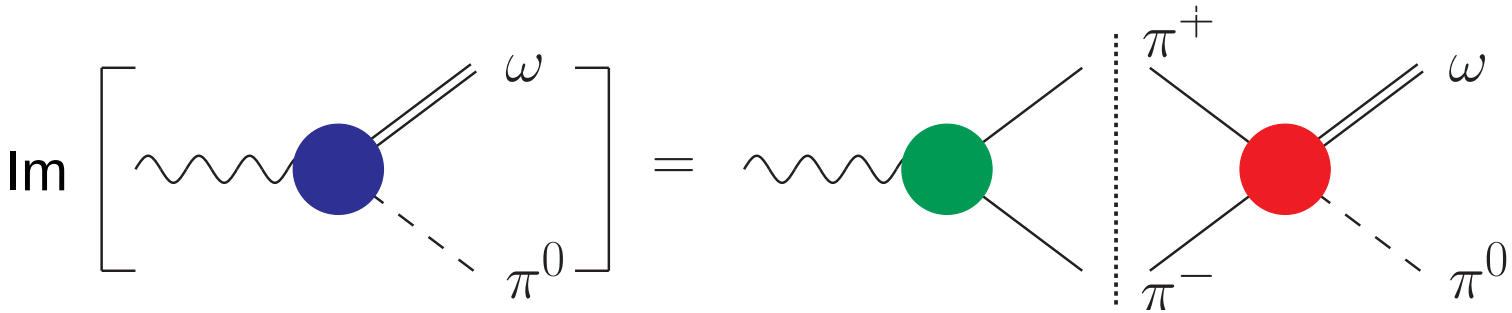


- ω transition form factor related to

  pion vector form factor $\times$ $\underbrace{\omega \to 3\pi \text{ decay amplitude}}_{\text{calculate like } \phi \to 3\pi}$

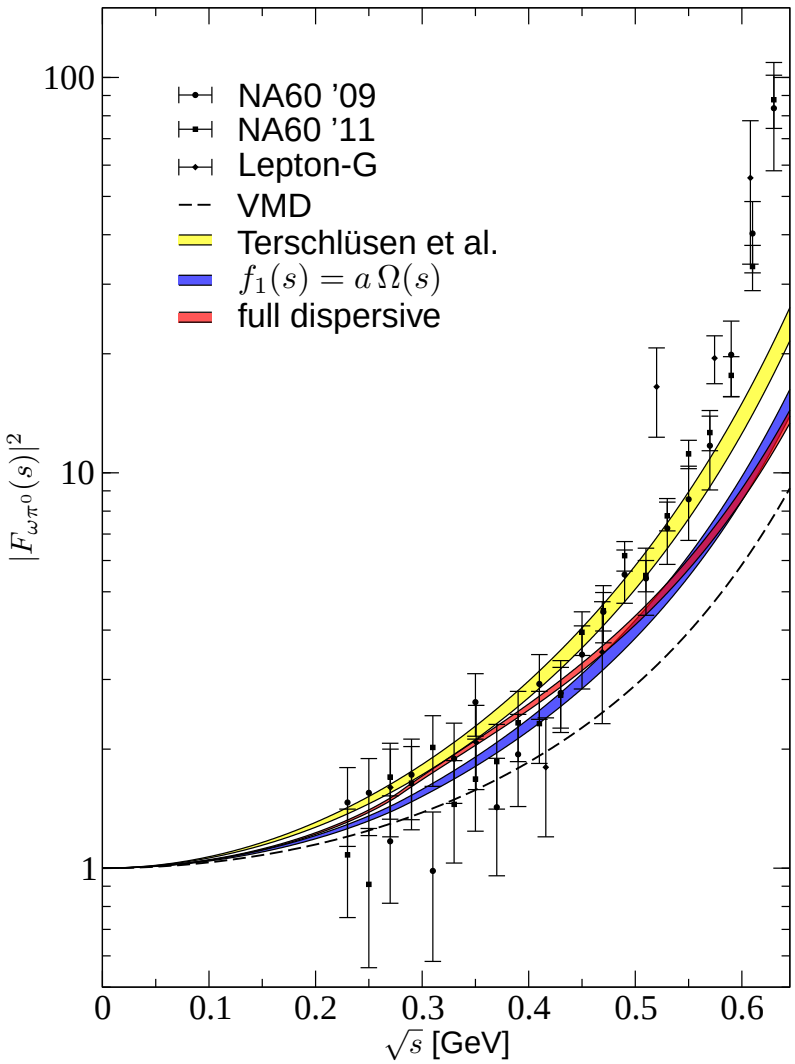
- form factor normalization yields rate $\Gamma(\omega \to \pi^0 \gamma)$

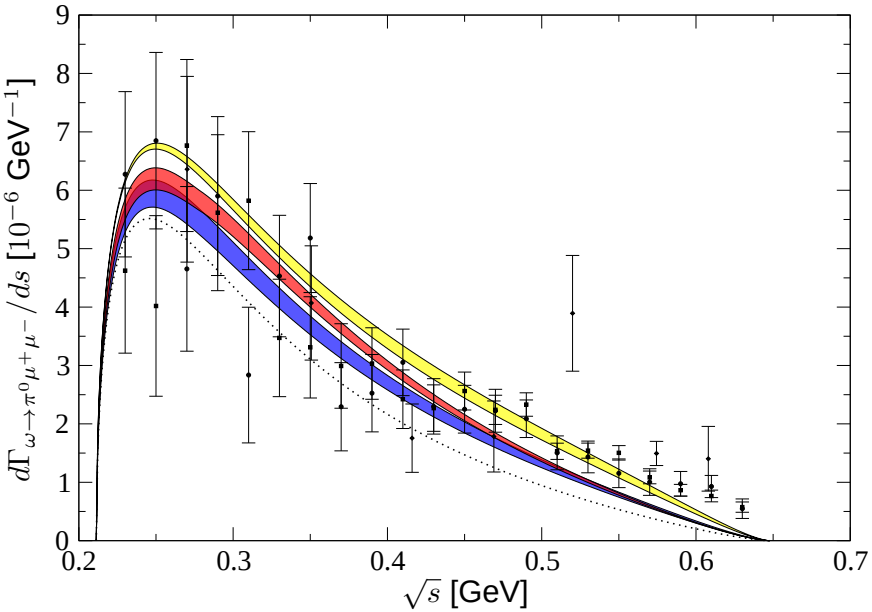
  (2nd most important ω decay channel)

  $\longrightarrow$ works at 95% accuracy

Schneider, BK, Niecknig 2012

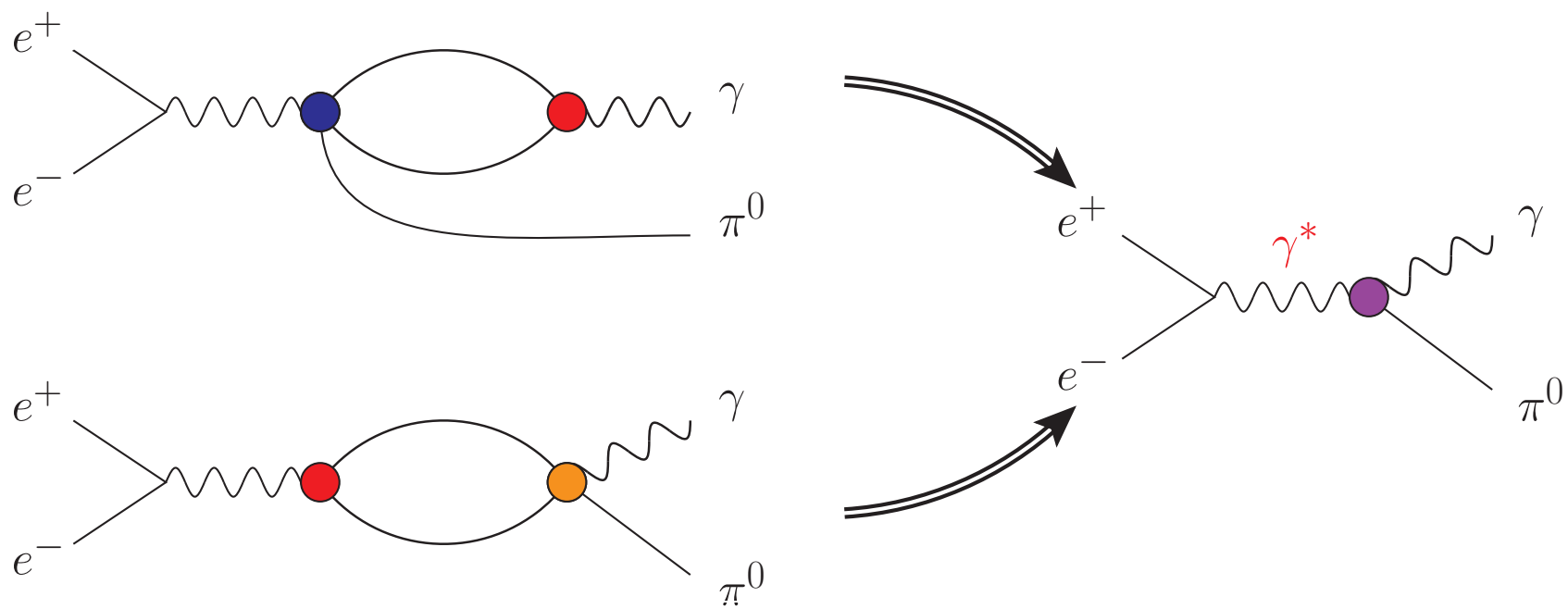
Numerical results: $\omega \to \pi^0 \mu^+ \mu^-$





- unable to account for steep rise in data (from heavy-ion collisions) NA60 2009, 2011
- more "exclusive" data?! CLAS?
- $\omega \to 3\pi$ Dalitz plot? CLAS?

Towards a dispersive analysis of $e^+e^- \to \pi^0\gamma$



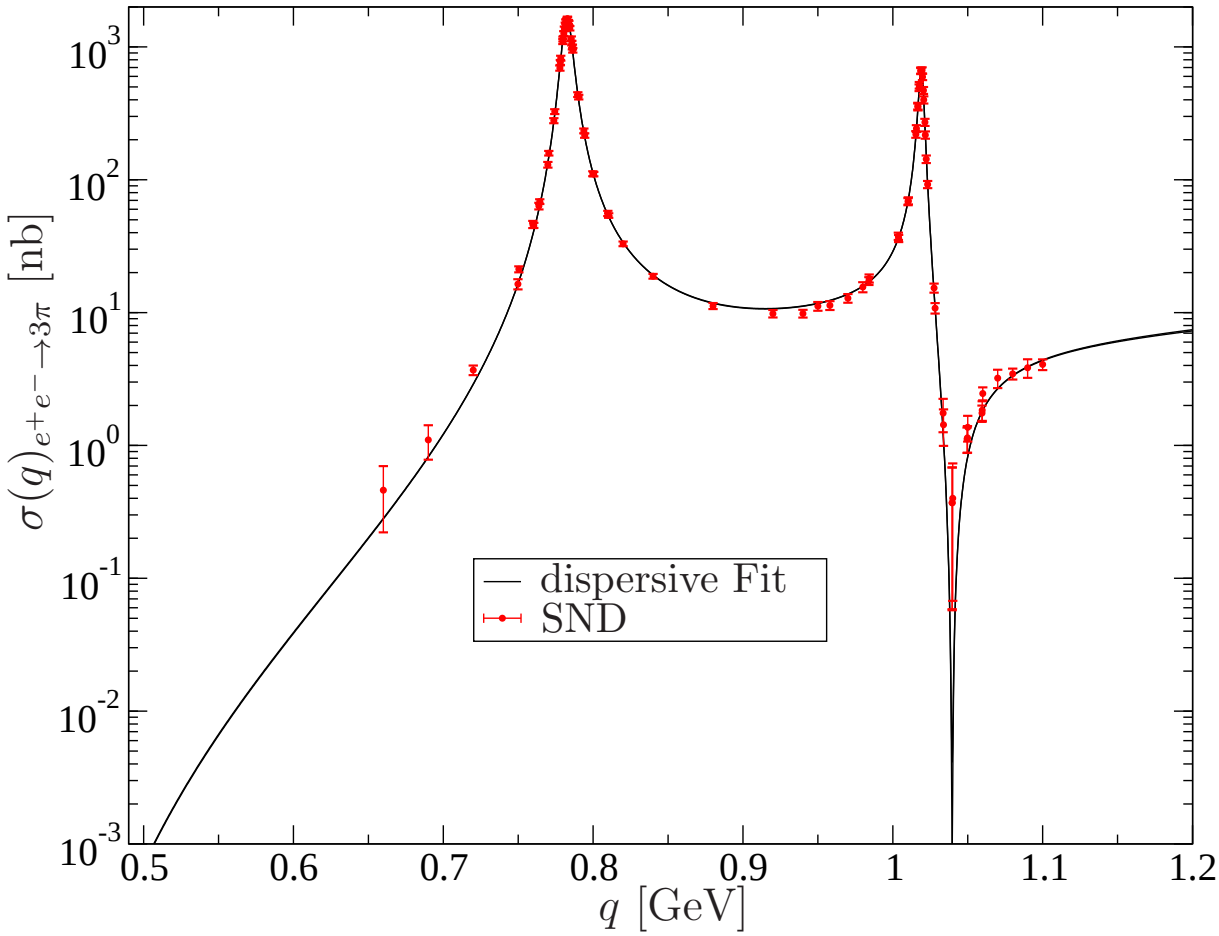
- combine isoscalar and isovector contribution to $e^+e^- \to \pi^0\gamma$:
  - ▷ parameterise $e^+e^- \to 3\pi$

    e.g. in terms of (dispersively improved)

    $\omega + \phi$ Breit–Wigner propagators with good analytic properties

    Lomon, Pacetti 2012; Moussallam 2013
  - ▷ prediction for $e^+e^- \to \pi^0\gamma$

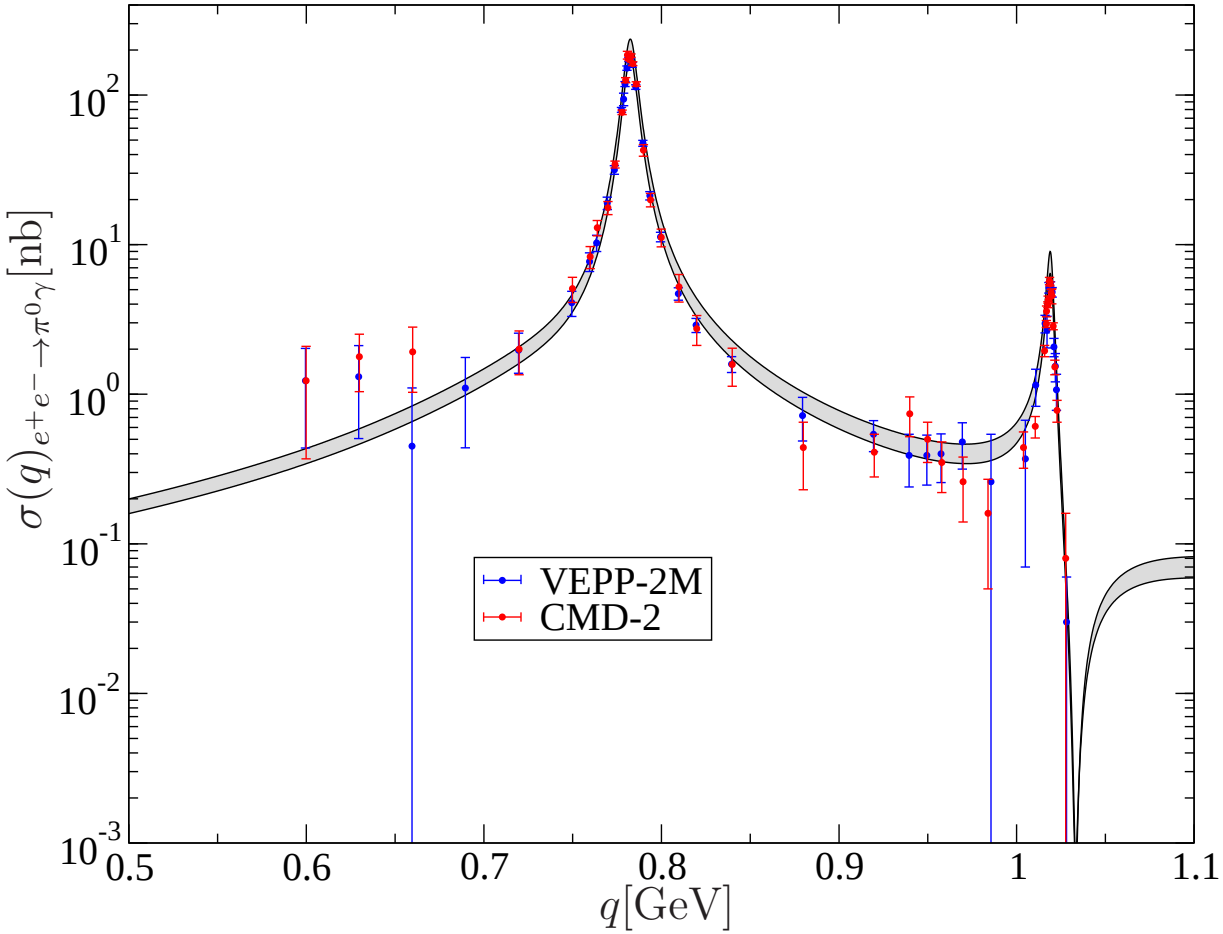
Fit to $e^+e^- \to 3\pi$ data



Hoferichter, BK, Leupold, Niecknig, Schneider preliminary

- one subtraction/normalisation at $q^2 = 0$ fixed by $\gamma \to 3\pi$
- fitted: ω, ϕ residues, one additional (linear) subtraction

Comparison to $e^+e^- \to \pi^0\gamma$ data



Hoferichter, BK, Leupold, Niecknig, Schneider preliminary

- "prediction"—no further parameters adjusted
- data well reproduced

Summary

2 pieces of modern strong-interaction physics using dispersion relations:

- Dalitz plot analyses
  - ▷ rigorous using modern phase shift input
  - ▷ allow to understand ad-hoc "background"
  - ▷ methods applicable for many kinds of "amplitude analyses": CP violation, spectroscopy issues

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2 pieces of modern strong-interaction physics using dispersion relations:

- Dalitz plot analyses
  - ▷ rigorous using modern phase shift input
  - ▷ allow to understand ad-hoc "background"
  - ▷ methods applicable for many kinds of "amplitude analyses": CP violation, spectroscopy issues
- The muon anomalous magnetic moment
  - ▷ hadronic physics needs to be constrained well before claiming "new physics"
  - ▷ interrelate as much experimental information as possible