MESON PHENOMENOLOGY IN A BETHE-SALPETER EQUATION APPROACH

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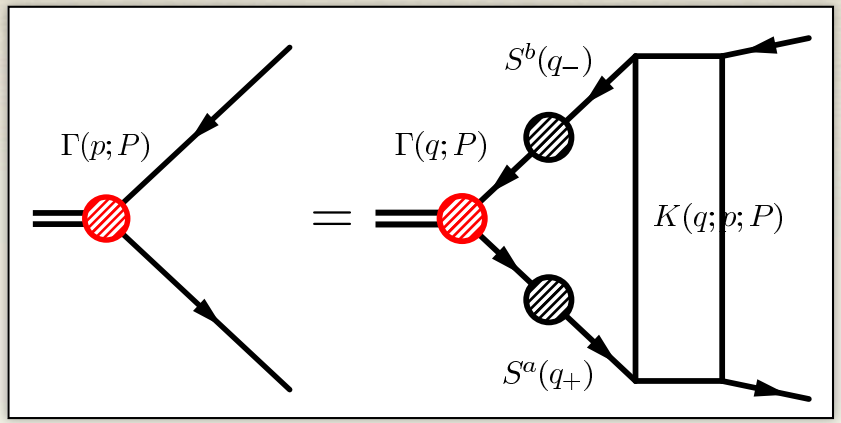
www.ModelsofHadrons.com

Motivation

- Goal: develop a model that is applicable from chiral limit up to $m_q = m_b$
  - sophisticated Landau gauge effective interaction
  - simple setup (rainbow ladder truncation)
- Calculate observables
  - masses, splittings
  - excitations of both radial and angular quantum numbers
  - deal with exotic states **[T. Hilger]**
- Should also include heavy-light systems **[M. Gomez-Rocha]**
- Strategy: start by describing bottomonium spectrum, continue to charmonium
- So far: bottomonium spectrum agrees very well with experiment

Homogeneous Bethe-salpether Equation



$$\Gamma(p;P) = \int_q S^a(q+\eta P)\,\Gamma(q;P)\,S^b(q-(1-\eta)P)\;K(q,p;P)$$

- Bound state mass $\boldsymbol{P^2 = -M^2}$
- Input:
  - dressed quark propagator ***S(q)*** (from the gap equation)
  - quark-antiquark scattering kernel ***K(q,p;P)*** (one gluon exchange)
  - ***Effective quark-gluon interaction*** employed in both DSE and BS kernel

Rainbow Ladder truncation

- DSE/BSE build up an infinite system of coupled integral eqs
  $\Rightarrow$ ***truncate the system***

- RL approximation satisfies Ward-Takahashi identity

(from AVWTI it follows that the *rainbow* truncated kernel of the quark DSE corresponds to the *ladder* truncated kernel of the meson BSE)

- Well suited for heavy quarks (e.g. Coulomb gauge studies)

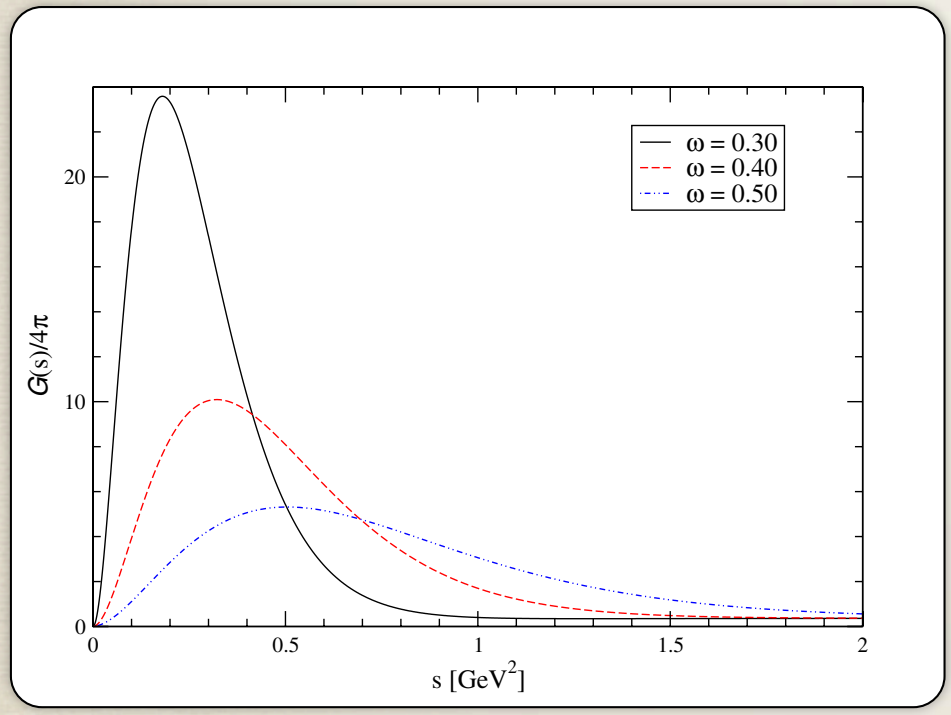
- Successfully employed in earlier investigations of meson spectra in Landau gauge

Blank, Krassnigg PRD84 2011; Krassnigg PRD80 2009

EFFECTIVE INTERACTION

$$\mathcal{D}(p^2) = \frac{D}{2}\,\frac{(2\pi)^2}{\omega^6}\,p^2 e^{-p^2/\omega^2} + \mathcal{F}_{UV}(p^2)$$

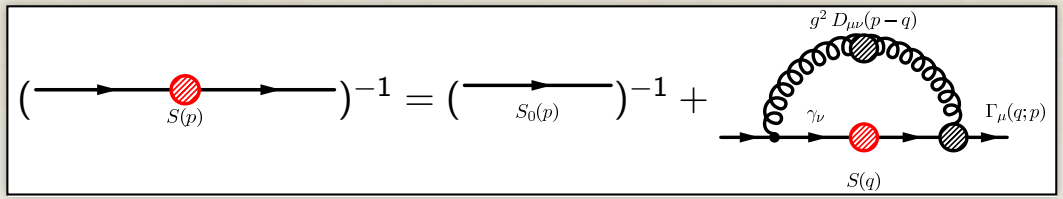
[Maris, Tandy 1999]



[Krassnigg PRD 80 2009]

- preserves confinement
- gives the right amount of χSB
- provides the correct one-loop renormalization group behaviour of QCD via $\mathcal{F}_{UV}$
  - intermediate momentum range (i.e., parameters ω, D) relevant for phenomenology

Nummerical solutions: DSE



$$S^{-1}(p) = S_0^{-1}(p) + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-q)\,\gamma_\nu\, S(q)\,\Gamma_\mu(q;p)$$

- Coupled nonlinear integral equations for quark propagator dressing functions

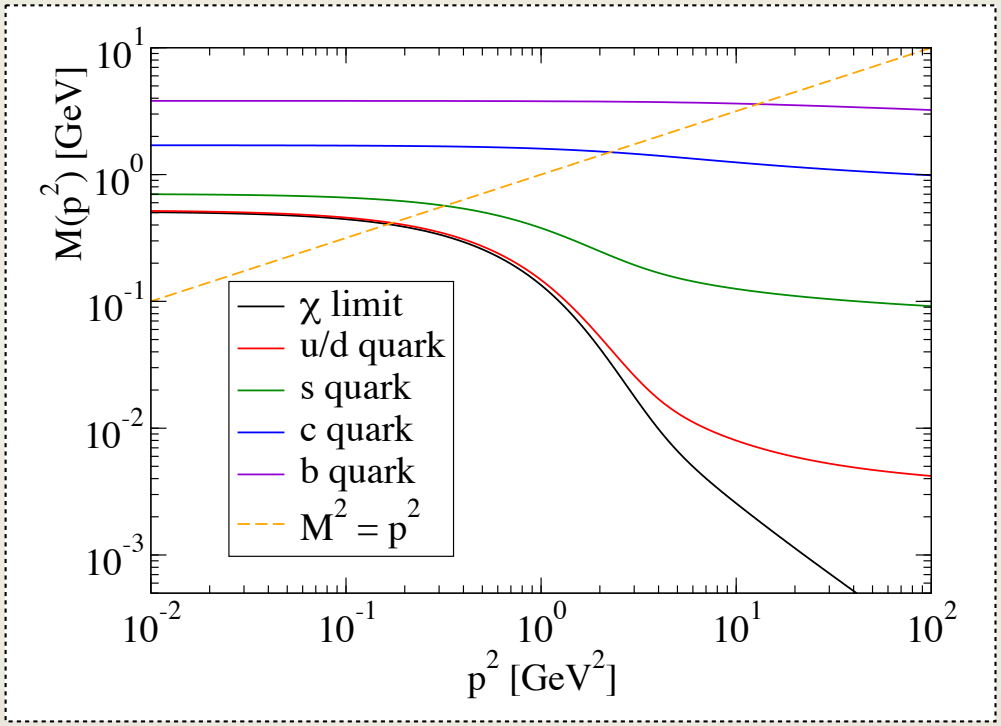
$$S^{-1}(p) = i\,\gamma\cdot p\; A(p^2) + B(p^2)$$

- Solve iteratively on the real p^2 axis

- On-shell condition $P^2 = -M^2$ leads to imaginary momentum
  $\Rightarrow$ analytical continuation of $A(p^2)$ and $B(p^2)$ into complex plane

Nummerical solutions: DSE

- Quark mass function $M(p^2) = B(p^2)/A(p^2)$



[Blank, PhD Thesis, 2011]

- Dynamical chiral symmetry breaking

Nummerical solutions: BSE

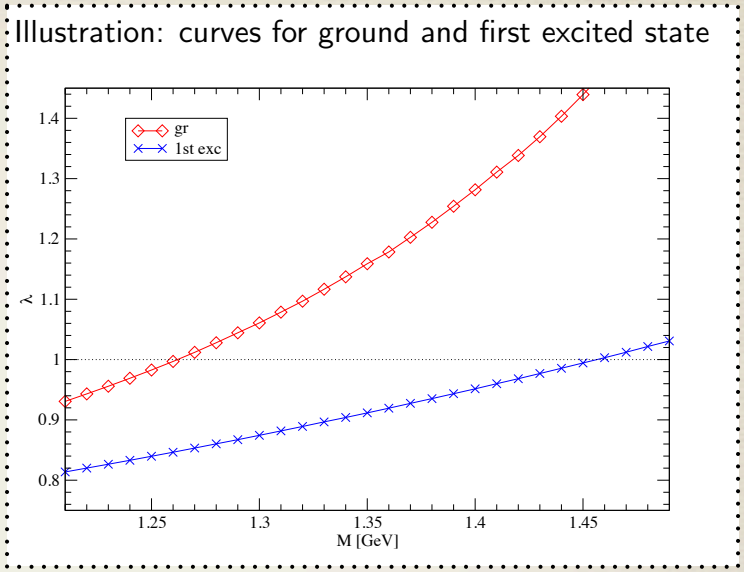
- Recast BSE into an eigenvalue problem and solve it for the largest few eigenvalues

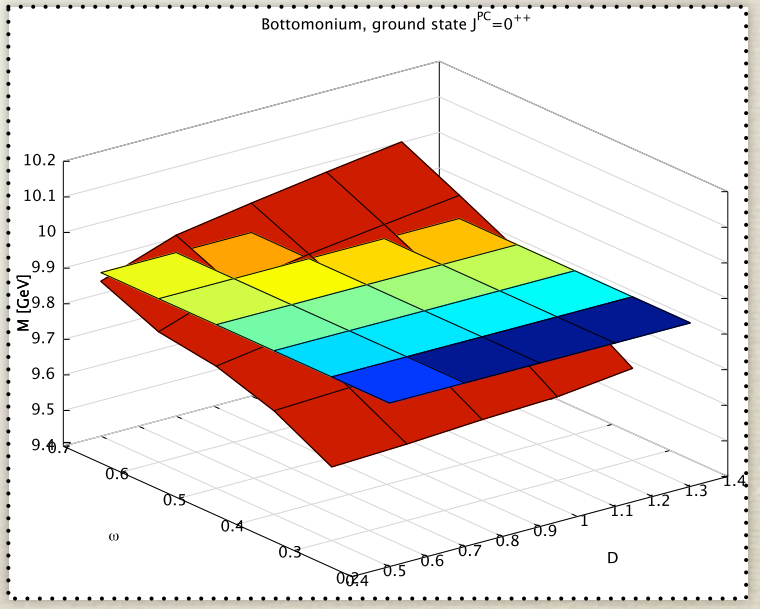
$$\lambda(P^2)\Gamma(p;P) = \tilde{K}(p;q;P)\Gamma(q;P)$$

- Obtain curves for the chosen eigenvalues
- Mass determined from the condition

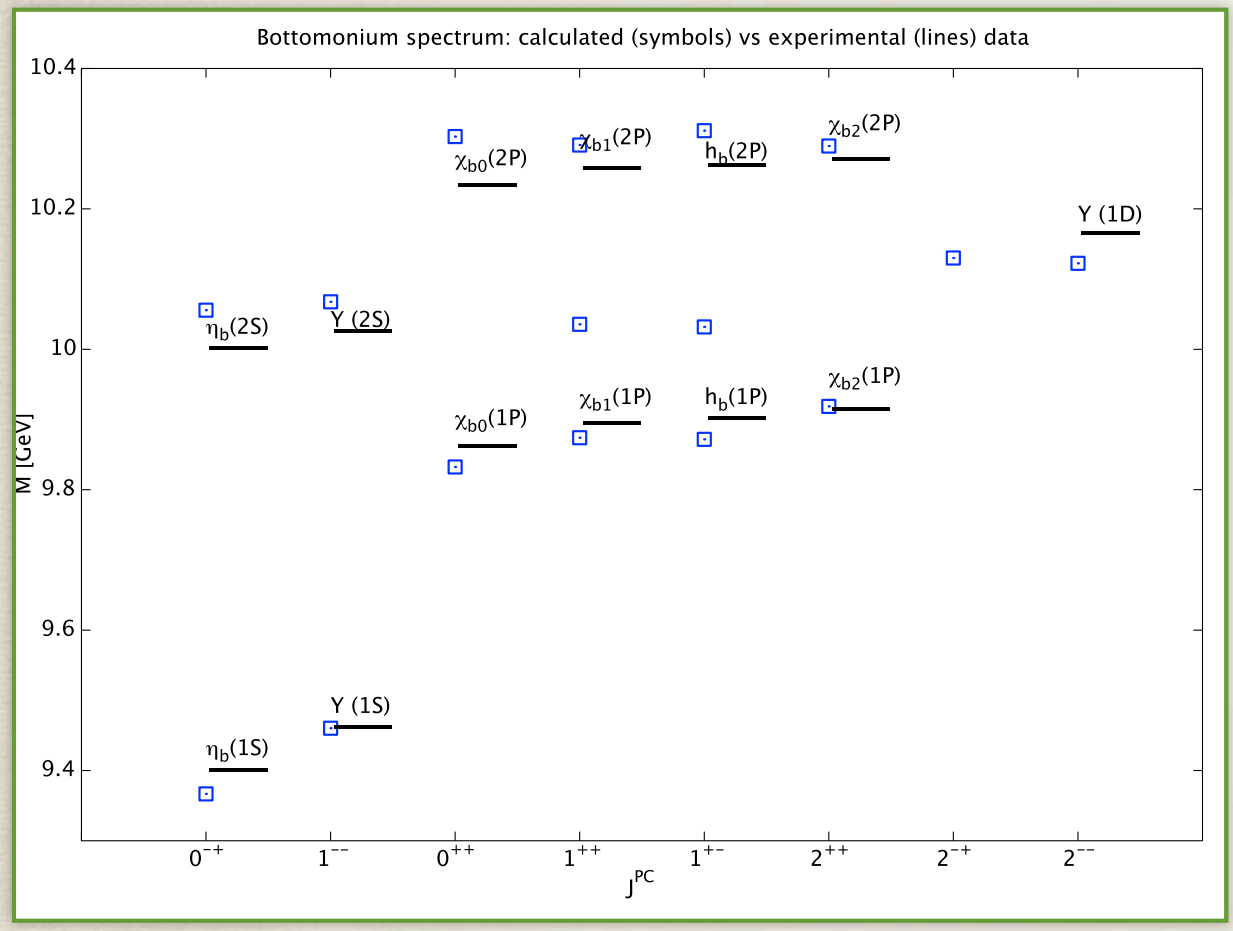
$$\lambda(P^2 = -M^2) = 1$$

- $\omega \cdot D$ not necessarily constant
- Read off the optimal set of parameters by intersecting the surface determined by (ω, D) with the experimental value (horizontal plane)





Bottomonium spectrum (Preliminary)



- Ground states and radial excitations
- Vector channel ground state fitted to experiment
- Good agreement with available experimental data
- Unclear whether all excitations appearing as solutions of BSE are physical $\Rightarrow$ spurious states?

Summary

- Comprehensive RL truncated model based on DSE/BSE
- Thorough analysis of the effective interaction
- Gap equation: solve iteratively on the real axis and perform analytical continuation in the complex plane
- BSE: Solve eigenvalue problem and obtain meson masses from the eigenvalues
- Bottomonium spectrum agrees remarkably well with experiment
- Charmonium in preparation

Outlook

- Meson states with exotic quantum numbers
- Hadronic, leptonic, electromagnetic transitions (from the BS amplitudes, i.e. eigenvectors of BSA)
- Other directions:
  - Finite temperature and chemical potential
  - Baryons