



Testing Hybrid Natural Inflation with BICEP2

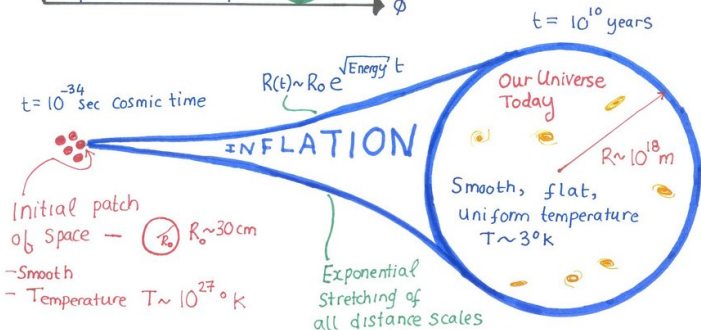
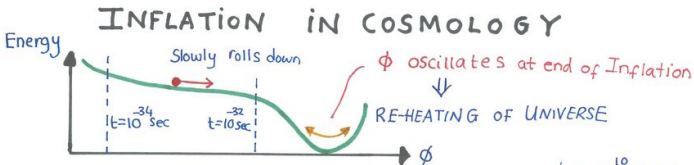
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Cake Seminar JLab, July 2014

Outline

- 1 Inflation
 - Slow-roll Inflation
- 2 Natural inflation
- 3 Observational constraints
 - Planck results
 - BICEP2 results
- 4 Hybrid natural inflation
 - Testing Hybrid Natural Inflation with BICEP2
 - Hybrid natural inflation at the maximum of the tensor index



Horizon problem

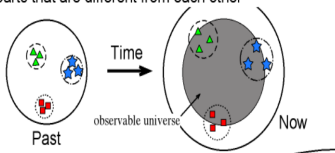
Homogeneous & isotropic

Flatness problem

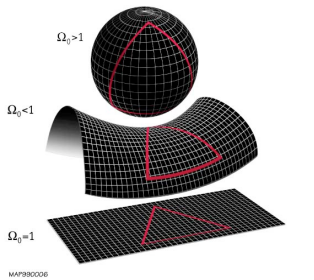
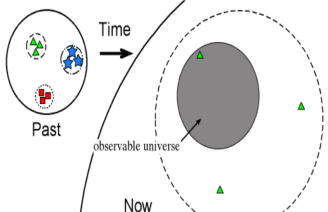
$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right)$$

$$(\Omega^{-1} - 1)\rho a = -3M_{Pl}k$$

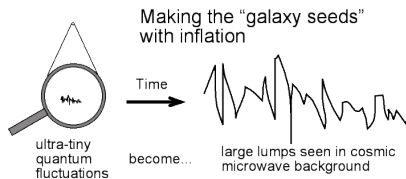
NO inflation: observable universe (shaded) includes parts that are different from each other



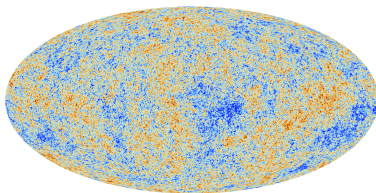
Inflation: observable universe (shaded) includes only one part that is the same throughout



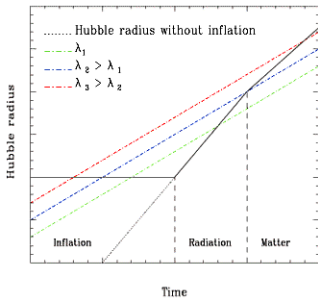
Inflation (ultra-fast expansion) flattens out the local curvature by making the universe extremely large.



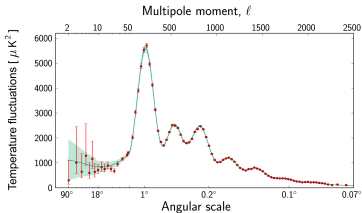
- **Primordial energy density perturbation.**
Inflaton perturbation, Scalar curvature perturbation.
- **Primordial gravitational waves.**
Spatial metric perturbation, transverse and traceless.



Primordial power spectra of perturbations



$$\mathcal{P}_{\mathcal{R}} = A_s \left(\frac{k}{k_*} \right)^{n_s - 1 + \frac{1}{2} \frac{dn_s}{d \ln k} \ln \frac{k}{k_*} + \dots}$$



Primordial quantities

$$\mathcal{R}(\mathbf{k}) = \frac{1}{4} \left(\frac{a}{k} \right)^2 R^{(3)}(\mathbf{k})$$

$$h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$$

$$\mathcal{P}_t = A_t \left(\frac{k}{k_*} \right)^{n_t + \frac{1}{2} \frac{dn_t}{d \ln k} \ln \frac{k}{k_*} + \dots}$$

Slow-roll inflation

EVOLUTION OF INFLATON FIELD

Equation of motion $\ddot{\phi} + 3H\dot{\phi} + V' = 0; \quad H = \dot{a}/a$

Friedmann equation $H^2 = \frac{1}{3M_{Pl}^2} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right)$

SLOW ROLL APPROXIMATION

Equation of motion $3H\dot{\phi} = -V'$

Friedmann equation $H^2 = \frac{V(\phi)}{3M_{Pl}^2}$

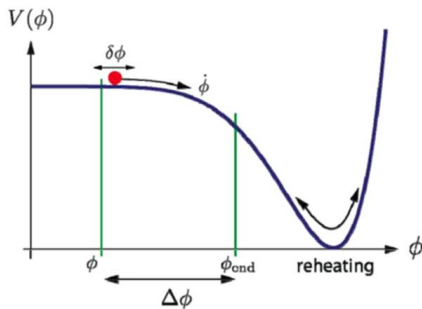
Slow-roll approximation

$$\epsilon \ll 1; \quad |\eta| \ll 1$$

Slow-roll parameters

$$\epsilon = \frac{V'^2}{2V^2} \quad \eta = \frac{V''}{V}$$

$$\xi^2 = \frac{V'V'''}{V^2} \quad \omega^3 = \frac{V'^2V''''}{V^3}$$



Observable quantities

$$\delta_H^2 = \frac{4}{25} A_s = \frac{1}{150\pi^2} \frac{V}{\epsilon_H}; \quad r \equiv \frac{A_t}{A_R} = 16\epsilon = -8n_t$$

$$N \equiv \int H dt = \int_{\phi_e}^{\phi_*} \frac{V}{V'} d\phi = \int_{\phi_e}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon}}$$

$$n_s = 1 + 2\eta - 6\epsilon$$

$$n_{sk} \equiv \frac{dn_s}{d \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2$$

$$n_{tk} \equiv \frac{dn_t}{d \ln k} = 4\epsilon\eta - 8\epsilon^2$$

Constraint equation

$$\rightarrow n_{tk} = \frac{r}{64} (r - 8\delta_{ns})$$

where $\delta_{ns} \equiv 1 - n_s$.

Natural Inflation

Fine tuning. Flat potential requires: $\frac{\Delta V}{\Delta \phi^4} \leq \frac{\text{height}}{\text{width}^4} \leq 10^{-6}$

$$V(\phi) = \Lambda^4 \left(1 \pm \cos \frac{\phi}{f}\right)$$

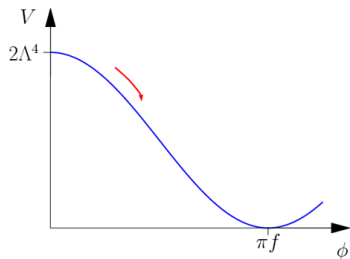
$$\epsilon = \frac{1}{2f^2} + \eta$$

$$\xi^2 = -2 \frac{1}{f^2} \epsilon \quad \omega^3 = 2 \frac{1}{f^2} \epsilon \eta$$

$$f = \frac{2}{\sqrt{4\delta_{ns} - r}} \longrightarrow r \leq 4\delta_{ns} \approx 0.16$$

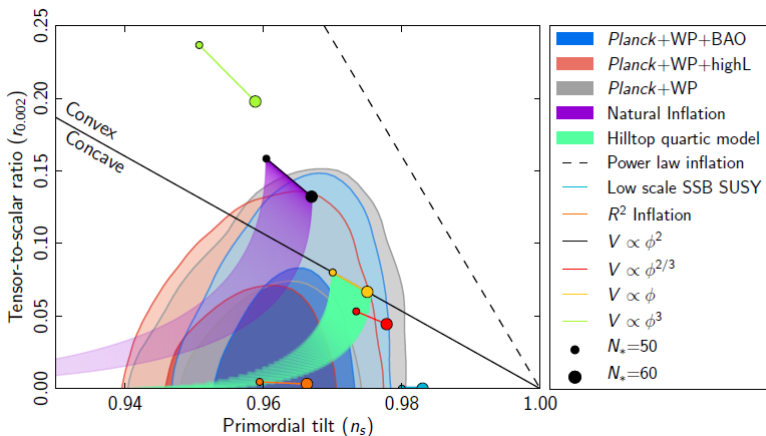
Large symmetry breaking scale!!

$$r = 0.15 \longrightarrow f = 20M_{Pl}$$

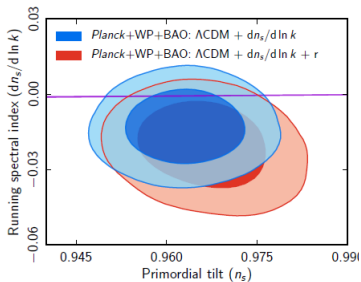


(Freese, Frieman, Olinto;
1990)

Planck collaboration



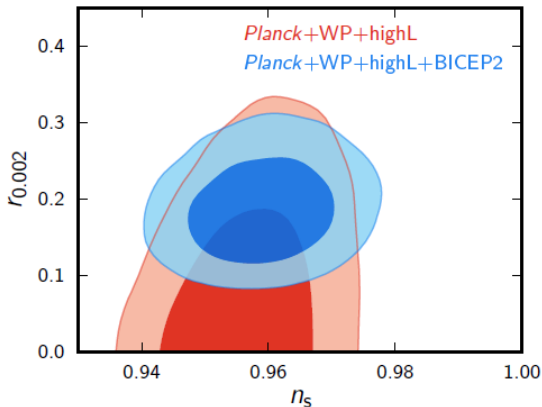
Model	Parameter	<i>Planck</i> +WP	<i>Planck</i> +WP+lensing	<i>Planck</i> + WP+high- ℓ	<i>Planck</i> +WP+BAO
Λ CDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	0.9643 ± 0.0059
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12



Model	Parameter	<i>Planck</i> +WP
Λ CDM + $dn_s/d \ln k$	n_s	0.9561 ± 0.0080
	$dn_s/d \ln k$	-0.0134 ± 0.0090
Λ CDM + $dn_s/d \ln k$ + $d^2 n_s/d \ln k^2$	n_s	$0.9514^{+0.087}_{-0.090}$
	$dn_s/d \ln k$	$0.001^{+0.016}_{-0.014}$
	$d^2 n_s/d \ln k^2$	$0.020^{+0.016}_{-0.015}$
Λ CDM + r + $dn_s/d \ln k$	n_s	0.9583 ± 0.0081
	r	< 0.25
	$dn_s/d \ln k$	-0.021 ± 0.012

BICEP2

Detection of B-mode Polarization at degree angular scales. $r = 0.20^{+0.07}_{-0.05}$



Hybrid natural inflation

Searching a lower symmetry breaking scale.

$$V(\phi) = V_0 \left(1 + a \cos \frac{\phi}{f} \right)$$

- Inflaton is a PNCB associated with spontaneous breaking of approximate continuous symmetry $U(1)$.
- Spontaneously broken global non-Abelian discrete symmetry $D_4 = Z_2 \times Z_2'$.
 → A 2nd field is responsible for ending inflation.
- Two models:
 - Non supersymmetric.
 - Supersymmetric.

(G. G. Gross, G. German; 2010)

Testing Hybrid Natural Inflation with BICEP2

(M. Carrillo-Gonzalez, G. German, A. Herrera-Aguilar, C. Hidalgo, R. Sussman; 2014)

Slow-roll parameters for hybrid natural inflation

$$\epsilon = \frac{a^2 \sin^2\left(\frac{\phi}{f}\right)}{2f^2 \left(1 + a \cos\left(\frac{\phi}{f}\right)\right)^2} \quad \xi^2 = -2 \frac{1}{f^2} \epsilon$$

$$\eta = -\frac{a \cos\left(\frac{\phi}{f}\right)}{f^2 \left(1 + a \cos\left(\frac{\phi}{f}\right)\right)} \quad \omega^3 = 2 \frac{1}{f^2} \epsilon \eta$$

$$\epsilon = \frac{a^2}{2f^2} + a^2 \eta - \frac{(1 - a^2)f^2}{2} \eta^2$$

Observables for hybrid natural inflation

$$n_{sk} = \frac{r}{32} \left(3r - 16\delta_{ns} + \frac{8}{f^2} \right)$$

$$f = \frac{\sqrt{8r}}{\sqrt{32n_{sk} - r(3r - 16\delta_{ns})}}$$

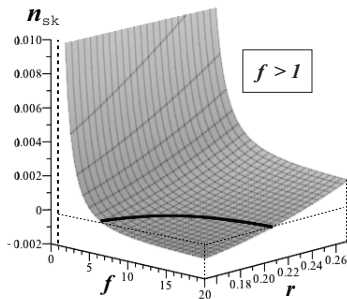
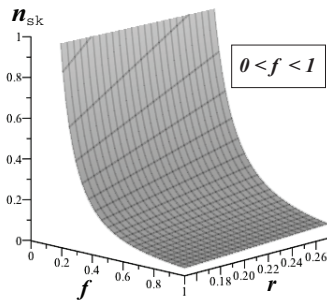
Constraints for n_{sk}

$$n_{sk} \geq -9 \times 10^{-4} \quad \text{for } r \geq 0.15$$

$$n_{sk} > 0 \quad \text{for } r > \frac{16}{3}\delta_{ns} \approx 0.21$$

$$N_* = \frac{f^2}{2a} \left((1+a) \ln \left(\frac{1-c_e}{1-c_*} \right) - (1-a) \ln \left(\frac{1+c_e}{1+c_*} \right) \right)$$

Running of the spectral index



Natural Inflation vs Hybrid Natural Inflation

	f	a	n_s	r	n_{sk}	n_{skk}	n_{tk}	Λ (GeV)	N
<i>NI</i>	8.9	—	0.96	0.11	-7.2×10^{-4}	-2.9×10^{-5}	-3.6×10^{-4}	1.88×10^{16}	51
<i>NI</i>	20	—	0.96	0.15	-8×10^{-4}	-3.2×10^{-5}	-4.0×10^{-4}	2.04×10^{16}	50
<i>NI</i>	∞	—	0.96	0.16	—	—	—	—	—
<i>HNI</i>	1	0.117	0.96	0.11	2.6×10^{-2}	2.2×10^{-3}	-3.6×10^{-4}	1.88×10^{16}	N_χ
<i>HNI</i>	1	0.136	0.96	0.15	3.7×10^{-2}	3.6×10^{-3}	-4.0×10^{-4}	2.04×10^{16}	N_χ
<i>HNI</i>	1	0.140	0.96	0.16	3.9×10^{-2}	4.0×10^{-3}	-4.0×10^{-4}	2.07×10^{16}	N_χ
<i>HNI</i>	1	0.156	0.96	0.20	5.0×10^{-2}	5.8×10^{-3}	-3.8×10^{-4}	2.19×10^{16}	N_χ
<i>HNI</i>	1	0.181	0.96	0.27	6.9×10^{-2}	9.8×10^{-3}	-2.1×10^{-4}	2.36×10^{16}	N_χ

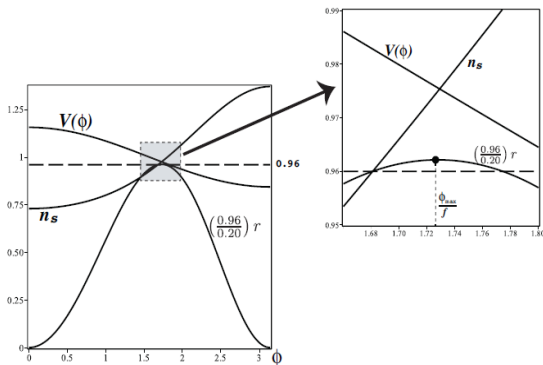
Maximum of the tensor index

Universality

$$\cos \frac{\phi_{max}}{f} = -a$$

$$r_{critical} = 8\delta_{ns} \simeq 0.32;$$

$$\epsilon_{critical} = \frac{1}{2}\eta_{critical}$$



Conclusions

Natural Hybrid Inflation

- Lower symmetry breaking scale.
- Allows the BICEP2 reported values for r .
- Predicts a “large” value for n_{sk} :
 - could be observable in the near future.
 - possible resolution to the apparent tension between high values of r and previous limits.

Observable inflation doesn't seem to occur at the maximum of the tensor index.