Bound states – from QED to QCD

Toward $O(\alpha_s^0)$ hadron states

Light Cone 2014

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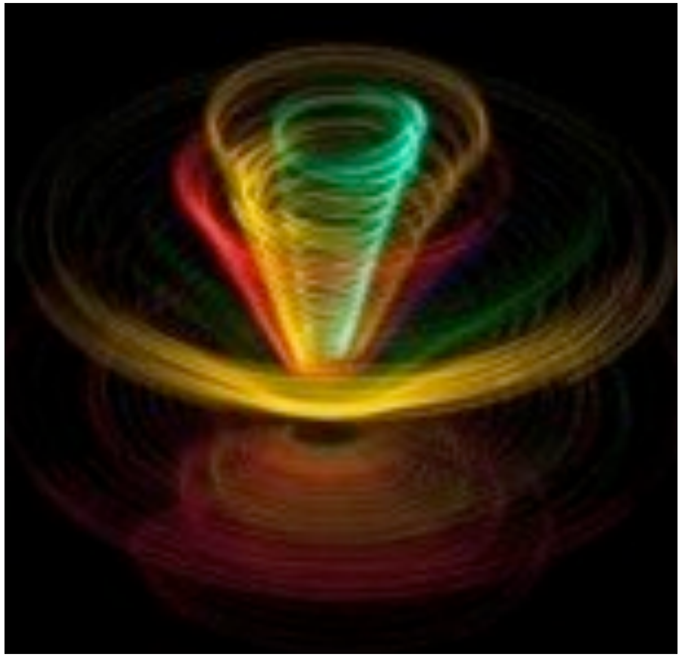


Work done with: D. D. Dietrich and M. Järvinen

The unexpected simplicities of hadrons

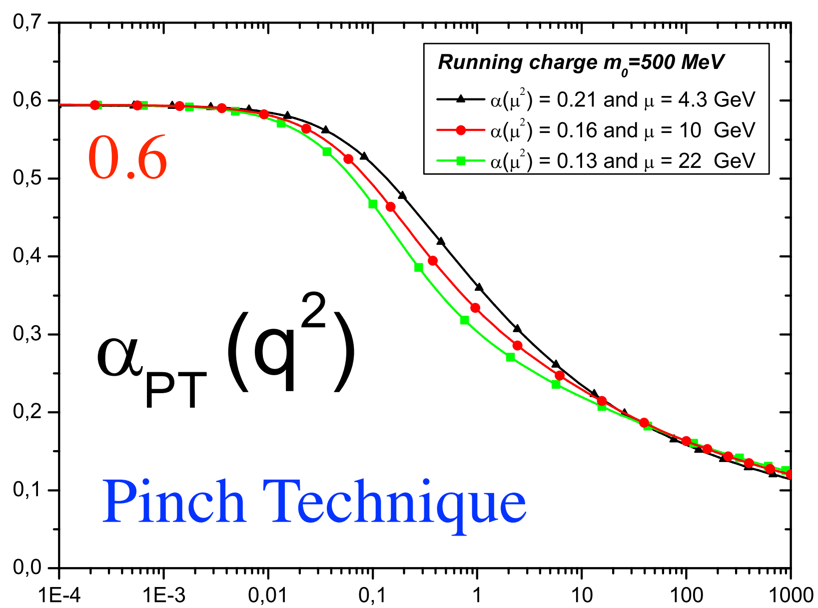
The spectra and interactions of hadrons reveal some surprising features.

These suggest that perturbation theory may be applicable even at low Q^2.

The strong interaction and confinement effects would then be limited to an $O(\alpha_s{}^0)$ sector of QCD.

What if: Can such a scenario be ruled out?

Lecture notes: arXiv:1402.5005



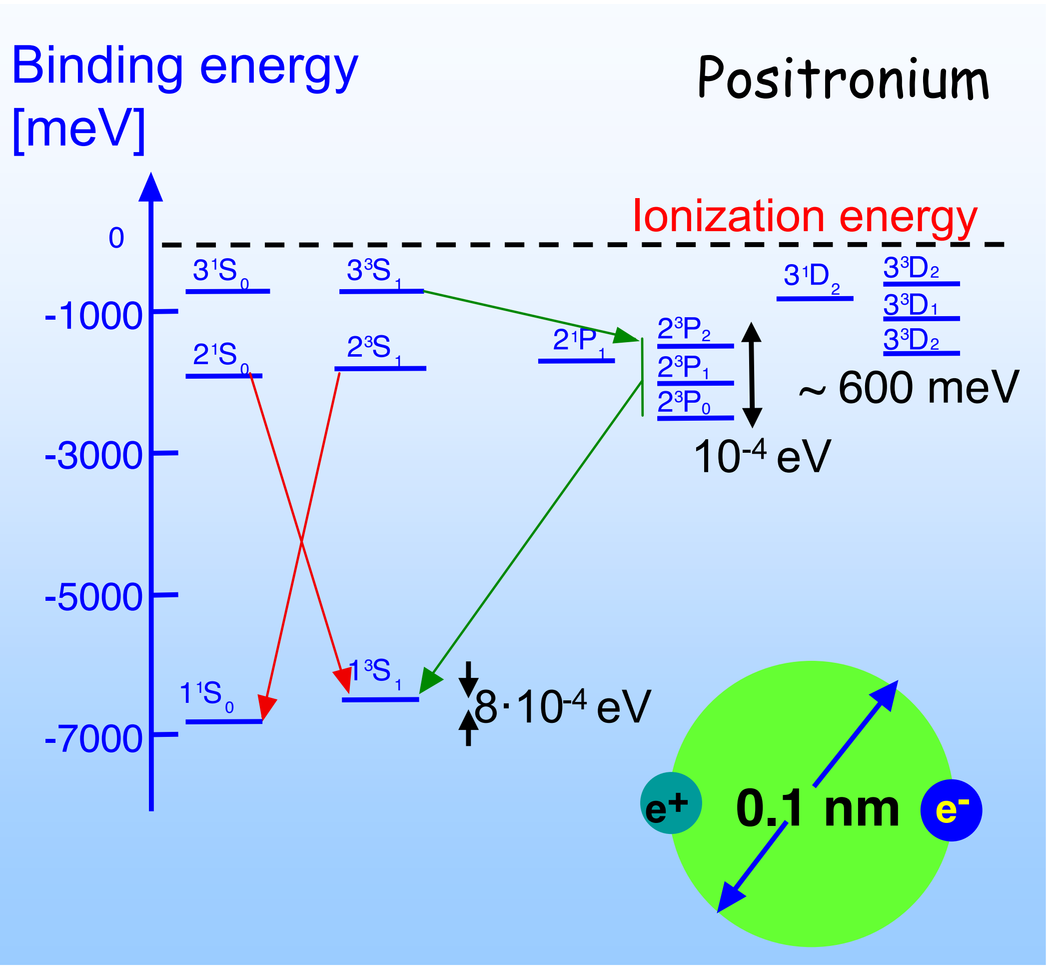
α_s may freeze

E.g.:

A. C. Aguilar, D. Binosi, J. Papavassiliou, J. Rodriguez-Quintero, PRD 80 (2009) 085018

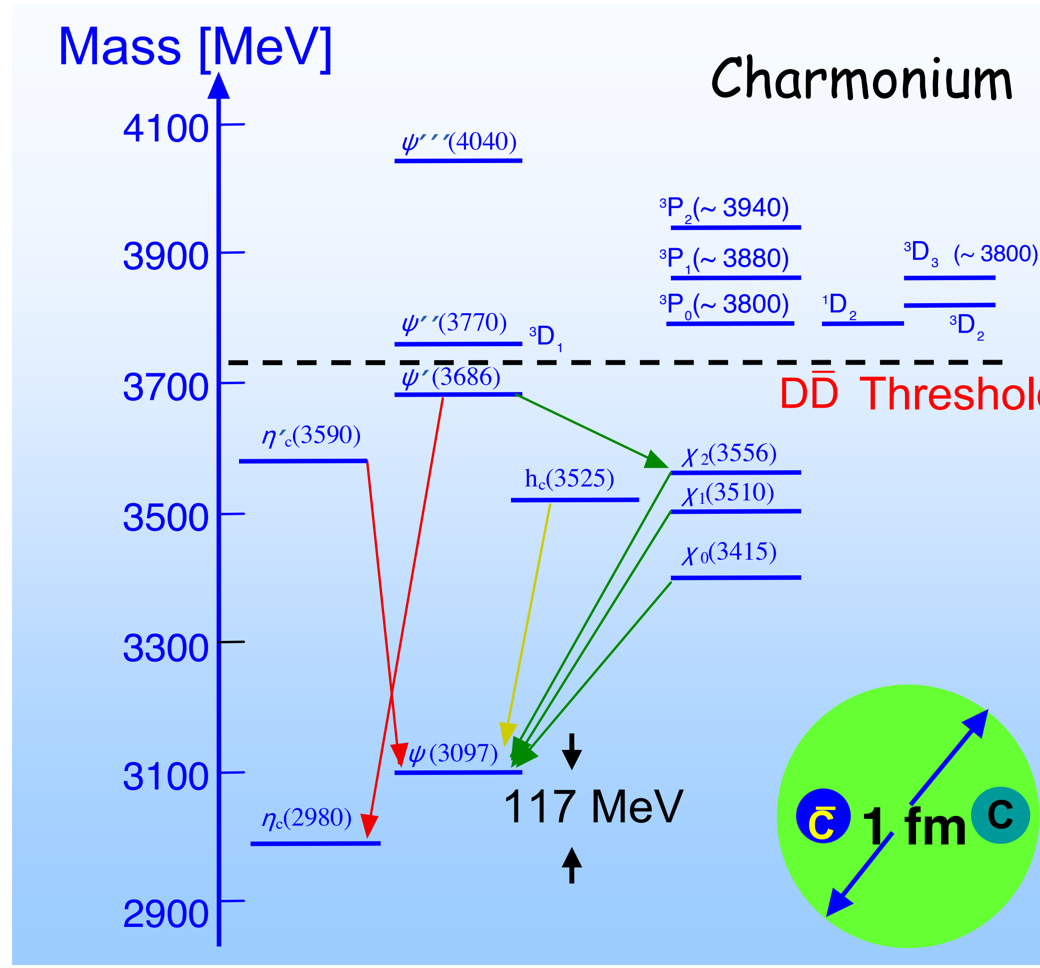
"The J/ψ is the Hydrogen atom of QCD"

QED



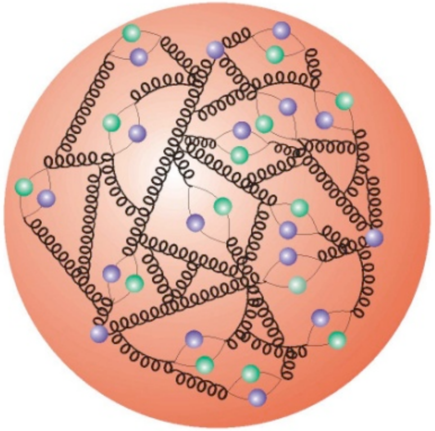
$$V(r) = -\frac{\alpha}{r}$$

QCD



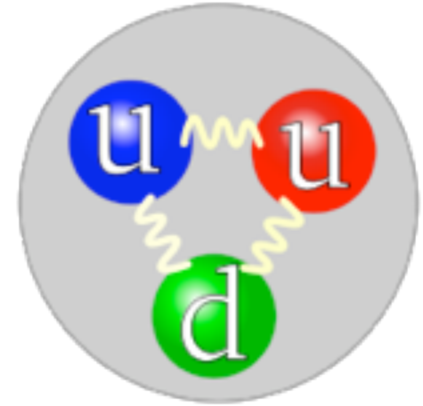
$$V(r) = cr - \frac{4}{3}\frac{\alpha_s}{r}$$

Dichotomy of Proton structure



Parton Picture

DIS and QFT require an infinite # of constituents:
Sea quarks and gluons



Valence Picture

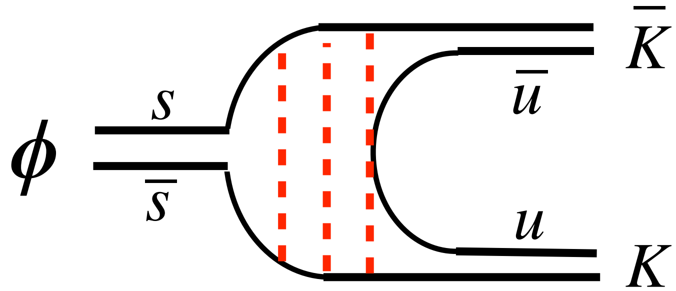
The hadron spectrum shows valence quark degrees of freedom only.

Relativistic bound states have multiparton Fock states and a valence quark spectrum

Okuba-Zweig-Iizuka Rule

$\phi(1020) \to K\bar{K}$

"Connected diagram"

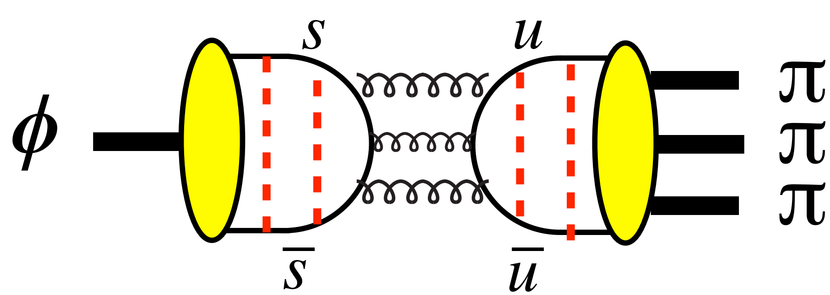


	ΔE	Br
$\phi(1020) \to K\bar{K}$	26 MeV	83.1 %
$\phi(1020) \not\to \pi\pi\pi$	610 MeV	15.3 %

String breaking caused by confining potential

$\phi(1020) \not\to \pi\pi\pi$

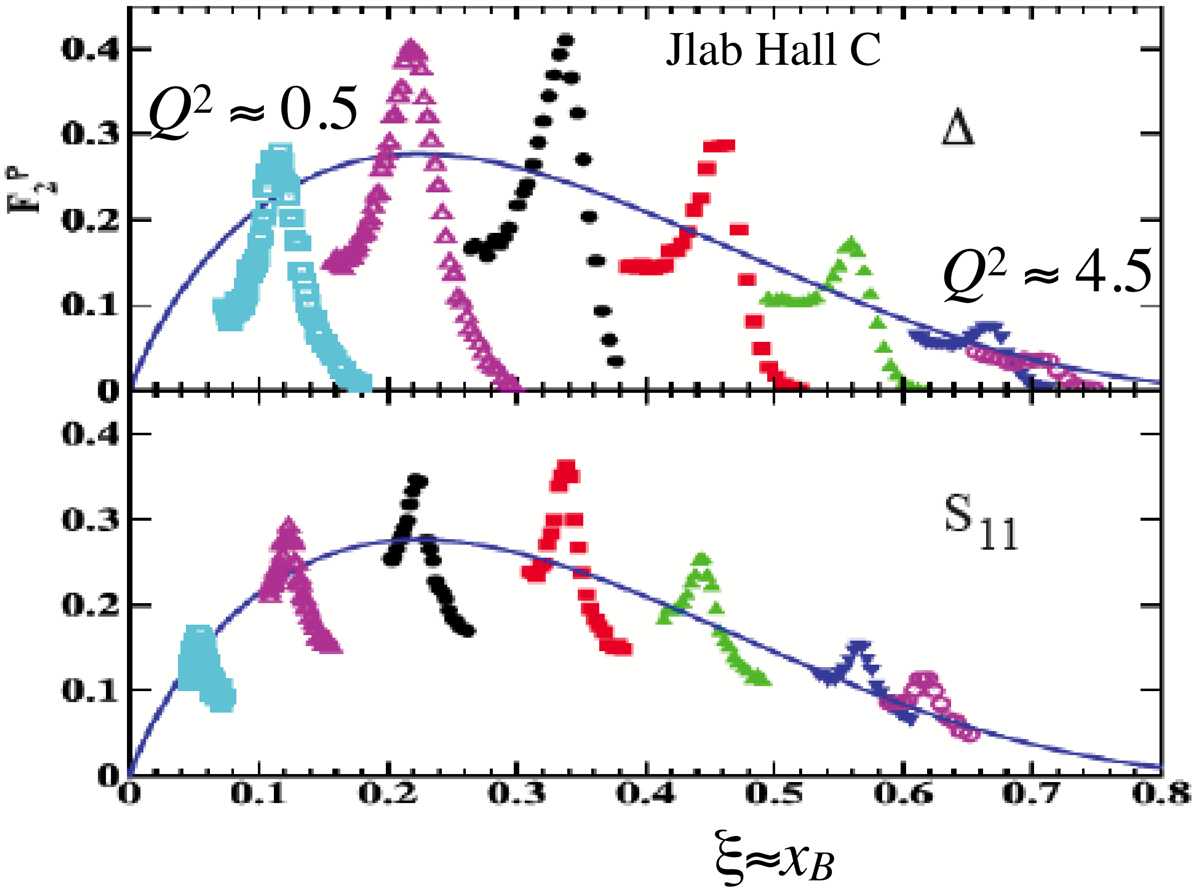
"Disconnected diagram"



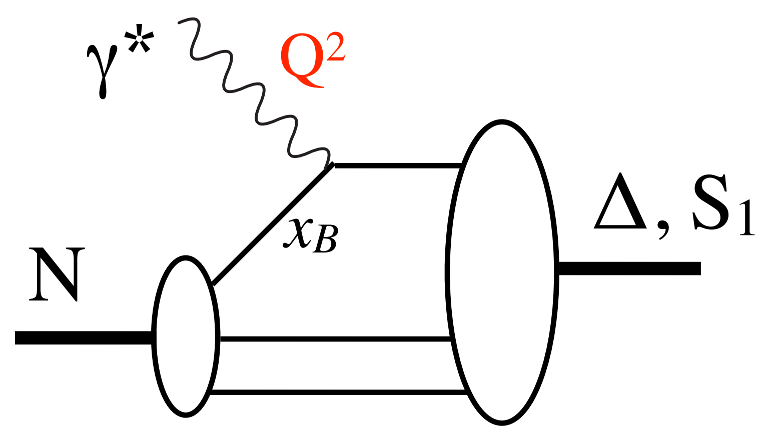
Perturbative gluon contributions are suppressed, even at low Q^2

Bloom-Gilman Duality

W. Melnitchouk et al, Phys. Rep. 406 (2005) 127



Resonance contributions
$ep \to eN^*$
build DIS scaling in
$ep \to eX$



Hadron wave functions describe ultra-relativistic (plane wave) partons.

We must consider bound states in an arbitrary frame.

Positronium from QED

The Coulomb potential A^0 may be expressed in terms of the electron fields,

$$-\boldsymbol{\nabla}^2 \hat{A}^0(t,\boldsymbol{x}) = e\psi^\dagger(t,\boldsymbol{x})\psi(t,\boldsymbol{x}) \quad\Rightarrow\quad \hat{A}^0(t,\boldsymbol{x}) = \int d^3\boldsymbol{y}\,\frac{e}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\psi^\dagger\psi(t,\boldsymbol{y})$$

In the rest frame we may neglect **A** (at lowest order in α).
The Hamiltonian can then be expressed in terms of the fermion fields only:

$$H_{QED}(\boldsymbol{A}=0) = \int d^3\boldsymbol{x}\bar{\psi}(-i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma} + \tfrac{1}{2}e\gamma^0\hat{A}^0 + m)\psi$$

An e^+e^- state at rest can be expressed as

$$\left|e^+e^-,t\right\rangle = \int d^3\boldsymbol{x}_1\,d^3\boldsymbol{x}_2\,\bar{\psi}_\alpha(t,\boldsymbol{x}_1)\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_\beta(t,\boldsymbol{x}_2)\left|0\right\rangle$$

where $\Phi_{\alpha\beta}(\boldsymbol{x}_1 - \boldsymbol{x}_2)$ is a 4 x 4 c-numbered, equal-time wave function.

Positronium from QED (cont.)

Denoting the binding energy by E_b the positronium state satisfies

$$H_{QED}\left|e^+e^-,t\right\rangle = (2m+E_b)\left|e^+e^-,t\right\rangle$$

Using $\{\psi_\alpha^\dagger(t,\boldsymbol{x}),\psi_\beta(t,\boldsymbol{y})\} = \delta_{\alpha\beta}\delta^3(\boldsymbol{x}-\boldsymbol{y})$ this gives the BSE for $\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\} + m\left[\gamma^0,\Phi(\boldsymbol{x})\right] = \left[2m+E_b-V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

Writing the wave function in 2 x 2 block form: $\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}$

and taking the non-relativistic limit as in the Dirac equation,

$$m = \mathcal{O}\left(\alpha^0\right) \qquad \boldsymbol{\nabla} = \mathcal{O}\left(\alpha\right) \qquad E_b,\ V = \mathcal{O}\left(\alpha^2\right)$$

we find the Schrödinger equation: $\left(-\dfrac{\boldsymbol{\nabla}^2}{m}+V\right)\Phi_{12} = E_b\Phi_{12}$

Positronium at relativistic CM momentum

For Positronium in motion wave functions defined at equal time t (IF) differ from the frame independent, equal LF time $x^+ = t+z$ wave functions.

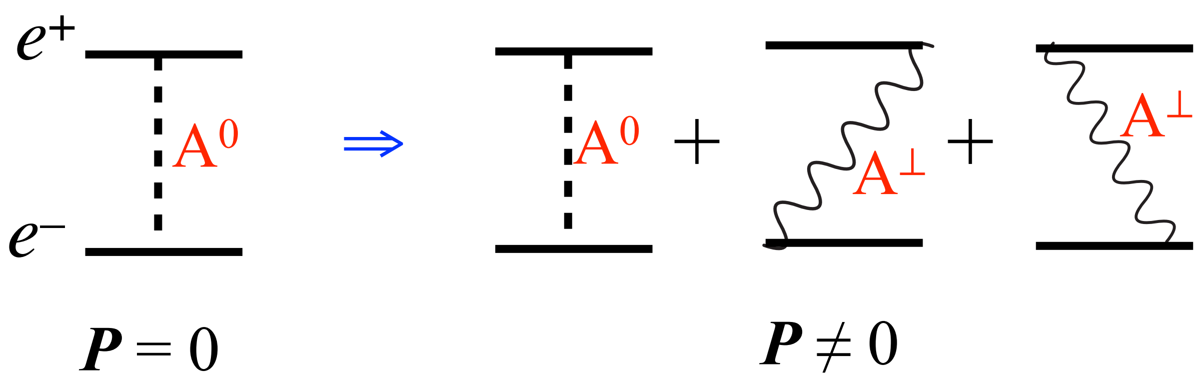
The IF wf's are (classically) expected to Lorentz contract.

In QFT their boost dependence is non-trivial (dynamical).

M. Järvinen, hep-ph/0411208

Coulomb (A^0) exchange dominates the kernel only in the rest frame, $\boldsymbol{P} = 0$.

When $\boldsymbol{P} \neq 0$ also transverse photon exchange contributes:



Positronium in motion is thus described by two ET Fock states:

$$\left|Pos., \boldsymbol{P} = 0\right\rangle = \Phi_0 \left|e^+ e^-\right\rangle + \Phi_\gamma \left|e^+ e^- \gamma\right\rangle$$

Positronium in the $P \to \infty$ frame might serve as a model for LF spin effects.

Hamiltonian formulation of the Dirac state

Similarly to Positronium, define the Dirac state as

$$|M,t\rangle = \int d^3\boldsymbol{x}\,\psi^\dagger(t,\boldsymbol{x})\Psi(\boldsymbol{x})|0\rangle$$

c-numbered spinor

The QED Hamiltonian for a fixed external field $A^0{}_Z$ is

$$H(t) = \int d^3\boldsymbol{x}\psi^\dagger(t,\boldsymbol{x})\big[-i\boldsymbol{\nabla}\cdot\gamma^0\boldsymbol{\gamma} + eA_Z^0(\boldsymbol{x}) + m\gamma^0\big]\psi(t,\boldsymbol{x})$$

$$H|M,t\rangle = \int d^3\boldsymbol{x}\,\big[H,\psi^\dagger(t,\boldsymbol{x})\big]\,\Psi(\boldsymbol{x})|0\rangle = M|M,t\rangle$$

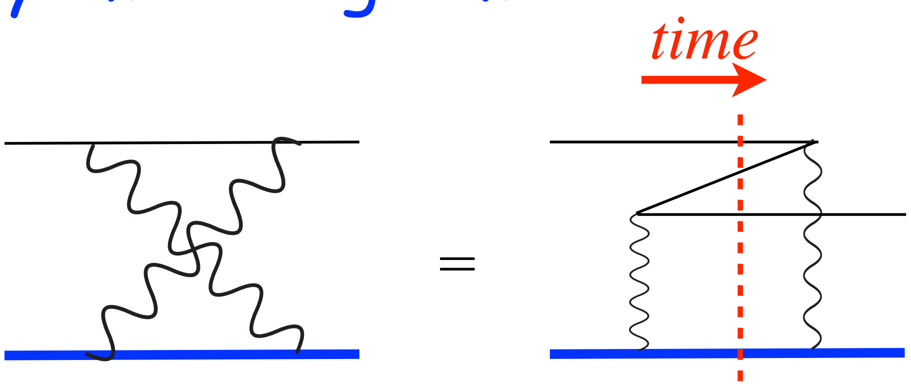
$\Rightarrow$ $(-i\boldsymbol{\nabla}\cdot\gamma^0\boldsymbol{\gamma} + eA_Z^0 + m\gamma^0)\Psi(\boldsymbol{x}) = M\Psi(\boldsymbol{x})$ Dirac eq. for Ψ

This required: $H|0\rangle = 0$ No pair production in vacuum!

Nevertheless: The Dirac state contains e^+e^- pairs (*cf*. Klein paradox)

Dirac equation from Feynman diagrams

Crossed, instantaneous Coulomb exchange corresponds to intermediate states with particle pairs.



For states with $M > 0$ the $i\varepsilon$ prescription at the $p^0 < 0$ pole of the electron propagator is irrelevant: We may use retarded boundary conditions $|0\rangle_R$

$$S_R(p^0, \boldsymbol{p}) = i\frac{\not{p} + m_e}{(p^0 - E_p + i\varepsilon)(p^0 + E_p + i\varepsilon)}$$

!

Also $p^0 < 0$ components move forward in time

$\Rightarrow$ • Only single electron intermediate states: $H|0\rangle_R = 0$

• $\Psi^\dagger\Psi(\boldsymbol{x})$ is an *inclusive* particle density.

The infinite number of pairs is described by a single electron wave function.

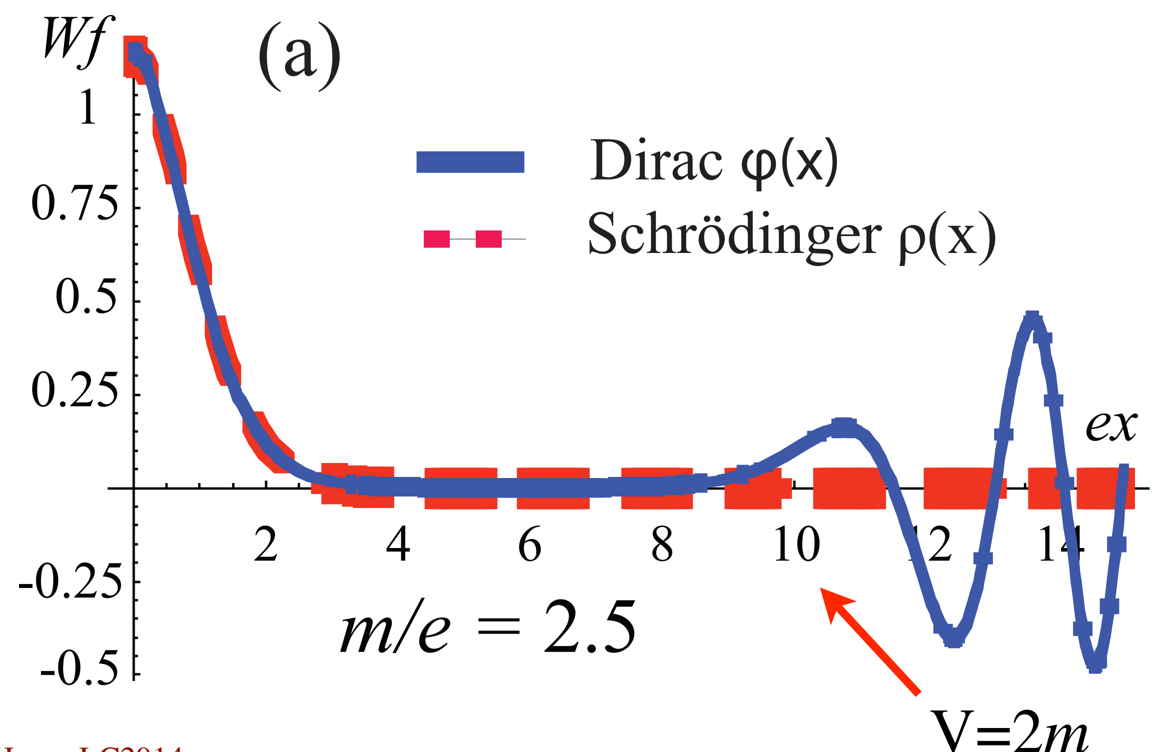
Dirac vs. Schrödinger wf's in D=1+1

Representing the Dirac matrices as 2x2 Pauli matrices, the Dirac eq. is:

$$\left[-i\sigma_1\partial_x + \tfrac{1}{2}e^2|x| + m\sigma_3\right]\begin{bmatrix}\varphi(x)\\ \chi(x)\end{bmatrix} = M\begin{bmatrix}\varphi(x)\\ \chi(x)\end{bmatrix}$$

The wf's $\phi(x)$, $\chi(x)$ are given by ${}_1F_1$-functions. For large m, they approach the Schrödinger wf's in the region of x where $V(x) \ll m$.

Pair contributions are manifest for $V(x) = \frac{1}{2}e^2|x| \geq 2m$



For polynomial potentials the Dirac wave function is not normalizable, and the mass spectrum M is continuous.

Its normalizability for the $V(r) = -\alpha/r$ potential in D=3+1 is an exception.

AUGUST 1, 1932 PHYSICAL REVIEW VOLUME 41

The Dirac Electron in Simple Fields*

By MILTON S. PLESSET
Sloane Physics Laboratory, Yale University

(Received June 6, 1932)

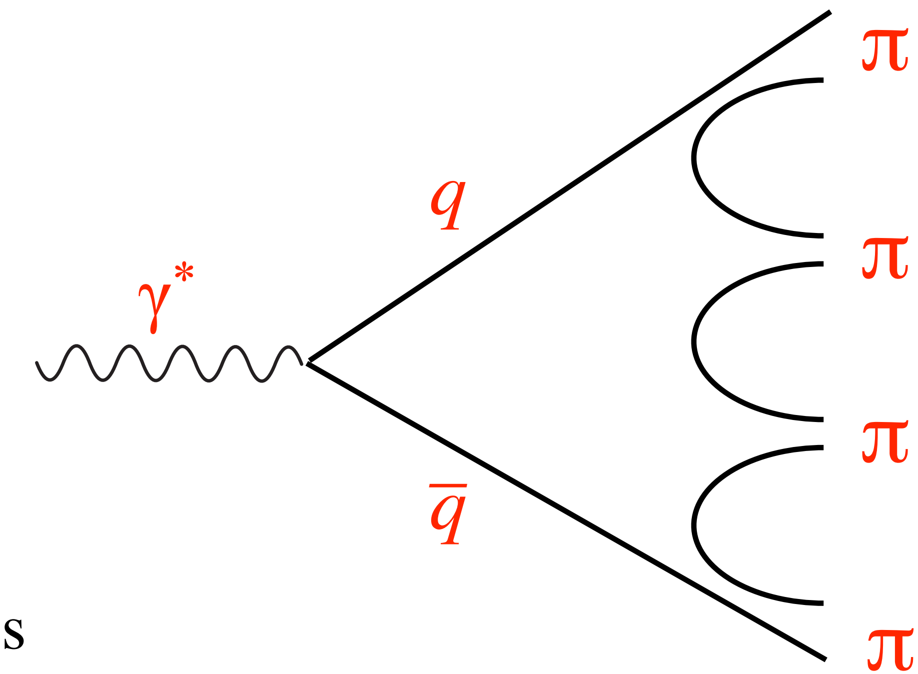
The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in x, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in r, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

See also: E. C. Titchmarsh, Proc. London Math. Soc. (3) 11 (1961) 159 and 169; Quart. J. Math. Oxford (2), 12 (1961), 227.

Constant particle density for $x \to \infty$

$$\Psi(x \to \infty) \sim \exp(\pm i x^2/4) \quad \Rightarrow \quad \Psi^\dagger \Psi(x \to \infty) \sim const.$$

We expect a constant particle density for the (virtual) pairs created by a linear potential.



The above approach allows also to discuss relativistic $e^+ e^-$ bound states (without an external potential)

I first consider D = 1+1 dimensions, where the Coulomb potential is linear.

In contrast to the Dirac states, we can define momentum eigenstates,

and they are found to have discrete mass spectra.

$f\,\bar{f}$ bound states in D=1+1

A state with two fermions of energy E and momentum $P^1 = P$:

$$|E,P\rangle = \int dx_1 dx_2\, \bar\psi(t,x_1)\exp\left[\tfrac{1}{2}iP(x_1+x_2)\right]\Phi(x_1-x_2)\psi(t,x_2)|0\rangle$$

With $\hat{P}^\mu|0\rangle = 0$, these are eigenstates of the translation generators:

$\hat{P}^1|E,P\rangle = P|E,P\rangle$ Bound state has momentum P (by construction)

$\hat{P}^0|E,P\rangle = E|E,P\rangle$ Gives bound state equation for $\Phi(x)$:

$$i\partial_x\left\{\sigma_1,\Phi(x)\right\} + \left[-\tfrac{1}{2}P\sigma_1 + m\sigma_3,\Phi(x)\right] = \left[E-V(x)\right]\Phi(x)$$

where $V(x) = \frac{1}{2}e^2|x|$ and $\gamma^0 = \sigma_3,\quad \gamma^1 = i\sigma_2,\quad \gamma^0\gamma^1 = \sigma_1$

Here the CM momentum P is a parameter, thus E and Φ depend on P .

Boost covariance

It is essential and non-trivial that the state is covariant under boosts:

$$|E+d\xi P, P+d\xi E\rangle = (1-id\xi\hat{M}^{01})|E,P\rangle$$

M^{01} is the QED_2 boost generator

This holds only for a linear potential and ensures that $E(P)=\sqrt{P^2+M^2}$

The correct dependence $E(\boldsymbol{P})$ also holds in D = 3+1, for the linear potential.

The P-dependence of the wave function Φ can be explicitly given:

$$\Phi^P(\sigma) = e^{\gamma_0\gamma_1\zeta/2}\Phi^{(P=0)}(\sigma)e^{-\gamma_0\gamma_1\zeta/2}$$

where $dx = -\dfrac{d\sigma}{E-V(x)}$ and $\tanh\zeta = -\dfrac{P}{E-V}$

Solutions of the bound state equation (D=1+1, $m_1=m_2$)

For a linear $V(x)$ the "invariant length" $\sigma = \Pi^2$

where the "kinetic 2-momentum" is $\Pi^\mu(x) \equiv (P-eA)^\mu = (E-V(x), P)$

and thus $\Pi^2 \equiv \sigma \equiv (E-V)^2 - P^2 = M^2 - 2EV + V^2$

Expanding the 2x2 wave function as $\Phi = \Phi_0+\sigma_1\Phi_1+\sigma_2\Phi_2+\sigma_3\Phi_3$ the bound state equation reduces to two coupled, frame-independent equations:

$$-2i\partial_\sigma \Phi_1(\sigma) = \Phi_0(\sigma) \qquad -2i\partial_\sigma \Phi_0(\sigma) = \left[1 - \frac{4m^2}{\sigma}\right]\Phi_1(\sigma)$$

with the general solution

$$\Phi_1(\sigma) = \sigma\, e^{-i\sigma/2}\left[a\,{}_1F_1(1-im^2, 2, i\sigma) + b\,U(1-im^2, 2, i\sigma)\right]$$

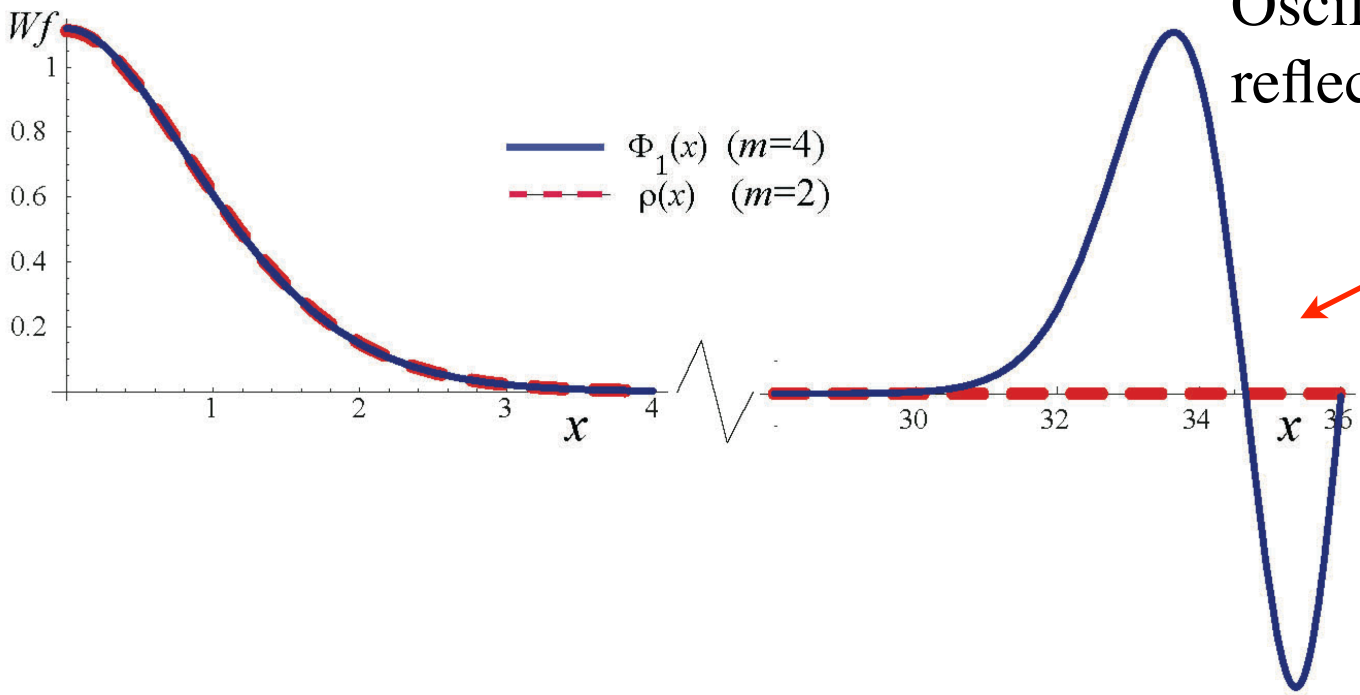
If $b \neq 0$ the wf Φ is singular at $\sigma = 0$. Requiring $b = 0$ the spectrum is discrete.

Note: This constraint only applies for $m \neq 0$.

Some numerical results

Nearly non-relativistic case: $m = 4.0e$

Schrödinger (Airy fn.) wf. $\varrho(x)$.



Oscillations for $V(x) > 2m$ reflect virtual pair production

In the limit of small fermion mass m:

$$M_n^2 = \pi n + \mathcal{O}\left(m^2\right) \;\; ; \qquad n = 0, 1, 2, \ldots$$

Parity $= (-1)^{n+1}$ No parity doublets for $m \neq 0$

Infinite Momentum Frame (IMF)

The wf is frame invariant as fn of $\sigma = (E-V)^2 - P^2$. Since $V(x) = \frac{1}{2}|x|$:

$$x = 2\left(E \pm \sqrt{P^2+\sigma}\,\right)$$

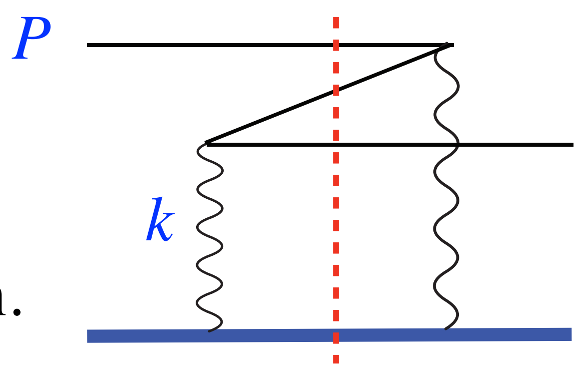
For $P \to \infty$ at fixed σ: $$x \simeq 2(E \pm P) \pm \frac{\sigma}{P} \simeq \begin{cases} 4P + \sigma/P \\ (M^2-\sigma)/P \end{cases}$$

Upper solution: $x \approx 4P \to \infty$ Pair production moves to infinite x.

Lower solution: $x \propto 1/P$ Lorentz-contracted "valence" region.

Perturbatively: "Z-diagrams" get infinite energy ($k \to \infty$) in the $P \to \infty$ limit.

This seems related to $H|0\rangle = 0$ in LF quantization.



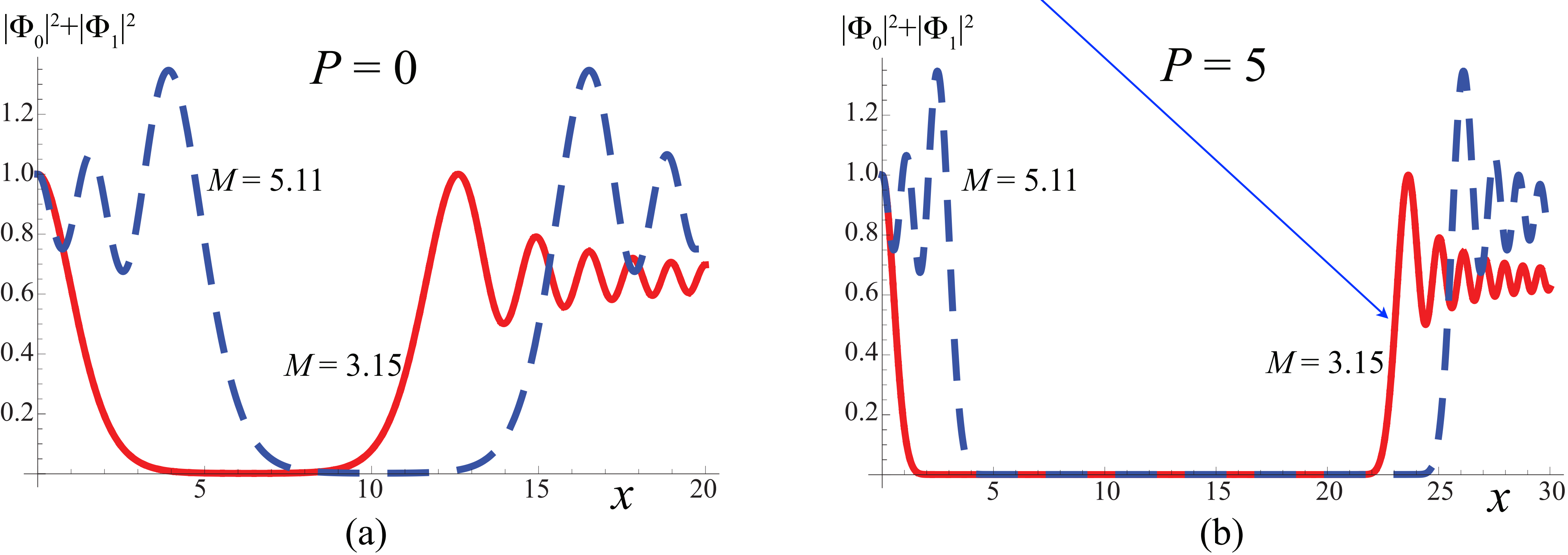
Explicitly: $$\Phi_{P\to\infty}(\sigma) = 2am\,P\gamma^+ e^{-i\sigma/2}\,{}_1F_1(1-im^2, 2, i\sigma)$$

Frame (P) dependence of the solutions ($m_1 \neq m_2$)

Comparison of ground and excited state wave functions for P=0 (CM frame) and for $P = 5e$.

m_1=1.0e m_2=1.5e

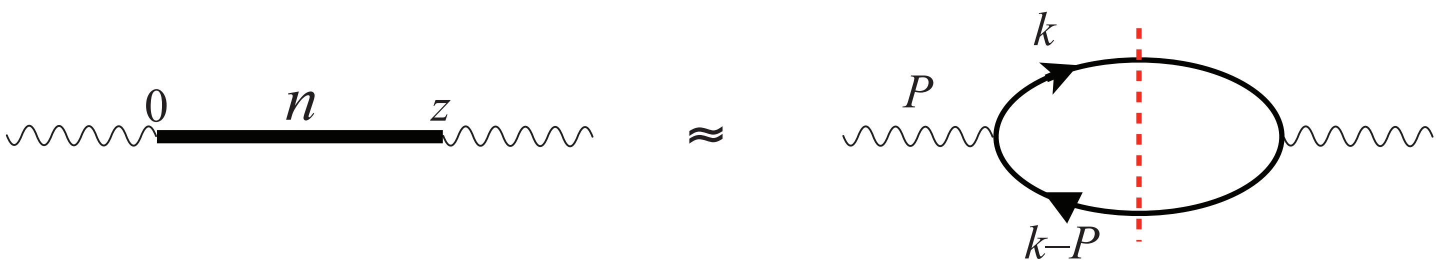
Moves away in IMF ($P \to \infty$ limit)



Note: In the IMF limit, only the normalizable, low x part of the wf remains.

Quark - Hadron duality

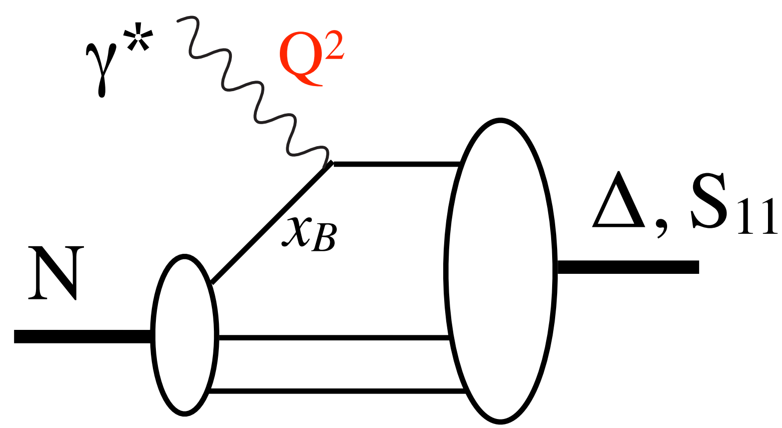
The wave functions of highly excited (large mass M) bound states can be normalized by comparison with free parton loop contributions to current propagators. All currents give consistent results.



$$\Rightarrow \quad |\Phi_0(x{=}0)|^2 = |\Phi_1(x{=}0)|^2 = \pi/2$$

Consistency with Bloom-Gilman duality:

At large M, and for $V(x) << M$,
the wave function reduces to a free $f\bar{f}$ pair
with momenta $k = \pm M/2$ (in the CM).



B-G Duality

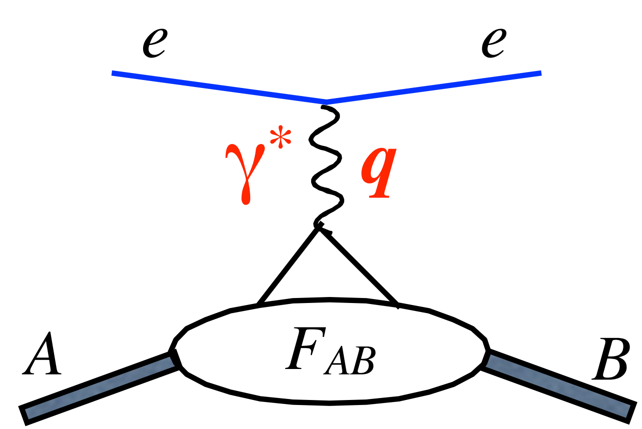
EM Form Factor

$$F^\mu_{AB}(z) = \langle B(P_B); t=+\infty | j^\mu(z) \, | A(P_A); t=-\infty \rangle$$

A, B: *in* & *out* states

EM current:

$$j^\mu(z) = \bar\psi(z)\gamma^\mu\psi(z) = e^{i\hat{P}\cdot z} j^\mu(0) e^{-i\hat{P}\cdot z}$$



Using anticommutators of fields:

$$F^\mu_{AB}(z) \;=\; e^{i(P_B-P_A)\cdot z}$$

$$\times \int_{-\infty}^{\infty} dx\, e^{i(P_B^1-P_A^1)x/2} \left\{ \mathrm{Tr}\left[\Phi_B^\dagger(x)\gamma^\mu\gamma^0\Phi_A(x)\right] - \eta_A\eta_B \mathrm{Tr}\left[\Phi_B(x)\gamma^0\gamma^\mu\Phi_A^\dagger(x)\right] \right\}$$

Gauge invariance is valid: $\partial_\mu F^\mu_{AB}(z) = 0$ (also in D = 3+1)

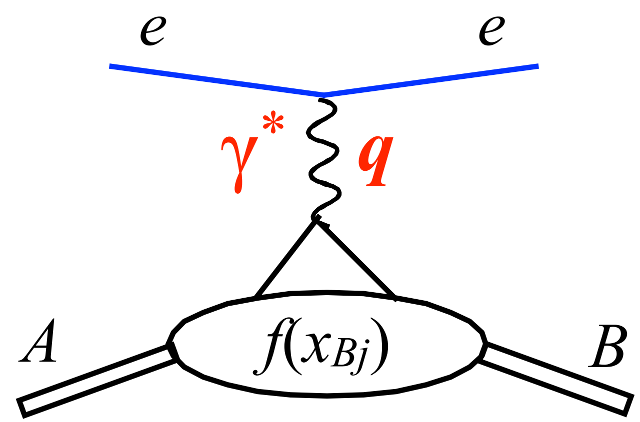
The invariant form factor is frame independent (was checked numerically):

$$F_{AB}(Q^2) = -4i\frac{1-\eta_A\eta_B}{q^1}\int_0^\infty dx\, \sin\left(\frac{q^1 x}{2}\right)$$

$$\times \left[\Phi_{0B}^*(x)\Phi_{0A}(x) + \Phi_{1B}^*(x)\Phi_{1A}(x)\left(1+\frac{4m^2}{\sigma_A\sigma_B}\tilde{\Pi}_A\cdot\Pi_B\right)\right]$$

Parton Distribution: γ*A → B

$$x_{Bj} = \frac{Q^2}{2p_A \cdot q} \qquad \text{fixed}$$



$$M_B^2 = Q^2\left(\frac{1}{x_{Bj}} - 1\right) \to \infty$$

From analogy to D=3+1:
$$f(x_{Bj}) = \frac{1}{8\pi m^2}\frac{1}{x_{Bj}}|Q^2 F_{AB}(Q^2)|^2$$

For large M_B use asymptotic form of Φ_B .

Result scales in $v = xQ/2$ (Breit frame)
$$\sigma_A \simeq M_A^2 - \frac{|v|}{x_{Bj}} \equiv \tau_A$$

$$Q^2 F_{AB}(\eta_B = -) \simeq -4i\sqrt{2\pi}(1+\eta_A)$$

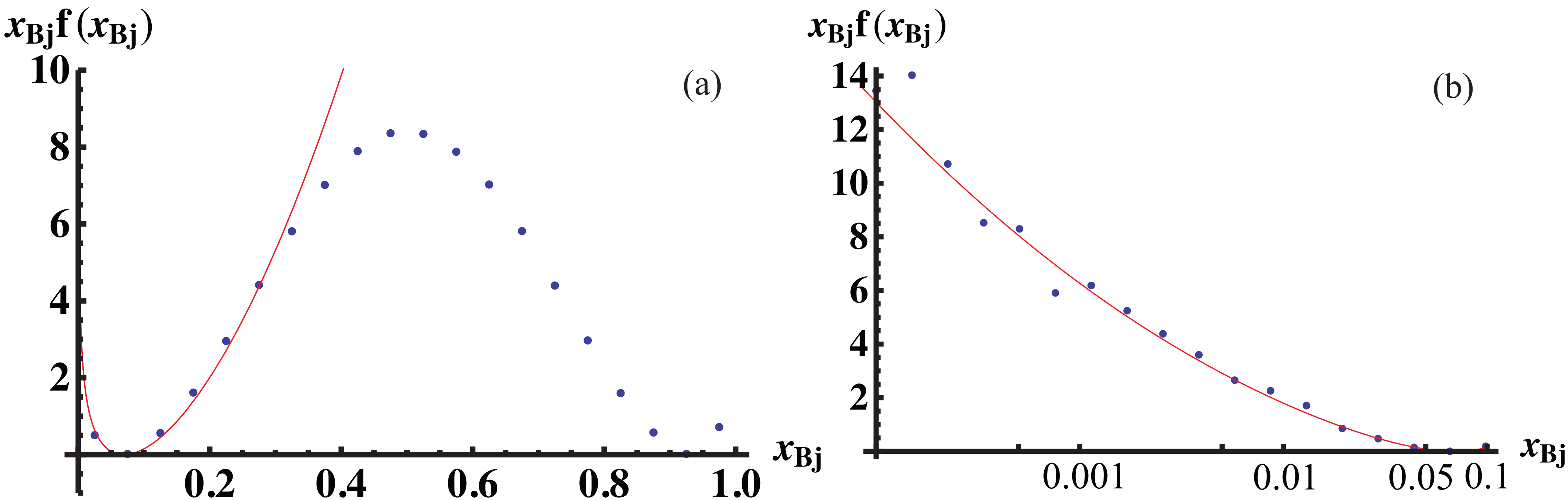
$$\times \int_0^\infty dv\, \sin v \left[\cos\left(\frac{v}{2x_{Bj}}\right) i\Phi_{0A}(\tau_A) - \sin\left(\frac{v}{2x_{Bj}}\right)\Phi_{1A}(\tau_A)\left(1 + \frac{2m^2}{x_{Bj}\tau_A}\right)\right]$$

An analytic/numerical evaluation shows a sea quark distribution at low x_{Bj}

Result for the parton distribution

The parton distribution of the ground state has a sea component at low m/e :

$$m/e = 0.1$$



(log scale in x_{Bj})

The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

Note: Enhancement at low x is not due to Φ_A^{IMF}

A linear potential in D=3+1 QCD

Dokshitzer: Confinement in QCD is governed by classical fields (2013)

Zwanziger: No confinement without Coulomb confinement (2003)

Gribov: Coulomb interaction rearranges the vacuum for $\alpha > \alpha^{crit}$ (1997):

$$\alpha^{crit}(\mathrm{QED}) = \pi\left(1 - \sqrt{\frac{2}{3}}\right) \simeq 0.58 \qquad \gg \frac{1}{137}$$

$$\alpha_s^{crit}(\mathrm{QCD}) = \frac{\pi}{C_F}\left(1 - \sqrt{\frac{2}{3}}\right) \simeq 0.43 \quad \gtrsim \alpha_s(m_\tau^2) \simeq 0.33$$

The Coulomb field is instantaneous, thus consistent with valence Fock states.

Gauss' law allows to express A^0 in terms of the propagating fields.

A homogeneous solution of Gauss' law

$$\hat{A}_a^0(t,\boldsymbol{x}) \propto \int d^3\boldsymbol{y}\,\psi_A^\dagger(t,\boldsymbol{y})T_a^{AB}\psi_B(t,\boldsymbol{y})\,\boldsymbol{x}\cdot\boldsymbol{y}$$

$\nabla^2 A^0(t,\boldsymbol{x}) = 0$

This solution is of $O(g^0)$, and has no Feynman diagram equivalent.

Only color singlet bound states are invariant under space translations.

Much of the analysis in D=1+1 can be repeated (work in progress).

The above A^0 leads to a linear potential for color singlet mesons:

$$V_{\mathcal{M}}(\boldsymbol{x}_1-\boldsymbol{x}_2) = \tfrac{1}{2}\sqrt{C_F}\,g\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|$$

The corresponding potential for color singlet baryons is:

$$V_{\mathcal{B}}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \tfrac{1}{2\sqrt{2}}\sqrt{C_F}\,g\Lambda^2\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+(\boldsymbol{x}_2-\boldsymbol{x}_3)^2+(\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$$

Note: $V_{\mathcal{B}}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) = V_{\mathcal{M}}(\boldsymbol{x}_1-\boldsymbol{x}_2)$

The Meson state

A meson state of CM momentum $\boldsymbol{P}$ is expressed as

$$|\mathcal{M};E,\boldsymbol{P}\rangle = \int d^3\boldsymbol{x}_1\, d^3\boldsymbol{x}_2\, e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\, \bar{\psi}_A(t,\boldsymbol{x}_1)\Phi_{\mathcal{M}}^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_B(t,\boldsymbol{x}_2)\,|0\rangle$$

with the color structure $\Phi_{\mathcal{M}}^{AB}(\boldsymbol{x}_1-\boldsymbol{x}_2) = \frac{1}{\sqrt{N_C}}\delta^{AB}\Phi_{\mathcal{M}}(\boldsymbol{x}_1-\boldsymbol{x}_2)$

$H|\mathcal{M};E,\boldsymbol{P}\rangle = E|\mathcal{M};E,\boldsymbol{P}\rangle$ for the $O(g)$ Hamiltonian imposes the BSE:

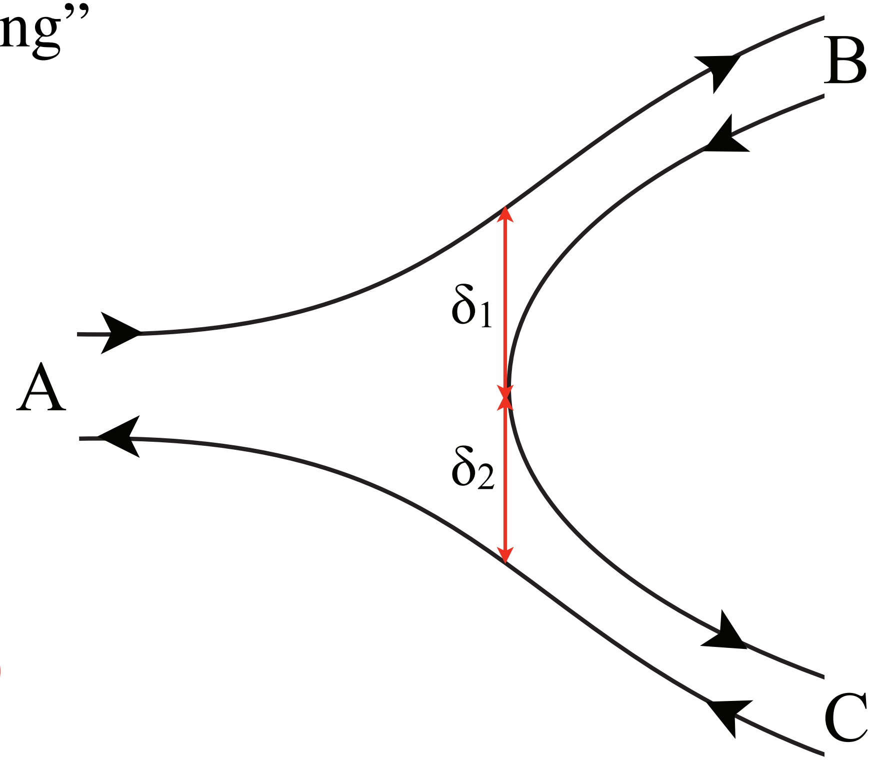
$$i\boldsymbol{\nabla}\cdot\{\gamma^0\boldsymbol{\gamma},\Phi_{\mathcal{M}}(\boldsymbol{x})\} - \tfrac{1}{2}\boldsymbol{P}\cdot\left[\gamma^0\boldsymbol{\gamma},\Phi_{\mathcal{M}}(\boldsymbol{x})\right] + m\left[\gamma^0,\Phi_{\mathcal{M}}(\boldsymbol{x})\right]$$

$$= \left[E - V_{\mathcal{M}}(\boldsymbol{x})\right]\Phi_{\mathcal{M}}(\boldsymbol{x})$$

In the rest frame ($\boldsymbol{P} = 0$) the radial and angular variables can be separated.

String breaking: $A \to B+C$

The linear potential induces "string breaking" at large separations of the quarks. The Poincaré invariant amplitude is given by the wave functions:



$$\langle B,C|A\rangle = -\frac{(2\pi)^3}{\sqrt{N_C}}\delta^3(\boldsymbol{P}_A-\boldsymbol{P}_B-\boldsymbol{P}_C)$$

$$\times\int d\boldsymbol{\delta}_1 d\boldsymbol{\delta}_2\, e^{i\boldsymbol{\delta}_1\cdot\boldsymbol{P}_C/2-i\boldsymbol{\delta}_2\cdot\boldsymbol{P}_B/2}\mathrm{Tr}\left[\gamma^0\Phi_B^\dagger(\boldsymbol{\delta}_1)\Phi_A(\boldsymbol{\delta}_1+\boldsymbol{\delta}_2)\Phi_C^\dagger(\boldsymbol{\delta}_2)\right]$$

When squared, this gives a hadron loop unitarity correction.