

# Excited meson spectroscopy and transitions from lattice QCD

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With Jo Dudek, Robert Edwards, Mike Peardon,  
David Richards and the *Hadron Spectrum Collaboration*



# Outline

- Introduction and motivation
- Excited spectra from LQCD – method outline
- Isovector meson spectra
- Multi-meson states and scattering
- Photocouplings in charmonium
- Summary and outlook

PR D79 094504 (2009)  
PRL 103 262001 (2009)  
PR D82 034508 (2010)  
arXiv:1011.6352

# Motivation

Renaissance in excited charmonium spectroscopy

BABAR, Belle, BES, CLEO-c, ...

Upcoming experimental efforts (charmonium and light mesons)

GlueX (JLab), BESIII, PANDA, ...

Exotics ( $J^{PC} = \mathbf{1}^{-+}, \mathbf{2}^{+-}, \dots$ )? – can't just be a  $q\bar{q}$  pair

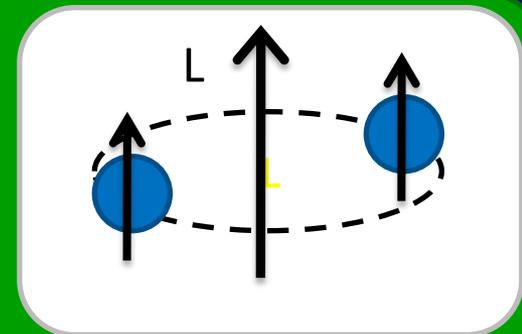
e.g. hybrids, multi-mesons

Quark-antiquark pair:  $2S+1L_J$

Parity:  $P = (-1)^{(L+1)}$

Charge Conj Sym:  $C = (-1)^{(L+S)}$

$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \dots$



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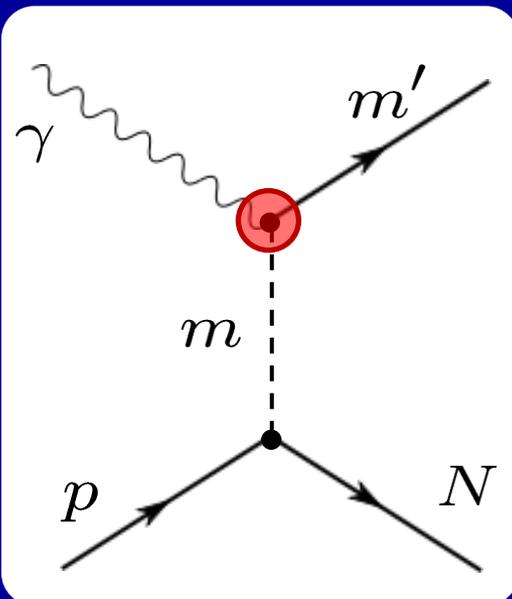
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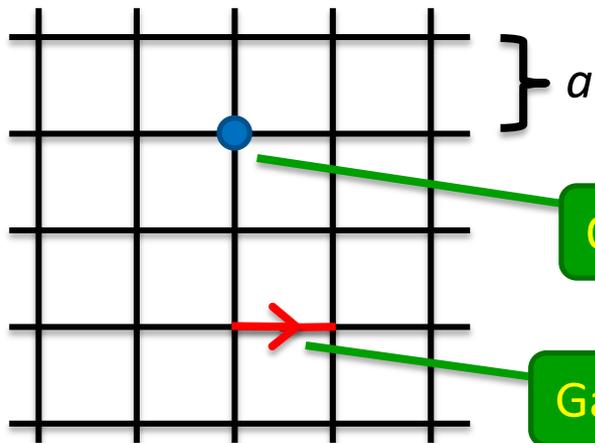
Photoproduction at GlueX (JLab 12 GeV upgrade)

Use Lattice QCD to extract excited spectrum...

... and photocouplings (tested in charmonium)



# QCD on a Lattice



Discretise on a grid (spacing =  $a$ ) – regulator

Finite volume  $\rightarrow$  finite no. of d.o.f.

Quarks fields on lattice sites

$$\psi(x) \rightarrow \psi_x$$

Gauge fields on links

$$A_\mu(x) \rightarrow U_{x,\mu} = e^{-aA_{x,\mu}}$$

Path integral formulation

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U f(\psi, \bar{\psi}, U) e^{iS[\psi, \bar{\psi}, U]}$$

Euclidean time:  $t \rightarrow i\tau$

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U f(\psi, \bar{\psi}, U) e^{-\tilde{S}[\psi, \bar{\psi}, U]}$$

Do fermion integral analytically then use importance sampling Monte Carlo

# Spectroscopy on the lattice

Calculate **energies** and **matrix elements** (“overlaps”,  $Z$ 's)  
from correlation functions of meson interpolating fields

$$C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$$

Construct operators which  
only overlap on to one spin  
in the continuum limit

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi}(x) \Gamma_i \overleftrightarrow{D}_j \overleftrightarrow{D}_k \dots \psi(x)$$

‘Distillation’ technology for constructing  
on lattice PR D80 054506 (2009)

( $p = 0$ )

definite  $J^{PC}$

$$Z_i^{(n)} \equiv \langle 0 | O_i | n \rangle$$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n t}}{2 E_n} \langle 0 | O_i(0) | n \rangle \langle n | O_j(0) | 0 \rangle$$

# Variational Method

Large basis of operators  $\rightarrow$  matrix of correlators

$$C_{ij}(t) = \langle 0 | O_i(t) O_j(0) | 0 \rangle$$

Generalised eigenvector problem:

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

Eigenvalues  $\rightarrow$  energies

$$\lambda^{(n)}(t) \rightarrow e^{-E_n(t-t_0)}$$

$$(t \gg t_0)$$

Eigenvectors  $\rightarrow$  optimal linear combination of operators to overlap on to a state

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

$Z^{(n)}$  related to eigenvectors

$$Z_i^{(n)} \equiv \langle 0 | O_i | n \rangle$$

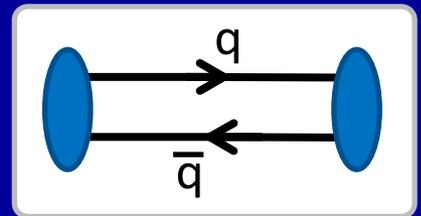
Var. method uses orthog of eigenvectors; don't just rely on separating energies

# Isovector spectra

- Dynamical (unquenched) calculation
- Anisotropic – finer in temporal dir ( $a_s/a_t = 3.5$ ),  $a_s \sim 0.12$  fm
- Two volumes:  $16^3$ ,  $20^3$  ( $L_s \approx 2.0, 2.4$  fm)

Lattice details in: PR D78 054501, PR D79 034502

- Only connected diagrams – Isovectors ( $I=1$ ) and kaons
- As an example: three degenerate ‘light’ quarks ( $N_f = 3$ ,  $M_\pi \approx 700$  MeV)
- Also ( $N_f = 2+1$ )  $M_\pi \approx 520, 440, 400$  MeV

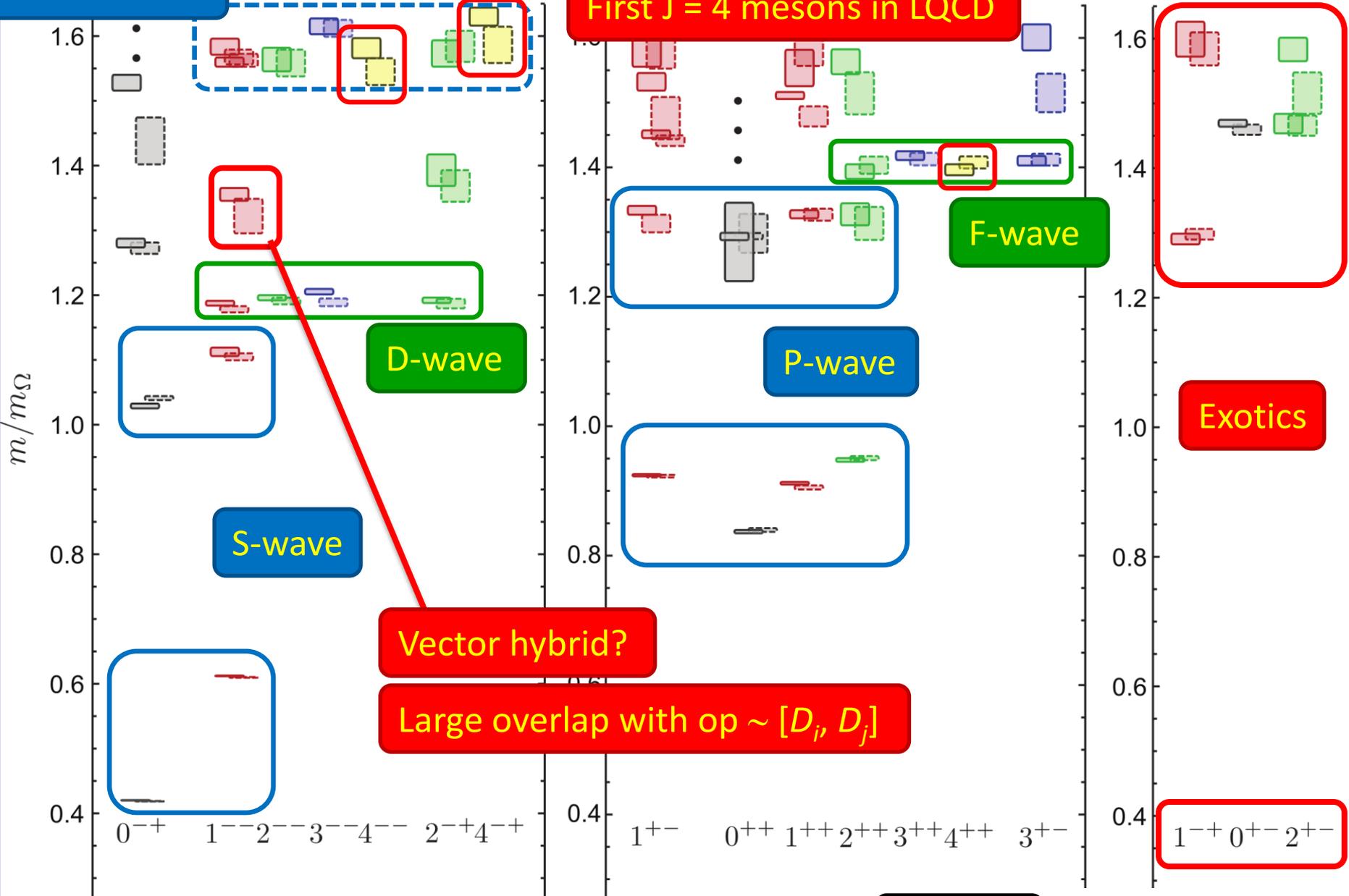


SU(3) sym

Method details and results: PRL 103 262001 (2009), PR D82 034508 (2010)

D + G-waves

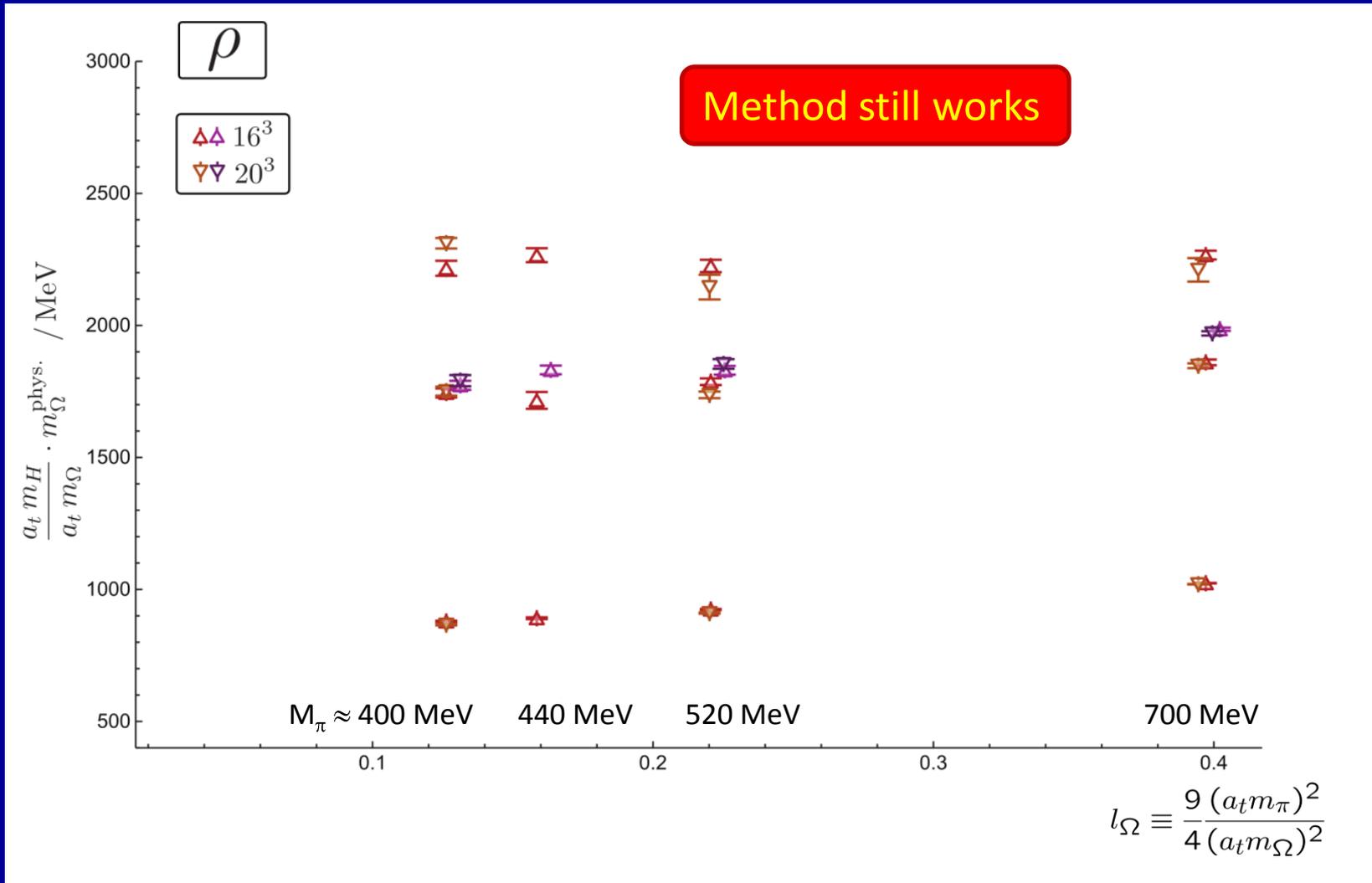
First J = 4 mesons in LQCD

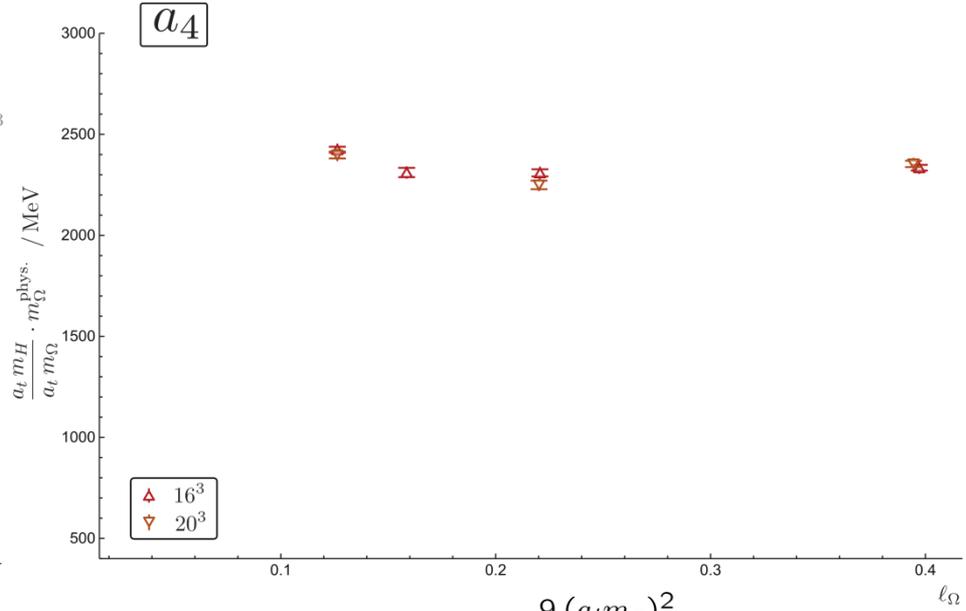
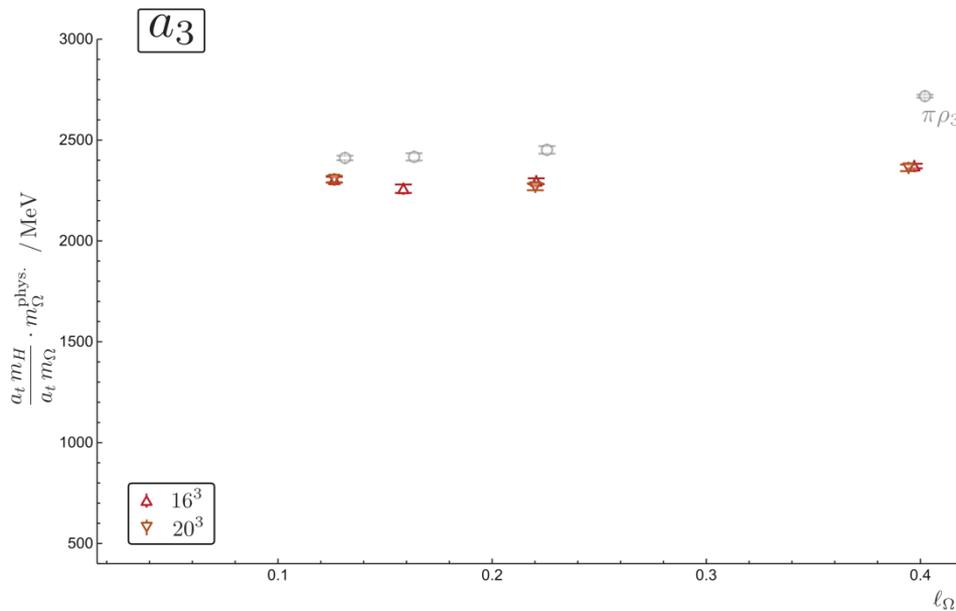
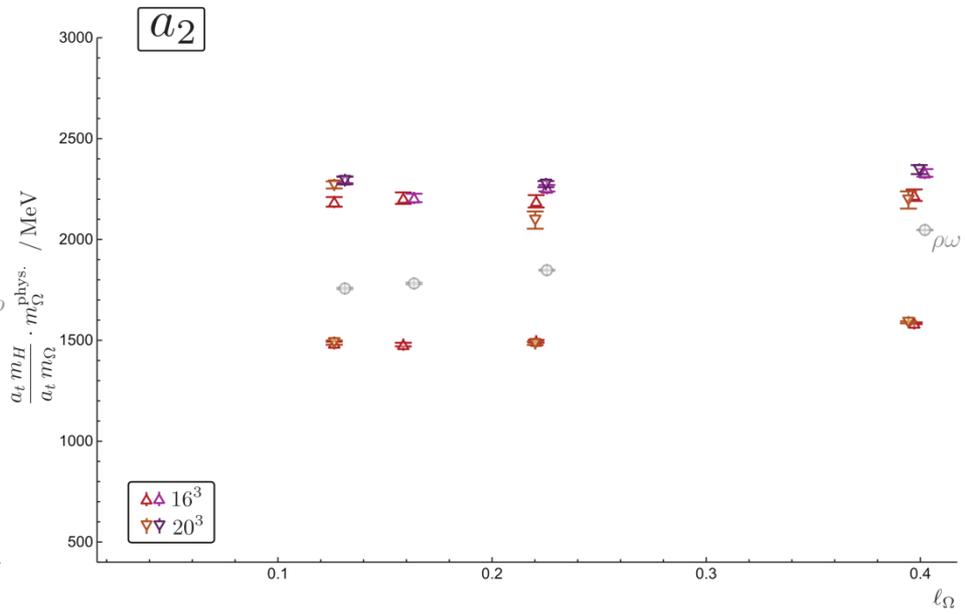
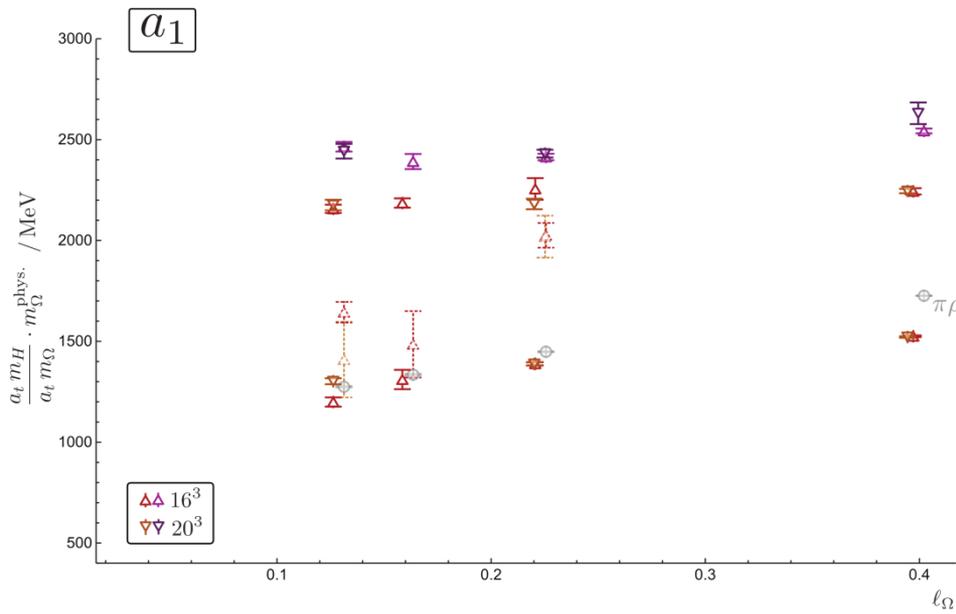


$N_f = 3$  isovectors

$16^3$  (~2 fm) and  $20^3$  (2.4 fm)

# Lower pion masses

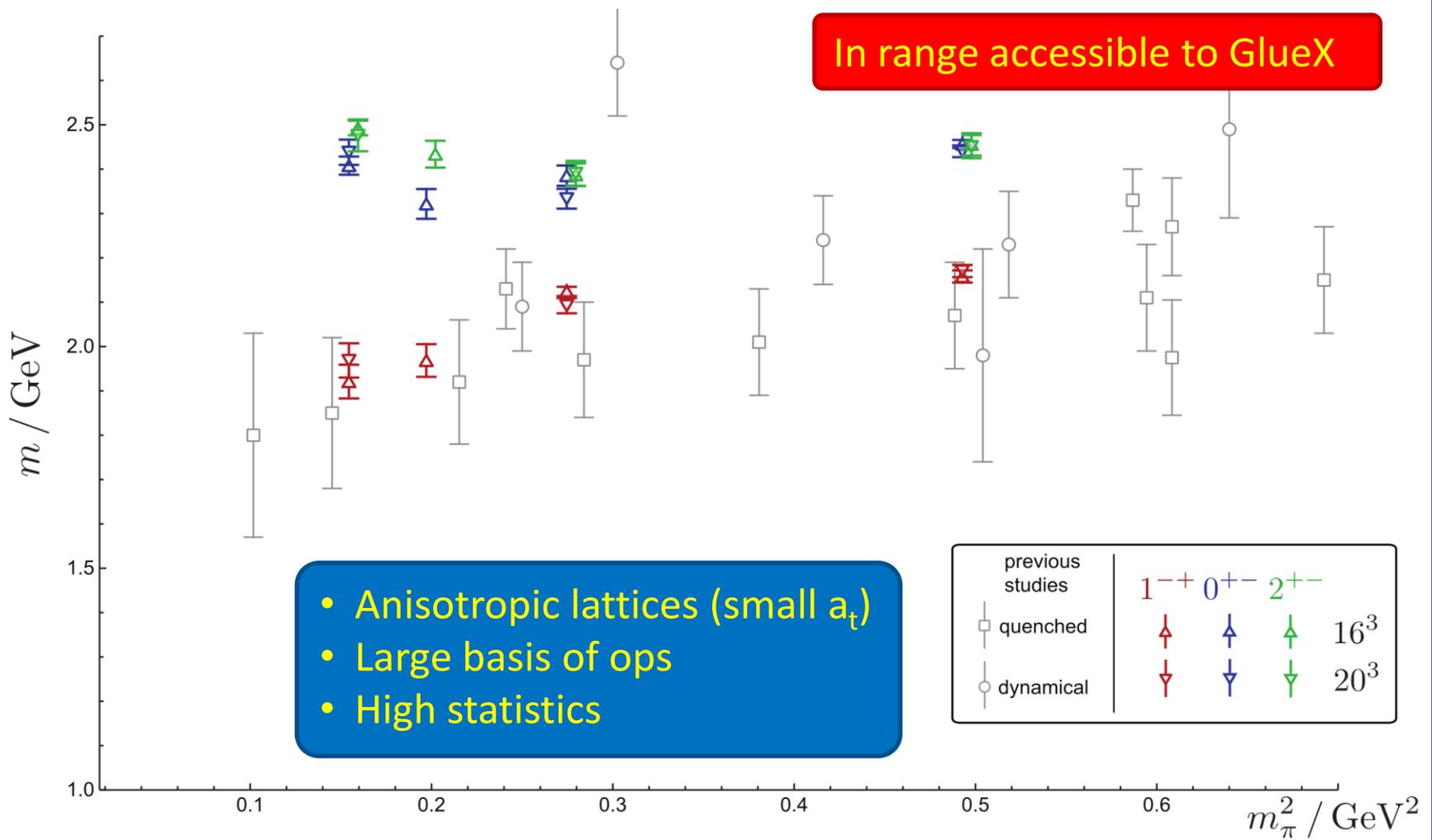




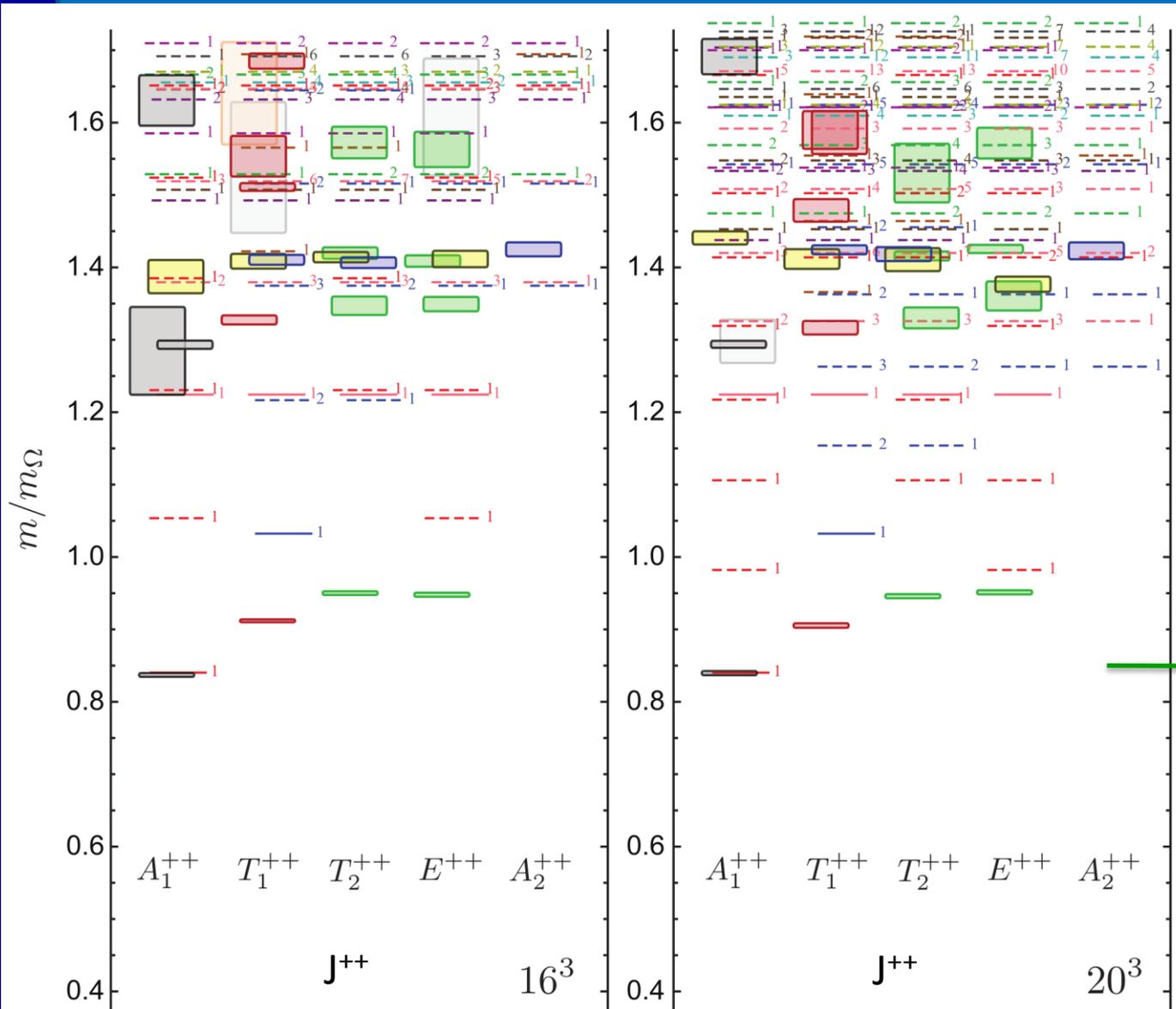
$$l_\Omega \equiv \frac{9 (a_t m_\pi)^2}{4 (a_t m_\Omega)^2}$$

# Exotics summary

In range accessible to GlueX



# Multi-particle states?



Finite box  
 → discrete allowed momenta  
 → discrete spectrum of multiparticle states

Expect two-meson states above  $2m_{\pi}$

$2m_{\pi} \sim 0.85 m_{\Omega}$

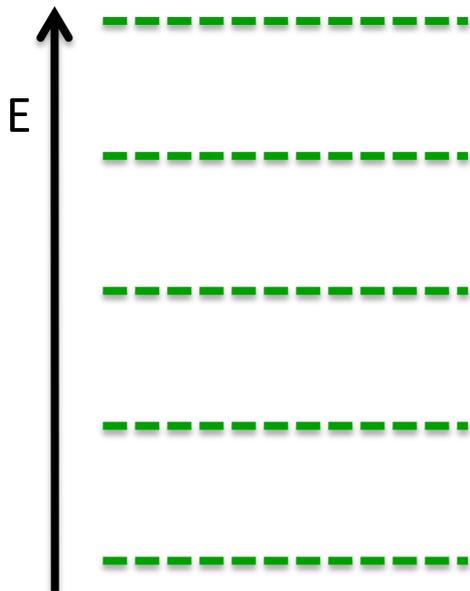
Where are they?

# Multi-particle states

Euclidean time: can't directly study dynamical properties like widths

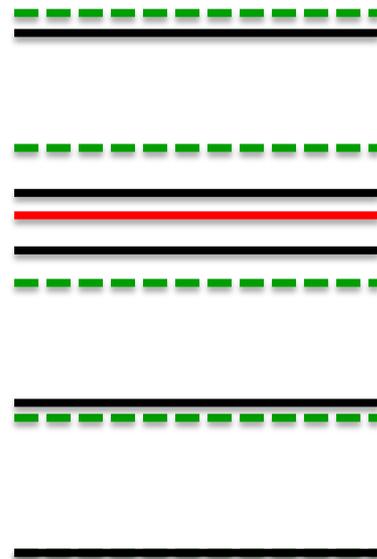
Lüscher: energy shifts in finite volume  $\rightarrow$  phase shift

Free 2-particle levels



$$E = 2\sqrt{m^2 + \vec{p}^2}$$

Measured levels

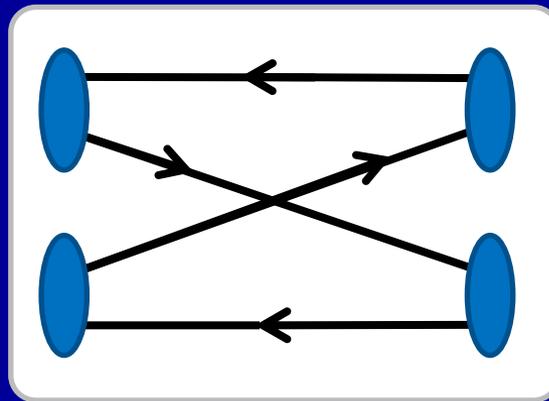
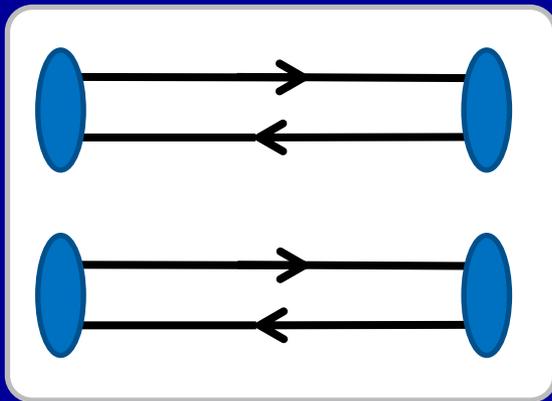


$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

Extract phase shift  
at discrete  $E$   
– Lüscher method

$$\Delta E(L_s) \rightarrow \delta(E, L_s)$$

# $\pi\pi$ isospin-2

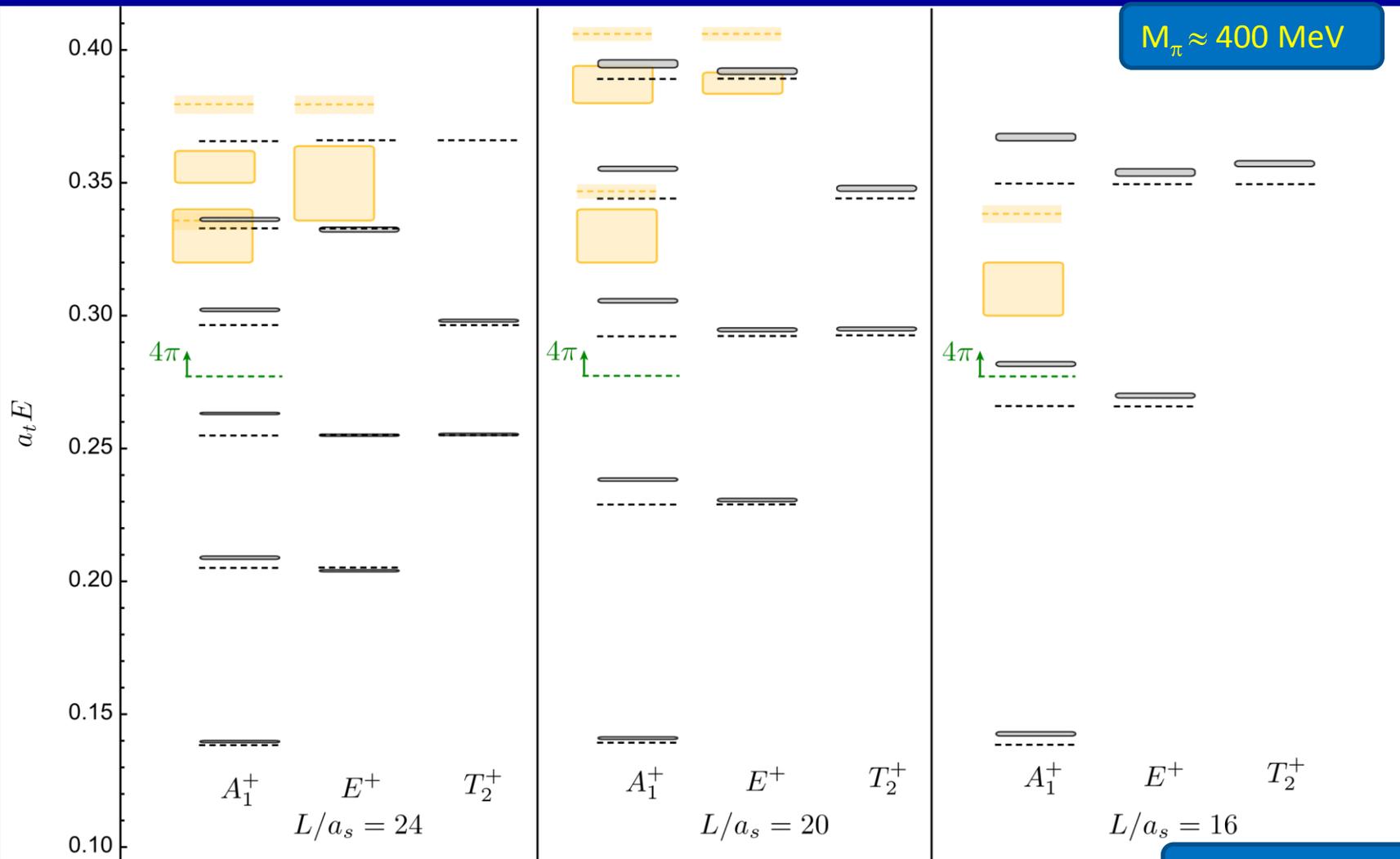


+ similar diagrams

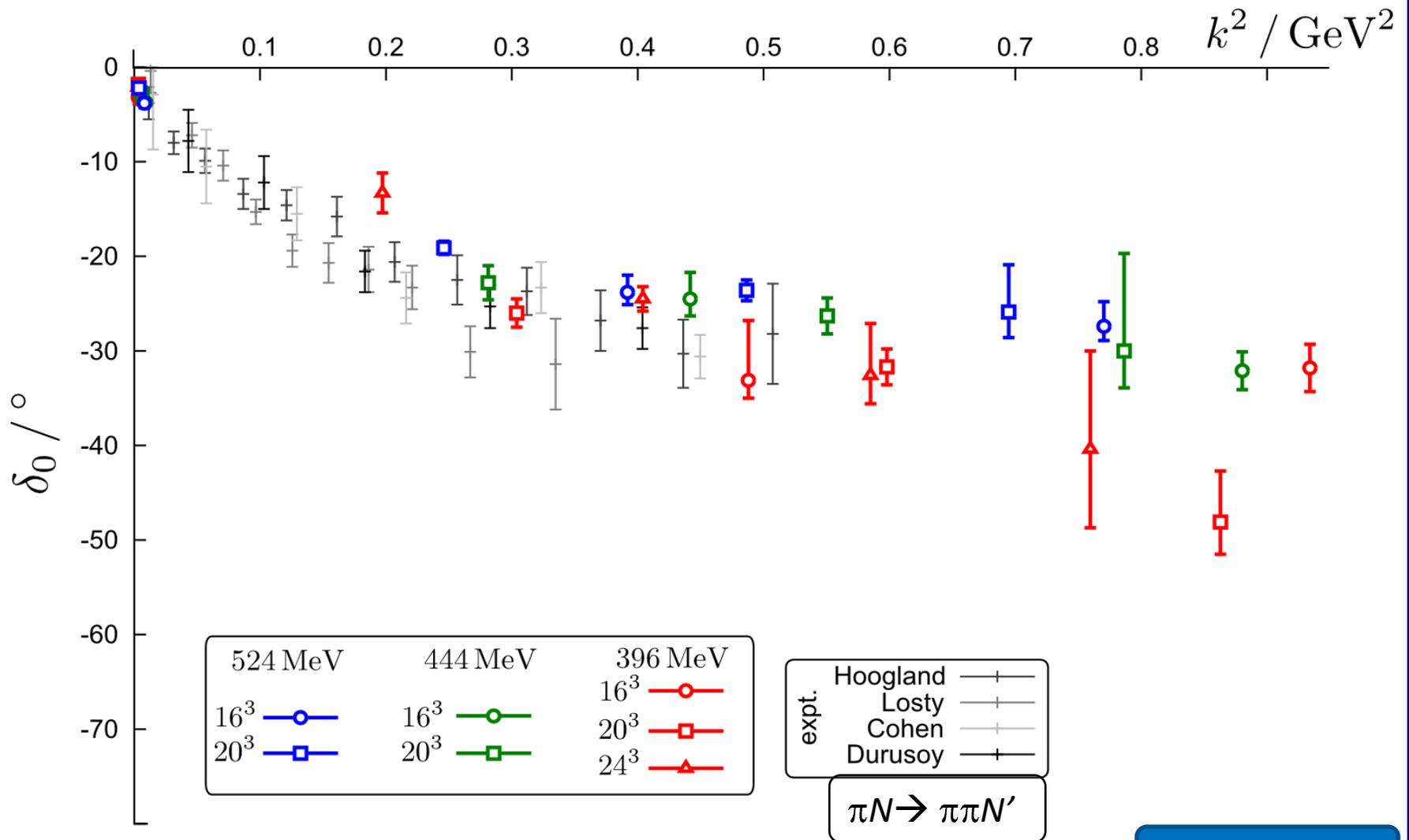
Operators

$$\mathcal{O}_{\pi\pi} = \sum_{\Omega_{\vec{p}}} Y_L^M(\Omega_{\vec{p}}) O_{\pi}(\vec{p}) O_{\pi}(-\vec{p})$$

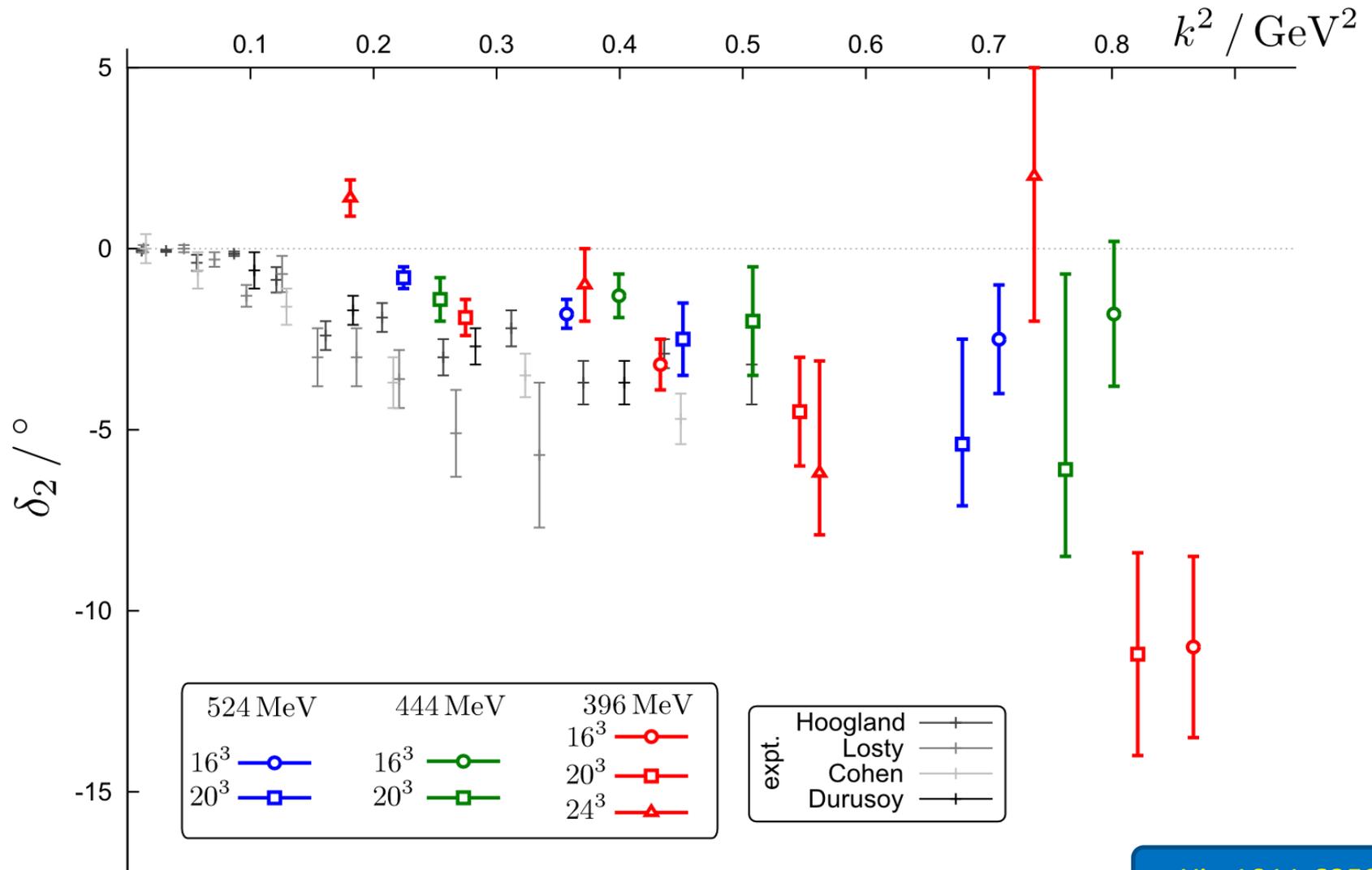
# $\pi\pi$ isospin-2



# $\pi\pi$ $l=2$ phase shift



# $\pi\pi$ $l=2$ phase shift



# Charmonium

“Hydrogen atom” of meson spectroscopy

Potential models, effective field theories, QCD sum rules, ...

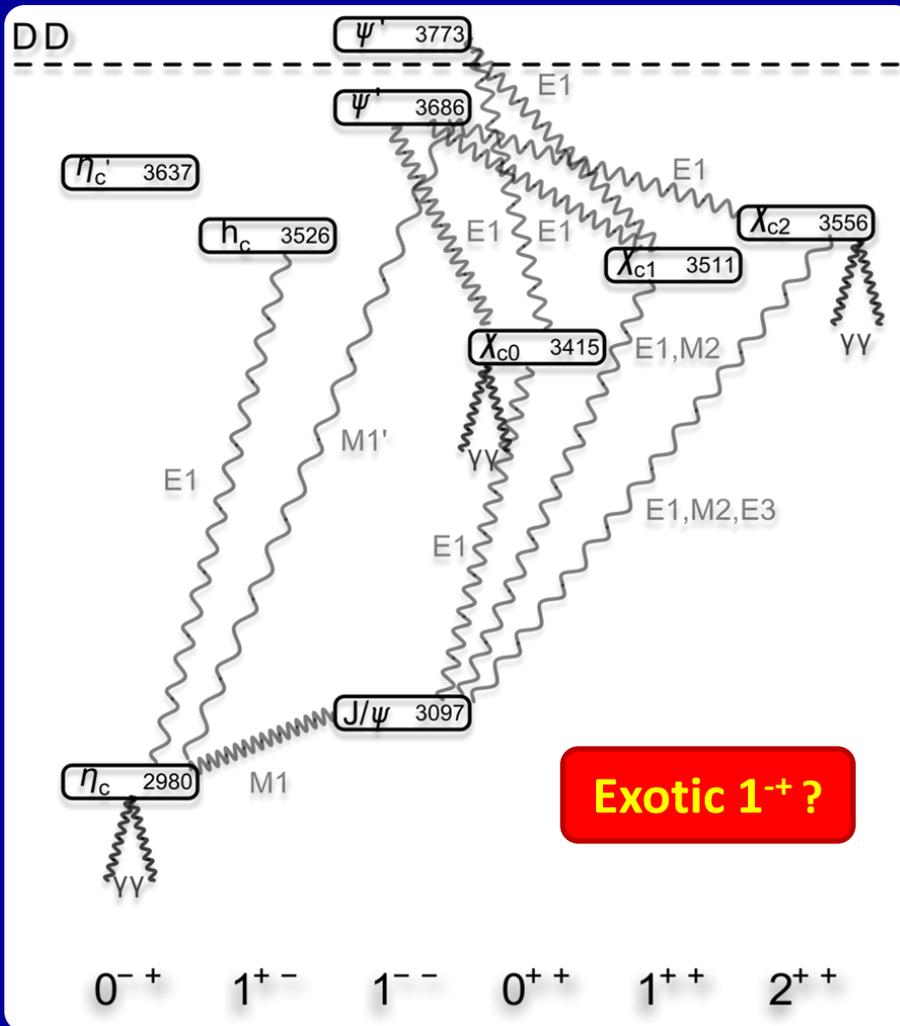
New and improved measurements at BABAR, Belle, BES, CLEO-c

New resonances not easily described by quark model

Theoretical speculation: hybrids, multiquark/molecular mesons, ...

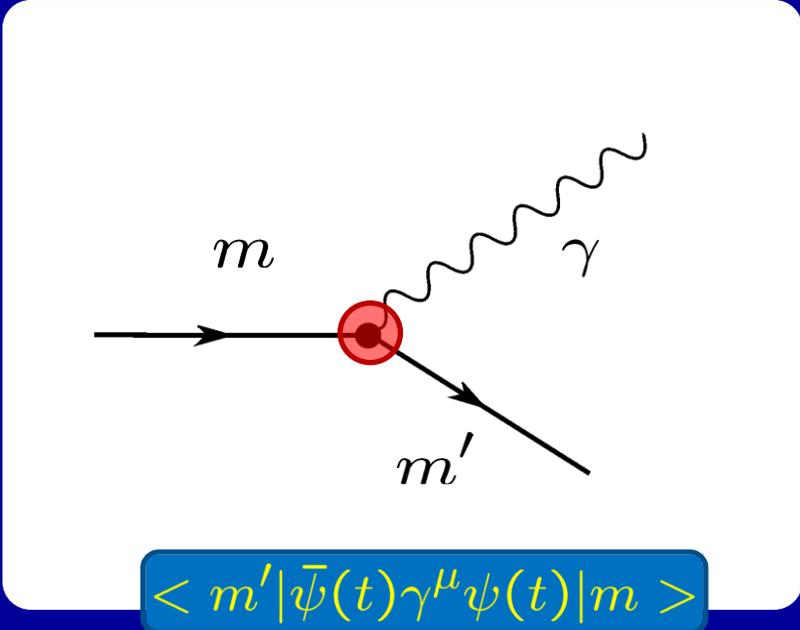
As yet, no exotic  $J^{PC}$  observed ( $1^{-+}$ ,  $0^{+-}$ ,  $2^{+-}$ )

# Charmonium radiative transitions



Below DD threshold radiative transitions have significant BRs

Meson – Photon coupling

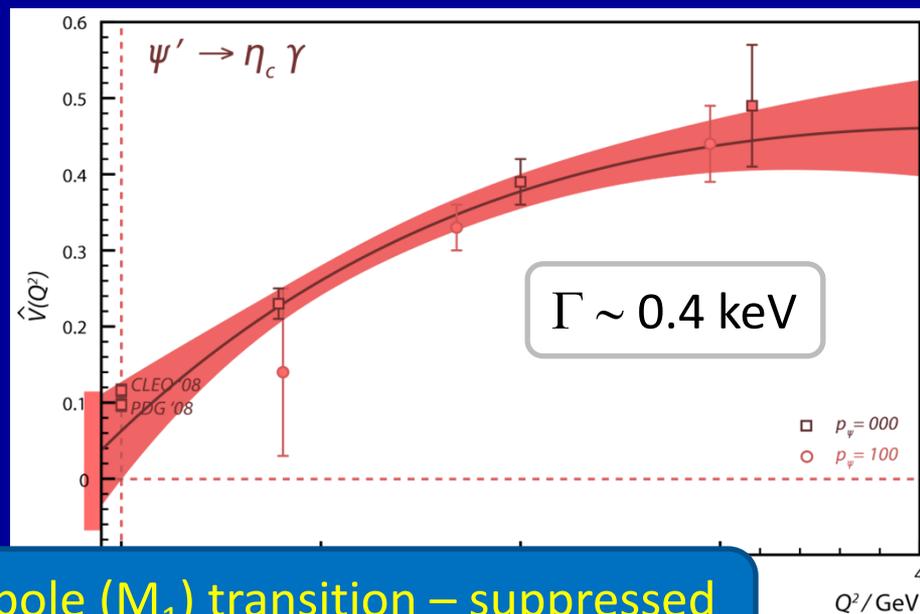
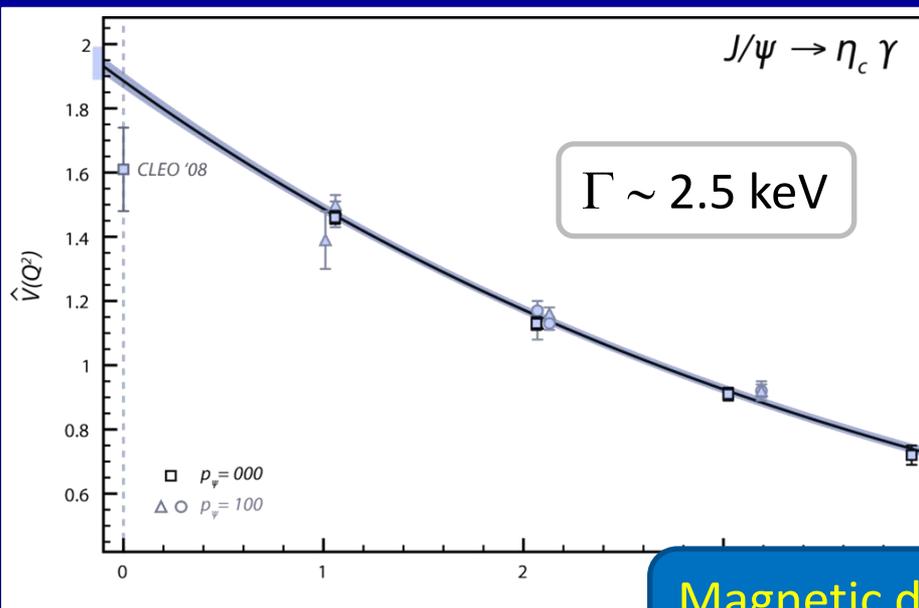
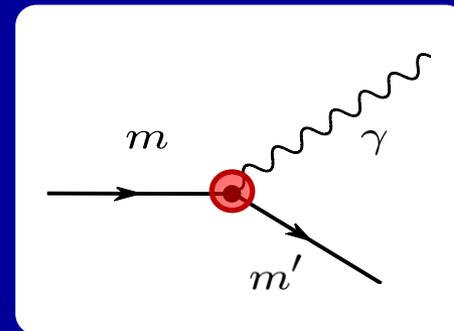


# Photocouplings

Charmonium (quenched) – testing method

$$C_{ij}(t_f, t, t_i) = \langle 0 | O_i(t_f) \bar{\psi}(t) \gamma^\mu \psi(t) O_j(t_i) | 0 \rangle$$

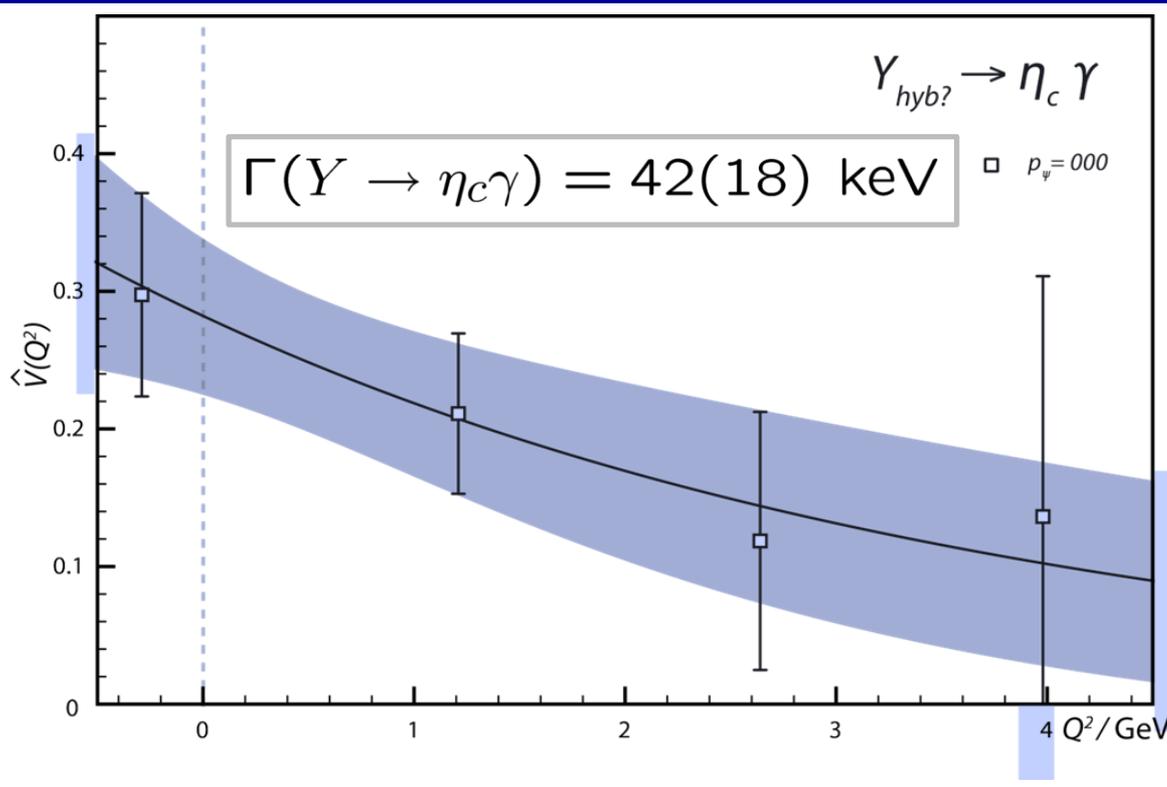
Conventional vector – pseudoscalar transition



Magnetic dipole ( $M_1$ ) transition – suppressed  
(in quark model spin flip  $\sim 1/m_c$ )

PR D79 094504 (2009)

# Photocouplings



Much larger than other  
 $1^{--} \rightarrow 0^{-+} M_1$  transitions

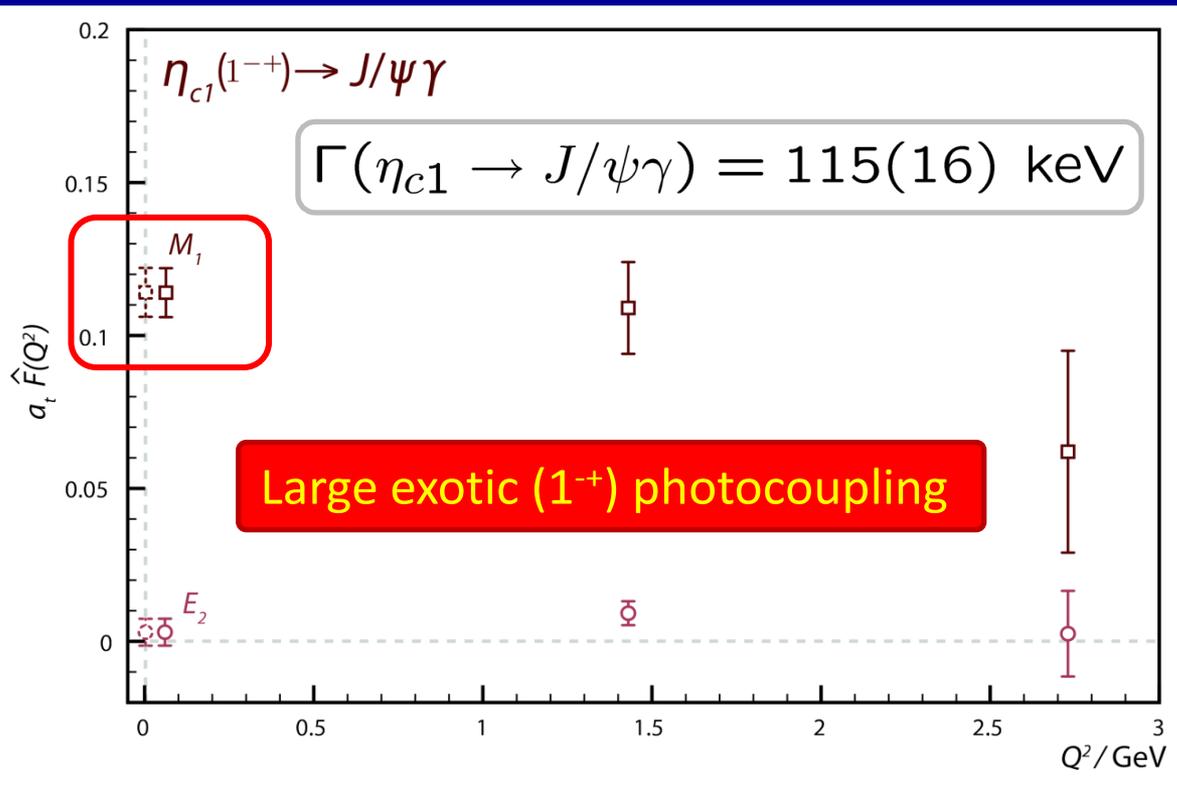
$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV}$

Spectrum analysis  
 suggests a vector hybrid  
 (spin-singlet)

c.f. flux tube model  
 30 – 60 keV

- Usually  $M_1 \rightarrow$  spin flip (e.g.  $^3S_1 \rightarrow ^1S_0$ )  $\rightarrow 1/m_c$  suppression
- Spin-singlet hybrid  $\rightarrow$  extra gluonic degrees of freedom  
 $\rightarrow M_1$  transition without spin flip  $\rightarrow$  not suppressed

# Exotic meson photocoupling



Same scale as many measured conventional charmonium transitions

BUT very large for an  $M_1$  transition

$\Gamma(J/\psi \rightarrow \eta_c \gamma) \sim 2 \text{ keV}$

Suggests a spin-triplet hybrid

- Usually  $M_1 \rightarrow$  spin flip (e.g.  $^3S_1 \rightarrow ^1S_0$ )  $\rightarrow 1/m_c$  suppression
- Spin-triplet hybrid  $\rightarrow$  extra gluonic degrees of freedom  
 $\rightarrow M_1$  transition without spin flip  $\rightarrow$  not suppressed

# More charmonium results

Tensor – Vector transitions  $\chi_{c2}, \chi'_{c2}, \chi''_{c2} \rightarrow J/\psi\gamma$

Identify  $1^3P_2, 1^3F_2, 2^3P_2$  tensors from hierarchy of multipoles  $E_1, M_2, E_3$

Vector – Pseudoscalar  $J/\psi, \psi', \psi'' \rightarrow \eta_c\gamma$

Scalar – Vector  $\chi_{c0} \rightarrow J/\psi\gamma \quad \psi', \psi'' \rightarrow \chi_{c0}\gamma$

Axial – Vector  $\chi_{c1}, \chi'_{c1} \rightarrow J/\psi\gamma$

# Summary and Outlook

## Summary

- **Technology and method work; spin identification** is possible
- **First spin 4 meson** extracted and confidently identified on lattice
- Isovectors and kaons: **exotics** and non-exotic **hybrids**
- Our first results on **scattering** – map out  $E$ -dependent phase shift
- **Photocouplings** of excited charmonia

## Outlook – ongoing work

- Disconnected diagrams – isoscalars and multi-mesons ( $I=0,1$ )
- Multi-meson operators – map out **resonances**
- Baryons
- Photocouplings
- Lighter pion masses and larger volumes

