Covariant dynamical models of photo-and electro-production of pions

JLab N* workshop, October 14, 2008

Franz Gross

JLab and W&M

- Goals:
  - Definition of the N* problem (from a 2003 talk)
  - Issues
- The Covariant Spectator Theory (CST)
  - Light front vs CST
  - Outline of planned CST calculation (with the EBAC group)
- Concluding remarks

Definition of the N* "problem" (??)

- "the treatment of multi-particle final states in a manner consistent with the constraints of unitarity and analyticity … are essential for a reliable interpretation" of JLab data (proposal for the establishment of an N* analysis center at JLab)
- the problem is two-fold:
  1. to develop an effective partial wave theory that can be used to analyze two and three body final states in a model independent way, and
  2. to extract N* masses and widths, undressed by pion rescattering, from the analyzed data (maybe only widths)
- the masses and widths can then be interpreted as "bare" masses and coupling constants suitable for comparison with a quark based theory that neglects pion rescattering effects (IF we choose a SCHEME)

Reproduced from my summary talk at the Ohio Workshop, June, 2003

Issues:

- ★ It is possible to separate the quark-gluon structure of resonances (called the "bare" term) from the dressing by pions (and other mesons)?

  The EBAC program assumes that this is possible.

- ★ Even if the separation can be done at low Q^2, will it still be meaningful at high Q^2?

- ★ An exact calculation of the pion dressings would preserve:
  - Relativistic Invariance
  - Unitarity
  - Crossing symmetry
  - Current conservation

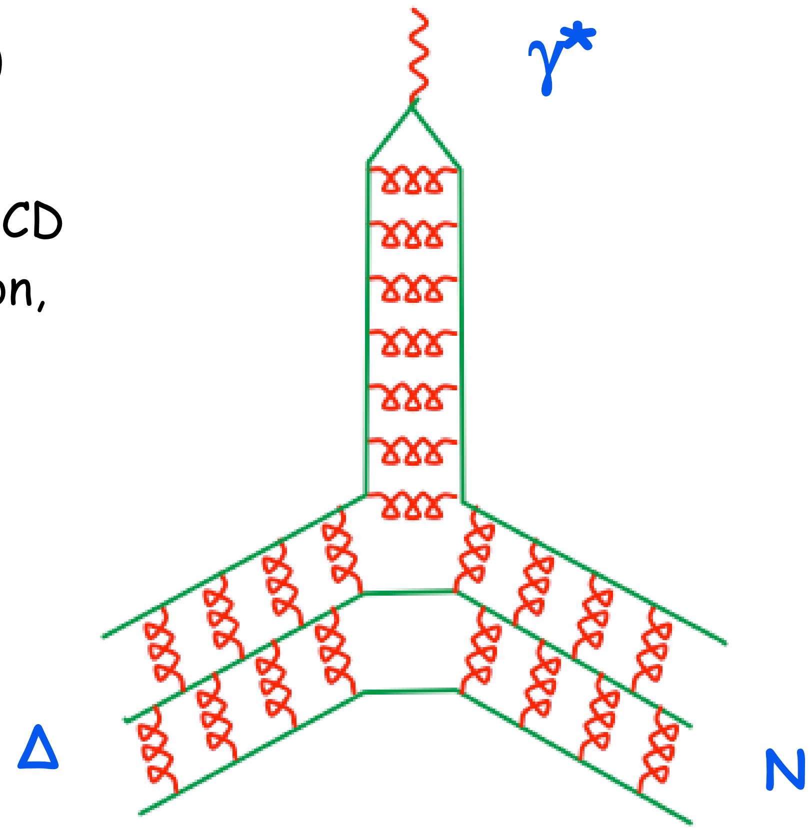
  No model can preserve all of these. Which one(s) can we give up?

What model to use?

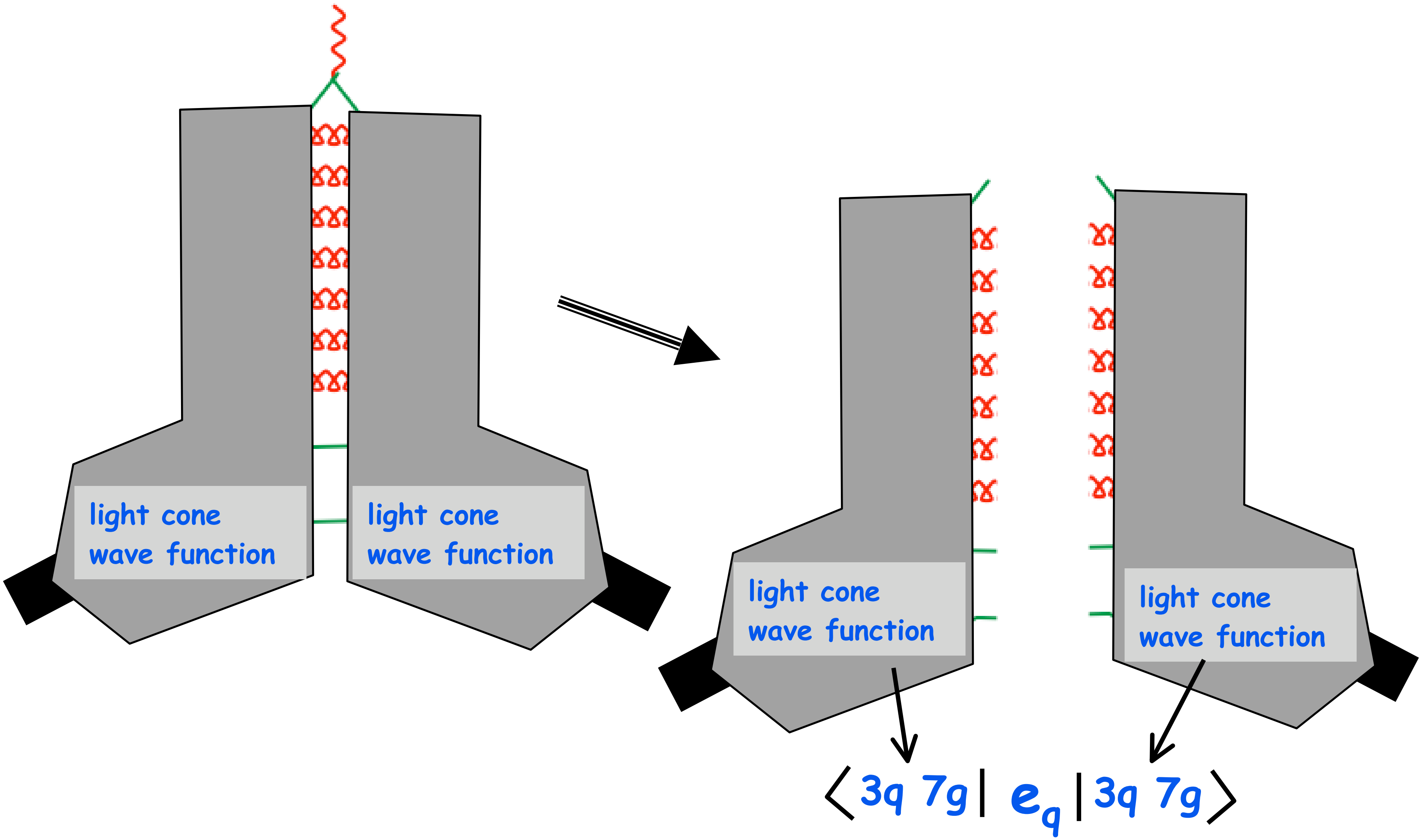
Two differing pictures -- with equivalent physics

- ★ Light front field theory (or quantum mechanics)
- ★ Covariant Spectator Theory (CST)

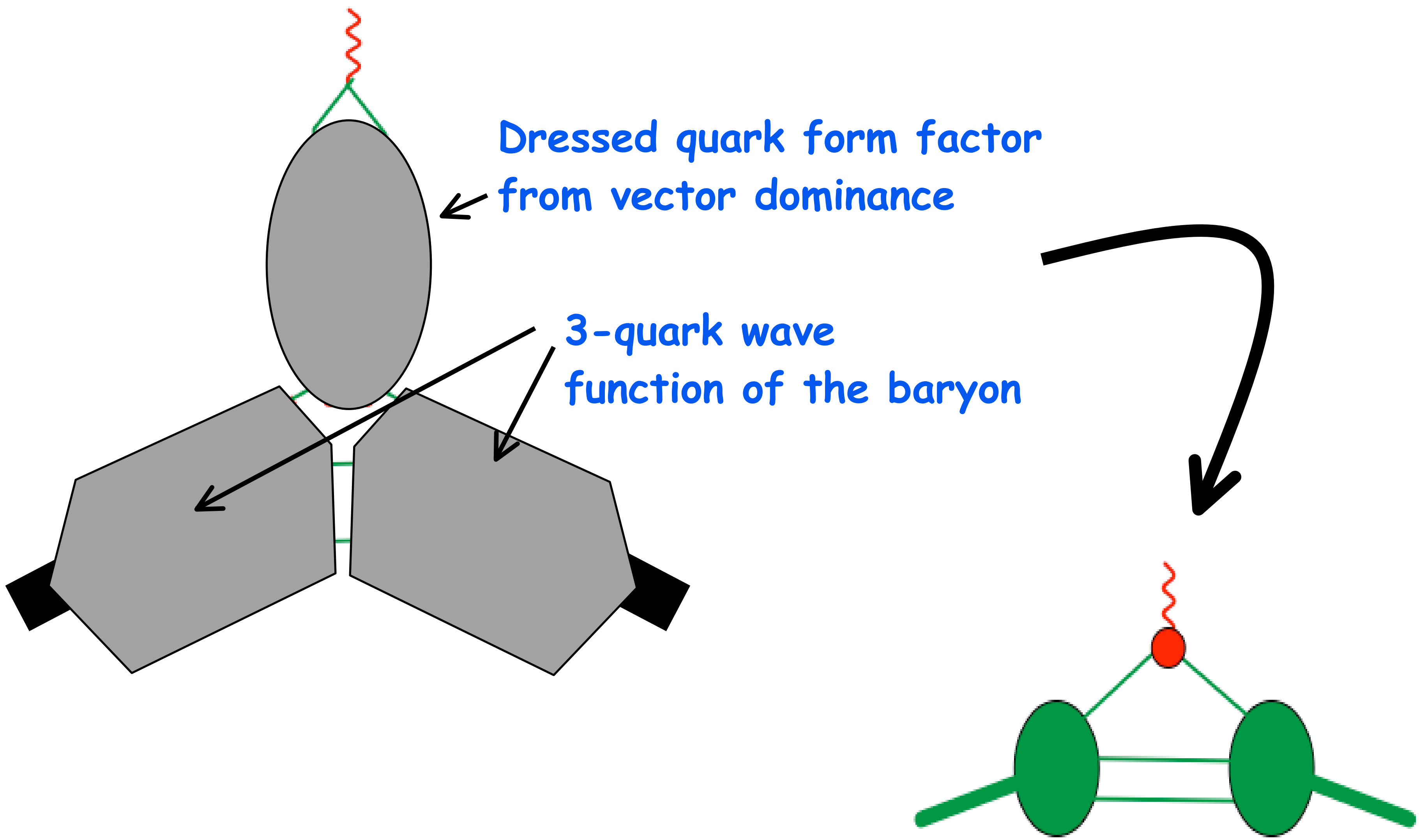
How do they describe a "typical" QCD diagram? (For the N $\rightarrow$ Δ transition, for example)



"Typical QCD diagram" : Light-Front interpretation



"Typical QCD diagram": CST interpretation



Assumptions & features of Covariant Spectator Theory (CST)

★ Exact (but not unique) treatment of:

- Poincare covariance
- Unitarity
- Current conservation
- (Crossing symmetry is only approximate)

CST is the only model with all three ??

★ Form factors ⇔ separation of scales:

- Strong interactions regularized by πNN, $\pi N\Delta$, etc form factors linked to few body physics*.
- Hadronic structure: electromagnetic form factors (pion, nucleon Delta, …) also linked to few body physics*.
- Bare resonance form factors from the quark model.

★ The virtual nucleon is on-mass shell, simplifying the physics

- (previous work by Surya & FG** placed the pion on shell, but showed that the CST can work)

***A few references next slide**

****Yohanes Surya and FG, PRC 47, 703 (1993); PRC 53, 2422 (1996)**

Successes of hadron descriptions in few-body physics - 1

- ★ Recent high precision fits to 2007 NN data base (below 350 MeV lab energy)*
  - Excellent fit with $\chi^2/N_{data} = 1.06$ (Model WJC-1)
    - as good as any fit ever produced; with fewer parameters
    - 27 parameters with 6 meson exchanges and two contact interactions
  - Very good fit with $\chi^2/N_{data} = 1.12$ (Model WJC-2)
    - As good as the famous PWA93 analysis of Nijmegen
    - Only 15 parameters and 6 boson exchanges
  - These fits fix the strong form factors ?!
- ★ Excellent explanation of the deuteron form factors in terms of deuteron wave functions and measured nucleon form factors (separation of scales)**

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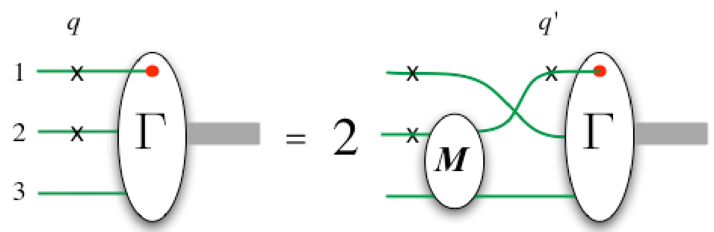
*FG and Alfred Stadler, PLB 657, 176 (2007); PRC 78, 014005 (2008)

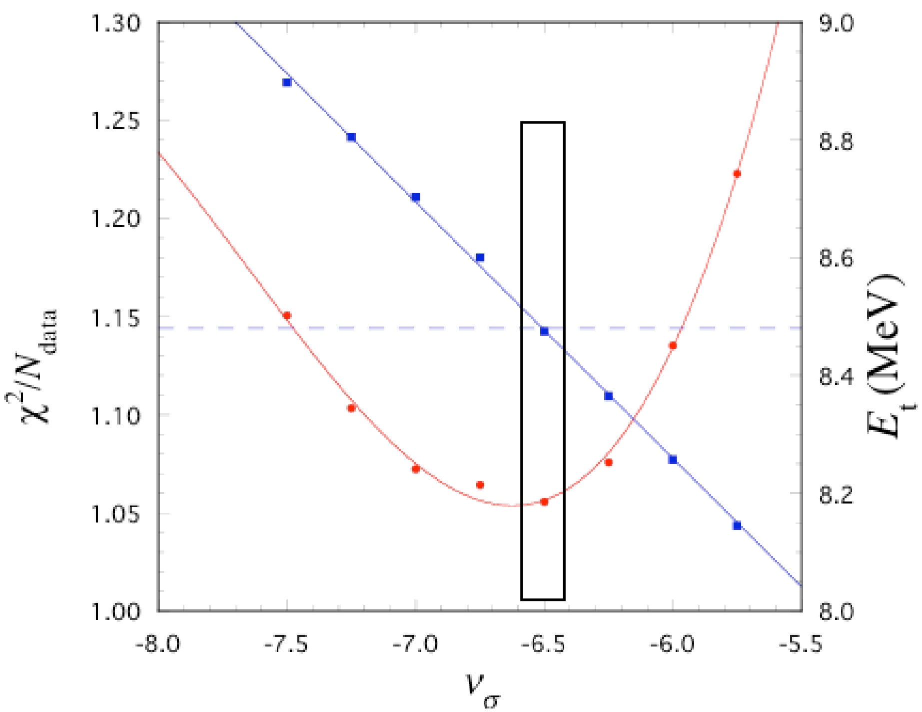
**J. W. Van Orden, N. Devine, and FG, PRL 75, 4369 (1995); R. Gillman and FG, J. Phys. G, 28, R37 (2002) [review].

Successes of hadron descriptions in few-body physics - 2

- ★ Beautiful connection between the NN and NNN sectors:
- ★ NNN interaction given entirely in terms of NN scattering -- three body bound state equation is:



- ★ Best fit to two body data gives the correct three body binding energy



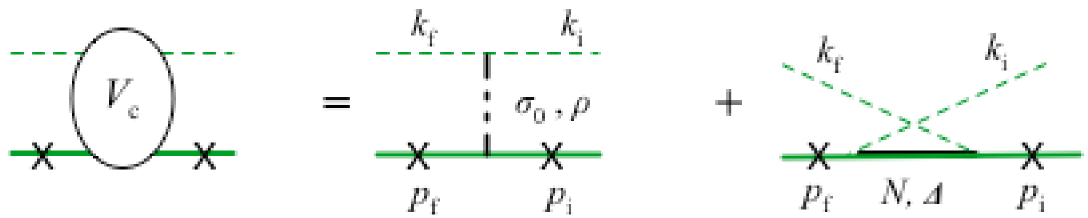
CST Model of πN scattering

New program planned with EBAC team

First phase: πN scattering below the 2πN threshold (1)

★ Dynamical model of background:

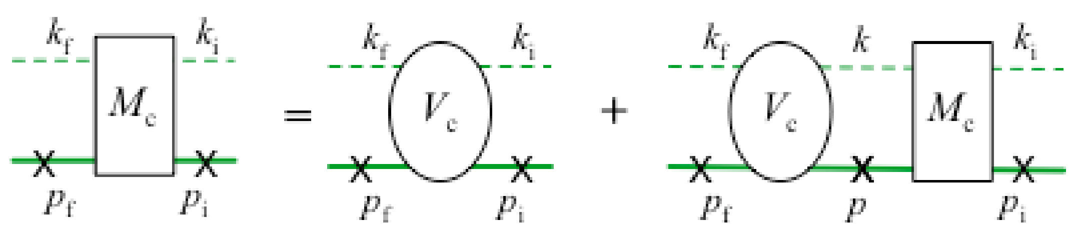
V_c =



t-channel poles
isoscalar σ and
isovector ρ

u-channel poles
nucleon and Δ
and other baryons (if present)

Iterate background kernel to all orders to insure unitarity (see next slide)



Emergence of Unitarity

- If the scattering is described by an infinite sum with a real V_c and complex propagator G:

$$M_c = V_c + V_c G V_c + V_c G V_c G V_c + \cdots \quad = \frac{V_c}{1 - G V_c}$$

- then the amplitude is automatically unitary

$$\operatorname{Im} M_c = \frac{1}{2i}\left\{\frac{V_c}{1 - G V_c} - \frac{V_c}{1 - G^* V_c}\right\} = \left(\frac{G - G^*}{2i}\right)\frac{V_c^2}{\left|1 - G V_c\right|^2} = \operatorname{Im} G \left|M_c\right|^2$$

- If the sum is NOT infinite, it is NOT unitary. Example: interaction to third order,

$$M_c = V_c + V_c G V_c + V_c G V_c G V_c$$
$$M_c^* = V_c + V_c G^* V_c + V_c G^* V_c G^* V$$

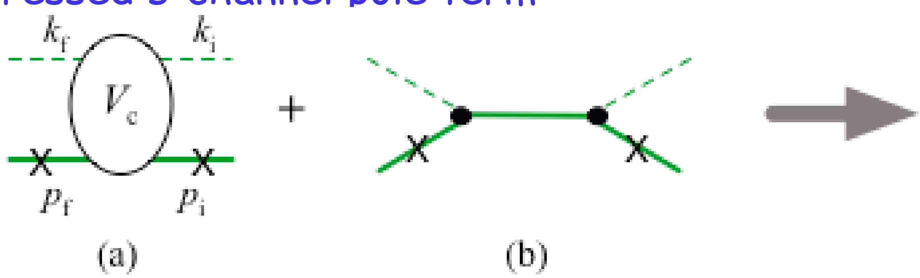
- Then the imaginary part of M_c is not proportional to $\left|M_c\right|^2$

$$\operatorname{Im} M_c = \operatorname{Im} G \left\{V_c^2 + 2 \operatorname{Re} G V_c^3\right\}$$
$$\left|M_c\right|^2 = M_c^* M_c = V_c^2 + 2 \operatorname{Re} G V_c^3 + \mathrm{O}\left(V_c^4\right)$$

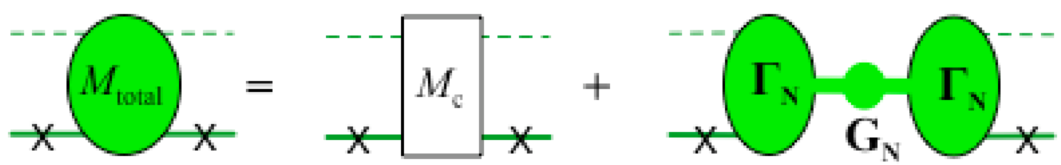
First phase: πN scattering below the 2πN threshold (2)

★ Addition of the direct *s*-channel poles leads to scattering with

- a background term
- a dressed *s*-channel pole term

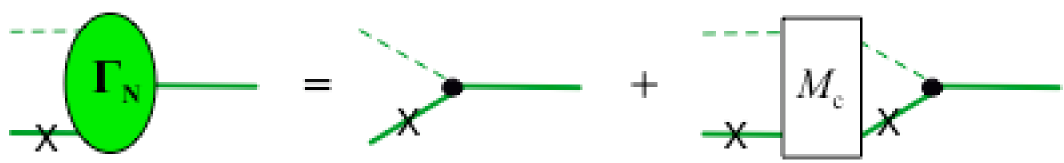


Exact solution of CST equation gives:



- with a dressed propagator G_N and a dressed vertex function:

$$G_N = \frac{1}{m_R - \not{P} - \Sigma(P)}$$



Electroproduction of Pions

First phase: Electroproduction of pions (1)

- ★ Add electromagnetic interactions to the hadronic model*
- ★ Conserve current exactly using the methods of Riska and FG**

  Restrictions (not serious)

  - Separable strong form factors [$F(p_1, p_2, p_3) = f_1(p_1)\, f_2(p_2)\, f_2(p_3)$], so that they may be reinterpreted as a dressing of the hadron propagators
  - Only one strong form factor for each hadron, i.e. π, ρ, N, Δ (but each hadron's form factor may be different) SLIDES

  Two steps:

  - Couple photon to hadrons in all possible places (including momentum dependent couplings, if there are any).
  - Use generalized electromagnetic couplings that satisfy Ward-Takahashi identities.
- ★ Each type of hadron may have a different electromagnetic form factor (i.e. $F_\pi \neq F_N$, for example)

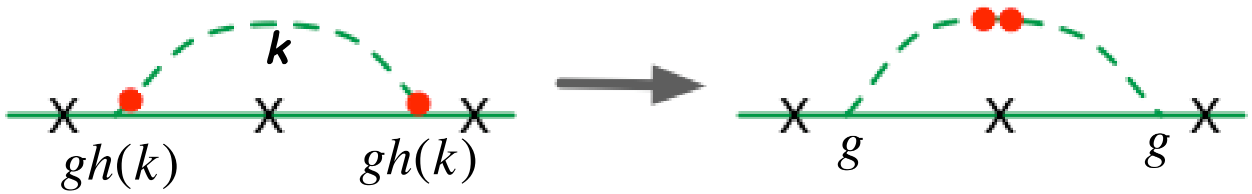
*Yohanes Surya and FG, PRC **47**, 703 (1993); PRC **53**, 2422 (1996)

FG and D. O. Riska, PRC **36, 1928 (1987)

First phase: Electroproduction of pions (2)

- ★ Sketch of how the diagrams are organized
- ★ Example 1: nucleon bubble diagram

$$gh(k) \quad \frac{1}{m_\pi^2 - k^2 - i\varepsilon} \quad gh(k) \qquad\qquad g^2 \frac{h^2(k)}{m_\pi^2 - k^2 - i\varepsilon}$$



A diagram with dressed couplings is converted into a diagram with point couplings and a dressed propagator.

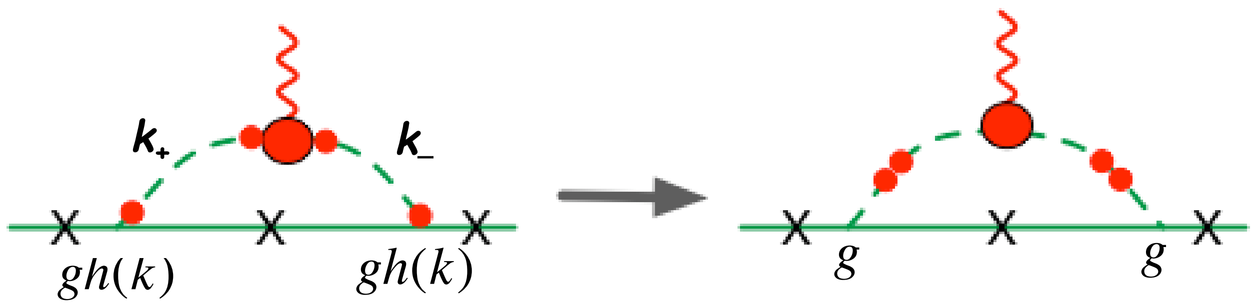
- ★ Same thing for the electromagnetic diagram

First phase: Electroproduction of pions (3)

- Example 2: pion coupling to nucleon bubble diagram

$$gh(k_+)\frac{1}{m_\pi^2-k_+^2-i\varepsilon}\overbrace{h(k_+)\;j_\pi^\mu\;h(k_-)}^{\text{full current}}\frac{1}{m_\pi^2-k_-^2-i\varepsilon}gh(k_-)$$

$$g^2\frac{h^2(k_+)}{m_\pi^2-k_+^2-i\varepsilon}\overbrace{j_\pi^\mu}^{\text{reduced current}}\frac{h^2(k_-)}{m_\pi^2-k_-^2-i\varepsilon}$$



A diagram with dressed couplings and a *full* current is converted into a diagram with point couplings, dressed propagators and a *reduced* current.

First phase: Electroproduction of pions (4)

- Add electromagnetic interactions to the hadronic model*
- Conserve current exactly using the methods of Riska and FG**

  Restrictions (not serious)

  - Separable strong form factors [$F(p_1, p_2, p_3) = f_1(p_1)\, f_2(p_2)\, f_2(p_3)$], so that they may be reinterpreted as a dressing of the hadron propagators
  - Only one strong form factor for each hadron, i.e. π, ρ, N, Δ (but each hadron's form factor may be different)

  Two steps:

  - Couple photon to hadrons in all possible places (including momentum dependent couplings, if there are any).
  - Use generalized electromagnetic couplings with *reduced* currents that satisfy Ward-Takahashi identities.
- Each type of hadron may have a different electromagnetic form factor (i.e. $F_\pi \neq F_N$, for example)

*Yohanes Surya and FG, PRC **47**, 703 (1993); PRC **53**, 2422 (1996)

FG and D. O. Riska, PRC **36, 1928 (1987)

First phase: Electroproduction of pions (5); an example

- Example: *reduced* pion current for the pion bubble contribution to the nucleon form factor

$$j_\pi^\mu = \Bigg[\underbrace{\left(F_\pi\left(Q^2\right)-1\right)\left\{ \left(k_+ + k_-\right)^\mu - \frac{\left[\left(k_+ + k_-\right)\cdot q\right]q^\mu}{q^2} \right\}}_{\text{EM form factor is purely transverse}} + \left(k_+ + k_-\right)^\mu \Bigg] \left(\frac{\boxed{\Delta^{-1}\left(k_-^2\right)} - \Delta^{-1}\left(k_+^2\right)}{k_+^2 - k_-^2} \right)$$

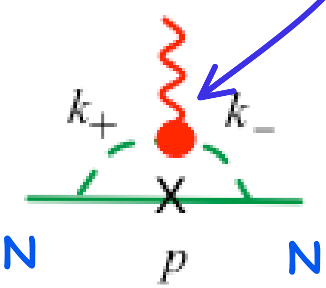
inverse of (dressed) pion propagator

- Current satisfies the WT identity: $q = k_+ - k_-$

$$q_\mu j_\pi^\mu = \Delta^{-1}\left(k_-^2\right) - \Delta^{-1}\left(k_+^2\right) = \frac{m_\pi^2 - k_-^2}{h^2\left(k_-^2\right)} - \frac{m_\pi^2 - k_+^2}{\boxed{h^2\left(k_+^2\right)}} \rightarrow k_+^2 - k_-^2$$

if h = 1

strong pion form factor



- When the q^μ term is discarded (contraction into a conserved current):

$$j_\pi^\mu = F_\pi\left(Q^2\right)\left(k_+ + k_-\right)^\mu \boxed{\left(\frac{\Delta^{-1}\left(k_-^2\right) - \Delta^{-1}\left(k_+^2\right)}{k_+^2 - k_-^2} \right)}$$

New current has an off-shell correction

First phase: Electroproduction of pions (6); an example

★ The pion bubble diagram is

on-shell nucleon energy

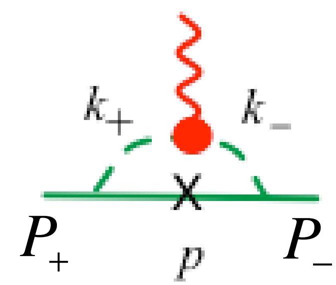
on-shell nucleon projection operator

$$J_\pi^\mu = -2\tau_3 g_{\pi NN}^2 \int \frac{d^3p}{(2\pi)^3 2E_p}\, \bar{u}(P_+,\lambda_+)\gamma^5 \boxed{(m+\hat{\not{p}})}\gamma^5 u(P_-,\lambda_-)\; \Delta(k_+^2)\Delta(k_-^2)\, j_\pi^\mu$$

remember that $j_\pi^\mu = F_\pi\left(Q^2\right)\left(k_+ + k_-\right)^\mu \left(\dfrac{\Delta^{-1}\left(k_-^2\right) - \Delta^{-1}\left(k_+^2\right)}{k_+^2 - k_-^2}\right)$

$$J_\pi^\mu = -2\tau_3 g_{\pi NN}^2 F_\pi(Q^2) \int \frac{d^3p}{(2\pi)^3 2E_p}\, \bar{u}(P_+,\lambda_+)(m-\hat{\not{p}})u(P_-,\lambda_-)$$

$$\times \boxed{\frac{\Delta(k_+^2)-\Delta(k_-^2)}{k_+^2-k_-^2}\left(k_+ + k_-\right)^\mu}$$



significant modification due to the off-shell current

Relativistic treatment of the spin 3/2 Δ* -- 1

*Pascalutsa, Phys. Rev. D **58**, 096002 (1998)
Pascalutsa and Timmermans, Phys. Rev. C **60**, 042201 (1999)
Pascalutsa and Phillips, Phys. Rev. C **68**, 055205 (2003)
Pascalutsa and Vanderhaeghen, Phys.Lett. **B63**, 31 (2006)

★ The Δ propagator includes spurious spin 1/2 components

$$S_{\mu\nu}(P) = \frac{1}{M - \not{P} - i\varepsilon}\left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3M^2}P_\mu P_\nu - \frac{1}{3M}\left(\gamma_\mu P_\nu - P_\mu\gamma_\nu\right)\right)$$

$$= \underbrace{\boxed{\frac{1}{M - \not{P} - i\varepsilon}P^{(3/2)}_{\mu\nu}}}_{\text{“good” part}} + \underbrace{\frac{2}{3M^2}(M + \not{P})P^{(1/2)}_{22,\mu\nu} + \frac{1}{\sqrt{3}M}\left(P^{(1/2)}_{12,\mu\nu} + P^{(1/2)}_{21,\mu\nu}\right)}_{\text{“bad” part}}$$

where
$$P^{(3/2)}_{\mu\nu} = g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3P^2}\left(\not{P}\gamma_\mu P_\nu + P_\mu\gamma_\nu\not{P}\right)$$

$$P^{(1/2)}_{22,\mu\nu} = \frac{P_\mu P_\nu}{P^2}; \quad P^{(1/2)}_{12,\mu\nu} = \frac{P^\rho i\sigma_{\mu\rho}P_\nu}{\sqrt{3}P^2}; \quad P^{(1/2)}_{21,\mu\nu} = \frac{P_\mu P^\rho i\sigma_{\rho\nu}}{\sqrt{3}P^2}$$

Note that spin 1/2 parts are all linear in P_μ or P_ν

Relativistic treatment of the spin 3/2 Δ -- 2

- ★ Pascalutsa considers the strong gauge invariance of the spin 3/2 field (needed to reduce the number of degrees of freedom to 4X2=8 to 4)
- ★ Conclusion is that strong gauge invariant couplings are needed
  - the couplings often used in the past were

$$\bar{\psi}_N \Theta^{\mu\nu} \Psi_{\Delta\mu} \partial_\nu \phi_\pi \quad \text{with} \quad \Theta^{\mu\nu} = g^{\mu\nu} - \left(z + \tfrac{1}{2}\right)\gamma_\mu \gamma_\nu$$

where z is the "off-shell" parameter

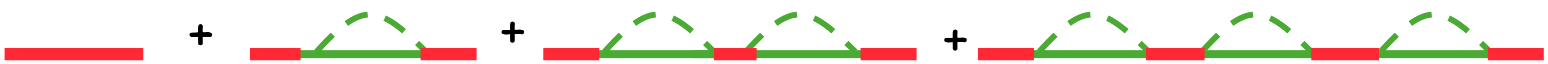
  - strong gauge invariance requires $\Theta^{\mu\nu} P_\nu = 0$. This constraint insures that all spin 1/2 parts of the propagator vanish, and is not satisfied by previous couplings
  - Pascalutsa uses

$$\bar{\psi}_N \Theta^{\mu\nu} \Psi_{\Delta\mu} \partial_\nu \phi_\pi = \bar{\psi}_N \gamma_5 \gamma_\mu \varepsilon^{\mu\nu\rho\sigma} \left(\partial_\rho \Psi_{\Delta\sigma} - \partial_\sigma \Psi_{\Delta\rho}\right) \partial_\nu \phi_\pi$$

$$\Rightarrow \bar{\psi}_N \gamma_5 \gamma_\mu k_{\pi\nu} \varepsilon^{\mu\nu\rho\sigma} \left(P_\rho \Psi_{\Delta\sigma} - P_\sigma \Psi_{\Delta\rho}\right)$$

Relativistic treatment of the spin 3/2 Δ -- Conclusion

- The strong gauge invariant treatment solves a long standing problem -- the Δ can now be treated and a pure spin 3/2 particle.
- Technical simplifications abound. The bubble sum can be computed easily:



$$P_{\mu\nu} + \Theta_{\mu\rho} B g^{\rho\rho'} \Theta_{\rho'\nu} + \Theta_{\mu\rho} B g^{\rho\rho'} \Theta_{\rho'\sigma} B g^{\sigma\sigma'} \Theta_{\sigma'\nu} + \Theta_{\mu\rho} B g^{\rho\rho'} \Theta_{\rho'\sigma} B g^{\sigma\sigma'} \Theta_{\sigma'\omega} B g^{\omega\omega'} \Theta_{\omega'\nu} + \cdots$$

$$= \frac{\Theta_{\mu\nu}}{1-B}$$

See: FG and Surya,
Phys. Rev. ***C*** 47, 703 (1993)

- A relativistic Effective Field Theory can be (and has been) developed

Near term goals

- ★ Repeat the previously successful programs (Sato-Lee & DMT & ⋯)
  - Complete πN scattering calculation below 2πN threshold with bare Δ. Use the new Pascalutsa πNΔ couplings. Fit all data.
  - Insert electromagnetic coupling and compute current conserving pion electroproduction reaction for the same physics. Fit all data.
  - Can we extract the "bare" Δ→N transition form factors, including the (very) small G_E and G_C [perhaps the bare (quark part) of these transition form factors is zero, showing that they are due entirely to the pion dressing]?
- ★ Study model dependence by comparing with other calculations
- ★ Unify the NN fits with the πN fits (are the form factor masses the same)?
- ★ Extend to higher energy by adding the next resonances, and prepare the way for adding 2πN processes.

Comments on the workshop questions

★ This program can provide a determination of the bare (quark) resonance parameters independent of other analyses; it will test the EBAC hypothesis.

★ The quark part of the wave functions of the baryon resonances are covariant and can be extracted phenomenologically.

★ CST has attractive features

- Exact covariance, unitarity, and conservation of current
- Smooth connection to field theory and the successful few-body program
- Inclusive inelastic electron scattering is characterized by two variables: Q^2 and $W^2=(P+q)^2$, the mass of the final state. As long as W is small (i.e. the Δ and low lying resonances), CST suggests that the SAME physics can describe BOTH the small Q^2 and large Q^2 regions (at least for $Q^2 < 10$ GeV^2).

END