



Nuclear modification of nucleon structure functions

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Outline

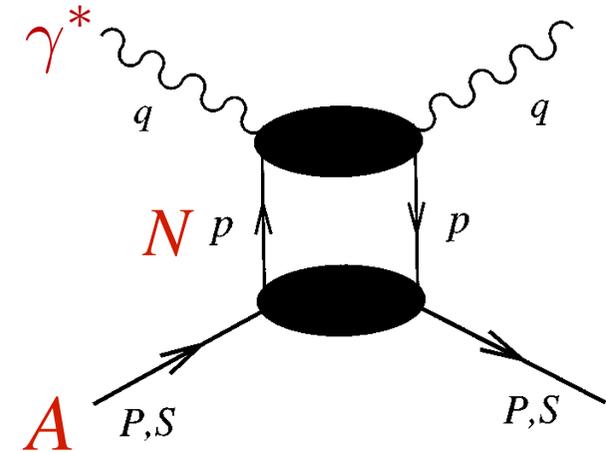
- Nuclear (→ deuteron) structure functions
 - smearing functions & quasielastic scattering
 - nucleon off-shell effects
 - nuclear effects on neutron structure function
- Extraction of neutron resonance structure from nuclear data
 - new “unsmearing” method
 - quark-hadron duality
 - constraints on off-shell effects from local duality, SRCs
- Outlook

Nuclear structure functions

Nuclear structure functions

- Incoherent scattering from nucleons in nucleus A ($x \gg 0$)

$$W_{\mu\nu}^A(P, q) = \int d^4p \text{Tr} \left[\hat{\mathcal{A}}(P, p) \cdot \hat{W}_{\mu\nu}^N(p, q) \right]$$



→ truncated (off-shell) nucleon tensor

$$\hat{W}_{\mu\nu}^N(p, q) = g_{\mu\nu} \left(I \hat{W}_0 + \not{p} \hat{W}_1 + \not{q} \hat{W}_2 \right)$$

bound nucleon “structure functions”

→ (off-shell) nucleon–nucleus scattering amplitude

$$\hat{\mathcal{A}}(P, p) = I \mathcal{A}_S + \gamma_\alpha \mathcal{A}_V^\alpha$$

scalar, vector amplitudes include sum over residual $A-1$ nuclear states

Nuclear structure functions

- (Spin-averaged) structure function of nucleus given by sum of *three* terms

$$F_2^A = \int d^4p \left(\mathcal{A}_S \widehat{W}_0 + p \cdot \mathcal{A}_V \widehat{W}_1 + q \cdot \mathcal{A}_V \widehat{W}_2 \right)$$

→ cannot be written (in general) as 1-dimensional convolutions:

factorization of amplitudes
≠ factorization of structure functions

WM, Schreiber, Thomas, PRD 49, 1183 (1994)

- Taking selective on-shell or nonrelativistic limits, one *can* identify convolution component *plus* off-shell corrections

$$F_2^A = \sum_N f_{N/A} \otimes F_2^N + \delta^{(\text{off})} F_2$$

← nucleon (light-cone) momentum distribution
("smearing function")

Deuteron structure functions

- For *deuteron*, nucleon momentum distribution can be computed “exactly” (relativistically or nonrelativistically), and form of off-shell corrections identified

$$F_2^d(x) = \int dy f_{N/d}(y, \gamma) F_2^N(x/y) + \delta^{(\text{off})} F_2^d(x)$$

WM, Schreiber, Thomas, PLB 335, 11 (1994)
Kulagin, Piller, Weise, PRC 50, 1154 (1994)

with relativistic (“MST”) smearing function

$$f_{N/d}(y) = \frac{M_d^2}{2M} \int \frac{d^3p}{(2\pi)^3} \frac{y}{p_0} \theta(p_0) \delta\left(y - \frac{p_0 + p_z}{M}\right) |\psi_d(p)|^2$$

→ at $Q^2 \rightarrow \infty$, function of light-cone momentum fraction y of d carried by N , with normalization $\int dy f_{N/d}(y) = 1$

Deuteron structure functions

- Expanding in powers of \mathbf{p}^2/M^2 and binding energy ε_d/M (“weak binding approximation”), smearing function reduces to

$$f_{N/d}(y, \gamma) = \int \frac{d^3p}{(2\pi)^3} \left(1 + \frac{\gamma p_z}{M}\right) \mathcal{C}(y, \gamma) |\psi_d(p)|^2 \delta\left(y - 1 - \frac{\varepsilon + \gamma p_z}{M}\right)$$

Kulagin, Petti, NPA 765, 126 (2006)

→ at finite Q^2 , additional dependence on photon “velocity”

$$\gamma = |\mathbf{q}|/q_0 = \sqrt{1 + 4M^2 x^2 / Q^2}$$

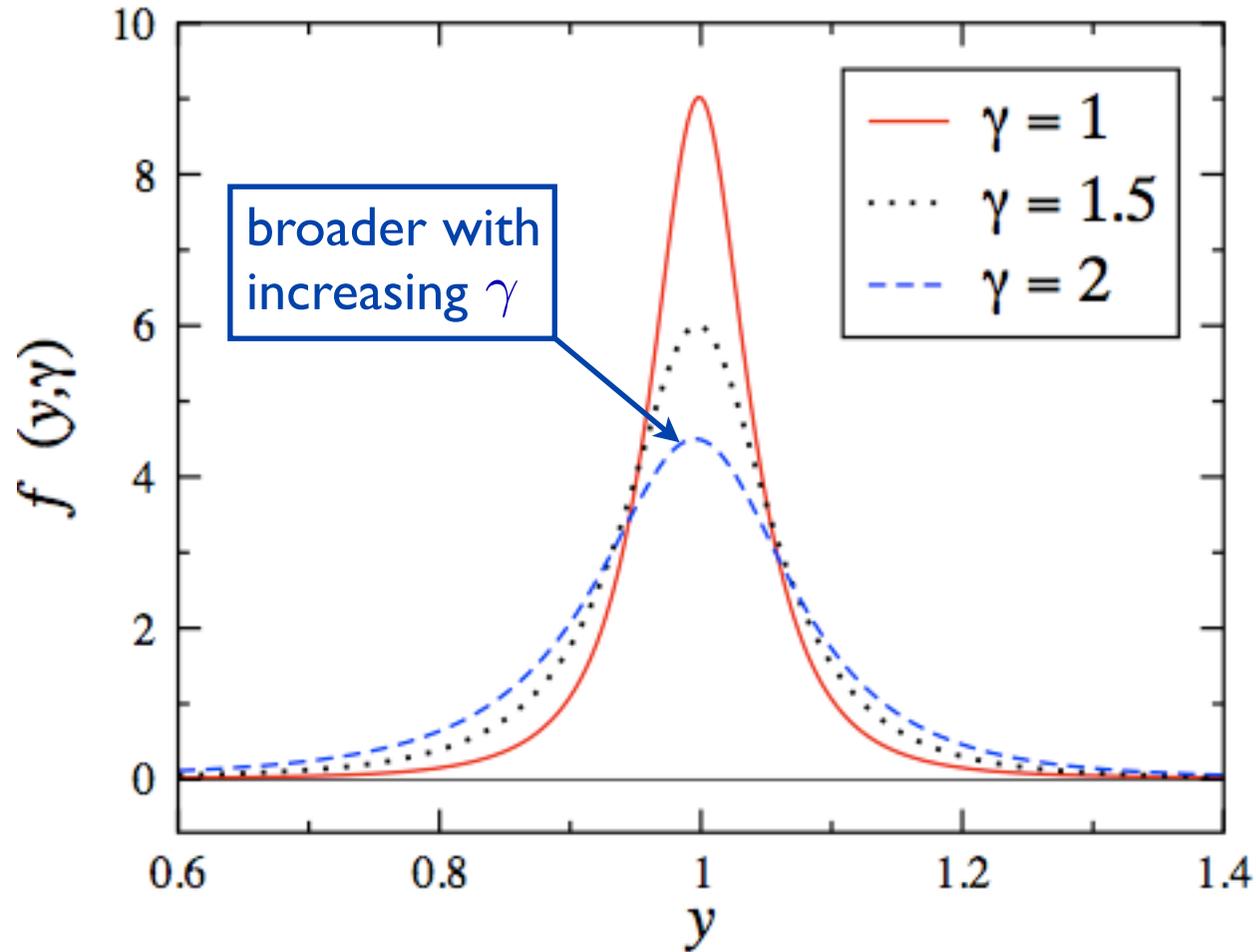
→ finite- Q^2 correction factor

$$\mathcal{C}(y, \gamma) = \frac{1}{\gamma^2} \left[1 + \frac{(\gamma^2 - 1)}{y^2} \left(1 + \frac{2\varepsilon}{M} + \frac{3p_{\perp}^2 - 2\mathbf{p}^2}{2M^2} \right) \right]$$

→ 1 for $\gamma \rightarrow 1$

separation energy $\varepsilon = p_0 - M$
 $\approx \varepsilon_d - \mathbf{p}^2/2M$

Finite- Q^2 smearing function



Kahn, WM, Kulagin, PRC 79, 035205 (2009)

→ for most kinematics $\gamma \lesssim 2$

→ effectively more smearing for larger x or lower Q^2

Finite- Q^2 smearing function

- Can we measure / constrain smearing function (and its Q^2 dependence) in other reactions?

→ quasielastic electron–deuteron scattering (in IA) directly probes nucleon distribution function (multiplied by nucleon form factor)

$$F_2^{N(\text{el})}(x/y, Q^2, p^2) = \left[\frac{G_{EN}^2 + \tau G_{MN}^2}{1 + \tau} \right] y \delta(y - x/x_{\text{th}})$$

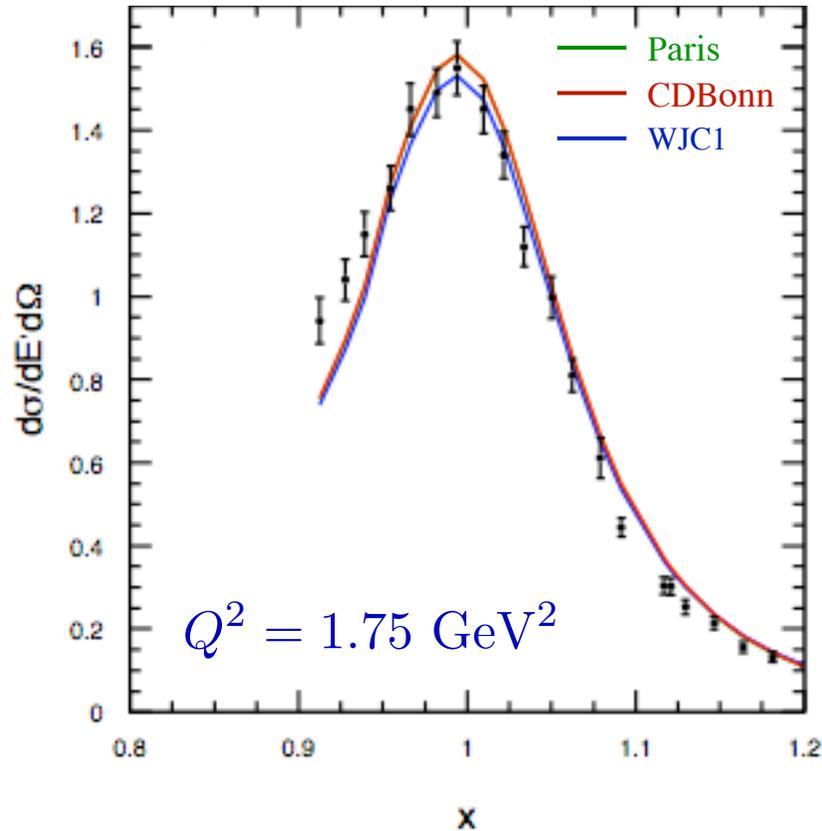
with $x_{\text{th}} = [1 - (p^2 - M^2)/Q^2]^{-1}$, $\tau = Q^2/4M^2$

→ quasielastic contribution to deuteron structure function

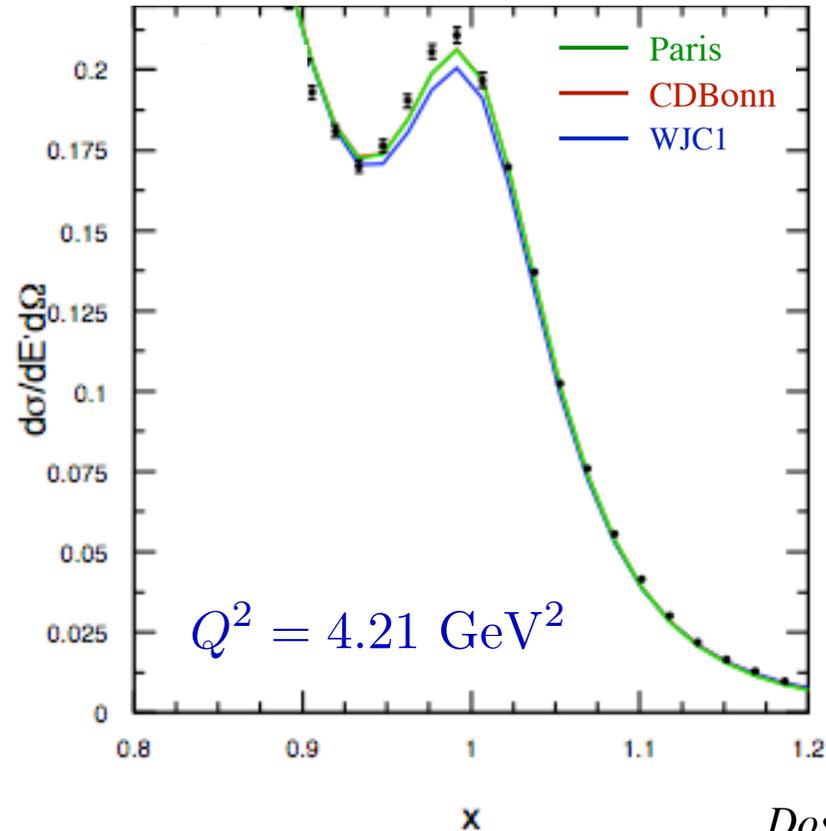
$$F_2^{d(\text{QE})}(x, Q^2) \rightarrow \sum_N \left[\frac{G_E^{N2} + \tau G_M^{N2}}{1 + \tau} \right] x f_{N/d}(x, \gamma)$$

Finite- Q^2 smearing function

Arnold et al., PRL 61, 806 (1988)



Arrington et al., PRL 82, 2056 (1999)

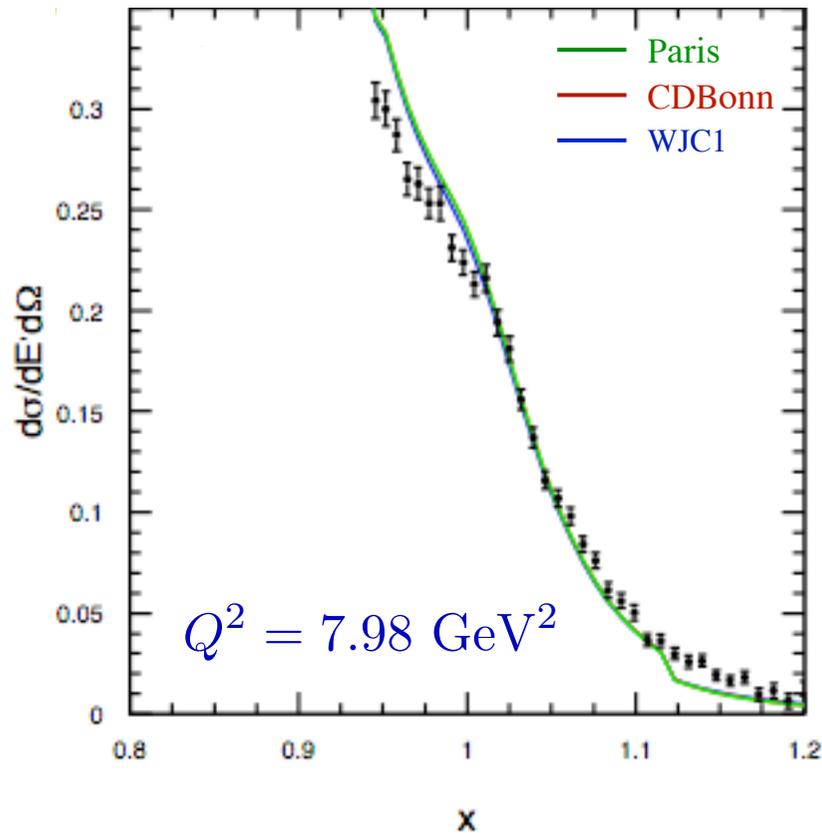


Doshi, WM (2012)

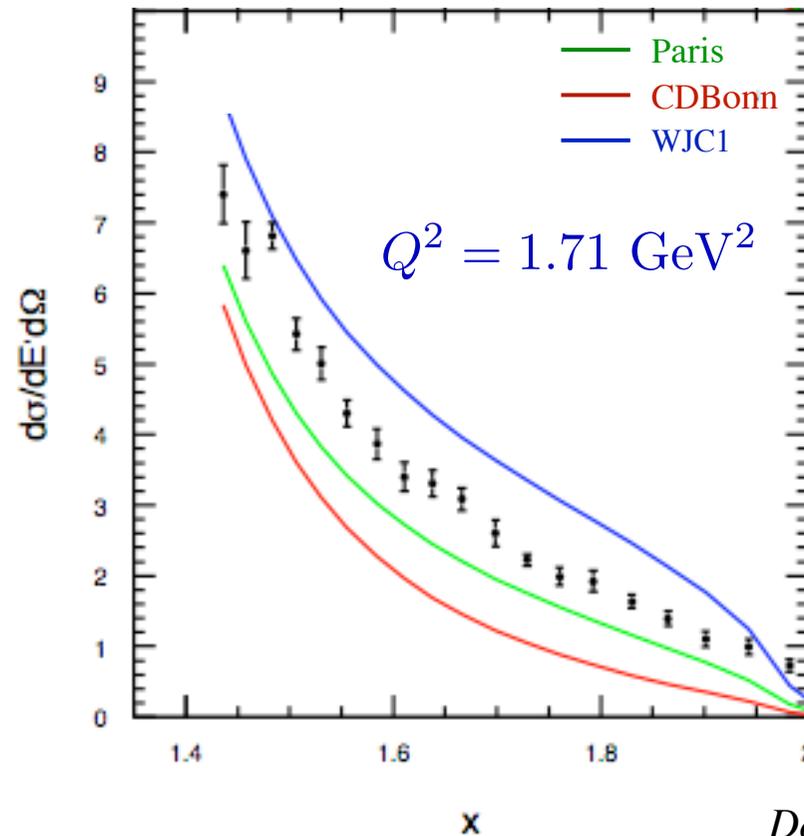
- most data can be described by WBA smearing function
- γ dependence crucial for describing Q^2 variation

Finite- Q^2 smearing function

Rock et al., PRD 46, 24 (1992)



Schutz et al., PRL 38, 259 (1977)



Doshi, WM (2012)

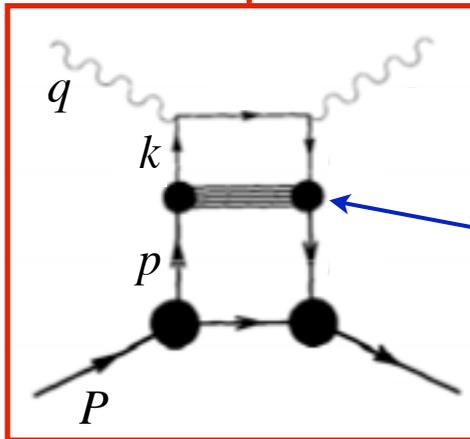
- most data can be described by WBA smearing function
- γ dependence crucial for describing Q^2 variation
- wave function dependence mild, except for $x \gg 1$ tails

Nucleon off-shell corrections

- In relativistic (MST) model, two sources of off-shellness

$$\delta^{(\text{off})} F_2^d \longrightarrow \delta^{(\Psi)} F_2^d \quad \text{negative energy components of } \psi_d$$

$$\longrightarrow \delta^{(p^2)} F_2^d \quad \text{off-shell } N \text{ structure function}$$



$$\Phi(p, k) = N(p^2) \frac{k^2 - m_q^2}{(k^2 - \Lambda^2)^n}$$

quark-diquark vertex functions

Nucleon off-shell corrections

- In relativistic (MST) model, two sources of off-shellness

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$$\longrightarrow \delta^{(p^2)} F_2^d \quad \text{off-shell } N \text{ structure function}$$

$$\begin{aligned} \delta^{(\Psi)} F_2^d \sim & \int dy \int dp^2 \left\{ \left[\frac{1}{2} (1 - E_p/p_0) F_2^N(x/y) \left(\frac{E_p}{M_d} \widehat{W}_1^{\text{on}} - \frac{P \cdot q}{M_d^2} \widehat{W}_2^{\text{on}} \right) \underline{(p^2 - M^2)} \right] |\psi_d(p)|^2 \right. \\ & + \left[-2M \widehat{W}_0^{\text{on}} + 2\mathbf{p}^2 \widehat{W}_1^{\text{on}} + \left(1 - y - \frac{E_p}{M_d} \right) P \cdot q \widehat{W}_2^{\text{on}} \right] \underline{(v_t^2 + v_s^2)} \\ & \left. + \left[M \widehat{W}_0^{\text{on}} + M^2 \widehat{W}_1^{\text{on}} + \frac{M^2}{\mathbf{p}^2} \left(1 - y - \frac{E_p}{M_d} \right) P \cdot q \widehat{W}_2^{\text{on}} \right] \frac{2|\mathbf{p}|}{\sqrt{3}M} \left(\underline{u(v_s - \sqrt{2}v_t)} + \underline{w(v_t + \sqrt{2}v_s)} \right) \right\} \end{aligned}$$

$$\delta^{(p^2)} F_2^d \sim \int dy \int dp^2 \left\{ \underline{\mathcal{A}_s \widehat{W}_0^{\text{off}}} + \underline{\mathcal{A}_v \cdot p \widehat{W}_1^{\text{off}}} + \underline{\mathcal{A}_v \cdot q \widehat{W}_2^{\text{off}}} \right\}$$

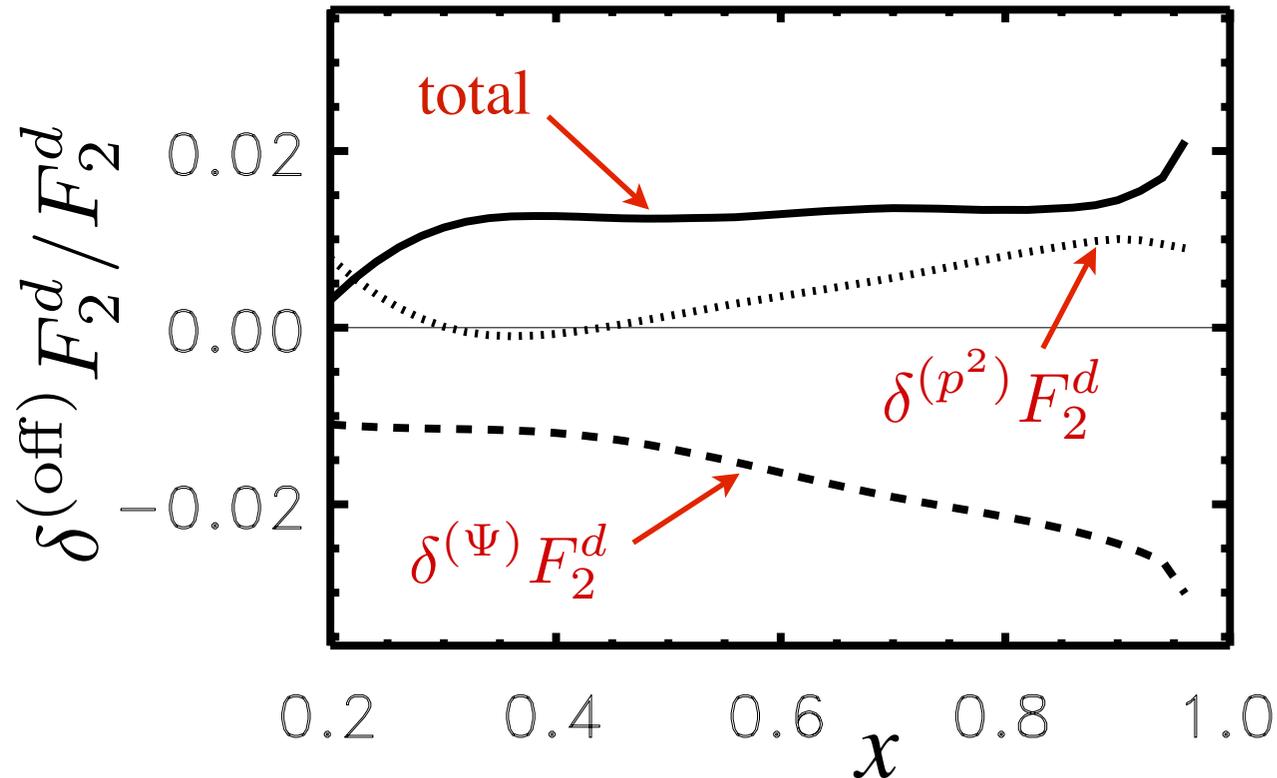
$$\widehat{W}_i^{\text{off}} = \widehat{W}_i - \widehat{W}_i^{\text{on}}$$

$$\mathcal{A}_{s,v} = \mathcal{A}_{s,v}(u, w, v_s, v_t)$$

Nucleon off-shell corrections

- In relativistic (MST) model, two sources of off-shellness

$$\begin{aligned} \delta^{(\text{off})} F_2^d &\longrightarrow \delta^{(\Psi)} F_2^d && \text{negative energy components of } \psi_d \\ &\longrightarrow \delta^{(p^2)} F_2^d && \text{off-shell } N \text{ structure function} \end{aligned}$$



WM, Schreiber, Thomas
PLB 335, 11 (1994)

→ $\lesssim 1 - 2\%$ effect

Nucleon off-shell corrections

- In relativistic (MST) model, two sources of off-shellness

$$\begin{aligned} \delta^{(\text{off})} F_2^d &\longrightarrow \delta^{(\Psi)} F_2^d && \text{negative energy components of } \psi_d \\ &\longrightarrow \delta^{(p^2)} F_2^d && \text{off-shell } N \text{ structure function} \end{aligned}$$

- High degree of model dependence

→ require *quark*-level description of nuclear (bound nucleon) structure

→ *see Cloët talk*

→ in practice easier to implement off-shellness at *hadron* level through WBA approach

Nucleon off-shell corrections

- In WBA model, nuclear structure function factorizes to $\mathcal{O}(p^2/M^2)$ into a 2-dimensional convolution (in y & p^2)

→ expand off-shell structure function about $p^2 = M^2$

$$F_2^{N(\text{off})}(x, p^2) = F_2^N(x) \left(1 + \delta f(x) \frac{p^2 - M^2}{M^2} \right)$$

with coefficient function

$$\delta f(x) = \left. \frac{\partial \log F_2^{N(\text{off})}}{\partial \log p^2} \right|_{p^2=M^2}$$

- “Phenomenological” model

→ assume δf is independent of nucleus, and fit to F_2^A/F_2^d data

$$\delta f^{(\text{fit})} = C_N(x - 0.05)(x - x_0)(1 + x_0 - x)$$

→ see Petti talk

Kulagin, Petti, NPA 765, 126 (2006)

Nucleon off-shell corrections

- “Microscopic” model – off-shell spectral function $\Phi(k^2, \Lambda(p^2))$

→ in valence approximation $F_2^N(x) \sim xq_v(x)$

$$\delta f(x) \rightarrow \frac{\partial \log q_v}{\partial x} h(x) + \int dx \frac{\partial q_v}{\partial x} h(x)$$

$$h(x) = x(1-x) \frac{(1-\lambda)(1-x)M^2 + \lambda s_0}{(1-x)M^2 - s_0}$$

parameter
 $s_0 \sim 2 \text{ GeV}^2$

$$\lambda = \left. \frac{\partial \log \Lambda^2}{\partial \log p^2} \right|_{p^2=M^2} \rightarrow \Lambda^{-1} \sim \text{confinement radius } R_N$$

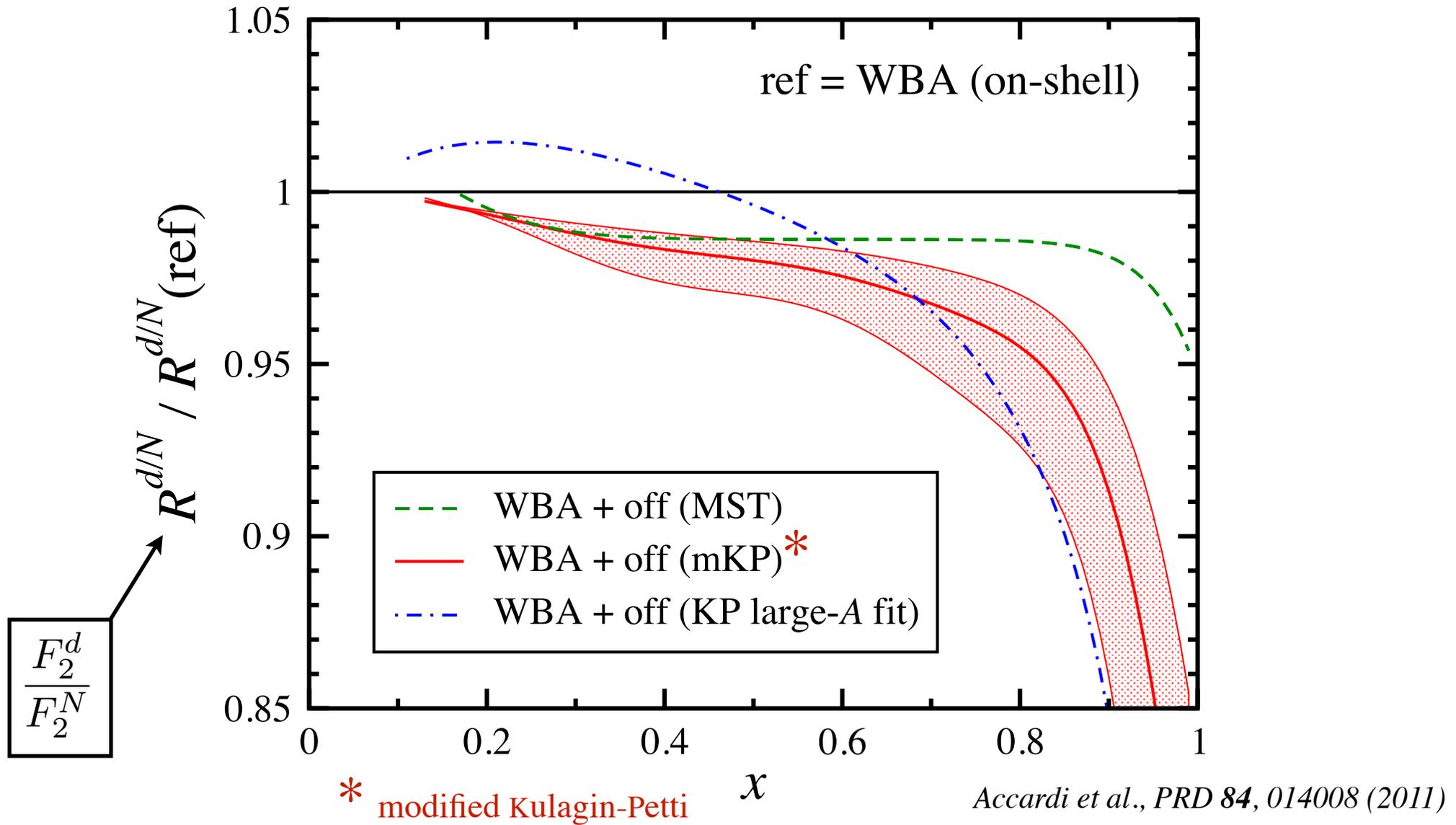
$$= -\frac{2 \delta R_N}{R_N} \frac{\delta p^2}{M^2} \rightarrow \underline{\underline{\delta R_N / R_N}} = 1.5\% - 1.8\%$$

Close et al., PRD 31, 1004 (1985)

$$\sim 0.46 - 1.00 \quad \underline{\underline{\delta p^2 / M^2}} \approx -3.7\% \text{ to } -6.2\%$$

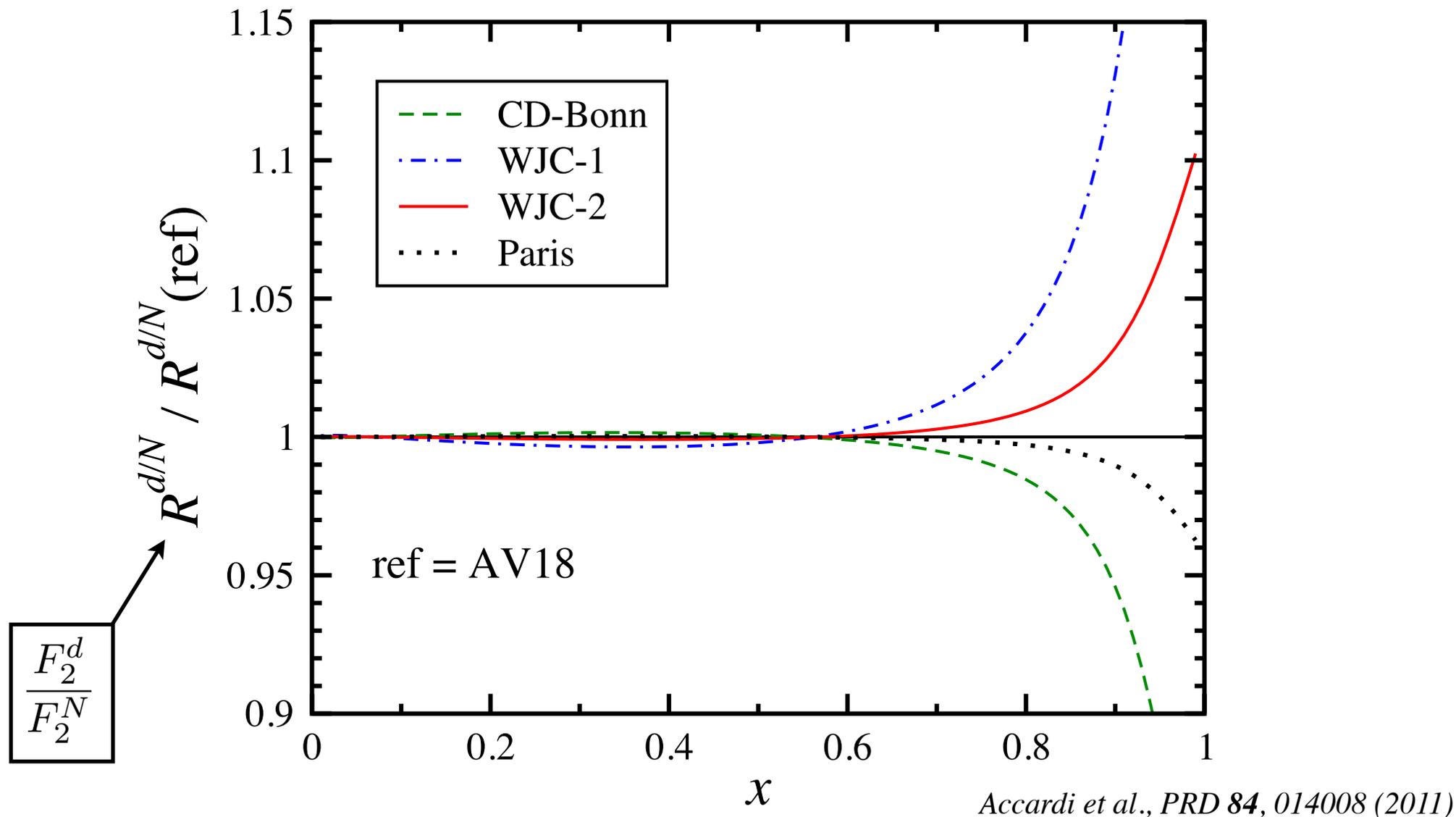
AV18, CD-Bonn, WJC

Model dependence: off-shell corrections



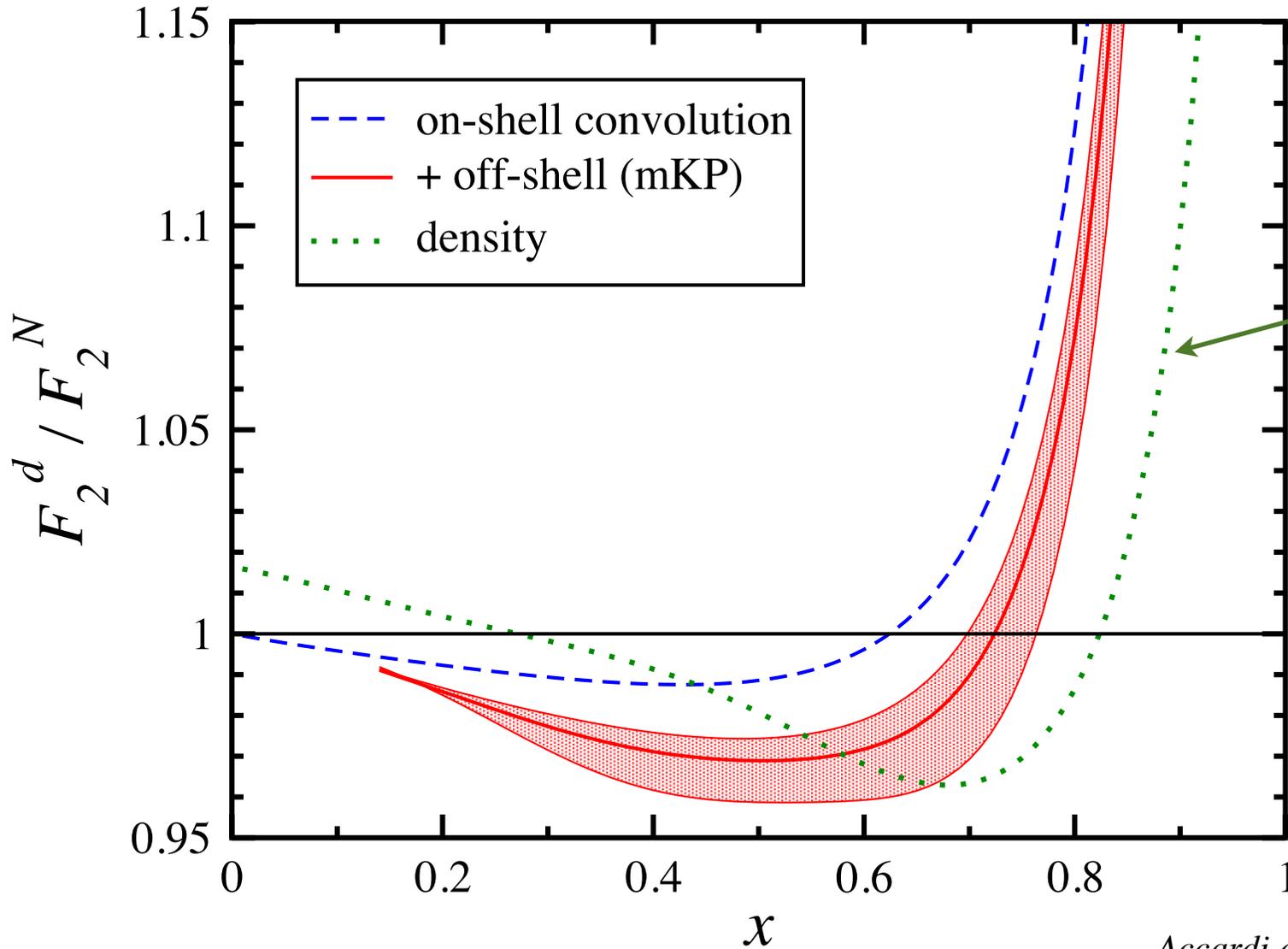
→ increasing nucleon off-shell suppression at large x

Model dependence: deuteron wave function



→ uncertainty in large- y tail of momentum distribution
(short-range NN interaction)

EMC effect in deuteron



nuclear density

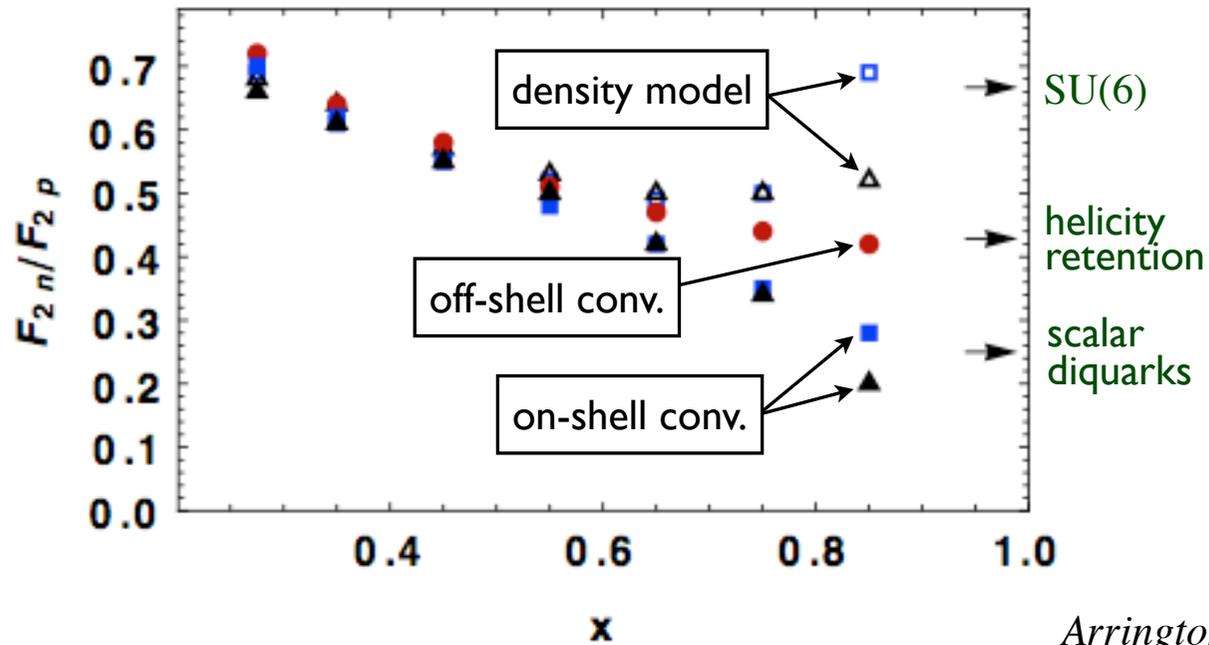
$$\frac{F_2^d}{F_2^N} - 1 \approx \frac{1}{4} \left(\frac{F_2^{\text{Fe}}}{F_2^d} - 1 \right)$$

assumes EMC effect scales with density; extrapolated from Fe \rightarrow deuterium

Accardi et al., PRD 84, 014008 (2011)

$\rightarrow \approx 2-4\%$ depletion at $x \sim 0.4-0.6$, depending on model

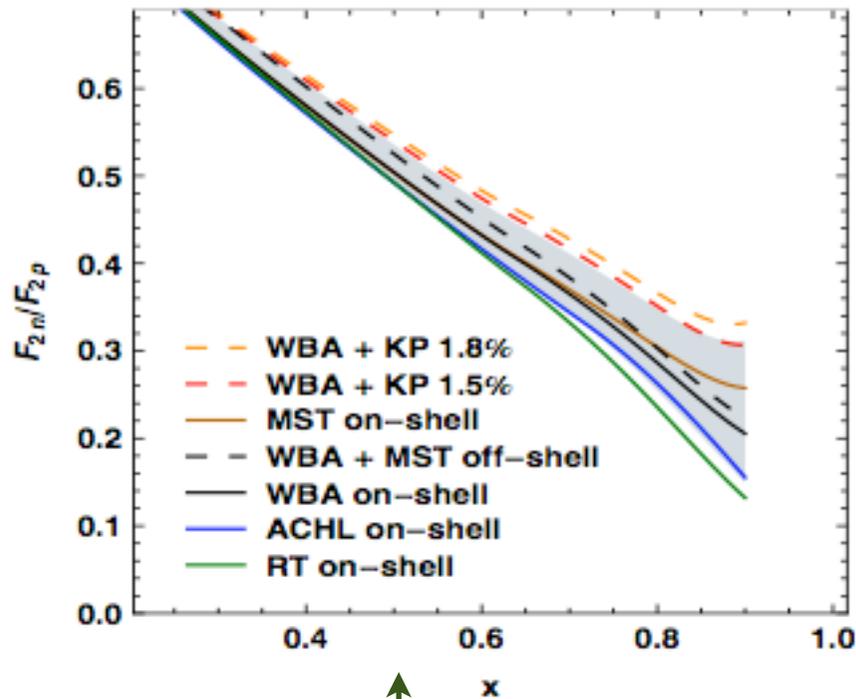
Nuclear effects on neutron structure



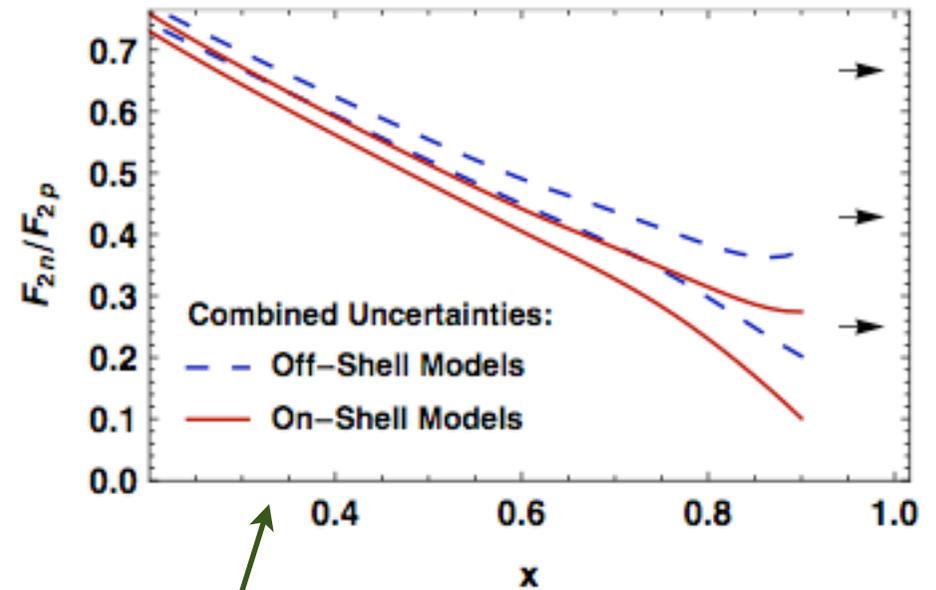
*Arrington, Rubin, WM
PRL (2012) [arXiv:1110.3362]*

- large effect of nuclear model uncertainty on extracted neutron structure function at high x
- cannot discriminate between predictions for $x \rightarrow 1$ behavior of F_2^n / F_2^p ratio

Nuclear effects on neutron structure



microscopic
deuteron models

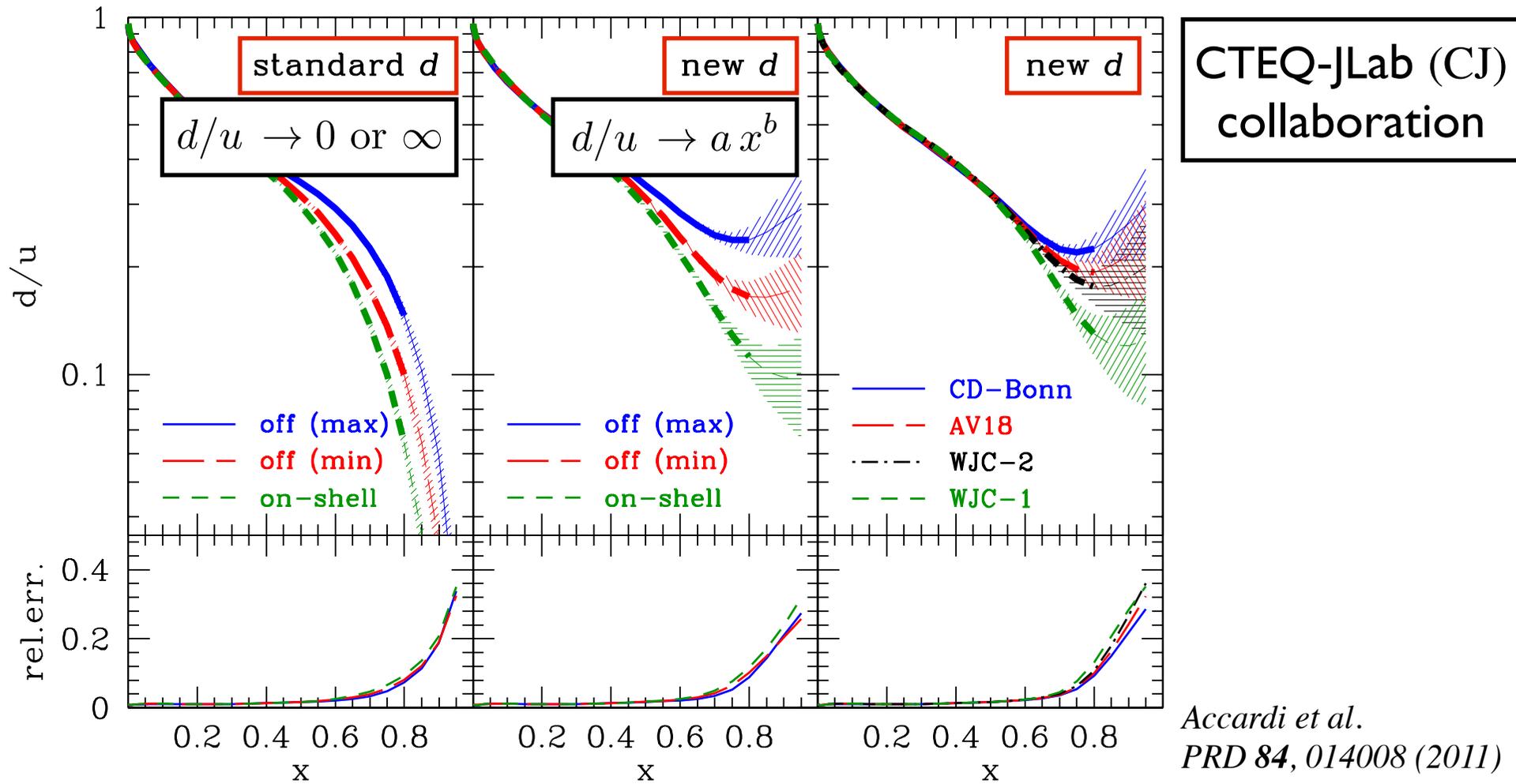


on-shell vs.
off-shell models

Arrington, Rubin, WM
PRL (2012) [arXiv:1110.3362]

- SU(6) prediction disfavored by microscopic models
- to disentangle *leading twist* need global pQCD analysis

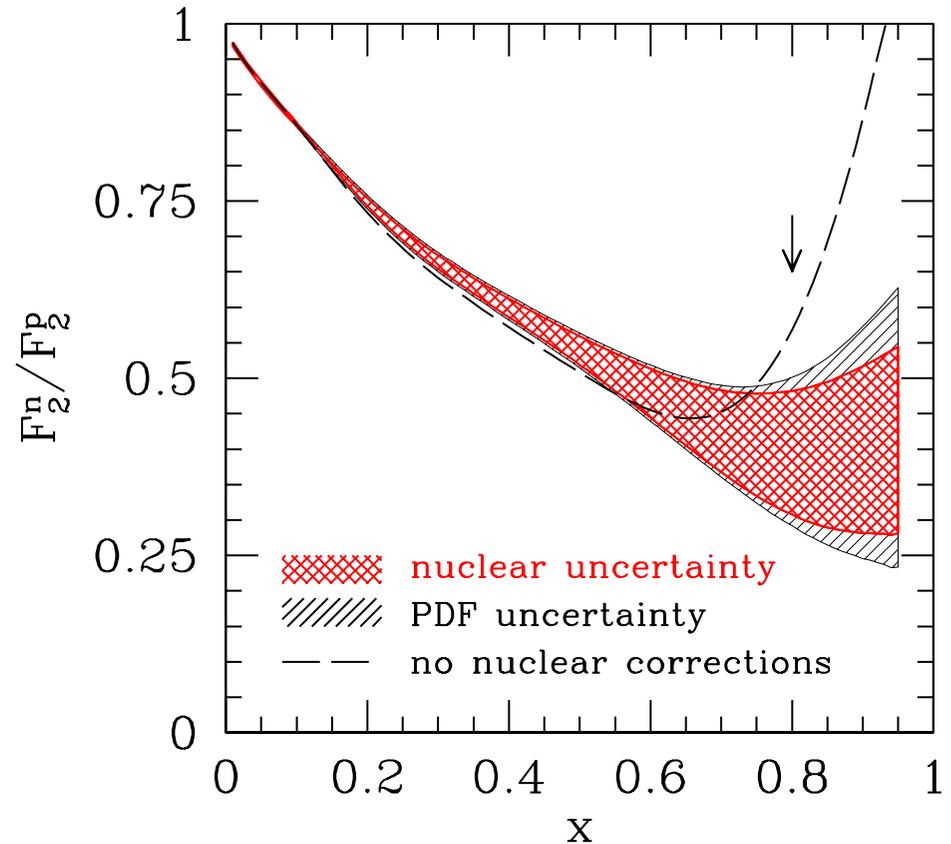
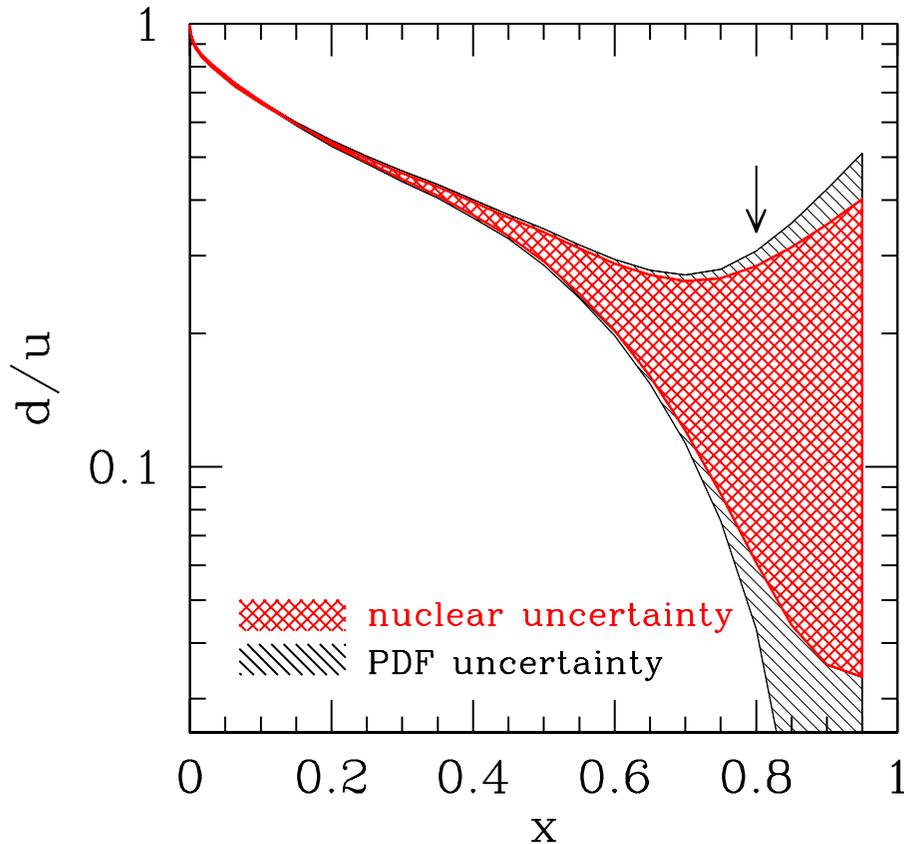
Nuclear effects on PDF analysis



→ larger off-shell effects → larger d/u ratio

→ precise $x \rightarrow 1$ limit depends on parametrization

Nuclear effects on PDF analysis



→ see Accardi talk

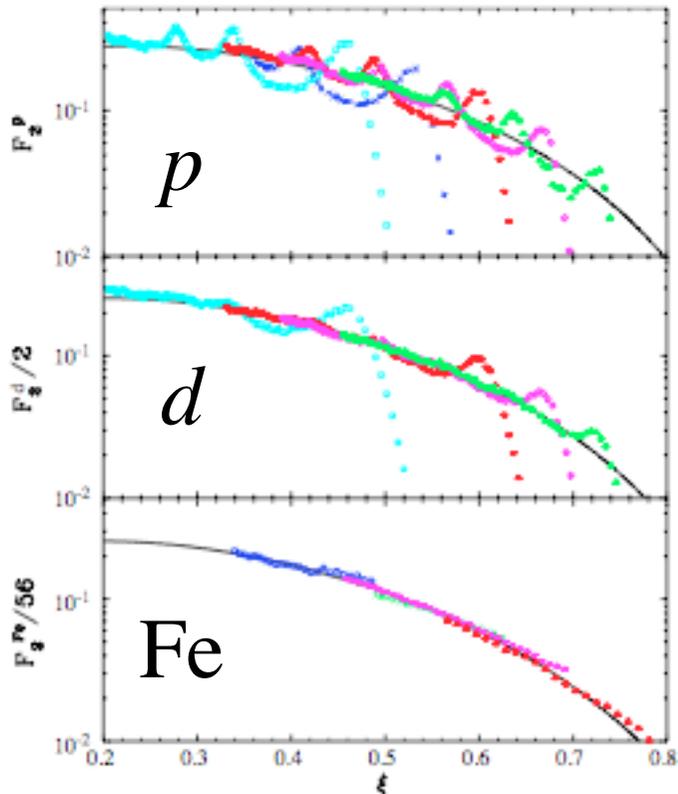
Accardi et al.
PRD 84, 014008 (2011)

- combined nuclear correction uncertainties sizable at $x > 0.5$
- $x \rightarrow 1$ limiting value depends critically on deuteron model
- n/p ratio smaller at large x *cf.* no nuclear corrections fit

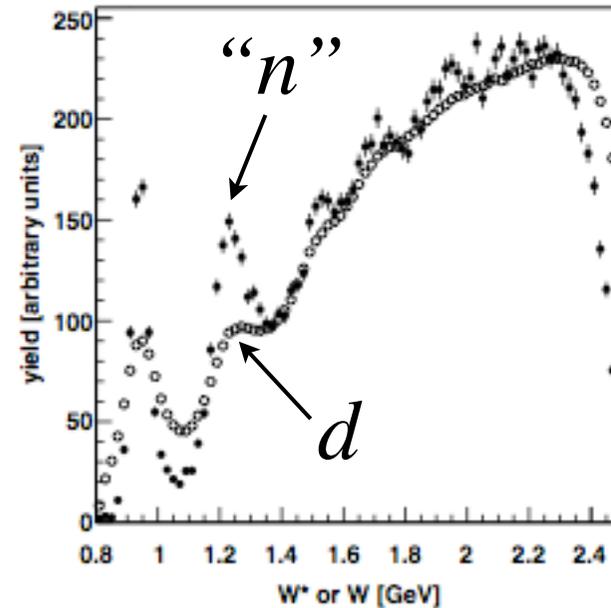
Nuclear effects on resonances

Neutron resonances

- Extraction of neutron information in *resonance* region is highly problematic
 - nuclear Fermi motion smears out resonance structures in neutrons bound in nuclei



Arrington et al., PRC **64**, 014602 (2001)



Baillie et al., PRL **108**, 142001 (2012)

Neutron resonances – unsmearing

- Calculated F_2^d depends on input F_2^n
→ extracted n depends on input n ... cyclic argument

- Solution: (additive) iteration procedure

0. subtract $\delta^{(\text{off})} F_2^d$ from d data: $F_2^d \rightarrow F_2^d - \delta^{(\text{off})} F_2^d$

1. define difference between smeared and free SFs

$$F_2^d - \tilde{F}_2^p = \tilde{F}_2^n \equiv f \otimes F_2^n \equiv F_2^n + \Delta$$

2. first guess for $F_2^{n(0)} \rightarrow \Delta^{(0)} = \tilde{F}_2^{n(0)} - F_2^n$

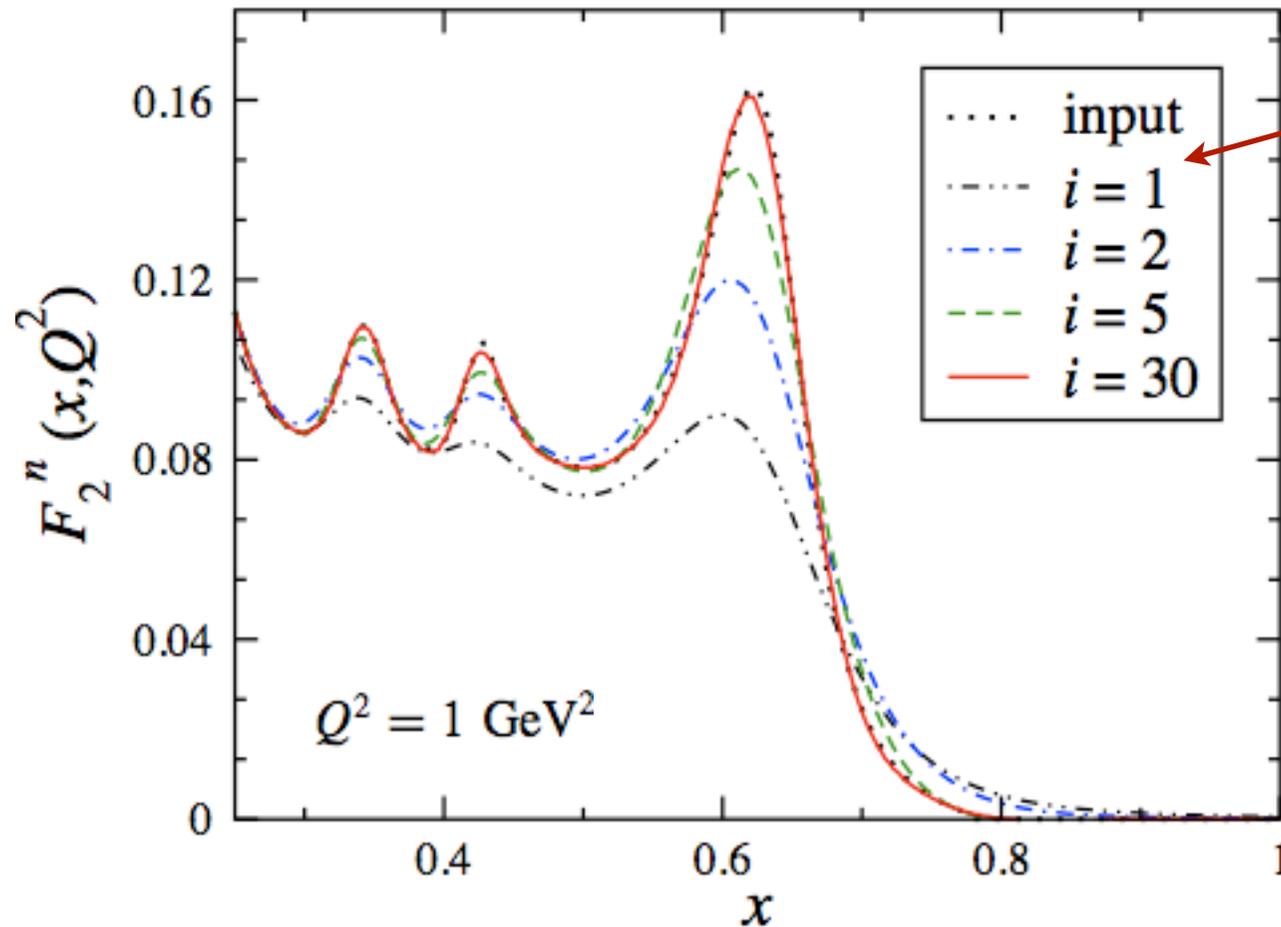
3. after one iteration, gives

$$F_2^{n(1)} = F_2^{n(0)} + (\tilde{F}_2^n - \tilde{F}_2^{n(0)})$$

4. repeat until convergence

Neutron resonances – unsmearing

- F_2^d constructed from known F_2^p and F_2^n inputs
(using MAID resonance parameterization)



initial guess

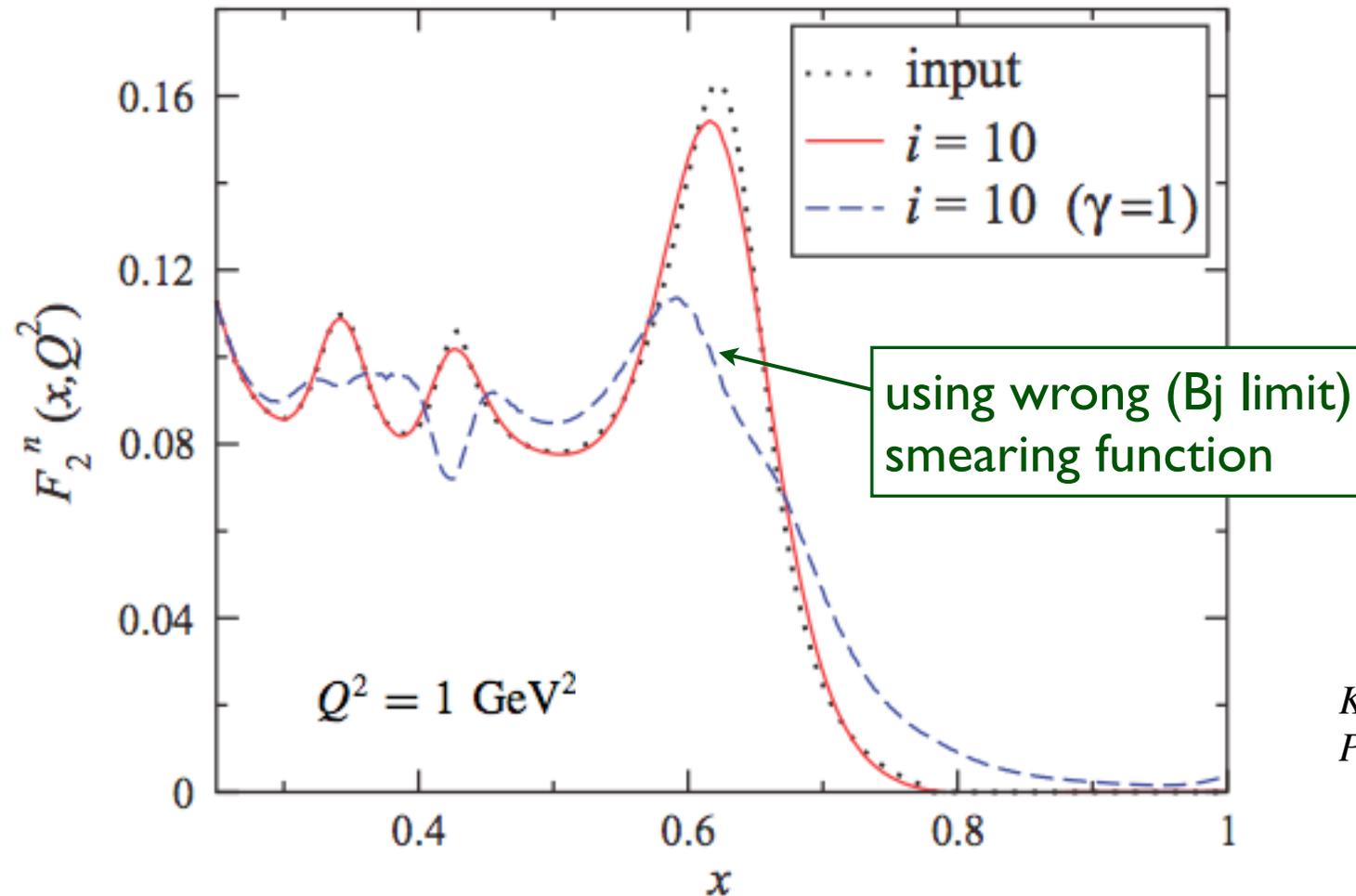
$$F_2^{n(0)} = 0$$

→ can reconstruct almost arbitrary shape

*Kahn, WM, Kulagin
PRC 79, 035205 (2008)*

Neutron resonances – unsmearing

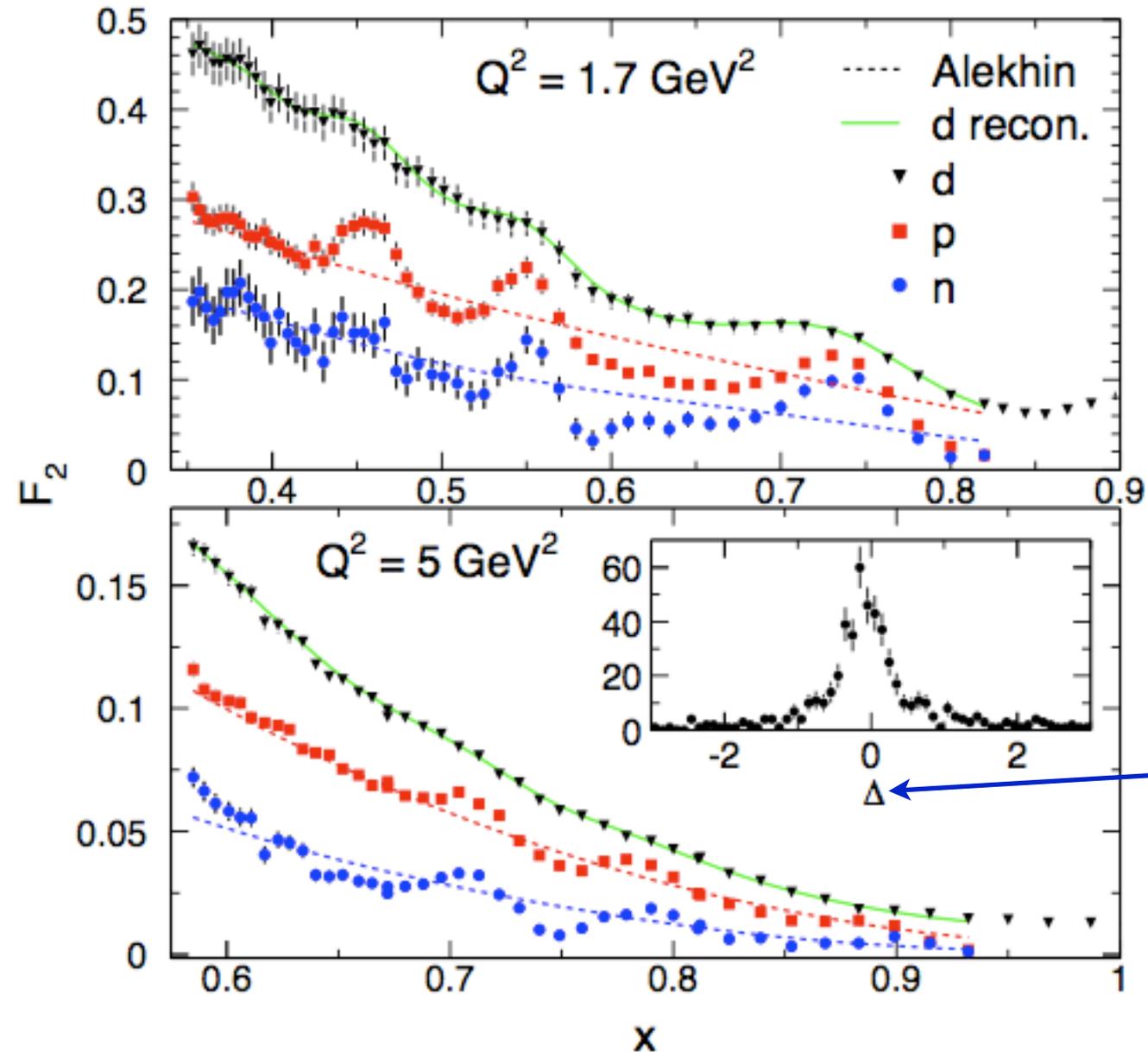
- F_2^d constructed from known F_2^p and F_2^n inputs
(using MAID resonance parameterization)



*Kahn, WM, Kulagin
PRC 79, 035205 (2008)*

→ vital to use correct (Q^2 -dependent) smearing function

Neutron resonances – unsmearing



→ JLab & SLAC data

→ 2 iterations
with $n(0)=p$

■ clear neutron resonance structure observed

■ weak dependence on input neutron

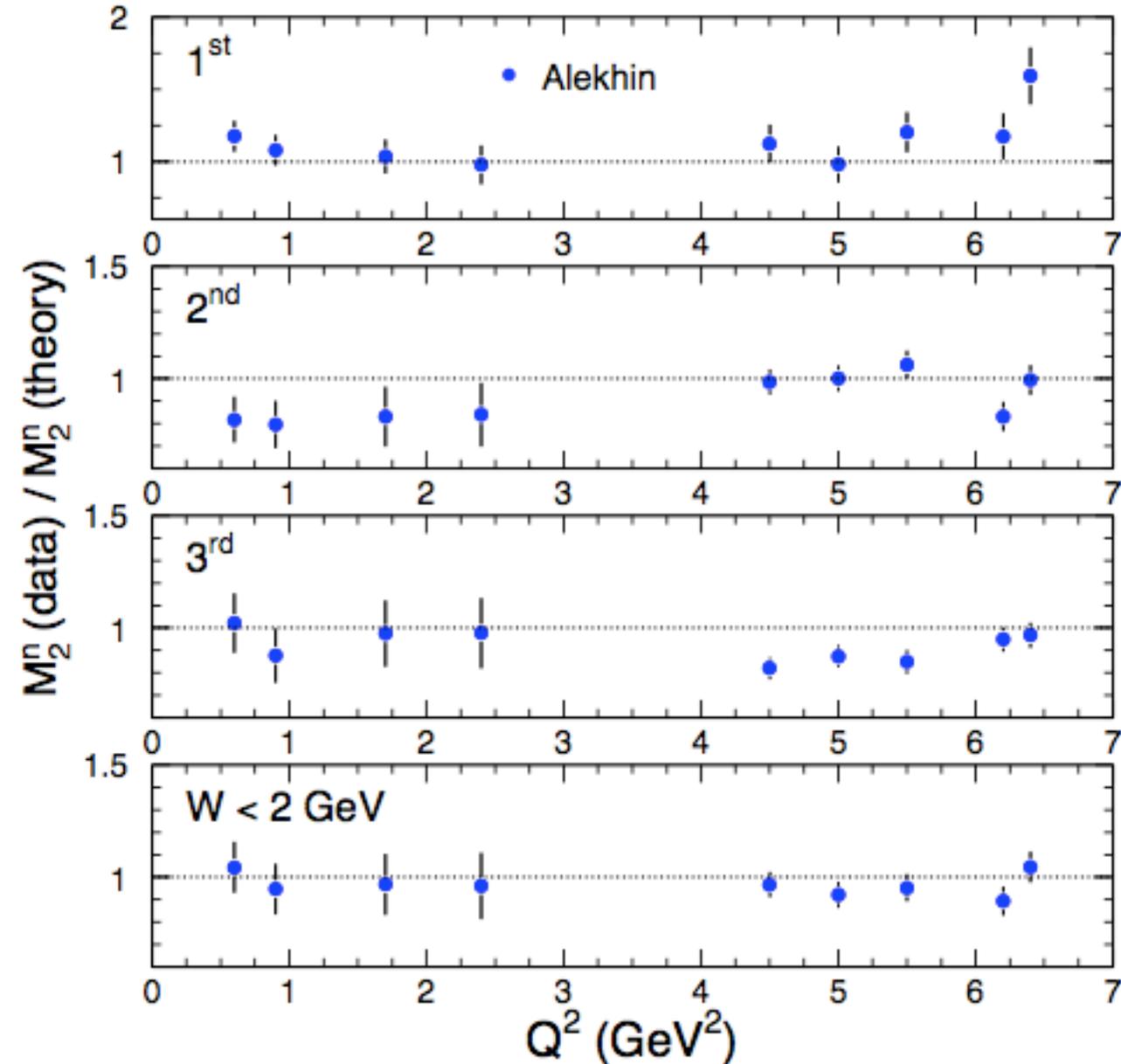
$$[F_2^n(n(0) = p) - F_2^n(n(0) = p/2)] / \sigma(F_2)$$

Δ

Malace, Kahn, WM, Keppel
PRL **104**, 102001 (2010)

→ striking similarity with QCD fit to DIS data!

Neutron resonances – duality



→ “theory” is QCD fit to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

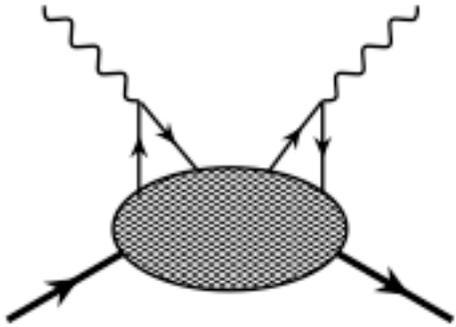
- *locally*, deviations in individual resonance regions < 15–20%
- *globally*, deviations generally < 10%

*Malace, Kahn, WM, Keppel
PRL 104, 102001 (2010)*

→ duality is *not accidental*, but a general feature of resonance–scaling transition!

Neutron resonances – duality

■ Accidental cancellations of charges?



cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i e_j \sim \left(\sum_i e_i \right)^2 - \sum_i e_i^2$$

↑ *coherent* ↑ *incoherent*

proton HT $\sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9} \right) = 0 !$

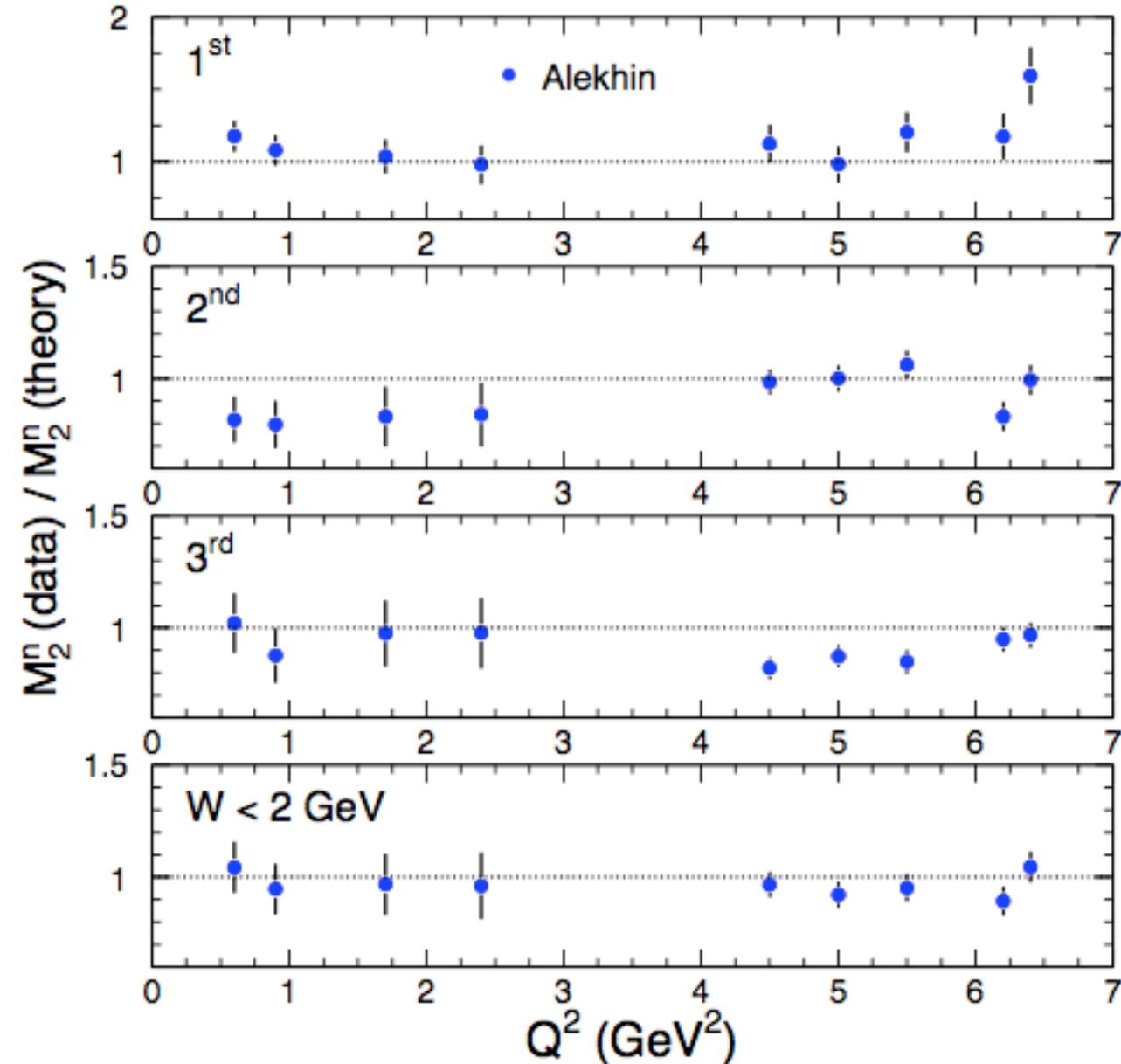
neutron HT $\sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9} \right) \neq 0$

→ duality in proton a *coincidence!*

→ should not hold for neutron

Brodsky, hep-ph/0006310

Neutron resonances – duality



→ “theory” is QCD fit
to $W > 2$ GeV data

Alekhin et al., 0908.2762 [hep-ph]

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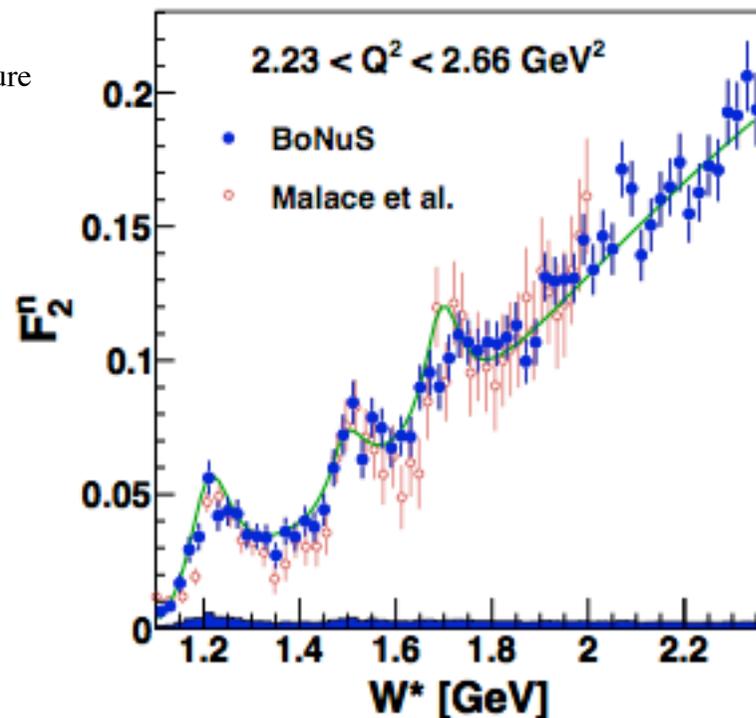
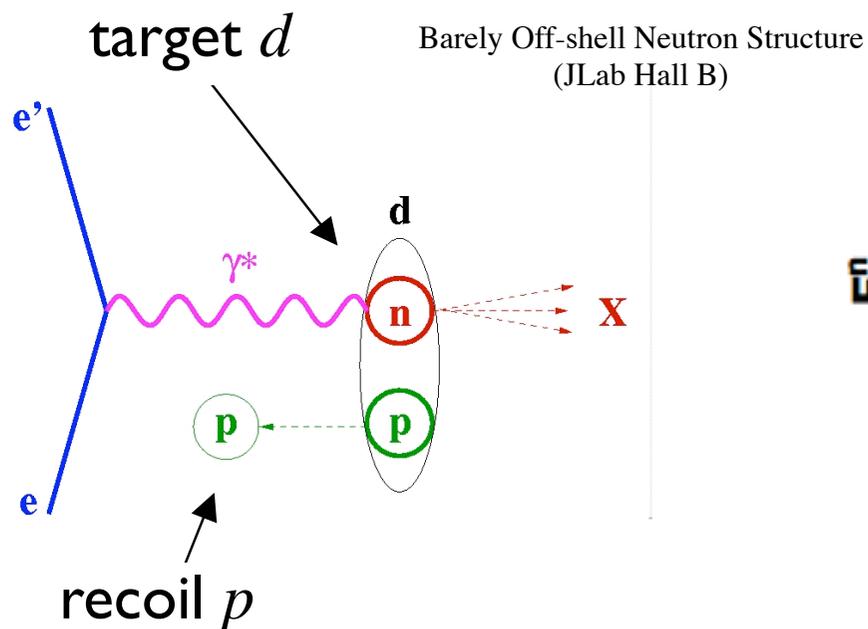
■ *globally*, deviations
generally < 10%

Malace, Kahn, WM, Keppel
PRL 104, 102001 (2010)

→ use resonance region data to learn about
leading twist structure functions?

Neutron resonances – BoNuS

- (Almost) free neutron structure function extracted from semi-inclusive scattering from d with spectator p tagging
 - slow, backward-moving proton ensures neutron is nearly on-shell, minimizes rescattering

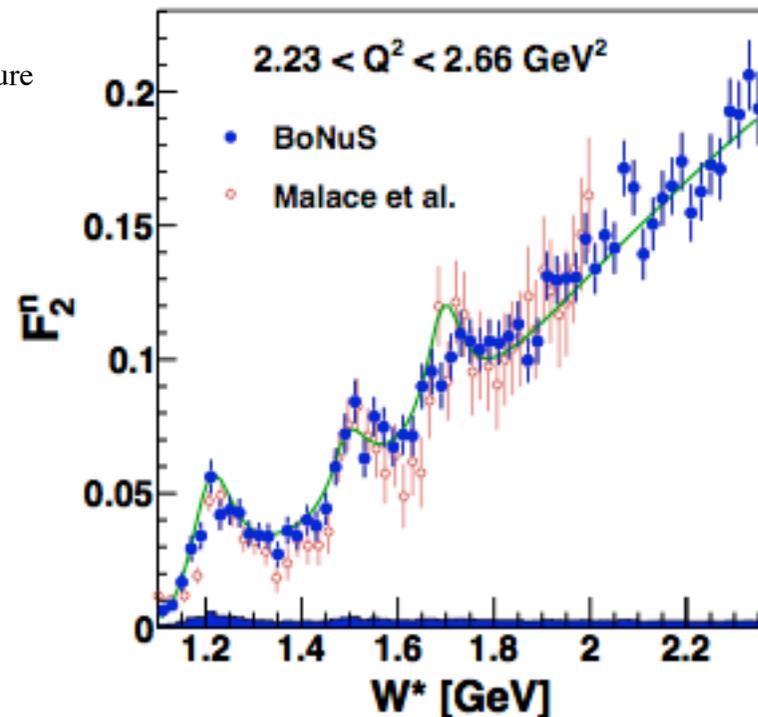
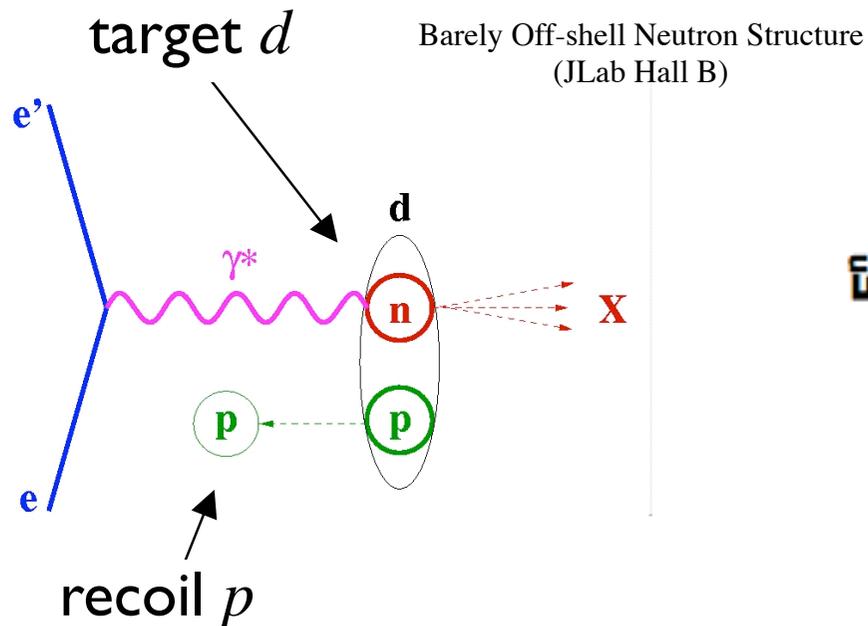


Baillie et al., PRL 108, 142001 (2012)

- agreement between extracted (inclusive) data and (more precise) BoNuS results

Neutron resonances – BoNuS

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 - slow, backward-moving proton ensures neutron is nearly on-shell, minimizes rescattering



Baillie et al., PRL 108, 142001 (2012)

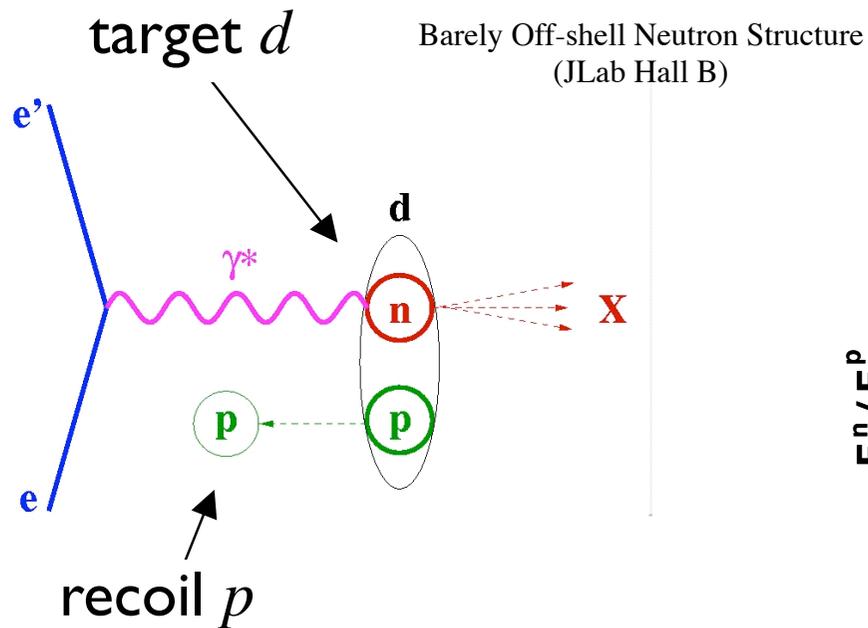
- duality analysis in progress

Niculescu, WM et al. (2012)

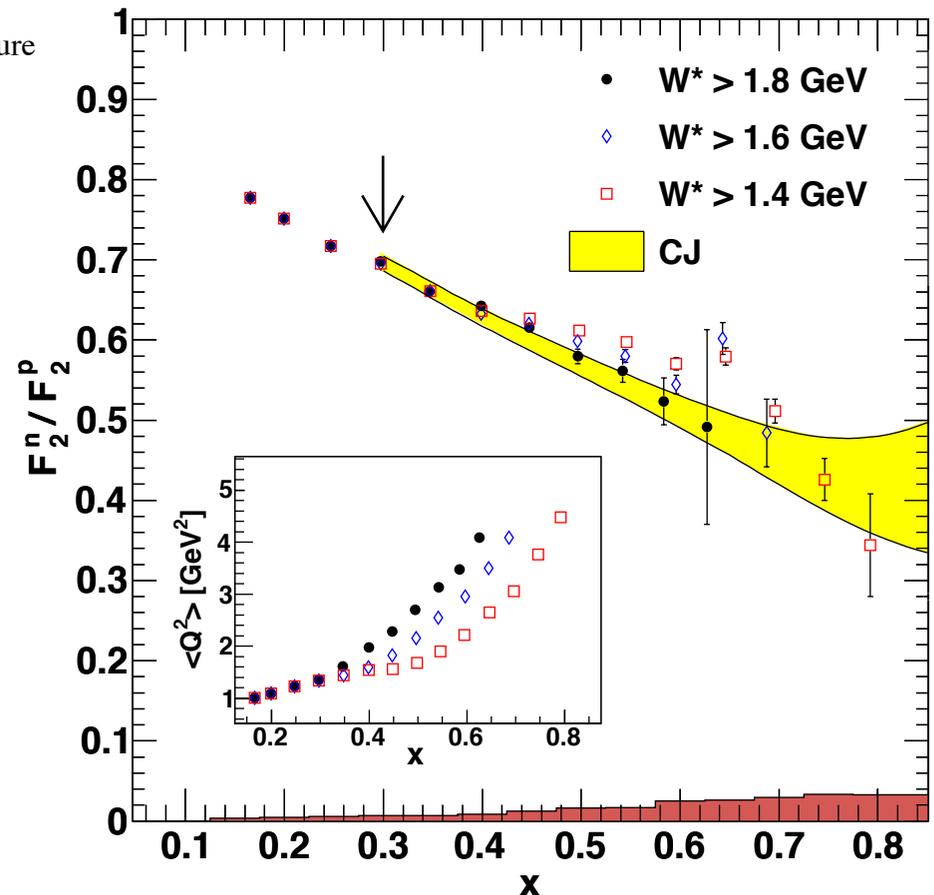
Neutron resonances – BoNuS

- (Almost) free neutron structure function extracted from semi-inclusive scattering from d with spectator p tagging

→ slow, backward-moving proton ensures neutron is nearly on-shell, minimizes rescattering



→ 12 GeV experiment will extend (DIS) range to $x \sim 0.8$



Constraints from local duality

- If validity (at ~15–20% level) of local duality holds also for extreme subthreshold region $M \leq W \leq M + m_\pi$

→ relate $x \rightarrow 1$ behavior of structure functions with $Q^2 \rightarrow \infty$ behavior of elastic (magnetic) form factors *

$$\frac{F_2^{N*}}{F_2^N} \longleftrightarrow \frac{d(G_M^{N*})^2/dQ^2}{d(G_M^N)^2/dQ^2}$$

Bloom, Gilman, PRL 16, 1140 (1970)
WM, PRL 86, 35 (2001)

* at finite Q^2 , corrections also from G_E^N

→ F_2^{N*}, G_M^{N*} = off-shell N structure function, form factor

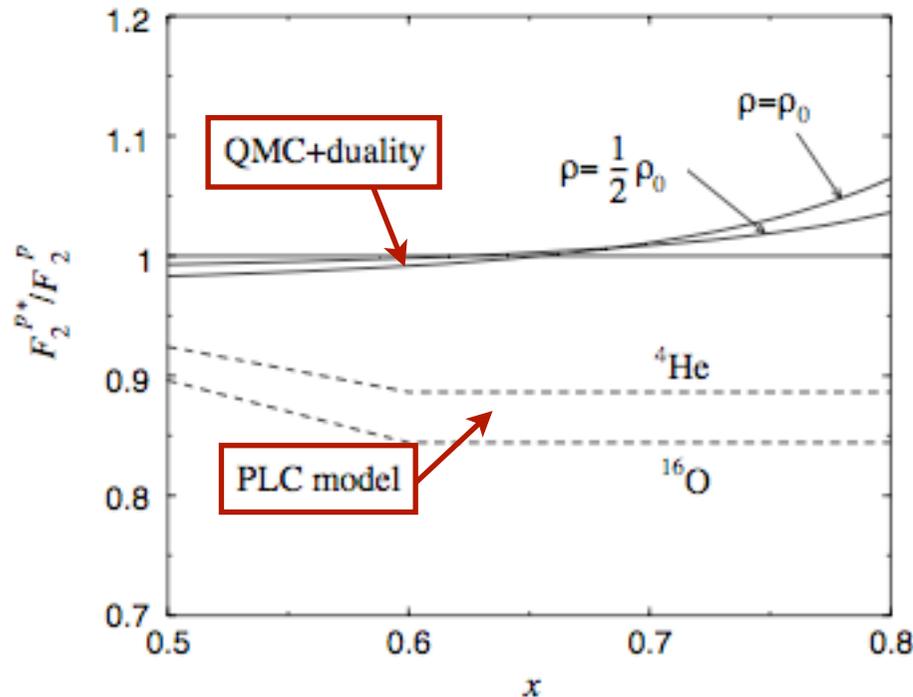


calculated in quark-meson coupling (QMC) model

Guichon, Thomas, Saito, Tsushima, ...

Constraints from local duality

- If validity (at $\sim 15\text{--}20\%$ level) of local duality holds also for extreme subthreshold region $M \leq W \leq M + m_\pi$
 - relate $x \rightarrow 1$ behavior of structure functions with $Q^2 \rightarrow \infty$ behavior of elastic (magnetic) form factors



- point-like configuration (PLC) suppression model ($x > 0.6$)

$$\frac{F_2^{N*}}{F_2^N} = 1 - \frac{2(k^2/2M + \epsilon_A)}{\Delta E_A}$$

- most of EMC effect attributed to off-shell nucleon modification

Frankfurt, Strikman, NPB 250, 1585 (1985)

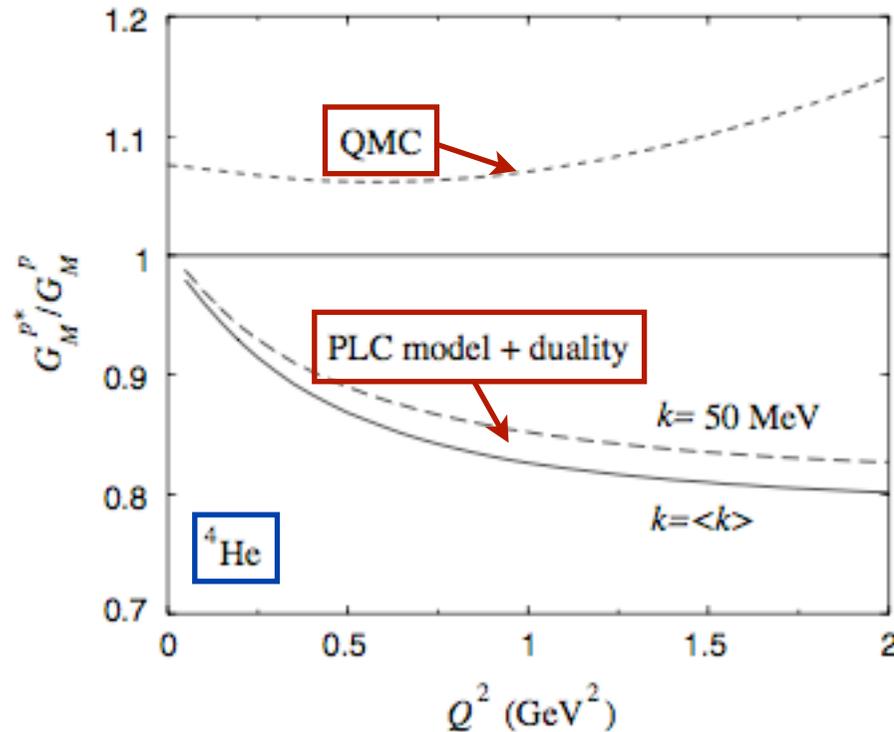
- sign of off-shell effect in models *opposite* at large x

Constraints from local duality

- Using duality in reverse, extract elastic form factor from integral of structure function below threshold

$$(G_M^p)^2 \approx \frac{(2 - \xi_0)}{\xi_0^2} \frac{(1 + \tau)}{(1/\mu_p^2 + \tau)} \int_{\xi_{th}}^1 d\xi F_2^p(\xi)$$

$\xi_0 = \xi(x=1)$ 



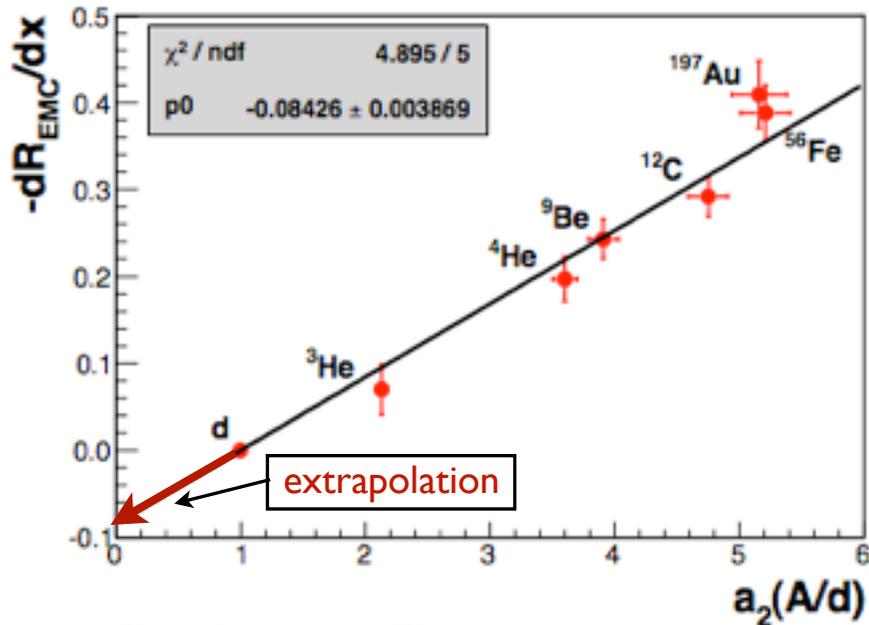
- PLC model predicts in-medium *suppression* of G_M^p
- QMC (consistent with $(\vec{e}, e'\vec{p})$ data) implies small *enhancement*

WM, Tsushima, Thomas
EPJA 14, 105 (2002)

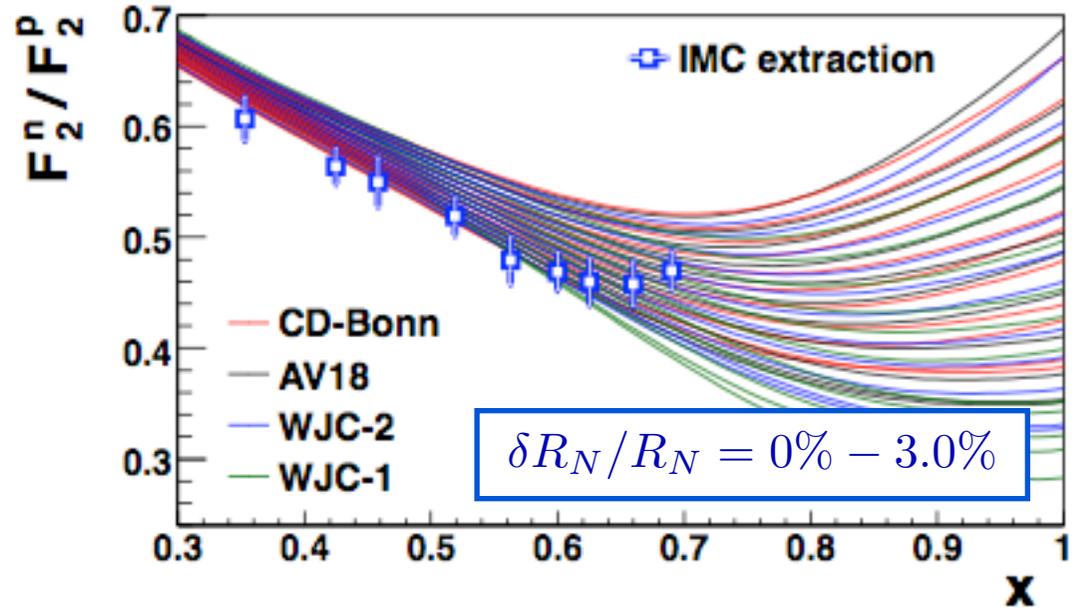
- disfavors models with *large* medium modifications of SFs

Constraints from $x > 1$

- “IMC” extraction of F_2^n / F_2^p , assuming extrapolation of EMC–SRC correlation to $A=1$



Hen, Piasezky, Weinstein
arXiv:1202.3452

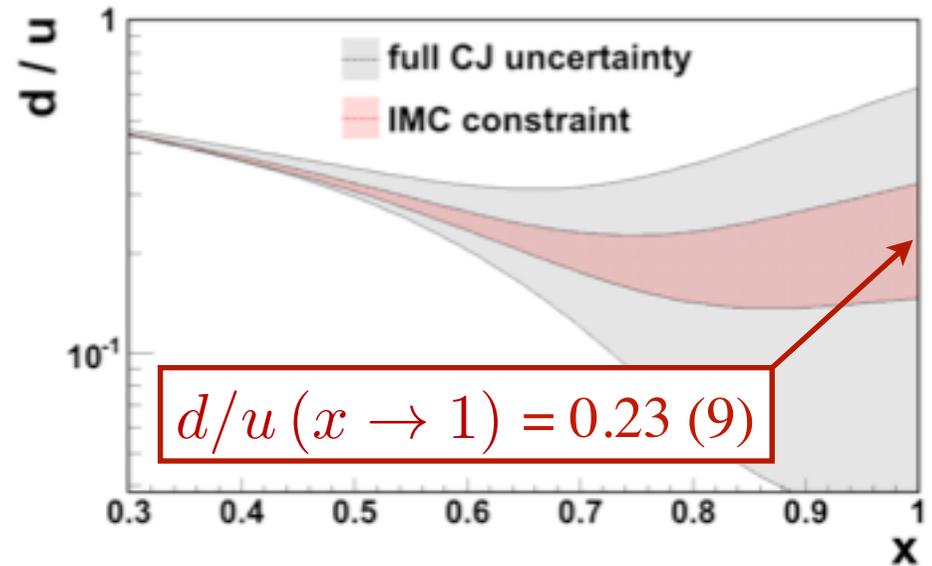
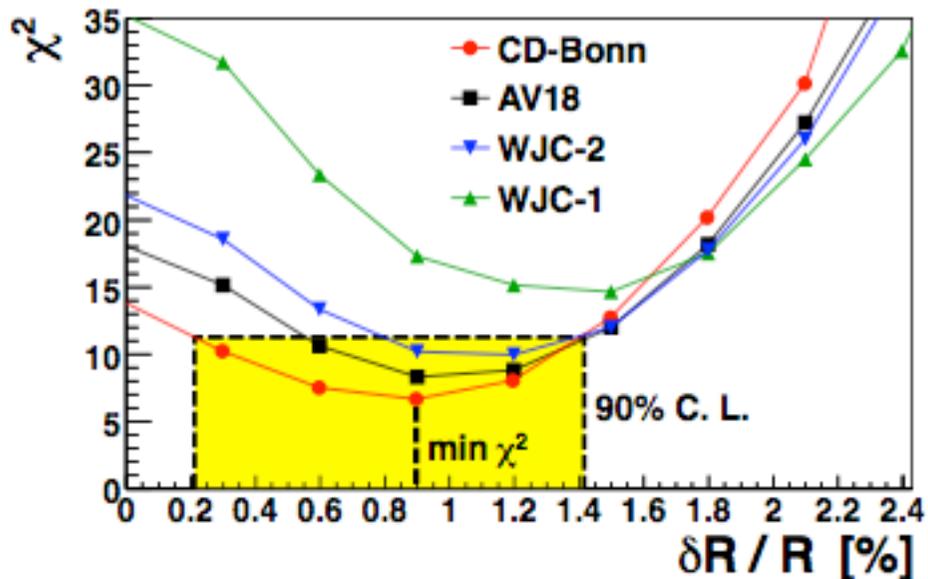


Hen, Accardi, WM, Piasezky
PRD 84, 117501 (2011)

→ assuming validity of extrapolation, how would IMC F_2^n / F_2^p data constrain deuteron correction models?

Constraints from $x > 1$

- “IMC” extraction of F_2^n / F_2^p , assuming extrapolation of EMC–SRC correlation to $A=1$



*Hen, Accardi, WM, Piasezky
PRD 84, 117501 (2011)*

- χ^2 fit constrains combination of d wave function and off-shell nucleon parameters (nucleon swelling)
- WJC-1 disfavored, $\delta R_N / R_N = 0.2\% - 1.4\%$ (at 90% C.L.)

Future methods of determining d/u

- $e d \rightarrow e p_{\text{spec}} X^*$
 “BoNuS” semi-inclusive DIS from d
→ tag “spectator” protons

- $e {}^3\text{He}({}^3\text{H}) \rightarrow e X^*$
 “MARATHON” ${}^3\text{He}$ -tritium mirror nuclei

- $e p \rightarrow e \pi^\pm X^*$ semi-inclusive DIS as flavor tag

- $e^\mp p \rightarrow \nu(\bar{\nu}) X$
 $\nu(\bar{\nu}) p \rightarrow l^\mp X$
 $p p(\bar{p}) \rightarrow W^\pm X, Z^0 X$
 $\vec{e}_L(\vec{e}_R) p \rightarrow e X^*$
 “PVDIS / SOLID” } weak current
as flavor probe

* planned for JLab at 12 GeV

Future methods of determining d/u



“BoNuS”

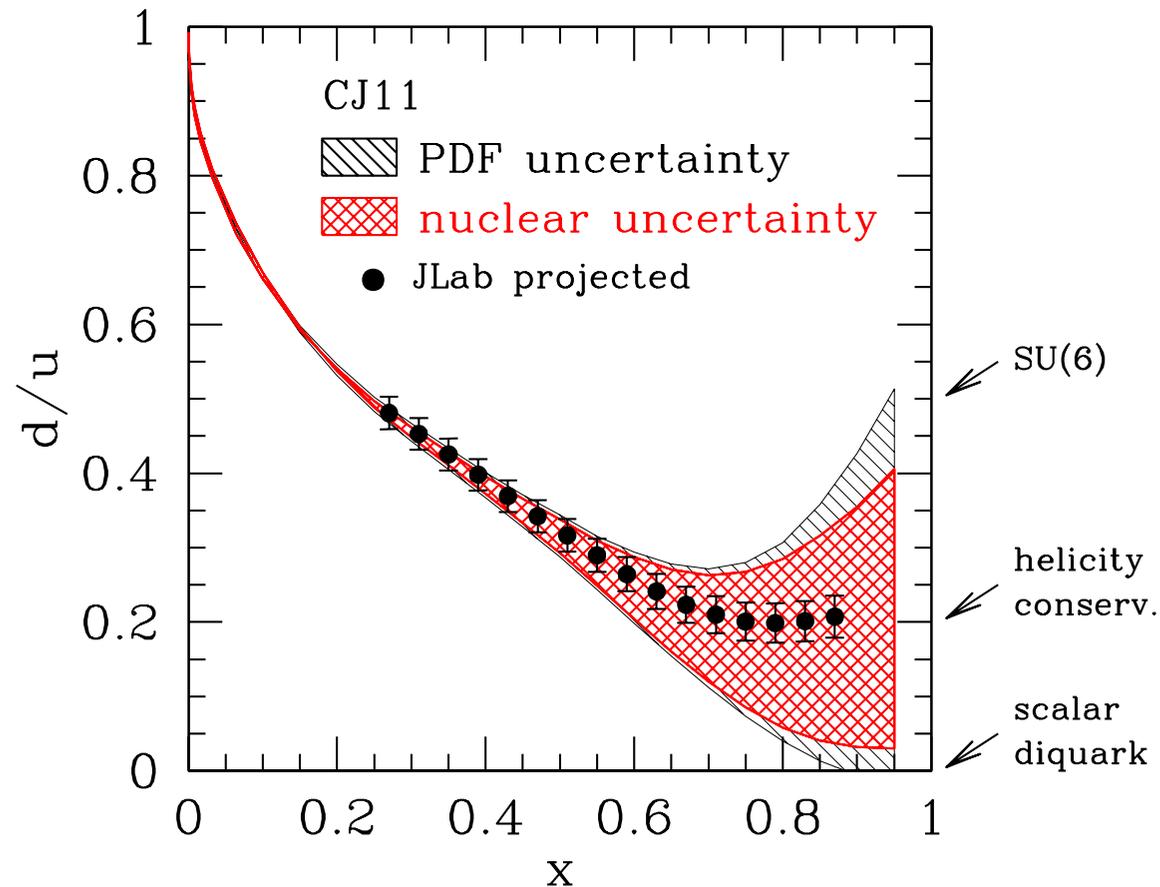
semi-inclusive DIS from d

→ tag “spectator” protons



“MARATHON”

${}^3\text{He}$ -tritium mirror nuclei



Summary

- Neutron structure at large x has remained elusive for > 40 years
→ impact of nuclear effects on PDF analysis (CJ collaboration)
- First (model-independent) glimpse of neutron resonance spectrum from BoNuS experiment
→ strong indication of validity of quark-hadron duality
(*not* result of accidental cancellations)
- Model-dependent constraints give conflicting evidence for magnitude of nucleon off-shell effects
(local duality → small, IMC → large)
→ definitive tests will require JLab 12 GeV data
(~ insensitive to nuclear effects)