Correlations in Nuclei: the Old and the New

- Tensor forces and ground state structure
- Tensor correlations in nuclei:
  1. Reviewing the circumstantial evidence for their presence
  2. Probing them directly via two-nucleon knockout processes
- Observing *pp* short-range correlations: the Coulomb sum rule
- Summary

Joe Carlson (LANL)
Steve Pieper (ANL)
Rocco Schiavilla (JLAB/ODU)
Bob Wiringa (ANL)
Juerg Jourdan (Basel)
Ingo Sick (Basel)

Preeminent features of v_{ij}:

- <u>short-range repulsion</u>
- <u>intermediate- to long-range tensor character</u>

These produce strongly anisotropic femtometer structures in T=0,S=1 channel in all nuclei:

$$\begin{aligned} \rho^{M_S}_{T=0,S=1}(\mathbf{r}) &\propto \rho^{M_S}_{d}(\mathbf{r}) \\ \rho^{M_S=0}_{T=0,S=1}(\mathbf{r}) &\neq \rho^{M_S=\pm 1}_{T=0,S=1}(\mathbf{r}) \end{aligned}$$

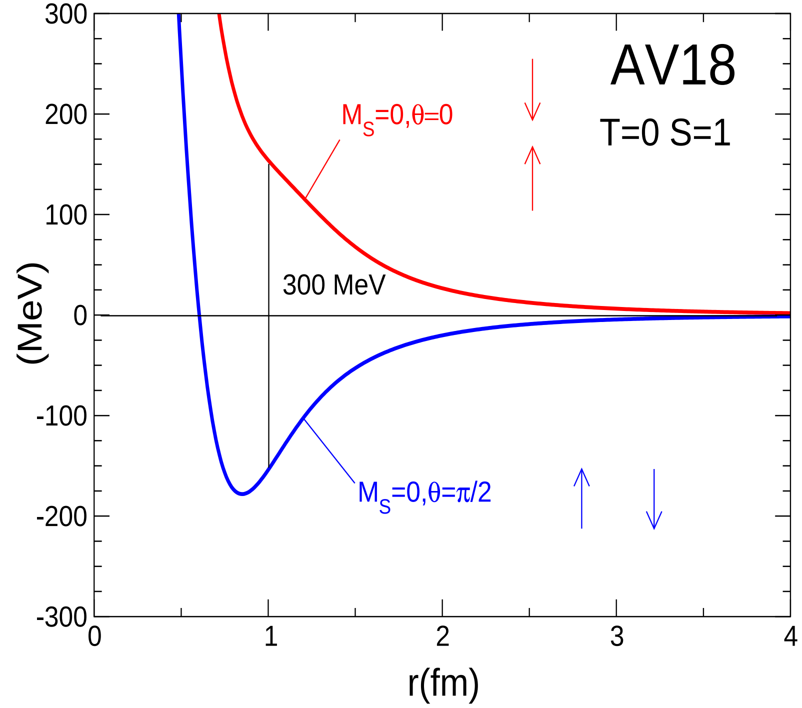
Two-nucleon density function:

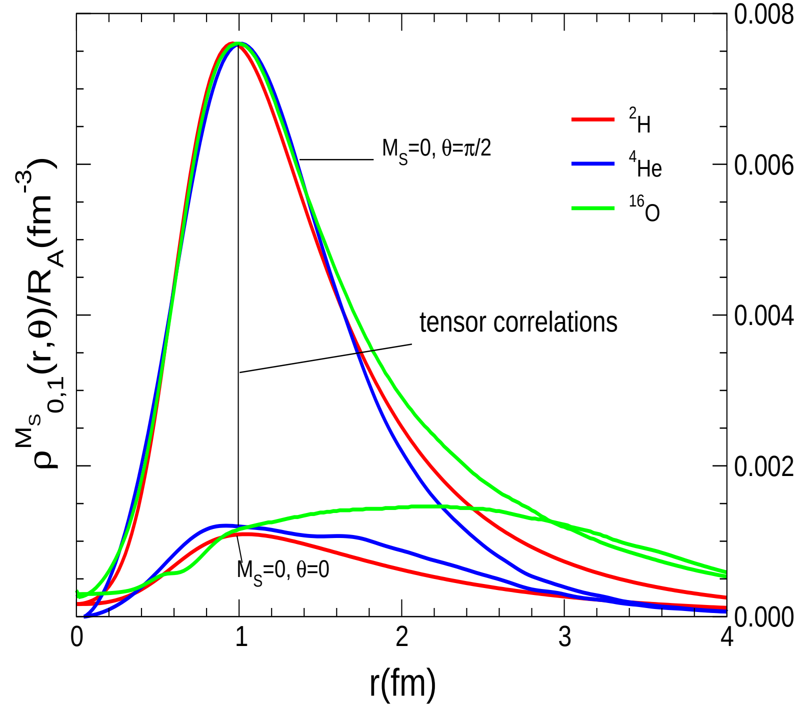
$$\rho^{M_S}_{T,S}(\mathbf{r}) = \frac{1}{2J+1} \sum_{M_J} \langle JM_J | \sum_{i<j} P^{T,SM_S}_{ij}(\mathbf{r}) | JM_J \rangle$$

$$P^{T,SM_S}_{ij}(\mathbf{r}) \equiv \delta(\mathbf{r} - \mathbf{r}_{ij})\, P^{T}_{ij}\ | SM_S, ij \rangle \langle SM_S, ij |$$

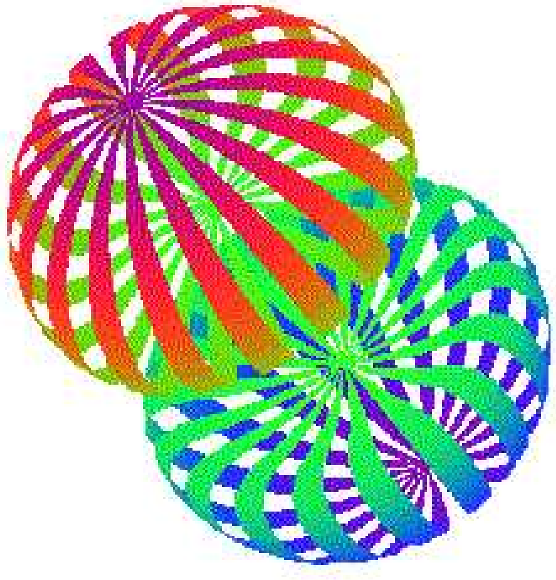
Coupling of Spatial and Spin Variables

Forest *et al.*, PRC**54**, 646 (1996)

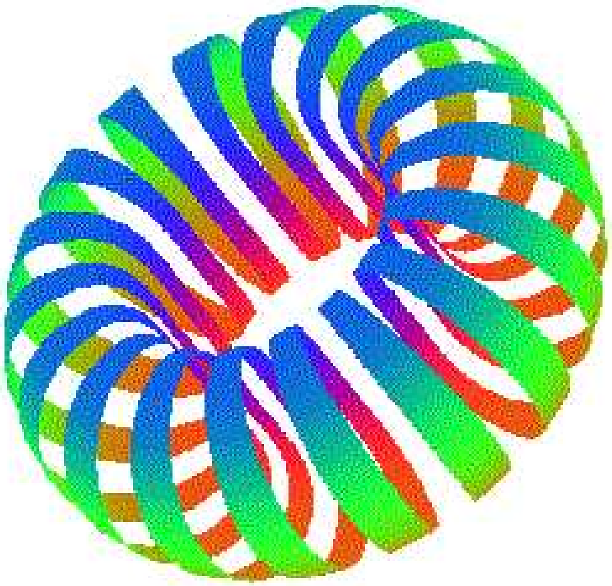




Two-Nucleon Density Profiles in T, S=0,1 States



$M_S = \pm 1$



$M_S = 0$

- Hole due to short-range repulsion
- Angular confinement due to tensor force
- Size of torus: $d \simeq 1.4$ fm, $t \simeq 0.9$ fm (at $\approx$ half-max density)

- At small separation, np relative w.f. in a nucleus $\propto$ deuteron w.f., but scaling factor $R_A >$ number of T, S=0,1 pairs

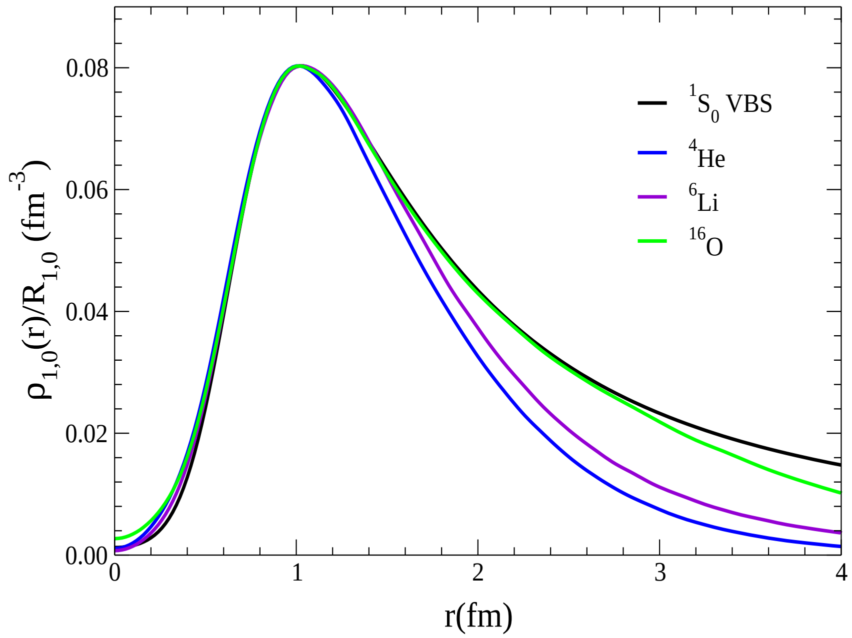
- $\langle O\rangle_A \simeq R_A \,\langle O\rangle_d$, where O is any short-range operator effective in the $T=0, S=1$ channel (*e.g.*, m.e. of axial two-body currents in pp weak capture and ^{3}H β-decay are proportional to each other $\rightarrow$ model independent prediction of pp cross section [Schiavilla *et al.*, PRC**58**, 1263 (1998)])

Scaling

	R_A	$\langle v^\pi\rangle_A/\langle v^\pi\rangle_d$	$\sigma^\pi_A/\sigma^\pi_d$	$\sigma^\gamma_A/\sigma^\gamma_d$
^{3}He	2.0	2.1	2.4(1)	$\simeq 2$
^{4}He	4.7	5.1	4.3(6)	$\simeq 4$
^{6}Li	6.3	6.3		
^{7}Li	7.2	7.8		$\simeq 6.5(5)$

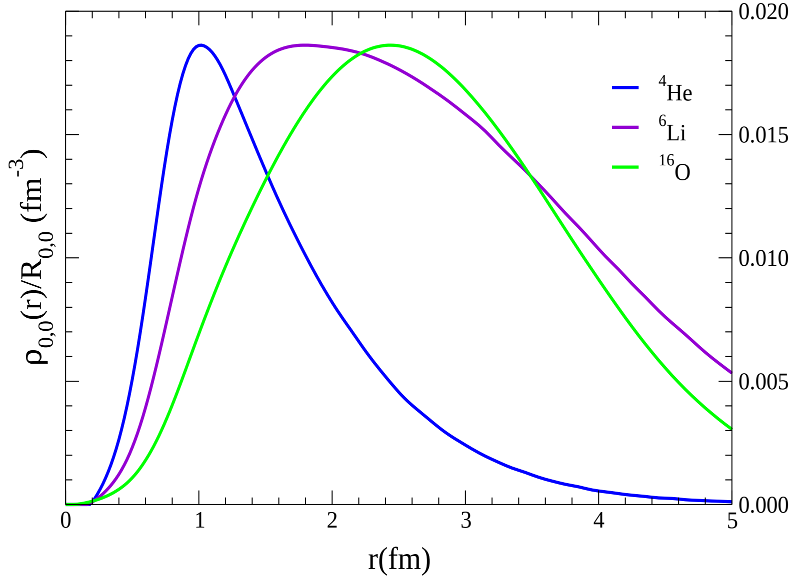
Two-Nucleon Density Profiles in other ($T, S \neq$0,1) States

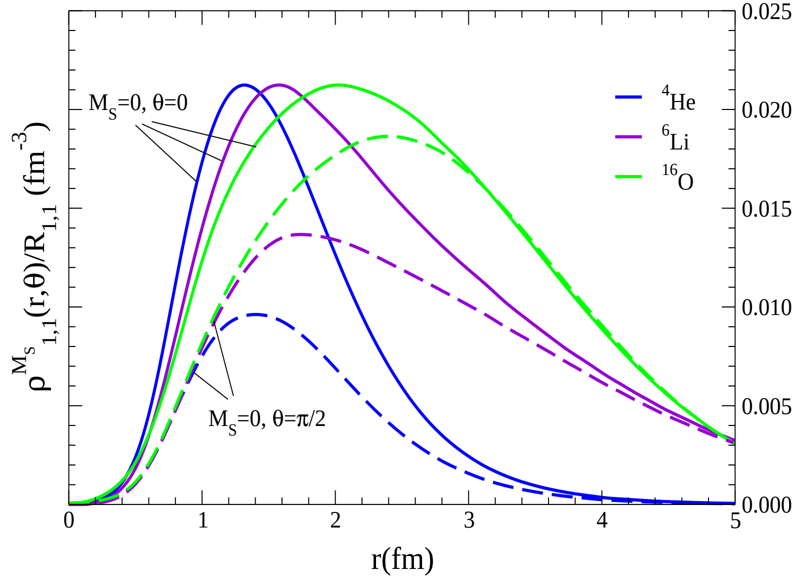
- Scaling occurs in T, S=1,0 channel (quasibound $^1S_0$ state) for $r \leq 2$ fm



- But <u>no scaling</u> in remaining channels (interaction either repulsive or weakly attractive)

T, S=0,0 $\qquad\qquad$ T, S=1,1





Experimental Evidence for Tensor Correlations (I): Deuteron

- Deuteron is a special case:

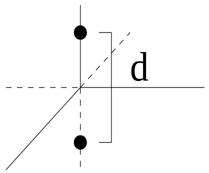
$$\rho_{T=0,S=1}^{M_S}(\mathbf{r})|_d \propto \rho_d^M(\mathbf{r}' = \mathbf{r}/2)$$

where $\rho_d^M(\mathbf{r}')$ is the one-nucleon density

- $\rho_d^M(\mathbf{r}')$ "measured" in elastic e-scattering:

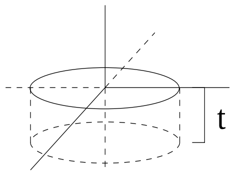
$$F_M(q) = \int d\mathbf{r}' \mathrm{e}^{\mathrm{i}qz'} \rho_d^M(\mathbf{r}')$$

$\rho_d^{M=\pm 1} \simeq$



$F_{M=\pm 1}(q) = \cos\left(\frac{qd}{2}\right)$

$\rho_d^{M=0} \simeq$



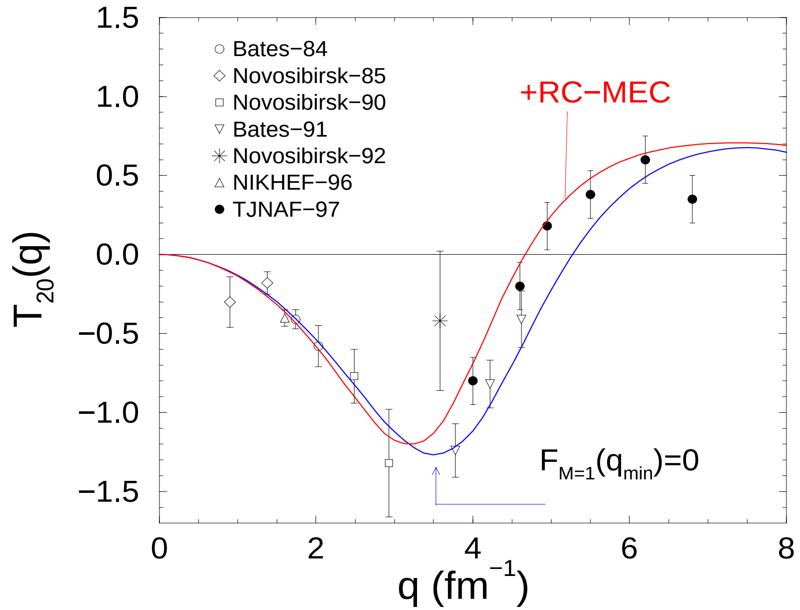
$F_{M=0}(q) = \frac{\sin(qt/2)}{(qt/2)}$

- Map out $M = 0$ and $M = \pm 1$ densities:

$$A(q) \simeq |F_{M=0}(q)|^2 + 2\,|F_{M=1}(q)|^2$$

$$T_{20}(q) \simeq -\sqrt{2}\,\frac{|F_{M=0}(q)|^2 - |F_{M=1}(q)|^2}{|F_{M=0}(q)|^2 + 2\,|F_{M=1}(q)|^2}$$

- There are RC and MEC corrections, but gross features confirmed by experiment



Experimental Evidence for Tensor Correlations (II): $A > 2$ Nuclei

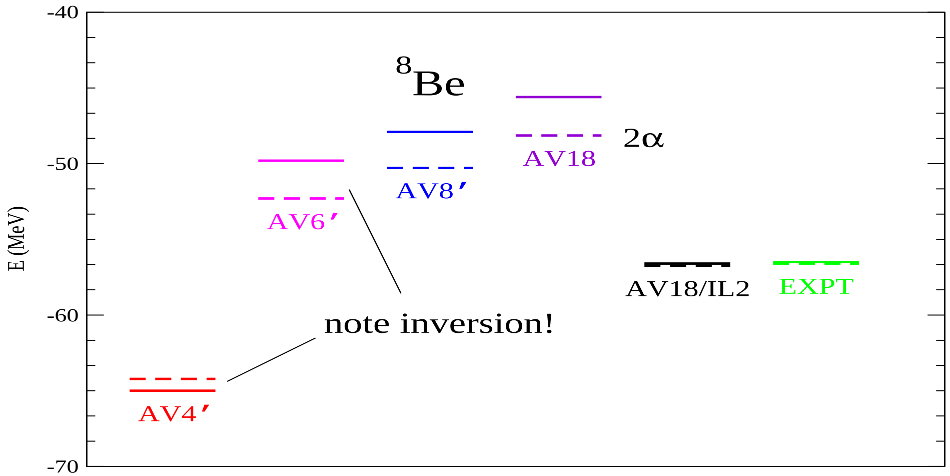
Several nuclear properties influenced by tensor correlations including:

- Ordering of levels in low-energy spectra of light nuclei and absence of stable A=8 nuclei
- Radiative (and weak) capture processes involving few-nucleon systems, *e.g.* $^2\mathrm{H}(n,\gamma)^3\mathrm{H}$, $^3\mathrm{He}(n,\gamma)^4\mathrm{He}$, $^2\mathrm{H}(d,\gamma)^4\mathrm{He}$, ...
- Distribution of strength in response to electromagnetic and hadronic probes, such as (e,e') scattering and (p,n) reactions
- Momentum distributions $N(k)$ and spectral functions $S(k,E)$ at high k and E

However, effects of tensor correlations are generally subtle, and are not easily isolated in the experimental data

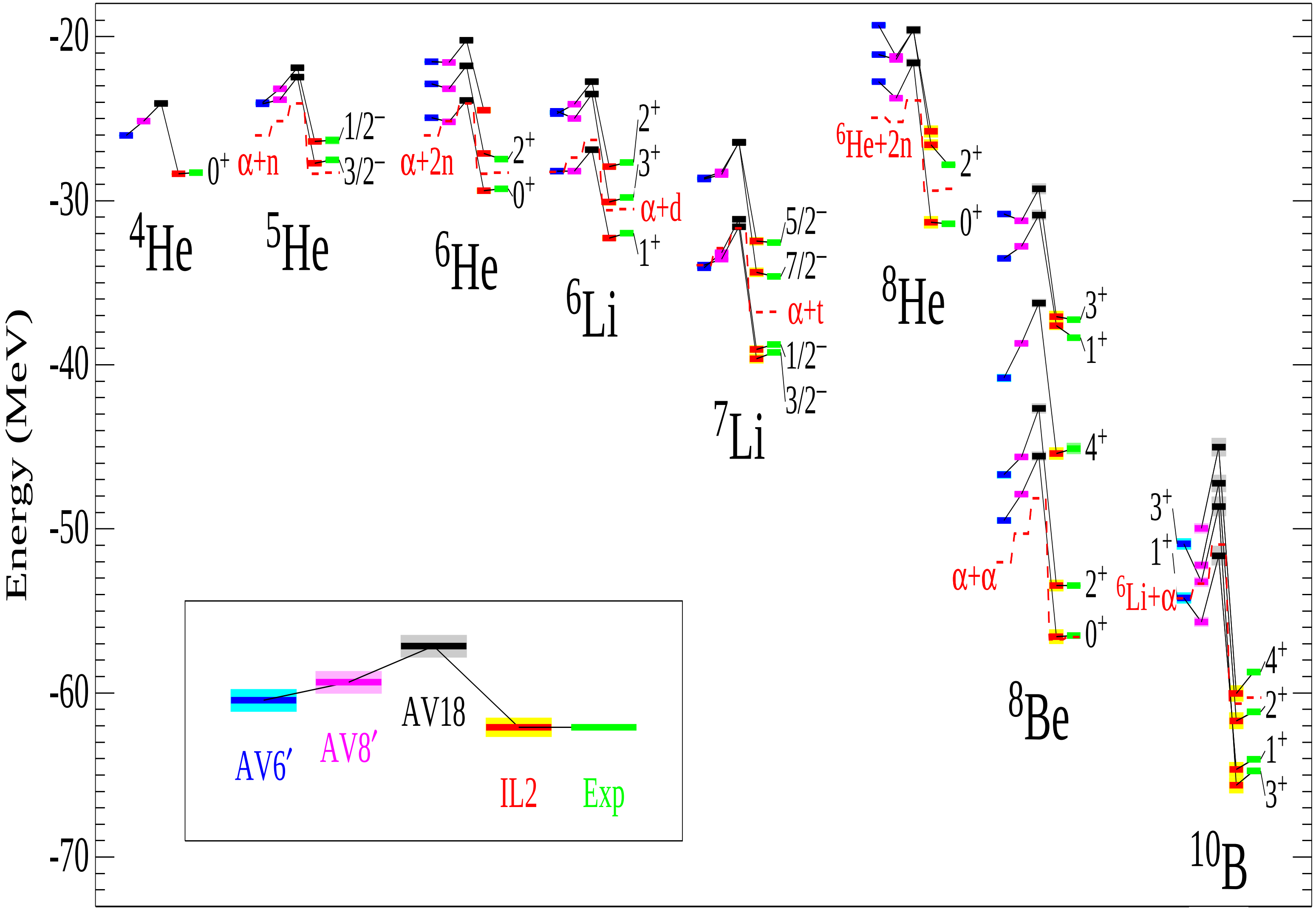
Build series of potentials designed to reproduce as many features of the deuteron and elastic NN scatttering as feasible at each stage:

1. $AV4' = [1\,,\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2] \otimes [1\,,\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$
2. $AV6' = AV4'$+tensor
3. $AV8' = AV6'$+spin-orbit, ...



Wiringa and Pieper, PRL**89**, 182501 (2002)

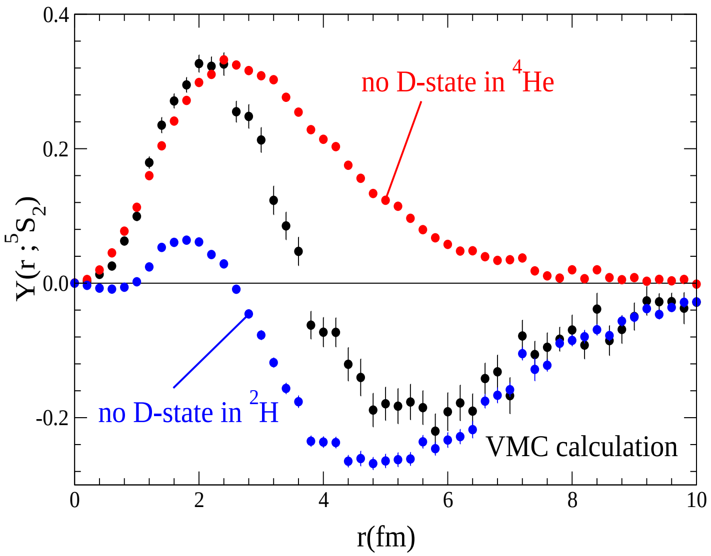
Wiringa and Pieper, PRL**89**, 182501 (2002)

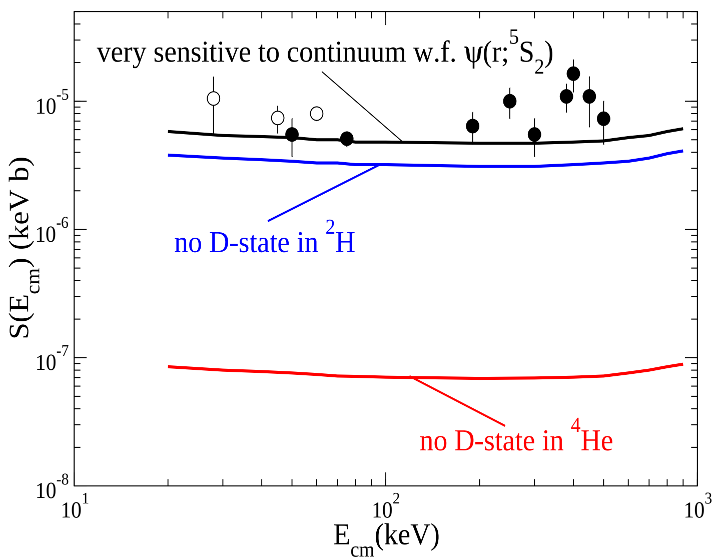


The ^{2}H$(d,\gamma)^4$He Radiative Capture at Low Energies

$$\langle ^4\text{He} \mid E_2 \mid dd \rangle \equiv \int_0^\infty \mathrm{d}r\, \psi(r;{}^5\,\mathrm{S}_2)\, Y(r;{}^5\,\mathrm{S}_2)$$

where $\psi(r)$ is the dd $^5\mathrm{S}_2$ scattering w.f. (the only channel relevant at $E_{\mathrm{cm}} \lesssim 500$ keV), and $Y(r)$ is the E_2-folded dd overlap in ^{4}He





Arriaga, Pandharipande, Schiavilla, PRC**43**, 983 (1991)

Tensor Correlations and Two-Nucleon Momentum Distributions

$$\rho^{NN}(\mathbf{q},\mathbf{Q}) = \frac{1}{2J+1}\sum_{M_J}\langle\psi_{JM_J} \mid \sum_{i<j} P_{ij}^{NN}(\mathbf{q},\mathbf{Q}) \mid \psi_{JM_J}\rangle$$

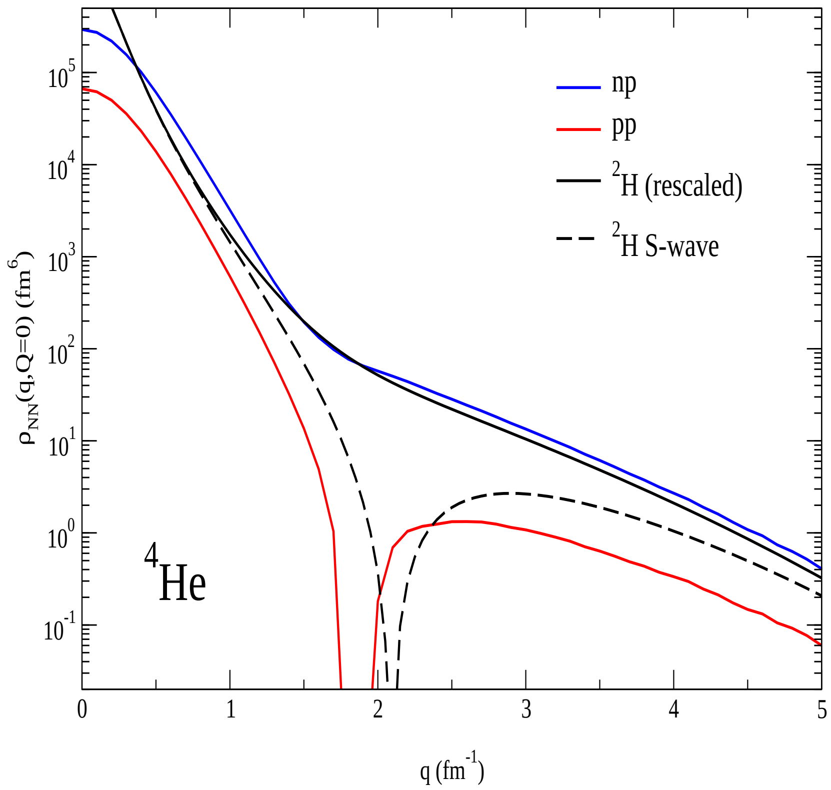
where $\mathbf{q}$ and $\mathbf{Q}$ are respectively the relative and total momenta of the NN pair, and

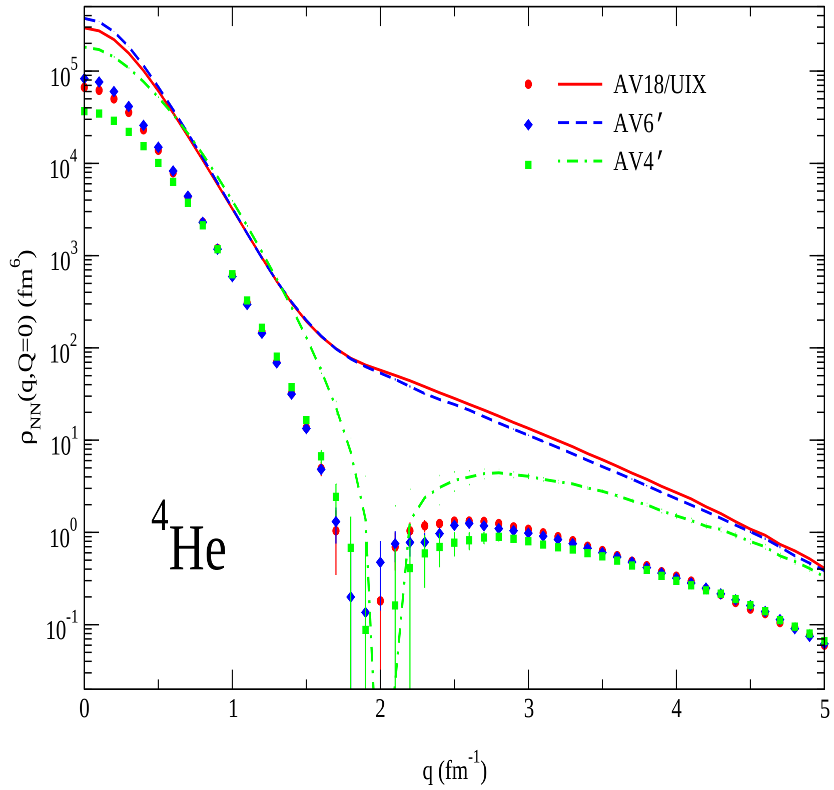
$$P_{ij}^{NN}(\mathbf{q},\mathbf{Q}) \equiv \delta(\mathbf{k}_{ij}-\mathbf{q})\delta(\mathbf{K}_{ij}-\mathbf{Q})\,P_{NN}(ij)$$

- np (pp) pairs predominantly in T=0 deuteron-like (T=1 quasi-bound) state→ large differences between ρ^{np} and ρ^{pp}
- These differences should be seen in $A(e,e'np)$ and $A(e,e'pp)$ (back-to-back kinematics)
- ρ^{NN} can be calculated exactly with QMC

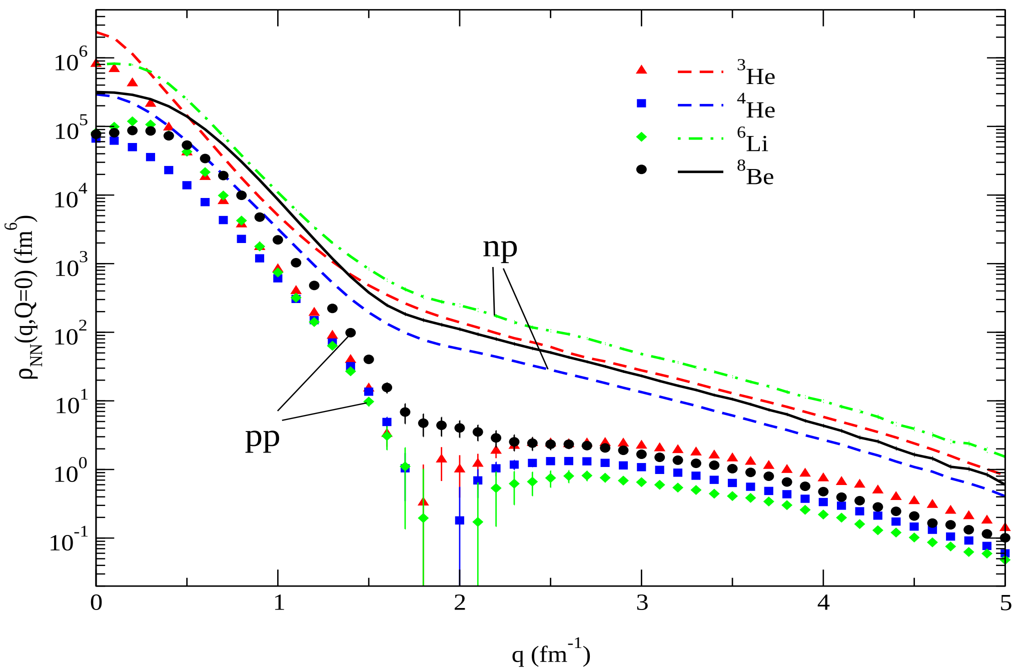
NN momentum distributions at Q=0

Schiavilla, Wiringa, Pieper, and Carlson, PRL**98**, 132501 (2007)





- Universal feature
- First indications from: i) analysis of ^{12}C (p, pp) and (p, ppn) BNL data, and ii) JLab measurements of ^{12}C$(e, e'pp)$ and ^{12}C$(e, e'pn)$: $P_{pp}/P_{np} \lesssim 0.04^{+0.09}_{-0.04}$ [Piasetzky *et al.*, PRL **97**, 162504 (2006)]



Coulomb Sum Rule

Schiavilla, Pandharipande, and Fabrocini, PRC**40**, 1484 (1989)

Carlson, Jourdan, Schiavilla, and Sick, PLB**553**, 191 (2003)

$$S_L(q) = \frac{1}{Z}\int_{\omega_{\rm th}}^{\infty} {\rm d}\omega\, \frac{R_L(q,\omega)}{G_{Ep}^2(Q^2)} = \frac{1}{Z}\langle 0 |\, \rho_L^\dagger(\mathbf{q})\, \rho_L(\mathbf{q})\, | 0\rangle - Z\, |\, F_L(q)\, |^2$$

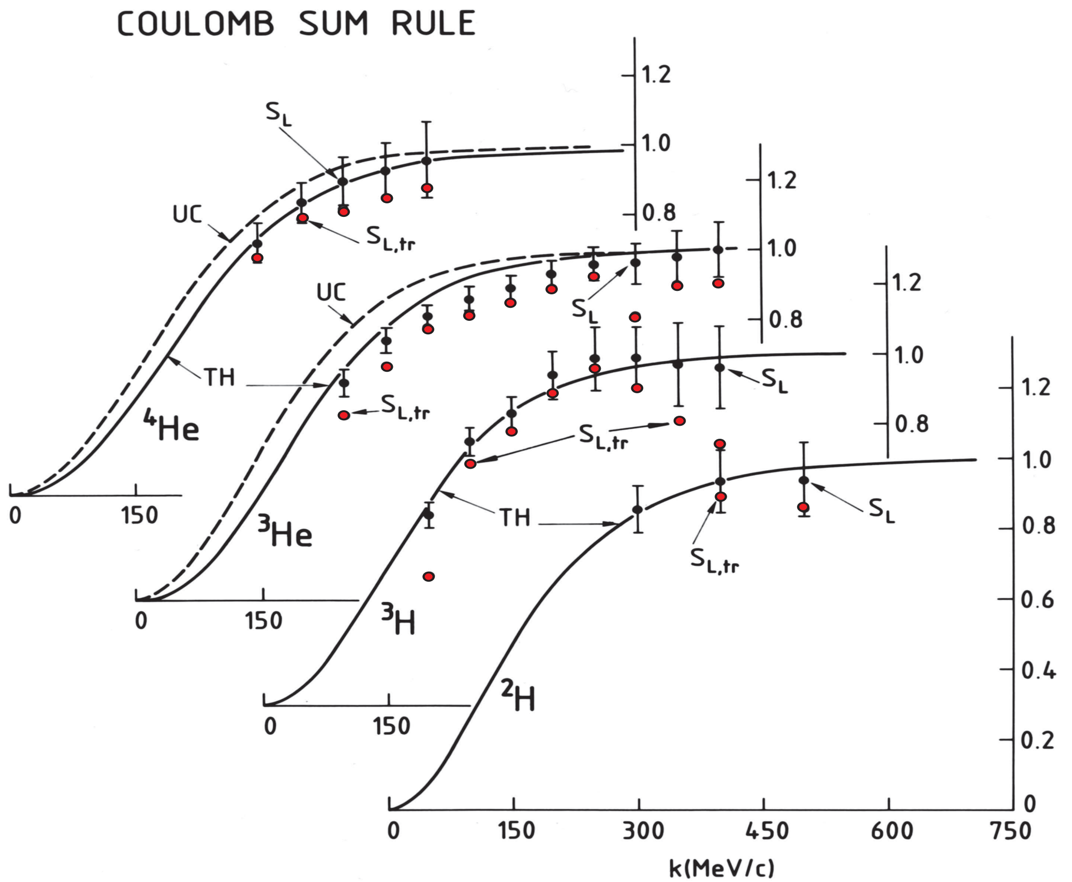
- $\rho_L(\mathbf{q})$=nuclear charge operator

$$\rho_L(\mathbf{q}) = \sum_i {\rm e}^{{\rm i}\,\mathbf{q}\cdot\mathbf{r}_i}\,(1+\tau_{z,i})/2 + \text{ neutron } + \text{RC/MEC}$$

- $R_L(q,\omega)$ measured by (e,e') up to $\omega_{\rm max}$ ($\omega_{\rm max} \leq q$): parameterize $R_L(q,\omega>\omega_{\rm max})$ constraining it to reproduce

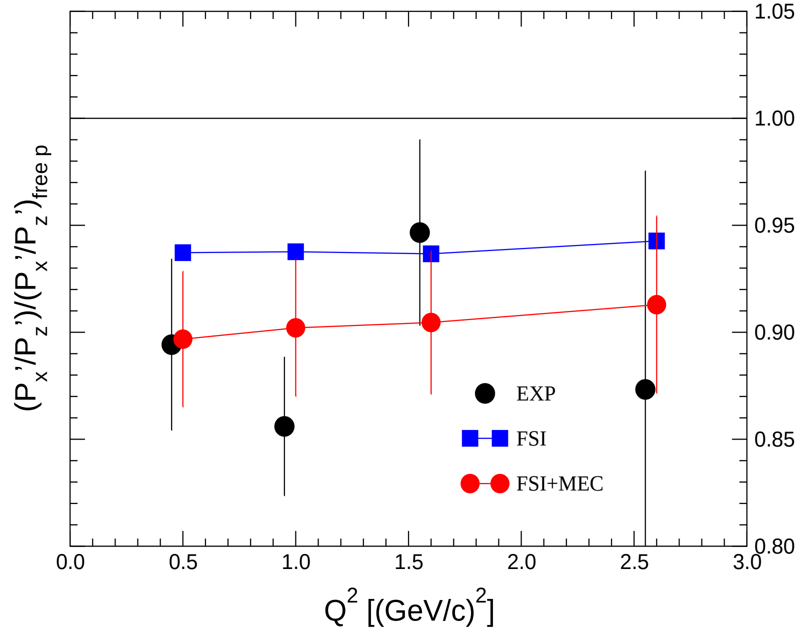
$$W_L(q) = \frac{1}{Z}\int_{\omega_{\rm th}}^{\infty} {\rm d}\omega\, \omega\, \frac{R_L(q,\omega)}{G_{Ep}^2(Q^2)} = \frac{1}{2Z}\langle 0 |\, \left[\rho_L^\dagger(\mathbf{q})\,,\, \left[H\,,\, \rho_L(\mathbf{q})\right]\right]\, | 0\rangle$$

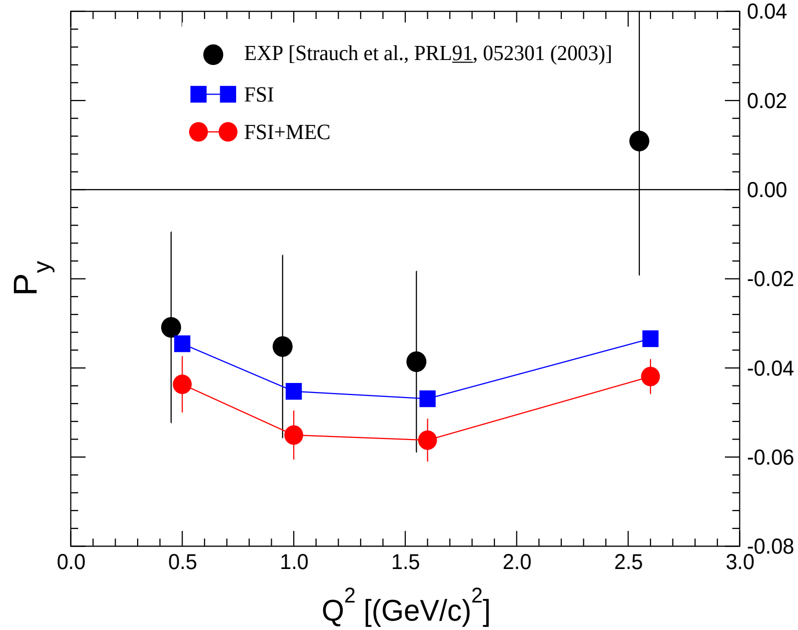
- RC/MEC contributions to $S_L(q)$ tend to cancel out
- Theory and experiment in agreement when using free proton f.f.



On the Issue of Medium-Modified p f.f.: the $^4\mathrm{He}(\vec{e}, e'\vec{p})^3\mathrm{H}$ process

- In PWIA: $P'_x/P'_z \propto (G_{Ep}/G_{Mp})$
- FSI effects and MEC contributions explain ratio: no medium modification is required





Schiavilla, Benhar, Kievsky, Marcucci, and Viviani, PRL**94**, 072303 (2005)

Short-Range *pp* Correlations

- Obtain from "measured" $S_L(q)$ and $F_L(q)$

$$\begin{aligned} \rho_{LL}(q) &= S_L(q) - 1 + \mid F_L(q) \mid^2 \\ &= \frac{1}{Z} \langle 0 | \, \rho_L^\dagger(\mathbf{q}) \, \rho_L(\mathbf{q}) \, | 0 \rangle - 1 \end{aligned}$$

- If RC/MEC corrections in $\rho_L(\mathbf{q})$ were negligible, then

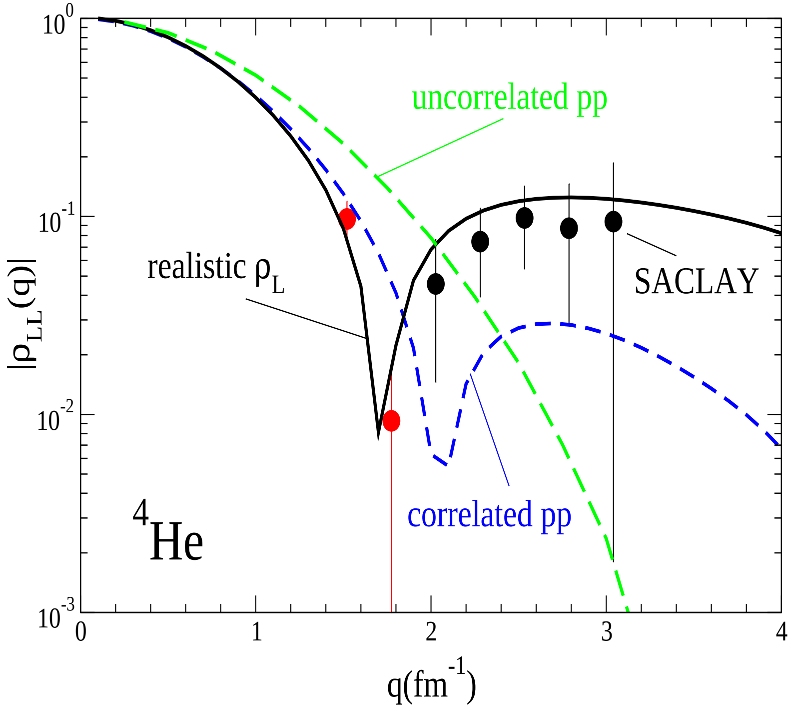
$$\rho_{LL}(q) \to (Z-1) \int \mathrm{d}\mathbf{r}_1 \mathrm{d}\mathbf{r}_2 \, \mathrm{e}^{\mathrm{i}\,\mathbf{q}\cdot(\mathbf{r}_1 - \mathbf{r}_2)} \rho_{pp}(\mathbf{r}_1, \mathbf{r}_2)$$

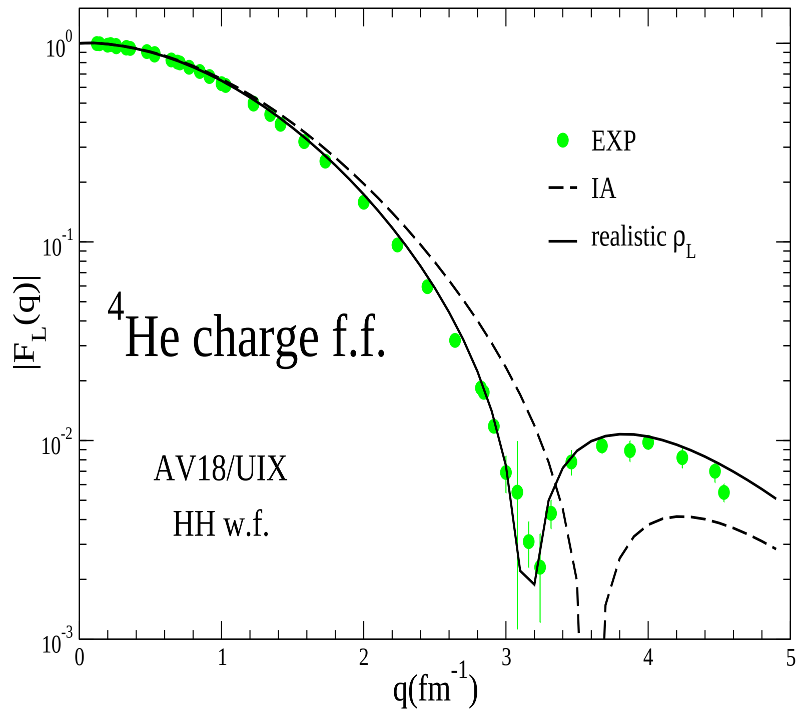
- In the limit of non-interacting nucleons:

$$\rho_{pp}^{\mathrm{unc}}(\mathbf{r}_1, \mathbf{r}_2) \simeq (Z-1) \, \rho_p(\mathbf{r}_1) \, \rho_p(\mathbf{r}_2)$$

$$\rho_p(\mathbf{r}) = \frac{1}{2\,Z} \sum_i \langle 0 | \, \delta(\mathbf{r} - \mathbf{r}_i) \, (1 + \tau_{z,i}) \, | 0 \rangle$$

Schiavilla, Wiringa, and Carlson, PRL**70**, 3856 (1993)





Error on $\rho_{LL}(q)$ mostly due to tail correction

Summary

- Tensor correlations affect a variety of nuclear properties: two-nucleon densities, energy spectra, radiative captures, ...
- They also lead to order of magnitude differences between the (back-to-back) *np*- and *pp*-pair momentum distributions
- This isospin dependence should be easily observable in *np*- or *pp*-knockout processes (already "seen" in BNL and JLab data)
- The Coulomb sum rule is a "clean" probe of *pp* short-range correlations (RC and/or MEC contaminations under control)