HUPD- 9504
January 1995

Perturbative QCD and Nucleon Structure Functions

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Abstract

We review the basic aspects of the perturbative QCD based on the operator product expansion to analyze the nucleon structure functions in a pedagogical way. We explain the non-trivial relation between the QCD results and the parton model especially to understand the polarized nucleon structure functions which deserve much attentions in recent years.

Invited talk at the
YITP Workshop on "From Hadronic Matter to Quark Matter: Evolving View of Hadronic Matter"
YITP, Kyoto Japan, October 1994

to be published in the proceedings

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1 Introduction

The quantum chromodynamics (QCD) is a theory of the strong interaction. So far all experimental data are consistent with the predictions of QCD. Especially the high energy behavior of QCD is believed to be described by the perturbation theory thanks to the asymptotically free nature of QCD. Among many interesting and important problems, the spin structure of nucleon has been one of the most exciting subjects in recent years. The data on the polarized deep inelastic scattering by the EMC collaboration [1] result in excitement of not only particle physicists but also nuclear physicists since the data seem to indicate that the nucleon's spin is not carried by quarks (partons). This EMC experiment has incited many (particle and nuclear) physicists to challenge the so-called "spin crisis" problem in QCD. After a flood of theoretical papers as well as new experiments [2]-[5], our understanding on this problem is now much more improved: the interpretation of the QCD results in terms of the parton model is never obvious and simpleminded one may fail: the axial anomaly plays an important role: etc.

The aim of this talk is to provide a pedagogical introduction to the perturbative QCD to study the nucleon structure through the deep inelastic process for non-experts of QCD who are interested in the deep structure of the nucleon. Those who are familiar with QCD and interested in recent progress in this field are referred to recent nice article [6] and reviews [7].

In Sect.2 we review the kinematics of the deep inelastic lepton nucleon scattering process. In Sec.3 the basic approach of the perturbative QCD based on the operator product expansion and the renormalization group equation will be explained. The relation between the QCD results and the parton model is discussed. It will be stressed that the parton density is a "conception" which depends on the renormalization scheme. In Sec.4 we consider the polarized structure functions. Concluding remarks including some subtleties and/or controversial aspects which deserve more investigations to understand recent experimental data are given in Sec.5.

2 The structure functions

The cross section for the deep inelastic lepton ($l\,(k)$) nucleon ($N\,(p)$) scattering $l\,(k) + N\,(p) \to l\,(k') + X$ (Fig.1) is given in terms of the leptonic and the hadronic

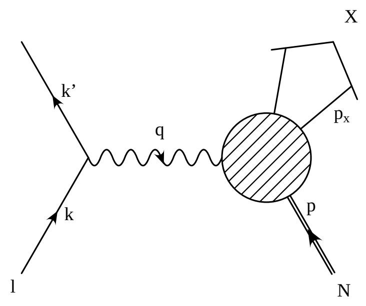

Figure 1: Kinematic variables in inelastic lepton-nucleon scattering.

tensors according to the standard procedure in the field theory.

$$k_0' \frac{d\sigma}{d^3k'} = \frac{1}{k \cdot p} \left(\frac{e^2}{4\pi Q^2} \right)^2 L^{\mu\nu} W_{\mu\nu} \ ,$$

where we consider only the QED interaction between the lepton and nucleon and keep only the lowest order in α_{QED}. q is the momentum transfer from the lepton to the nucleon and $q^2 \equiv -Q^2 = (k - k')^2$. The leptonic ($L_{\mu\nu}$) and the hadronic ($W_{\mu\nu}$) tensors are defined as follows;

$$L_{\mu\nu} = \frac{1}{2} \sum_{s'} \langle k, s | j_\mu(0) | k', s' \rangle \langle k', s' | j_\nu(0) | k, s \rangle \ ,$$

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2\pi} \sum_X \langle p, S | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p, S \rangle (2\pi)^4 \delta^4(p_X - p - q) \\ &= \frac{1}{2\pi} \int d^4x e^{iq\cdot x} \langle p, S | [J_\mu(x)\, ,\, J_\nu(0)] | p, S \rangle \ . \end{aligned} \qquad (1)$$

with the lepton's (hadron's) electromagnetic current j_μ (J_ν). $s(S)$ is the spin of the lepton (nucleon).

In general, the spin 4-vector of the fermion with mass m and momentum k is defined as $\vec{s}^{\,2} = 1$, $s^2 = -1$, $s \cdot k = 0$. So, for the longitudinally polarized (helicity $\pm$) states, we get,

$$s^\mu = \pm \frac{1}{m}(k\,,\,0\,,\,0\,,\,k^0) \quad , \quad k = |\vec{k}|^2 \ .$$

We can use the following approximation for leptons $ms^\mu \simeq \pm k^\mu$ since $m_{\rm lepton} \simeq 0$. Using this approximation, we get for the leptonic tensor,

$$L^\pm_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu + \frac{q^2}{2} g_{\mu\nu} \mp i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda k^\sigma \ .$$

On the other hand, the hadronic tensor contains all the information of the strong interaction (QCD). Taking into account the various symmetries, namely the Lorentz invariance, current conservation of the QED current, T and P invariance, we can write down the general form for $W_{\mu\nu}$,

$$W_{\mu\nu} \equiv W^S_{\mu\nu} + i\, W^A_{\mu\nu} \ ,$$

with

$$\begin{aligned} W^S_{\mu\nu} &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2} q_\nu\right)\frac{W_2}{M^2} \ , \\ W^A_{\mu\nu} &= \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ S^\sigma M G_1 + (p\cdot q S^\sigma - q\cdot S p^\sigma)\frac{G_2}{M}\right\} \ . \end{aligned}$$

where M is the mass of the nucleon. $W^S_{\mu\nu}$ ($W^A_{\mu\nu}$) is the symmetric (antisymmetric) part in $\mu\nu$ and relevant to the unpolarized (polarized) process as shown below. Usually we define the following dimensionless structure functions (scaling functions):

$$F_1 \equiv W_1 \ , \ F_2 \equiv \frac{\nu}{2M} W_2 \ , \ g_1 \equiv \frac{M\nu}{2} G_1 \ , \ g_2 \equiv \frac{\nu^2}{2} G_2 .$$

with $M\nu \equiv p \cdot q$. These structure functions depend on the Q^2 and ν or Q^2 and x (Bjorken variable) $x \equiv \frac{1}{\omega} = \frac{Q^2}{2M\nu}$.

The explicit formula for the cross sections for this process in the Laboratory frame corresponding to the configuration in Fig.2 is easily calculated to be [8],

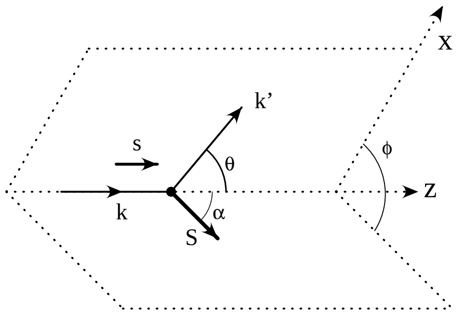

Figure 2: Momentum and spin configuration in Lab. frame.

$$\frac{d\sigma^{\pm,S}}{dE'd\Omega} = \frac{d\bar{\sigma}}{dE'd\Omega} \pm \frac{d\sigma^{A}}{dE'd\Omega},$$

with

$$\frac{d\bar{\sigma}}{dE'd\Omega} = \frac{2\alpha^2 E'^2}{Q^4 M}\left(2\,W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}\right),$$

$$\begin{aligned}\frac{d\sigma^{A}}{dE'd\Omega} &= -\frac{\alpha^2 E'}{MQ^2 E}\Big[\cos\alpha\left\{(E+E'\cos\theta)\,MG_1 - Q^2 G_2\right\} \\ &\qquad + \sin\alpha\cos\phi\; E'\sin\theta\{MG_1 + 2EG_2\}\Big].\end{aligned}$$

The superscript $\pm$ refers to the lepton's helicity and $E' \equiv k'_0$ and $E \equiv k_0$. From these formulae, we can derive the expressions , for example, for the longitudinal asymmetry which is the difference between the cross section for the nucleon's spin being parallel to the lepton's ($\uparrow\uparrow$) and the nucleon's spin being anti-parallel to the lepton's ($\uparrow\downarrow$) :

$$\frac{d\sigma^{\uparrow\downarrow}}{dE'd\Omega} - \frac{d\sigma^{\uparrow\uparrow}}{dE'd\Omega} = \frac{2\alpha^2 E'}{MQ^2 E}\left\{MG_1(E+E'\cos\theta) - Q^2 G_2\right\}.$$

It is traditional and sometimes convenient to express the structure functions in terms of the virtual photoabsorption cross sections (in Lab. frame). The definition of the virtual photoabsorption cross section is given by,

$$\sigma_\lambda \equiv \frac{\pi e^2}{2M(\nu - Q^2/2M)} \epsilon_\lambda^{\mu *}(q) W_{\mu\nu} \epsilon_\lambda^\nu(q),$$

with the photon's polarization vector ϵ_λ^μ. We have adopted the Hand-Berkelman's convention. Taking the direction of photon's momentum q to be z-axis (note that $\vec{q} \neq \vec{k}$), we have the following polarization vector for photons in the nucleon's rest frame:

$$q^\mu = (\nu,\ 0,\ 0,\ \sqrt{\nu^2 + Q^2})\ ,$$

$$\epsilon_{\substack{R \\ L}} = \frac{1}{\sqrt{2}}(0,\ \mp i,\ 1,\ 0)\ ,\ \epsilon_S = \frac{1}{\sqrt{Q^2}}(\sqrt{\nu^2 + Q^2},\ 0,\ 0,\ \nu)\ .$$

For the unpolarized structure functions W_1 and W_2, we get the relations:

$$\begin{aligned} \sigma_S &= \frac{\pi e^2}{2M(\nu - Q^2/2M)} \left[W_2 \left(1 + \frac{\nu^2}{Q^2} \right) - W_1 \right], \\ \sigma_T &= \frac{\pi e^2}{2M(\nu - Q^2/2M)} W_1. \end{aligned}$$

where $T = R$ and/or L. The unpolarized lepton-nucleon scattering cross section is given in terms of σ_S and σ_T,

$$\frac{d\bar{\sigma}}{dE' d\Omega} = \Gamma_T(\sigma_T + \varepsilon \sigma_S)\ ,$$

where

$$\varepsilon^{-1} = 1 + 2\left(1 + \frac{\nu^2}{Q^2}\right)\tan^2\frac{\theta}{2}\ ,\quad \Gamma_T = \frac{\alpha}{2\pi^2}\frac{E'}{E}\frac{\nu - Q^2/2M}{Q^2(1-\varepsilon)}.$$

Here we note that ε means the ratio of T- and S-photon present in the virtual photon .

To get the expressions for the polarized structure functions G_1 and G_2, let us consider the three types of the photoabsorption processes: $\sigma_{1/2}$ ($\epsilon_{\substack{L \\ R}}$ with $S^\mu = (0, 0, 0, \pm 1)$); $\sigma_{3/2}$ ($\epsilon_{\substack{R \\ L}}$ with $S^\mu = (0, 0, 0, \pm 1)$); σ_{TS} (the interference between T- and

S- photon with $S_y = \pm 1$). It is easy to obtain,

$$\begin{aligned}
\sigma_{1/2} &= \frac{\pi e^2}{2M(\nu - Q^2/2M)}[W_1 + M\nu G_1 - Q^2 G_2] \ , \\
\sigma_{3/2} &= \frac{\pi e^2}{2M(\nu - Q^2/2M)}[W_1 - M\nu G_1 + Q^2 G_2] \ , \\
\sigma_{TS} &= \frac{\pi e^2}{2M(\nu - Q^2/2M)}\sqrt{Q^2}[M G_1 + \nu G_2] \ .
\end{aligned}$$

Note that, $\sigma_T = \frac{1}{2}(\sigma_{1/2} + \sigma_{3/2})$. By defining the asymmetries A_1 and A_2:

$$A_1 \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} = \frac{M\nu G_1 - Q^2 G_2}{W_1} \ , \ \ A_2 \equiv \frac{\sigma_{TS}}{\sigma_T} = \sqrt{Q^2}\ \frac{M G_1 + \nu G_2}{W_1},$$

we can write the longitudinal asymmetry A as;

$$A \equiv \frac{\frac{d\sigma^{\uparrow\downarrow}}{dE' d\Omega} - \frac{d\sigma^{\uparrow\uparrow}}{dE' d\Omega}}{\frac{d\sigma^{\uparrow\downarrow}}{dE' d\Omega} + \frac{d\sigma^{\uparrow\uparrow}}{dE' d\Omega}} = D(A_1 + \eta A_2).$$

where

$$D = \frac{1 - (E'/E)\varepsilon}{1 + \varepsilon R} \ , \ \ \eta = \frac{\varepsilon \sqrt{Q^2}}{E - E'\varepsilon} \ , \ \ R \equiv \frac{\sigma_S}{\sigma_T} = \frac{\left(1 + \frac{\nu^2}{Q^2}\right) W_2 - W_1}{W_1}.$$

D is called as the depolarization factor and its physical meaning is obvious in Fig.3. It is easily verified that,

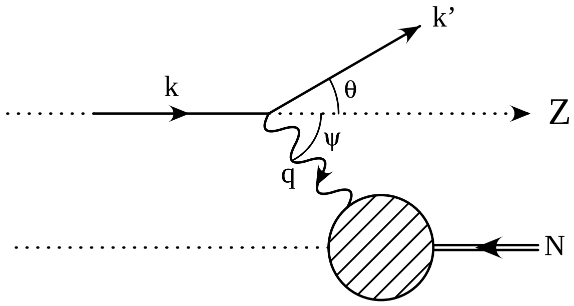

Figure 3: Momentum configuration in inelastic lepton-nucleon scattering.

$$D = D_1 \times D_2 \ ,$$

with

$$
\begin{aligned}
D_1 &\equiv \cos\psi = \frac{1-(E'/E)\varepsilon}{\sqrt{1-\varepsilon^2}} \ , \\
D_2 &\equiv \text{probability to have } T\text{−photon} = \frac{\sqrt{1-\varepsilon^2}\ \sigma_T}{\sigma_T+\varepsilon\sigma_S}.
\end{aligned}
$$

The structure functions g_1 and g_2 can be expressed in terms of A_1 and A_2.

$$
g_1 = \frac{F_2}{2x(1+R)}\left(A_1 + \frac{\sqrt{Q^2}}{\nu}A_2\right) \ , \ g_2 = \frac{F_2}{2x(1+R)}\left(\frac{\nu}{\sqrt{Q^2}}A_2 - A_1\right) \ ,
$$

where the following relation has been used,

$$
F_1 = W_1 = \frac{F_2}{x(1+R)}\left(1+\frac{Q^2}{\nu^2}\right) \ .
$$

Here it is to be noted that we have the inequalities from the unitarity arguments ; $|A_1| < 1$, $|A_2| < \sqrt{R}$. Furthermore if we consider the scaling region (Bjorken limit), $\frac{Q^2}{\nu^2} = \frac{4M^2x^2}{Q^2} \ll 1$, g_1 is given by,

$$
g_1 \cong \frac{A_1 F_2}{2x(1+R)}.
$$

This is the basic formula on which the experimental determination of g_1 is based.

3 The perturbative QCD

In this section, we review the fundamental aspects of QCD to analyze the structure functions introduced above. At first, we discuss the general strategy of QCD based on the operator product expansion (OPE) and the renormalization group equation (RGE). Next, we consider the relation between the QCD results and the parton model interpretations of the process we are considering.

3.1 Formal approach in the perturbative QCD

The kinematical region which we are interested in is the Bjorken limit, namely Q^2, $\nu \to \infty$ with $x = \frac{Q^2}{2M\nu}$ fixed. In this limit, we can easily recognize that the hadronic tensor Eq.(1) is governed by the behavior of the current products near the light-cone. So the light-cone expansion, which is a variant of the OPE, of two currents might be applied to this precess. However the OPE can not be used directly here by the following reason. The OPE makes sense in the short distance limit and this limit corresponds to the region where all component of q_μ become infinity. Therefore, $Q^2 \geq 2p\cdot q$. On the other hand, the physical region for the deep-inelastic process is $Q^2 \leq 2p \cdot q$. Fortunately we can overcome this dilemma by using the dispersion relation [9] which relates the short distance limit to the Bjorken limit for the deep-inelastic process.

Consider the time-ordered product of two currents which corresponds to the forward Compton scattering of the virtual photon with "mass q^2 " , (the Lorentz and the spin structure being neglected),

$$T = i\int dx\, e^{iq\cdot x}\langle p|TJ(x)J(0)|p\rangle,$$

the physical region of which is $2p\cdot q/|q^2| = 2M\nu/Q^2 \leq 1$. For this process, we can use the OPE. In general, the product of two operators (currents) can be expanded as follows;

$$TJ(x)J(0) \sim \sum_{i,n} C_i^n(x^2 - i\varepsilon)x^{\mu_1}x^{\mu_2}\cdots x^{\mu_n}O^i_{\mu_1\cdots\mu_n}(0), \qquad (2)$$

where C_i^n is a c-number function called Wilson's coefficient function and $O^i_{\mu_1\mu_2\cdots\mu_n}$ are local composite operators labeled by the index i. The dimensional argument tells us that $C_i^n(x^2)$ behaves like,

$$C_i^n(x^2) \sim (x^2)^{(d_O^n - n - d_J - d_J)/2},$$

in the concerned limit of $x^2 \to 0$ with x^μ small but $\neq 0$, where d is the dimension of the corresponding operator. So the operators with the lower twist (τ_N):

$$\tau_N \equiv \text{dim.} - \text{spin} = d_O^n - n$$

dominate. The Fourier transform of Eq.(2) assumes the following form with an appropriate normalization to define $C_i^n(Q^2)$,

$$i \int d^4x e^{iq\cdot x} T J(x) J(0) \sim \sum_{i,n} C_i^n(Q^2) \left(\frac{2}{Q^2}\right)^n q^{\mu_1} \cdots q^{\mu_n} O^i_{\mu_1 \cdots \mu_n}.$$

By defining the matrix element of O^i ,

$$\langle p | O^i_{\mu_1 \cdots \mu_n}(0) | p \rangle = 2\, A^i_n p_{\mu_1} \cdots p_{\mu_n} - \text{trace terms},$$

(note that the operators should have definite twists, so must be traceless) the forward Compton scattering amplitude T is written as,

$$T(\nu, Q^2) = 2 \sum_{i,n} \omega^n A^i_n C^n_i(Q^2) \quad , \quad \omega = \frac{2M\nu}{Q^2} = \frac{1}{x}.$$

To get information for the structure functions in the Bjorken limit, we rely on the analytic structure of T in the complex ω-plane. There are cuts going out to infinity from $\omega = \pm 1$ and the discontinuity of T is related to the structure function W: $W = \frac{1}{\pi}\,\text{Im}T$. The next step is just to use the Cauchy's theorem. The contour integral of T around the origin in ω-plane picks up its n-th coefficient:

$$\frac{1}{2\pi i} \oint d\omega \omega^{-n-1} T = 2 \sum_i A^i_n C^n_i(Q^2).$$

Deforming the contour to pick up the discontinuity of T, the left hand side becomes,

$$\text{LHS} = \frac{2}{\pi} \int_1^\infty d\omega \omega^{-n-1} \text{Im}T(\omega, Q^2) = 2 \int_0^1 dx x^{n-1} W(x, Q^2),$$

where we have used the crossing symmetry,

$$T(q,p) = T(-q,p)\ ,\ \ W(q,p) = -\,W(-q,p).$$

Finally we get the so-called moment sum rule,

$$\int_0^1 dx x^{n-1} W(x, Q^2) = \sum_i A_n^i C_i^n(Q^2).$$

(In the above derivation, it is assumed that the integral is convergent at infinity. Since this region corresponds to the Regge limit, the problem of convergence is controlled by the Regge behavior of the corresponding amplitude.)

Now the Wilson's coefficient functions $C_i^n(Q^2)$ depend on only the large momentum Q^2. So the RGE is effectively applied to $C_i^n(Q^2)$ and they are calculated in the perturbation theory due to the asymptotic freedom of QCD. On the other hand, the hadronic matrix element of the composite operator A_n^i can not be estimated perturbatively (long distance physics) and treated as input parameters in the perturbative QCD. The RGE for the coefficient functions is easily derived as follows. Consider the Green's function of currents and fundamental fields ϕ;

$$\begin{aligned} G_{JJ}^k &\equiv \langle 0|TJ(x)J(0)\phi(x_1)\cdots\phi(x_k)|0\rangle \\ &= \sum_n C^n(x)\langle 0|TO_n(0)\phi(x_1)\cdots\phi(x_k)|0\rangle \equiv \sum_n C^n G_n^k, \end{aligned}$$

where we write the OPE symbolically as $J(x)J(0) = \sum_n C^n(x) O_n(0)$. The RGE for LHS reads,

$$[\mathcal{D} + 2\,\gamma_J - k\gamma]\, G_{JJ}^k = 0$$

where γ_J , γ the anomalous dimensions of J , ϕ . $\mathcal{D}$ is the well-known operator in the obvious notation,

$$\mathcal{D} = \mu\frac{d}{d\mu} = \mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_m(g) m\frac{\partial}{\partial m} \ ,$$

with m the quark mass. The RGE for RHS becomes,

$$[\mathcal{D} + \gamma_O^n - k\gamma]\, G_n^k = 0,$$

with γ_O^n the anomalous dimension of O_n .

$$\gamma_O^n = \mu \frac{d}{d\mu} \ln Z_O^n \quad , \quad O_n = (Z_O^n)^{-1} O_n^0 \ .$$

Therefore RGE for the coefficient functions is given by,

$$[\mathcal{D} + 2\,\gamma_J - \gamma_O^n]\, C^n = 0.$$

It is easy to solve the RGE and the solution for the Fourier transformed coefficient function becomes,

$$\begin{aligned} C^n(Q^2) \quad &= \quad C^n\left(\frac{Q^2}{\mu^2}\,,\, g(\mu)\,,\, m(\mu)\right) = C^n(\,1\,, \bar{g}(t)\,, \bar{m}(t)\,) \\ &\qquad\qquad \times \exp \int_0^t dt' \,[2\,\gamma_J(g(t')) - \gamma_O^n(g(t'))]\,, \end{aligned}$$

where,

$$t = (1/2)\ln(Q^2/\mu^2)\ ,$$

$$\frac{d\bar{g}}{dt} = \beta\,(\bar{g}) \quad , \quad \frac{d\bar{m}}{dt} = -(\,1 + \gamma_m\,(\bar{g}))\bar{m}\ ,$$

with $\bar{g}(0) = g(\mu)\,,\, \bar{m}(0) = m(\mu)$. μ is the renormalization point. It is to be noted that the above solution takes, in general, a matrix form because the operators with the same quantum number and twist will mix under the renormalization.

Here let us consider how to calculate the anomalous dimension γ and the "coefficient function" , $C^n(1, \bar{g}(t), \bar{m}(t))$. The anomalous dimension γ can be obtained from the renormalization constant for the composite operator O_n. To obtain $C^n(\,1, \bar{g}, \bar{m})$, we use the fact that the OPE is an operator relation. So, consider a Green's function which can be explicitly calculated, for example,

$$\begin{aligned} \Gamma(p,q) \quad &= \quad i\int d^4x e^{iq\cdot x}\langle 0|T\phi(-p)J(x)J(0)\phi^\dagger(p)|0\rangle \\ &= \quad \sum_n C^n(Q^2)\left(\frac{2}{Q^2}\right)^n q^{\mu_1}\cdots q^{\mu_n}\ \langle 0|T\phi(-p)O_n\phi^\dagger(p)|0\rangle. \end{aligned}$$

The LHS, the current correlation function, will be calculated to be,

$$\Gamma(p,q) = 2\sum_n t_n \omega^n \quad , \quad t_n = t_n^0 + g^2 t_n^1 + \cdots .$$

The RHS becomes,

$$\Gamma(p,q) = 2\sum_n a_n C^n(Q^2)\omega^n \quad , \quad a_n = a_n^0 + g^2 a_n^1 + \cdots ,$$

where a_n is obtained from

$$\langle 0|T\phi(-p)O_n\phi^\dagger(p)|0\rangle = 2a_n p_{\mu_1}\cdots p_{\mu_n} - \text{trace term}.$$

Therefore,

$$t_n = a_n C^n \ . \tag{3}$$

Expand $C^n(Q^2)$ in powers of g,

$$\begin{aligned} C^n(Q^2) &= C^n(\,1,0\,) + g^2 \left.\frac{\partial C^n(\,1,\bar{g})}{\partial \bar{g}^2}\right|_{\bar{g}=0} \\ &\quad -\frac{1}{2}g^2\gamma_O^n \ln\frac{Q^2}{\mu^2}C^n(\,1,0\,) + \mathcal{O}(g^4)\ , \end{aligned}$$

where $\gamma_O^n = \gamma_0^n g^2 + \mathcal{O}(g^4)$ and, for simplicity, we neglect the mass m and take into account the fact that the anomalous dimension of the conserved current J vanishes. By comparing terms of the same power of g in Eq.(3), we will get $C^n(\,1,\bar{g})$.

Now the realistic case of QCD. The OPE of the electromagnetic currents looks like,

$$\begin{aligned} & i\int d^4x e^{iq\cdot x} T J_\mu(x) J_\nu(0) \\ \sim\ & \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\sum_{i,n} C_{L,i}^n(Q^2)\left(\frac{2}{Q^2}\right)^n q^{\mu_1}\cdots q^{\mu_n} O^i_{\mu_1\cdots\mu_n} \\ & + (-g_{\mu\lambda}g_{\nu\sigma}q^2 + g_{\mu\lambda}q_\nu q_\sigma + g_{\nu\sigma}q_\mu q_\lambda - g_{\mu\nu}q_\lambda q_\sigma) \\ & \qquad\qquad \times \sum_{i,n} C_{2,i}^n(Q^2)\left(\frac{2}{Q^2}\right)^n q^{\mu_1}\cdots q^{\mu_{n-2}} O^i_{\lambda\sigma\mu_1\cdots\mu_{n-2}} \\ & -i\varepsilon_{\mu\nu\lambda\sigma}q^\lambda \sum_{i,n}\left(\frac{2}{Q^2}\right)^n q^{\mu_1}\cdots q^{\mu_{n-1}} \\ & \qquad\qquad \times [E_{1,i}^n(Q^2)R^{1,i}_{\sigma\mu_1\cdots\mu_{n-1}} + E_{2,i}^n(Q^2)R^{2,i}_{\sigma\mu_1\cdots\mu_{n-1}}]. \end{aligned}$$

The explicit forms of the composite operators with the lowest twist which contribute to the unpolarized structure functions [10] read :

$$
\begin{aligned}
O^i_{\mu_1\cdots\mu_n} &= i^{n-1} S\bar{\psi}\gamma_{\mu_1} D_{\mu_2}\cdots D_{\mu_n}\frac{1}{2}\lambda^i\psi \ , \\
O^F_{\mu_1\cdots\mu_n} &= i^{n-1} S\bar{\psi}\gamma_{\mu_1} D_{\mu_2}\cdots D_{\mu_n}\psi \ , \\
O^G_{\mu_1\cdots\mu_n} &= \frac{1}{2}\, i^{n-2} S G_{\mu_1\alpha} D_{\mu_2}\cdots D_{\mu_{n-1}} G^{\alpha}_{\mu_n} \ ,
\end{aligned}
$$

where λ^i are the $SU(f)$ generators, D is the gauge covariant derivative, and S means the symmetrization on the Lorentz indices. To the spin dependent structure functions, the following twist-2 (R_1) [11] and the twist-3(R_2) [12] operators contribute [13].

$$
\begin{aligned}
R^{1,i}_{\sigma\mu_1\cdots\mu_{n-1}} &= i^{n-1} S\bar{\psi}\gamma_5\gamma_\sigma D_{\mu_1}\cdots D_{\mu_{n-1}}\frac{1}{2}\lambda^i\psi \ , \\
R^{1,F}_{\sigma\mu_1\cdots\mu_{n-1}} &= i^{n-1} S\bar{\psi}\gamma_5\gamma_\sigma D_{\mu_1}\cdots D_{\mu_{n-1}}\psi \ , \\
R^{1,G}_{\sigma\mu_1\cdots\mu_{n-1}} &= \frac{1}{2}\, i^{n-1} S\varepsilon_{\sigma\alpha\beta\gamma} G^{\beta\gamma} D_{\mu_1}\cdots D_{\mu_{n-2}} G^{\alpha}_{\mu_{n-1}} \ ,
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
R^{2,F}_{\sigma\mu_1\cdots\mu_{n-1}} &= \frac{i^{n-1}}{n}\Big[(n-1)\bar{\psi}\gamma_5\gamma_\sigma D_{\{\mu_1}\cdots D_{\mu_{n-1}\}}\psi \\
&\qquad -\sum_{l=1}^{n-1}\bar{\psi}\gamma_5\gamma_{\mu_l} D_{\{\sigma} D_{\mu_1}\cdots D_{\mu_{l-1}} D_{\mu_{l+1}}\cdots D_{\mu_{n-1}\}}\psi\Big] \ , \\
R^{2,m}_{\sigma\mu_1\cdots\mu_{n-1}} &= i^{n-2} m\bar{\psi}\gamma_5\gamma_\sigma D_{\{\mu_1}\cdots D_{\mu_{n-2}}\gamma_{\mu_{n-1}\}}\psi \ , \\
R^{2,k}_{\sigma\mu_1\cdots\mu_{n-1}} &= \frac{1}{2n}(V_k - V_{n-1-k} + U_k + U_{n-1-k}) \ ,
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
V_k &= i^n g\bar{\psi}\gamma_5 D_{\mu_1}\cdots G_{\sigma\mu_k}\cdots D_{\mu_{n-2}}\gamma_{\mu_{n-1}}\psi \ , \\
U_k &= i^{n-3} g\bar{\psi} D_{\mu_1}\cdots \widetilde{G}_{\sigma\mu_k}\cdots D_{\mu_{n-2}}\gamma_{\mu_{n-1}}\psi \ .
\end{aligned}
$$

We have shown only the flavor non-singlet operators for R_2.

Defining the matrix element of composite operators by,

$$
\langle p,S|O^j_{\mu_1\cdots\mu_n}|p,S\rangle = 2\,A^j_n p_{\mu_1}\cdots p_{\mu_n} \ ,
$$

$$
\begin{aligned}
\langle p,S|R^{1,j}_{\sigma\mu_1\cdots\mu_{n-1}}|p,S\rangle &= -2\,Ma^j_n S_{\{\sigma}p_{\mu_1}\cdots p_{\mu_{n-1}\}} \\
\langle p,S|R^{2,F}_{\sigma\mu_1\cdots\mu_{n-1}}|p,S\rangle &= -2\,M\frac{n-1}{n}d_n(S_\sigma p_{\mu_1}-S_{\mu_1}p_\sigma)p_{\mu_2}\cdots p_{\mu_{n-1}}\ , \\
\langle p,S|R^{2,m}_{\sigma\mu_1\cdots\mu_{n-1}}|p,S\rangle &= -2\,Me_n(S_\sigma p_{\mu_1}-S_{\mu_1}p_\sigma)p_{\mu_2}\cdots p_{\mu_{n-1}}\ , \\
\langle p,S|R^{2,k}_{\sigma\mu_1\cdots\mu_{n-1}}|p,S\rangle &= -2\,Mf^k_n(S_\sigma p_{\mu_1}-S_{\mu_1}p_\sigma)p_{\mu_2}\cdots p_{\mu_{n-1}}\ ,
\end{aligned}
$$

The moment sum rules are written as,

$$
\begin{aligned}
\int dx x^{n-1}F_L(x,Q^2) &= \sum_j A^j_n C^n_{L,j}(Q^2)\ ,\ \int dx x^{n-2}F_2(x,Q^2) = \sum_j A^j_n C^n_{2,j}(Q^2)\ , \\
\int dx x^{n-1}g_1(x,Q^2) &= \frac{1}{2}\sum_j a^j_n E^n_{1,j}(Q^2)\ , \\
\int dx x^{n-1}g_2(x,Q^2) &= -\frac{n-1}{2n}\Big[a^i_n E^n_{1,i}(Q^2) - d_n E^n_{2,F}(Q^2)\Big] \\
&\qquad +\frac{1}{2}\Big[e_n E^n_{2,m}(Q^2) + \sum_k f^k_n E^n_{2,k}(Q^2)\Big]\ .
\end{aligned}
$$

The each coefficient function (E^n or C^n) takes the following form neglecting the operator mixing,

$$
E^n(Q^2) = E^n\left(1,\bar{g}(t)\right)\exp\left[-\int_0^t dt'\gamma^n(\bar{g}(t'))\right].
$$

If we insert the perturbative results,

$$
\begin{aligned}
E^n\left(1,\bar{g}(t)\right) &= e^n_0 + e^n_1\bar{g}^2 + \cdots\ , \\
\gamma^n(g) &= \gamma^n_0 g^2 + \gamma^n_1 g^4 + \cdots\ , \\
\beta(g) &= -\beta_0 g^3 - \beta_1 g^5 + \cdots,
\end{aligned}
\tag{6}
$$

we can get the final answer at the one-loop level,

$$
E^n(Q^2) = N_n\left[e^n_0 + \bar{g}^2\left\{e^n_1 + e^n_0\left(\frac{\gamma^n_1}{2\beta_0} - \frac{\beta_1\gamma^n_0}{2\beta_0^2}\right)\right\}\right]\bar{g}^{\gamma^n_0/\beta_0}, \tag{7}
$$

where

$$
N_n = g^{-\gamma^n_0/\beta_0}\left(1+\frac{\beta_1}{\beta_0}g^2\right)^{-\gamma^n_1/2\beta_1+\gamma^n_0/2\beta_0}.
$$

In this way, the Q^2 dependence of the structure functions can be predicted in the perturbative QCD based on the OPE.

3.2 Parton pictures in the perturbative QCD

It is now a widespread belief that the results of the perturbative QCD (at least at the lowest twist level) can be understood in terms of the "parton "language. In fact, the idea of the QCD improved parton model [14] has been justified for various processes and has produced a great deal of progresses in many hard reactions, especially those to which we can not apply the OPE directly. As far as the deep inelastic process, which we are interested in, is concerned, we do not need the parton model at all since we can make a definite prediction based only on the OPE and RGE. To relate the QCD results based on the OPE with the parton picture, we must define the parton distribution function in a appropriate way [15].

Let us define the parton distribution function from the previous results based on the formal approach of QCD. Consider, for example, the moment of F_2 in the singlet channel.

$$M_n \equiv \int dx x^{n-2} F_2 \ .$$

The QCD says,

$$M_n(Q^2) = A_n^F(Q_0^2) C_F^n(Q^2/Q_0^2\,,\, g_0) + A_n^G(Q_0^2) C_G^n(Q^2/Q_0^2\,,\, g_0) \ ,$$

where

$$C_i^n(Q^2/Q_0^2\,,\, g_0) = \left[T \exp\left\{ -\int_0^t dt' \gamma^n(\bar{g}(t')) \right\} \right]^{ij} C_j^n(1, \bar{g}(t)) \ ,$$

and $A_n(Q_0^2)$ is the nucleon matrix element of the composite operator renormalized at Q_0^2. The Q_0^2 dependence is (should be) cancelled between $A_n(Q_0^2)$ and the coefficient function. Since Q_0 is arbitrary, let us put $Q_0^2 = Q^2$.

$$M_n(Q^2) = A_n^F(Q^2) C_F^n(1, \bar{g}(Q^2)) + A_n^G(Q^2) C_G^n(1, \bar{g}(Q^2)) \ .$$

Now we can define the "Q^2 *dependent parton distribution function*" of the quark and gluon; $q(x,Q^2)$, $g(x,Q^2)$ as,

$$A_n^F(Q^2) \equiv \int dx x^{n-1} q(x,Q^2) \ , \ A_n^G(Q^2) \equiv \int dx x^{n-1} g(x,Q^2) \ .$$

Expanding C^n perturbatively ,

$$C_F^n(1,\bar{g}) = 1 + c_{1,F}^n \bar{g}^2 + \cdots \ , \ C_G^n(1,\bar{g}) = c_{1,G}^n \bar{g}^2 + \cdots \ ,$$

and making the inverse Mellin transformations, we get,

$$\begin{aligned} \frac{1}{x} F_2(x,Q^2) &= q(x,Q^2) + \bar{g}^2 \int_x^1 \frac{dy}{y} K_F \left(\frac{x}{y}\right) q(y,Q^2) \\ &\quad + \bar{g}^2 \int_x^1 \frac{dy}{y} K_G \left(\frac{x}{y}\right) g(y,Q^2) + \mathcal{O}(\bar{g}^4), \end{aligned}$$

where

$$\int dx x^{n-1} K_F(x) = c_{1,F}^n \ , \ \int dx x^{n-1} K_G(x) = c_{1,G}^n .$$

We can interpret the above expressions as: the first term describes the interaction of the charged parton (quark) at the lowest order and other terms indicate the radiative corrections for quarks and gluons at the higher orders. So, the QCD results can be interpreted in terms of the parton model with the Q^2 dependent parton distribution functions. Furthermore we must take into account the presence of the radiative corrections.

It is here to be noticed that the above definition of the parton distribution function (consequently, the corresponding coefficient functions too) has a degree of arbitrariness. Since the parton distribution functions are defined as the nucleon matrix element of the composite operator renormalized at Q^2 , they depend on how to renormalize the composite operators. To understand the source of the arbitrariness in the definition of parton distribution functions, let us remember the procedure of obtaining the coefficient functions. In general, the quantities in Eq.(3) will be given as,

$$
\begin{aligned}
t_n &= 1 + g^2 \left(d_n - \frac{1}{2}\gamma_0^n \ln \frac{Q^2}{-p^2} - \gamma_F \ln \frac{-p^2}{\mu^2} \right) + \mathcal{O}(g^4) \ , \\
a_n &= 1 + g^2 \left\{ b_n + \left(\frac{1}{2}\gamma_0^n - \gamma_F \right) \ln \frac{-p^2}{\mu^2} \right\} + \mathcal{O}(g^4) \ , \\
C^n &= 1 + g^2 \left(c_1^n - \frac{1}{2}\gamma_0^n \ln \frac{Q^2}{\mu^2} \right) + \mathcal{O}(g^4) \ ,
\end{aligned}
$$

where we assume $t_n^0 = a_n^0 = c_0^n = 1$. γ_F is the anomalous dimension of the external field with which one estimates the both sides of the OPE. So, from Eq.(3) we get; $c_1^n = d_n - b_n$. Now b_n depends on the renormalization scheme adopted. Therefore c_1^n also depends on the scheme. The scheme ambiguity comes from how to renormalize the composite operator. Suppose that the bare expression for a_n to be,

$$
(a_n)_{\text{bare}} = 1 + g^2 \left\{ \left(\gamma_F - \frac{1}{2}\gamma_0^n \right) \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{-p^2} \right) + f_n \right\} + \mathcal{O}(g^4) \ .
$$

To renormalize a_n, we multiply $Z_2 Z_{O_n}^{-1}$;

$$
Z_2 = 1 - g^2 \gamma_F \frac{1}{\varepsilon} + \mathcal{O}(g^4) \ ,
$$

$$
Z_{O_n}^{(j)} = 1 + g^2 \left(z_n^j - \frac{1}{2}\gamma_0^n \frac{1}{\varepsilon} \right) + \mathcal{O}(g^4) \ ,
$$

where j discriminate various renormalization schemes. Therefore,

$$
b_n = f_n - z_n^j \ ,
$$

and this means,

$$
c_{1(j)}^n = d_n - (f_n - z_n^j).
$$

The ambiguity in Z_2 cancels between t_n and a_n, however z_n^j is quite arbitrary. So, the composite operators and also the coefficient functions turn out to depend the renormalization scheme.

Physical quantities like moments should not depend on the renormalization scheme adopted. How does the above ambiguity cancel in the final expression ? The difference between the schemes for the composite operator;

$$Z^{(j)}_{O_n} = \left[1 + g^2(z^j_n - z^k_n) + \mathcal{O}(g^4)\right] Z^{(k)}_{O_n} \ ,$$

affects the anomalous dimension of composite operator.

$$\begin{aligned} \gamma^{n(j)} &= \mu\frac{d}{d\mu}\ln\left[1 + g^2(z^j_n - z^k_n) + \mathcal{O}(g^4)\right] + \gamma^{n(k)} \\ &= -2(z^j_n - z^k_n)\beta_0 g^4 + \mathcal{O}(g^6) + \gamma^{n(k)} \ . \end{aligned}$$

Then,

$$\gamma^{n(j)}_1 = -2(z^j_n - z^k_n)\beta_0 + \gamma^{n(k)}_1 \ .$$

Therefore the ambiguity cancels in the final answer through the combination,

$$c^n_1 + \frac{\gamma^n_1}{2\beta_0} \quad .$$

in $C^n(Q^2)$. But each term can depend on the renormalization scheme. We can say schematically,

$$\begin{aligned} JJ &\sim \sum\left[C^n_q O^q_n + C^n_G O^G_n\right] \\ &= \sum\left[\widetilde{C}^n_q \widetilde{O}^q_n + \widetilde{C}^n_G \widetilde{O}^G_n\right] \ . \end{aligned}$$

LHS does not depend on the scheme. However the arbitrariness remains in the definition of the parton distribution functions. The point is that the use of the parton language is helpful to interpret the QCD results intuitively and/or economically. So, the definition of parton distribution functions can depend on one's convention. It should be also mentioned that even if we start from the parton model without using the OPE, the same ambiguities appear at the stage of the factorization.

4 The spin structure functions

In this section, we will give a little bit more detailed analyses for the spin structure functions g_1 and g_2. We will not mention to the numerical studies of the recent experimental data [7, 16, 17]. Let us remember the moment sum rules for g_1 and g_2 explained in sec.3 [18, 19, 20],

$$\int dx x^{n-1} g_1(x,Q^2) = \frac{1}{2}\sum_j a_n^j E_{1,j}^n(Q^2)\ , \tag{8}$$

$$\begin{aligned}\int dx x^{n-1} g_2(x,Q^2) &= -\frac{n-1}{2n}\Big[a_n^i E_{1,i}^n(Q^2) - d_n E_{2,F}^n(Q^2)\Big] \\ &\quad +\frac{1}{2}\Big[e_n E_{2,m}^n(Q^2) + \sum_k f_n^k E_{2,k}^n(Q^2)\Big].\end{aligned} \tag{9}$$

The first moments $n=1$ of the above equations are related to the interesting sum rules: Bjorken sum rule [21](the flavor nonsinglet part of g_1), Ellis-Jaffe sum rule [22] (the flavor singlet part of g_1) and Burkhardt-Cottingham sum rule [23] (g_2).

At first let us consider g_2. For general n, the QCD analysis is not so straightforward for g_2 as for g_1. The fact that the twist-3 operators also contribute to g_2 in the leading order of $1/Q^2$ produces new aspects which do not appear in the analyses of other structure functions. The appearance of composite operators which are proportional to the equation of motion makes the operator mixing problems rather complicated. This is a general feature of the higher twist operators. This problem has been discussed in Ref.[20] and references therein. However, as far as the first moment ($n=1$) of g_2 is concerned, the above complexity is irrelevant since there is no operators corresponding to $n=1$ (see Eq.(5)). So, eq.(9) for $n=1$ predicts the Burkhardt-Cottingham sum rule:

$$\int dx g_2(x,Q^2) = 0.$$

Recently the validity of this sum rule was questioned [24] and explicit calculations of g_2 (current correlation functions) at the one-loop level have been performed [25, 26].

The results confirm that the Burkhardt-Cottingham sum rule does not receive any radiative corrections in the (at least) perturbative QCD.

Next we turn to g_1 Eq.(8). Three operators in Eq.(4) contribute to the moment of g_1. For convenience of explanation, let us rewrite the operators and their proton's matrix elements in the following way:

$$R_n^{1,i} \equiv i^{n-1} S\bar{\psi}_i \gamma_5 \gamma_\sigma D_{\mu_1} \cdots D_{\mu_{n-1}} \psi_i \ , \tag{10}$$

$$R_n^{1,0} \equiv R^{1,F}_{\sigma\mu_1\cdots\mu_{n-1}} \quad , \quad R_n^{1,G} \equiv R^{1,G}_{\sigma\mu_1\cdots\mu_{n-1}} \ , \tag{11}$$

where i is the the flavor index (we consider the case of 3 flavor $i =$ u, d, s):

$$\langle p, S | R_n^{1,j} | p, S \rangle \equiv -2\, M \Delta_n^j S_{\{\sigma} p_{\mu_1} \cdots p_{\mu_{n-1}\}}.$$

Here j runs over i, 0 and G. In this notation, the moment sum rule for g_1 Eq.(8) reads,

$$\begin{aligned} I^n(Q^2) &\equiv \int dx x^{n-1} g_1(x, Q^2) \\ &= \frac{1}{2}\left[\sum_i e_i^2 \Delta_n^i E_{NS}^n(Q^2) + \left\langle e^2 \right\rangle \left[\Delta_n^0 E_S^n(Q^2) + \Delta_n^G E_G^n(Q^2)\right]\right], \end{aligned} \tag{12}$$

after taking into account the quark charge factor. $\Delta_n^0 = \Delta_n^{\rm u} + \Delta_n^{\rm d} + \Delta_n^{\rm s}$ and $\langle e^2 \rangle = \sum_i e_i^2/f = 2/9$ for the number of flavors being $f = 3$. $E_{NS}^n \equiv E_{1,i}^n$ is the coefficient function for the flavor non-singlet operator Eq.(10) and does not depend on i. On the other hand $E_S^n \equiv E_{1,0}^n$ and $E_G^n \equiv E_{1,G}^n$ correspond to the flavor singlet channel Eq.(11) and get mixed under the renormalization.

The QCD calculation of the coefficient functions at one loop level has been done many years ago for both the non-singlet [18, 27] and singlet [19] parts. In the following, we restrict our discussions only to the first ($n = 1$) moment. For a nucleon target (p: proton, n: neutron), the first moment becomes from Eq.(12),

$$I_{p,n}^1(Q^2) = \frac{1}{12}\left[E_{NS}^1(Q^2)\left[(+-)(\Delta_1^{\rm u} - \Delta_1^{\rm d}) + \frac{1}{3}(\Delta_1^{\rm u} + \Delta_1^{\rm d} - 2\Delta_1^{\rm s})\right] + \frac{4}{3} E_S^1(Q^2) \Delta_1^0\right], \tag{13}$$

Since the gluon operator (corresponding coefficient function) with $n = 1$ does not exist (see Eq.(4)), only the fermion bilinear operators contribute which turn out to be the axial vector currents. Note that the anomalous dimension of the non-singlet axial current vanishes because of the current conservation in (massless) QCD. Therefore the nucleon's matrix elements of the non-singlet axial currents are scale independent and given by,

$$\begin{aligned}\Delta_1^{\mathrm{u}} - \Delta_1^{\mathrm{d}} = \frac{G_A}{G_V} &\cong 1.26 \ , \\ \Delta_1^{\mathrm{u}} + \Delta_1^{\mathrm{d}} - 2\Delta_1^{\mathrm{s}} &\cong 0.58 \ ,\end{aligned}$$

using the flavor SU(3) symmetry. The corresponding coefficient function has been calculated in the perturbative QCD up to the three loop in $\overline{\mathrm{MS}}$ scheme [30],

$$E^1_{NS}(Q^2) = 1 - \frac{\alpha_S}{\pi} - 3.58\left(\frac{\alpha_S}{\pi}\right)^2 - 20.2\left(\frac{\alpha_S}{\pi}\right)^3 + \cdots,$$

where $\alpha_S \equiv \bar{g}^2(Q^2)/4\pi$. Since the Bjorken sum rule receives the contribution only from the non-singlet channel, we can make a definite QCD prediction for this sum rule in the sense that the matrix element is known.

$$I^1_p - I^1_n = \frac{1}{6}\frac{G_A}{G_V}\left[1 - \frac{\alpha_S}{\pi} - 3.58\left(\frac{\alpha_S}{\pi}\right)^2 - 20.2\left(\frac{\alpha_S}{\pi}\right)^3 + \cdots\right].$$

The flavor singlet axial current is not conserved due to the Adler-Bell-Jackiw anomaly [28]. This fact makes the matrix element scale dependent [19, 29]. The perturbative result for $E^1_S(Q^2)$ is given as (see Eqs.(6,7)),

$$E^1_S(1, \bar{g}(t)) = 1 - \frac{\alpha_S}{\pi} + \cdots.$$

Due to the anomaly, the anomalous dimension γ starts at the two loop [19],

$$\gamma^1_0 = 0 \quad , \quad \gamma^1_1 = \frac{1}{(16\pi^2)^2} 24 C_2(R) T(R).$$

(For the higher order corrections, see Refs.[31].) Using these results, we write the singlet parts as follows. At first, let us define $\Delta\Sigma(Q^2)$ by (see section 3.2),

$$E_S^1(Q^2)\Delta_1^0(\mu) \equiv E_S^1(1,\bar{g}(t))\Delta\Sigma(Q^2).$$

Then,

$$\begin{aligned}\Delta\Sigma(Q^2) &= exp\left[-\int_0^t dt'\gamma^1(\bar{g}(t'))\right]\Delta\Sigma(\mu^2) \\ &= \left(1+\frac{\alpha_S}{\pi}\frac{6f}{33-2f}+\cdots\right)\Delta\Sigma(\infty)\end{aligned}$$

The final expression becomes,

$$E_S^1(1,\bar{g}(t))\Delta\Sigma(Q^2) = \left(1-\frac{\alpha_S}{\pi}\frac{33-8f}{33-2f}+\cdots\right)\Delta\Sigma(\infty).$$

In the remainder of this section, we will mention to the so called "spin crisis"or "spin deficit" [17] problem. At first we want to stress that this is never the problem of QCD. From the viewpoint of the formal approach, we have already finished all tasks to predict the moment of the structure functions. What we should do next is just to compare the above results with the experimental data. On the other hand, it is also true that the interpretation of QCD in terms of the parton language is very helpful and convenient to understand the hard processes. As explained in section 3.2, the definition of parton densities depends on the scheme; then is ambiguous. In most cases like the unpolarized structure functions, however, this ambiguity produces only small differences and a naive parton model interpretation holds rather well (of course, with the Q^2 parton densities). If it is the case also for g_1, we will have,

$$\Delta\Sigma = \sum_i \int_0^1 dx \Delta q_i(x),$$

where

$$\Delta q_i(x) \equiv q_{i+}(x) + \bar{q}_{i+}(x) - q_{i-}(x) - \bar{q}_{i-}(x),$$

with $q_{i\pm}$ ($\bar{q}_{i\pm}$) being the density of flavor i quark (antiquark) parton with helicities $\pm 1/2$. So if the nucleon's spin is carried by charged partons, we will expect $\Delta\Sigma \sim 1$. The first EMC result [1] was $\Delta\Sigma \sim 0.1$ ($\Delta_1^s \sim -0.2$)!! (for the recent status of data, see Ref.[16].)

A resolution to this problem was proposed in Ref.[32]. In the approach of the QCD improved parton model, they have derived the relation,

$$\Delta\Sigma(Q^2) = \sum_i \int_0^1 dx \left(\Delta q_i(x, Q^2) - f \frac{\alpha_S}{2\pi} \Delta g(x, Q^2) \right),$$

with

$$\Delta g(x, Q^2) \equiv g_+(x, Q^2) - g_-(x, Q^2).$$

$g_\pm$ is the gluon density with helicities ± 1. This result says that $\Delta\Sigma$ does not measure the spin fraction carried by quarks. Furthermore, it was also shown that the second term is independent of the scale Q^2 in the leading logarithmic approximation. (There may be some nonperturbative corrections to the above relation. See Ref.[7] for the details.) So if the second term assumes some finite value, a smallness of $\Delta\Sigma$ is not necessarily in contradiction with our naive expectation. However the story is not so simple because we can reach a different conclusion even if we work in the the QCD improved parton model depending on the regularization of soft singularities which turn out to be the scheme dependence [33].

Now how does the above result reconcile with the approach based on the OPE ? We do not have a gauge invariant operator composed from the gluon field which may correspond to Δg. The answer is the following: we just introduce the "Chern-Simons current" k_σ as $n = 1$ gluonic operator,

$$k_\sigma = \varepsilon^{\sigma\mu\nu\lambda} A_\mu^a (\partial_\nu A_\lambda^a - \tfrac{1}{3} g f_{abc} A_\nu^b A_\lambda^c).$$

In this case, we can make a finite renormalization of the composite operators between the axial current $R_1^{1,0}$ and k_σ. And we can get under a particular scheme that the coefficient function E_G^1 which is associated with k_σ takes the value,

$$E_G^1(1, \bar{g}) = -\frac{\alpha_S}{\pi} T(R) = -\frac{\alpha_S}{\pi}\frac{f}{2}.$$

Therefore,

$$\Delta\Sigma(Q^2) \sim \Delta^q - f\frac{\alpha_S}{2\pi}\Delta^G,$$

with,

$$\langle P, S| \left[R_1^{1,0}\right]_R |P, S\rangle = -2M\Delta^q S_\sigma \ , \ \langle P, S| \left[k_\sigma\right]_R |P, S\rangle = -2M\Delta^G S_\sigma,$$

where the subscript R denotes the renormalized operator. If we identify Δ^q (Δ^G) to be the moment of $\sum_i \Delta q_i(x)$ ($\Delta g(x)$), there is no contradiction. In the OPE of the electromagnetic currents, only the gauge (BRS) invariant operators can appear. The Chern-Simons current, however, is gauge dependent. This gap will be filled, if one notes the following trick. In the OPE, only the operator $R_1^{1,0}$ in fact appears. But we can always write as,

$$R_1^{1,0} = \left(R_1^{1,0} + f\frac{g^2}{8\pi^2}k_\sigma\right) - f\frac{g^2}{8\pi^2}k_\sigma.$$

The first (second) term will be identified as quark (gluon) densities. In fact, in the above renormalization scheme,

$$\left[R_1^{1,0}\right]_R = R_1^{1,0} + f\frac{g^2}{8\pi^2}k_\sigma,$$

at the lowest order.

Now ambiguities can come in the above game. Even if we introduce k_σ, we can get different answer depending on how one renormalizes the composite operators. Namely we can put an arbitrary number in the front of the Chern-Simons current by choosing a different scheme. This fact exactly corresponds to the ambiguities which appear also in the QCD improved parton model approach mentioned before.

In this way, the definition of the parton densities inevitably depends on the scheme. So it is a waste of time to make a discussion without specifying the scheme one adopted. The point is : Which scheme will be convenient to parametrize the parton densities and easy to understand the physics ? And once one defines the parton distribution functions, whether one can explain other processes ? (The definition in Ref.[32] seems to be reasonable in the sense that the quark helicity Δ_q is conserved to all orders since the corresponding operator is conserved current by construction.)

5 Concluding Remarks

We have explained, in this review, only the first stage of the approach in the perturbative QCD to analyze the structure of nucleons. New experiments at CERN [2, 3] and SLAC [4, 5] and detailed theoretical investigations have enormously improved our understanding on the (spin) structure of nucleon. Now all data seem to be consistent with the expectations from the perturbative QCD. It is impressive that we can make a precision test of QCD to the accuracy of about 10% [16, 34] for the spin structure of nucleons. Furthermore, the problem of "spin crisis" became less exciting. We can construct a consistent picture in terms of the parton language at least qualitatively for the spin structure function [7, 35].

On the other hand, we would like to emphasize that there still remain many subtleties and controversial aspects when one tries to make a much more precisely quantitative prediction of QCD. We will list up below several points which will need more investigations. To compare the experimental data with the QCD prediction, we must take into account many theoretical corrections and sometimes make several assumptions. Among those problems are : the small x (and large x) behavior of the structure functions [36]: theoretical treatments of the "wee" partons including the role of the s quark and the iso-spin SU(2) violation [37]: the Q^2 dependence of the structure functions both in theories and experiments [38]: etc. Since the experimental data on g_1 are taken at rather small values of Q^2, the power corrections in Q^2 to the sum rule may give some modifications. The power corrections come from the higher twist terms as well as the target mass effects [39]. For the higher twist effects, many authors have tried to reveal their theoretical behavior and numerical importance in the analyses of data [40]. However it has been pointed out recently that there is not only phenomenological but also theoretical problem in including the higher twist contributions [41]. Due to the renormalon singularities, there appears some degree of arbitrariness in the twist expansion. These problems require much more elaborate theoretical investigations. Forthcoming precision measurements will also provide us with much information and clues to answer the above questions.

As explained in the text, parton densities depend on the scheme. Therefore to confirm the parton model interpretation of the spin structure function, we need independent experiments which could measure the parton (quark and gluon) densities. If we consider a quite different process (e.g. Drell-Yan process), we encounter other spin-dependent structure functions. The experimental and theoretical studies [42] of these structure functions must help us to fully understand the spin structure of nucleons. Nuclear effects in measuring the neutron structure functions and nuclear structure functions are also interesting and important. We need more investigations [43, 44] on these subjects.

Finally, we hope that various kinds of new experiments and more detailed theoretical investigations will be able to clarify the not only the perturbative but also the nonperturbative aspects of QCD related to the Spin Physics.

Acknowledgements

The author is indebted to S.Matsuda, K.Sasaki, T.Uematsu and Y.Yasui for invaluable discussions and for fruitful collaborations on various topics discussed here. He also thanks H.Kawamura and T.Nasuno for useful comments in preparing this talk. This work is supported in part by the Monbusho Grant-in-Aid for Scientific Research No. C-05640351.

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