Spin Structure Function of the Deuteron in the Resonance Region and the GDH Sum Rule for the Neutron[1]

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Abstract

The nuclear effects in the spin-dependent structure function g_1 of the deuteron are studied in the kinematics of future experiments at CEBAF, ($\nu \leq 3\ GeV$, $Q^2 \leq 2\ GeV^2$). The magnitude of the nuclear effects is found to be significantly larger than the one occurring in deep inelastic scattering ($\nu \to \infty$, $Q^2 \to \infty$). We discuss the mechanism leading to large effects in the region of the nucleon resonances. A possibility to measure the neutron structure functions in the CEBAF experiments with deuterium is analysed, and conclusions about the experimental study of the Q^2 dependence of the Gerasimov-Drell-Hearn Sum Rule for the neutron are drawn.

I. Recently it has been proposed [1] at CEBAF to experimentally study the spin-dependent structure function (SF) of the neutron, g_1^n, in a wide interval of energy, ν ($0.2-3\ GeV$), and momentum transfers, Q^2 ($0.15-2\ GeV^2$), using polarized deuterium and ^3He targets. These experiments will shed light on a number of quantum chromodynamics (QCD) sum rules and will help establish a connection between results predicted by low energy theorems ($Q^2 \to 0$) and perturbative QCD ($Q^2 \gg m^2$, m being the nucleon mass).

Of particular interest is the Q^2 dependence of the Gerasimov-Drell-Hearn sum rules [2] for the neutron and proton, and the connection of those sum rules with the Bjorken and Ellis-Jaffe sum rules (see review [3] and references therein).

Keeping in mind the lessons of the EMC-effect, one might expect that nuclear corrections could play an important role in estimating the neutron SF from the combined nuclear and proton data [4, 5]. In the region of finite $Q^2 \sim m^2$ and $\nu \sim m$, nuclear corrections are much more important than those in the deep inelastic limit [6, 7]. In this talk the role of nuclear structure effects in electron-deuteron scattering in the resonance region

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will be discussed, paying special attention to the procedure of extracting the neutron SF from the deuteron data in the kinematics of future experiments at CEBAF.

II. The nucleon contribution to the deuteron structure functions is usually calculated by weighting the amplitude of electron scattering on the nucleon with the wave function of the nucleon in the deuteron (for recent developments see e.g. [8, 9, 10, 11] and references therein). For the spin-dependent SF the most important effects are those of the Fermi motion and depolarizing effect of the D-wave. Additional effects, such as off-mass-shell effects or nucleon deformation, are found to be small [12, 13]. For finite values of Q^2 and ν, the deuteron SF $g_1^D(x, Q^2)$ reads as follows:

$$g_1^D(x, Q^2) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{m\nu}{kq} g_1^N\left(x^*, Q^2\right)\left(1 + \frac{\xi(x, Q^2)k_3}{m}\right)\left(\Psi_D^{M+}(\boldsymbol{k}) S_z \Psi_D^M(\boldsymbol{k})\right)_{M=1} \quad (1)$$

$$= \int_{y_{min}(x,Q^2)}^{y_{max}(x,Q^2)} \frac{dy}{y} g_1^N(x/y, Q^2)\vec{f}_D(y, \xi(x, Q^2)), \quad (2)$$

where $g_1^N = (g_1^p + g_1^n)/2$ is the isoscalar nucleon SF and $\Psi_D^M(\boldsymbol{k})$ the deuteron wave function with spin projection M. In the rest-frame of the deuteron, with $\boldsymbol{q}$ opposite the z-axis, kinematical variables are defined as:

$$kq = \nu(k_0 + \xi(x, Q^2)k_3), \quad k_0 = m + \epsilon_D - \boldsymbol{k}^2/2m, \quad (3)$$

$$\xi \equiv q_3/\nu = |\boldsymbol{q}|/\nu = \sqrt{1 + 4m^2x^2/Q^2}, \quad Q^2 \equiv -q^2, \quad x^* = Q^2/2kq, \quad (4)$$

where $\epsilon_D = -2.2246\ MeV$ is the deuteron binding energy. The limits $y_{min(max)}(x, Q^2)$ are defined to provide an integration over the physical region of momentum in (1) and to take into account the pion production threshold in the virtual photon-virtual nucleon scattering[2]. Since both $y_{min}(x, Q^2)$ and $y_{max}(x, Q^2)$ are solutions of a transcendent equation, explicit expressions for them cannot be given. However, in our numeric calculations they are accurately taken into account.

Eqs. (1)-(2) have the correct limit in the deep inelastic kinematics ($Q^2 \to \infty, \quad \nu \to \infty$). In this case: $\xi(x, Q^2) \to 1, \quad y_{min} \to x, \quad y_{max} \to M_D/m$, and the usual convolution formula for the deuteron SF [8, 11] is recovered:

$$g_1^D(x, Q^2) = \int_x^{M_D/m} \frac{dy}{y} g_1^N(x/y, Q^2)\vec{f}_D(y). \quad (5)$$

Equation (5) defines the spin-dependent "effective distribution of the nucleons", $\vec{f}_D$, which describes the bulk of the nuclear effects in g_1^D. The main features of the distribution function, $\vec{f}_D(y)$, are a sharp maximum at $y = 1 + \epsilon_D/2m \approx 0.999$ and a normalization given by $(1 - 3/2P_D)$ (P_D being the weight of the D-wave in the deuteron). As a result, in the region of medium values of $x \sim 0.2 - 0.6$, the deuteron SF $g_1^D(x)$ is slightly suppressed

[2]For x not too close to the limit of single-nucleon kinematics, $x \to 1$, the quasi elastic contribution can be disregarded

by a depolarization factor, $(1-3/2P_D)\times g_1^N(x)$, compared to the free nucleon SF. However, the magnitude of this suppression is small ($\sim 1\%$) and this is why it is phenomenologically acceptable to extract the neutron SF from the deuteron and proton data by making use of the following approximate formula:

$$g_1^D(x,Q^2) \approx \left(1-\frac{3}{2}P_D\right)(g_1^n(x,Q^2)+g_1^p(x,Q^2))/2. \tag{6}$$

In addition, when integrated over x, eqs. (5) and (6) give exactly the same results ($\Gamma = \int dx g_1(x)$), i.e.

$$\Gamma_D(Q^2) = \left(1-\frac{3}{2}P_D\right)(\Gamma_n(Q^2)+\Gamma_p(Q^2))/2, \tag{7}$$

which allows one to define *exactly* the integral of the neutron SF Γ_n from the deuteron and proton integrals, without solving (5).

Eqs. (1)-(2) at finite values of Q^2 and ν are more sophisticated than the corresponding equations in the deep inelastic limit. In particular, they do not represent a "convolution formula" in the usual sense, since the effective distribution function $\vec{f}_D$ and the integration limits are also functions of x. This circumstance immediately leads to the conclusion that, in principle, when integrals of the SF are considered, the effective distribution can not be integrated out to get the factor similar to $(1-3/2P_D)$ in (7). Another interesting feature of formulae (1)-(2) is the Q^2-dependence of $\vec{f}_D$ and $y_{min,(max)}(x,Q^2)$. If we again limit ourselves to the discussion of the integrals of SF, one concludes that the Q^2-dependence of such an integral is governed by both the QCD-evolution of the nucleon SF and the kinematical Q^2-dependence of the effective distribution of nucleons.

Thus, we have established that in the non-asymptotic regime, equation (7), in principle, does not hold. Furthermore, it is not clear whether an equation similar to (6) could be applied in this region. Indeed, we are discussing the kinematical conditions pertaining to nucleon resonances, where the "elementary" nucleon SF explicitly exhibits Breit-Wigner resonance structures corresponding to the excitations of the nucleon by the photon and one expects that the Fermi motion and binding of nucleons will result in a shift and smearing of the resonance structures. However, one can hope the actual effects will be quantitatively small so that eqs. similar to (7) and (6) can phenomenologically still be valid.

III. In our numerical estimates we use a reliable parametrization of the proton and neutron SF given by Burkert [14], which takes into account several nucleon excitations and provides a reasonable description of the available nucleon data in the resonance region. Using the Bonn potential model for the deuteron wave function [15], we carry out a realistic calculation of the deuteron SF, $g_1^D(x,Q^2)$ in the region of nucleon resonances.

In Fig. 1 and 2 the results of the calculation of the deuteron SF, $g_1^D(x,Q^2)$ and $F_2^D(x,Q^2)$ at $Q^2 = 0.1$ GeV and 1.0 GeV, are compared with the input of the calculation, i.e. the isoscalar nucleon SF, $g_1^N(x,Q^2)$ and $F_2(x,Q^2)$. It can be seen that the role of nuclear effects in the resonance region is much larger (up to $\sim 50\%$ in the maxima of the resonances), than in the deep inelastic regime ($\sim 7-9\%$, depending upon the models [8, 9, 5, 11], resulting in $\sim 6-7\%$ from the depolarization factor $(1-3/2P_D)$

and $\sim 1 - 2\%$ from the binding effects and Fermi motion). Such a drastic effect is a consequence of the presence of the narrow resonance peaks in the nucleon SF.

Indeed, let us write the elementary nucleon SF as a sum of smooth background contribution, $g_1^{N,bg}$, and several resonances, ϕ_i, which can be both positive and negative:

$$g_1^N(x,Q^2) = g_1^{N,bg}(x,Q^2) + \sum_i \phi_i(x,Q^2). \qquad (8)$$

The size of the effect of the Fermi motion and binding, for the smooth function, $g_1^{N,bg}$, is similar to the one in the deep inelastic regime. The smearing of the nucleon resonances in the deuteron SF is estimated by formula:

$$\tilde{\phi}(x,Q^2) = \phi_i(x_i,Q^2)\frac{1}{< y >}\int\limits_{x/(x_i+\Delta_i)}^{x/(x_i-\Delta_i)} dy \vec{f}(y,x_i,Q^2), \qquad (9)$$

where x_i and $2\Delta_i$ are the position and width of the i-th resonance, $< y > \simeq 1$. In deriving formula (9) we approximated the resonances by the rectangles of height $\phi(x_i)$ and width $2\Delta_i$. This estimation shows that resonance is smeared over the entire region of x and is strongly suppressed everywhere, if limits of integrations in r.h.s. of eq. (9) are close or, the same, if $2\Delta_i$ is small. Formula (9) predicts that resonances are more suppressed if Δ_i is smaller and if x_i is larger. We present in Fig. 3 the results of pedagogical calculations, aiming to illustrate the features of the formula (9). One can explicitly see that narrow resonance structures at high x are strongly suppressed by the convolution (see also behaviour of resonances on Figs. 1 and 2).

Fig. 4 shows the results of the extraction of the neutron SF from the deuteron and proton data by using the approximate formula (6), which we believe to give an upper limit of the possible errors in this extraction. To emphasize the role of nuclear effects in the region of finite Q^2, the extracted neutron SF is compared with the original (input in the calculation) parametrization of the neutron SF. The use of the approximate formula (6) appears to be in some regions completely unreliable. This can be easily understood as follows: the proton and neutron SF have similar behavior in the resonance region, in that the positions of the nucleon resonances are the same for both of them, whereas the resonances in the resulting deuteron SF are smeared and shifted, compared to the isoscalar SF. Therefore, the subtraction of the proton SF from the deuteron one, in the maximum of the former, can result in a minimum for the neutron SF, instead of a maximum. The conclusion of our analysis is that nuclear effects in the resonance region are very specific and the approximate formula (6) does not work, even for the crude extraction of the neutron SF. Obviously, another method of extracting the neutron SF should be used.

IV. In ref. [16] a rigorous method of solving eq. (5) for the unknown neutron SF has been proposed and applied in the deep inelastic region. It has been shown that this method, which works for both spin-independent and spin-dependent SF's, in principle allows one to extract the neutron SF exactly, requiring only the analyticity of the SF. It can also be applied by a minor modification to the extraction of the SF at finite Q^2, which is our present aim.

The basic idea is to replace the integral equation (2) by a set of linear algebraic equations, $K\mathcal{G}_N = \mathcal{G}_D$, where K is a square matrix (depending upon the deuteron model), $\mathcal{G}_D$ is a vector of the experimentally known deuteron SF and $\mathcal{G}_N$ is a vector of an unknown solution. Changing the integration variable in (2), $\tau = x/y$, we get

$$g_1^D(x,Q^2) = \int\limits_{\tau_{min}(x,Q^2)}^{\tau_{max}(x,Q^2)} d\tau g_1^N(\tau,Q^2)\frac{1}{\tau}\vec{f}_D(x/\tau,\xi(x,Q^2)), \tag{10}$$

where $\tau_{min}(x,Q^2) = x/y_{max}(x,Q^2)$, $\tau_{max}(x,Q^2) = x_{max}(Q^2)/y_{min}(x,Q^2)$ and $x_{max}(Q^2)$ is defined by the pion production threshold in virtual photon-nucleon scattering. Let us assume that the deuteron SF has been measured experimentally in the interval (x_1,x_2) and a reasonable parametrization for the SF is found in this interval. Then, dividing both intervals (x_1,x_2) and (τ_{min},τ_{max}) into N small parts, one may write:

$$g_1^D(x_i,Q^2) \approx \sum_{j=1}^{N} g_1^N(\tilde{\tau}_j,Q^2) \int\limits_{\tau_j}^{\tau_{j+1}} \frac{1}{\tau}\vec{f}_D(x_i/\tau,Q^2)d\tau, \quad i = 1\dots N, \tag{11}$$

where $\tilde{\tau}_j = \tau_{min} + h(j-1/2)$ and $h = (\tau_{max} - \tau_{min})/N$. Equation (11) is already explicitly of the form $\mathcal{G}_D = K\mathcal{G}_N$, therefore the usual linear algebra methods can be applied to solve it.

Note that the range of variation of τ is larger than the one for x. Therefore, in principle, the SF of the deuteron, experimentally known in the interval (x_1,x_2), contains information about neutron SF in wider interval (for example, in deep inelastic regime $\tau_{min} \approx x/2$ and $\tau_{max} = 1$). However, extracting information beyond the interval $\tilde{\tau}_{min} = x_1$ to $\tilde{\tau}_{max} = x_2$ is almost impossible in view of the structure of the kernel of eq. (10) and the kinematical condition of planned experimental data [16]. We have to redefine the kernel of eq. (10) to incorporate new limits of integration $\tilde{\tau}_{min} = x_1$ and $\tilde{\tau}_{max} = x_2$ [16].

The procedure of solving the eq. (2) in the kinematical region of finite Q^2 and ν will be presented elsewhere in details; here we only stress that the method works with good accuracy. To check it, we calculated the deuteron SF by formula (2) with the nucleon SF $g_1^N(x,Q^2)$ from ref. [14] and the deuteron wave function of the Bonn potential [15]. Then the obtained $g_1^D(x,Q^2)$ has been used as "experimental" data to calculate the vector $\mathcal{G}_D$ in (11); the matrix K has been calculated by using the same deuteron wave function. Equation (11) has been solved numerically for various "experimental" situations (changing the "measured" interval (x_1,x_2), for different Q^2, etc.). The obtained solution, i.e. the extracted neutron SF, has been compared point by point with the input to the calculation of g_1^D. We found that method is stable and allows one to unfold the neutron SF with errors not larger than 10^{-4}, which is much smaller than the expected experimental errors. (Note, that all results and conclusions are valid for both polarized and unpolarized SF.)

V. In this section the role of nuclear corrections in the analysis of the integrals of the SF, such as the GDH Sum Rule will be discussed. A very important observation has been made in the deep inelastic limit, the *exact* formula (5) and the *approximate* formula (6) give the same result for the integral of the neutron structure function, $g_1^n(x,Q^2 \gg m^2)$

(see eq. (7)). Likewise, we consider a possibility to apply the eq. (6) to the estimate of the integral of the nucleon SF from the deuteron data. The applicability of the approximate formula in the deep inelastic region is based on the conservation of the norm of the distribution, $\vec{f}(y)$, by the convolution formula. This circumstance can not be immediately extended to the case of the resonanse region, (i) the convolution is broken in eq. (2) and (ii) the normalization of the function $\vec{f}(y, x, Q^2)$ is different from one of $\vec{f}(y)$. However, the size of the effects is not too large and they should not lead to large errors if we use eq. (7).

In order to understand the deviations of the integral of the deuteron SF in the resonances region from eq. (7), let us evaluate the deuteron SF, (2), using the presentation (8) of the nucleon SF:

$$g_1^D(x,Q^2) \approx \frac{1}{\langle y\rangle} g_1^{N,bg}(x/\langle y\rangle, Q^2) \int\limits_{y_{min}(x,Q^2)}^{y_{max}(x,Q^2)} \vec{f}_D(y,x,Q^2)dy + \sum_i \tilde{\phi}_i(x,Q^2), \tag{12}$$

where the first term in the r.h.s. of eq. (12) is obtained using an expansion at the point of sharp maximum of distribution function, $\vec{f}$, and the second term is defined by (9). Next, we define an auxiliary function n_{eff} as:

$$n_{eff}(x,Q^2) \equiv \left(1 - \frac{3}{2}P_D\right)^{-1} \int\limits_{y_{min}(x,Q^2)}^{y_{max}(x,Q^2)} \vec{f}_D(y,x,Q^2)dy. \tag{13}$$

Thus, it can be seen that n_{eff} represents the "effective number" of nucleons "seen" by the virtual photon in the process when the virtual photon is absorbed by the nucleon and at least one pion is produced in the final state. Obviously, in the deep inelastic limit, $n_{eff} = 1$ corresponds to the normalization of the deuteron wave function. The effect of Q^2 on n_{eff} consists in narrowing the interval on x from 0 to $x_{max}(Q^2)$. At finite Q^2, n_{eff} is close to 1, in so far as x (or ν) is far from the threshold, and n_{eff} rapidly falls down only very close to the threshold. Figure 5 illustrates the behaviour of $n_{eff}(x,Q^2)$ as a function of x. The arrows indicate the kinematical limits x_{max} at a given Q^2. The function $n_{eff}(x,Q^2)$ significantly differs from unity only when $x \to x_{max}$ where we expect the nucleon SF to be rather small (see Figs. 1,2).

Then, integrating (12) we get:

$$\Gamma^D(Q^2) \approx \left(1 - \frac{3}{2}P_D\right) \left\{ \int\limits_0^{x_{tr}} dx g_1^{N,bg}(x,Q^2) n_{eff}(x,Q^2) + \sum_i 2\Delta\phi_i(x_i,Q^2) n_{eff}(x_i,Q^2) \right\}. \tag{14}$$

Noting that the integral of the nucleon SF, (8), is

$$\Gamma^N(Q^2) = \int\limits_0^{x_{tr}} dx g_1^{N,bg}(x,Q^2) + \sum_i 2\Delta\phi_i(x_i,Q^2), \tag{15}$$

and $n_{eff(x)}$ is close to 1, we expect the quantity in curly brackets in the r.h.s. of eq. (14) is close to the integral of the nucleon SF, (15).

Therefore, we expect only small corrections to the integral of the deuteron SF compared to the (7). This effect can be accounted for by a new equation:

$$\Gamma_D(Q^2) = \left(1 - \frac{3}{2}P_D\right) N_{eff}(Q^2)(\Gamma_n(Q^2) + \Gamma_p(Q^2))/2, \qquad (16)$$

where $N_{eff}(Q^2) \neq \int dx n_{eff}(x, Q^2)$; eq. (16) and the integral of eq. (2) represent the definition of the effective number $N_{eff}(Q^2)$, which depends upon the form of the nucleon SF, g_1^N; and, since this is expected to strongly oscillates (see Fig. 1), even the sign of the correction can vary. For instance, we obtain using the SF from [14],

$$N_{eff}(Q^2 = 0.1 \text{ GeV}^2) = 1.02, \quad N_{eff}(Q^2 = 1.0 \text{ GeV}^2) = 0.997, \qquad (17)$$

i.e. a rather small effect (+2% and −0.3% correspondly). Therefore eq. (16) appears to be reliable for estimating the integrals of the SF: setting $N_{eff}(Q^2) = 1$ does not lead to errors larger than 3% for $Q^2 = 0.1 - 2.0$ GeV^2.

VI. In conclusion, we have shown that the effects of nuclear structure in the extraction of the neutron SF in the resonance region are much more important than in the deep inelastic scattering. We have explained how the correct neutron SF can be firmly extracted from the combined deuteron and proton data. At the same time, we have found that the integrals of the SF, such as the GDH Sum Rule, can be estimated with accuracy better than 3% by the simple formula (7) which is also valid in deep inelastic region.

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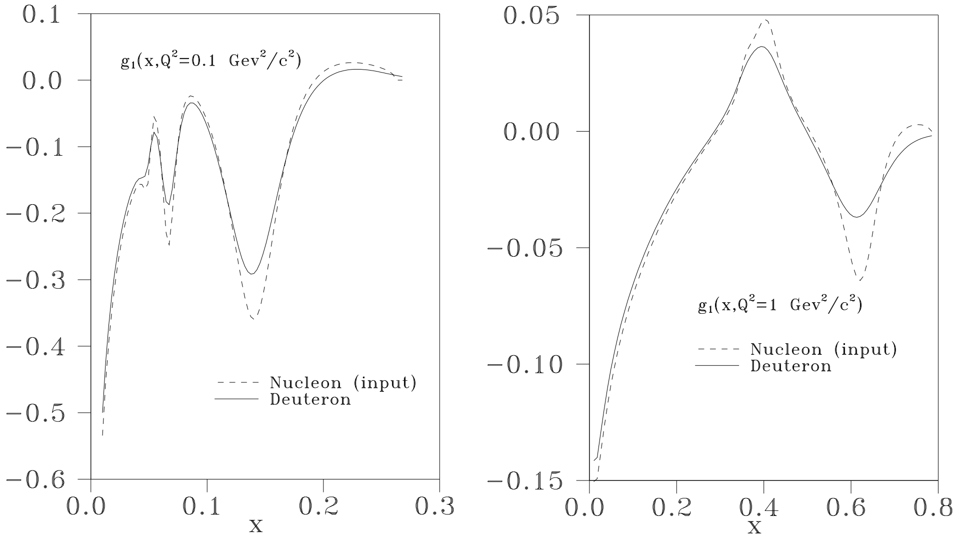

Figure 1. The spin dependent structure functions $g_1(x, Q^2)$ for two values of Q^2. The deuteron SF (solid line) is compared with the isoscalar nucleon SF (dotted line) used as input into the calculation in eq. (2).

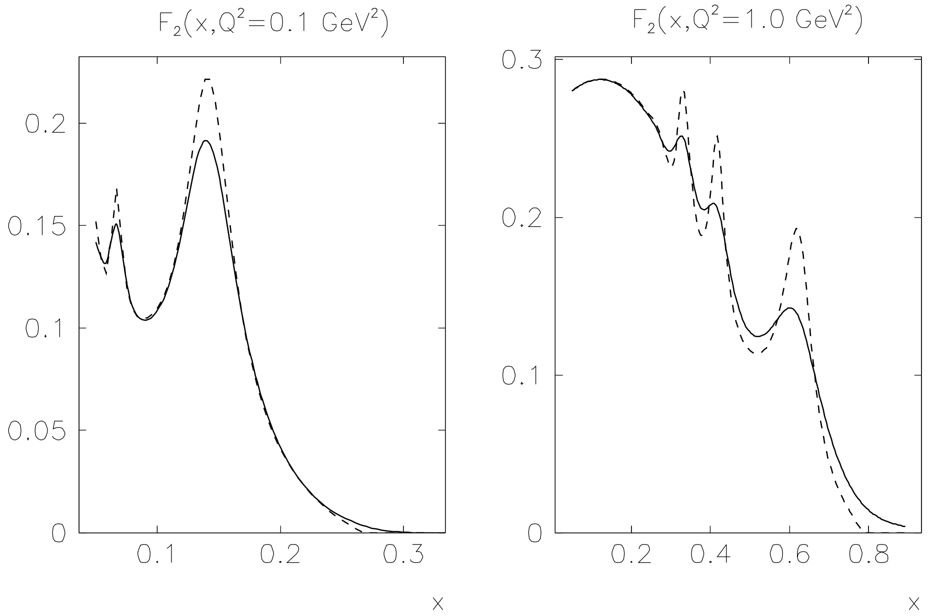

Figure 2. The spin independent structure functions $F_2(x, Q^2)$ for two values of Q^2. The deuteron SF (solid line) is compared with the isoscalar nucleon SF (dotted line) used as input into the calculation in similar to eq. (2).

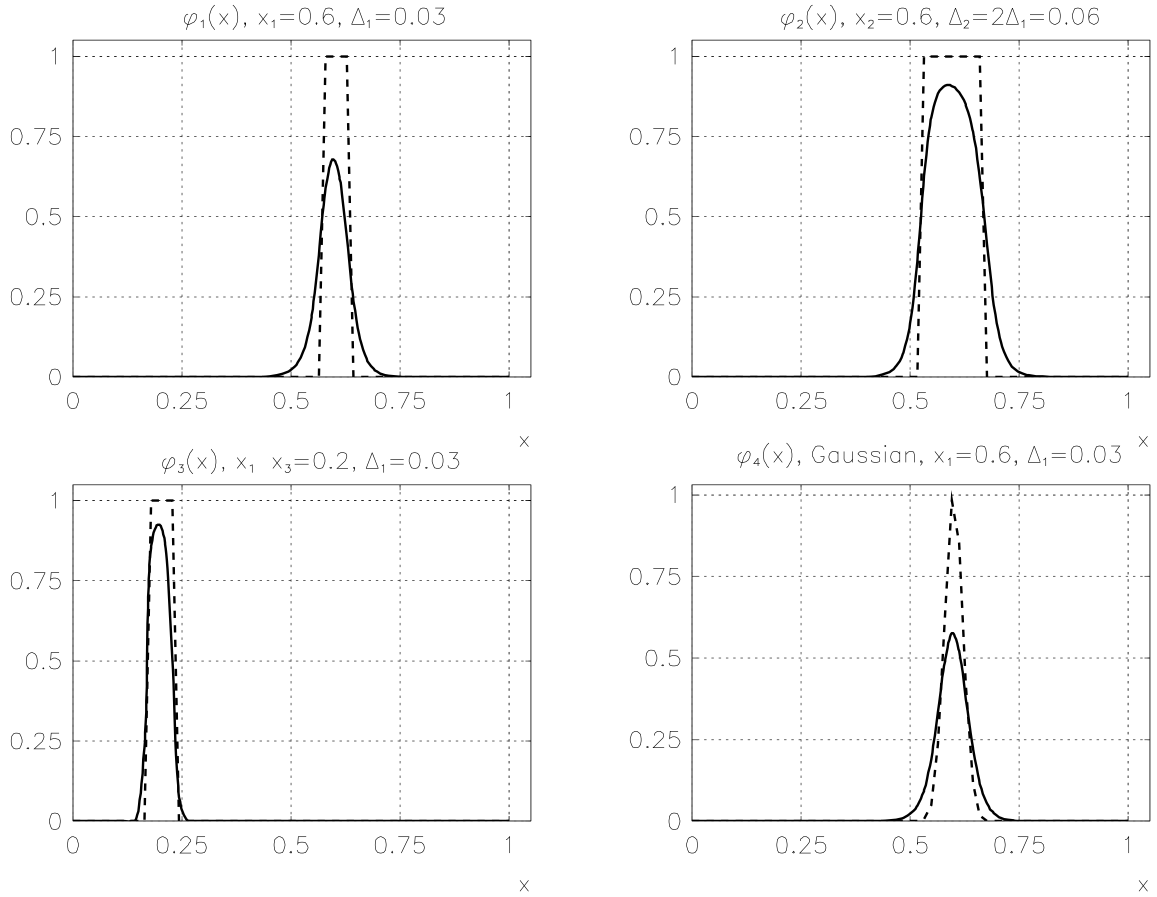

Figure 3. Illustration for formula (9). Examples of calculation of the convolution (5), solid lines, with narrow resonance structures, dashed lines.

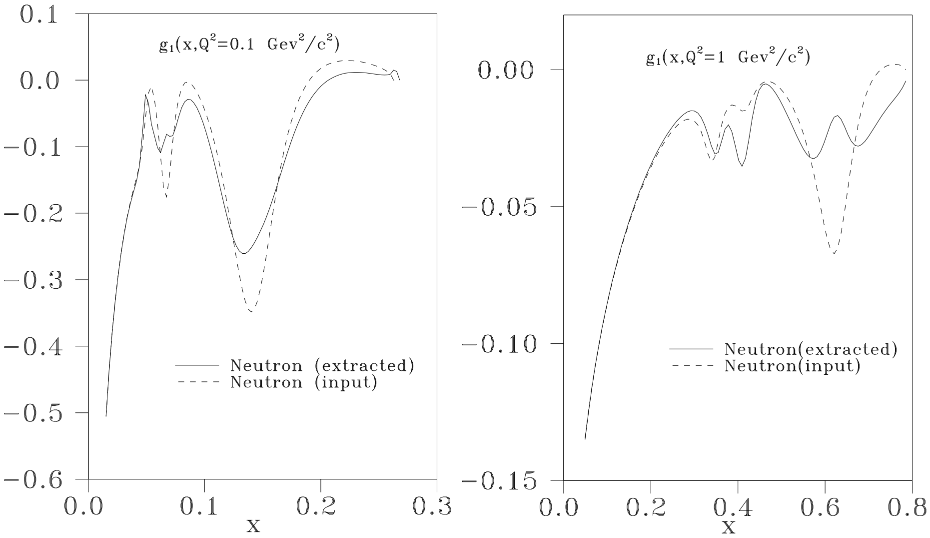

Figure 4. The extracted neutron SF (dotted line) by approximate formula eq. (6) in comparison with the original parametrization (solid line) used into the convolution formula (2).

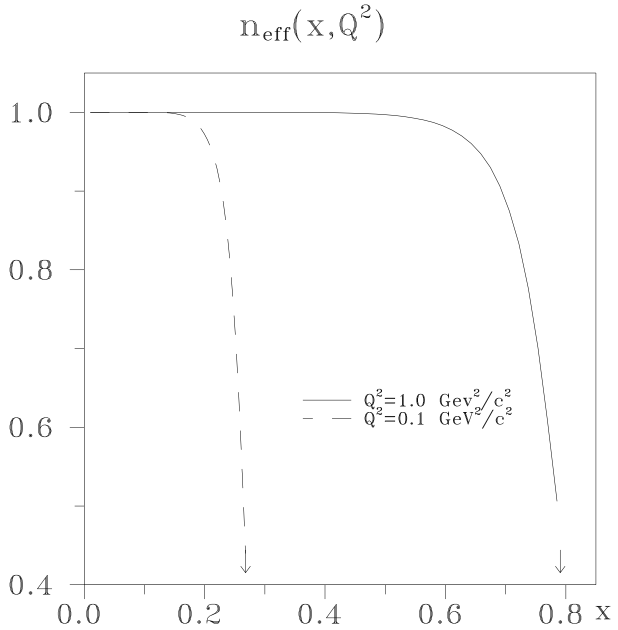

Figure 5. The effective number $n_{eff}(x, Q^2)$ characterizing the additional nuclear effects in the GDH integral, eq. (13).