Measurement of the Neutron (^{3}He) Spin Structure Function at Low Q^2 : a connection between the Bjorken and Drell-Hearn-Gerasimov sum rules.

Thesis by

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or

How Steffen got his Thesis

Starring

Emlyn "The Hammer" Hughes

David "Phone Call Time" Pripstein

Connie "Puts up with me" Jensen

Jang Ping "No error is small enough" Chen

Gordon "Story Time" Cates

Zein-Eddine "Cross section while we're at it" Meziani

Alex "The French Connection" Deur

Ioannis "Come on Man" Kominis

Karl "Impression Master" Sliffer

Pibero "Run for the Border" Djawotho

Seonho "With Child" Choi

"M.C." Xiaodong Jiang

Wolfgang "Put a label on everything" Korsch

Ed "Magic Oil" Folts and the entire Hall A team

John "Physica Master" Arrington

and

Various Kelloggites

Abstract

The spin dependent cross sections, $\sigma^T_{1/2}$ and $\sigma^T_{3/2}$, and asymmetries, $A_{||}$ and $A_{\perp}$, for ^{3}He have been measured at the Jefferson Lab's Hall A facility. The inclusive scattering process $^3\vec{\mathrm{He}}(\vec{e},e)X$ was performed for initial beam energies ranging from 0.86 to 5.1 GeV, at a scattering angle of 15.5°. Data includes measurements from the quasi-elastic peak, resonance region, and the deep inelastic regime. An approximation for the extended Gerasimov-Drell-Hearn integral is presented at a 4-momentum transfer Q^2 of 0.2-1.0 GeV^2.

Also presented are results on the performance of the polarized ^{3}He target. Polarization of ^{3}He was achieved by the process of spin-exchange collisions with optically pumped rubidium vapor. The ^{3}He polarization was monitored using the NMR technique of adiabatic fast passage (AFP). The average target polarization was approximately 35% and was determined to have a systematic uncertainty of roughly ±4% relative.

Contents

List of Figures

List of Tables

Chapter 1 Introduction

1.1 Polarized Electron Scattering

In 1988 the European Muon Collaboration (EMC) at CERN published results on the spin structure of the proton [5, 6]. They measured the proton spin structure function by scattering high energy polarized muons by a polarized ammonia target. Before the EMC result it was expected that the three valence quarks (up, up, and down) could account for the proton's spin. Such a line of thinking did not seem unreasonable since the naive quark model appeared to do quite well at explaining the charge, the magnetic moment, parity, isospin, and symmetry properties of hadrons [7]. Indeed early experiments at the Stanford Linear Accelerator Center (SLAC) confirmed the premise that the valence quarks were the dominate contribution to the proton's spin [8, 9]. By measuring the quark contribution over a wider kinematic region, the European Muon Collaboration found that the quark contribution was $12\% \pm 9\%_{stat} \pm 14\%_{sys}$, a small value consistent with zero. The resulting situation was dubbed the "proton spin crisis."

What followed was an explosion in theoretical work. In addition a number of experimental programs began dedicated to understanding the spin structure of the proton, neutron, and deuteron. The European Muon Collaboration reorganized into the Spin Muon Collaboration (SMC) and became dedicated to studying both the proton and deuteron spin structure functions [10]. Meanwhile, at the Stanford Linear Accelerator Center (SLAC) a series of experiments began using a polarized electron beam and polarized ^{3}He, ammonia, and Lithium Deuteride (LiD) targets to extract the spin structure functions of the neutron, proton, and deuteron: E142 [11], E143 [12], E154 [3], and E155 [13]. In Europe, at DESY, the HERMES collaboration began a program using polarized electrons and polarized hydrogen, deuterium, and ^{3}He [14]. For a recent review of these experiments, see [15].

Much of this recent work focused on measuring the Bjorken sum rule:

$$\int_0^1 (g_1^p - g_1^n)dx = \frac{1}{6}\frac{g_A}{g_V}, \tag{1.1}$$

where $g_1^{p(n)}$ is the spin structure function for the proton (neutron), $\frac{g_A}{g_V}$ is the ratio of axial and vector coupling constants, and the integration is taken over the full range of the kinematic variable x (Eq. 1.8). Because of the surprising EMC proton result, experimenters raced to check this fundamental relation by measuring the neutron. If found, a violation of the Bjorken sum rule would represent a major problem for perturbative quantum chromodynamics (pQCD). With the large amount of data from the polarized scattering experiments listed above, the Bjorken sum rule was verified and the quark contribution towards a nucleon's spin was determined to be 30%±10% [15].

One condition of the Bjorken sum rule is that it only holds true at high energies. That is, the electron four-momentum transfer to the target nucleon must be large. Interactions at high momentum transfers are governed by perturbative QCD. For the case of the Bjorken sum rule, pQCD corrections are made for the fact that experiments were not conducted at infinite momentum transfer.

At low energies the validity of pQCD comes into question. No longer are interactions strictly between the electron and a constituent quark, as modeled at high energy. More of the nucleon's macroscopic structure is expected to play a role. This structure is complicated by the addition of sea quarks (quark-antiquark pairs) and gluons (carriers of the strong interaction). There is currently no way to predict or calculate accurately the spin structure functions at low energy without directly measuring them. The hope is that someday lattice QCD will be able to derive the nucleon structure functions from first principals.

Experiments in the low energy region can use the Gerasimov-Drell-Hearn (GDH) sum rule as a lighthouse to study the evolution of the nucleon spin structure functions:

$$\int_{thr}^{\infty}(\sigma_{1/2}-\sigma_{3/2})\frac{d\nu}{\nu}=-\frac{2\pi^2\alpha}{M^2}\kappa^2. \tag{1.2}$$

The GDH sum rule relates the difference in polarized cross sections ($\sigma_{1/2(3/2)}$), for the scattering of real photons, to the target's anomalous magnetic moment (κ), mass (M), and the electro-magnetic coupling constant (α). There is now a large effort to study the nucleon spin structure in the low energy region, much of it focused on testing the GDH sum rule. This thesis focuses on experiment E94-010 conducted at Jefferson Lab in Virginia. The experiment measured the neutron's spin structure functions in both the low and high energy regimes in order to provide data to connect the Bjorken and GDH sum rules. The remainder of this chapter deals with the formalism of polarized electron scattering and describes in detail the GDH and Bjorken sum rules.

1.2 Formalism

The purpose of the E94-010 experiment was to measure the extended Gerasimov-Drell-Hearn sum rule for ^{3}He and the neutron. From the detection of the scattered electrons with different polarization configurations, the polarized ^{3}He cross sections were determined. These were in turn used to calculate the GDH sum rule. The sum rule requires that data be recorded over a wide range of energies. Therefore, E94-010 measured the four principal regions of the cross section: the elastic, quasi-elastic, resonance, and deep inelastic regions (Figure 1.1).

These distinct regions are a consequence of the ^{3}He structure. The virtual photon probes the target and has a wavelength that depends upon its energy. The shorter the wavelength, the greater the resolution of the probe (Figure 1.2). At low energies the ^{3}He appears as one object. Scattering occurs elastically, leaving the ^{3}He nucleus intact. As the energy increases, the constituent nucleons are resolved and scattering can occur elastically from the protons or the neutron. This is called the quasi-elastic

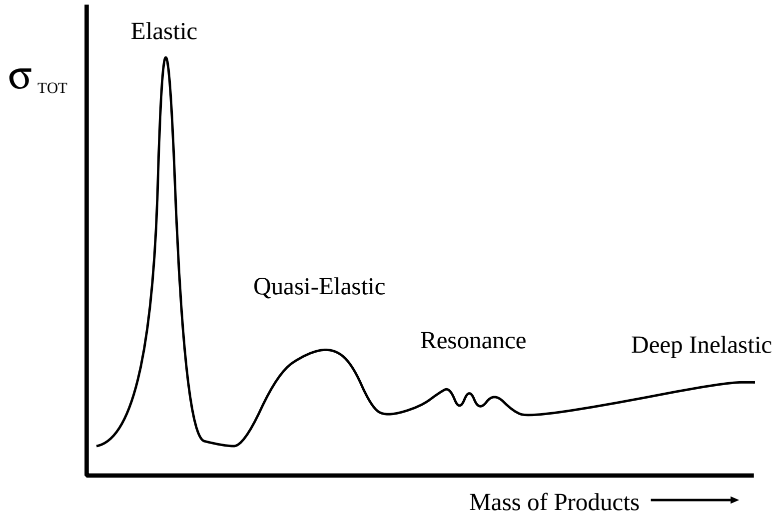

Figure 1.1: Schematic of the ^{3}He cross section. The relative peak heights change depending upon the momentum of the virtual photon.

regime. The cross-sectional peak is smeared due primarily to Fermi motion. Increasing the energy allows for the excitation of the nucleons. There are numerous types of resonances in this region. For example, a proton, having interacted with the scattered electron, becomes a Δ particle which then decays into a pion and a baryon. Finally, when the energy is high enough, the interaction probes the individual quarks. This is known as deep inelastic scattering.

The purpose of this chapter is to introduce the formalism used to describe the extended Gerasimov-Drell-Hearn sum rule. Since E94-010 measures polarized scattering cross sections, particular emphasis is placed on defining the scattering asymmetries and their relation to total cross sections in each region.

There are a number of ways to describe the kinematics of electron scattering by a target. In the experimental hall (lab frame) the three parameters measured are the electron's incident energy, E, the scattered electron's energy, E', and the scattering angle, θ (Figure 1.3). The electron's initial helicity and the target polarization are also known. These three lab variables are related to variables used when considering

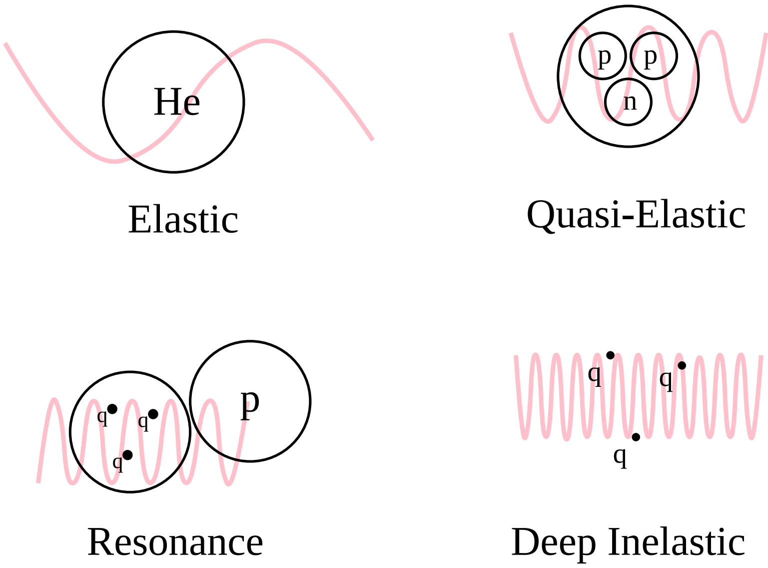

Figure 1.2: The increasing resolution of the ^{3}He nucleus. As the virtual photon energy increases so too does its ability to probe the internal structure of the ^{3}He nucleus and constituent nucleons.

the interaction of the electron and the target.

The interaction between the electron and the target can be described as an electron emitting a virtual photon, which in turn interacts with the target (bottom of Figure 1.3). The electron has an initial four vector k^μ and a final four vector k'^μ. The initial spin of the electron is s^μ and the final spin is not measured in inclusive scattering. For the target, the initial momentum and spin are P^μ and S^μ. The hadronic products have an invariant mass squared given by W^2. The virtual photon has a four vector given by $q^\mu = (\nu, \vec{q})$. Some useful quantities related to the lab frame variables are

$$\nu = E - E', \tag{1.3}$$

$$Q^2 = -q^\mu q_\mu = 4EE' \sin^2\frac{\theta}{2}, \tag{1.4}$$

$$|\vec{q}| = \sqrt{Q^2 + \nu^2}, \tag{1.5}$$

$$\cos\theta^* = -\frac{E - E'\cos\theta}{|\vec{q}|}, \tag{1.6}$$

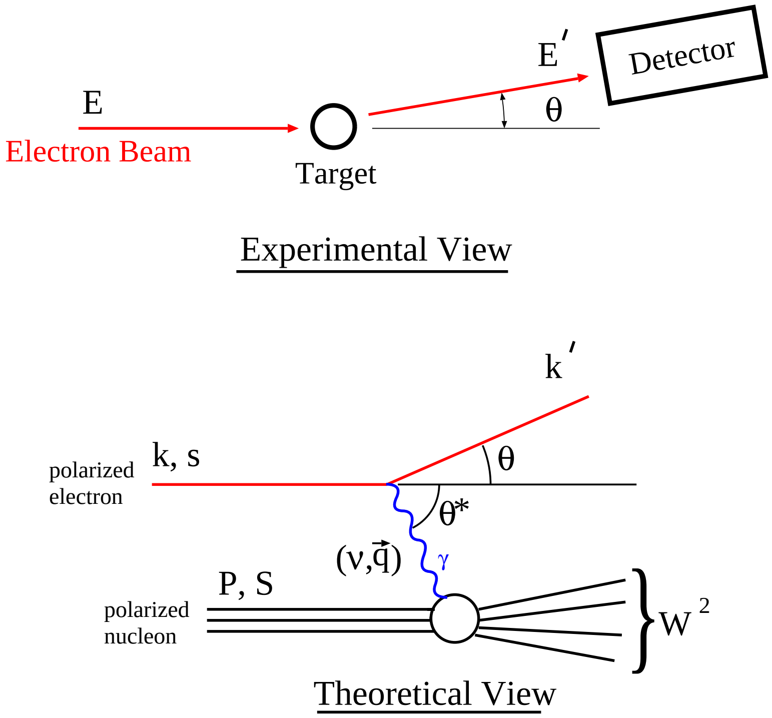

Figure 1.3: The scattering of an electron by a nucleus. The top picture shows scattering in the laboratory frame. The bottom picture depicts how scattering occurs as an exchange of a virtual photon.

$$W^2 = (q^\mu + P^\mu)^2 = M_T^2 + 2M_T\nu - Q^2, \tag{1.7}$$

$$x = \frac{Q^2}{2M_T\nu}, \tag{1.8}$$

$$\tau = \frac{Q^2}{4M_T^2}, \tag{1.9}$$

$$\epsilon = (1 + 2\frac{|\vec{q}|^2}{Q^2}\tan^2\frac{\theta}{2})^{-1}, \tag{1.10}$$

where M_T is the target mass (either the ^{3}He or nucleon mass), $Q^2 > 0$, θ^* is the angle between $\vec{k}$ and $\vec{q}$, x is related to the fractional momentum carried by the struck parton, $\tau > 0$, and ϵ is related to the virtual photon's polarization.

Now that some common variables have been defined, the particular formalism for each region is described.

Elastic Scattering

Following the formalism presented in [16], the differential cross section for elastic scattering ($Q^2 = 2M\nu$) of an electron by a nucleus or nucleon is given by

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\left[\left(F_1^2(Q^2) + \tau\kappa^2 F_2^2(Q^2)\right) + 2\tau(F_1(Q^2) + \kappa F_2(Q^2))^2 \tan^2\frac{\theta}{2}\right] \quad (1.11)$$

where κ is the anomalous magnetic moment in Bohr magnetons, F_1 and F_2 are the Dirac and Pauli form factors, and σ_M is the Mott cross section. The Mott cross section describes scattering of an electron from a point-like particle:

$$\sigma_M = \frac{\alpha^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}} \quad (1.12)$$

where $\alpha = \frac{1}{137}$ is the fine structure constant. The Dirac and Pauli form factors are functions that account for our lack of knowledge regarding the photon-hadron vertex. At $Q^2 = 0$ they have the following values for the proton and neutron: $F_1^p(0) = 1$, $F_2^p(0) = 1$, $F_1^n(0) = 0$, and $F_2^n(0) = 1$. Typically, for elastic scattering, a new set of form factors are defined:

$$G_E(Q^2) = F_1(Q^2) - \tau\kappa F_2(Q^2), \quad (1.13)$$

$$G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2). \quad (1.14)$$

These are known as the Sachs form factors, and they have a direct physical interpretation. If ν is small compared to the nucleon mass, then G_E is the Fourier transform of the charge density and G_M is related to the magnetization density. At $Q^2 = 0$, $G_E^p = 1$, $G_M^p = \mu_p$, $G_E^n = 0$, and $G_M^n = \mu_n$, where $\mu_p = 2.79$ and $\mu_n = -1.91$ in Bohr magnetons. In terms of the Sachs form factors, the elastic scattering cross section is [17]

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\frac{1}{\epsilon(1+\tau)}[\epsilon G_E^2 + \tau G_M^2]. \quad (1.15)$$

Because the target and initial electron spins are known, three polarized cross sections can be measured: the case when the spins are aligned, the case when they are anti-aligned, and the case when the target spin is perpendicular to the electron spin. By measuring the scattering rates for the different combinations, two asymmetries are measured:

$$A_{||} = \frac{d\sigma^{\downarrow\uparrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\downarrow\uparrow} - d\sigma^{\uparrow\uparrow}} \qquad \text{and} \qquad A_{\perp} = \frac{d\sigma^{\downarrow\leftarrow} - d\sigma^{\uparrow\leftarrow}}{d\sigma^{\downarrow\leftarrow} - d\sigma^{\uparrow\leftarrow}}. \tag{1.16}$$

The first arrow refers to the beam helicity and the second refers to the direction of the target's polarization. The up and down arrows represent spin directions parallel and anti-parallel to the beam axis. A left arrow means that the spins of the target are perpendicular to the beam momentum direction.

In the case of elastic scattering from ^{3}He these asymmetries are related to the ^{3}He form factors in the following way [18]:

$$A = -\frac{2\tau v_{T'} G_M^2 \cos\psi - 2v_{TL'}\sqrt{2\tau(1+\tau)}G_E G_M \sin\psi}{v_L(1+\tau)G_E^2 + 2v_T \tau G_M^2}, \tag{1.17}$$

where ψ is the angle between the target spin vector and $\vec{q}$. For $A_{||}$, $\psi = \theta^*$. For $A_{\perp}$, $\psi = \theta^* \pm 90°$. The quantities v_X are kinematic variables and are given by:

$$v_{T'} = \tan\frac{\theta}{2}\sqrt{\tan^2\frac{\theta}{2} + \frac{1}{1+\tau}}, \tag{1.18}$$

$$v_{TL'} = -\frac{1}{\sqrt{2}}\frac{1}{1+\tau}\tan\frac{\theta}{2}, \tag{1.19}$$

$$v_L = \left(\frac{1}{1+\tau}\right)^2, \tag{1.20}$$

$$v_T = \tan^2\frac{\theta}{2} + \frac{1}{1+\tau}. \tag{1.21}$$

Measured values for the ^{3}He electric and magnetic form factors can be found in [19][1].

[1] Dunn uses the form factors F_C and F_M which are related to G_E and G_M: $G_E = ZF_C$ and $G_M = \mu_{^3He}F_M$ and $F_C(0) = F_M(0) = 1$.

The size of $A_{||}$ can easily be estimated. For an incident electron energy of 0.86 GeV and a scattering angle of 15.5°, $|A_{||}| \approx 0.02\frac{G_M}{G_E}$. If the assumption is made that the form factors for ^{3}He are just the addition of proton and neutron form factors, then[2]

$$G_E^{He} = G_E^p + G_E^p + G_E^n \overset{Q^2 \to 0}{\approx} 1 + 1 + 0 = 2, \tag{1.22}$$

$$G_M^{He} = G_M^p + G_M^p + G_M^n \overset{Q^2 \to 0}{\approx} \mu_p - \mu_p + \mu_n = -1.913, \tag{1.23}$$

and $|A_{||}| \approx 2 \times 10^{-2}$ which is correct to within a factor of two.

Quasi-Elastic Scattering

Quasi-elastic scattering is essentially elastic scattering from a nucleus' constituent nucleons. The scattering cross section for a nucleon is given by Eq.(1.11). The quasi-elastic peak is not as sharp as the ^{3}He elastic peak. This is because the nucleons carry a large momentum dispersion due to their confinement in Δx ($\Delta p \Delta x \sim h$). The Fermi motion smears the peak [20]. As for the spin-dependent asymmetry in this region, the contributions from the two proton spins should cancel due to the Pauli exclusion principle. This dilutes the asymmetry contributed by the neutron. Dilution from the fact that the ^{3}He wavefunction is not only in a ground state also occurs, but the contribution to the asymmetry is small [21]. The spread due to Fermi motion also dilutes the asymmetry. Therefore, the measured quasi-elastic asymmetry is expected to be smaller than the elastic asymmetry for ^{3}He.

Resonance Region

When the energy of the virtual photon is large enough to distinguish the constituent quarks of a nucleon, but not large enough to scatter individually off quarks, resonant scattering can occur. In such an interaction the photon excites the nucleon's internal

[2]The magnetic moments of the two protons in ^{3}He should anti-align due to the Pauli Exclusion Principal. That's why the second μ_p has a negative sign.

constituents. Shortly after excitation (10^{-23}s) the excited nucleon decays into a nucleon and a meson (Figure 1.4). The lowest energy resonance is the Δ particle or P_{33} state (the 3's represent the (isospin, J) (3/2,3/2) state). Because of the $\frac{1}{\nu}$ weighting in the GDH integral (Eq. 1.63), this lowest energy resonance is the dominate contribution to the sum rule for the neutron.

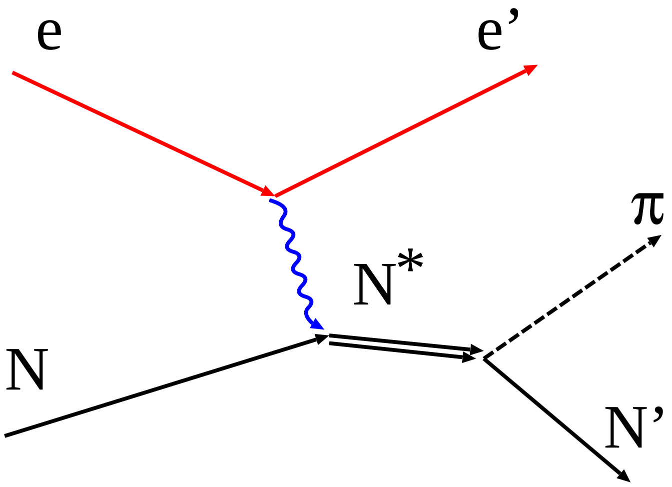

Figure 1.4: Resonance scattering. The virtual photon excites the nucleon (N) into an excited state (N*) which then decays into a nucleon (N') and a meson (π).

Polarized scattering in this regime is characterized by the total photo-absorption cross sections. These cross sections describe the interaction between the virtual photon and the target. The virtual photon can have transverse and longitudinal polarizations. This will yield four photo-absorption cross sections when the target has a spin of 1/2. In terms of |(initial photon projection), (initial target spin projection)> they are:

$$
\begin{aligned}
\sigma_{1/2} &\sim |(1,-1),(1/2,1/2)>, \\
\sigma_{3/2} &\sim |(1,1),(1/2,1/2)>, \\
\sigma_{L} &\sim |(1,0),(1/2,1/2)>, \\
\sigma_{TL} &\sim |(1,0),(1/2,1/2)>|(1,-1),(1/2,1/2)>.
\end{aligned}
$$

The total transverse photo-absorption cross section is $\sigma_T = \sigma_{1/2} + \sigma_{3/2}$, where $\sigma_{3/2(1/2)}$ is for the case when the photon and target spins are aligned(anti-aligned). The difference in these cross sections is $\sigma_{TT} = \sigma_{1/2} - \sigma_{3/2}$. The total longitudinal cross section is σ_L. There is also an interference term between the transverse and longitudinal interactions given by σ_{TL}.

Two virtual photon asymmetries can be formed from these cross sections:

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_T} \quad \text{and} \quad A_2 = \frac{\sigma_{LT}}{\sigma_T}. \tag{1.24}$$

The boundary on their values are [22]

$$-1 \leq A_1 \leq 1 \quad \text{and} \quad A_2 \leq \sqrt{\frac{\sigma_L}{\sigma_T}}. \tag{1.25}$$

The asymmetries A_1 and A_2 are related to $A_{||}$ and $A_\perp$ [23]:

$$A_{||} = D(A_1 + \eta A_2) \quad \text{and} \quad A_\perp = d(A_2 - \zeta A_1), \tag{1.26}$$

where

$$D = \frac{1 - E'\epsilon/E}{1 + \epsilon R}, \tag{1.27}$$

$$\eta = \epsilon \frac{\sqrt{Q^2}}{E - E'\epsilon}, \tag{1.28}$$

$$d = D\sqrt{\frac{2\epsilon}{1+\epsilon}}, \tag{1.29}$$

$$\zeta = \frac{\eta(1+\epsilon)}{2\epsilon}, \tag{1.30}$$

$$R = \frac{\sigma_L}{\sigma_T}. \tag{1.31}$$

If the incoming electron transfers all its energy, $D = 1$ and $d = 0$ and therefore $A_{||} = A_1$.

The P_{33} resonance, or Δ particle, is a proton (neutron) with both up (down) quark spins pointing in the same direction and with a total angular momentum of $L_z = 0$ for all three quarks. The P_{33} asymmetry can be predicted by examining the following transitions:

$$|u(\uparrow)u(\downarrow)d>_p \rightarrow |u(\uparrow)u(\uparrow)d>_\Delta \qquad \text{or} \qquad |d(\uparrow)d(\downarrow)u>_n \rightarrow |d(\uparrow)d(\uparrow)u>_\Delta . \tag{1.32}$$

Relating these transitions to their Clebsch-Gordan coefficients, the ratio $\sigma_{1/2}/\sigma_{3/2} = \frac{1}{3}$, and therefore $A_1 = -\frac{1}{2}$, which is in good agreement with the data [24].

The double polarized differential cross section in terms of the virtual photon asymmetries and photo-absorption cross sections is [25]

$$\frac{d^2\sigma}{d\Omega dE'} = \Gamma_T(\sigma_T + \epsilon\sigma_L \pm \sqrt{1-\epsilon^2}\cos\psi\sigma_T A_1 \pm \sqrt{2\epsilon(1-\epsilon^2)}\sin\psi\sigma_T A_2), \tag{1.33}$$

where Γ_T is the virtual photon flux:

$$\Gamma_T = \frac{\alpha}{4\pi Q^2}\frac{(W^2 - M^2)E'}{ME}\frac{1}{1-\epsilon}. \tag{1.34}$$

The unpolarized portion of the differential cross section $(\sigma_T + \epsilon\sigma_L)$ can be approximated by σ_T since the ratio σ_L/σ_T is fairly small in the P_{33} resonance region. This, in turn, is broken down into resonant and non-resonant contributions:

$$\sigma_T + \epsilon\sigma_L \approx \sigma_T = \sigma_R + \sigma_{NR}. \tag{1.35}$$

The non-resonant background is generally parameterized by a polynomial [25]:

$$\sigma_{NR} = \sqrt{W - W_{thr}}\sum_{i=0}^{m} a_i(W - W_{thr})^i, \tag{1.36}$$

where a_i are free parameters fit to the data and W_{thr} is the value of $\sqrt{W^2}$ at the threshold of Δ production. The resonant cross section for P_{33} can be expressed in terms of electric and magnetic form factors for the Δ and the total width of the resonance, Γ:

$$\sigma_R(P_{33}) = \frac{4\pi\alpha}{\Gamma}\frac{|\vec{q}|^2}{W(W^2-M^2)}(|G_M^\Delta|^2 + 3|G_E^\Delta|^2). \tag{1.37}$$

The cross section for P_{33} is strongly excited at Q^2 near zero, and G_M^Δ, the dominate form factor, falls off rapidly with increasing Q^2 [1]. This implies that the GDH sum rule will have a strong Q^2 dependence at low Q^2 (Figure 1.5).

Deep Inelastic Scattering

The scattering of electrons in the deep inelastic region is described by the contraction of two tensors. The electron virtual photon vertex is described by the lepton tensor $L_{\mu\nu}$ and the nucleon virtual photon vertex is described by the hadron tensor $W^{\mu\nu}$. Each is comprised of a spin-independent part and a spin-dependent part. The differential cross section is given by [20]:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2}{Q^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}. \tag{1.38}$$

Following the form of Hughes and Kuti [22], the lepton tensor, when the final spin state of the scattered electron is summed over, is given by

$$L_{\mu\nu} = \underbrace{k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu}k'\cdot k}_{spin-independent} + \underbrace{im_e\varepsilon_{\mu\nu\alpha\beta}s^\alpha q^\beta}_{spin-dependent}, \tag{1.39}$$

where $\varepsilon_{\mu\nu\alpha\beta}$=1 for even permutations of 1234, $g_{\mu\nu}$ is the metric tensor, and m_e is the mass of the electron. The hadron tensor is represented by

$$W^{\mu\nu} = W^{\mu\nu}_{(S)} + W^{\mu\nu}_{(A)}, \tag{1.40}$$

where

$$W^{\mu\nu}_{(S)} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1 + \frac{1}{M^2}\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)W_2 \tag{1.41}$$

and

$$W^{\mu\nu}_{(A)} = iM\varepsilon^{\mu\nu\alpha\beta}q_\alpha S_\beta G_1 + \frac{i}{M}\varepsilon^{\mu\nu\alpha\beta}q_\alpha[(P\cdot q)S_\beta - (S\cdot q)P_\beta]G_2. \tag{1.42}$$

Here, four new parameters have been introduced: W_1, W_2, G_1, and G_2. They are the inelastic structure functions, and they represent the interaction between the virtual photon and the nucleon. The first two, W_1 and W_2, characterize unpolarized scattering, and G_1 and G_2 are relevant for polarized scattering. All four of the structure functions depend on ν and Q^2 (not simply Q^2 as in the case of elastic scattering).

For unpolarized scattering, the inclusive cross section, when the final electron spin state is averaged over, is given by

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4}\left(\cos^2\frac{\theta}{2}W_2 + 2\sin^2\frac{\theta}{2}W_1\right). \tag{1.43}$$

Polarized scattering is characterized by measuring the asymmetries A_1 and A_2. The virtual photon cross sections have the following relations to the structure functions [20, 23]:

$$\begin{aligned}
\sigma_{3/2} \sim \quad & <1,1/2|1,1/2> && = W_1 + \nu M G_1 - Q^2 G_2, && (1.44)\\
\sigma_{1/2} \sim \quad & <1,-1/2|1,-1/2> && = W_1 - \nu M G_1 + Q^2 G_2, && (1.45)\\
\sigma_L \sim \quad & <0,1/2|0,1/2> && = (1+\frac{\nu^2}{Q^2})W_2 - W_1, && (1.46)\\
\sigma_{TL} \sim \quad & <1,1/2|0,1/2> && = \sqrt{Q^2}(MG_1 + \nu G_2). && (1.47)
\end{aligned}$$

When the scattering energies are sufficiently high (Q^2 large) the structure functions become dependent upon only one variable: x. This behavior is known as scaling, and it arises from the fact that the nucleon is comprised of point-like particles. The relationship between the general structure functions and those used when the scattering is in this regime are given by

$$\frac{1}{M^2}W_1(\nu,Q^2) = F_1(x), \qquad \frac{\nu}{M^3}W_2(\nu,Q^2) = F_2(x), \tag{1.48}$$

$$\frac{\nu}{M}G_1(\nu, Q^2) = g_1(x), \quad \text{and} \quad \frac{\nu^2}{M^2}G_2(\nu, Q^2) = g_2(x). \tag{1.49}$$

Furthermore, F_1 and F_2 are related to one another by a modification of the Callan-Gross relationship:

$$F_1(x) = F_2(x)\frac{1 + \frac{Q^2}{\nu^2}}{2x(1 + R)}. \tag{1.50}$$

The confirmation of this relationship indicates that the quarks have a spin of 1/2. As $Q^2 \to \infty$ for constant x, $R \to 0$, and $F_2 = 2xF_1$. The relation between the structure functions and the virtual photon asymmetries are given by

$$g_1 = \frac{F_1}{\left(1 + \frac{Q^2}{\nu^2}\right)}\left(A_1 + \sqrt{\frac{Q^2}{\nu^2}}A_2\right) \quad \text{and} \quad g_2 = \frac{F_1}{\left(1 + \frac{Q^2}{\nu^2}\right)}\left(\sqrt{\frac{\nu^2}{Q^2}}A_2 - A_1\right). \tag{1.51}$$

At high energies, the spin structure functions have a simple physical interpretation in the quark-parton model [22]. For F_1 and g_1, the interpretation is that they are equal to the sum over longitudinally polarized quark types:

$$F_1(x) = \frac{1}{2}\sum_i e_i^2[f_i^{\uparrow}(x) + f_i^{\downarrow}(x)], \tag{1.52}$$

$$g_1(x) = \frac{1}{2}\sum_i e_i^2[f_i^{\uparrow}(x) - f_i^{\downarrow}(x)], \tag{1.53}$$

where $f_i^{\uparrow(\downarrow)}(x)$ is the probability of finding a quark with its spin aligned (anti-aligned) with the nucleon's spin and with a fraction x of the nucleon's total momentum. The charge of the quark type is given by e_i.

1.3 Sum Rules

Theoretical knowledge of the spin structure functions is limited to integrals over some kinematic region (e.g., $\int_0^1 g_1(x)dx$). These types of integrals, or combinations thereof, are known as sum rules. Measurement of these sum rules allow for tests of their assumptions. Some of these sum rule studies represent fundamental tests of QCD.

1.3.1 Bjorken Sum Rule

In 1966 Bjorken [26] derived a sum rule using current algebra in the limit of $Q^2 \to \infty$ (Eq. 1.61). Integrating Eq. 1.53 over the full range of x gives, what is called, the first moment:

$$\Gamma_1 = \int_0^1 g_1(x)dx = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9} + \Delta d \frac{1}{9}\Delta s\right). \tag{1.54}$$

The quantities Δq are the helicity carried by each quark type. They can be calculated from the following matrix element:

$$2\Delta q s^\mu =< N|\overline{q}\gamma^\mu\gamma^5 q|N >, \tag{1.55}$$

where N represents the nucleon, γ^μ and γ^5 are the Dirac matrices. The Bjorken sum rule relates the first moment to the weak current matrix element for neutron decay:

$$< \bar{p}|J_w|n >=< \bar{p}|\tau^+\gamma^\mu(1-\gamma^5)|n >= \bar{U}\gamma^\mu\left(1 - \frac{g_A}{g_V}\right)U = 2\left(p^\mu - \frac{g_A}{g_V}s^\mu\right), \tag{1.56}$$

where U represents a Dirac spinor and τ is a Pauli matrix for isospin. Equation 1.56 can be converted to a matrix element between proton states:

$$< \overline{p}|\tau^+\gamma^\mu\gamma^5|n >=< \overline{p}|\tau^3\gamma^\mu\gamma^5|p > . \tag{1.57}$$

This, in turn, is related to the helicity carried by the quarks:

$$< \overline{p}|\tau^3\gamma^\mu\gamma^5|p >=< \overline{p}|\overline{u}\gamma^\mu\gamma^5 u - \overline{d}\gamma^\mu\gamma^5 d|p >= 2s^\mu(\Delta u_p - \Delta d_p). \tag{1.58}$$

Now, a portion of the matrix element in Eq. 1.56 is related to the quark helicities in Eq. 1.58:

$$< \overline{p}|\tau^+\gamma^\mu\gamma^5|n >= 2s^\mu\frac{g_A}{g_V} = 2s^\mu(\Delta u_p - \Delta d_p) \tag{1.59}$$

and the Bjorken sum rule is realized:

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}(\Delta u_p - \Delta d_p) = \frac{1}{6}\frac{g_A}{g_V}. \tag{1.60}$$

At finite Q^2, gluon interactions modify the sum rule. The modification is calculated using pQCD and amounts to a correction by a power series in $\alpha_s(Q^2)$:

$$\int_0^1 (g_1^p(x) - g_1^n(x))dx = \frac{1}{6}\frac{g_A}{g_V}\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right). \tag{1.61}$$

The strong coupling constant, $\alpha_s(Q^2)$, has the following form

$$\alpha_s(Q^2) = \frac{1}{\left(\frac{33-2f}{12\pi}\right)\ln Q^2/\Lambda^2}, \tag{1.62}$$

where f is the number of quark flavors, and Λ ($\sim$100 MeV) characterizes the strength of the strong coupling constant. Today the Bjorken sum rule corrections have been calculated up to third order in α_s [27].

1.3.2 Gerasimov-Drell-Hearn Sum Rule

In 1966, S.B. Gerasimov [28] and, independently, S.D. Drell and A.C. Hearn [29] derived a sum rule for the absorption of polarized real photons on a polarized nucleon. The sum rule takes the photon energy weighted difference of two spin dependent cross sections and relates it to the nucleon's anomalous magnetic moment:

$$\int_{thr}^{\infty} \frac{\sigma_{1/2} - \sigma_{3/2}}{\nu} d\nu = -\frac{2\pi^2\alpha}{M^2}\kappa^2. \tag{1.63}$$

Here, $\sigma_{3/2(1/2)}$ is the absorption cross section when the photon and nucleon spins are aligned (anti-aligned), ν is the photon energy, κ is the anomalous magnetic moment, α is the electromagnetic coupling constant, and M is the mass of the nucleon. The integration is carried out from the pion threshold ($\frac{Q^2+2m_\pi M+m_\pi^2}{2M}$) to infinity.

Derivation of the sum rule starts by using the results of Gell-Mann, Goldberger, and Thirring [30] for the forward scattering of photons. They derived the Compton scattering amplitude for a photon scattering at zero degrees:

$$f(\nu) = f_1(\nu)\vec{\epsilon}'^* \cdot \vec{\epsilon} + f_2(\nu) i\vec{\sigma} \cdot \vec{\epsilon}'^* \times \vec{\epsilon}. \tag{1.64}$$

Here $\vec{\epsilon}'^*$ and $\vec{\epsilon}$ are the final and incident photon polarization vectors, ν is the photon energy, and $\vec{\sigma}$ is the target spinor. The functions $f_1(\nu)$ and $f_2(\nu)$ are scalar invariants. The optical theorem relates the imaginary parts of $f_1(\nu)$ and $f_2(\nu)$ to the polarized cross sections $\sigma_{3/2}$ and $\sigma_{1/2}$:

$$\mathrm{Im} f_1(\nu) = \frac{\nu}{4\pi}\frac{\sigma_{3/2} + \sigma_{1/2}}{2} \quad \text{and} \quad \mathrm{Im} f_2(\nu) = \frac{\nu}{4\pi}\frac{\sigma_{3/2} - \sigma_{1/2}}{2}. \tag{1.65}$$

For the GDH sum rule, the interest is in the real part of $f_2(\nu)$. This real part can be related to the imaginary part using a non-subtractive dispersion relation [31]:

$$\mathrm{Re} f_2(\nu) = \frac{2}{\pi}\int_{thr}^{\infty} \frac{\nu' \mathrm{Im} f_2(\nu')}{\nu'^2 - \nu^2} d\nu'. \tag{1.66}$$

By differentiating with respect to ν and then letting $\nu \to 0$, Eq. 1.66 becomes

$$f_2'(0) = \frac{2}{\pi}\int_{thr}^{\infty} \frac{\mathrm{Im} f_2(\nu)}{\nu^2} d\nu. \tag{1.67}$$

In the same year, 1954, as Gell-Mann, Goldberger, and Thirring published their results for forward Compton scattering, F.E. Low [32] and Gell-Mann and Goldberger [33] derived a result for the scattering of low energy light by a system of spin $\frac{1}{2}$ particles. For the case of forward scattering in the low energy limit, they found that the derivative of the scattering amplitude was related to the anomalous magnetic moment:

$$f_2'(0) = \frac{\alpha}{2M^2}\kappa^2. \tag{1.68}$$

The Gerasimov-Drell-Hearn (GDH) sum is completed by taking Eqs. 1.68 and 1.65 and substituting them into Eq. 1.67.

Extended GDH Sum Rule

The GDH sum is only valid for real photons ($Q^2 = 0$). In the case of electron scattering we have virtual photons ($Q^2 \neq 0$). While there is no sum rule for virtual photons, we can construct a related integral by considering the difference of polarized cross sections:

$$\sigma_{TT} = \sigma_{1/2} - \sigma_{3/2} = \frac{8\pi^2\alpha}{M^3(\nu - Q^2/2M)}(M\nu G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2)). \tag{1.69}$$

The integral [34]

$$I(Q^2) = -\int_{thr}^{\infty} \frac{d\nu}{\nu}(\sigma_{1/2} - \sigma_{3/2}) \tag{1.70}$$

converges to the GDH sum rule result as $Q^2 \rightarrow 0$:

$$I(0) = -\frac{2\pi^2\alpha}{M^2}\kappa^2, \tag{1.71}$$

which is equal to $-204\mu b$ for protons, $-232.8\mu b$ for neutrons, and $-496\mu b$ for ^{3}He. For a ^{3}He GDH sum rule, integration begins just above the elastic peak and goes to infinity. For a neutron GDH sum rule, integration is from the pion threshold to infinity. The extent to which a polarized ^{3}He can be modeled as a polarized neutron and two unpolarized protons can be tested by integrating $\sigma_{TT'}$ for ^{3}He over the neutron range:

$$\underbrace{\int_{\nu>elastic}^{\infty} \frac{\sigma_{TT}^{^3He}(\nu, Q^2=0)}{\nu} d\nu}_{-496\mu b} = \underbrace{\int_{\nu>elastic}^{\nu=pion} \frac{\sigma_{TT}^{^3He}(\nu, Q^2=0)}{\nu} d\nu}_{-263.2\mu b} + \underbrace{\int_{\nu>pion}^{\infty} \frac{\sigma_{TT}^{^3He}(\nu, Q^2=0)}{\nu} d\nu}_{-232.8\mu b(\text{free neutron})} \tag{1.72}$$

1.3.3 Motivation for Measurement of Nucleon Spin Structure for Intermediate and Low Values of Q^2

The early motivation to study the spin structure functions in the low Q^2 regime was to study in more detail the 'proton spin crisis'. Anselmino and others [35] proposed that higher twist effects could alter the value of the Bjorken sum rule for finite Q^2 beyond pQCD corrections (Eq. 1.61). The size of the effect could be estimated using the GDH sum rule. By doing this, they found that the quark contribution to the Bjorken sum rule could be as high as 50%. Not only did the GDH sum rule bring the quark contribution up to something "reasonable", but it also suggested dramatic spin structure at intermediate values of Q^2 (.02 $\leftrightarrow$ 1 GeV^2). With measurements taken after EMC, the Bjorken sum rule for finite Q^2 is entirely accounted for using pQCD corrections implying that higher twist effects are small. However, dramatic spin structure effects at low energy are still possible.

Polarized data in this region ($Q^2 < 1$) is nonexistent. For the extended GDH sum rule, unpolarized data in the resonance region has been used to try and understand its Q^2 evolution. Because of the $1/\nu$ weighting, the largest contribution to the neutron is expected to come from the P_{33} resonance (Figure 1.5).

Current interest in the intermediate region has been spurred on by the development of a generalized GDH sum rule proposed by Ji and Osborne [36]. This sum rule relates the spin structure function $G_1(\nu, Q^2)$ to the forward virtual Compton scattering amplitude $S_1(\nu, Q^2)$:

$$4\int_{el}^{\infty} \frac{G_1(\nu, Q^2)}{\nu} d\nu = S_1(0, Q^2). \tag{1.73}$$

At $Q^2 = 0$, Eq. 1.73 is related to the GDH sum rule:

$$S_1(\nu, 0) \overset{\nu\to 0}{\longrightarrow} -\frac{\kappa^2}{M^2}. \tag{1.74}$$

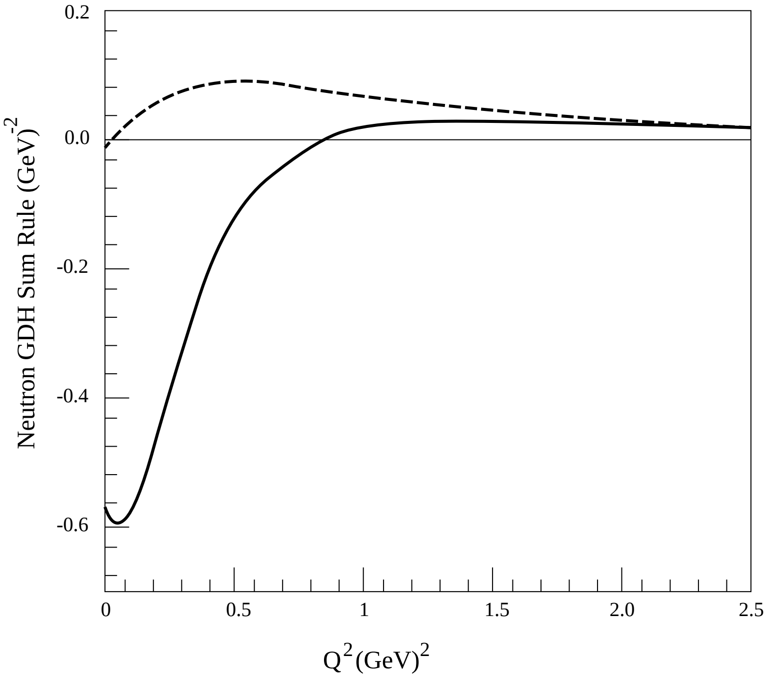

Figure 1.5: The extended GDH sum rule as presented in [1]. The solid line is the integral $I^n(Q^2)$ with the P_{33} resonance. The long-dashed line is without the P_{33} contribution.

For $Q^2 \to \infty$, Eq. 1.73 is connected to the Bjorken sum rule by the use of the dispersion relation for $S_1(\nu, Q^2)$:

$$S_1^{(p)}(0, Q^2) - S_1^{(n)}(0, Q^2) = \frac{4}{3Q^2}\frac{g_A}{g_V}. \tag{1.75}$$

The forward virtual Compton Scattering amplitude can be extended from $Q^2 = 0$ using chiral perturbation theory and from $Q^2 \to \infty$ using Operator Product Expansion. These theoretical predictions can be tested against the measured value of $G_1(\nu, Q^2)$.

Experiment E94-010 marked the first time the spin structure functions will have been measured for ^{3}He and the neutron in the low and intermediate energy regimes.