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Perturbative QCD study of the polarized structure function g_1 at Next-to-Leading-Order

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Abstract

We report on a NLO perturbative study of all available data on polarized structure function $g_1(x, Q^2)$. We first explain how the Q^2 evolution was performed using the DGLAP equations, which used with the present data, allow to constrain the behaviour of polarized parton distributions. We emphasize all along on the very strong dependency of the result upon the initial assumptions made to fit the data. Finally, we discuss implications of such a study and tentatively derive important quantities such as an estimation of the quark and gluon polarizations in the nucleon.

1 Introduction

The study of the internal structure of the nucleon has received a definite attention lately, both unpolarized with the recent HERA measurements and polarized with the new results from SLAC (E154 and forthcoming results from E155) which dramatically improved the measurements of g_1 on neutron, proton and deuteron. Extensive QCD analysis of the unpolarized data has been performed at Next-to-Leading Order (NLO) and a fairly good level of accuracy has been reached testing the different theoretical approaches.

However, the polarized case requires more work and attention for several reasons. The first being that no experiment so far has provided us with enough data at small enough x. The immediate consequence is that the data just doesn't constrain the behaviours of parton distributions such as the glue or the sea, which seem to dominate at small x. Also, the NLO evolution kernel has been calculated only recently, allowing the polarized case to have the same accuracy as the unpolarized case.

We will first explain the method used to perform the NLO perturbative analysis of the polarized data. We will pay particular attention to the determination and handling of errors, statistical, systematic and theoretical. Among the various systematic checks, we will choose different initial conditions, such as the input parton distributions. Also, we will study the effects of several theoretical assumptions giving additional contraints on the fit to polarized data. Finally, we will tentatively estimate the first moments of the parton distributions, focusing on the quark and gluon contributions to the nucleon spin.

2 The Q^2 evolution

2.1 Parametrization of initial polarized parton distributions

In order to perform a Q^2 evolution of the polarized structure function g_1, one has to start with a parametrization of the polarized parton distributions at some starting scale Q_0^2.

There is no way to predict the shape of the polarized parton distributions, hence one has to choose a reasonable and data compatible parametrization. Following the arguments of GRSV [1], we parametrize the polarized parton distributions at some low scale Q_0^2 as a function of the corresponding unpolarized distributions, accurately constrained using the unpolarized data:

$$
\begin{aligned}
\delta u_v(x,Q_0^2) &= N_{u_v}\, x^{a_{u_v}}\, u_v(x,Q_0^2) && (1)\\
\delta d_v(x,Q_0^2) &= N_{d_v}\, x^{a_{d_v}}\, d_v(x,Q_0^2) && (2)\\
\delta \bar q(x,Q_0^2) &= (\delta\bar d+\delta\bar u)/2 = N_{\bar q}\, x^{a_{\bar q}}(1-x)^{b_{\bar q}}\, \bar q(x,Q_0^2) && (3)\\
\delta \Delta(x,Q_0^2) &= (\delta\bar d-\delta\bar u)/2 = N_{\Delta}\, x^{a_{\Delta}}(1-x)^{b_{\Delta}}\, \Delta(x,Q_0^2) && (4)\\
\delta s(x,Q_0^2) &= \delta\bar s(x,Q_0^2) = N_s\, \delta\bar q(x,Q_0^2) && (5)\\
\delta g(x,Q_0^2) &= N_g\, x^{a_g}(1-x)^{b_g}\, g(x,Q_0^2) && (6)
\end{aligned}
$$

where $u_v, d_v, \bar u, \bar d, \bar q, s$ and $\bar s$ stand for the unpolarized parton distributions. This parametrization has 14 real parameters[1]:

$$N_{u_v},\ N_{d_v},\ N_{\bar q},\ N_{\Delta},\ N_s,\ N_g,\ a_{u_v},\ a_{d_v},\ a_{\bar q},\ a_{\Delta},\ a_s,\ a_g,\ b_{\bar q},\ b_{\Delta} \text{ and } b_g$$

In order to have the complete set of parametrizations, one has to choose a set of unpolarized parton distributions at the same initial scale Q_0^2. One possible choice is the unpolarized GRV parametrization [2], which gives a very good fit to the unpolarized data at any scale from the initial scale $Q_0^2 = 0.34$ GeV^2. The only inconvenient of such a choice is the very low value of $\alpha_s(M_z)$ chosen by this group at that time, 0.109 against 0.119 from the latest reports [3]. To test the dependence of the fit upon the choice of the unpolarized parametrization, we will also use the MRS unpolarized parametrization [4], using their fit corresponding to $\alpha_s(M_z) = 0.120$, very close to the current world average. Their initial scale value is $Q_0^2 = 1$ GeV^2.

- Unpolarized GRV parton distributions[2] at $Q_0^2 = 0.34$ GeV^2:

$$
\begin{aligned}
xu_v(x,Q_0^2) &= 0.988\, x^{0.543}(1+1.58\sqrt{x}+2.58\,x+18.1\,x^{3/2})(1-x)^{3.380} && (7)\\
xd_v(x,Q_0^2) &= 0.182\, x^{0.316}(1+2.51\sqrt{x}+25\,x+11.4\,x^{3/2})(1-x)^{4.113} && (8)\\
x\bar q(x,Q_0^2) &= (\bar d+\bar u)/2 = 0.545\, x^{0.30}(1+2.65\,x)(1-x)^{8.33} && (9)\\
x\Delta(x,Q_0^2) &= (\bar d-\bar u)/2 = 0.0263\, x^{0.381}(1+15.2\,x+132\,x^{3/2})(1-x)^{8.65} && (10)\\
xg(x,Q_0^2) &= 26.2\, x^{1.9}(1-x)^{4.0} && (11)\\
xs(x,Q_0^2) &= x\bar s(x,Q_0^2) = 0 && (12)
\end{aligned}
$$

[1] We do not assume a light sea isospin symmetry ($\delta\bar u = \delta\bar d$) at the initial scale.

[2] Let us point out that GRV chooses $s(x,Q_0^2) = x\bar s(x,Q_0^2) = 0$ and $\delta s(x,Q_0^2) = \delta\bar s(x,Q_0^2) \neq 0$, which seems contradictory with the positivity constraint $|\delta f| \leq f$. However, this inequality is satisfied for perturbatively relevant $Q^2 \geq 0.8$ GeV^2. According to our polarized parametrization choice, it is clear that the assumption $s(x,Q_0^2) = x\bar s(x,Q_0^2) = 0$ has no impact on the radiative generation of the polarized distributions using experimental data measured at $Q^2 \geq 1$ GeV^2.

- Unpolarized MRS parton distributions[3] at $Q_0^2 = 1$ GeV^2:

$$\begin{aligned}
xu_v(x,Q_0^2) &= A_{u_v}\, x^{0.61}(1-x)^{3.54}(1-0.98\sqrt{x}+6.51\,x) && (13)\\
xd_v(x,Q_0^2) &= A_{d_v}\, x^{0.24}(1-x)^{4.21}(1+7.37\sqrt{x}+29.9\,x) && (14)\\
x\bar{q}(x,Q_0^2) &= (\bar{d}+\bar{u})/2 = 0.07\,x^{-0.15}(1-x)^{8.27}(1+1.13\sqrt{x}+14.4\,x) && (15)\\
x\Delta(x,Q_0^2) &= (\bar{d}-\bar{u})/2 = 0.018\,x^{0.3}(1-x)^{8.27}(1+64.9\,x) && (16)\\
xg(x,Q_0^2) &= 14.4\,x^{0.51}(1-x)^{5.51}(1-4.20\sqrt{x}+6.47\,x) && (17)
\end{aligned}$$

in which the normalization coefficients A_{u_v} and A_{d_v} of the valence distributions are calculated using the valence quark counting rules in the proton:

$$\int_0^1 u_v(x,Q_0^2)\,dx = 2 \tag{18}$$

$$\int_0^1 d_v(x,Q_0^2)\,dx = 1 \tag{19}$$

2.2 The DGLAP evolution kernel at NLO

The DGLAP equations allow one to evolve the parton distributions, either polarized or not, from an initial Q^2 to another. The evolution equations are perturbative by nature, hence they have to be used at a perturbative scale [1, 2], typically more than a few hundred MeV. Practically speaking, the experimental groups usually don't use data below 1 GeV^2 and we will assume it is the perturbative region already. This hypothesis is well verified in the unpolarized case where perturbative predictions are valid at low scales. The formulation of the DGLAP equations depends on the flavor singlet or non-singlet nature of the distribution combination to evolve:

- **Non-singlet (NS) parton distribution evolution**

$$\begin{aligned}
\frac{d}{d\ln Q^2}(\delta q+\delta\bar{q}-\delta q'-\delta\bar{q}') &= \frac{\alpha_s}{2\pi}(\delta P_{qq}+\delta P_{q\bar{q}})\otimes(\delta q+\delta\bar{q}-\delta q'-\delta\bar{q}') && (20)\\
\frac{d}{d\ln Q^2}(\delta q-\delta\bar{q}) &= \frac{\alpha_s}{2\pi}(\delta P_{qq}-\delta P_{q\bar{q}})\otimes(\delta q-\delta\bar{q}) && (21)
\end{aligned}$$

where $\delta q = \delta q_{valence} + \delta q_{sea}$ and $\delta\bar{q} = \delta\bar{q}_{sea}$ for $q = u, d$. α_s is the QCD strong coupling constant, evolved at NLO using the parametrization given in appendix A. The convolution product $\otimes$ of two parton distributions f et g is defined as:

$$(f\otimes g)(x,Q^2) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z},Q^2\right) g(z,Q^2) \tag{22}$$

[3] The MRS group does not parametrize the unpolarized strange quark distribution; it has no impact on our parametrization of the polarized strange quark distribution.

- **Singlet (S) parton distribution evolution**

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\delta\Sigma\\ \delta G\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}\delta P_{qq}+\delta P_{q\bar{q}}+\delta P_{qq,PS} & \delta P_{qg}\\ \delta P_{gq} & \delta P_{gg}\end{pmatrix}\otimes\begin{pmatrix}\delta\Sigma\\ \delta G\end{pmatrix} \tag{23}$$

with $\delta\Sigma = \sum_q(\delta q + \delta\bar{q})$. The parton distributions depend on x and Q^2.

δP_{ij} are the polarized splitting functions and have recently been calculated at next-to-leading order [5, 6].

2.3 Evolution in the Mellin space

Rather than using the DGLAP equations as they are, which requires solving the differential equations by a small-step integration method in the "Bjorken-x" space, it is convenient to work in the so-called Mellin n-moment space in which the parton distributions f are represented by their n^{th} order moment defined as:

$$\delta f(n) = \int_0^1 dx\, x^{n-1}\delta f(x) \tag{24}$$

Then the parton distributions depend on n and Q^2. The unique advantage of such a variable change is that it leads to a complete analytical solution of the DGLAP equations, and needs no integration. For instance, the convolution product of two functions can be written in the Mellin space as the simple product of both functions:

$$(f\otimes g)(n) = \int_0^1 dx\, x^{n-1}(f\otimes g)(x) = f(n)g(n) \tag{25}$$

After evolution, the solutions are transformed back to the "Bjorken-x" space using a simple numerical integration: [1].

We have calculated the n^{th} order moments of the polarized parton distributions using the FORTRAN 90 version of the "Uncomplete Gamma Functions" $B_1(p,q)$ defined as:

$$B_1(p,q) = \int_0^1 dt\, t^{p-1}(1-t)^{q-1} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \tag{26}$$

in which p and q are complex numbers and $\Gamma(p)$ is the Gamma function with a complex argument evaluated using the CERNLIB function WLGAMMA [7].

The polarized parton distributions are evolved from the initial scale Q_0^2 to the final Q^2 using the DGLAP equations written in the Mellin space:

- **Non-singlet (NS) parton distributions evolution**

$$\begin{aligned}
\delta q^{\eta}_{NS}(Q^2) &= \left[1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\left(\frac{\delta\gamma^{(1)\eta}_{NS}}{2\beta_0}-\frac{\beta_1\,\delta\gamma^{(0)}_{qq}}{2\beta_0^2}\right)\right] \\
&\cdot\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\delta\gamma^{(0)}_{qq}/(2\beta_0)}\delta q^{\eta}_{NS}(Q_0^2)
\end{aligned} \tag{27}$$

The η coefficient is either equal to $+1$ or -1.

- $\eta = +1$ is used for the combinations $\delta u - \delta\bar{u} = \delta u_v$ and $\delta d - \delta\bar{d} = \delta d_v$.
- $\eta = -1$ is used for the combinations $(\delta u + \delta\bar{u}) - (\delta d + \delta\bar{d}) = \delta q_3$ and $(\delta u + \delta\bar{u}) + (\delta d + \delta\bar{d}) - 2(\delta s + \delta\bar{s}) = \delta q_8$.

The functions $\delta\gamma_{ij}$ are called polarized anomalous dimensions. They are the Mellin space expression of the polarized splitting functions, and are linked to them by the following expressions:

$$\delta\gamma^{(0)}_{ij}(n) = -4\int_0^1 dx\, x^{n-1}\delta P^{(0)}_{ij}(x) \tag{28}$$

$$\delta\gamma^{(1)}_{ij}(n) = -8\int_0^1 dx\, x^{n-1}\delta P^{(1)}_{ij}(x) \tag{29}$$

The exponent $^{(0)}$ stands for the leading order term while the exponent $^{(1)}$ stands for the next-to-leading order term. The full expressions can be found in the appendix B.

The β_0 and β_1 coefficients are given as a function of the number of active flavors f:

$$\beta_0 = 11 - 2f/3 \text{ and } \beta_1 = 102 - 38f/3 \tag{30}$$

f is chosen to be 3 for the computation of the anomalous dimensions. Then, $\beta_0 = 9$ and $\beta_1 = 64$.

- **Singlet (S) parton distributions evolution**

Let us define the (2×1) vector:

$$\Gamma(Q^2) = \begin{pmatrix}\delta\Sigma(Q^2)\\ \delta g(Q^2)\end{pmatrix} \tag{31}$$

We can now write the corresponding evolution equation:

$$\Gamma(Q^2) = \left\{\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\lambda_-/(2\beta_0)}\left[\boldsymbol{\mathcal{P}}_- - \frac{1}{2\beta_0}\frac{\alpha_s(Q_0^2)-\alpha_s(Q^2)}{4\pi}\boldsymbol{\mathcal{P}}_-\boldsymbol{\gamma}\boldsymbol{\mathcal{P}}_+\right.\right.$$

$$
- \left(\frac{\alpha_s(Q_0^2)}{4\pi} - \frac{\alpha_s(Q^2)}{4\pi} \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)} \right)^{(\lambda_+ - \lambda_-)/(2\beta_0)} \right) \frac{\boldsymbol{\mathcal{P}}_- \boldsymbol{\gamma} \boldsymbol{\mathcal{P}}_+}{2\beta_0 + \lambda_+ - \lambda_-} \Bigg]
$$

$$
+(+ \Leftrightarrow -) \Bigg\} \Gamma(Q_0^2) \tag{32}
$$

where $(+ \Leftrightarrow -)$ stands for the exchange of the $+$ and $-$ indices.

The (2×2) matrix $\boldsymbol{\gamma}$ is defined as:

$$
\boldsymbol{\gamma} \;=\; \boldsymbol{\gamma}^{(1)} - \frac{\beta_1}{\beta_0} \boldsymbol{\gamma}^{(0)} \tag{33}
$$

and the (2×2) matrices as:

$$
\boldsymbol{\gamma}^{(p)} = \begin{pmatrix} \delta\gamma_{qq}^{(p)} & \delta\gamma_{qg}^{(p)} \\ \delta\gamma_{gq}^{(p)} & \delta\gamma_{gg}^{(p)} \end{pmatrix}, \; p = 0, 1 \tag{34}
$$

where $p = 0$ stands for the matrix expression at LO, while $p = 1$ is at NLO.

The matrices $\boldsymbol{\mathcal{P}}_\pm$ and the eigenvalues $\lambda_\pm$ of the $\boldsymbol{\gamma}$ matrix are given by:

$$
\boldsymbol{\mathcal{P}}_\pm \;=\; \pm \frac{1}{\lambda_+ - \lambda_-} \left(\boldsymbol{\gamma}^{(0)} - \lambda_\mp \boldsymbol{\mathcal{I}} \right) \tag{35}
$$

$$
\lambda_\pm \;=\; \frac{1}{2} \left(\delta\gamma_{qq}^{(0)} + \delta\gamma_{gg}^{(0)} \pm \sqrt{(\delta\gamma_{gg}^{(0)} - \delta\gamma_{qq}^{(0)})^2 + 4\delta\gamma_{qg}^{(0)} \delta\gamma_{gq}^{(0)}} \right) \tag{36}
$$

where $\boldsymbol{\mathcal{I}}$ is the (2×2) identity matrix .

2.4 Back to the "Bjorken-x" space

In order to obtain the results in the "Bjorken-x" space, it is necessary to Mellin-invert the moments of the polarized parton distributions once evolved in the Mellin space. This is done using a complex integral:

$$
\delta f(x, Q^2) = \frac{1}{\pi} \int_0^\infty dz \mathrm{Im} \left(e^{i\phi} x^{-c - z e^{i\phi}} \delta f(n = c + z e^{i\phi}, Q^2) \right) \tag{37}
$$

where n is now a complex number. The upper integration limit is chosen to be $5 + 10/\ln(1/x)$ if $x \leq 0.4$ and 45 otherwise. A possible choice for the constants is $c = 1.8$ and $\phi = 1.9$ which allows for stable results at high x. The integral is evaluated using the N point Gauss-Legendre quadratic method. Practically speaking, we have used the CERNLIB function RGQUAD [8] with $N = 24$ points, this choice leading to the optimal calculation time for the needed result accuracy.

3 Validation of the evolution code

The GRSV group provided us with a table which allowed us to compute the values of the polarized parton distributions at any x and Q^2 from their parametrization at $Q_0^2 = 0.34$ GeV^2:

$$\begin{aligned}
\delta u_v(x, Q_0^2) &= 0.6586\, x^{0.18}\, u_v(x, Q_0^2) && (38)\\
\delta d_v(x, Q_0^2) &= -0.3392\, d_v(x, Q_0^2) && (39)\\
\delta \bar{q}(x, Q_0^2) &= \delta \bar{d} = \delta \bar{u} = -0.525\, x^{0.51}\, \bar{q}(x, Q_0^2) && (40)\\
\delta s(x, Q_0^2) &= \delta \bar{s}(x, Q_0^2) = \delta \bar{q}(x, Q_0^2) && (41)\\
\delta g(x, Q_0^2) &= 10.47\, x^{1.1} (1-x)^{4.3}\, u_v(x, Q_0^2) && (42)
\end{aligned}$$

Choosing the same initial parametrizations[4], we can compare the polarized parton distributions, evolved for example to $Q^2 = 100$ GeV^2 using our code with the values given by their table at $Q^2 = 100$ GeV^2. The comparison is shown on figure 1, and demonstrates a perfect agreement between both evolution codes.

Our code evolves directly the non-singlet distributions δu_v, δd_v, δq_3 and δq_8 and the singlet distributions $\delta\Sigma$ and δg, where:

$$\begin{aligned}
\delta q_3 &= \delta u_v - \delta d_v - 4\delta\Delta && (43)\\
\delta q_8 &= \delta u_v + \delta d_v + 4\delta\bar{q} - 4\delta\bar{s} && (44)\\
\delta\Sigma &= (4 + 2N_S)\delta\bar{q} + \delta u_v + \delta d_v && (45)
\end{aligned}$$

After evolution, those combinations give us the distributions $\delta\bar{q}$, $\delta\Delta$ and $\delta\bar{s}$ using the relations:

$$\begin{aligned}
\delta\bar{q} &= \frac{1}{12}(2\delta\Sigma + \delta q_8 - 3\delta u_v - 3\delta d_v) && (46)\\
\delta\Delta &= \frac{1}{4}(\delta q_3 - \delta u_v - \delta d_v) && (47)\\
\delta\bar{s} &= \delta s = \frac{1}{6}(\delta\Sigma - \delta q_8) && (48)
\end{aligned}$$

[4]The GRSV group assumes the light sea isospin symmetry at $Q_0^2 = 0.34$ GeV^2.

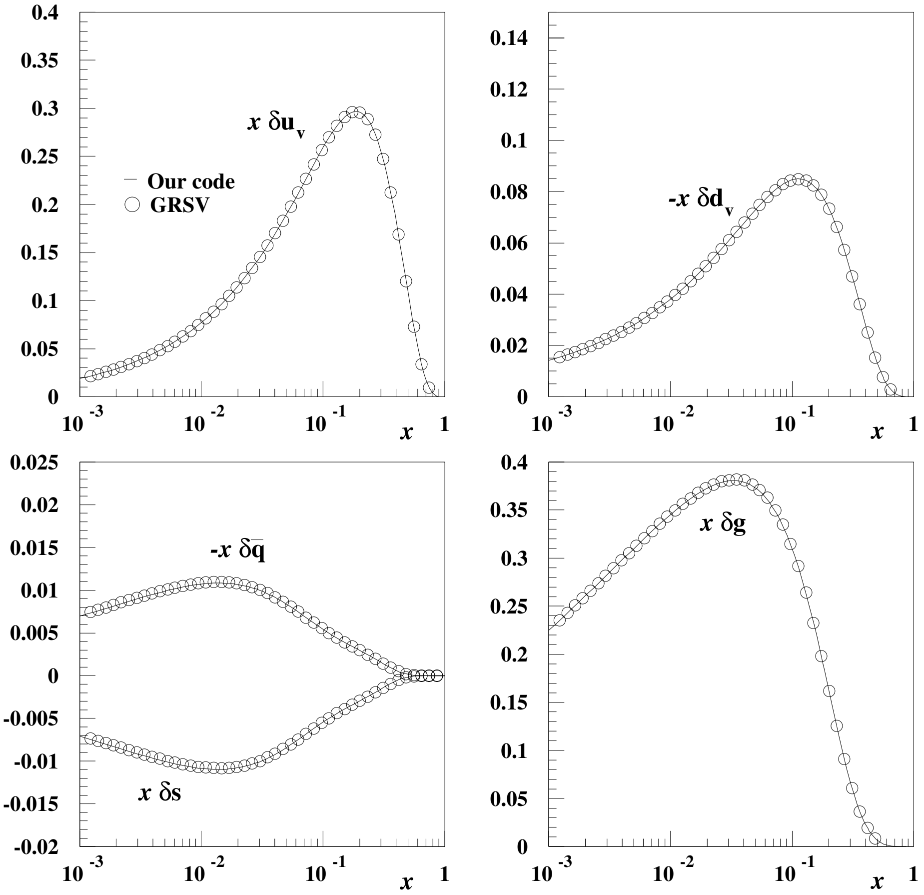

Figure 1: *Comparison of the polarized distributions evolved to $Q^2 = 100\ GeV^2$. The circles show the results given by our code and the solid lines show the results of GRSV's table.*

4 Fitting the polarized distributions using the experimental data

We will describe here how the experimental measurements of the nucleon structure function g_1 allow us to estimate the value of the 14 parameters used in the expressions of the polarized parton distributions, as defined in 2.1.

4.1 Evolution of the g_1 structure function

The structure function g_1 can be directly determined using the polarized parton distributions and using the Wilson coefficients, at any scale Q^2. The principle of the evolution method is rather simple: after evolving the polarized distributions to the desired scale in the Mellin space, the structure functions g_1^p for the proton, g_1^n for the neutron and g_1^d for the deuteron are built in this space using the following expressions:

$$g_1^p = \left[\frac{1}{12}\delta q_3 + \frac{1}{36}\delta q_8 + \frac{1}{12}\delta\Sigma\right]\left[1 + \frac{\alpha_s}{2\pi}\delta C_q^{(1)}\right] + \frac{1}{9}\frac{\alpha_s}{2\pi}\delta C_g^{(1)}\delta g \tag{49}$$

$$g_1^n = \left[-\frac{1}{12}\delta q_3 + \frac{1}{36}\delta q_8 + \frac{1}{12}\delta\Sigma\right]\left[1 + \frac{\alpha_s}{2\pi}\delta C_q^{(1)}\right] + \frac{1}{9}\frac{\alpha_s}{2\pi}\delta C_g^{(1)}\delta g \tag{50}$$

$$g_1^d = \frac{1}{2}\left(\left[\frac{1}{18}\delta q_8 + \frac{2}{9}\delta\Sigma\right]\left[1 + \frac{\alpha_s}{2\pi}\delta C_q^{(1)}\right] + \frac{2}{9}\frac{\alpha_s}{2\pi}\delta C_g^{(1)}\delta g\right)\left(1 - \frac{3}{2}\omega_D\right) \tag{51}$$

where $\omega_D = 0.05$ stands for the deuteron probability to be in the D state [9].

The Wilson coefficients $\delta C_q^{(1)}$ and $\delta C_g^{(1)}$ at NLO in the Mellin space and in the $\overline{\mathrm{MS}}$ scheme are expressed as in [6]:

$$\begin{aligned}\delta C_q^{(1)} &= C_F\left[-S_2(n) + S_1(n)\left(S_1(n) + \frac{3}{2} - \frac{1}{n(n+1)}\right)\right. \\ &\quad \left. + \frac{1}{n^2} + \frac{1}{2n} + \frac{1}{n+1} - \frac{9}{2}\right] \end{aligned} \tag{52}$$

$$\delta C_g^{(1)} = 2T_f\left[-\frac{n-1}{n(n+1)}\left(S_1(n) + 1\right) - \frac{1}{n^2} + \frac{2}{n(n+1)}\right] \tag{53}$$

The C_F and T_f coefficients and the S_1 function are given in appendix B.

4.2 Principle of the code used to constrain the parameters and determination of their value

The value of each parameter is calculated by minimizing the χ^2 between the experimental measurements of g_1 and the corresponding theoretical estimation from the polarized parton distributions evolved to the experimental Q^2. The χ^2 function to minimize depends on the 14 parameters and can be written as:

$$\chi^2 = \sum_{i,j}(g^i_{1,exp}(x_i, Q_i^2) - g^i_{1,NLO}(x_i, Q_i^2))V^{-1}_{i,j}(g^j_{1,exp}(x_j, Q_j^2) - g^j_{1,NLO}(x_j, Q_j^2)) \tag{54}$$

where the double summation on the i, j indices is performed on all the experimental points. $g_{1,exp}$ stands for an experimental measurement of g_1 (proton, neutron or deuteron) at its mesured x and Q^2 values; $g_{1,NLO}$ represents the g_1 NLO prediction for the same values of x and Q^2. This prediction is calculated using the polarized distributions evolved from Q_0^2 to the experimental Q^2 and depends on the value of the 14 parameters. V^{-1}_{ij} stands for the element located at row i and at column j of the inversed error matrix, also called covariance matrix.

The covariance matrix has been calculated taking into account both statistical and systematic errors on the experimental measurements of g_1. The systematic errors on the g_1 measurements which belong to the same experiment are assumed to be correlated at 100%, because the largest contribution to the systematic error comes from 100% correlated factors, such as the beam and the target polarizations and the dilution factor. We will detail briefly the method used to build the error matrix. Let us consider two experiments, quoted (I) and (II). Each of them has made two measurements $g^a_{1,exp}$, $g^b_{1,exp}$ for experiment (I) and $g^c_{1,exp}$, $g^d_{1,exp}$ for experiment (II). Each measurement has a statistical error quoted σ_{stat} and a systematic error quoted σ_{syst}. The error matrix V is calculated by adding a purely statistical diagonal matrix to a purely systematic matrix which has a zero determinant [16]:

$$
\begin{aligned}
V \;=\; & \begin{pmatrix} (\sigma^a_{stat})^2 & 0 & 0 & 0 \\ 0 & (\sigma^b_{stat})^2 & 0 & 0 \\ 0 & 0 & (\sigma^c_{stat})^2 & 0 \\ 0 & 0 & 0 & (\sigma^d_{stat})^2 \end{pmatrix} \\
& + \begin{pmatrix} (\sigma^a_{syst})^2 & \sigma^a_{syst}\sigma^b_{syst} & 0 & 0 \\ \sigma^a_{syst}\sigma^b_{syst} & (\sigma^b_{syst})^2 & 0 & 0 \\ 0 & 0 & (\sigma^c_{syst})^2 & \sigma^c_{syst}\sigma^d_{syst} \\ 0 & 0 & \sigma^c_{syst}\sigma^d_{syst} & (\sigma^d_{syst})^2 \end{pmatrix}
\end{aligned} \qquad (55)
$$

4.3 The scenarios

The data used by the fitting procedure are listed in table 1.

Experiment	E154	E142	HERMES	E143	SMC	E155
Reference	[10]	[11]	[12]	[13]	[14]	[15]
Target	n	n	n	p, d	p, d	p
Nb of points	18	32	9	80	24	54

Table 1: *Summary of the experimental measurements of g_1 used to constrain the polarized parton distributions.*

It is interesting to study the dependence of the result as a function of different theoretical hypothesis:

- Choice of the unpolarized parton distribution parametrization (GRV at 0.34 GeV^2 or MRS at 1 GeV^2)
- Assume the light sea isospin symmetry at the initial scale ($\delta\bar{u} = \delta\bar{d}$)

- Choice of the factorization scheme: $\overline{\mathrm{MS}}$ or AB
- Use the helicity retention property [17], which leads to set all the b coefficients appearing in the $(1-x)^b$ factors to zero
- Fix the value of Δq_3 to $F+D$, which is equivalent to assume that the Bjorken sum rule is satisfied
- Fix the value of Δq_8 to $3F-D$
- Use the complete covariance matrix for the minimization[5]
- Include the E155 data: the data are preliminary and can lead to a significative difference; it is therefore interesting to estimate their effect on the result.

The table 2 summmarizes the features of the six most significative scenarios we have studied.

Scenario	**A**	**B**	**C**
Using E155 data	no	no	yes
Parametrization	GRV	GRV	GRV
Δq_8	free	free	free
Errors	stat.	complete	complete
Factorization scheme	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$
Scenario	**D**	**E**	**F**
Using E155 data	yes	yes	yes
Parametrization	GRV	GRV	GRV
Δq_8	fixed	free	fixed
Errors	complete	complete	complete
Factorization scheme	$\overline{\mathrm{MS}}$	AB	AB

Table 2: *Features of the most significative scenarios we have studied.*

4.4 Error computation

The errors on the parameters and the errors on the polarized parton distribution moments can be classified into three groups:

[5]All the other world analysis performed either do not include the systematic errors to estimate the χ^2, or assume that the systematic errors between the experiments and between the different points of the experiments are uncorrelated, which is wrong. To compare with other analysis, we keep the possibility to use the more traditional approach consisting in weighting the χ^2 by the statistical error.

- a *statistical* error, corresponding to the statistical errors on the experimental measurements of g_1
- a *systematic* error, corresponding to the systematic errors on the experimental measurements of g_1
- a *theoretical* error, corresponding to the uncertainties on the analysis theoretical parameters, such as α_s or the quark masses

The statistical error is calculated by randomly generating 800 values of g_1 taken in a gaussian distribution: the standard deviation of the gaussian distribution is assumed to be equal to the statistical error on the measurement. The corresponding distributions of the parameter values are not necesseraly symmetrical. Therefore, we have to define two asymmetrical errors σ_+ and σ_- such as 68% of the events are within an asymmetrical standard deviation (*cf.* figure 2).

The systematic error is calculated by performing the quadratic summation on all the experiments of the variations of the parameters, when we increase or decrease all the measurements belonging to the same experiment by their systematic error. We assume that the systematic errors of the points within an experiment are 100% correlated.

The theoretical error is computed by performing the quadratic summation of the systematic errors due to each single source of theoretical error, those being calculated by studying the variations of the parameters when we add or subtract to each source its sytematic error. The theoretical error sources are:

- errors on the b and c quark masses: $m_c = 1.5 \pm 0.5$ GeV and $m_b = 4.5 \pm 0.5$ GeV
- errors on the parameters $\Lambda^{(3,4,5)}$ used to calculate α_s (*cf.* appendix A)
- errors on Δq_3 and/or Δq_8 if we choose to fix their value: $\Delta q_3 = 1.257 \pm 0.003$ and $\Delta q_3 = 0.579 \pm 0.032$
- error on the probability of the deuteron to be in the D state: $\omega_D = 0.05 \pm 0.01$

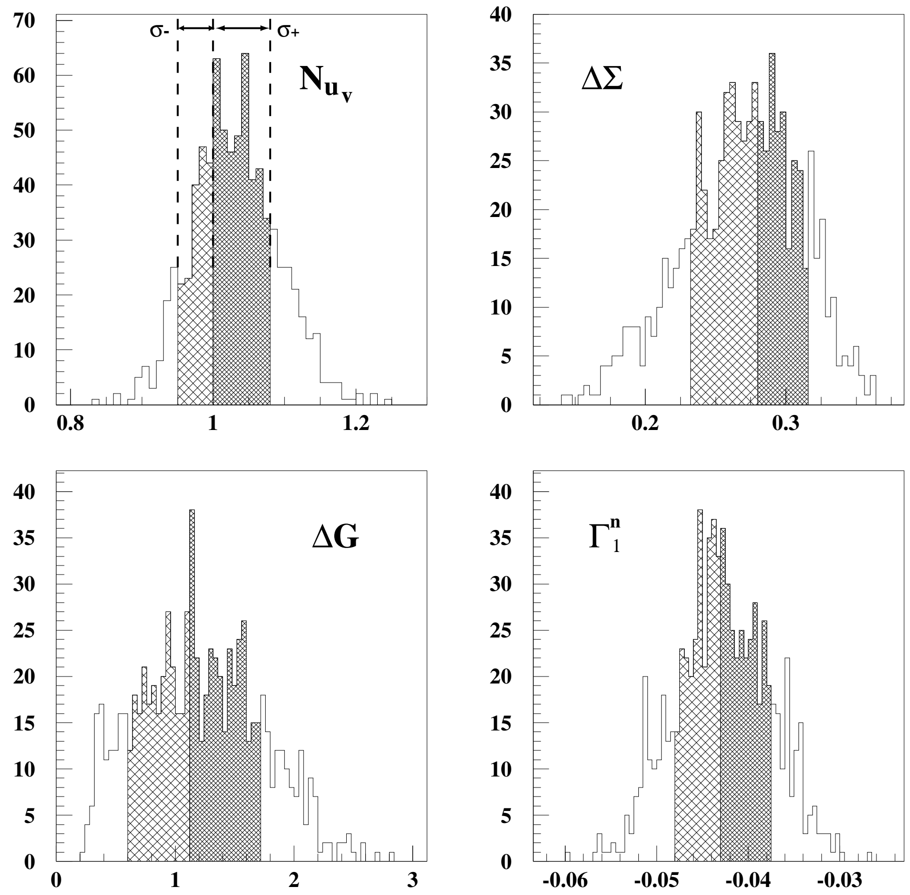

Figure 2: *Example of distributions of the parameter N_{u_v}, of the first moments $\Delta\Sigma$, ΔG and Γ_1^n for a test scenario. The hatched areas represent the asymmetrical errors σ_- and σ_+. They have been determined by including 68% of the randomly generated values, on each side of the central value. Here, the scenario used shows a broad ΔG distribution, illustrating how the present experimental data do not sufficiently constrain that distribution.*

5 The results

5.1 Tables

We summarize in the following tables the values of the parameters of the polarized parton distributions at the initial scale Q_0^2 as well as the first moments of those distributions at 5 GeV^2. Each value is followed by asymmetrical statistical, systematic and theoretical errors.

The tables referenced as 3, 4, 5, 6, 7 and 8 correspond to the six scenarios.

Scenario A				
parameter	**value**	**stat.**	**syst.**	**theo.**
N_{u_v}	1.02	$^{+0.08}_{-0.04}$	$^{+0.03}_{-0.04}$	$^{+0.70}_{-0.27}$
a_{u_v}	0.63	$^{+0.05}_{-0.04}$	$^{+0.02}_{-0.02}$	$^{+0.37}_{-0.19}$
N_{d_v}	-0.75	$^{+0.07}_{-0.06}$	$^{+0.04}_{-0.03}$	$^{+0.17}_{-0.35}$
a_{d_v}	0.29	$^{+0.04}_{-0.10}$	$^{+0.01}_{-0.03}$	$^{+0.21}_{-0.13}$
$N_{\bar{q}}$	-0.02	$^{+0.01}_{-0.01}$	$^{+0.03}_{-0.01}$	$^{+0.00}_{-0.02}$
$a_{\bar{q}}$	0.20	$^{+0.00}_{-0.19}$	$^{+0.17}_{-0.17}$	$^{+0.65}_{-0.65}$
N_s	0.21	$^{+0.70}_{-0.24}$	$^{+0.79}_{-0.34}$	$^{+0.74}_{-0.59}$
N_g	1.36	$^{+0.71}_{-0.36}$	$^{+0.25}_{-0.20}$	$^{+0.37}_{-0.37}$
A_g	0.96	$^{+0.22}_{-0.33}$	$^{+0.16}_{-0.14}$	$^{+1.40}_{-0.62}$

1$^{\text{st}}$ moment	**value**	**stat.**	**syst.**	**theo.**
Δu_v	0.69	$^{+0.02}_{-0.03}$	$^{+0.04}_{-0.04}$	$^{+0.03}_{-0.02}$
Δd_v	-0.37	$^{+0.03}_{-0.03}$	$^{+0.03}_{-0.04}$	$^{+0.00}_{-0.01}$
$\Delta\bar{q}$	-0.01	$^{+0.04}_{-0.01}$	$^{+0.01}_{-0.01}$	$^{+0.00}_{-0.00}$
Δs	-0.01	$^{+0.01}_{-0.01}$	$^{+0.00}_{-0.01}$	$^{+0.00}_{-0.01}$
ΔG	1.33	$^{+0.64}_{-0.90}$	$^{+0.14}_{-0.10}$	$^{+0.31}_{-0.00}$
$\Delta\Sigma$	0.26	$^{+0.05}_{-0.05}$	$^{+0.05}_{-0.07}$	$^{+0.00}_{-0.02}$
Δq_3	1.07	$^{+0.03}_{-0.03}$	$^{+0.07}_{-0.06}$	$^{+0.04}_{-0.02}$
Δq_8	0.29	$^{+0.04}_{-0.04}$	$^{+0.07}_{-0.01}$	$^{+0.02}_{-0.03}$
Γ_1^p	0.117	$^{+0.005}_{-0.005}$	$^{+0.003}_{-0.006}$	$^{+0.003}_{-0.003}$
Γ_1^d	0.032	$^{+0.005}_{-0.005}$	$^{+0.006}_{-0.007}$	$^{+0.001}_{-0.002}$
Γ_1^n	-0.048	$^{+0.006}_{-0.006}$	$^{+0.004}_{-0.006}$	$^{+0.000}_{-0.002}$

Table 3: *Values and uncertainties associated with the parameters and first moments for scenario A.*

Scenario B				
parameter	**value**	**stat.**	**syst.**	**theo.**
N_{u_v}	1.01	$^{+0.08}_{-0.06}$	$^{+0.05}_{-0.01}$	$^{+0.58}_{-0.24}$
a_{u_v}	0.64	$^{+0.05}_{-0.05}$	$^{+0.03}_{-0.00}$	$^{+0.32}_{-0.18}$
N_{d_v}	-0.70	$^{+0.08}_{-0.12}$	$^{+0.05}_{-0.04}$	$^{+0.19}_{-0.10}$
a_{d_v}	0.30	$^{+0.10}_{-0.08}$	$^{+0.03}_{-0.06}$	$^{+0.08}_{-0.16}$
$N_{\bar{q}}$	-0.02	$^{+0.01}_{-0.02}$	$^{+0.02}_{-0.00}$	$^{+0.00}_{-0.06}$
$a_{\bar{q}}$	0.20	$^{+0.01}_{-0.20}$	$^{+0.02}_{-0.24}$	$^{+1.60}_{-0.10}$
N_s	0.98	$^{+0.02}_{-0.19}$	$^{+0.02}_{-0.02}$	$^{+0.02}_{-1.01}$
N_g	1.25	$^{+0.85}_{-0.54}$	$^{+0.19}_{-0.54}$	$^{+0.14}_{-1.14}$
A_g	0.92	$^{+0.70}_{-0.15}$	$^{+0.17}_{-0.32}$	$^{+0.97}_{-0.13}$

1$^{\text{st}}$ moment	**value**	**stat.**	**syst.**	**theo.**
Δu_v	0.68	$^{+0.03}_{-0.03}$	$^{+0.02}_{-0.01}$	$^{+0.02}_{-0.01}$
Δd_v	-0.34	$^{+0.03}_{-0.04}$	$^{+0.01}_{-0.02}$	$^{+0.02}_{-0.01}$
$\Delta\bar{q}$	-0.01	$^{+0.01}_{-0.03}$	$^{+0.01}_{-0.01}$	$^{+0.01}_{-0.01}$
Δs	-0.01	$^{+0.01}_{-0.00}$	$^{+0.01}_{-0.00}$	$^{+0.01}_{-0.00}$
ΔG	1.31	$^{+0.64}_{-0.82}$	$^{+0.06}_{-0.28}$	$^{+0.21}_{-0.54}$
$\Delta\Sigma$	0.27	$^{+0.08}_{-0.03}$	$^{+0.04}_{-0.02}$	$^{+0.04}_{-0.01}$
Δq_3	1.02	$^{+0.02}_{-0.04}$	$^{+0.03}_{-0.01}$	$^{+0.02}_{-0.01}$
Δq_8	0.34	$^{+0.00}_{-0.05}$	$^{+0.00}_{-0.05}$	$^{+0.04}_{-0.07}$
Γ_1^p	0.115	$^{+0.006}_{-0.006}$	$^{+0.005}_{-0.003}$	$^{+0.005}_{-0.003}$
Γ_1^d	0.034	$^{+0.005}_{-0.005}$	$^{+0.003}_{-0.002}$	$^{+0.004}_{-0.002}$
Γ_1^n	-0.042	$^{+0.005}_{-0.005}$	$^{+0.001}_{-0.002}$	$^{+0.003}_{-0.002}$

Table 4: *Values and uncertainties associated with the parameters and first moments for scenario B.*

Scenario C				
parameter	**value**	**stat.**	**syst.**	**theo.**
N_{u_v}	0.92	$^{+0.06}_{-0.04}$	$^{+0.04}_{-0.02}$	$^{+0.55}_{-0.23}$
a_{u_v}	0.60	$^{+0.02}_{-0.03}$	$^{+0.05}_{-0.03}$	$^{+0.37}_{-0.19}$
N_{d_v}	-0.59	$^{+0.07}_{-0.10}$	$^{+0.08}_{-0.02}$	$^{+0.08}_{-0.10}$
a_{d_v}	0.24	$^{+0.03}_{-0.08}$	$^{+0.09}_{-0.09}$	$^{+0.14}_{-0.15}$
$N_{\bar{q}}$	0.00	$^{+0.40}_{-0.00}$	$^{+0.02}_{-0.00}$	$^{+0.00}_{-0.03}$
$a_{\bar{q}}$	1.45	$^{+0.90}_{-0.37}$	$^{+0.00}_{-1.04}$	$^{+1.01}_{-1.52}$
N_s	0.49	$^{+0.62}_{-0.13}$	$^{+0.51}_{-0.85}$	$^{+0.95}_{-0.90}$
N_g	0.05	$^{+0.06}_{-0.20}$	$^{+0.45}_{-0.17}$	$^{+0.29}_{-0.07}$
A_g	1.58	$^{+0.45}_{-0.68}$	$^{+0.13}_{-0.04}$	$^{+0.00}_{-1.47}$

1^{st} moment	**value**	**stat.**	**syst.**	**theo.**
Δu_v	0.66	$^{+0.02}_{-0.02}$	$^{+0.01}_{-0.01}$	$^{+0.01}_{-0.01}$
Δd_v	-0.32	$^{+0.02}_{-0.03}$	$^{+0.02}_{-0.00}$	$^{+0.02}_{-0.01}$
$\Delta\bar{q}$	-0.01	$^{+0.01}_{-0.00}$	$^{+0.00}_{-0.01}$	$^{+0.00}_{-0.01}$
Δs	-0.01	$^{+0.00}_{-0.00}$	$^{+0.00}_{-0.01}$	$^{+0.00}_{-0.01}$
ΔG	0.47	$^{+1.22}_{-0.30}$	$^{+0.13}_{-0.03}$	$^{+0.41}_{-0.03}$
$\Delta\Sigma$	0.31	$^{+0.02}_{-0.04}$	$^{+0.02}_{-0.00}$	$^{+0.02}_{-0.04}$
Δq_3	0.99	$^{+0.02}_{-0.03}$	$^{+0.00}_{-0.02}$	$^{+0.01}_{-0.02}$
Δq_8	0.34	$^{+0.03}_{-0.04}$	$^{+0.03}_{-0.00}$	$^{+0.02}_{-0.02}$
Γ_1^p	0.118	$^{+0.003}_{-0.006}$	$^{+0.002}_{-0.001}$	$^{+0.001}_{-0.005}$
Γ_1^d	0.038	$^{+0.004}_{-0.005}$	$^{+0.003}_{-0.000}$	$^{+0.002}_{-0.004}$
Γ_1^n	-0.037	$^{+0.003}_{-0.005}$	$^{+0.004}_{-0.000}$	$^{+0.003}_{-0.004}$

Table 5: *Values and uncertainties associated with the parameters and first moments for scenario C.*

Scenario D				
parameter	**value**	**stat.**	**syst.**	**theo.**
N_{u_v}	0.79	$^{+0.06}_{-0.06}$	$^{+0.04}_{-0.02}$	$^{+0.57}_{-0.23}$
a_{u_v}	0.39	$^{+0.07}_{-0.04}$	$^{+0.04}_{-0.01}$	$^{+0.44}_{-0.26}$
N_{d_v}	-0.72	$^{+0.10}_{-0.16}$	$^{+0.05}_{-0.48}$	$^{+0.21}_{-0.49}$
a_{d_v}	0.62	$^{+0.12}_{-0.00}$	$^{+0.39}_{-0.03}$	$^{+0.37}_{-0.06}$
$N_{\bar{q}}$	-0.84	$^{+0.80}_{-0.15}$	$^{+0.08}_{-0.26}$	$^{+1.17}_{-0.81}$
$a_{\bar{q}}$	0.83	$^{+0.10}_{-0.10}$	$^{+0.12}_{-0.07}$	$^{+1.51}_{-1.10}$
N_s	1.00	$^{+0.00}_{-0.02}$	$^{+0.00}_{-0.01}$	$^{+0.00}_{-0.01}$
N_g	0.03	$^{+0.00}_{-0.03}$	$^{+0.22}_{-0.05}$	$^{+0.78}_{-0.08}$
A_g	1.66	$^{+0.00}_{-0.85}$	$^{+0.58}_{-1.57}$	$^{+0.59}_{-0.74}$

1st moment	**value**	**stat.**	**syst.**	**theo.**
Δu_v	0.77	$^{+0.01}_{-0.02}$	$^{+0.01}_{-0.02}$	$^{+0.02}_{-0.01}$
Δd_v	-0.20	$^{+0.02}_{-0.01}$	$^{+0.02}_{-0.00}$	$^{+0.01}_{-0.02}$
$\Delta\bar{q}$	-0.05	$^{+0.05}_{-0.04}$	$^{+0.00}_{-0.01}$	$^{+0.06}_{-0.04}$
Δs	-0.05	$^{+0.04}_{-0.05}$	$^{+0.00}_{-0.01}$	$^{+0.06}_{-0.05}$
ΔG	0.43	$^{+0.43}_{-0.08}$	$^{+0.06}_{-0.06}$	$^{+0.47}_{-0.20}$
$\Delta\Sigma$	0.29	$^{+0.02}_{-0.03}$	$^{+0.01}_{-0.03}$	$^{+0.04}_{-0.04}$
Δq_3	0.98	$^{+0.03}_{-0.03}$	$^{+0.02}_{-0.04}$	$^{+0.04}_{-0.03}$
Δq_8	0.579	$^{+0.000}_{-0.000}$	$^{+0.00}_{-0.00}$	$^{+0.032}_{-0.032}$
Γ_1^p	0.121	$^{+0.004}_{-0.003}$	$^{+0.003}_{-0.005}$	$^{+0.005}_{-0.005}$
Γ_1^d	0.040	$^{+0.004}_{-0.003}$	$^{+0.001}_{-0.003}$	$^{+0.002}_{-0.002}$
Γ_1^n	-0.031	$^{+0.004}_{-0.003}$	$^{+0.002}_{-0.002}$	$^{+0.002}_{-0.002}$

Table 6: *Values and uncertainties associated with the parameters and first moments for scenario D.*

Scenario E				
parameter	**value**	**stat.**	**syst.**	**theo.**
N_{u_v}	0.93	$^{+0.06}_{-0.06}$	$^{+0.00}_{-0.07}$	$^{+0.52}_{-0.25}$
a_{u_v}	0.58	$^{+0.06}_{-0.05}$	$^{+0.01}_{-0.02}$	$^{+0.37}_{-0.19}$
N_{d_v}	-0.67	$^{+0.09}_{-0.14}$	$^{+0.23}_{-0.02}$	$^{+0.38}_{-0.13}$
a_{d_v}	0.32	$^{+0.13}_{-0.11}$	$^{+0.02}_{-0.24}$	$^{+0.01}_{-0.33}$
$N_{\bar{q}}$	-0.01	$^{+0.01}_{-0.03}$	$^{+0.02}_{-0.01}$	$^{+0.02}_{-0.86}$
$a_{\bar{q}}$	1.09	$^{+0.00}_{-0.15}$	$^{+0.80}_{-0.58}$	$^{+0.99}_{-0.14}$
N_s	0.04	$^{+0.68}_{-0.26}$	$^{+0.96}_{-0.38}$	$^{+1.71}_{-0.31}$
N_g	0.60	$^{+0.50}_{-0.53}$	$^{+0.40}_{-0.69}$	$^{+0.00}_{-1.19}$
A_g	1.19	$^{+0.44}_{-0.42}$	$^{+0.62}_{-0.08}$	$^{+0.55}_{-0.70}$

1$^{\text{st}}$ moment	**value**	**stat.**	**syst.**	**theo.**
Δu_v	0.67	$^{+0.01}_{-0.03}$	$^{+0.01}_{-0.04}$	$^{+0.00}_{-0.04}$
Δd_v	-0.31	$^{+0.03}_{-0.04}$	$^{+0.00}_{-0.03}$	$^{+0.06}_{-0.04}$
$\Delta\bar{q}$	0.00	$^{+0.01}_{-0.01}$	$^{+0.00}_{-0.00}$	$^{+0.00}_{-0.05}$
Δs	0.00	$^{+0.02}_{-0.01}$	$^{+0.00}_{-0.00}$	$^{+0.00}_{-0.05}$
ΔG	0.88	$^{+0.91}_{-0.38}$	$^{+0.48}_{-0.63}$	$^{+0.00}_{-0.92}$
$\Delta\Sigma$	0.36	$^{+0.02}_{-0.03}$	$^{+0.01}_{-0.07}$	$^{+0.00}_{-0.07}$
Δq_3	0.99	$^{+0.03}_{-0.03}$	$^{+0.02}_{-0.02}$	$^{+0.01}_{-0.02}$
Δq_8	0.36	$^{+0.05}_{-0.06}$	$^{+0.01}_{-0.07}$	$^{+0.00}_{-0.07}$
Γ_1^p	0.115	$^{+0.003}_{-0.006}$	$^{+0.000}_{-0.005}$	$^{+0.003}_{-0.004}$
Γ_1^d	0.035	$^{+0.004}_{-0.004}$	$^{+0.000}_{-0.005}$	$^{+0.002}_{-0.003}$
Γ_1^n	-0.039	$^{+0.003}_{-0.004}$	$^{+0.001}_{-0.005}$	$^{+0.002}_{-0.003}$

Table 7: *Values and uncertainties associated with the parameters and first moments for scenario E.*

Scenario F

parameter	**value**	**stat.**	**syst.**	**theo.**
N_{u_v}	0.84	$^{+0.01}_{-0.05}$	$^{+0.01}_{-0.08}$	$^{+0.90}_{-0.26}$
a_{u_v}	0.43	$^{+0.04}_{-0.04}$	$^{+0.00}_{-0.06}$	$^{+0.52}_{-0.22}$
N_{d_v}	-0.85	$^{+0.23}_{-0.11}$	$^{+0.60}_{-0.03}$	$^{+0.67}_{-0.23}$
a_{d_v}	0.73	$^{+0.09}_{-0.17}$	$^{+0.04}_{-0.55}$	$^{+0.24}_{-0.67}$
$N_{\bar{q}}$	-0.94	$^{+0.12}_{-0.06}$	$^{+0.08}_{-0.08}$	$^{+2.05}_{-0.91}$
$a_{\bar{q}}$	0.96	$^{+0.07}_{-0.73}$	$^{+0.07}_{-0.68}$	$^{+0.89}_{-0.73}$
N_s	1.00	$^{+0.01}_{-0.01}$	$^{+0.00}_{-0.00}$	$^{+0.00}_{-0.01}$
N_g	1.62	$^{+0.01}_{-0.12}$	$^{+1.79}_{-2.03}$	$^{+0.68}_{-2.34}$
A_g	1.26	$^{+0.96}_{-0.57}$	$^{+0.57}_{-1.21}$	$^{+0.71}_{-2.11}$

1^{st} moment	**value**	**stat.**	**syst.**	**theo.**
Δu_v	0.77	$^{+0.01}_{-0.04}$	$^{+0.01}_{-0.01}$	$^{+0.04}_{-0.00}$
Δd_v	-0.19	$^{+0.03}_{-0.01}$	$^{+0.01}_{-0.02}$	$^{+0.03}_{-0.05}$
$\Delta\bar{q}$	-0.04	$^{+0.05}_{-0.01}$	$^{+0.00}_{-0.01}$	$^{+0.08}_{-0.04}$
Δs	-0.04	$^{+0.00}_{-0.01}$	$^{+0.00}_{-0.00}$	$^{+0.08}_{-0.04}$
ΔG	1.49	$^{+0.37}_{-0.14}$	$^{+0.36}_{-1.41}$	$^{+0.86}_{-0.92}$
$\Delta\Sigma$	0.37	$^{+0.00}_{-0.03}$	$^{+0.02}_{-0.06}$	$^{+0.07}_{-0.02}$
Δq_3	0.97	$^{+0.02}_{-0.07}$	$^{+0.02}_{-0.02}$	$^{+0.08}_{-0.02}$
Δq_8	0.579	$^{+0.00}_{-0.00}$	$^{+0.000}_{-0.000}$	$^{+0.032}_{-0.032}$
Γ_1^p	0.113	$^{+0.004}_{-0.003}$	$^{+0.009}_{-0.003}$	$^{+0.004}_{-0.004}$
Γ_1^d	0.035	$^{+0.003}_{-0.004}$	$^{+0.008}_{-0.003}$	$^{+0.004}_{-0.004}$
Γ_1^n	-0.038	$^{+0.004}_{-0.003}$	$^{+0.010}_{-0.004}$	$^{+0.006}_{-0.006}$

Table 8: *Values and uncertainties associated with the parameters and first moments for scenario F.*

5.2 Discussion

Table 9 gives the number of data points fitted as well as the resulting χ^2 for each scenario.

Scenario	**A**	**B**	**C**	**D**	**E**	**F**
f	163-9	163-9	217-9	217-8	217-9	217-8
χ^2/f	0.77	0.82	0.97	1.21	0.89	1.18

Table 9: *Number of experimental measurements of g_1 minus the number of fitting parameters used for each scenario and associated reduced χ^2.*

In addition to the six scenarios, we have tried to use the MRS parametrization as our unpolarized parton distribution set. In this case, the starting scale is $Q_0^2 = 1$ GeV^2. We have observed no major effect due to this change of unpolarized parametrization. Another systematic check consists in including heavier flavors such as c and b in our analysis. We have included them purely radiatively, that is, assigning for instance $\delta c = 0$ before and at the charm threshold and letting the Q^2 evolution radiatively generate them at Q^2 greater than the charm threshold. We observed a very small contribution to the nucleon spin from c or b, typically of the order of 0.1%, compatible with 0.

Once we have got the full parametrization at the starting scale, we can evolve them and have them at any Q^2. For instance, figure 4 shows the polarized parton distributions obtained with our six scenarios at $Q^2 = 5$ GeV^2. It is obvious that the behavior of the sea and the gluons depends heavily on which scenario is considered. The $\delta\Delta$ distribution is always negligible in this analysis. In fact, g_1 is not directly sensitive to the sea distributions, hence the polarized structure function cannot constrain those distributions. The only constraint comes from the Q^2 evolution itself.

The results for the polarized structure functions g_1^p, g_1^n and g_1^d are shown in figures 5, 6, 7, 8 and 9 and are compared with experimental data evolved to 5 GeV^2. Very generally, all the scenarios give very good fits in the data region. Unfortunately, they give very different results at very low x where their behavior is driven by the theoretical assumptions made for each scenario. The Q^2 dependence of g_1^p is presented in figure 10, compared to the experimental measurements. It is very interesting to see that the Q^2 dependence is very consistent for the different scenarios. The contributions from the sea plus the gluons and the valence quarks to the polarized structure functions are represented in figures 11 and 12 for the proton, neutron and deuteron respectively, at $Q^2 = 5$ GeV^2. Let us point out that the relative contribution from the sea+gluons is about the same size than the contribution from valence quarks at x lower than 0.01.

The results on the 1st moments of the polarized parton distributions clearly indicate that the valence densities are rather well determined ($\Delta u_v \sim 0.68$ and $\Delta d_v \sim 0.64$ in $\overline{\mathrm{MS}}$ at 5 GeV^2). However, the sea and gluon distributions are very poorly constrained and give very different results depending which scenario is considered. The apparent disagreement between the "theoretical" values of Δq_3 and Δq_8 and the fitted values can be explained by the very high sensibility of those quantities to higher orders. Let us keep in mind that fixing or not fixing Δq_8 at $3F - D$ has a huge influence on the low x behavior of the g_1 structure function.

Figure 3 gives our result concerning the quark and gluon contributions to the nucleon spin $\Delta\Sigma$ and ΔG. The fraction of spin carried by quarks is very consistent for the scenarios considered among the same scheme: an average of $\Delta\Sigma = 0.29 \pm 0.06$ in the $\overline{\mathrm{MS}}$ scheme at $Q^2 = 5$ GeV^2 and 0.37 ± 0.07 in the AB scheme. The difference between both schemes leads to a ΔG of the order of 0.4, compatible with the other measurements. It is still hazardous to quote a value for ΔG, since it is very different depending which scenario is considered, not to mention the sizeable associated error bar. Nevertheless, the observed spread seems to indicate that ΔG lies between 0 and 2.

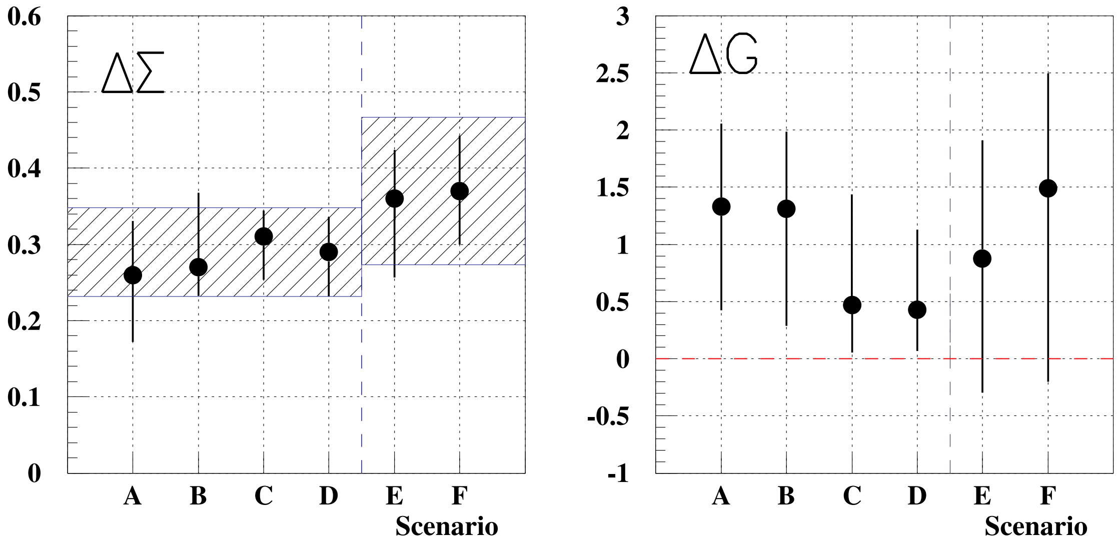

Figure 3: *Fraction of spin carried by quarks (left) and gluons (right) for the 6 scenarios. The dashed vertical line separates scenarios calculated in the* $\overline{\mathrm{MS}}$ *scheme from those calculated in the AB scheme. The error bars are calculated as the quadractic sum of the statistical, systematic and theoretical errors.*

6 Figures

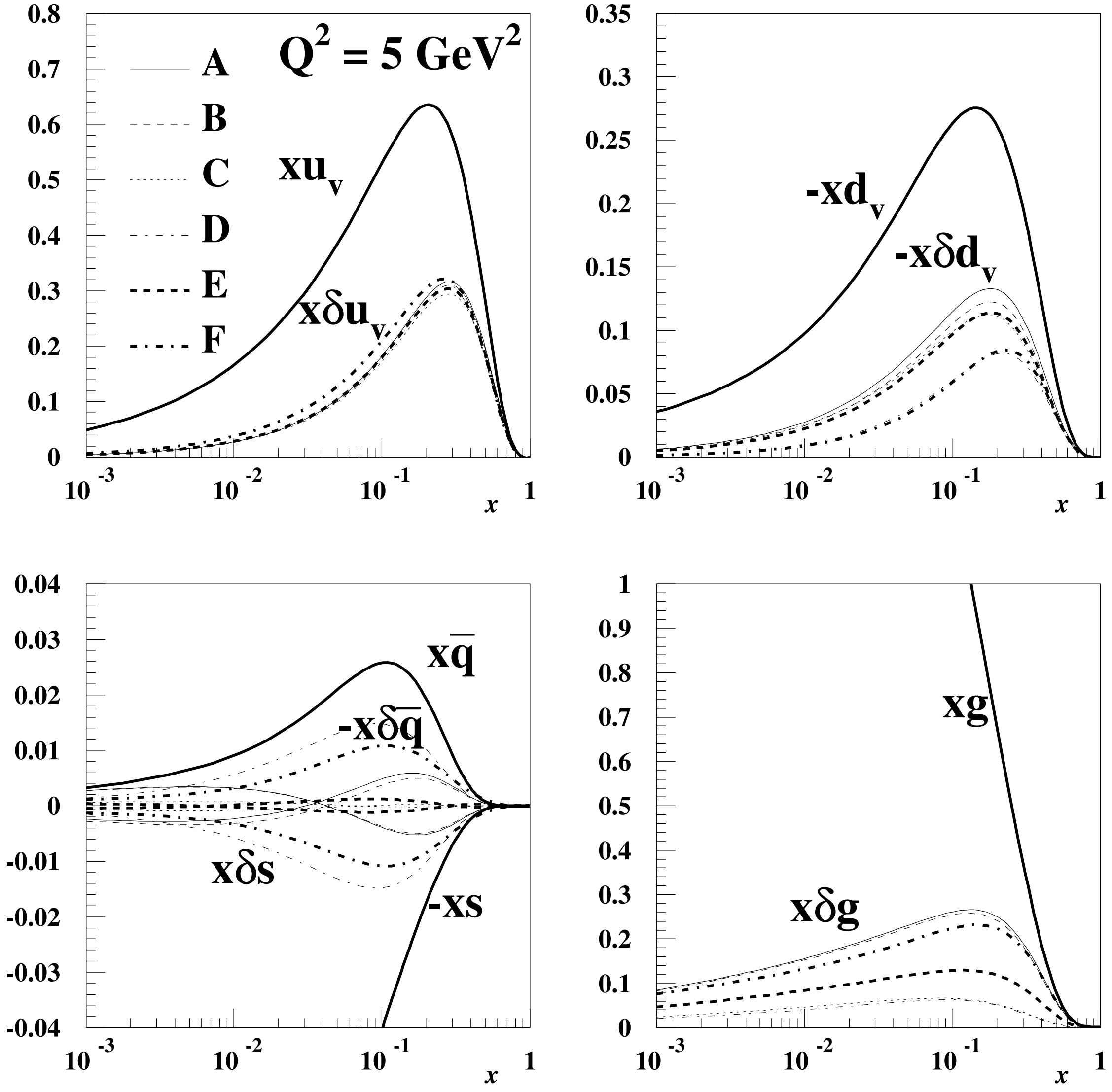

Figure 4: *Polarized parton distributions for different scenarios at $Q^2 = 5\ GeV^2$.*

7 Acknowledgments

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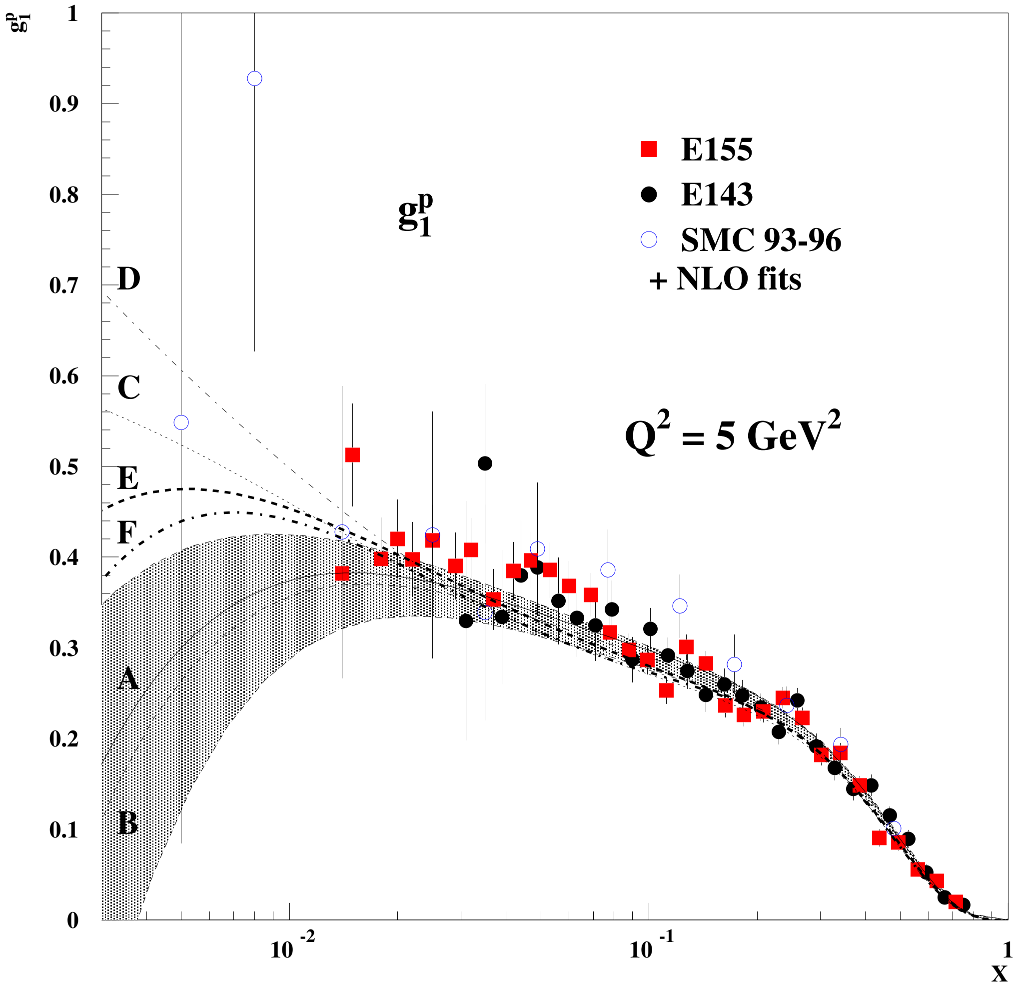

Figure 5: *Fits to the polarized structure function g_1^p for the six scenarios at $Q^2 = 5$ GeV^2. The experimental measurements from the E143, E155 et SMC experiments are evolved using the DGLAP evolution code. Scenario B is represented with its associated statistical error bar (shaded area).*

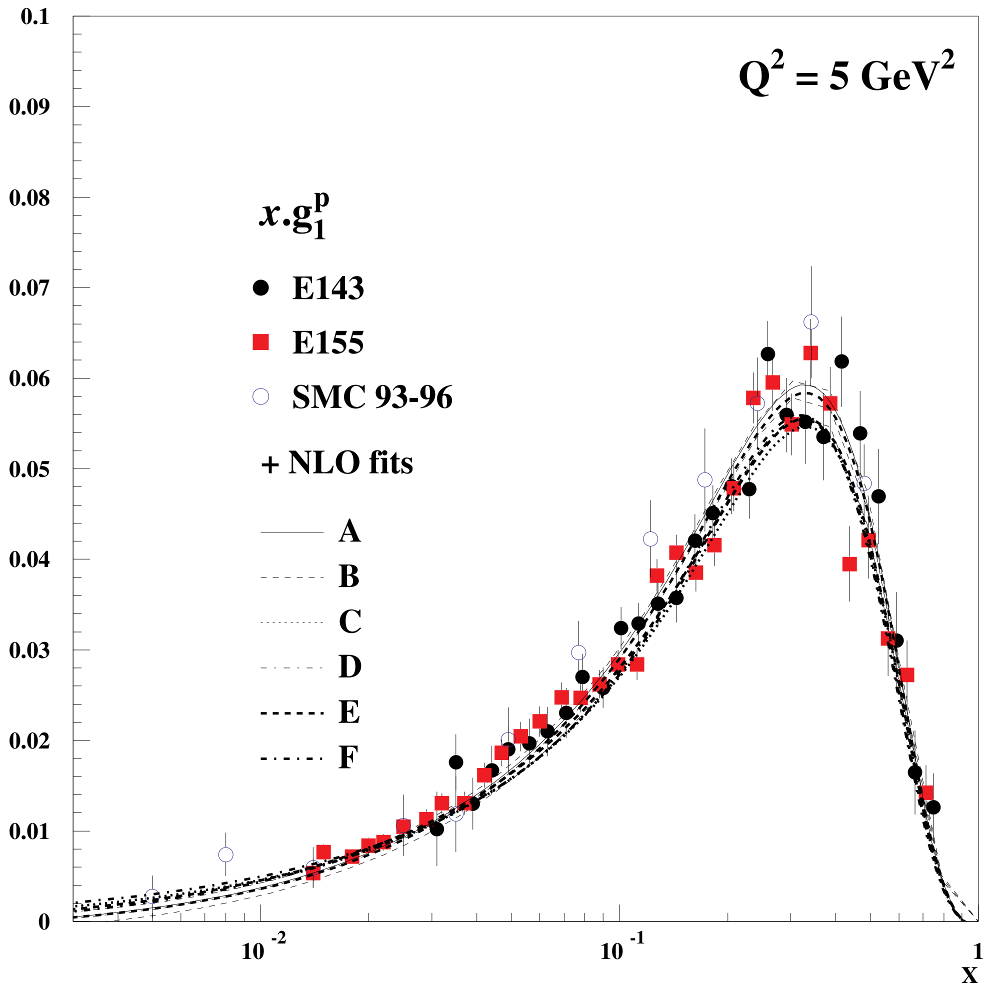

Figure 6: *Fits to the polarized structure function* $x.g_1^p$ *for the six scenarios at* $Q^2 = 5\ GeV^2$. *The experimental measurements from the E143, E155 et SMC experiments are evolved using the DGLAP evolution code.*

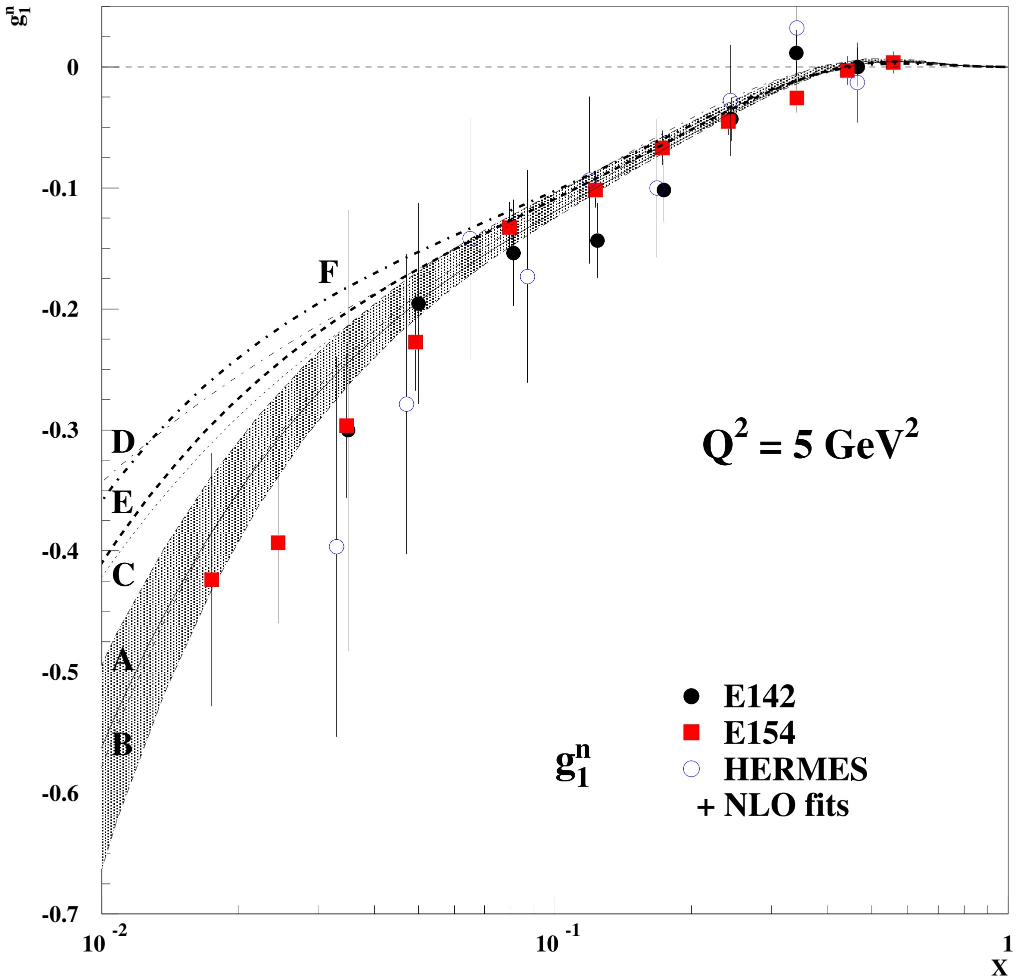

Figure 7: *Fits to the polarized structure function g_1^n for the six scenarios at $Q^2 = 5\ GeV^2$. The experimental measurements from the E143, E155 et SMC experiments are evolved using the DGLAP evolution code. Scenario B is represented with its associated statistical error bar (shaded area).*

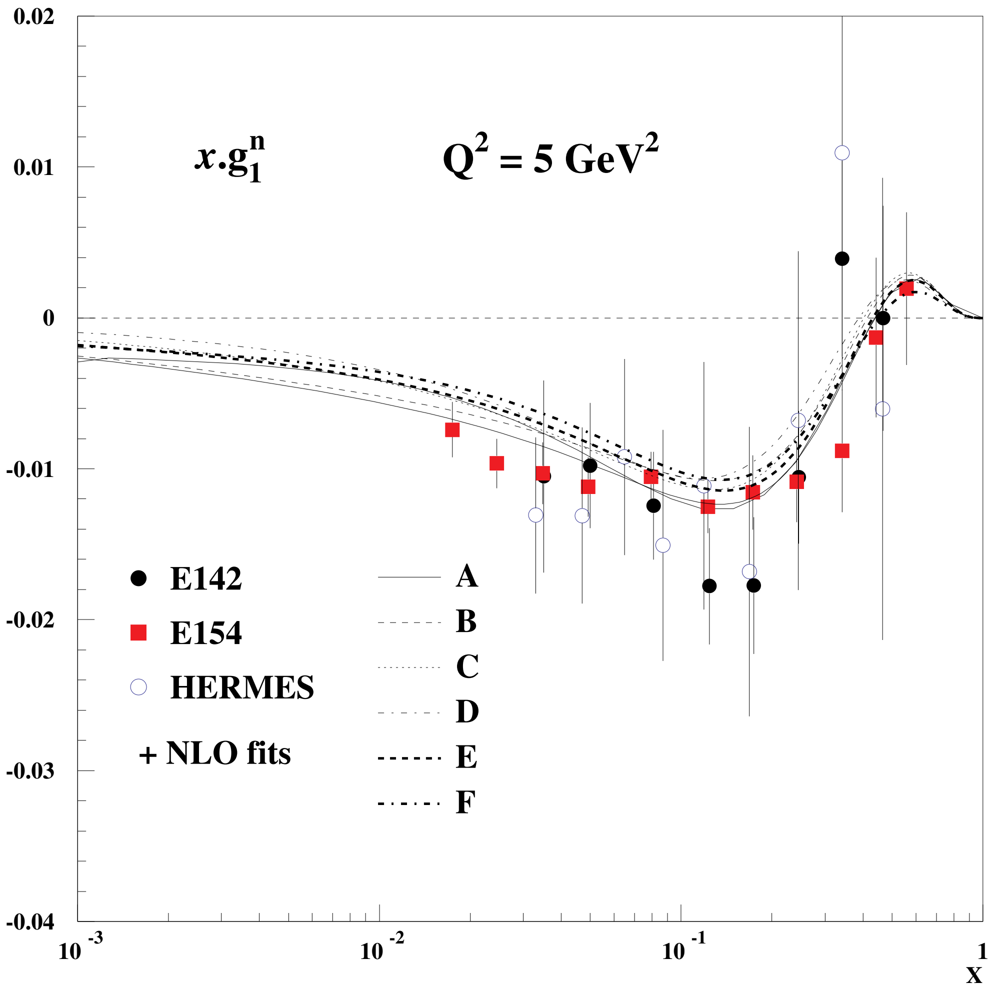

Figure 8: *Fits to the polarized structure function $x.g_1^n$ for the six scenarios at $Q^2 = 5\ GeV^2$. The experimental measurements from the E143, E155 et SMC experiments are evolved using the DGLAP evolution code.*

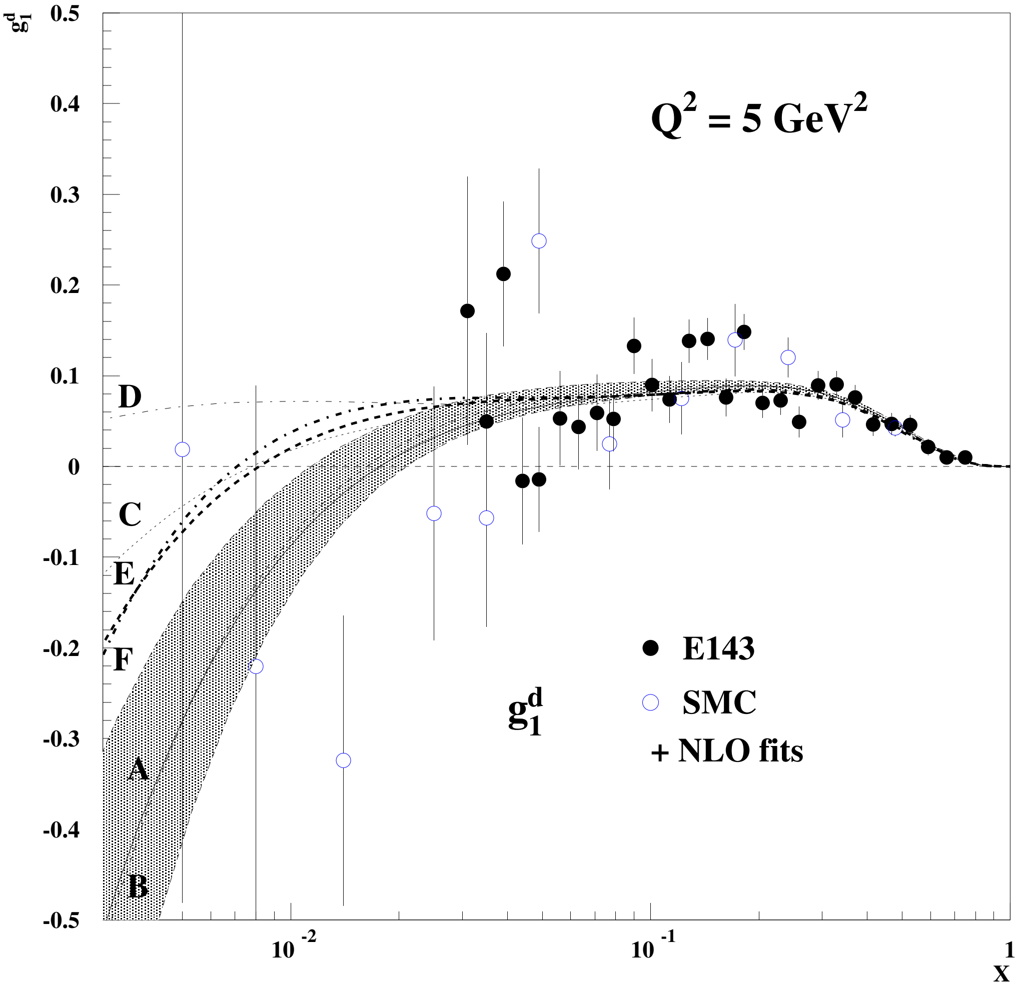

Figure 9: *Fits to the polarized structure function g_1^d for the six scenarios at $Q^2 = 5\ GeV^2$. The experimental measurements from the E143, E155 et SMC experiments are evolved using the DGLAP evolution code. Scenario B is represented with its associated statistical error bar (shaded area).*

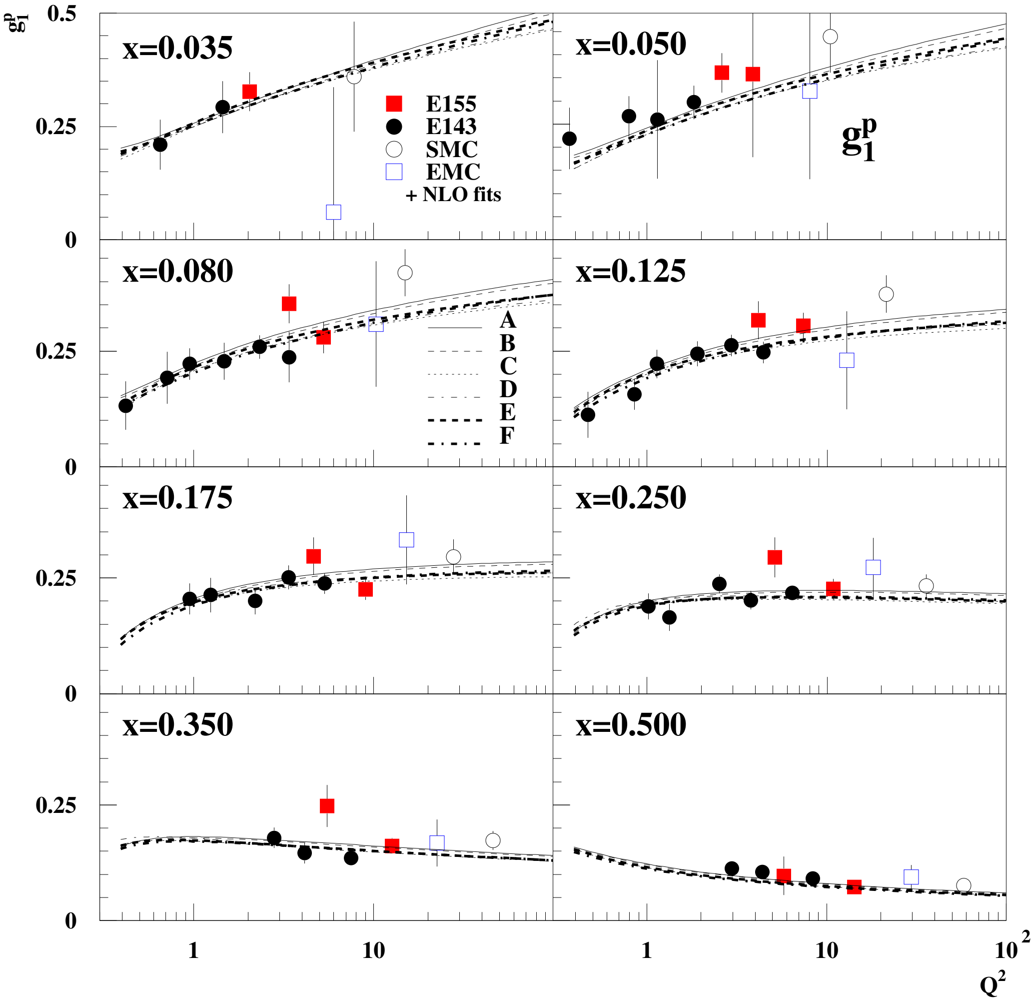

Figure 10: Q^2 *dependence of the polarized structure function* g_1^p *for the six scenarios. The experimental measurements from the E143, E155, SMC and EMC experiments are also shown, and are in very good agreement with our fits.*

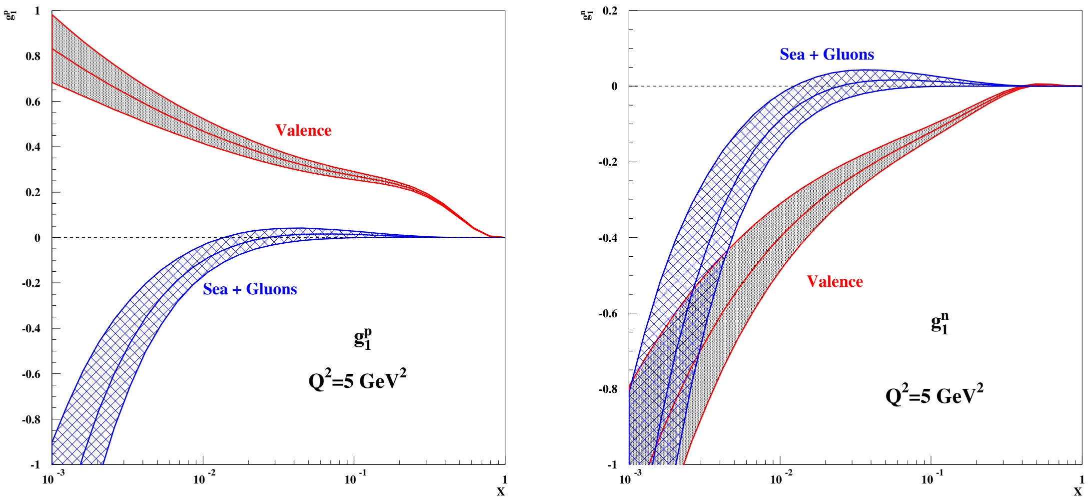

Figure 11: *Contribution from the sea+gluons and the quarks to the g_1^p structure function (left) and g_1^n structure function (right) at $Q^2 = 5\ GeV^2$. The hatched area represents the statistical error.*

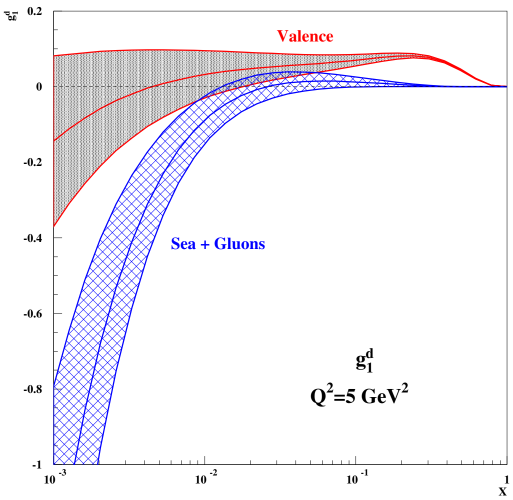

Figure 12: *Contribution from the sea+gluons and the quarks to the g_1^d structure function at $Q^2 = 5\ GeV^2$. The hatched area represents the statistical error.*

A α_s evolution

We evolve the QCD strong coupling constant to any Q^2 value using the following parametrization:

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{(f)})} - \frac{\beta_1}{\beta_0^3} \frac{\ln\left(\ln(Q^2/\Lambda^2_{(f)})\right)}{\left(\ln(Q^2/\Lambda^2_{(f)})\right)^2} \tag{56}$$

The β_0 et β_1 coefficients are given by:

$$\beta_0 = 11 - 2f/3 \text{ and } \beta_1 = 102 - 38f/3 \tag{57}$$

f being the number of active flavors at the considered Q scale.

The $\Lambda_{(f)}$ parameter is given by the recursive relation:

$$\Lambda_{(f+1)} = m_h(\Lambda_{(f)}/m_h)^{(33-2f)/(31-2f)} \tag{58}$$

where m_h stands for the heavy quark mass of flavor h=c, b,... determining the Q threshold above which the number of active flavors to take into account must change from f to $f+1$. This recursive relation is derived from the coupling constant continuity $\alpha_s(Q^2)$ at the threshold $Q = m_h$. Choosing $\overline{\mathrm{MS}}$: $\Lambda_{(4)} = 200 \pm 50$ MeV, we get $\Lambda_{(3)} = 248 \pm 60$ MeV and $\Lambda_{(5)} = 131 \pm 40$ MeV.

To evolve α_s from $Q_0^2 = 0.34$ GeV^2 to a given $Q^2 > Q_0^2$ value, we only need to know the number of active flavors f, given by:

- if $Q^2 > m_b^2 = 20.25$ GeV^2, then f=5, active flavors: u, d, s, c and b
- if $Q^2 > m_c^2 = 2.25$ GeV^2, then f=4, active flavors: u, d, s and c
- otherwise f=3, active flavors: u, d and s

B Expressions of the polarized anomalous dimensions at leading order

In the $\overline{\mathrm{MS}}$ renormalization scheme, the four polarized anomalous dimensions $\delta\gamma^{(0)}_{qq}$, $\delta\gamma^{(0)}_{qg}$, $\delta\gamma^{(0)}_{gq}$ and $\delta\gamma^{(0)}_{gg}$ are:

$$\delta\gamma^{(0)}_{qq}(n) = 4C_F\left[2S_1(n) - \frac{1}{n(n+1)} - \frac{3}{2}\right] \tag{59}$$

$$\delta\gamma_{qg}^{(0)}(n) = -8T_f \frac{n-1}{n(n+1)} \tag{60}$$

$$\delta\gamma_{gq}^{(0)}(n) = -4C_F \frac{n+2}{n(n+1)} \tag{61}$$

$$\delta\gamma_{gg}^{(0)}(n) = 4C_A \left[2S_1(n) - \frac{4}{n(n+1)} - \frac{11}{6}\right] + \frac{8}{3}T_f \tag{62}$$

where $C_F = 4/3$, $C_A = 3$, $T_f = f/2 = 3/2$, f is the number of active quark flavors fixed to 3 for the computation of the anomalous dimensions. The non-singlet polarized anomalous dimension $\delta\gamma_{qq}^{(0)}$ is equal to the unpolarized anomalous dimension $\gamma_{qq}^{(0)}$. Moreover,

$$S_k(n) = \sum_{j=1}^{n} \frac{1}{j^k} \tag{63}$$

and

$$S_1(n) = \gamma_E + \psi(n+1)\,, \quad \gamma_E = 0.577216 \tag{64}$$

$$S_2(n) = \zeta(2) - \psi'(n+1)\,, \quad \zeta(2) = \pi^2/6 \tag{65}$$

$$S_3(n) = \zeta(3) + \frac{1}{2}\psi''(n+1)\,, \quad \zeta(3) = 1.202057 \tag{66}$$

with

$$\psi^{(k)}(n+1) = \psi^{(k)}(n) + \frac{(-)^k\ k!}{n^{k+1}} \tag{67}$$

When the real part of n is greater or equal to 10, the ψ functions and their derivatives have the asymptotic expression:

$$\psi(n) \approx \ln(n) - \frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} - \frac{1}{256n^6} \tag{68}$$

$$\psi'(n) \approx \frac{1}{n} + \frac{1}{2n^2} + \frac{1}{6n^3} - \frac{1}{30n^5} + \frac{1}{42n^7} - \frac{1}{30n^9} \tag{69}$$

$$\psi''(n) \approx -\frac{1}{n^2} - \frac{1}{n^3} - \frac{1}{2n^4} + \frac{1}{6n^6} - \frac{1}{6n^8} + \frac{3}{10n^{10}} - \frac{5}{6n^{12}} \tag{70}$$

On the other hand, if the real part of n is stricly less than 10, it is necessary to use the recursive relation:

$$\psi^{(k)}(n+1) = \psi^{(k)}(n) + \frac{(-)^k\ k!}{n^{k+1}} \tag{71}$$

until the real part of the $\psi^{(n)}$ function argument becomes greater or equal to 10. We choose c less than 10 and will use that formula to evaluate the ψ functions.

C Polarized anomalous dimensions at next-to-leading order

In the $\overline{\mathrm{MS}}$ renormalization scheme, the four polarized anomalous dimensions $\delta\gamma_{qq}^{(1)}$, $\delta\gamma_{qg}^{(1)}$, $\delta\gamma_{gq}^{(1)}$ and $\delta\gamma_{gg}^{(1)}$ are:

$$\delta\gamma_{qq}^{(1)}(n) = \delta\gamma_{NS}^{(1)\ \eta=-1}(n) + \delta\gamma_{PS,qq}^{(1)}(n) \tag{72}$$

with

$$\begin{aligned}
\delta\gamma_{NS}^{(1)\ \eta}(n) = \quad & C_F^2 \quad \left[\frac{16S_1(n)(2n+1)}{n^2(n+1)^2} + 24S_2(n) + 64\tilde{S}(n,\eta) - 8S_3'(n/2,\eta) - 3 + \right.\\
& 16\left(2S_1(n) - \frac{1}{n+1}\right)\left(S_2(n) - S_2'(n/2,\eta)\right) - \\
& \left. 8\frac{3n^3+n^2-1+2\eta(2n^2+2n+1)}{n^3(n+1)^3}\right] + \\
& C_F C_A \quad \left[\frac{536}{9}S_1(n) - 8\left(2S_1(n) - \frac{1}{n(n+1)}\right)\left(2S_2(n) - S_2'(n/2,\eta)\right) - \right.\\
& \frac{88}{3}S_2(n) - 32\tilde{S}(n,\eta) + 4S_3'(n/2,\eta) - \frac{17}{3} - \\
& \left. \frac{\frac{4}{9}(151n^4+236n^3+88n^2+3n+18) - 8\eta(2n^2+2n+1)}{n^3(n+1)^3}\right] + \\
& C_F T_f \quad \left[-\frac{160}{9}S_1(n) + \frac{32}{3}S_2(n) + \frac{4}{3} + \frac{16}{9}\frac{11n^2+5n-3}{n^2(n+1)^2}\right]
\end{aligned} \tag{73}$$

and

$$\delta\gamma_{PS,qq}^{(1)}(n) = 16C_F T_f \frac{n^4+2n^3+2n^2+5n+2}{n^3(n+1)^3} \tag{74}$$

the PS index standing for a "pure flavor singlet" quantity. The polarized anomalous dimension $\delta\gamma_{NS}^{(1)\eta}$ is equal to the corresponding unpolarized anomalous dimension $\gamma_{NS}^{(1)\eta}$.

The three other polarized anomalous dimensions are:

$$\begin{aligned}
\delta\gamma_{qg}^{(1)}(n) = \quad & 8C_F T_f \quad \left[2\frac{n-1}{n(n+1)}\left(S_2(n) - S_1^2(n)\right) + 4\frac{n-1}{n^2(n+1)}S_1(n) - \right.\\
& \left. \frac{5n^5+5n^4-10n^3-n^2+3n-2}{n^3(n+1)^3}\right] + \\
& 16C_A T_f \quad \left[\frac{n-1}{n(n+1)}\left(-S_2(n) + S_2'(n/2,-1) + S_1^2(n)\right) - \right.
\end{aligned} \tag{75}$$

$$\frac{4}{n(n+1)^2}S_1(n) - \frac{n^5+n^4-4n^3+3n^2-7n-2}{n^3(n+1)^3}\Bigg]$$

$$\begin{aligned}
\delta\gamma_{gq}^{(1)}(n) = \quad & 32C_F T_f \quad \left[-\frac{n+2}{3n(n+1)}S_1(n) + \frac{5n^2+12n+4}{9n(n+1)^2}\right] + \qquad (76)\\
& 4C_F^2 \quad \Bigg[2\frac{n+2}{n(n+1)}\left(S_2(n)+S_1^2(n)\right) - 2\frac{3n^2+7n+2}{n(n+1)^2}S_1(n) + \\
& \qquad \frac{9n^5+30n^4+24n^3-7n^2-16n-4}{n^3(n+1)^3}\Bigg] + \\
& 8C_A C_F \quad \Bigg[\frac{n+2}{n(n+1)}\left(-S_2(n)+S_2'(n/2,-1)-S_1^2(n)\right) + \\
& \qquad \frac{11n^2+22n+12}{3n^2(n+1)}S_1(n) - \\
& \qquad \frac{76n^5+271n^4+254n^3+41n^2+72n+36}{9n^3(n+1)^3}\Bigg]
\end{aligned}$$

$$\begin{aligned}
\delta\gamma_{gg}^{(1)}(n) = \quad & 8C_F T_f \quad \left[\frac{n^6+3n^5+5n^4+n^3-8n^2+2n+4}{n^3(n+1)^3}\right] + \qquad (77)\\
& 32C_A T_f \quad \left[-\frac{5}{9}S_1(n) + \frac{3n^4+6n^3+16n^2+13n-3}{9n^2(n+1)^2}\right] + \\
& 4C_A^2 \quad \Bigg[-S_3'(n/2,-1) - 4S_1(n)S_2'(n/2,-1) + 8\tilde{S}(n,-1) + \\
& \qquad \frac{8}{n(n+1)}S_2'(n/2,-1) + \\
& \qquad 2\frac{67n^4+134n^3+67n^2+144n+72}{9n^2(n+1)^2}S_1(n) - \\
& \qquad \frac{48n^6+144n^5+469n^4+698n^3+7n^2+258n+144}{9n^3(n+1)^3}\Bigg]
\end{aligned}$$

with the definitions:

$$T_f = T_R \qquad (78)$$

$$\begin{aligned}
S_k'(n/2,\eta) &= 2^{k-1}\sum_{j=1}^{n}\frac{1+(-)^j}{j^k} \qquad (79)\\
&= \frac{1}{2}(1+\eta)S_k\left(\frac{n}{2}\right) + \frac{1}{2}(1-\eta)S_k\left(\frac{n-1}{2}\right)\\
\tilde{S}(n,\eta) &= \sum_{j=1}^{n}\frac{(-)^j}{j^2}S_1(j) \qquad (80)\\
&= -\frac{5}{8}\zeta(3) + \eta\left[\frac{S_1(n)}{n^2} - \frac{\zeta(2)}{2}G(n) + \int_0^1 dx\ x^{n-1}\frac{\mathrm{Li}_2(x)}{1+x}\right]
\end{aligned}$$

and

$$G(n) = \psi\left(\frac{n+1}{2}\right) - \psi\left(\frac{n}{2}\right) \qquad (81)$$

with the dilog function $\mathrm{Li}_2(x) = -\int_0^x dz\ \ln(1-z)/z$ which can be approximated using:

$$\int_0^1 dx\ x^{n-1}\frac{\mathrm{Li}_2(x)}{1+x} \approx \frac{1.010}{n+1} - \frac{0.846}{n+2} + \frac{1.155}{n+3} - \frac{1.074}{n+4} + \frac{0.550}{n+5} \tag{82}$$

D Change of renormalization scheme

To use the evolution equations in the AB scheme, it is necessary to correct the Wilson coefficients and the anomalous dimensions using the Z matrix defined in [18]:

$$Z = \begin{pmatrix} Z_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} = \begin{pmatrix} 0 & 2T_f/n \\ 0 & 0 \end{pmatrix} \tag{83}$$

The Wilson coefficients in the AB scheme are then deduced from the ones in the $\overline{\mathrm{MS}}$ scheme by:

$$\delta C_q(n) \ \rightarrow\ \delta C_q(n) - Z_{qq}(n) \tag{84}$$

$$\delta C_g(n) \ \rightarrow\ \delta C_g(n) - Z_{qg}(n) \tag{85}$$

The only coefficient which needs to be modified is δC_g. The anomalous dimensions in AB are deduced from the ones in $\overline{\mathrm{MS}}$ using:

$$\delta\gamma_{NS}^{(1)\eta}(n) \ \rightarrow\ \delta\gamma_{NS}^{(1)\eta}(n) + 4\beta_0 Z_{qq}(n) \tag{86}$$

$$\delta\gamma_{ij}^{(1)}(n) \ \rightarrow\ \delta\gamma_{ij}^{(1)}(n) + 2[Z, \boldsymbol{\gamma}^{(0)}]_{i,j} + 4\beta_0 Z_{ij}(n) \tag{87}$$

where the $\boldsymbol{\gamma}^{(0)}$ matrix is defined as:

$$\boldsymbol{\gamma}^{(0)} = \begin{pmatrix} \delta\gamma_{qq}^{(0)} & \delta\gamma_{qg}^{(0)} \\ \delta\gamma_{gq}^{(0)} & \delta\gamma_{gg}^{(0)} \end{pmatrix} \tag{88}$$

E Fixing the values of Δq_3 and/or Δq_8

The Q^2 evolution of the non-singlet flavor distributions δq_3 and δq_8, defined by the relations (43) and (44), is described by the evolution equation (27). Their 1st moments, quoted respectively Δq_3 et Δq_8, are given by:

$$\Delta q_3 = \int_0^1 dx\ \delta q_3(x, Q_0^2) \tag{89}$$

$$\Delta q_8 = \int_0^1 dx\ \delta q_8(x, Q_0^2) \tag{90}$$

Those moments of order $n = 1$ do not depend on Q^2: their value is always the same at any scale. To convince oneself, one can check in the evolution equation (27) that the terms $\delta\gamma_{NS}^{(1)\eta=-1}(n=1)$ and $\delta\gamma_{qq}^{(0)}(n=1)$ are equal to zero. Those moments are then fixed to their values:

$$\Delta q_3 = F + D = \frac{g_A}{g_V} \tag{91}$$

$$\Delta q_8 = 3F - D \tag{92}$$

Equation (91) is simply the Bjorken sum rule.

We can choose or not to fix the value of several parameters in the polarized distributions, so that Δq_3 and/or Δq_8 keep the same value, at any Q^2. Let us examine successively the four possibilities:

- **Only Δq_3 is fixed**

  We fix the normalization constant N_{u_v} of the valence distribution such as the value of Δq_3 remains the same. According to equation (43), we can compute the first moment of δq_3 which value is known, and extract N_{u_v}:

$$N_{u_v} = \frac{1}{\alpha_{u_v}(Q_0^2)} \left[\Delta q_3 + \int_0^1 dx\ \delta_{d_v}(x, Q_0^2) + 4\int_0^1 dx\ \delta\Delta(x, Q_0^2)\right] \tag{93}$$

$$\alpha_{u_v}(Q_0^2) = \int_0^1 dx\ x^{a_{uv}} u_v(x, Q_0^2) \tag{94}$$

  where the $u_v(x, Q_0^2)$ parametrization is given by (1). Then, Δq_3 will keep the value $F + D$ if we calculate N_{u_v} using this expression.

  Fixing Δq_3 is equivalent to assume that the Bjorken sum rule is satisfied.

- **Only Δq_8 is fixed**

  We can fix the normalization constants N_{u_v} or N_{d_v} of the valence distributions, such as the value of Δq_8 remains the same. According to (44), we can compute the first moment of δq_8 which value is known, and extract N_{u_v} or N_{d_v}:

$$\begin{aligned} N_{u_v} = {} & \frac{1}{\alpha_{u_v}(Q_0^2)} \left[\Delta q_8 - \int_0^1 dx\ \delta_{d_v}(x, Q_0^2) - 4\int_0^1 dx\ \delta\bar{q}(x, Q_0^2)\right. \\ & \left. +4\int_0^1 dx\ \delta\bar{s}(x, Q_0^2)\right] \end{aligned} \tag{95}$$

$$\alpha_{u_v}(Q_0^2) = \int_0^1 dx\ x^{a_{uv}} u_v(x, Q_0^2) \tag{96}$$

where the $u_v(x, Q_0^2)$ parametrization is given by (1). Then, Δq_8 will keep the value $3F - D$ if we use this formula to calculate N_{u_v}.

On the other hand, if we wish to fix N_{d_v} instead of N_{u_v}:

$$\begin{aligned}
N_{d_v} &= \frac{1}{\alpha_{d_v}(Q_0^2)} \Big[\Delta q_8 - \int_0^1 dx \ \delta_{u_v}(x, Q_0^2) - 4 \int_0^1 dx \ \delta\bar{q}(x, Q_0^2) \qquad (97) \\
&\quad +4 \int_0^1 dx \ \delta\bar{s}(x, Q_0^2) \Big] \\
\alpha_{d_v}(Q_0^2) &= \int_0^1 dx \ x^{a_{d_v}} d_v(x, Q_0^2) \qquad (98)
\end{aligned}$$

where the parametrization of $d_v(x, Q_0^2)$ is given by (2). Then, Δq_8 will keep the value $3F - D$ if we use this formula to calculate N_{d_v}

- **Δq_3 and Δq_8 are both fixed**

  If we fix simultaneously the values of Δq_3 and Δq_8, it is necessary to solve the system of two equations containing two unknowns N_{u_v} and N_{d_v}, given by the relations (93) and (98) . The solutions are:

$$\begin{aligned}
N_{u_v} &= \frac{1}{2\alpha_u} \left(\Delta q_8 + \Delta q_3 + 4 \int_0^1 dx \ \delta\Delta - 4 \int_0^1 dx \ \delta\bar{q} + 4 \int_0^1 dx \ \delta\bar{s} \right) \qquad (99) \\
N_{d_v} &= \frac{1}{2\alpha_d} \left(\Delta q_8 - \Delta q_3 - 4 \int_0^1 dx \ \delta\Delta - 4 \int_0^1 dx \ \delta\bar{q} + 4 \int_0^1 dx \ \delta\bar{s} \right) \qquad (100)
\end{aligned}$$

  One again, fixing the value of Δq_3 is equivalent to assume that the Bjorken sum rule is satisfied.

- **Δq_3 and Δq_8 are not fixed**

  In that case, we do not fix the value of the normalization constants N_{u_v} et N_{d_v}. They will be computed during the minimization procedure.

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