Spin Transport Effect at TJNAF

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1 Introduction

Consider an electron moving into a uniform magnetic field whose direction is perpendicular to the initial electron momentum. We know from classical electromagnetic theory that the electron will rotate in the plane perpendicular to the magnetic field, and its energy will not change. But in the context of Quantum electrodynamics, we will find that due to radiation effect, the electron energy will decrease instead of remaining constant. And there is another sutlety concerning the electron spin. Suppose the spin is in the moving plane, we will find that the spin will not remain the same angle with the momentum, instead, it will rotate faster than the momentum. The marvel is that given the rotated angle of the electron momentum, then independent of the details of the magnetic field, the rotation angle of the spin in the rest frame is a definite value which is easily calculable. This is the famous "$g-2$" effect. And the spin rotation angle is given by the following formular:

$$\Delta\theta_{spin} = \frac{E}{m_e} \times \frac{g_e - 2}{2} \times \Delta\theta_{rotation} \tag{1}$$

where $\Delta\theta_{spin}$ is the spin rotation angle in the rest frame of the electron, with respect to the electron momentum; E is the electron energy, m_e is the electron mass, $\Delta\theta_{rotation}$ is the rotation angle of the electron momentum in the lab frame, and g_e is the electron g-factor. We notice from the above formular that the two angles are always in the same direction(clockwise or counter-clockwise). If we look in the lab frame, we will find the spin will rotate with the momentum, but quit a bit faster. To give a feeling about this, we consider an example. Suppose an electron with energy 1Gev rotates half a circle in the lab frame, substitute $m_e = 0.510999$ Mev and $g_e = 2.002319304$ into Equation 1, we get the spin rotation angle in the electron rest frame to be $\Delta\theta_{spin} = 2.269\pi$, which is a very large value. This spin rotation angle with respect to the momentum in the rest frame is called spin tranport angle.

2 Spin Transports at TJNAF

At CEBAF, the typical electron energy is around 4 Gev, so the spin transport effect is very large. A simple drawing of the beam line is provided in the following.

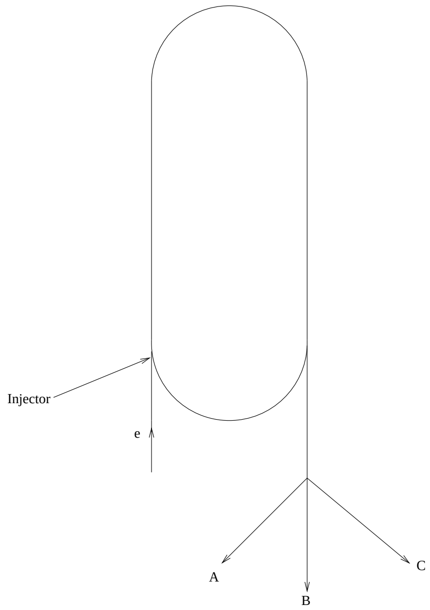

Figure 1: Beam Line at CEBAF

In principle, we can begin from Equation 1, consider all the rotations before the electron gets to the halls and calculate the transport angle. But there is a sutlety here. Since the energy is so large, we must take into account the retardation effect. Thus, we should divide each rotation into a number of steps, for each step use Equation 1 to calculate the transport angle, and then add them up together. When we divide the rotation into smaller and smaller steps, we can get a better and better result, which will eventually converge to the physical transport angle. We write some C codes to perform this. The most important one is spin_abc.c, which you can find in subdirectory ~maple/spin. In that code, you enter the single linac energy(the injector energy is proportional to this energy), the pass numbers for each hall, and the number of steps you want to divide the rotation into,then it will give you both the final energies and the spin transport angles for all the three halls. It converges very well. Note that the tranport angles have been moduled by 360^0, which is of course reasonable.

One goal of the accelerator group is to provide as much as possible simultaneously longitudinal polarization in all three halls. One way to do this is to adjust the linac energy, since the dependence of the transport angle on the linac energy is significant. This has already been discussed in some technique reports. Suppose they have choosen some definite linac energy and the pass numbers, as in the case of E94010, they can use similar code to calculate the three transport angles, moduled by 180^0, and choose the preprecessionn angle of Wein filter at the injector to satisfy all the halls as far as possible.

3 Effect of Spin_transport on the Physics of E94010

If we get the transport angle in Hall A for a specific final energy, and know the preprecession angle at the injector, we can get the final angle β of the electron spin with respect to the electron momentum in the rest frame. Combining this with the result of Enka et.al., we can get the effect of the spin transport on the physical results, namely, changing the statistic error of A_1, A_2 and g_1. But there is one thing to keep in mind. β is not exactly the λ angle used in their report. Instead, we have $\lambda = 360^0 - \beta$, as shown below. This is just due to different conventions.

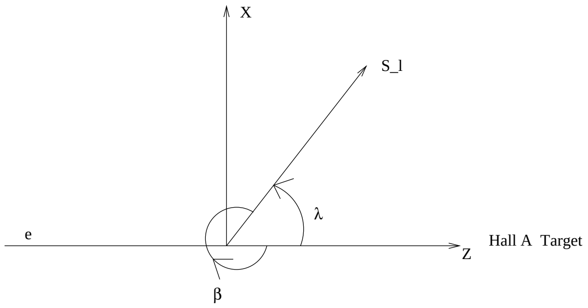

Figure 2: β andλ

Due to the spin transport, the final spin in the rest frame of the electron may form a relatively large angle(may be as large as 30^0) with the momentum, this may be a little bit misleading. In fact, since the electron is moving so fast($\gamma = E/m \sim 8000$), due to the relativistic effect, the spin angle is much more forward oriented in the lab frame than in the electron rest frame. Suppose β' is the spin angle in the lab frame, then we have:

$$\tan(\beta') = \frac{\tan(\beta)}{\gamma} \tag{2}$$

Thus the transverse component of the spin in the lab frame is almost negligible, unless the spin angle in the rest frame is close to 90^0, which is not the truth.

4 Acknowledgement

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