Jefferson Lab
E94010 Technical Note 04
August 1998

The Effect of the Transverse Component of the Electron Beam Polarization on Experimental Asymmetries

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Abstract

We present here a detailed calculation of the inclusive polarized electron scattering cross section asymmetries from the nucleon, considering a transverse component in the polarization of the electron beam.

1 Introduction

In this experiment, we investigate, among other things, the Q^2 evolution of $g_1^n\ (x, Q^2)$, the first component of neutron spin dependent structure function, and also the connection of Gerasimov-Drell-Hearn and the Björken sum rules. Thinking along those lines, we want to measure the longitudinal and transverse asymmetry for longitudinally polarized electrons to be scattered by longitudinally or transversely polarized ^{3}He target (denoted $\mathrm{A}_{\|}^{Jlab}$ and $\mathrm{A}_{\perp}^{Jlab}$ respectively), and then we extract A_1', A_2', and g_1^n from these two quantities. Availability of the proton data enables us to study the Q^2 evolution of the Björken sum rule. As a starting point, we consider only a perfectly longitudinally polarized electron beam. For TJNAF accelerator it becomes very difficult, almost impossible, to provide longitudinally polarized electron beams for all the three halls simultaneously. Therefore, in writing the spin dependent cross-se! ctions, we consider the somewhat more subtle and general case - the electron beam polarization with a LONGITUDINAL and TRANSVERSE COMPONENT.

2 Kinematics and Basic Formulae

To calculate the differential cross-section of an electron with energy approximately 4 GeV, scattered off a neutron, we may just consider the first-order Feynman diagram and only the elecromagnetic interaction (since $\mathrm{M}_z \sim 90$ GeV $\gg 4$ GeV, weak interaction is negligible). Hence, the electron with initial four momentum $k = (E, \vec{k})$ will emit a virtual photon with four momentum $q = (\nu, \vec{q})$, and the scattered electron will have $k' = (E', \vec{k'})$, $q = \vec{k} - \vec{k'}$, and $\nu = E - E'$. The initial neutron is at rest in the lab frame, thus $p = (M, 0, 0, 0)$. After absorbing the virtual photon, the neutron will break into many hadronic products, denoted by X [1].

3 Conventions

As a prelude to discussing the asymmetries, let us introduce some of the kinematic factors that will be used in the litterature. The electron beam is supposed to be ultrarelativistic moving along the z-axis.

- c : celerity of light, taken equal to 1
- E : incident electron energy in the lab
- E' : scattered electron energy in the lab
- θ : electron scattering angle in the lab
- ϕ and α : the two spherical angles that specify the direction of the neutron polarization
- $k = (E, 0, 0, E)$: incident electron quadrimomentum in the lab

- $k' = (E', E'\sin\theta\cos\phi, E'\sin\theta\sin\phi, E'\cos\theta)$: scattered electron quadrimomentum in the lab
- $\nu = E - E'$: electron energy loss in the lab
- M : target nucleon rest mass
- $p = (M, 0, 0, 0)$: nucleon quadrimomentum in the lab
- $q = k - k'$: virtual photon quadrimomentum
- $Q^2 = -q^2 = -(\vec{k} - \vec{k'})^2 = -[(E - E')^2 - (\vec{k} - \vec{k'})^2] = 2EE'(1 - cos\theta)$: virtual photon quadrimomentum squared (since $E, E' > m_e$, $|\vec{k}| \approx E$, $|\vec{k'}| \approx E'$
- $x = Q^2/2M\nu$: Björken variable
- $y = \nu/E$: fractional energy loss of the incident electron in the lab
- m : electron rest mass
- $H_l = \pm 1$: gives the electron spin orientation in the lab
- $H_h = \pm 1$: gives the nucleon spin orientation in the lab
- β_e : electron relativistic factor
- $\gamma_e = 1/\sqrt{1 - \beta_e^2}$
- λ : angle between the electron spin and the z axis in the rest frame of the electron
- s_l : incident electron polarization quadrivector
- s_h : nucleon target polarization quadrivector
- R : the ratio of the longitudinal over transverse component of the nucleon cross-section

4 Differential cross section general expression

For the inclusive polarized electron beam scattering off a polarized neutron, the differential cross-section has the following form [2]:

$$\begin{aligned}\frac{d^3\sigma}{dxdyd\phi} &= \frac{e^4 ME}{4\pi^2 Q^4}((xy^2)F_1 + (1 - y - \gamma^2\frac{y^2}{4})F_2 + (2xy^2\frac{s_h s_l}{pq} \\ &+ y^2\frac{qs_l}{pq}\frac{qs_h}{pq})g_1 + (2xy^2\frac{s_h s_l}{pq} - 2xy^2\frac{(qs_h)(ps_l)}{(pq)(pq)})g_2) \qquad (1)\end{aligned}$$

Here, s_h is the polarization quadrivector of the neutron target at rest. In E94-010 the spin of the polarized neutron is either perpendicular or parallel to the momentum of the electron beam and it has the general form:

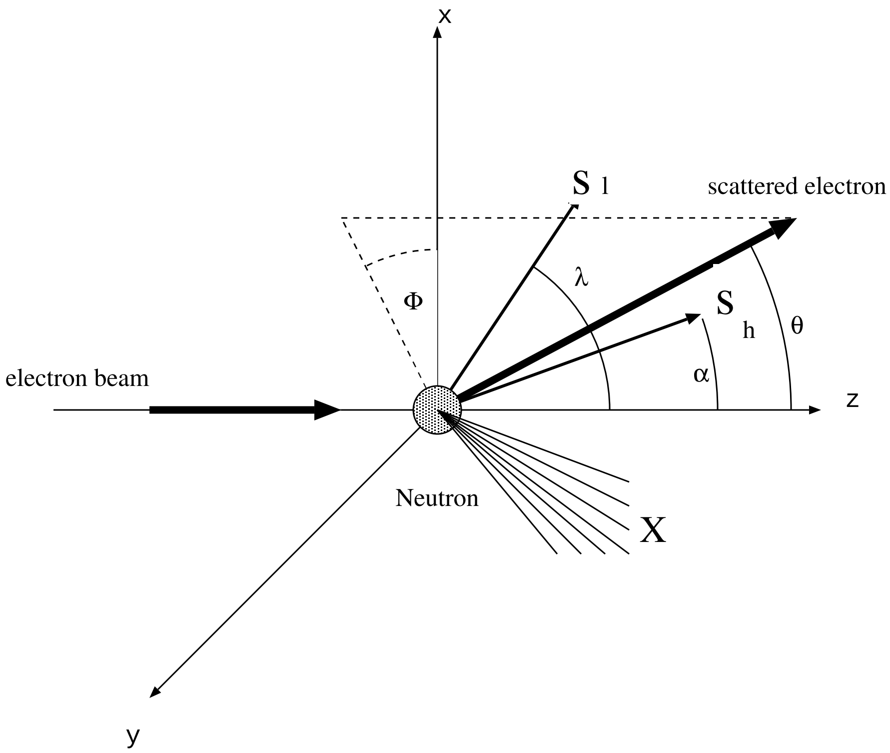

Figure 1: Kinematics diagram for the scattering of the electron off the polarized neutron target. The spin direction of the electron in its rest frame and that of the target in the lab frame are shown above.

$$s_h = M(0, \cos\phi \sin\alpha, \sin\phi \sin\alpha, \cos\alpha) \tag{2}$$

In the experiment, the target polarization is either longitudinal (z-direction) or transverse, (x-direction) therefore, $\phi = 0$. Hence, taking into account the spin flip we have :

$$s_h = MH_h(0, \sin\alpha, 0, \cos\alpha) \tag{3}$$

where $H_h = \pm 1$. Due to the spin transport of the electron beam and the requirement of simultaneous longitudinal polarization for all three halls at TJNAF, we cannot get a perfectly longitudinally polarized electron beam. Instead, it will have a transportation angle λ in the electron's rest frame. In the present context, we keep in mind that this spin transport occurs only in the scattering plane.

5 Polarization quadrivectors

What can we pick for the angle? λ is the transport angle in the rest frame of the electron. In the rest frame, we know:

$$L = \begin{pmatrix} \gamma_e & 0 & 0 & \beta_e\gamma_e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta_e\gamma_e & 0 & 0 & \gamma_e \end{pmatrix} \tag{4}$$

$$s_l^{Rest} = H_l \begin{pmatrix} 0 \\ m\sin\lambda \\ 0 \\ m\cos\lambda \end{pmatrix} \tag{5}$$

And in the lab frame,

$$s_l^{Lab} = H_l \begin{pmatrix} \gamma_e & 0 & 0 & \beta_e\gamma_e \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta_e\gamma_e & 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} 0 \\ m\sin\lambda \\ 0 \\ m\cos\lambda \end{pmatrix} = H_l \begin{pmatrix} m\gamma_e\beta_e\cos\lambda \\ m\sin\lambda \\ 0 \\ m\gamma_e\cos\lambda \end{pmatrix} \equiv H_l E \begin{pmatrix} cos\lambda \\ \frac{\sin\lambda}{\gamma_e} \\ 0 \\ \cos\lambda \end{pmatrix} \tag{6}$$

We have $\beta_e = \frac{v_e}{c} = 1/\sqrt{1-\gamma_e^2}$, $\gamma_e = \frac{E}{m_e} \approx 8000$ which implies that $\beta_e \approx 1$.

6 Asymmetries

Now, we sit happily in the lab frame.

$$p \quad = \quad M(1,0,0,0) \tag{7}$$

$$q \quad = \quad (\nu, -E'\sin\theta, 0, E - E'\cos\theta) \tag{8}$$

$$s_l s_h \quad = \quad H_h H_l E M(-\sin\alpha\frac{\sin\lambda}{\gamma_e} - \cos\lambda\cos\alpha) \tag{9}$$

$$\begin{aligned} q s_l \quad &= \quad H_l E(\nu\cos\lambda + \frac{\sin\lambda}{\gamma_e}E'\sin\theta - \cos\lambda(E - E'\cos\theta)) \\ &= \quad H_l E(\frac{Q^2}{2E}\cos\lambda + E'\frac{\sin\lambda}{\gamma_e}\sin\theta) \end{aligned} \tag{10}$$

$$
\begin{aligned}
qs_h &= H_h M[E' \sin\theta \sin\alpha - \cos\alpha(E - E'\cos\theta)] \\
&= H_h M[E' \sin\theta \sin\alpha - \cos\alpha(\nu + \frac{Q^2}{2E})]
\end{aligned} \tag{11}
$$

$$
ps_l = H_l M E cos\lambda \tag{12}
$$

$$
pq = M\nu \tag{13}
$$

Substituting the above into the expression of the cross-section (1), results:

$$
\begin{aligned}
\frac{d^3\sigma}{dxdyd\phi} &= \frac{e^4 ME}{4\pi^2 Q^4}(xy^2 F_1 + (1 - y - \frac{y^2\gamma^2}{4})F_2 + H_h H_l g_1[2xy(-\sin\alpha\frac{\sin\lambda}{\gamma_e} - \cos\alpha\cos\lambda) \\
&+ \frac{(\nu\cos\lambda + E'\sin\theta\frac{sin\lambda}{\gamma_e} - (E - E'\cos\theta)\cos\lambda)(\frac{E'\sin\theta\sin\alpha}{\gamma_e} - (E - E'\cos\theta)\cos\alpha)}{ME}] \\
&+ H_h H_l g_2[2xy(-\sin\alpha\frac{\sin\lambda}{\gamma_e} - \cos\alpha\cos\lambda) \\
&- 2x\frac{E'\sin\theta\sin\alpha - (E - E'\cos\theta)\cos\alpha}{E}\cos\lambda])
\end{aligned} \tag{14}
$$

The target is longitudinally polarized when $\alpha = 0^o$. The expression above reduces:

$$
\begin{aligned}
\frac{d^2\sigma}{dEd\Omega'} &= \frac{e^4 ME}{4\pi^2 Q^4}[xy^2 F_1 + (1 - y - \frac{y^2\gamma^2}{4})F_2 - H_h H_l g_1\frac{\sin\lambda}{\gamma_e}\frac{E'\sin\theta(\nu + \frac{Q^2}{2E})}{ME} \\
&+ \cos\lambda(2xy - \frac{Q^2(\nu + \frac{Q^2}{2E})}{2E^2 M}) + H_h H_l g_2\frac{xQ^2}{E^2}\cos\lambda]
\end{aligned} \tag{15}
$$

and transversely polarized when $\alpha = 90^o$, hence (14) simplifies to:

$$
\begin{aligned}
\frac{d^2\sigma}{dEd\Omega'} &= \frac{e^4 ME}{4\pi^2 Q^4}(xy^2 F_1 + (1 - y - \frac{y^2\gamma^2}{4})F_2 - H_h H_l g_1(\frac{\sin\lambda}{\gamma_e}(2xy - \frac{E'^2\sin\lambda^2}{ME}) \\
&+ \frac{Q^2 E'\sin\theta}{2E^2 M}\cos\lambda) - H_l H_h g_2(2xy\frac{\sin\lambda}{\gamma_e} + \cos\lambda\frac{2x\beta E'\sin\theta}{E}))
\end{aligned} \tag{16}
$$

where we have used $Q^2 = 2EE'(1 - \cos\theta)$, and $\nu = E - E'$.
The parallel asymmetry is defined as :

$$A_\parallel^{Jlab} = \frac{\frac{d^2\sigma}{d\Omega dE'}^{\swarrow\Uparrow} - \frac{d^2\sigma}{d\Omega dE'}^{\nearrow\Uparrow}}{\frac{d^2\sigma}{d\Omega dE'}^{\swarrow\Uparrow} + \frac{d^2\sigma}{d\Omega dE'}^{\nearrow\Uparrow}}$$

$$= \frac{g_1[(2xy - \frac{Q^2(\nu+\frac{Q^2}{2E})}{2ME^2})\cos\lambda + \frac{E'\sin\theta(\nu+\frac{Q^2}{2E})}{ME}\frac{\sin\lambda}{\gamma_e}] - g_2\frac{xQ^2}{E^2}\cos\lambda}{xy^2F_1 + (1-y-\frac{y^2\gamma^2}{4})F_2} \quad (17)$$

and the transverse asymmetry :

$$A_\perp^{Jlab} = \frac{\frac{d^2\sigma}{d\Omega dE'}^{\swarrow\Leftarrow} - \frac{d^2\sigma}{d\Omega dE'}^{\nearrow\Leftarrow}}{\frac{d^2\sigma}{d\Omega dE'}^{\swarrow\Leftarrow} + \frac{d^2\sigma}{d\Omega dE'}^{\nearrow\Leftarrow}}$$

$$= \frac{g_1[(2xy - \frac{E'^2\sin\theta^2}{ME})\frac{\sin\lambda}{\gamma_e} + \frac{Q^2E'\sin\theta}{2ME^2}\cos\lambda] + g_2[2xy\frac{\sin\lambda}{\gamma_e} + \cos\lambda\frac{2xE'\sin\theta}{E}]}{xy^2F_1 + (1-y-\frac{y^2\gamma^2}{4})F_2} \quad (18)$$

($\swarrow\Leftarrow$: $H_l = -1, H_h = 1$)

The polarized and unpolarized structure functions are:

$$g_1 = \frac{F_2}{2x(1+R)}(A_1 + \gamma A_2) \quad (19)$$

$$g_2 = \frac{F_2}{2x(1+R)}(\frac{A_2}{\gamma} - A_1) \quad (20)$$

$$F_1 = \frac{F_2(1+\gamma^2)}{2x(1+R)} \quad (21)$$

Substituting g_1 and g_2 into (18) and (20) we obtain $A_\parallel^{JLab}$ and $A_\perp^{JLab}$ in terms of A'_1 and A'_2:

$$\begin{aligned} A_\parallel^{Jlab} &= \frac{1}{xy^2(1+\gamma^2) + 2x(1-y-\frac{y^2\gamma^2}{4})(1+R)}((\frac{\sin\lambda}{\gamma_e}\frac{E'\sin\theta(\nu+\frac{Q^2}{2E})}{ME} \\ &+ \cos\lambda(xy(2-y) + \frac{xQ^2(1-\frac{y}{2})}{E^2}))A'_1 + (\frac{\sin\lambda}{\gamma_e}\frac{\gamma E'\sin\theta(\nu+\frac{Q^2}{2E})}{ME} \\ &+ 2xy\gamma(1-y-\frac{y^2\gamma^2}{4})\cos\lambda)A'_2) \end{aligned} \quad (22)$$

$$\begin{aligned} A_\perp^{Jlab} &= \frac{1}{xy^2(1+\gamma^2) + 2x(1-y-\frac{y^2\gamma^2}{4})(1+R)}((\frac{\sin\lambda}{\gamma_e}(\frac{-E'^2\sin\theta^2}{ME}) \\ &- \cos\lambda\frac{2xE'\sin\theta(1-\frac{y}{2})}{E})A'_1 + (\frac{\sin\lambda}{\gamma_e}(\gamma(2xy - \frac{E'^2\sin\theta^2}{ME}) + \frac{2xy}{\gamma}) \\ &+ \cos\lambda(\gamma\frac{Q^2E'\sin\theta}{2ME^2} + \frac{2xE'\sin\theta}{E\gamma}))A_2) \end{aligned} \quad (23)$$

The kinematic coefficients are defined as follows :

$$D' = \frac{xy(2-y)(1+\frac{y\gamma^2}{2})\cos\lambda + y\frac{E'(1+\frac{y\gamma^2}{2})\sin\theta\sin\lambda}{M\gamma_e}}{xy^2(1+\gamma^2)+2x(1-y-\frac{y^2\gamma^2}{4})(1+R)} \tag{24}$$

$$\eta' = \frac{2xy\gamma(1-y-\frac{y^2\gamma^2}{4})\cos\lambda + y\gamma\frac{E'\sin\theta\sin\lambda(1+\frac{y\gamma^2}{2})}{M\gamma_e}}{xy^2(1+\gamma^2)+2x(1-y-\frac{y^2\gamma^2}{4})(1+R)} \tag{25}$$

$$d' = \frac{\frac{2x}{E\gamma}(1+\frac{y\gamma^2}{2})E'\sin\theta\cos\lambda + \frac{\sin\lambda}{\gamma_e}(\gamma(2xy-\frac{E'^2\sin\theta^2}{ME})+\frac{2xy}{\gamma})}{xy^2(1+\gamma^2)+2x(1+R)(1-y-\frac{y^2\gamma^2}{4})} \tag{26}$$

$$\zeta' = \frac{\frac{\sin\lambda}{\gamma_e}(\frac{E'^2\sin\theta^2}{ME})+\frac{xE'(2-y)\sin\theta}{E}\cos\lambda}{\frac{2x}{E\gamma}(1+\frac{y\gamma^2}{2})E'\sin\theta\cos\lambda+(\gamma(2xy-\frac{E'^2\sin\theta^2}{ME})+\frac{2xy}{\gamma})\frac{\sin\lambda}{\gamma_e}} \tag{27}$$

Now, we write the virtual photon-nucleon asymmetries:

$$A'_1 = \frac{1}{D'(1+\eta'\zeta')}A_{\parallel}^{JLab} - \frac{\eta'}{d'(1+\eta'\zeta')}A_{\perp}^{JLab} \tag{28}$$

$$A'_2 = \frac{\zeta'}{D'(1+\eta'\zeta')}A_{\parallel}^{JLab} + \frac{1}{d'(1+\eta'\zeta')}A_{\perp}^{JLab} \tag{29}$$

When $\lambda = 0^o$ we have :

$$\begin{gathered} D' \to D \\ \eta' \to \eta \\ d' \to d \\ \zeta' \to \zeta \end{gathered}$$

¿From (22) and (23), we know that due to the $\frac{1}{\gamma_e}$ factor, the transverse component will have a non-negligible effect only when $\lambda \to 90^o$ or when $\theta \to 0^o$ (i.e. the small angle scattering, obviously seen from the form of $A_\perp$). For E94-010, $\frac{1}{\gamma_e}$ is always negligible.

Having the expressions for $A_{\parallel}^{Jlab}$ and $A_{\perp}^{Jlab}$ we can establish easily the polarized structure functions g_1 and g_2. Since g_2 is extremely tiny and very difficult to measure, we are interested in finding the first component of the polarized structure function only. Thus,

$$g_1 = \frac{[xy^2F_1+(1-y-\frac{y^2\gamma^2}{4})F_2][A_{\perp}^{Jlab}xy^2\gamma^2\cos\lambda + A_{\parallel}^{Jlab}(2xy\frac{\sin\lambda}{\gamma_e}+\frac{2xE'\sin\theta}{E}\cos\lambda)]}{2\cos\lambda\frac{\sin\lambda}{\gamma_e}(xy^3\gamma^2+\frac{x\nu E'^2\sin\theta^2}{ME^2}+x^2y^2(2-y-\frac{y^2\gamma^2}{2}))} \tag{30}$$

The green light is on now, and we can calculate the statistical errors using the above defined quantities.

7 Acknowledgements

We would like to express our gratitude to Prof. Zein-Eddine Meziani and Dr. Jian-Ping Chen for their comments and continuous support. In particular, we would like to express our appreciation to Alexandre Deur for his help and advice.

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