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Optimization of the ^{3}He Target Polarization

Sébastien L. Incerti
Thomas Jefferson National Laboratory
incerti@jlab.org

Enkeleida K. Lakuriqi
Temple University, PA
enka@euclid.math.temple.edu

Abstract

The optimization of the pick-up coils guarantees a better NMR signal. The ^{3}He polarization is determined by laser pumping and we only use the signal to deduce the polarization. Given a certain ^{3} He polarization, we shall try to optimize the signal by changing the pick-up coil geometry.

1 The Problem

NMR is one of the most useful and widely known methods to measure the ^{3}He polarization. Many aspects of this fascinating subject have been explored. Precise recipes for computing the flux in this framework have been presented in different theses from students in the E154 experiment. Considering that the polarization is oriented in the z-direction, as is the main-field, we apply a small RF field in the x-direction. If we view this system in the rotating frame with the same frequency as the RF field, the effective field is no longer along the z-direction. Moreover, the magnetic moment will also rotate around this new effective field. If we change the main field slowly, the effective field will also change slowly and the polarization will follow it. This is the famous Adiabatic Fast Passage (AFP) method. If we were to view the system from the lab frame, the polarization has non-zero component in x'-direction and another component in the xy-plane which is rotating with the frequency of the RF field. Therefore, if we put two pick-up coils along the z-direction, the change in polarization will produce a change in flux throughout the pick-up coils, and as the result a change in voltage across the two ends of the coils. This can be measured by a lock-in amplifier. The signal amplitude correponds to the moment when the effective field is along x'-direction, namely, the resonance. Since the signal is directly proportional to the polarization, the calibration of water can give us the value of ^{3}He polarization. The water signal is also dependent on the geometry of the pick-up coils, which is readily known. All of these facts encourage the idea of trying to maximize the signal by choosing the optimal size for the pick-up coils, which is the string subject of this technical note.

1.1 Magnetic Flux and Coil Modeling

The total magnetic flux that passes through the pick-up coils can be found by integrating the flux from a dipole of size $\mu P \rho dV$ in a volume element dV over the volume of the cell, where μ is the magnetic moment, P is the polarization, and n is the density of the dipoles. Before we compute the integral we assume that the cell is placed symmetrically between the two pick-up coils. To make the calculations easier the cell is considered to be a combination of a horizontal cylinder, a vertical cylinder and a sphere. Also the dimensions of the cell play a very important role in polarimetry analysis. When working with the H_2O cell we claim that it has uniform density of

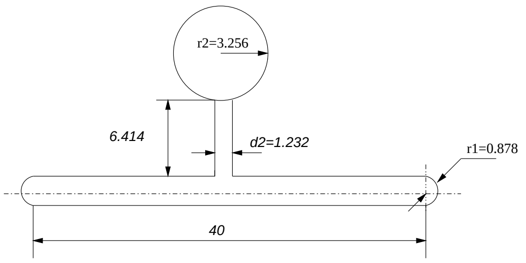

Figure 1: Dimensions of the target cell Cuervo, cm.

spins and polarization. The absolute polarization is given by the Boltzman distribution:

$$P_w = \tanh \frac{\mu_p B}{kT} \tag{1}$$

For the magnetic field we use the coordinate-free equation:

$$\vec{B} = \frac{\mu_0}{4\pi r^5}(3((\vec{m}\vec{r})\vec{r}) - \vec{m}r^2) \tag{2}$$

The flux produced by the coil's magnetic field is, (w stands for water):

$$\Phi_{tot} = \int_{V_w}\int_{A_{coil}} \mu_w B(\vec{r},\vec{r'}) P_w(\vec{r}) n_w(\vec{r}) d^3(\vec{r}) dA' \tag{3}$$

where μ_w magnetic moment for the proton, $B(\vec{r},\vec{r'})$ is the magnetic field flux from a dipole of unit magnitude at position $\vec{r}$ through a surface area dA' of the pick-up coils at position $\vec{r'}$, P_w the polarization.

The contribution of the target cell to the flux:

$$\Phi_{target} = \int_0^{-a/2}\int_0^{-b/2}\int_0^{l/2}\int_0^{R}\int_0^{\pi} \zeta_1 dx_0 dy_0 dy d\rho d\phi \tag{4}$$

and ζ_1:

$$\zeta_1 = \frac{8\rho(2(c-\rho\cos\phi)^2 - (x_0-\rho\sin\phi)^2 - (y_0-y)^2)}{((c-\rho\cos\phi)^2 + (x_0-\rho\sin\phi)^2 + (y_0-y)^2)^{\frac{5}{2}}} \tag{5}$$

The contribution of the pumping cell to the flux:

$$\Phi_{pumping} = \int_0^{-a/2}\int_0^{-b/2}\int_0^{l/2}\int_0^{R}\int_0^{\pi} \zeta_2 dx_0 dy_0 d\rho d\phi d\theta \tag{6}$$

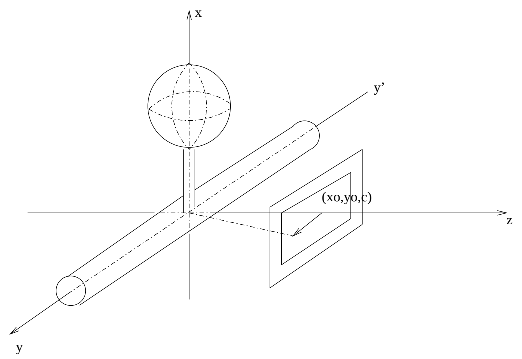

Figure 2: Coordiante system for Target Cell and the Pick-up Coils

where ζ_2 is:

$$\zeta_2 = \frac{4\rho^2 \sin\theta(2(c-\rho\cos\theta)^2 - (x_0 - d - \cos\phi\sin\theta)^2 - (y_0 - \rho\sin\phi\sin\theta)^2)}{((c-\rho\cos\theta)^2 + (x_0 - d - \rho\cos\phi\sin\theta)^2 + (y_0 - \rho\sin\phi\sin\theta)^2)^{\frac{5}{2}}} \tag{7}$$

The contribution of the transfer tube to the flux:

$$\Phi_{transfer} = \int_0^{-a/2}\int_0^{-b/2}\int_0^{l/2}\int_0^{R}\int_0^{\pi} \zeta_3 dx_3 dy_3 dx d\rho d\phi \tag{8}$$

$$\zeta_3 = \frac{4\rho(2(c-\rho\cos\phi)^2 - (y_3 + \rho\sin\phi)^2 - (x_3 - x)^2)}{((c-\rho\cos\phi)^2 + (y_3+\rho\sin\phi)^2 + (x_3 - x)^2)^{\frac{5}{2}}} \tag{9}$$

Putting equations (4), (6) and (8) together we obtain:

$$\begin{aligned} \Phi_{total} &= \int_{target}\int_{A_{coil}} \zeta_1 dx_0 dy_0 dy d\rho d\phi \\ &+ \int_{pumping}\int_{A_{coil}} \zeta_2 dx_0 dy_0 d\rho d\phi d\theta \\ &+ \int_{transfer}\int_{A_{coil}} \zeta_3 dx_3 dy_3 dx d\rho d\phi \end{aligned} \tag{10}$$

The five dimensional integral for each of the cells was computed using C Programing language. The flux numbers given in Table 1 were generated for each of the cells using carefully the cell dimensions and assuming that the pick-up coils are positioned parallel with respect to each other. From the

TARGET Cells	FLUX
Jin	41.53
Sysiphos	55.58
Nepheli	40.71
Armageddon	34.54
Be Happy	36.59
Don't Worry	37.22

Table 1: The cell fluxes in units of cm^2

generated flux values, it was observed that the target cell makes the major contribution to the total flux, whereas the contribution of the transfer tube is minuscule and that of the pumping cell is negative. This is in agreement with what we expected to find. Imagine our cell-pick-up coil system for a moment: the pick-up coils are near the target cell and the magnetic field lines of the dipole are directed towards the target cell whereas the magnetic field lines from the pumping cell are in the opposite direction. When computing the integral we take into consideration the vertical position of the cell with respect to the pick-up coils which improves the signal by approxiamtely 2 %.