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Calculation of the flux induced by a magnetized water cell

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Abstract

We present here the calculation and systematic studies of the flux induced into the pick-up coils by the magnetized water target at the NMR resonance.

1 Principle of the calculation

The calculation of the flux induced into the pick-up coils by the magnetized water target allows us to study systematic effects, such as the contribution of the different parts of the cell, the effect of the cell position between the pick-up coils, etc...

The water cell is made of three different parts : a spherical pumping cell, a cylindrical transfer tube and a cylindrical target chamber, as shown on Figure 1. The dimensions have been measured and are given in Table 1 (cf. [1]).

Table 1: *dimensions of the water cell and pick-up coils*

Part	**Dimension (cm)**
Pumping cell radius	3.256
Transfer tube radius	0.616
Transfer tube length	6.414
Target cell radius	0.878
Target cell length	40.000
Pick-up coil length	11.020
Pick-up coil heigth	1.974

The flux Φ through the pick-up coils is given by the double integral over the pick-up coil surface Σ :

$$\Phi = \int_\Sigma \vec{B}.\vec{dS} \tag{1}$$

where $\vec{B}$ is the macroscopic magnetic field created by the magnetized water and $\vec{dS}$ is a surface element, normal to the pick-up coil surface. The calculation is more easily performed using Green's theorem, where the integration over the whole surface is replaced by an integration along a closed path limiting the coil surface, taken as the external limit of the coil. The magnetic field $\vec{B}$ is then replaced by its corresponding potential vector $\vec{A}$:

$$\Phi = \oint_C \vec{A}.\vec{dl} \tag{2}$$

$\vec{A}$ is obtained from the integral of the single potential vectors created by each individual volume element dV of the target, located at the distance r from the pick-up coil, in Gaussian units (refer to [2]) :

$$\Phi = \oint_C \int_V \frac{\vec{m} \wedge \vec{r}}{\vec{r}^3}.\vec{dl}dV \tag{3}$$

$\vec{m}$ stands for the target density of magnetization. It is a common practice to assume m has no unit, instead of $A.m^{-1}$; the flux is then expressed in cm^2. We will assume the magnetization is aligned along the $-y$ axis of the cell, neglecting any polarization gradient at thermal equilibrium. To perform the integrations along the closed path and over the cell volume, we will divide the pick-up coils into mini elementary coils and the cell into cubical volume elements. Cartesian coordinates will be used. The central position of the cell is the default position, where both pick-up coils are centered on the middle of the target cell, as shown on Figure 1. All the flux given in this note will be normalized to the value of the flux in the default position. The position of each pick-up coil, thus the position of the water cell, can be adjusted as necessary. In the default position, the pick-up coils are parallel and 2 cm away from the center of the target cell, with respect to the y axis. Figure 2 illustrates the principle of the flux calculation and Figure 3 shows the sampling of the transfer tube, projected on the horizontal plane. Figure 4 summarizes the studies of the flux presented in this note.

2 Contribution of the different parts

Table 2 shows the flux contribution of each cell part (sphere, transfer tube and target cell). This is not a normalized flux, but the absolute flux in cm^2. The values have been obtained using the geometry given in Table 1, and sampling each volume element of the cell into 40 volume elements. 55 and 10 mini coils are used to integrate along the pick-up coil closed path. The spherical cell contribute negatively to the total flux.

Table 2: *contributions*

Part	**Flux (cm^2)**
Sphere	-4.13694
Transfer tube	0.24392
Target	42.29140
Total	38.39830

3 Effect of the cell vertical position

We show on Figure 5 the dependence of the flux value with the vertical position of the cell, along the x axis. Our result is in excellent agreement with the calculation by Ioannis Kominis, E94-010 Collaboration Meeting, July 17, 1999.

4 Effect of the cell vertical angle

Figure 6 shows the variation of the flux when the vertical inclination of the cell is changed, in the (xOz) plan.

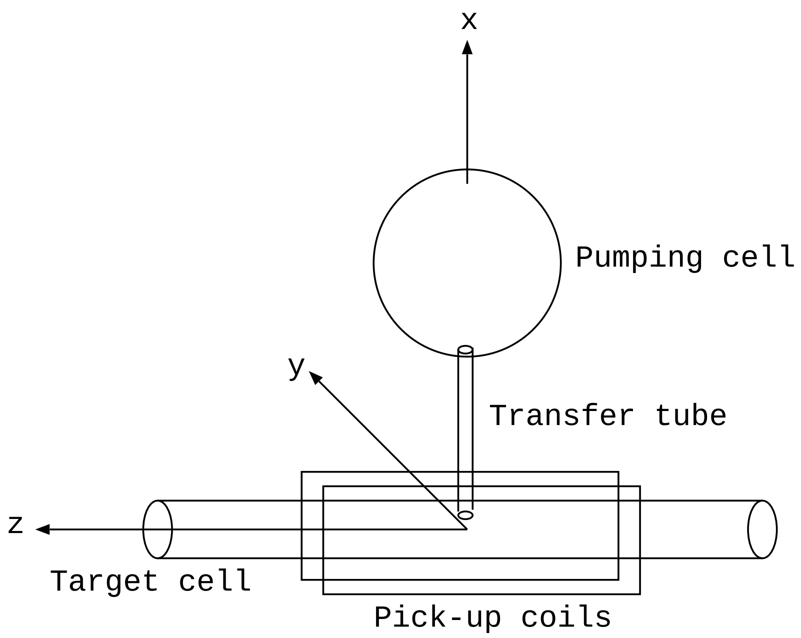

Figure 1: the water cell and the referential.

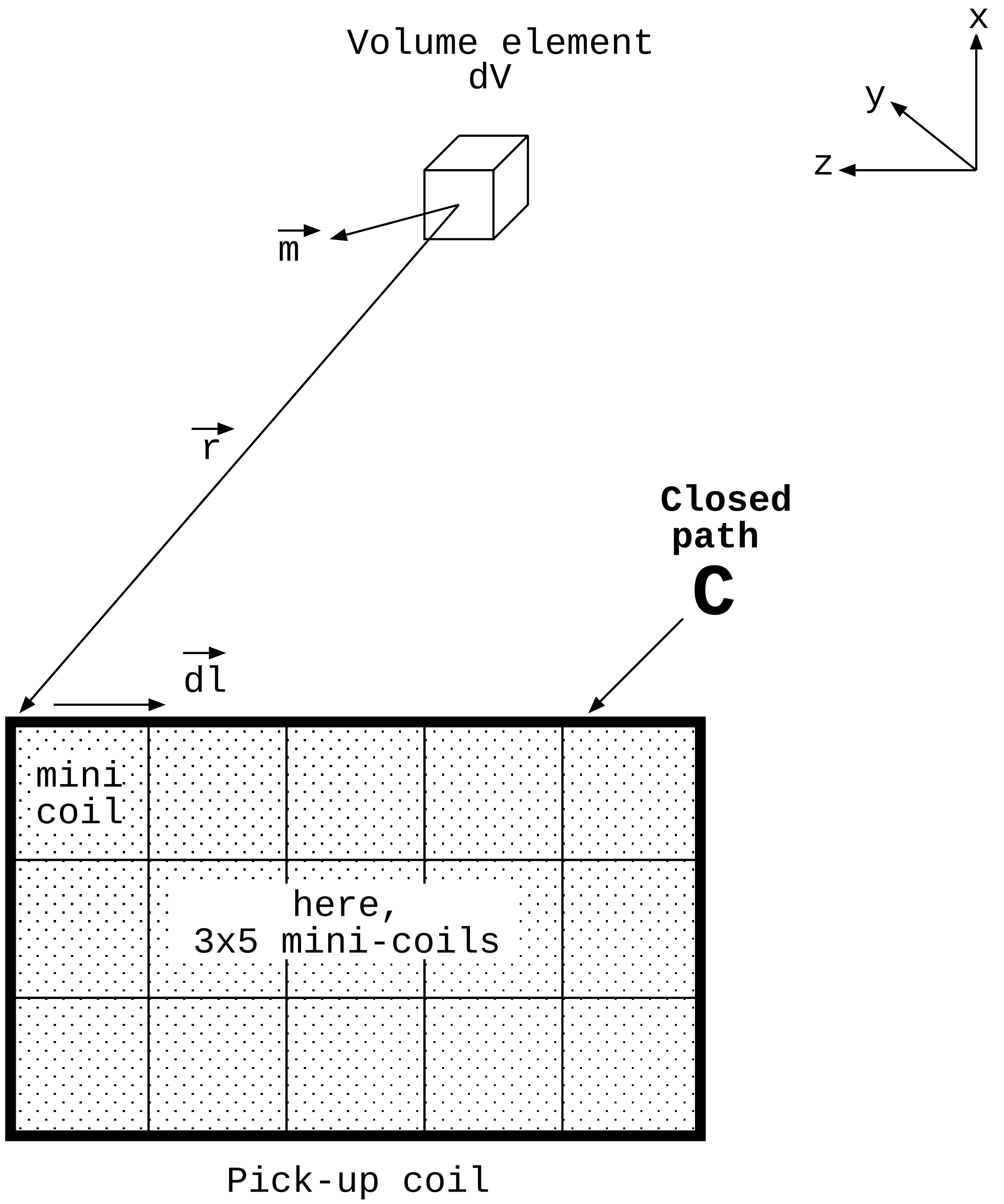

Figure 2: principle of the flux calculation.

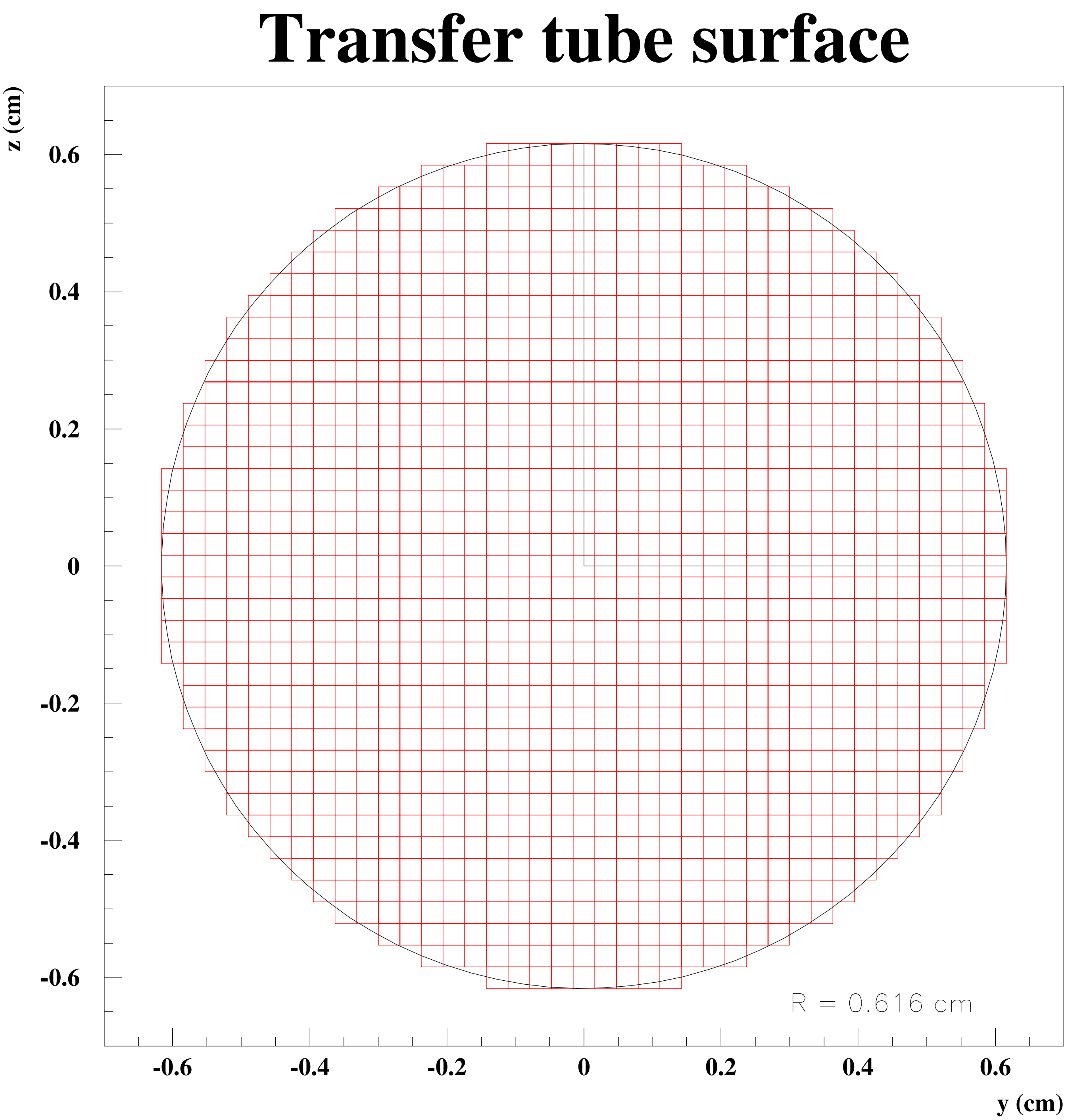

Figure 3: example of sampling of the transfer tube surface. 40 volume elements are spread along the diameter of the tube.

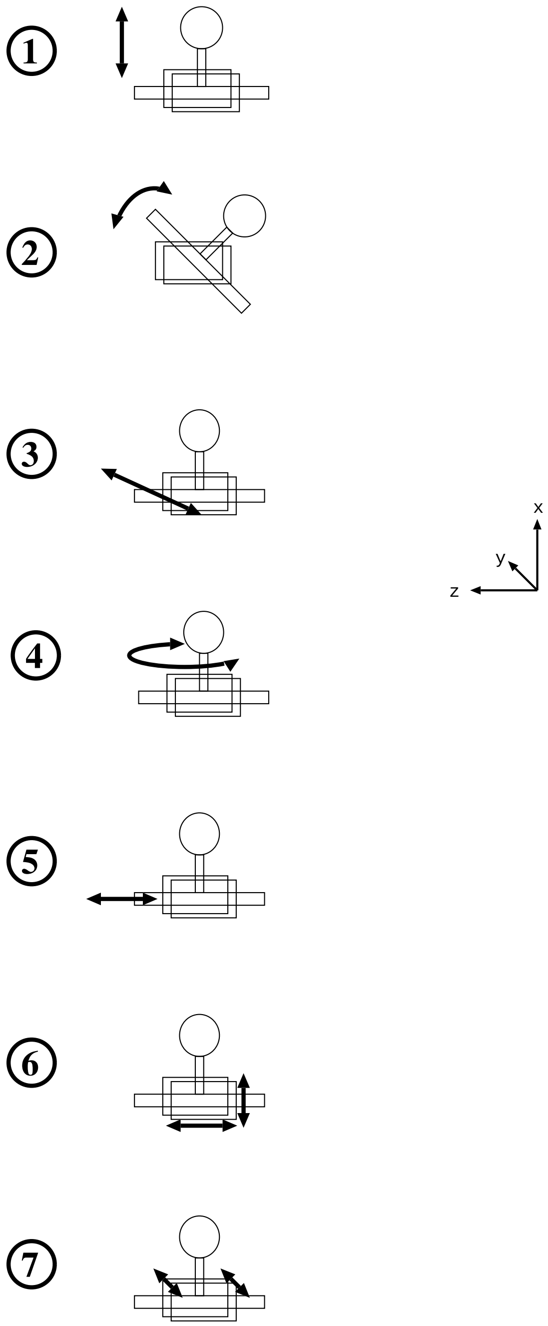

Figure 4: seven step flux study.

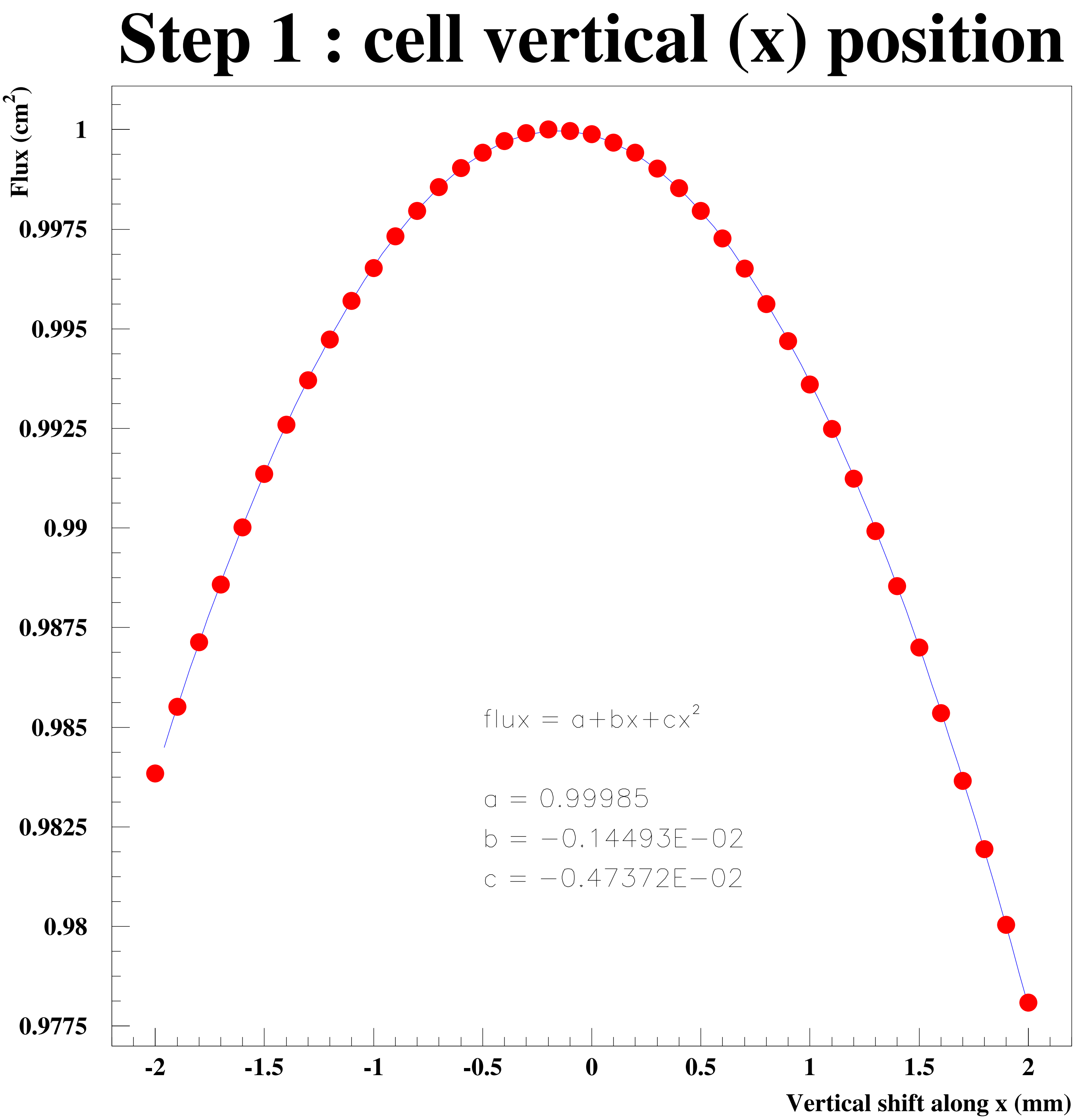

Figure 5: step 1 : flux versus cell vertical (x) position.

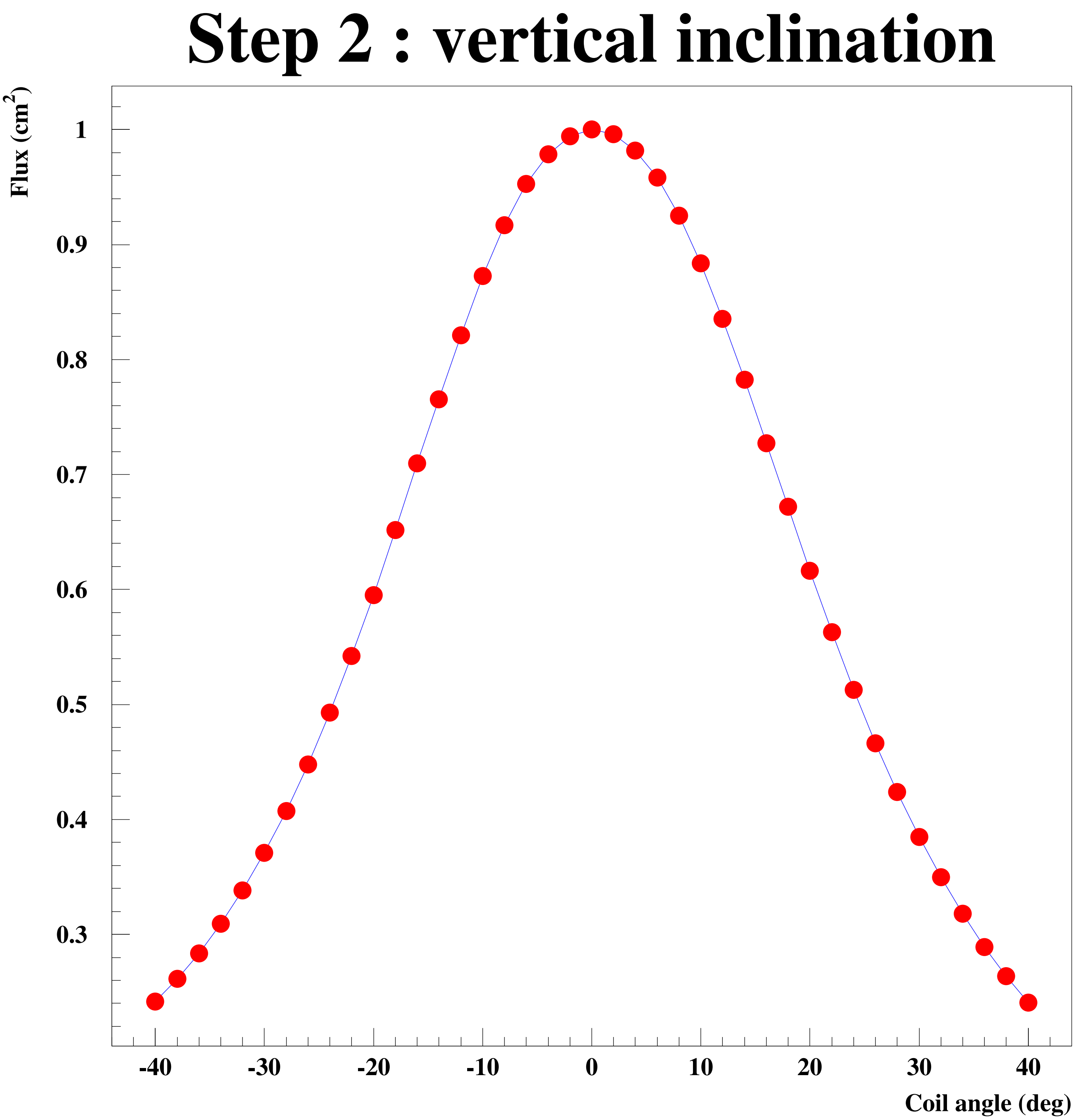

Figure 6: step 2 : flux versus cell vertical angle.

5 Effect of the cell horizontal position

We show on Figure 7 the dependence of the flux value with the horizontal position of the cell, along the y axis.

6 Effect of the cell horizontal angle

We show on Figure 8 the dependence of the flux value with the horizontal twist angle of the cell, in the (yOz) plan. The maximum horizontal angle the cell can reach is 11.3 deg.

7 Effect of the cell longitudinal position

We show on Figure 9 the dependence of the flux value with the horizontal position of the cell, along the z axis.

8 Optimization of the pick-up coils dimensions

Figure 10 shows the dependence of the flux value with the length and height of the pick-up coils. The figure has been obtained by choosing 40 length and height values for the pick-up coils. There is a clear maximum obtained when the length of both pick-up coils is maximum, 40 cm here, and when the height is close to 4 cm. during E94-010 and E95-001, the length was 11.020 cm and the height 1.974 cm. The optimum coil dimensions should be calculated from this study. Figure 11 shows the ratiop flux/surface, that is the ratio signal/noise if we assume the noise is proportional to the coil's surface.

9 Effect of the distance between the pick-up coils

Figure 12 shows the effect of the horizontal separation between the pick-up coils, along the y axis.

References

[1] Ioannis Kominis, kominis@jlab.org. Private communication.

[2] Classical Electrodynamics, J. D. Jackson, Ed. 1, p. 146 (1962)

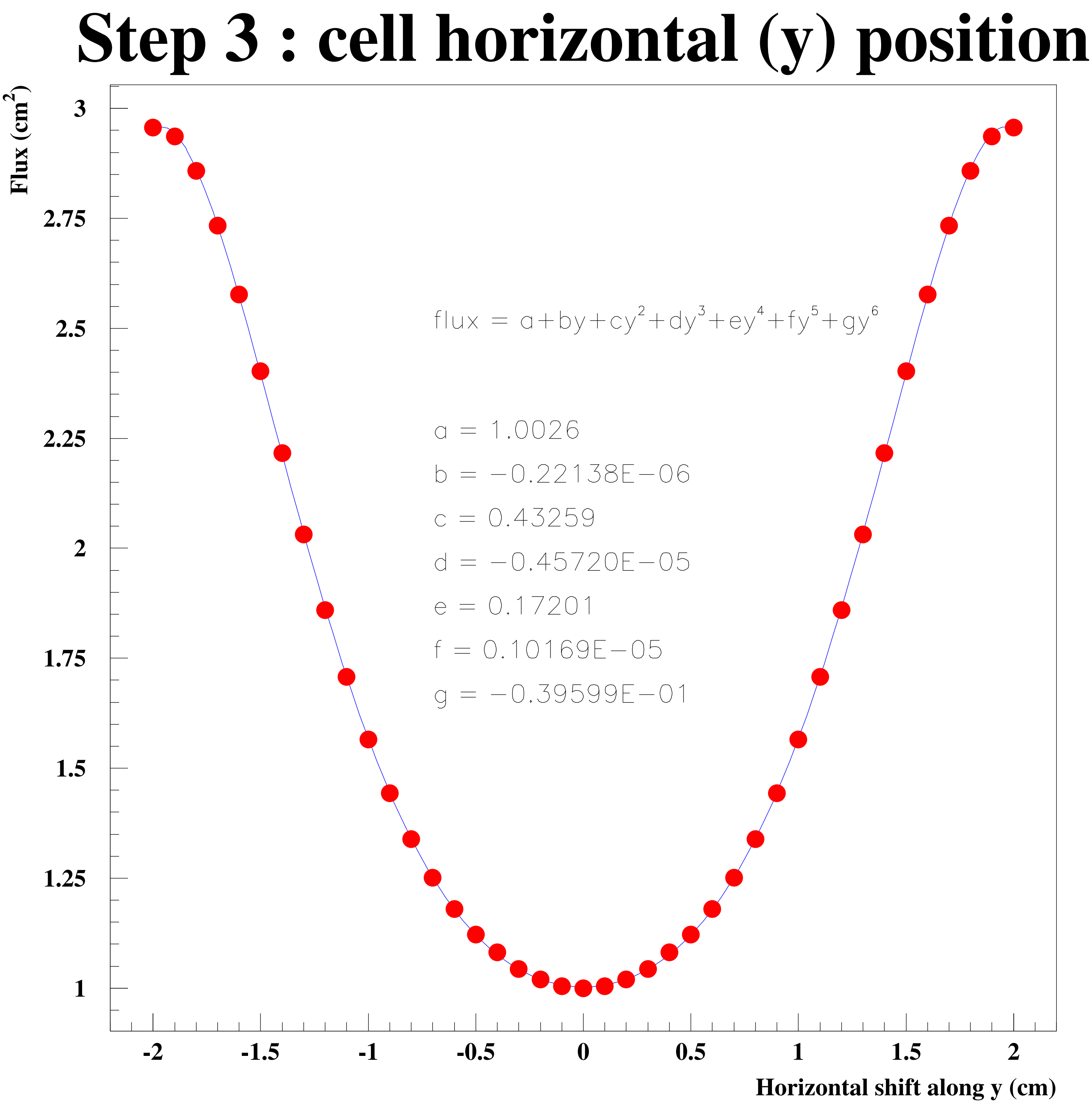

Figure 7: step 3 : flux versus cell horizontal (y) position.

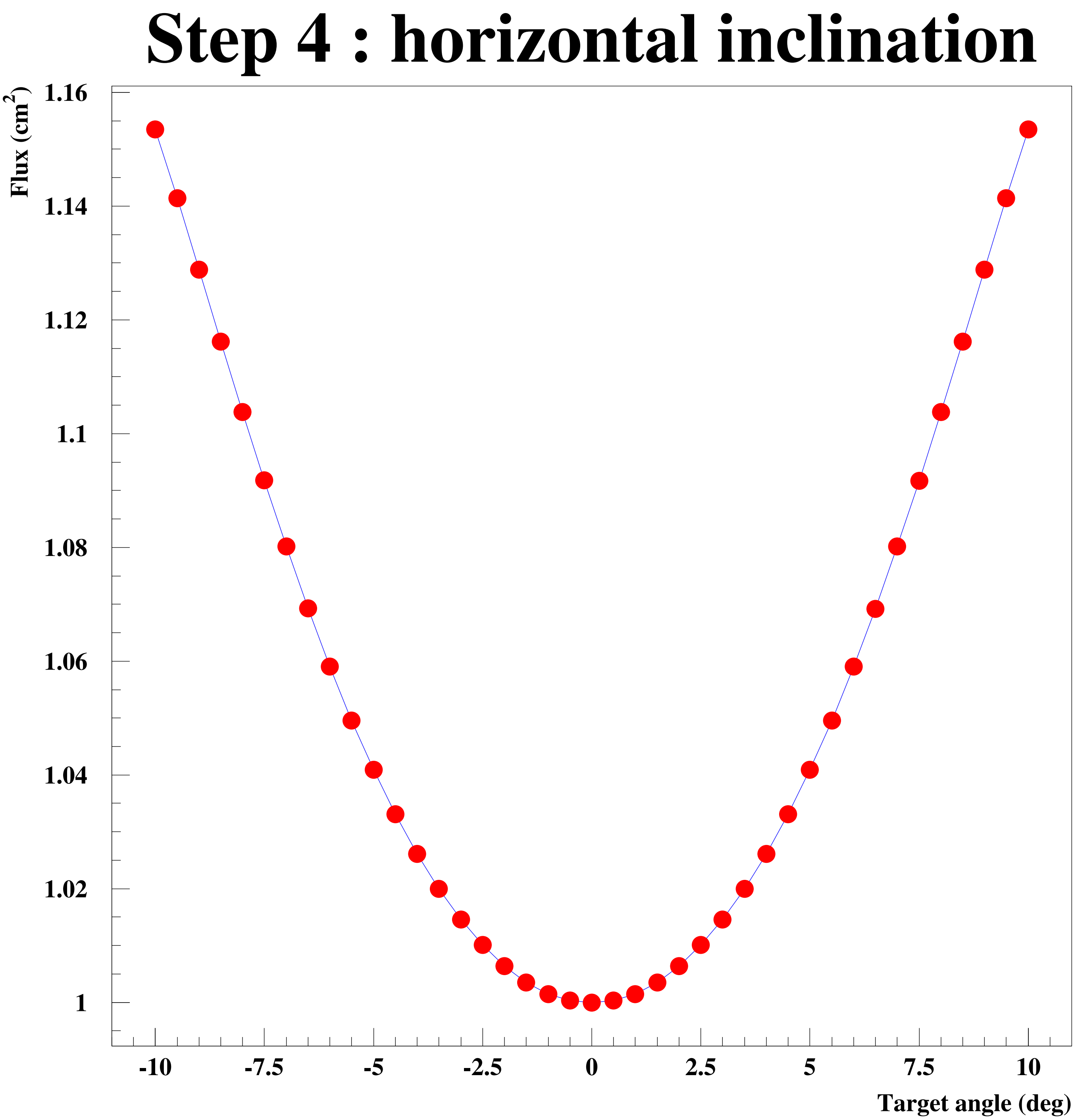

Figure 8: step 4 : flux versus cell horizontal angle.

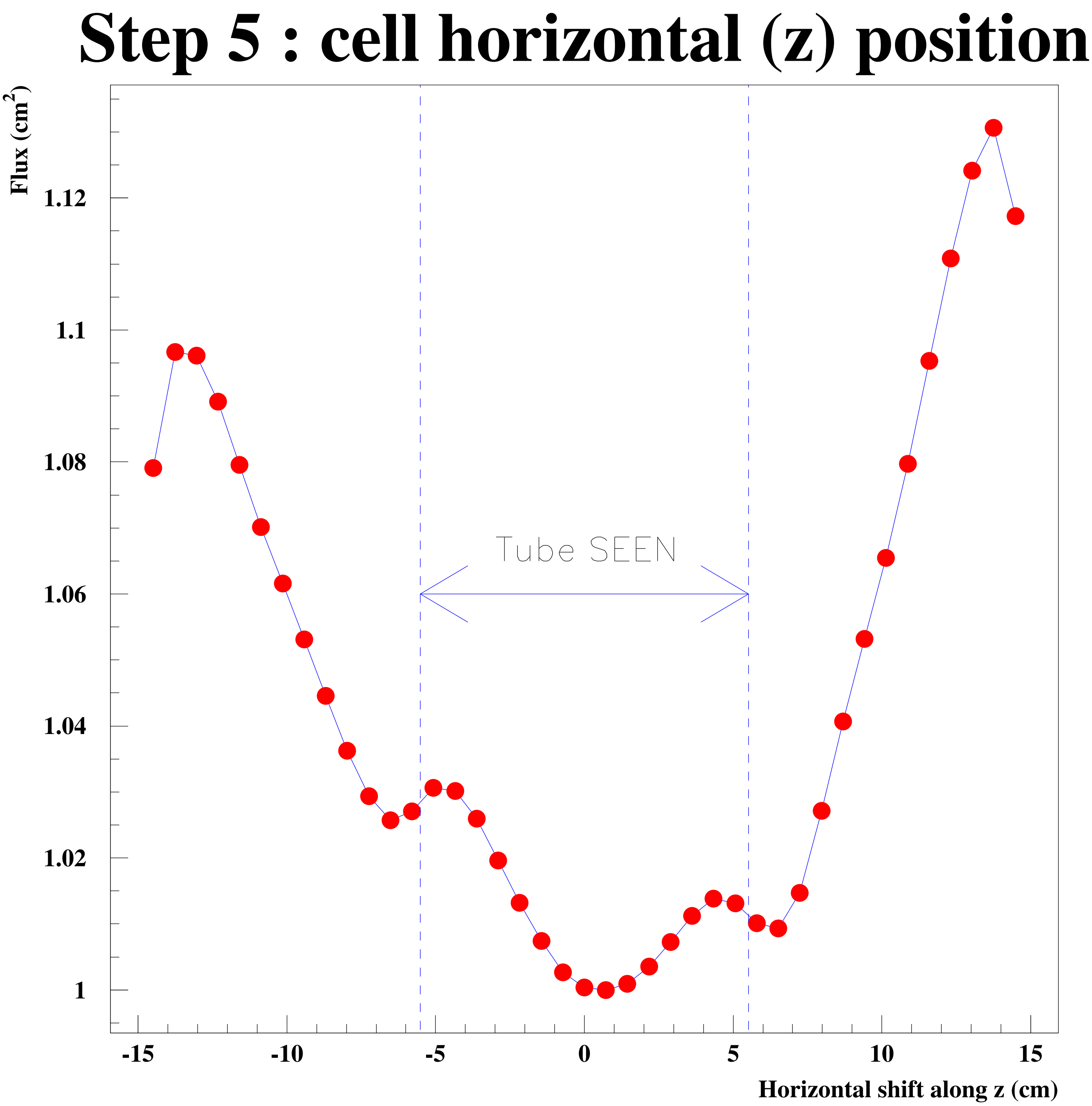

Figure 9: step 5 : flux versus cell horizontal (z) shift. The dashed lines show the region where the pick-up coils see part of the transfer tube.

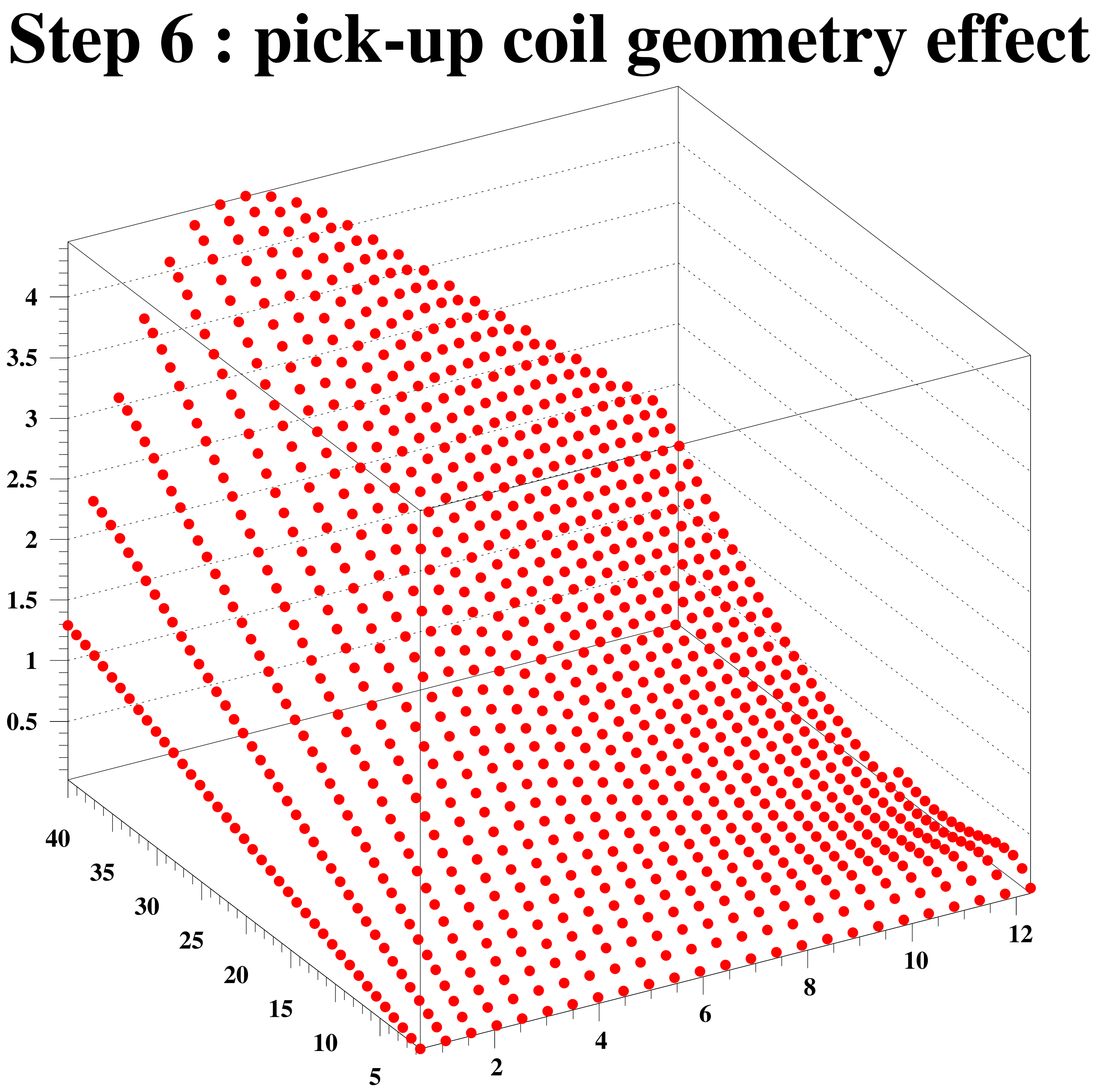

Figure 10: flux versus pick-up coil dimensions. Length (horizontal) is on the left side, height (vertical) is on the right side, given in cm.

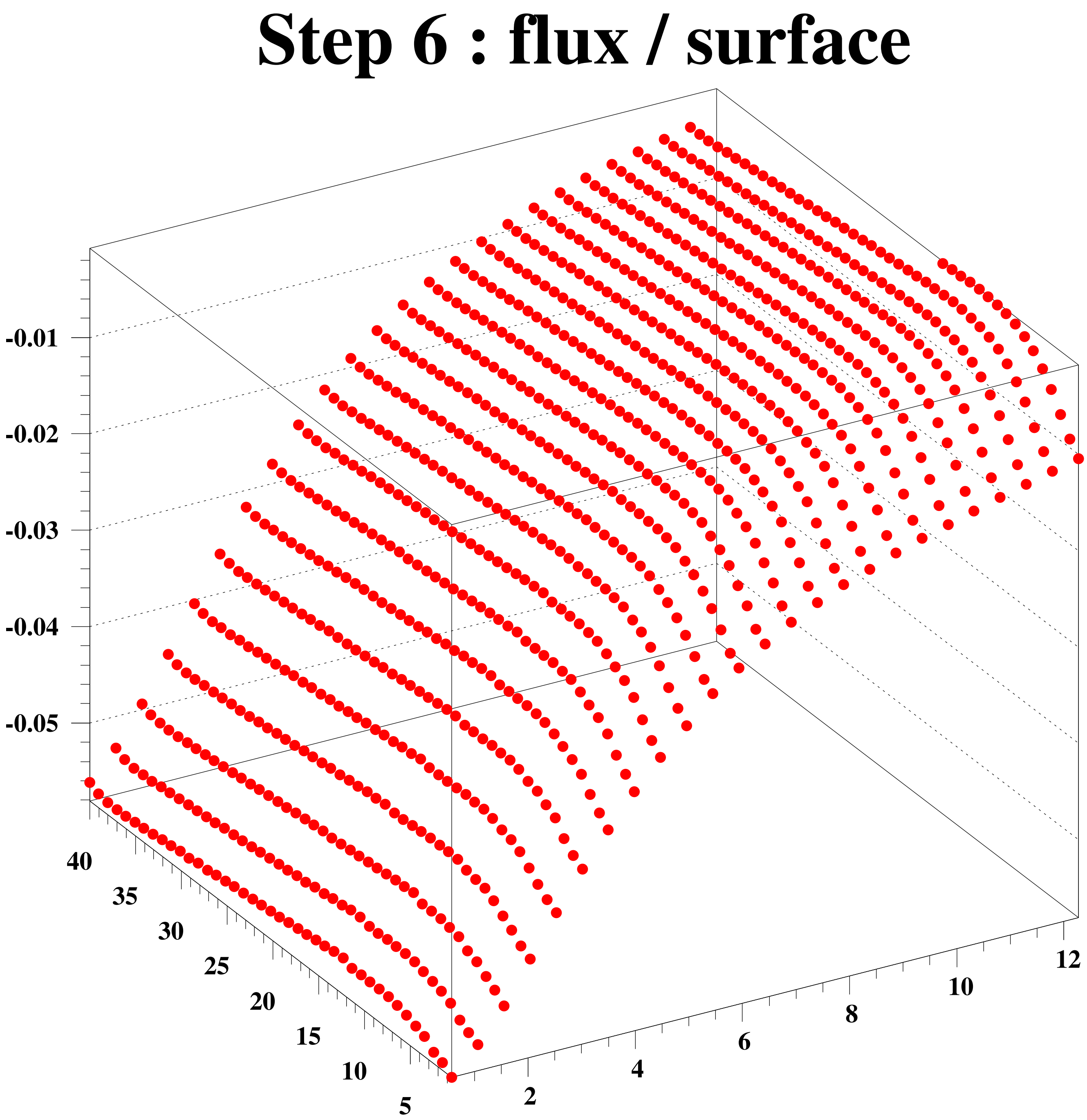

Figure 11: ratio flux over pick-up coil surface. Length (horizontal) is on the left side, height (vertical) is on the right side, given in cm. The vertical flux axis is negative for better viewing.

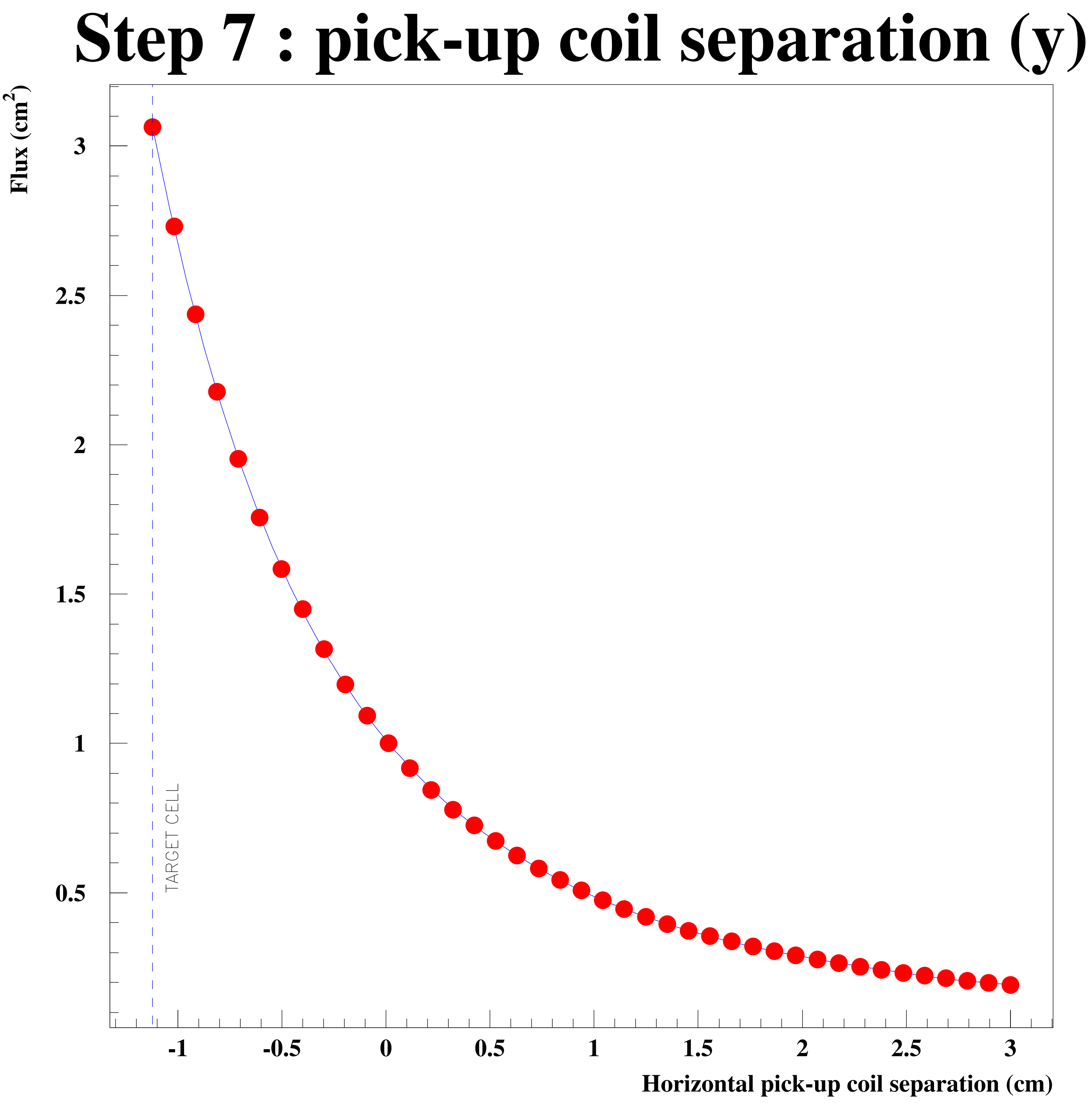

Figure 12: flux versus pick-up separation along the y axis. 0 corresponds to the default pick-up coil position, where they are parallel and respectively 2 cm away from the center of the target cell.