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Calculation of the water calibration constant for E94-010

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Abstract

We present in this note the calculation of the E94-010 polarized target calibration constant using the water target.

1 Principle of the water calibration

Three reliable water calibrations of the polarized Helium 3 target have been performed during E94-010. These water calibrations allow us to measure the well known water thermal equilibrium polarization, when the water sample is placed in a static magnetic holding field. The three calibrations are summarized in Table 1. Each calibration consists of many averaged NMR sweeps, like 500, performed one after the other. Noisy sweeps are swept away.

Table 1: *the three E94-010 water calibrations*

Date	**Number of sweeps**
09/25/98	123
11/04/98	271
11/25/98	213

The water calibration constant c is usually expressed in mV per percent per amagat per cm^2, unit of flux, mV/(% amg cm^2), so that the polarization P_{He} (in percent) of any Helium cell can be written :

$$P_{He} = \frac{S_{He}}{c\,[He]\,\Phi_{He}} \tag{1}$$

where S_{He} is the Helium NMR signal (in mV), Φ_{He} is the Helium target flux (in cm^2), which should be estimated for each single Helium NMR measurement from the Helium target position, and $[He]$ is the Helium density in the cylindrical target cell (in amagat). We recall : 1 amagat = 2.689E9 cm^{-3}. The constant c is given by :

$$c = \frac{1}{P_w \Phi_w} S_w \frac{G_{He}}{G_w} \frac{\mu_{He}}{\mu_p} \frac{1}{[p]} \tag{2}$$

- the value of P_w is calculated from our model of the the Bloch equations, knowing the T_1 and T_2 relaxation times, for the up and down sweeps. P_w takes different values for the up sweep and the down sweep.

- the flux induced into the pick-up coils Φ_w is calculated from our model of the water target. Refer to [8].

- S_w is the amplitude of the water signals extracted from the Bloch fit to the water NMR sweeps, as explained in [2]. S_w has been measured six times, three times for the up sweeps of the three water calibrations and three times for the down sweeps of the three water calibrations.

- G_{He} and G_w are the preamplifier gain and are set by hand once down in the Hall (usually, G_w=100 or 200 and G_{He}=1), before each water calibration.

- the density of protons in the water cell is estimated to be $[p] = 6.670.10^{22} \pm 0.005$ cm $^{-3} = 2482 \pm 2$ Amagats at 22^oC [2], which is the average temperature of Hall A during the NMR water measurements.
- the ratio μ_{He}/μ_p is well known from the literature and equal to $\mu_{He}/\mu_p = \frac{1}{1.313}$.

Be aware this definition of the water calibration constant is different from the one we gave in [1], but is of common usage.

2 Extraction of S_w

For each water calibration, we use the following procedure :

- selection of clean single sweeps, applying a noise cut on the X or Y lock-in channels Abs(maximum-minimum) less then cut value, for the up and down ramps, as shown on Figure 1. The cut value is given in Table 2.

Table 2: *noise cuts*

Date	**Cut (μV)**
09/25/98	50
11/04/98	30
11/25/98	300

- arithmetic average of the clean single sweeps.
- fit to the averaged sweeps, using the Bloch functional form, assuming $T_1 = T_2$. Refer to [2] for details.
- extract the ratio $R = S_w^{UP}/S_w^{DOWN}$ of the amplitudes measured for the up and down ramps.

The statistical combination of the three R values allows us to determine rather precisely the value of the longitudinal relaxation time T_1, using the method exposed in [3]. The extracted T_1 value is then introduced back into the Bloch functional form to fit all the up and down sweeps, until the extracted T_1 value from the ratio R value obtained with the new functional form is equal to the T_1 value introduced into the functional form. The error on the up and down NMR signals obtained from our fit is purely statistical. Assuming the R measurements are independent, we compute the mean value on R and its error using :

$$R = \frac{\sum_{i=1}^{i=3} \frac{R_i}{\left[\sigma_{R_i}^{stat}\right]^2}}{\sum_{i=1}^{i=3} \frac{1}{\left[\sigma_{R_i}^{stat}\right]^2}} \tag{3}$$

$$\sigma_R = \frac{1}{\sqrt{\sum_{i=1}^{i=3} \frac{1}{\left[\sigma_{R_i}^{stat}\right]^2}}} \tag{4}$$

where the sum is performed over each water calibration. The statistical error on each R measurement comes from the statistical errors on the amplitude of the up and down signals, obtained from the fit. The average statistical error affecting each value of the NMR up or down averaged signal, corresponding to each holding field value (373 values, ranging from 18 G to 25 G), has been calculated using the following procedure : each single sweep (among the 123 sweeps for the 09/25/98 water calibration, for example) is fit to a straight line. We determine the deviation δy_i between the fit and the measured sweep for the corresponding holding field value H_i, i ranging from 1 to 373, and we finally plot the distribution of δy_i for the 123 sweeps. The variance σ_i^2 of the corresponding parent population is obtained from the variance s_i^2 of the sample population through the approximation :

$$\sigma_i^2 \sim s_i^2 = \frac{1}{123-1} \sum_{j=1}^{123} \left[\delta y_{i,j} - \delta \bar{y}_i\right]^2 \tag{5}$$

where $\delta \bar{y}_i$ is the mean of the distribution, and the sample population error on the mean is given by :

$$s_{\mu,i} = \frac{s_i}{\sqrt{123}} \tag{6}$$

for each H_i value. Finally, the average statistical error affecting each value of the NMR up or down averaged signal, corresponding to each holding field value (373 values, ranging from 18 G to 25 G), is given by the mean of the distribution of the 373 values of $s_{\mu,i}$. We give the final average statistical error in Table 3.

Our final value for $R = S_w^{UP}/S_w^{DOWN}$ and its statistical error is then :

$$R = 0.839 \pm 0.010 \tag{7}$$

leading to a T_1 value of :

$$T_1 = 3.0 \pm 0.4s \tag{8}$$

in very good agreement with the world value $T_1 = 2.95$s at 20^{o}C. According to [4], we obtain for T_2 :

$$T_2 = 2.7 \pm 0.4s \tag{9}$$

working with a RF field H_1 of 90.8 ± 0.5 mG, as measured in [4]. We will assume T_1 and T_2 are 100% correlated.

We give in Table 4 the raw amplitudes obtained for the three water calibrations, with the corresponding preamplifier gain. Our Mathematica (with its precision !) model of the Bloch equations gives us the corresponding water sample polarization values :

Table 3: *statistical error on the averaged NMR signal heights*

Date	**Ramp**	**Error (μV)**
09/25/98	Up	0.49
09/25/98	Down	0.48
11/04/98	Up	0.27
11/04/98	Down	0.27
11/25/98	Up	1.87
11/25/98	Down	1.87

- UP signal : 6.39763418856248211E-9
- DOWN signal : 7.60871453305072265E-9

Table 4: *raw amplitudes*

Date	**Ramp**	**Gain**	**Amplitude $\pm$ statistical error (μV)**
09/25/98	Up	200	13.11 $\pm$ 0.22
09/25/98	Down	200	15.52 $\pm$ 0.22
11/04/98	Up	100	6.48 $\pm$ 0.12
11/04/98	Down	100	7.83 $\pm$ 0.13
11/25/98	Up	1000	66.85 $\pm$ 0.89
11/25/98	Down	1000	79.54 $\pm$ 0.93

3 Water calibration constant values

Several effects must be taken into account to obtain the experiment averaged water calibration constant :

- variations of the response of the pick-up coil circuit, monitored during the experiment through the Q curves (not only after cell explosions, cf. [6], but also due to temperature changes, cf. [7]). The Q curves are shown on Figure 6 ; the correction factor is given in Table 5 and is calculated from the relative variation of the circuit gain at 91 kHz, between two Q curves, which calculation is explained in [6]. The gain values and their relative variation are given on Figure 6.

Table 5: *Q curve multiplicative correction to S_w*

Date	Correction (no unit)
09/25/98	1.0000 (reference)
11/04/98	1.0033
11/25/98	1.0078

- position of the water cell and of the pick-up coils, affecting the flux Φ_w induced into the pick-up coils, cf. [8]. We use here the absolute positions of each corner of each pick-up coil, according to [9]. We give in Table 6 and in Table 7 the locations of each corner we have used for each pick-up coil, using the referential quoted in [8]. Refer to Figure 8. We give our flux estimations Φ_w for each calibration in Table 8. We also give the flux values for the default pick-up coils position, where they are perfectly parallel and each 2 cm away from the target cell center.

Table 6: *position of pick-up coil A corners (in cm)*

Date	Corner 1	Corner 2	Corner 3	Corner 4
09/25/98	x = 0.987	x = 0.987	x = -0.987	x = -0.987
09/25/98	y = 1.661	y = 1.734	y = 1.734	y = 1.661
09/25/98	z = 5.510	z = -5.510	z = -5.510	z = 5.510
11/04/98	x = 0.987	x = 0.987	x = -0.987	x = -0.987
11/04/98	y = 1.938	y = 1.953	y = 1.953	y = 1.938
11/04/98	z = 5.510	z = -5.510	z = -5.510	z = 5.510
11/25/98	x = 0.987	x = 0.987	x = -0.987	x = -0.987
11/25/98	y = 1.986	y = 2.034	y = 2.034	y = 1.986
11/25/98	z = 5.510	z = -5.510	z = -5.510	z = 5.510

- drift of the holding field during the water calibrations, which may be caused by temperature / current changes in the Helmholtz power supplies. The drift of the holding field is estimated by slicing each set of sweeps into 50 sweeps for the up and down ramp and is close to $\Delta H/H = 0.001$ per hour ; consequently it reduces the signal height. This corresponds to a drift of 1.167E-5 Gauss per sweep. To estimate the effect of the drift, we generate successive signal shapes from our Bloch functional form, separated by that amount of drift and estimate the height reduction of the average of the sweeps, as a function of the number of sweeps. The water up and down signal heights should be multiplied by the factors given in Table 9. The effect of the height reduction is shown on Figure 7.

Table 7: *position of pick-up coil B corners (in cm)*

Date	Corner 1	Corner 2	Corner 3	Corner 4
09/25/98	x = 0.987	x = 0.987	x = -0.987	x = -0.987
09/25/98	y = -1.849	y = -1.905	y = -1.905	y = -1.849
09/25/98	z = 5.510	z = -5.510	z = -5.510	z = 5.510
11/04/98	x = 0.987	x = 0.987	x = -0.987	x = -0.987
11/04/98	y = -1.442	y = -1.600	y = -1.600	y = -1.442
11/04/98	z = 5.510	z = -5.510	z = -5.510	z = 5.510
11/25/98	x = 0.987	x = 0.987	x = -0.987	x = -0.987
11/25/98	y = -1.409	y = -1.529	y = -1.529	y = -1.409
11/25/98	z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 8: *flux contributions* Φ_w *for each calibration (in* cm^2*)*

Date	Sphere	Tube	Target	Total (Φ_w)
09/25/98	-4.27	0.00	50.67	46.39
11/04/98	-4.30	-0.11	54.17	49.76
11/25/98	-4.29	-0.13	54.59	50.17
Default	-4.14	0.24	42.28	38.39

- correction to the fit height using the Bloch lineshape, due to the difference $T_1 \neq T_2$. From our Bloch model, we estimate that the height of the fit to the UP water signal must be reduced by 0.36% and the height of fit to the DOWN water signal must be reduced by 0.21% to account for $T_1 = 2.95$ s (world value) and $T_2 = 2.7$ s.

After taking into account those corrections, we obtain the calibration constant value for the experiment :

$$c = (6.578 \pm 0.039).10^{-4} mV/percent/amg/cm^2 \qquad (10)$$

4 Water calibration constant per cell

We give in Table 29 the value of the water calibration constant multiplied by the Helium flux for each Helium cell (given in Table 28). The Helium flux has been calculated assuming a constant magnetization distribution within the target (cf. [7]).

Table 9: *field drift correction*

Date	Number of sweeps	Multiplicative correction
09/25/98	123	1.0035
11/04/98	271	1.0156
11/25/98	213	1.0099

Table 10: *cell dimensions - DON'T WORRY - (in cm)*

Part	Dimension
Pumping cell radius	3.165
Transfer tube radius	0.680
Transfer tube length	5.910
Target cell radius	0.843
Target cell length	38.58

Table 11: *position of pick-up coil A corners - DON'T WORRY - (in cm)*

Corner 1	Corner 2	Corner 3	Corner 4
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = 1.598	y = 1.629	y = 1.629	y = 1.598
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 12: *position of pick-up coil B corners - DON'T WORRY - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = -1.888	y = -2.030	y = -2.030	y = -1.888
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 13: *cell dimensions - BE HAPPY - (in cm)*

Part	**Dimension**
Pumping cell radius	3.220
Transfer tube radius	0.600
Transfer tube length	5.334
Target cell radius	0.833
Target cell length	40.00

Table 14: *position of pick-up coil A corners - BE HAPPY - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = 2.110	y = 2.034	y = 2.034	y = 2.110
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 15: *position of pick-up coil B corners - BE HAPPY - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = -1.335	y = -1.577	y = -1.577	y = -1.335
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 16: *cell dimensions - ARMAGEDDON - (in cm)*

Part	**Dimension**
Pumping cell radius	3.050
Transfer tube radius	0.600
Transfer tube length	5.910
Target cell radius	0.805
Target cell length	39.27

Table 17: *position of pick-up coil A corners - ARMAGEDDON - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = 2.039	y = 2.065	y = 2.065	y = 2.039
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 18: *position of pick-up coil B corners - ARMAGEDDON - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = -1.389	y = -1.516	y = -1.516	y = -1.389
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 19: *cell dimensions - NEPHELI - (in cm)*

Part	**Dimension**
Pumping cell radius	3.089
Transfer tube radius	0.652
Transfer tube length	5.940
Target cell radius	0.860
Target cell length	39.69

Table 20: *position of pick-up coil A corners - NEPHELI - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = 2.012	y = 2.065	y = 2.065	y = 2.012
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 21: *position of pick-up coil B corners - NEPHELI - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = -1.425	y = -1.588	y = -1.588	y = -1.425
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 22: *cell dimensions - SYSIPHOS - (in cm)*

Part	**Dimension**
Pumping cell radius	3.165
Transfer tube radius	0.621
Transfer tube length	5.880
Target cell radius	0.902
Target cell length	39.60

Table 23: *position of pick-up coil A corners - SYSIPHOS - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = 1.816	y = 1.969	y = 1.969	y = 1.816
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 24: *position of pick-up coil B corners - SYSIPHOS - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = -1.644	y = -1.651	y = -1.651	y = -1.644
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 25: *cell dimensions - JIN - (in cm)*

Part	**Dimension**
Pumping cell radius	3.090
Transfer tube radius	0.740
Transfer tube length	6.030
Target cell radius	0.866
Target cell length	39.50

Table 26: *position of pick-up coil A corners - JIN - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = 1.909	y = 1.967	y = 1.967	y = 1.909
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 27: *position of pick-up coil B corners - JIN - (in cm)*

Corner 1	**Corner 2**	**Corner 3**	**Corner 4**
x = 0.987	x = 0.987	x = -0.987	x = -0.987
y = -1.561	y = -1.708	y = -1.708	y = -1.561
z = 5.510	z = -5.510	z = -5.510	z = 5.510

Table 28: *flux contributions for each Helium cell (in cm^2)*

Cell	**Sphere**	**Tube**	**Target**	**Total flux**
Don't Worry	-4.58	0.08	47.24	42.74
Be Happy	-5.55	0.08	48.67	43.20
Armageddon	-4.28	0.06	45.73	41.51
Nepheli	-4.30	-0.02	50.83	46.52
Sysiphos	-4.56	-0.03	54.49	49.89
Jin	-4.19	0.00	49.70	45.51

References

[1] E94-010 Technical Note 19.

[2] E94-010 Technical Note 10.

[3] E94-010 Technical Note 20.

[4] E94-010 Technical Note 12.

[5] E94-010 Technical Note 18.

[6] E94-010 Technical Note 22.

[7] E94-010 Technical Note 26.

[8] E94-010 Technical Note 27.

[9] Courtesy of Ioannis Kominis.

Table 29: *water calibration constant per Helium cell*

Helium cell	**Constant (in mV/%/amg)**
Don't Worry	0.0281 ± 0.0002
Be Happy	0.0284 ± 0.0002
Armageddon	0.0273 ± 0.0002
Nepheli	0.0306 ± 0.0002
Sysiphos	0.0328 ± 0.0002
Jin	0.0299 ± 0.0002

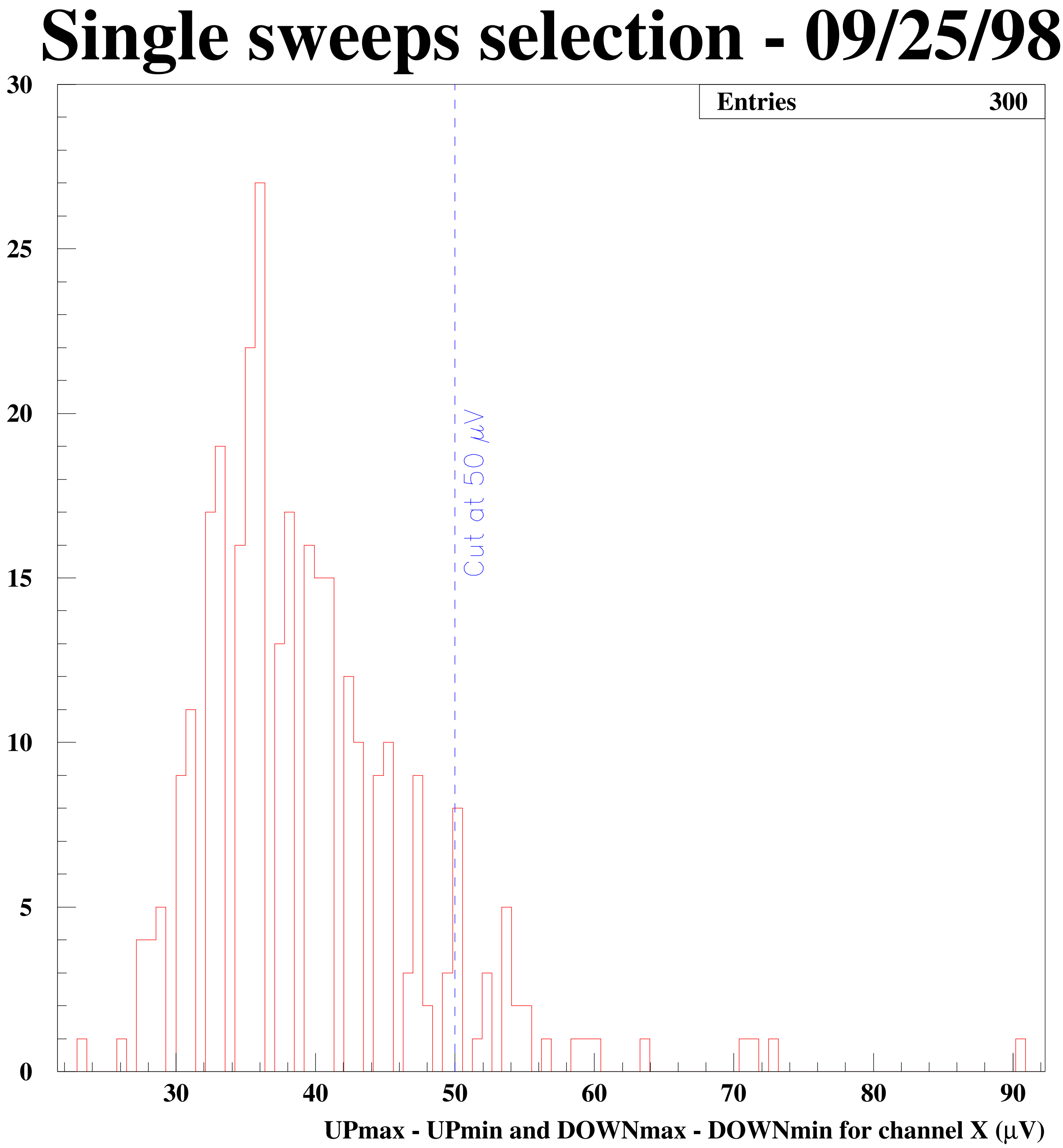

Figure 1: single sweeps distribution of the differences (maximum-minimum) amplitude for the UP ramp and Abs(maximum-minimum) amplitude for the DOWN ramp, for the lock-in channel X, containing the NMR signal. Example given for the 09/25/98 water calibration, before applying the cut on the maximum difference value, less than 50 μV.

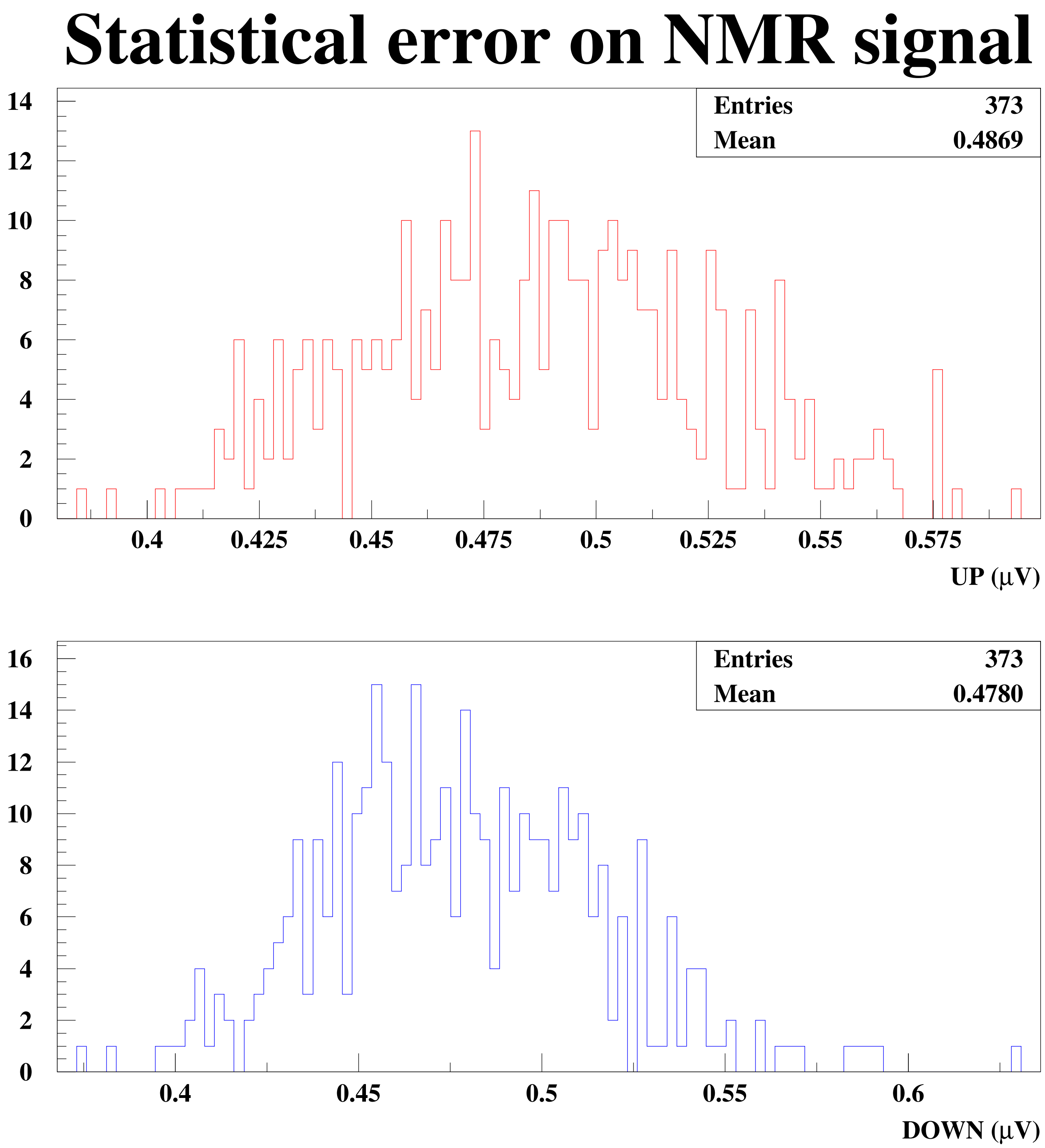

Figure 2: average statistical error on the averaged up and down NMR signals. The value is shown in the upper right corner, under the name Mean.

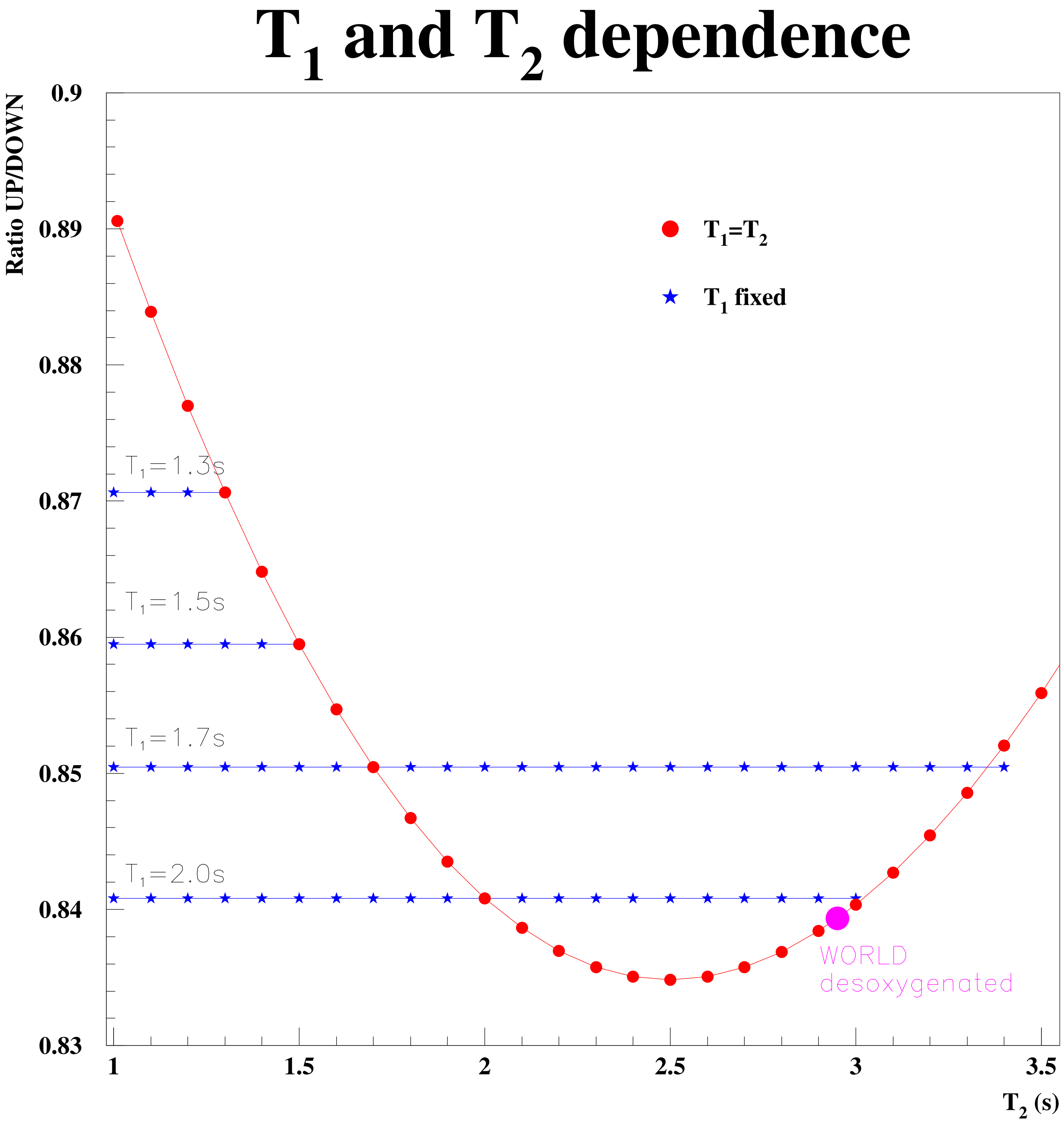

Figure 3: ratio UP/DOWN obtained from the Bloch equations. The world value is shown too for desoxygenated water, the kind of water this water cell has been filled with.

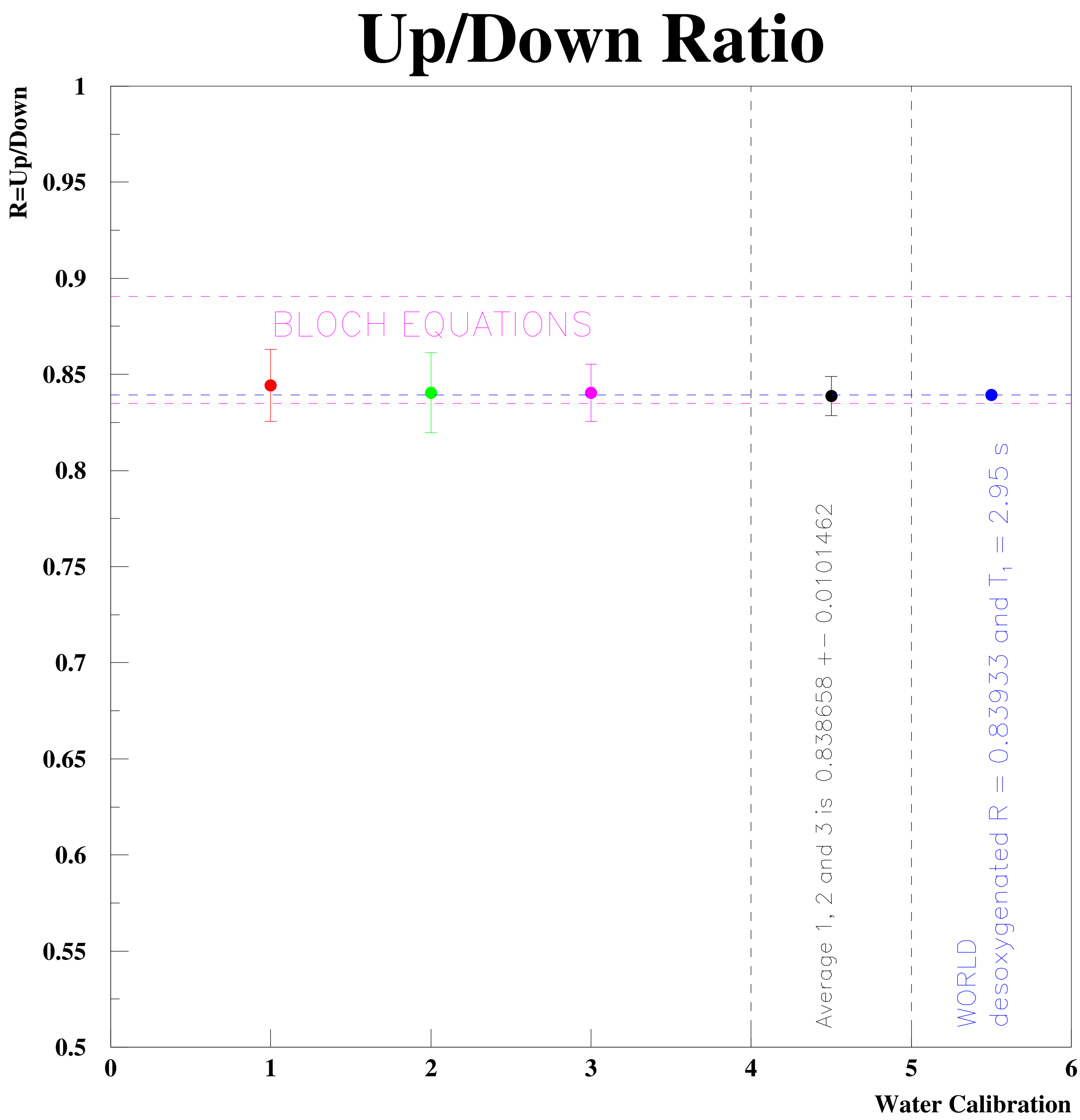

Figure 4: ratio UP/DOWN extracted from each water calibration.

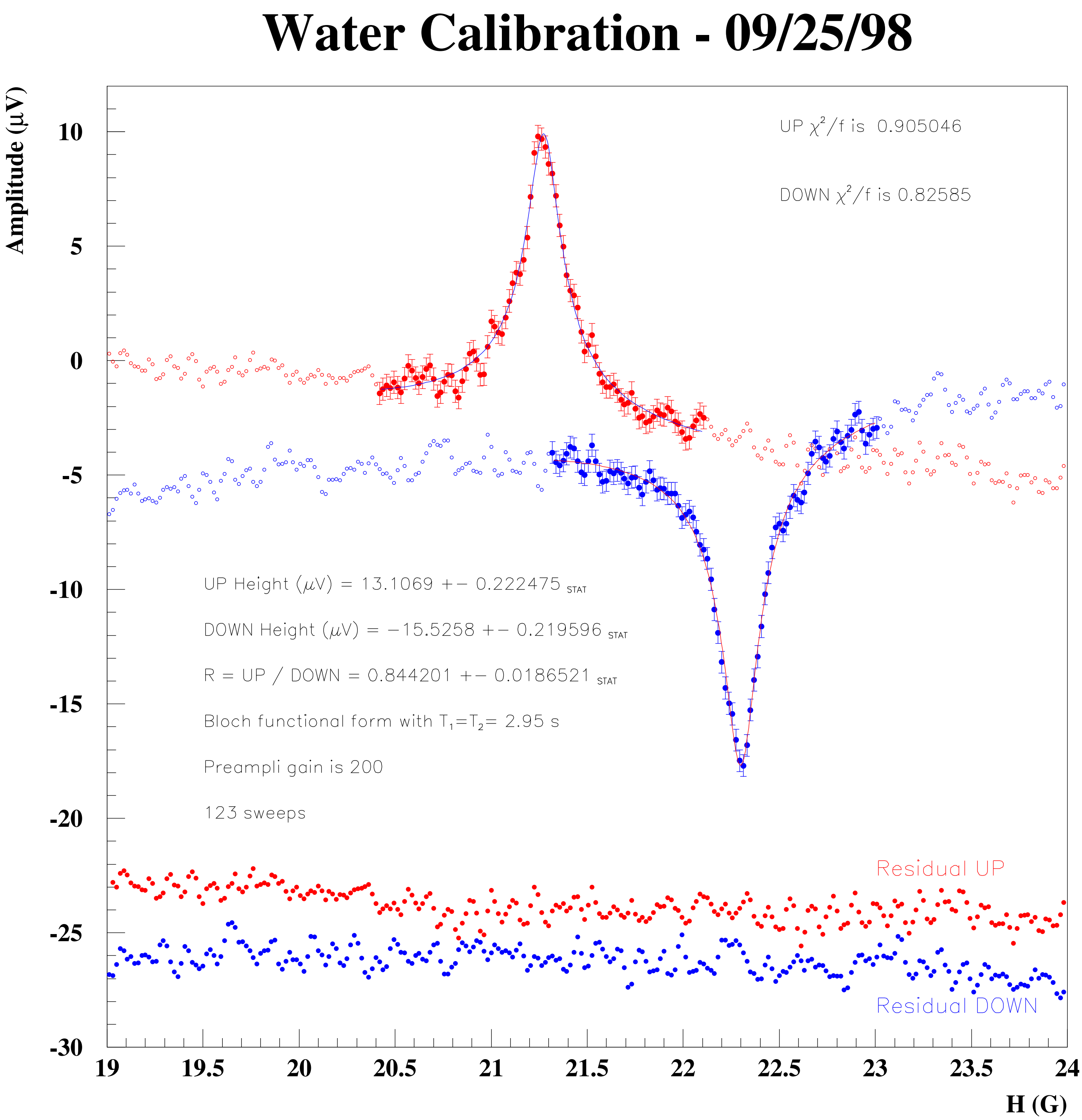

Figure 5: NMR water signal for the UP and DOWN ramps, 09/25/98. The error bars represent the average statistical error on the averaged sweep, which calculation has been estimated in the text. The residuals are also shown.

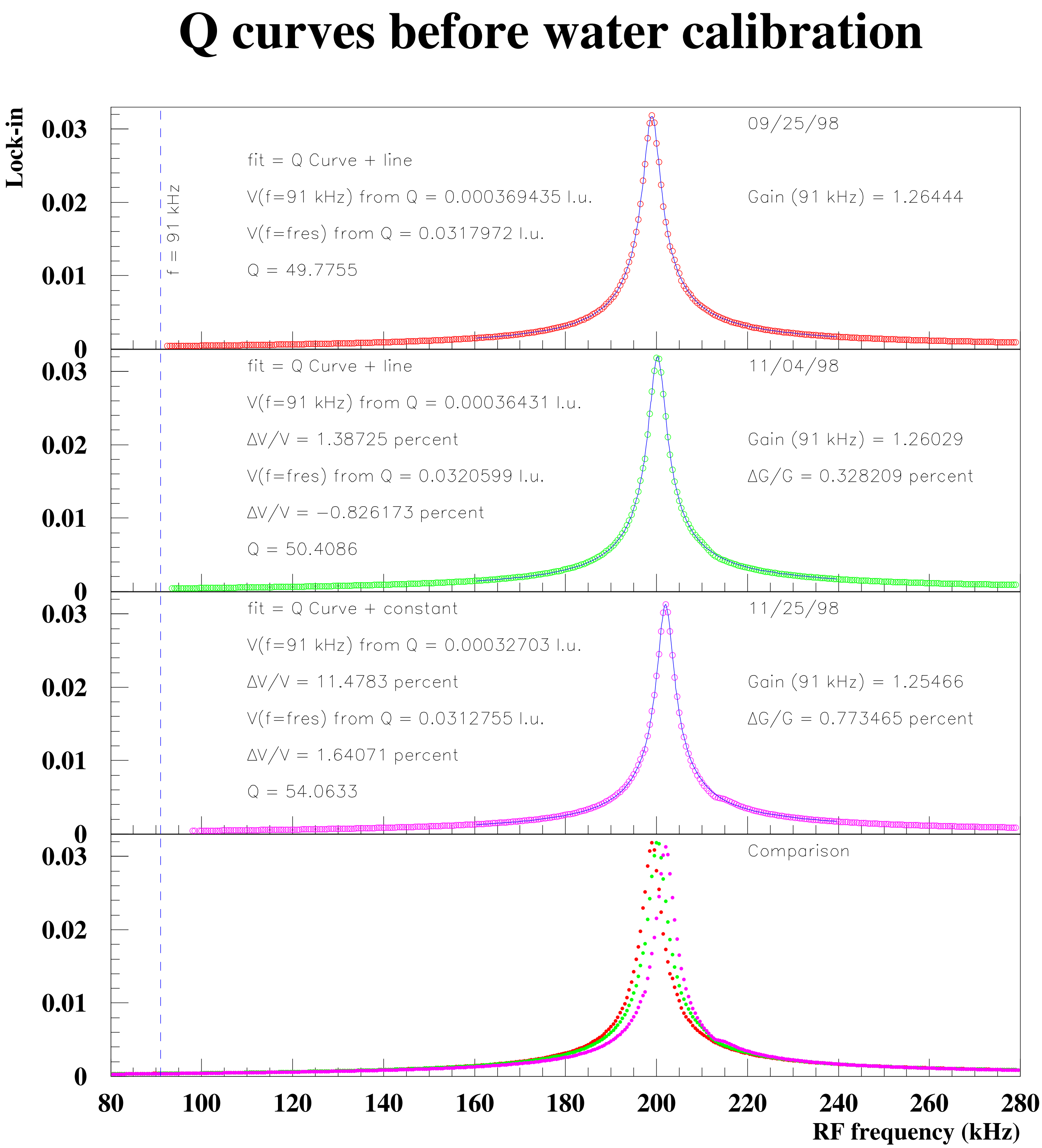

Figure 6: the three Q curves performed before each water calibration.

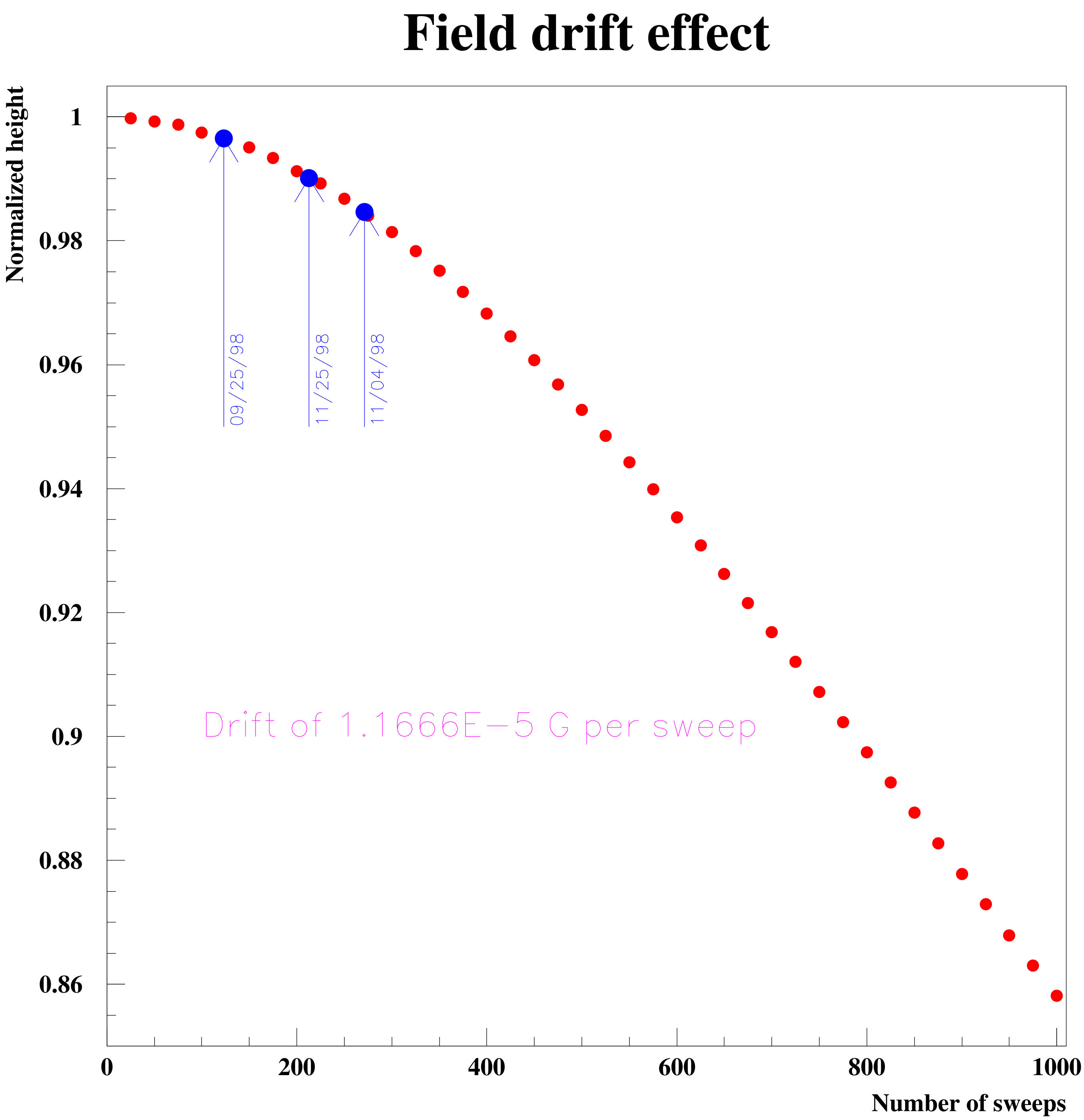

Figure 7: effect of the water signal height reduction due to the holding field drift.

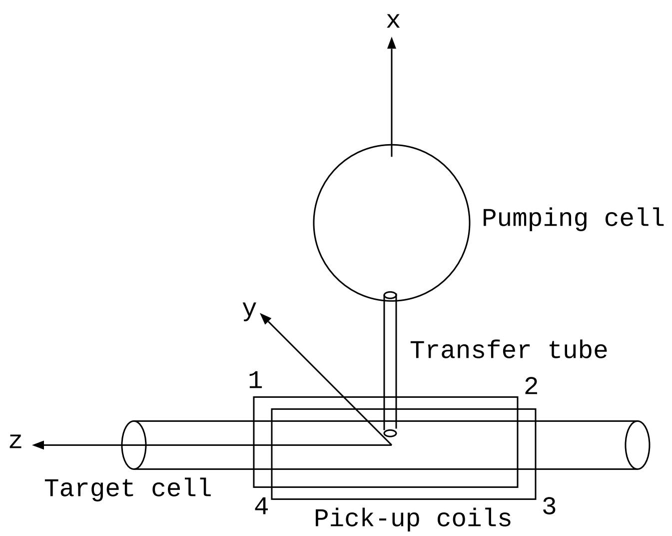

Figure 8: pick-up coil corners positions.

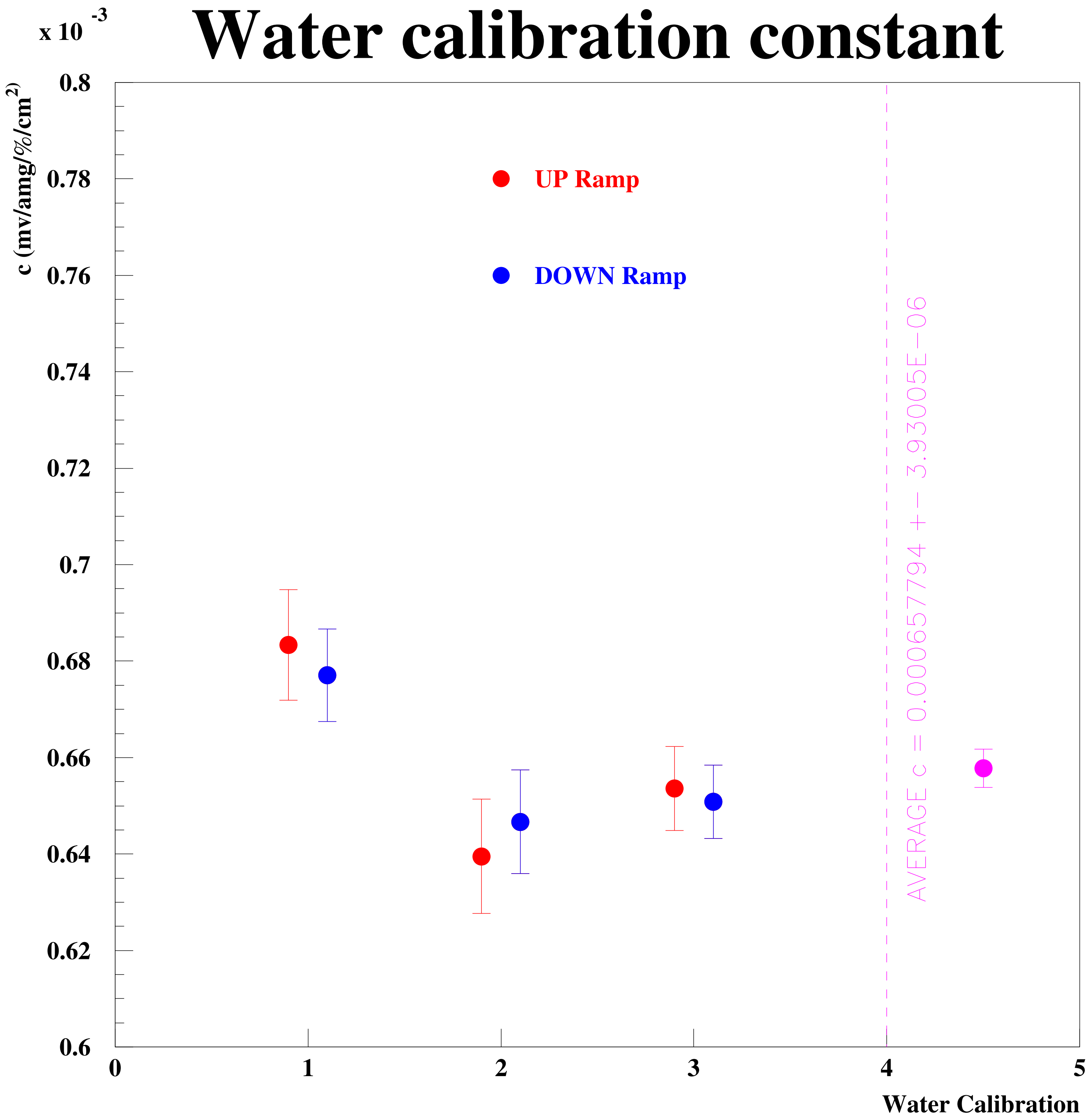

Figure 9: E94-010 water calibration constant and its statistical error.