**A New Proposal to Jefferson Lab PAC-24**

# Flavor Decomposition of Nucleon Spin Structure


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**Abstract:** We propose to measure the semi-inclusive double-spin asymmetries in deep-inelastic $\vec{p}(\vec{e}, e'h)X$ and $\vec{d}(\vec{e}, e'h)X$ reactions ($h = \pi^+, \pi^-, K^+$ and $K^-$) on longitudinally polarized $NH_3$ and LiD targets. The high precision data will allow a leading order five-flavor decomposition ($\Delta u$, $\Delta d$, $\Delta\bar{u}$, $\Delta\bar{d}$, $\Delta s = \Delta\bar{s}$) of the nucleon spin structure in the region of $x = 0.12 \sim 0.43$ at a $Q^2$ of $1.21 \sim 3.22$ GeV$^2$. A large calorimeter array, currently under construction for the Hall C $G_{Ep}$ experiment, will be used as the electron arm in conjunction with a new threshold gas Cherenkov. The HMS spectrometer will be used as the hadron arm. For the first time, the combined $\pi^+ + \pi^-$ spin-dependent yield ratios will be measured, since the acceptance and the hadron detection efficiencies of the HMS spectrometer can be accurately determined. Independent of the knowledge of fragmentation functions and parton distributions, the z-dependence ($z = 0.38 \sim 0.71$) of the combined ratios provides a clear test of factorization at the moderate $Q^2$. This experiment will probe the possible flavor asymmetry of $\Delta\bar{u} - \Delta\bar{d}$ with much improved statistical accuracies compare with that of HERMES. The kaon asymmetries will also provide limited access to the strange quark polarization. A total of 28 days of beam time at 6 GeV in Hall C is requested.

Contents

## 1 Introduction

The last decade has seen remarkable progress in the knowledge of the polarized quark distributions $\Delta q_f(x) \equiv q_f^{\uparrow\uparrow}(x) - q_f^{\uparrow\downarrow}(x)$, where $q_f^{\uparrow\uparrow}(x)$ represents the probability of finding a quark of any particular flavor $f$ with momentum fraction $x$ of the target nucleon, and with its spin parallel to that of the nucleon. The most precise and clearly interpreted experimental tool has been inclusive deep-inelastic lepton scattering (DIS) applied at the CERN and SLAC laboratories. However, the information available from inclusive DIS process has inherent limitations, as the cross sections are sensitive only to the square of the quark charge, quarks and anti-quarks are being probed on equal footing, and only one particular flavor non-singlet combination can be directly inferred from proton and "neutron" data: $\Delta q_3(x, Q2) = \Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d} = 6(g_1^p - g_1^n)$. Further inference from inclusive data requires an additional assumption of $\mathrm{SU}(3)_f$ flavor symmetry relating the spin structure of various hadrons. This allows the use of hyperon beta decay to constrain the first moment of another non-singlet flavor combination involving the strange sea. The celebrated result of this approach is that quark helicities seem to make a small net contribution to the nucleon spin, and the strange sea appears to be negatively polarized.

The sensitivity to each individual quark flavor is realized in semi-inclusive deep inelastic scattering (SIDIS) in which one of the leading hadrons is also detected. Since the leading hadrons produced by fragmentation of the struck quark carry information about the struck quark's flavor, identification of the type of hadrons detected together with the scattered lepton effectively "tags" the quark flavor. Therefore, SIDIS offers a great opportunity for determining the spin, flavor, and sea structure of the nucleon [1], as well as extracting information on new distributions [2], such as transversity, which are not accessible in inclusive scattering, thereby significantly enriching our understanding of QCD and the nucleon structure. High precision polarized SIDIS data on proton and neutron allows a complete flavor decomposition of nucleon spin structure, which could lead to the discovery of a possible flavor asymmetry in the light sea quark polarization and also provide an access to the strange quark polarization. Recently, the HERMES collaboration released the preliminary results of leading order spin flavor decomposition from polarized proton and deuteron data collected during 1996-2000, and for the first time separated the $\bar{u}, \bar{d}$ and $s = \bar{s}$ sea quark polarization [3]. Unlike the prediction of several theoretical models, HERMES found that within the available statistics $\Delta\bar{u} - \Delta\bar{d}$ is consistent with an unbroken $\mathrm{SU}(2)_f$ symmetry in the light polarized sea. In contrast to the results based on inclusive data alone, the strange quark polarization is reported as slightly positive, which was viewed as "almost impossible" since a non-negative value of $\Delta s$ would imply a total break down of $\mathrm{SU}(3)_f$ symmetry for the strong interactions [4].

At high enough energy transfer the scattering and production mechanisms factorize: the cross section at leading order in QCD becomes a simple product of the structure function, which gives the probability of finding a quark in the nucleon, and a quark fragmentation function, the probability that the quark hadronizes into the meson. The extent to which factorization applies at moderate energies is an open question. HERMES has demonstrated that, within the experimental precision, the semi-inclusive double-spin asymmetries at $\langle Q^2 \rangle = 2.5$ GeV$^2$ agree reasonably well with the SMC data which was obtained at $\langle Q^2 \rangle = 10$ GeV$^2$. This agreement indicated that the violation of factorization might be smaller than would be naively expected at $Q^2 = 2.5$ GeV$^2$. One noticeable point is that with the high luminosity available at Jefferson Lab, it is often possible to perform precision measurements at much larger scattering angles, which makes Q$^2$ not far away from that of HERMES while studying a similar $x$ region.

Jefferson Lab is actively pursuing the opportunity of an energy upgrade to 12 GeV. Semi-inclusive processes could be a very rich program with the 12 GeV upgrade if factorization can be demonstrated at a reasonable level. The question of how well factorization will hold at the kinematics accessible with JLab at 6-12 GeV is crucial for determining how clean and important the semi-inclusive program can become. To quantitatively answer the question of factorization at Jefferson Lab energies, it is crucial to perform precision measurements on observables that are sensitive to the violation of factorization. Recently, a schematic strategy of factorization tests has been suggested [5] which requires no prior knowledge of fragmentation functions nor parton distributions. The major experimental challenge in this strategy is to observe the appearance of the $z$-independent behavior of the combined $\pi^+ + \pi^-$ spin-dependent yield ratio. This $z$-independent behavior is caused by the exact cancellation of the fragmentation functions in the combined spin-dependent yield ratio – which is a direct consequence of factorization. The combined $\pi^+ + \pi^-$ spin-dependent yield ratio would have been very difficult to construct from the existing HERMES data since precise knowledge of phase spaces and detection efficiencies are required.

This experiment is specifically designed to have well controlled phase spaces and hadron detection efficiencies such that the combined spin-dependent yield ratios, in addition to asymmetries, can be constructed as a function of $z$ ($z = 0.38 \sim 0.71$) to serve as a clear test of factorization. Double-spin asymmetries of $(e, e'\pi^{\pm})$ and $(e, e'K^{\pm})$ will be determined with high precision as a function of $z$ rather than integrated over $z$, to give a better level-arm in the flavor decomposition. At $Q^2$ of $1.21 \sim 3.22$ GeV$^2$, a leading order five-flavor decomposition ($u$, $d$, $\bar{u}$, $\bar{d}$, $s = \bar{s}$) of the nucleon spin structure will be performed in the region of $x = 0.12 \sim 0.43$. The much improved statistics over the HERMES data will present us with the first opportunity to probe the possible flavor asymmetry of the light sea quark polarization. The kaon asymmetries will also allow limited access to the strange quark polarization. At the high-$x$ bins, the proposed experiment overlaps with the recent Hall A experiment [6] (E99117) which extracted ratios of $(\Delta u + \Delta\bar{u})/(u + \bar{u})$ and $(\Delta d + \Delta\bar{d})/(d + \bar{d})$ from the inclusive $A_{1n}$ measurements at high-$x$. The consistency check between data from this experiment and the inclusive data of E99117 provides the "Holy Grail" of the validity of semi-inclusive spin flavor decomposition.

## 2 Physics Motivation

The principle goal of spin-dependent SIDIS measurements is to perform flavor decomposition of nucleon spin structure taking the advantages of flavor tagging. At the leading order QCD analysis, factorization is assumed between the hard quark scattering and the non-perturbative fragmentation processes. In addition, isospin symmetry and charge conjugation is also assumed (i.e. $u_p(x) = d_n(x)$, $d_p(x) = u_n(x)$ *etc*). In this section, we summarize the HERMES preliminary results from the "purity method" of flavor decomposition. More details of the HERMES preliminary results are attached in Appendix–A. We then outline the new "$z$-fitting method" of flavor decomposition for this experiment and the "combined spin-dependent yield ratio method" of factorization test as suggested by Christova and Leader [5]. The exact detail and justification of these methods are attached in Appendix–B.

### *2.1 The factorization assumption in SIDIS*

At high enough energy transfer, independent factorization is assumed between the virtual photon-quark hard scattering process and the hadronization of the struck quark. This implies:

$$\sigma^h(x,z) \propto \sum_i e_f^2 q_f(x) \cdot D_{q_f}^h(z), \tag{1}$$

where $\sigma^h(x,z)$ is the cross section of hadron-$h$ production in deep-inelastic scattering, $e_f$ and $q_f(x)$ are the charge and distributions of quark with flavor $f$ ($f = u, d, \bar{u}, \bar{d}, s, \bar{s}$ neglecting heavy quark contributions). The functions $D_{q_f}^h(z)$ represent the probability that a quark of flavor $f$ fragments into a hadron of type-$h$.

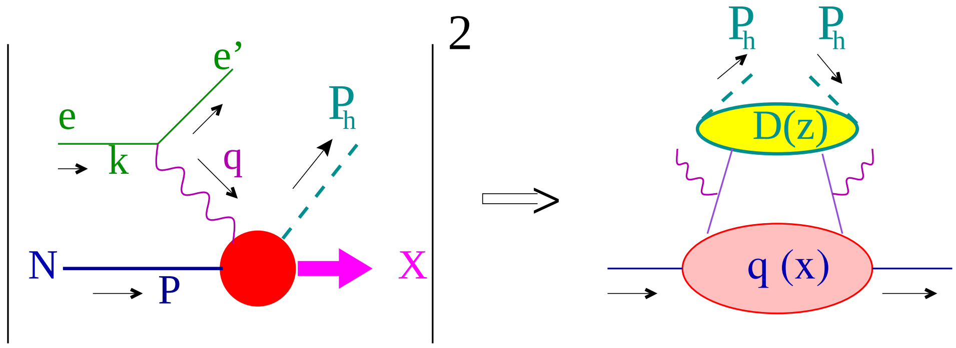


Figure 1: The assumption of factorization implies that the hard scattering and the soft fragmentation processes factorize.

Considering the beam and target polarization ($P_B$ and $P_T$), and the dilution factor ($f$), which is caused by the existence of unpolarized nucleons in the target ($f = 1$ for

HERMES), the experimental double-spin asymmetry [7] of a longitudinally polarized beam on a longitudinally polarized target is ($A_2^h \approx 0$):

$$A_{\parallel}^h = f P_B P_T \cdot \frac{1+R}{1+\gamma^2} \cdot \mathcal{D}(1+\gamma\eta) \cdot A_1^h, \tag{2}$$

in which

$$\eta = \frac{2\gamma(1-y)}{2-y}, \qquad \mathcal{D} = \frac{1-(1-y)\epsilon}{1+\epsilon \cdot R},$$
$$\epsilon^{-1} = 1 + 2(1+\nu^2/Q^2)\tan^2(\theta_e/2), \tag{3}$$

and $y = \nu/E_0$, $\gamma^2 = Q^2/\nu^2$, $R = \sigma_L/\sigma_T$ accounts for the longitudinal component of the virtual photon which has been assumed to be the same for semi-inclusive and the inclusive cases. For the deuteron case, an additional correction of $(1+\frac{3}{2}\omega_D)^{-1}$ with $\omega_D = 0.05 \pm 0.01$ is required to account for the $D$ state in deuteron [8]. In the current fragmentation regime, the leading hadron from the fragmentation is strongly correlated with the struck quark, and the virtual photon asymmetry $A_1^h$ is related to quark polarization at the leading order as:

$$A_1^h(x, Q^2, z) = \frac{\sum_f e_f^2 \Delta q_f(x, Q^2) \cdot D_f^h(z, Q^2)}{\sum_f e_f^2 q_f(x, Q^2) \cdot D_f^h(z, Q^2)}. \tag{4}$$

### *2.2 The "purity method" and HERMES flavor decomposition results*

The HERMES analysis used the so-called "purity method" for flavor decomposition [9]. In Eq. 4, a "purity matrix" $\mathcal{P}_f^h(x, Q^2, z)$ is defined such that:

$$A_1^h(x, Q^2, z) \equiv \sum_f \mathcal{P}_f^h(x, Q^2, z) \cdot \frac{\Delta q_f(x, Q^2)}{q_f(x, Q^2)}. \tag{5}$$

where

$$\mathcal{P}_f^h(x) = \frac{e_f^2 q_f(x) \int dz D_f^h(z)}{\sum_f' e_{f'}^2 q_{f'}(x) \int dz D_{f'}^h(z)}, \tag{6}$$

in which the integration over $z$ allows more events to be included, and the explicit $Q^2$ notation has been omitted for simplicity. The exact values of $\mathcal{P}_f^h(x, Q^2, z)$ in the HERMES analysis were obtained from detailed Monte Carlo simulations which was based on the Lund string fragmentation model [10] and taking into account of the experimental setup and detector efficiencies.

Integrating hadron events over $0.2 < z < 0.8$, HERMES extracted five flavor quark polarizations:

$$\vec{Q} = \left(\frac{\Delta u}{u}, \frac{\Delta d}{d}, \frac{\Delta \bar{u}}{\bar{u}}, \frac{\Delta \bar{d}}{\bar{d}}, \frac{\Delta s + \Delta \bar{s}}{s + \bar{s}}\right), \tag{7}$$

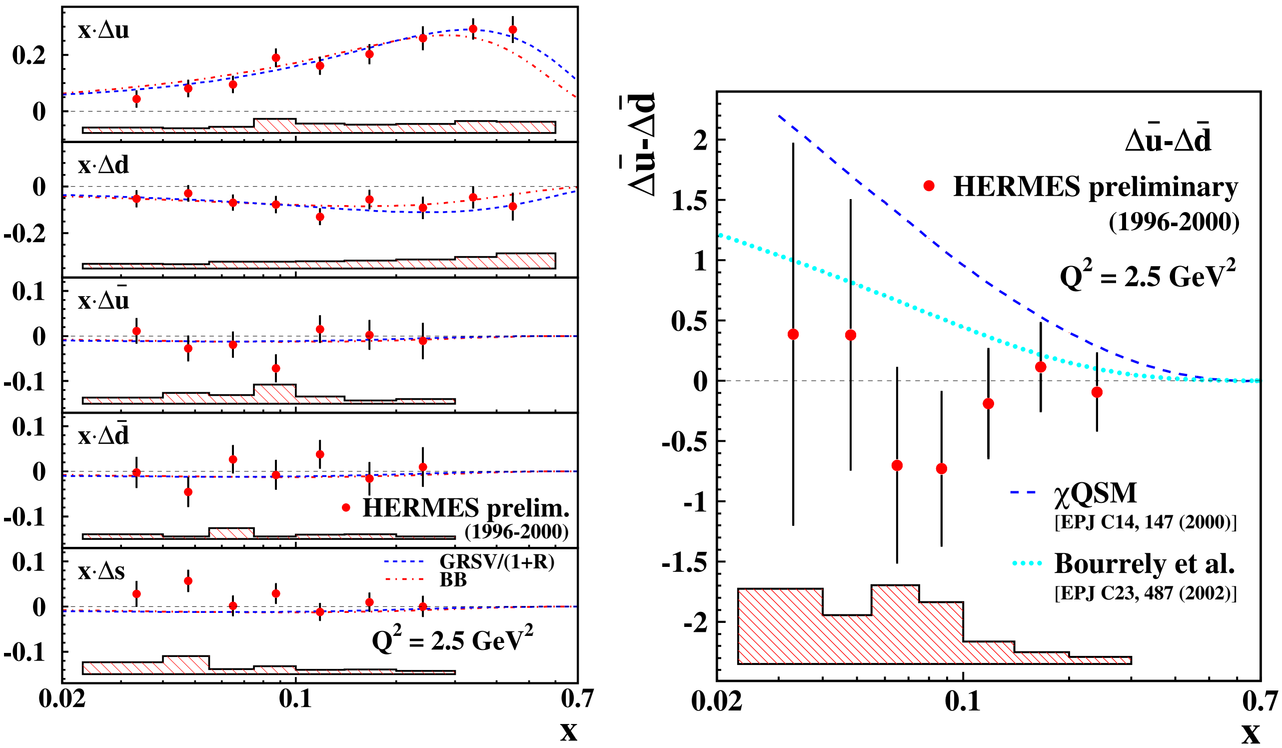


Figure 2: The left panel shows the HERMES preliminary results [3] of the polarized quark distributions at a common scale of $Q^2 = 2.5$ GeV$^2$ for $u$, $\bar{u}$, $d$, $\bar{d}$, and $s + \bar{s}$ quark flavors versus $x$ with two different parameterizations [11,12]. The right pannel shows the difference of the polarized light sea $\Delta\bar{u} - \Delta\bar{d}$ as a function of $x$ at a scale of $Q^2 = 2.5$ GeV$^2$. The error bars are statistical, while the shaded bands indicate the systematic uncertainties of the results.

from a data base of measured double-spin asymmetries

$$\vec{A} = \left( A_{1p}, A_{1d}, A_{1p}^{\pi^\pm}, A_{1d}^{\pi^\pm}, A_{1d}^{K^\pm} \right) \tag{8}$$

by solving the relations of $\vec{A} = \mathcal{P}_f^h(x) \cdot \vec{Q}$.

The detailed HERMES asymmetry results are attached in Appendix–A. The HERMES preliminary results of flavor decomposition are shown in Fig 2. As expected, the $u$-quarks are strongly polarized in a direction parallel to that of the proton polarization, while the $d$-quarks are weakly polarized in the opposite direction. The $u$ and $d$ sea quarks are seen to carry a small amount of spin. With a limited statistics, there is a hint of a small positive $s$-quark polarization at low-$x$. Fig. 2 also shows the HERMES measurement of $\Delta\bar{u}(x) - \Delta\bar{d}(x)$ together with predictions of a broken $\mathrm{SU}(2)_f$ symmetry [14,15]. The data are consistent with an unbroken $\mathrm{SU}(2)_f$ symmetry in the light polarized sea. Since the Fermilab Drell-Yan data demonstrated a sizable flavor asymmetry in the unpolarized light sea distributions [16,17] while $\bar{d}/\bar{u}$ peaked around $x = 0.2$, one speculates that a sizable flavor asymmetry might also appear in the polarized light sea distributions in the same $x$ region.

### *2.3 A new method of flavor decomposition and factorization tests*

The power of semi-inclusive reaction lies in its ability of flavor tagging. Once factorization can be demonstrated at a reasonable level, flavor decomposition of nucleon spin structure can be achieved based on data sets of double-spin asymmetry measurements, for example in this experiment we use $A_{1p}^{\pi^\pm}$, $A_{1p}^{K^\pm}$, $A_{1d}^{\pi^\pm}$, $A_{1d}^{K^\pm}$ and the better-known inclusive asymmetries $A_{1p}$ and $A_{1d}$. This experiment will follow a new method of flavor decomposition, which we label as the "$z$-fitting method". Rather than calculating the "purity matrix elements" as integrated quantities over a large range of $z$, this method uses a maximum likelihood fit to constrain polarized quark distributions through many high precision double-spin asymmetry measurements as functions of $z$. Within the same data set, the assumption of factorization can be tested in a self-consistent way by observing the $z$-dependencies of the combined $\pi^+ + \pi^-$ spin-dependent yield ratio. In this section, we briefly describe the "$z$-fitting method" of flavor decomposition and list the most important observables of factorization tests. The full details and the derivation of this method are attached in Appendix-B.

### **A new method of flavor decomposition**

For this experiment, which covers $0.12 < x < 0.43$, we will assume a symmetrical strange quark distribution and polarization ($s(x) = \bar{s}(x)$, $\Delta s(x) = \Delta\bar{s}(x)$) and neglect heavy quark contributions. Under the factorization assumption of Eq. 1, the $x$-dependencies of the double spin asymmetries, $A_{1N}^h(x, z) = \Delta\sigma_N^h/\sigma_N^h$, are determined by the quark distributions, while the $z$-dependencies are determined by the ratios of fragmentation functions. Introducing the short-hand notations for the fragmentation function ratios:

$$\begin{aligned}
\lambda_\pi(z) &= D_\pi^-(z)/D_\pi^+(z), &\qquad \xi_\pi(z) &= D_s^\pi(z)/D_\pi^+(z), \\
\lambda_K(z) &= D_K^-(z)/D_K^+(z), &\qquad \xi_K(z) &= D_d^K(z)/D_K^+(z).
\end{aligned} \tag{9}$$

We have semi-inclusive charge pion asymmetries:

$$\begin{aligned}
A_{1p}^{\pi^+}(x,z) &= \frac{4\Delta u + \Delta\bar{d} + (4\Delta\bar{u} + \Delta d)\,\lambda_\pi + 2\Delta s\xi_\pi}{4u + \bar{d} + (4\bar{u} + d)\,\lambda_\pi + 2s\ \xi_\pi}, \qquad (10)\\
A_{1p}^{\pi^-}(x,z) &= \frac{(4\Delta u + \Delta\bar{d})\lambda_\pi + 4\Delta\bar{u} + \Delta d + 2\Delta s\xi_\pi}{(4u + \bar{d})\lambda_\pi + 4\bar{u} + d\ \ + 2s\xi_\pi}, \\
A_{1d}^{\pi^+}(x,z) &= \frac{4(\Delta u + \Delta d) + \Delta\bar{u} + \Delta\bar{d} + \left(\Delta u + \Delta d + 4(\Delta\bar{u} + \Delta\bar{d})\right)\lambda_\pi + 4\Delta s\xi_\pi}{4(u+d) + \bar{u} + \bar{d} + \left(u + d + 4(\bar{u} + \bar{d})\right)\lambda_\pi + 4s\xi_\pi}, \\
A_{1d}^{\pi^-}(x,z) &= \frac{\left(4(\Delta u + \Delta d) + \Delta\bar{u} + \Delta\bar{d}\right)\lambda_\pi + \Delta u + \Delta d + 4(\Delta\bar{u} + \Delta\bar{d}) + 4\Delta s\xi_\pi}{\left(4(u+d) + \bar{u} + \bar{d}\right)\lambda_\pi + u + d + 4(\bar{u} + \bar{d}) + 4s\xi_\pi}.
\end{aligned}$$

and charge kaon asymmetries:

$$\begin{aligned}
A_{1p}^{K^+}(x,z) &= \frac{4\Delta u + \Delta s + (4\Delta\bar{u} + \Delta s)\,\lambda_K + (\Delta d + \Delta\bar{d})\xi_K}{4u + s + (4\bar{u} + s)\,\lambda_K + (d + \bar{d})\xi_\pi}, \qquad (11)\\
A_{1p}^{K^-}(x,z) &= \frac{(4\Delta u + \Delta s)\,\lambda_K + 4\Delta\bar{u} + \Delta s + (\Delta d + \Delta\bar{d})\xi_K}{4u + s + (4\bar{u} + s)\,\lambda_K + (d + \bar{d})\xi_\pi}, \\
A_{1d}^{K^+}(x,z) &= \frac{4(\Delta u + \Delta d) + 2\Delta s + \left(4(\Delta\bar{u} + \Delta\bar{d}) + 2\Delta s\right)\lambda_K + (\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d})\xi_K}{4(u+d) + 2s + \left(4(\bar{u} + \bar{d}) + 2s\right)\lambda_K + (u + \bar{u} + d + \bar{d})\xi_K}, \\
A_{1d}^{K^-}(x,z) &= \frac{(4(\Delta u + \Delta d) + 2\Delta s)\,\lambda_K + 4(\Delta\bar{u} + \Delta\bar{d}) + 2\Delta s + (\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d})\xi_K}{(4(u+d) + 2s)\,\lambda_K + 4(\bar{u} + \bar{d}) + 2s + (u + \bar{u} + d + \bar{d})\xi_K}.
\end{aligned}$$

For a given $x$-bin, at a fixed $z$-value, each asymmetry measurement provides an independent constraint on a linear combination of $\vec{Q} = (\Delta u, \Delta d, \Delta\bar{u}, \Delta\bar{d}, \Delta s)$. The relative variation of the coefficients in Eq. 10 and Eq. 11 as a function of $z$ provides additional sensitivity for flavor decomposition. A maximum likelihood fit through asymmetry data from the same $x$-bin but covers different $z$ values could determine the quark polarization $\vec{Q}$ once the linear coefficients are known. The determination of these coefficients only requires inputs from unpolarized quark distributions and ratios of fragmentation functions of Eq. 9. If all $(e, e'h)$ asymmetry data are obtained from exactly the same phase space, with the relative hadron detection efficiencies well-determined, the exact knowledge of the fragmentation functions becomes unnecessary, only their ratios are relevant in Eq. 10 and Eq. 11.

These fragmentation function ratios are better known than the fragmentation function themselves. The existing parameterizations [18] obtained from $e^+e^-$ data provide reasonable accuracies to start with. The existing data from HERMES had shown [19] that $\lambda_\pi(z) = D_\pi^-/D_\pi^+$ closely follows $(1-z)/(1+z)$, as first predicted by Field and Feynman [20]. Furthermore, it is the same form of $\lambda$ and $\xi$ appears in all $x$-bins, and once the factorization has been demonstrated, the $x$-independence requirement imposes strong constraints on their values [5].

In addition to the constraints of Eq. 10 and Eq. 11, the better known inclusive asymmetries $A_{1p}$ and $A_{1d}$ (or $A_{1n}$) also impose constraints on the sum of $\Delta u + \Delta\bar{u}$, $\Delta d + \Delta\bar{d}$ and $\Delta s + \Delta\bar{s} = 2\Delta s$ through:

$$
\begin{aligned}
A_{1p}(x) &= \frac{4(\Delta u + \Delta\bar{u}) + \Delta d + \Delta\bar{d} + 2\Delta s}{4(u+\bar{u}) + d + \bar{d} + 2s}, \\
A_{1d}(x) &= \frac{5(\Delta u + \Delta d + \Delta\bar{u} + \Delta\bar{d}) + 4\Delta s}{5(u + d + \bar{u} + \bar{d}) + 4s}.
\end{aligned}
\tag{12}
$$

**Spin observables for factorization test**

As discussed in detail in Appendix–B and Ref[5], the $z$-dependencies of the combined $\pi^+ + \pi^-$ spin-dependent yield ratio indicate how well factorization holds. For this experiment, the most important observables for a self-consistent factorization test are:

$$
\begin{aligned}
&A_{1p}^{\pi^+ + \pi^-}(x, Q^2, z) = \frac{\Delta\sigma_p^{\pi^+} + \Delta\sigma_p^{\pi^-}}{\sigma_p^{\pi^+} + \sigma_p^{\pi^-}} \\
&\approx A_{1p}\left[1 + \frac{2s}{4(u+\bar{u}) + d + \bar{d}} - \frac{2\Delta s}{4(\Delta u + \Delta\bar{u}) + \Delta d + \Delta\bar{d}} + \mathcal{O}(\alpha)\right]
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&A_{1d}^{\pi^+ + \pi^-}(x, Q^2, z) = \frac{\Delta\sigma_d^{\pi^+} + \Delta\sigma_d^{\pi^-}}{\sigma_d^{\pi^+} + \sigma_d^{\pi^-}} \\
&\approx A_{1d}\left[1 + \frac{4s}{5(u + \bar{u} + d + \bar{d})} - \frac{4\Delta s}{5(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d})} + \mathcal{O}(\beta)\right],
\end{aligned}
\tag{14}
$$

where the small $z$-dependent terms can be safely neglected in this experiment, they are of the order of:

$$
\alpha = \frac{2\xi_\pi}{1+\lambda_\pi} \cdot \frac{2s}{4u + d} \ll 1, \qquad \beta = \frac{2\xi_\pi}{1+\lambda_\pi} \cdot \frac{4s}{5(u+d)} \ll 1.
$$

Independent of the fragmentation functions and the strange quark effects, the combined proton and deuteron data can provide a strong test of factorization through the relation of:

$$
\begin{aligned}
A_{1(2p-d)}^{\pi^+ + \pi^-}(x, Q^2, z) &= \frac{2(\Delta\sigma_p^{\pi^+} + \Delta\sigma_p^{\pi^-}) - (\Delta\sigma_d^{\pi^+} + \Delta\sigma_d^{\pi^-})}{2(\sigma_p^{\pi^+} + \sigma_p^{\pi^-}) - (\sigma_d^{\pi^+} + \sigma_d^{\pi^-})}, \\
&= \frac{(\Delta u + \Delta\bar{u}) - (\Delta d + \Delta\bar{d})}{(u + \bar{u}) - (d + \bar{d})} = \frac{g_1^p - g_1^n}{F_1^p - F_1^n},
\end{aligned}
\tag{15}
$$

## 3 The Proposed Measurement

### *3.1 Overview*

We plan to study the $\vec{p}(\vec{e}, e'h)X$ and $\vec{d}(\vec{e}, e'h)X$ reactions ($h = \pi^+, \pi^-, K^+$ and $K^-$) with longitudinally polarized $NH_3$ and LiD targets in Hall C with a 6 GeV polarized electron beam. Relative yields will be determined for different combinations of beam and target spin orientations and the combined $\pi^+ + \pi^-$ spin-dependent yield ratio will be constructed in addition to the various double-spin $(e, e'h)$ asymmetries. The existing HMS spectrometer is located at 10.8° beam right as the hadron arm detector. Three HMS momentum settings (2.00, 2.62 and 3.22 GeV/c) will be taken for both positive and negative polarities. Special efforts will be made on the HMS aerogel and gas Cherenkov detectors such that clear particle IDs for pions and kaons can be obtained over the entire momentum range. A large calorimeter array (BigCal), currently under construction for the $G_{Ep}$ experiment, will be centered at 30° beam left 3.5 meter from the target, as the electron arm in conjunction with a new threshold gas Cherenkov detector. Since this is a coincidence experiment, and the HMS can be used for target position reconstruction, from coincidence timing and HMS vertex cuts one can eliminate the majority of the background on the BigCal calorimeter array. In the proposal[21] of JLab-PAC23 P03002, " Spin Asymmetries on the Nucleon Experiment" (SANE proposal), a similar scheme of electron detection has been proposed. In this section, we only describe the relevant detectors: the BigCal and the gas Cherenkov. More technical details and the background rates can be found in the "SANE" proposal. The SOS spectrometer will be parked on the beam left, as an independent luminosity monitor.

### *3.2 Kinematics and phase space*

The definitions of the kinematics variables are the following: Bjorken-$x$, which indicates the fractional momentum carried by the struck quark, $x = Q^2/(2\nu M_N)$, $M_N$ is the nucleon mass. The momentum of the outgoing hadron is $p_h$ and the fraction of the virtual photon energy carried by the hadron is: $z = E_h/\nu$. $W$ is the invariant mass of the whole hadronic system and $W'$ is the invariant mass of the hadronic system without the detected pion. We have:

$$\begin{aligned} W^2 &= M_N^2 + Q^2(\frac{1}{x} - 1), \\ W'^2 &= (M_N + \nu - E_\pi)^2 - |\vec{q} - \vec{p}_\pi|^2. \end{aligned} \tag{16}$$

We have chosen to cover the highest possible $W$ with a 6 GeV beam, $2.27 < W < 3.09$ GeV, corresponds to $0.12 < x < 0.43$ and $1.21 < Q^2 < 3.22$ (GeV/c)$^2$. We have also chosen to detect the leading fragmentation hadron which carries $z = 0.38 \sim 0.72$ of the energy transfer to favor the current fragmentation. The value of $W'$ is also chosen to be as high as possible (1.50∼2.36 GeV) to avoid contributions from the