CEBAF Program Advisory Committee Nine Proposal Cover Sheet

This proposal must be received by close of business on Thursday, December 1, 1994 at:

CEBAF
User Liaison Office, Mail Stop 12 B
12000 Jefferson Avenue
Newport News, VA 23606

Proposal Title

Search for Narrow Excited States of the Proton

Contact Person

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Experimental Hall: C* **Days Requested for Approval:** 2

Hall B proposals only, list any experiments and days for concurrent running:

* Hall A would be OK, too.

CEBAF Use Only

Receipt Date: 12/15/94 PR 94-113

By: gp

LAB RESOURCES REQUIREMENTS LIST

CEBAF Proposal No.: _____ Date: _____
(For CEBAF User Liaison Office use only.)

List below significant resources — both equipment and human — that you are requesting *from CEBAF* in support of mounting and executing the proposed experiment. Do not include items that will be routinely supplied to all running experiments, such as the base equipment for the hall and technical support for routine operation, installation, and maintenance.

Major Installations *(either your equip. or new equip. requested from CEBAF)*

NONE

New Support Structures:

Data Acquisition/Reduction

Computing Resources:

New Software:

Major Equipment

Magnets

Power Supplies

Targets

Detectors

Electronics

Computer Hardware

Other

Other

HAZARD IDENTIFICATION CHECKLIST

CEBAF Proposal No.: _____ Date: _____
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Hall A or Hall C spectrometer system with 10 cm lqd. H2 target

Check all items for which there is an anticipated need.

Cryogenics
- _____ beamline magnets
- _____ analysis magnets
- _____ target
  - type: _____
  - flow rate: _____
  - capacity: _____

Electrical Equipment
- _____ cryo/electrical devices
- _____ capacitor banks
- _____ high voltage
- _____ exposed equipment

Radioactive/Hazardous Materials
List any radioactive or hazadorous/ toxic materials planned for use:
_____
_____
_____
_____

Pressure Vessels
- _____ inside diameter
- _____ operating pressure
- _____ window material
- _____ window thickness

Flammable Gas or Liquids
- type: _____
- flow rate: _____
- capacity: _____

Drift Chambers
- type: _____
- flow rate: _____
- capacity: _____

Other Target Materials
- ___ Beryllium (Be)
- ___ Lithium (Li)
- ___ Mercury (Hg)
- ___ Lead (Pb)
- ___ Tungsten (W)
- ___ Uranium (U)
- ___ Other (list below)

_____
_____

Vacuum Vessels
- _____ inside diameter
- _____ operating pressure
- _____ window material
- _____ window thickness

Radioactive Sources
- _____ permanent installation
- _____ temporary use
- type: _____
- strength: _____

Large Mech. Structure/System
- _____ lifting devices
- _____ motion controllers
- _____ scaffolding or
- _____ elevated platforms

Lasers
- type: _____
- wattage: _____
- class: _____

Installation:
- _____ permanent
- _____ temporary

Use:
- _____ calibration
- _____ alignment

Hazardous Materials
- ____ cyanide plating materials
- ____ scintillation oil (from)
- ____ PCBs
- ____ methane
- ____ TMAE
- ____ TEA
- ____ photographic developers
- ____ other (list below)

_____
_____

General:

Experiment Class:
- _____ Base Equipment
- _____ Temp. Mod. to Base Equip.
- _____ Permanent Mod. to Base Equipment
- _____ Major New Apparatus

Other: _____
_____

BEAM REQUIREMENTS LIST

CEBAF Proposal No.: _____ Date: _____
(For CEBAF User Liaison Office use only.)

List all combinations of anticipated targets and beam conditions required to execute the experiment. (This list will form the primary basis for the Radiation Safety Assesment Document (RSAD) calculations that must be performed for each experiment.)

Condition #	Beam Energy (MeV)	Beam Current (μA)	Polarization and Other Special Requirements (e.g., time structure)	Target Material (use multiple rows for complex targets — e.g., w/windows)	Target Material Thickness (mg/cm²)
1	300	< 60 μA		10 cm lqd. H2	
2	4000	< 60 μA		10 cm lqd. H2	

e beam energies, E_{Beam}, available are: $E_{Beam} = N \times E_{Linac}$ where N = 1, 2, 3, 4, or 5. For 1995, E_{Linac} = 800 MeV, i.e., available E_{Beam} are 800, 1600, 2400, 3200, and 4000 MeV. Starting in 1996, in an evolutionary way (and not necessarily in the order given) the following additional values of E_{Linac} will become available: E_{Linac} = 400, 500, 600, 700, 900, 1000, 1100, and 1200 MeV. The sequence and timing of the available resultant energies, E_{Beam}, will be determined by physics priorities and technical capabilities.

RESEARCH PROPOSAL TO CEBAF

Search for Narrow Excited States of the Proton

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and others

Abstract

We propose a measurement of the reaction $e+p \rightarrow e'+X$ to search for narrow excited states of the proton , p*, in the mass region:

$$M_p \langle M_x \langle M_p + 1250 \text{ MeV}$$

The sensitivity of the proposed measurements is more than a factor of 50 better than in previous experiments. We will use the Hall C HMS spectrometer to measure the scattered electron, e', at a 12.5^{o} angle. Beam energies of .3 GeV and 4 GeV and a maximum beam current of 60 μA are proposed, for a total of 50 hours. The experiment will use a 10 cm long liquid hydrogen target. The use of this thin walled target, together with the expected spectrometer resolutions and beam resolutions will provide excellent missing mass resolution. With very high sensitivity, at the lower beam energy, .3 GeV, the mass region $M_p \langle M_x \langle M_p + 140 \text{ MeV}$ will be covered. At the higher beam energy, 4. GeV, the mass region $M_p \langle M_x \langle M_p + 1250 \text{ MeV}$ will be scanned with very good sensitivity.

An indication of the proposed experiment's sensitivity is that any narrow state, p*, will be detected if its cross section relative to elastic scattering is greater than 2×10^{-5} in the region $M_p < M_x < M_p + 140$ MeV. There is a suggestion, by R. P. Feynman, concerning the possible existence of a colored proton, p_c, at a mass of about 990 MeV. In terms of coupling constants, this p_c is expected to be detected if the color carrying part of the photon has a strength α_c, relative to the electromagnetic coupling constant α_{EM}, of $\alpha_c / \alpha_{EM} \geq 2x\,10^{-5}$.

Requests: The Hall C HMS spectrometer operated for high resolution for electron measurements. Beams of a maximum current of 60 μA at .3 GeV and 4 GeV, each for 24 hours. 10 cm liquid hydrogen target. (This experiment might be run simultaneously with the proposed experiment: Search for Direct Conversion of Electrons into Muons)

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Motivation

The primary motivation for this proposal is the principle that experiments should be done whenever a dramatic improvement in sensitivity is possible over previous studies in a region of "physics phase space" of special interest. The conjecture, presented below, is discussed to point out such a region of special interest and the proposed experiment has a sensitivity of better than a factor of 50 over previous experiments.

The quark model, with color, has been very successful in explaining many features of hadron spectroscopy.[2)] In its basic form, the baryons, are made up of three quarks bound together by the exchange of gluons. Each of the quarks inside the physical baryon has a different color and the total color is zero. In this model, a physical proton is considered to be made up of two u quarks and a d quark. The simplest analysis of the proton and neutron magnetic moments implies[2)] that the masses of the u and d quarks are :

$$M_q \cong M_u \cong M_d \cong 336 \text{ MeV}$$

It follows that the binding energy, E, of the three quarks in the physical proton is :

$$E = 3M_q - M_p = 3(336) - 938 \cong 70 \text{ MeV}$$

or each quark-quark bond, B, can be considered to give rise to a binding energy E_B of :

$$E_B \cong E/3 \cong 23 \text{ MeV}.$$

Using this model, let us consider the properties of a physical hadron (colored proton) , p^*, made up to two u quarks and a d quark, where two of the quarks have the same color. These quarks inside the p^* will have two attractive bonds and one repulsive bond. If one assumes, in analogy to electrostatics, that the interaction energy of two like colored quarks is the opposite of the interaction energy for two unlike colored quarks in the same state, one estimates:

$$M_{p^*} \cong 3(336) - 23 \cong 985 \text{ MeV}.$$

R. P. Feynman's[1)] calculation used group theory to calculate the p* mass with the result: $M_{p^*} \cong 990$ MeV. Other calculations[3)] based on $M_q \cong M_u \cong M_d \cong 363$ MeV give $M_{p^*} \cong 1039$ MeV.

Obviously, the p* mass is not predicted exactly, even though the above model suggests that: $M_p \langle M_{p^*} \langle M_p + 140$ MeV. Generally the width of the p* state will probably be very narrow , $\Gamma \langle .1$ MeV, since it is expected to decay into a gamma and a proton. However, if the p* is high enough in mass to decay into a proton and a colored π, it probably would not be narrow. Since very little is known about colored π s, we will not speculate further, other than to say the entire p* mass region accessible at CEBAF should be searched with high sensitivity. Fortunately, such a search will take relatively little beam time, about 2 days.

In summary: the main importance of the above arguments is that they provide a plausibility argument for the existence of a narrow excited proton. At a minimum, the proposed experiment will search the CEBAF mass region for new excited states of the proton with greatly improved sensitivity over prior experiments.

The Experiment for $M_p \langle M_{p^*} \langle M_p + 140$ MeV

The physical background, in this mass region, is the radiative tail of elastic ep scattering. The calculations, below, are estimates of the sensitivity of the proposed experiment, which is determined by the size of the radiative tail and the missing mass resolution δM_X. The calculations, presented, are for $M_{p^*} \cong 985$ MeV and the results for other masses are given at the end of the proposal. As we shall show, our methods are in excellent agreement with actual data provided to us by K. Dow.[4)]

We plan to use the Hall C HMS spectrometer to detect the scattered electrons at 12.5^o . We will use the Hall A liquid hydrogen target, 10 cm length, "beer can" geometry with the beam traveling along the can's axis, .008 in Al walls. The Al windows each of have 2.3×10^{-3} radiation lengths, r.l. For an incident beam energy E_e = 300 MeV and a scattering angle of 12.5^o, the elastic scattered energy is $E_{el} \cong 298$ MeV. As discussed below, the level of the radiative tail is set by the total number

of radiation lengths, X_o, seen by the scattered electron[5] and by the elastic scattering cross section. X_o has three pieces:

$$X_o = X_{o.t.w.} + X_{o.l.h.} + X_{o.i.b.}$$

where $X_{o.t.w.}$ = the number of radiation lengths in the target windows = $4.6x10^{-3}$

$X_{o.l.h.}$ = the number of radiation lengths in the liquid hydrogen = $11.6x10^{-3}$.

$X_{o.i.b.}$ = the number of equivalent radiation lengths[5] when the electron scatters elasticly in the field of the proton = $3.0x10^{-2}$

Thus, the total equivalent radiation length is: $X_o \cong 4.6x10^{-2}$.

The counting rates and estimates of the sensitivity of the experiment are calculated following the methods of L. W. Mo and Y. S. Tsia[5] and are as follows:

Let N_R be the number of electrons per second, in a bin of width Γ_E in the radiative tail. Here Γ_E is the full width half maximum of the measurement uncertainty in the missing mass, M_X. To a good approximation[6]:

$$N_R \cong \frac{\Gamma_E X_o}{2*(E_{el} - E_{e'})} * \left[(E_{el} / E_{e'})^2 + 1 \right] * N_{el}$$

where: $N_{el} = (\#e's/sec)*(\#p's/cm^2)*\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep} * \Delta\Omega$. Here $\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep}$ is the elastic scattering cross section and $\Delta\Omega$ is the HMS solid angle. N_e = is the number of elastic scatterings detected by the HMS under the same running conditions. (See Ref. 6 for the above relationship expressed in terms of the cross sections.)
E_{el} = 298 MeV is the scattered energy at the elastic peak and $E_{e'}$ = 250 MeV is the scattered electron energy at M_x = 985 MeV.

Γ_E has four parts: $\Gamma_E = \Gamma_\Theta \oplus \Gamma_{El} \oplus \Gamma_{Eb} \oplus \Gamma_I$, where $\oplus$ implies that the terms are added in quadrature. Here: Γ_Θ = the gaussian resolution, full

width, contribution to the uncertainty of the M_X measurement due to the scattering angle measurement uncertainty. Γ_{El} = the resolution contribution from the measurement of the energy loss, E_{El}, the difference between the scattered electron's energy, $E_{e'}$ and incident beam energy, E_b. We plan to achieve $\Gamma_{El} = .001\ E_{e'}$. Γ_{Eb} = is the contribution from the beam energy spread where we plan to achieve Γ_{Eb} = .0001 or better. Γ_I = the resolution from the variation in energy loss by ionization due to different path lengths in the target.

For the kinematics given above:

$$\Gamma_E = \Gamma_\Theta \oplus \Gamma_{El} \oplus \Gamma_{Eb} \oplus \Gamma_I$$

$$\Gamma_E = 0.205 \oplus 0.246 \oplus 0.000 \oplus 0.000$$

$$\Gamma_E = 0.32 \text{ MeV}$$

(More complete details of these calculations are given following the References in the back of this proposal titled: "Calculation of the mass resolution." Also, similar calculations for carrying out the experiment using the a HRS spectrometer in Hall A gives: Γ_E=0.21 MeV. Thus, the sensitivity of the HRS would be a factor of 1.5 better than for the HMS in Hall C, as indicated below.)

$$\text{Thus,} \quad N_R \cong 4.1\text{X}10^{-4} * N_{el}$$

$$\text{and } N_{el} = (\#e's/\text{sec})*(\#p's/\text{cm}^2)*\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep} * \Delta\Omega\ .$$

The 10 cm diameter liquid hydrogen target, a beam current of .6μA, an HMS angular acceptance $\Delta\Omega$ = 10 msr and $\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep} = 3.9\text{x}10^{-28}\ \text{cm}^2/\text{sr}$, gives:

$$N_{el}=(0.4\text{x}10^{13})(0.4\text{x}10^{24})(4\text{x}10^{-28})(10^{-2})=0.6\text{x}10^{7}\ /\text{sec.}$$

Thus, $N_R = (4\text{x}10^{-4})*(0.6\text{x}10^{7}) \cong 2.4\text{x}10^{3}$ counts/(second Γ_E)

If we require that the p^* signal appear at least 5% above the radiative tail and be at least a 5 standard deviation affect, it follows that:

$$\frac{N_{p^*}}{N_R} \geq 5\text{x}10^{-2} \quad \text{and} \quad N_R \geq 10^4.$$

To accumulate 10^4 counts per Γ_E bin, for the setting indicated above, requires less than 1 minute, even at a beam current of .6 μA. For these parameters, the total scattered electron event rate at the focal plane will be approximately $2\text{x}10^4$ per second. This will probably saturate the data handling capabilities of the HMS. For a scan of 25 magnet settings of 20 minutes each, this implies a total beam time of 8 hours. Taking various inefficiencies into account, we conservatively ask for 24 hours.

It also follows that:

$$N_{p^*} \geq (5\text{x}10^{-2})*(4\text{x}10^{-4})*N_{el}=2\text{x}10^{-5}*N_{el}$$

if this p^* is to be detected in this experiment. This defines the sensitivity of the experiment for M_{p^*} = 985 MeV.

We get the cross section limit of $\dfrac{\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep^*}}{\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep}} \geq 2\text{x}10^{-5}$.

If the p^* form factor is the same as the proton, it follows that:

$\dfrac{\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep^*}}{\left.\frac{d\sigma}{d\Omega}\right|_{ep\to ep}} \cong \dfrac{\alpha_c}{\alpha_{EM}}$. Thus, our experiment typically requires $\left(\dfrac{\alpha_c}{\alpha_{EM}}\right) \geq 2x10^{-5}$ for the detection of a p^* in the mass region to be measured.

The sensitivity calculations, above, apply to the special kinematics case of where the beam energy is E_B=300 MeV, the scattering angle is 12.5^O and M_{p^*}=985 MeV. The results of the calculations for the full mass region to be covered at E_B=300 MeV are given in Figs.1 through 3. It is assumed that the momentum acceptance of the HMS is 10%. Our plan is to change the magnet current in 5% steps (50% overlap) accumulating at least 10^4

events per Γ_E for each magnet setting. Thus, each mass bin of width Γ_E will contain at least 10^4 events for each of two magnet settings and there will be good overlap between the adjacent magnet settings.

A Check On Our Calculations

The measurements of K. Dow[4)] at a beam energy of 289 MeV and an angle of 54° at Bates are shown in Fig. 3 along with our calculations. There are no free parameters in these calculations. The excellent agreement between Dow's measurements and the calculations indicates that our method for calculating the radiative tail is reliable. Furthermore, the elastic peak in Dow's data has Γ_E = 0.90 MeV, in agreement with our calculations.

Limits Set By Previous Experiments

The best previous experiments known to us are the works of K. Dow[4)] and A. Esaulov, et al.[7)] neither of which was designed to search for narrow states. However, both experiments did measure some of the mass region of interest. We estimate that the sensitivity of our proposed experiment is better than a factor of 50 more than that of K. Dow and more than a factor of 150 better than that of A. Esaulov, et al.

Additional Remarks

1) It is of interest to compare the above sensitivity limit to the limit implied by the agreement between the measured Z^0's width and its width predicted from the Standard Model. The Z^0 width has been measured to $\pm 1.0\text{x}10^{-2}$ GeV. [8)] If the Z^0, the heavy photon, has a colored piece, this will broaden the Z^0 beyond the Standard Model value. Thus, from the Z^0 decay width (LEP Experiments) one gets: $\frac{\delta\Gamma}{\Gamma} \approx \frac{1.0\text{x}10^{-2}}{2.487} \approx .4\text{x}10^{-2} \geq \frac{\alpha_c}{\alpha_{EM}}$.

Our experiment is sensitive to: $\alpha_c / \alpha_{EM} \geq 2x\,10^{-5}$ or nearly a factor of 200 more sensitive to a colored photon than the limits suggested by the Z^0 width.

The Experiment for $(M_p + 140\ \mathrm{MeV}) \langle M_{p*} \langle (M_p + 1250\ \mathrm{MeV})$

As discussed before, the mass of the p* cannot be predicted exactly. Therefore, we wish to scan the full mass region accessible at CEBAF. In this higher mass region the "background" is not the radiative tail but rather the scattering of electrons from the lowest lying N* states. For this scattering the ratio of the cross section for the N* scattering, in an interval Γ_E, in MeV, to the elastic ep scattering is approximately:[9),3)]

$$R(N^*/ep) = \Gamma_E/270$$

The sensitivity of our experiment versus Mx, derived from the above using the method developed for the lower mass radiative tail background, is presented in Fig. 4.

References

1) R. P. Feynman, Les Houches, Session XXIX, (1976)

2) A. De Rujula, et al., Phys. Rev. D 12 (1975) 147

3) D. J. Griffiths, Introduction to Elementary Particles, John Wiley & Sons, Inc. (1987)

4) Measurements made by K. Dow at Bates as part of her Ph. D. thesis research (unpublished.) We thank K. Dow for making the results of these measurements available to us.

5) L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. 41 (1969) 205

6) This formula $N_R \cong \frac{\Gamma_E X_0}{2*(E_{el}-E_{e'})}*\left[(E_{el}/E_{e'})^2+1\right]*N_{el}$ can be arrived at in the following way. First, one uses the approximation of L. W. Mo and Y. S. Tsia[5)] in which the interaction, bremsstrahlung, is approximated in two parts. One part in which the incident electron sees one half of the total radiation lengths, $X_0/2$, before it scatters and one part in which the electron scatters and then sees one half of the total radiation lengths. The first term of the above expression is the standard bremsstrahlung expression. The first term inside the brackets corresponds to beam bremsstrahlung followed by elastic scattering. This term is bigger than one since the degraded beam electron has a larger elastic cross section than does the electron at beam energy. The second term inside the brackets is just the bremsstrahlung after the beam electron has scattered elasticly.

The above formula, in terms of the cross sections, is:

$$\frac{\Gamma_E d\sigma}{d\Omega dM_X} \cong \frac{\Gamma_E X_0}{2*(E_{el}-E_{e'})}*\left[(E_{el}/E_{e'})^2+1\right]*\frac{d\sigma}{d\Omega_{el}}$$

7) A. S. Esaulov, et al., Nuc. Phys. B136 (1978) 511

8) Particle Data Group, Phys. Rev D11 (1992)

9) P.L. Pritchett, et al., Phys. Rev. 184 (1969)1825

Calculation of the mass resolution.

Now calculate the mass resolution:

First,the energy loss determination:

$$\Gamma El_j := \frac{1}{M_j} \cdot (Mp + Eb \cdot (1 - \cos(\theta))) \cdot .001 \cdot E_j$$

Second, the variation in energy loss due to differing path lengths in the target:

$$\Gamma l_j := \frac{1}{M_j} \cdot (Mp + Eb \cdot (1 - \cos(\theta))) \cdot .00$$

Third, the effect of the spread in the beam energy:

$$\Gamma Eb_j := \left(\frac{1}{M_j}\right) \cdot (Eb + E_j) \cdot (1 - \cos(\theta)) \cdot (.0001 \cdot Eb)$$

Fourth, the scattering of incident beam, b:

The quantities indicated by deltas are rms.

$$Xin := .0081 \quad Xout := .0081$$

$$\delta\theta b := \frac{13.6}{Eb} \cdot \sqrt{Xin} \cdot (1 + .038 \cdot \ln(Xin))$$

Fifth, the scattering of outgoing electron of energy E_j:

$$\delta\theta E_j := \frac{13.6}{E_j} \cdot \sqrt{Xout} \cdot (1 + .038 \cdot \ln(Xout))$$

Combine the two angles:

$$\delta\theta total_j := \sqrt{\delta\theta b^2 + (\delta\theta E_j)^2}$$

$$\delta\theta mrad_j := 1000 \cdot \delta\theta total_j$$

$$\Gamma\theta_j := \frac{1}{M_j} \cdot Eb \cdot E_j \cdot \sin(\theta) \cdot \delta\theta total_j \cdot 2.36$$

$$\Gamma total_j := \sqrt{(\Gamma\theta_j)^2 + (\Gamma El_j)^2 + (\Gamma Eb_j)^2 + (\Gamma l_j)^2}$$

Fig. 1. Mass resolution versus mass at 300 MeV, 12.5^{o}.
The units are MeV.

p* 300/12.5 deg

$$\Gamma total_j := \sqrt{(\Gamma\theta_j)^2 + (\Gamma El_j)^2 + (\Gamma Eb_j)^2 + (\Gamma\Pi_j)^2}$$

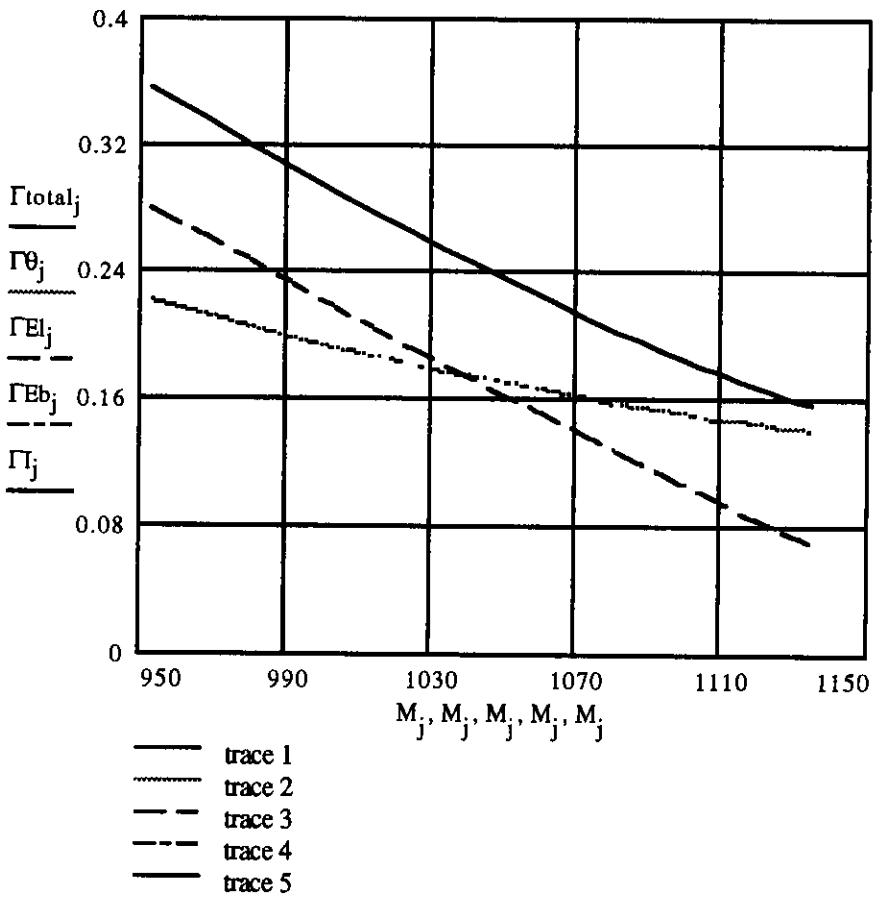


Fig. 2. Cross section sensitivity, relative to elastic scattering, versus mass in MeV at 300 MeV 12.5^{o}.

$$Xotot := Xin + Xout + 1.5 \cdot \frac{1}{137 \cdot 3.14} \cdot \left(\ln\left(\frac{QSQ}{.260}\right) - 1 \right) \qquad Xotot = 0.0465$$

$$\Gamma_j := \Gamma total_j \qquad Xo := Xotot \qquad Xofield := Xotot - Xin - Xout \qquad Xofield = 0.0303$$

$$R_j := \Gamma_j \cdot \frac{Xo}{2} \cdot \left(\frac{1}{Ef - E_j}\right) \cdot \left[\left(\frac{Ef}{E_j}\right)^2 + 1 \right]$$

Note, R_j is the probability, per elastically scattered electron, of degraded electron ending up in in the bin R_j.

$$S_j := .05 \cdot R_j$$

p* 300/12.5 deg dPb/Pb=.0001 dEe'/Ee'=.001

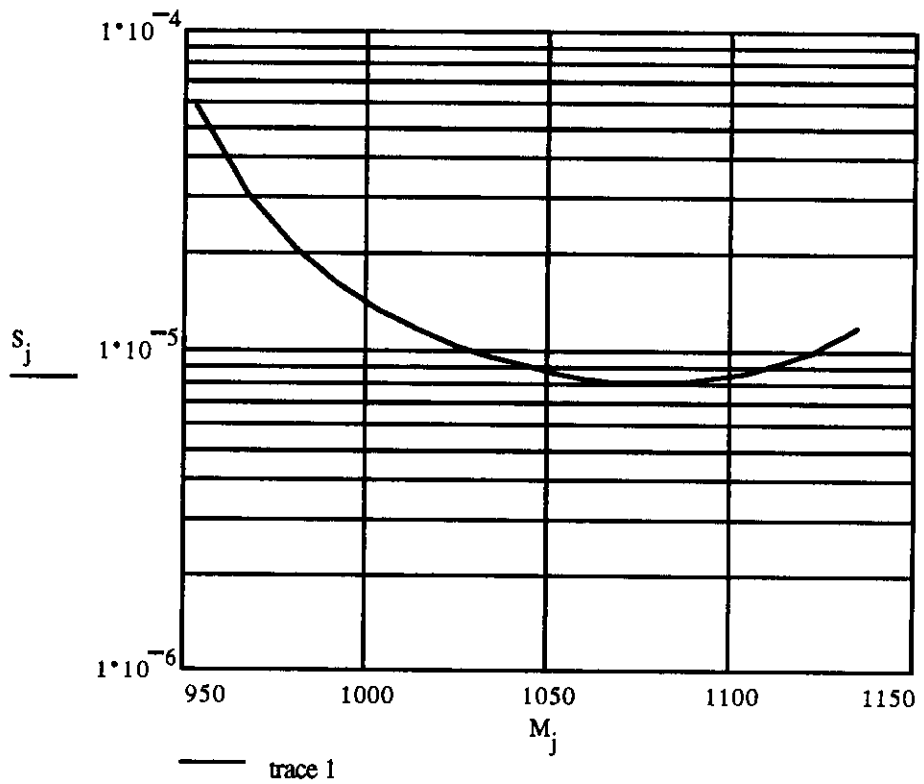


Fig. 3. A comparison of the radiative tail calculations to the measurements of K. Dow. The units of the cross section are nb/(MeV sr). There are no free parameters in the calculation. The mass units are MeV. (289 MeV, 54°.)

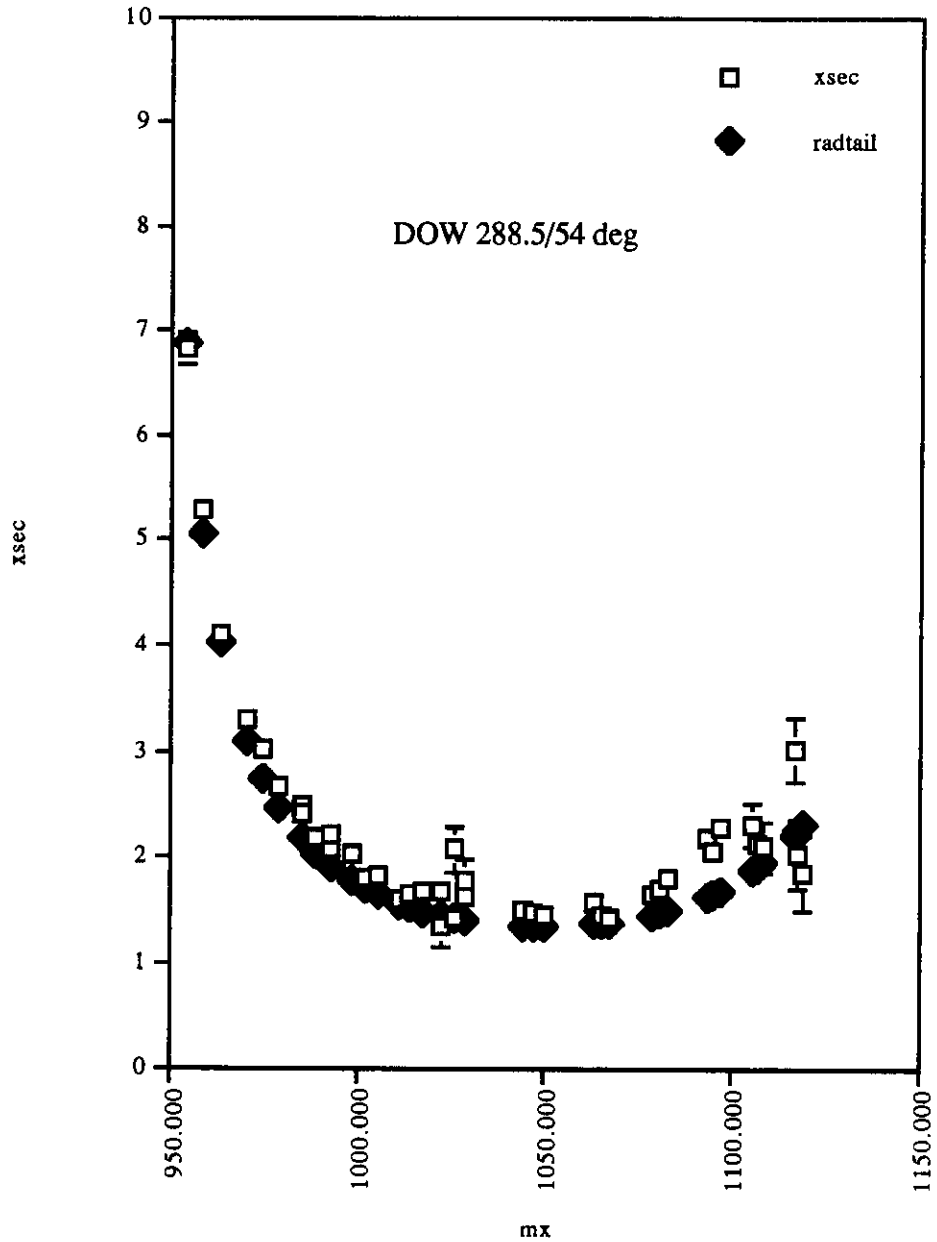
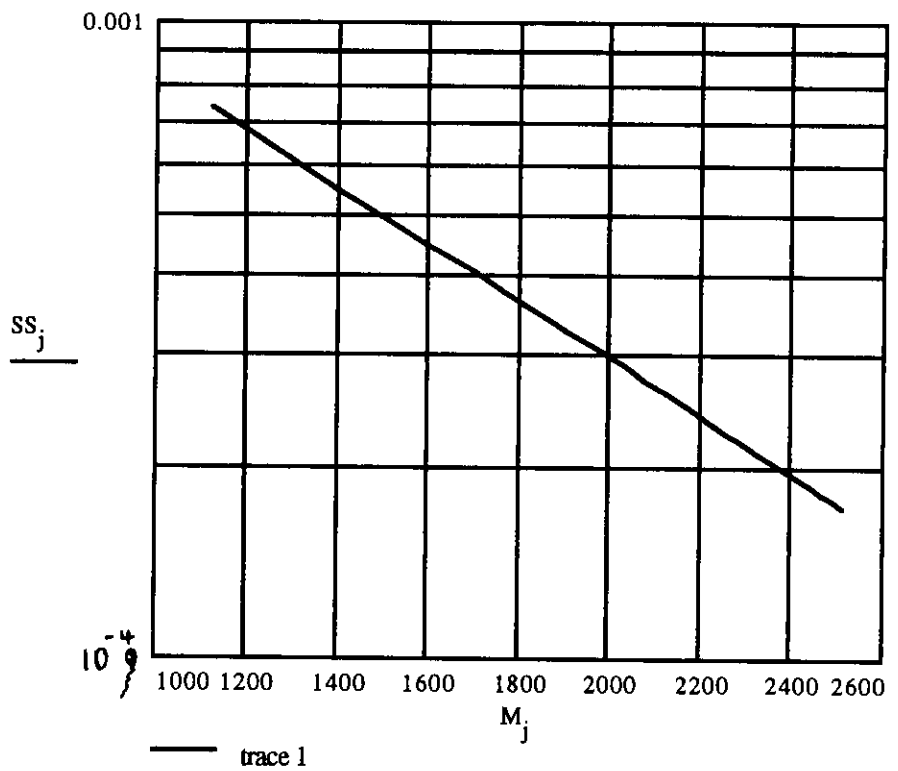


Fig. 4. Cross section sensitivity, relative to elastic scattering, versus mass in MeV at 4000 MeV, 12.5°.

$$\Gamma_j := \Gamma total_j$$

$$RR_j := \frac{\Gamma_j}{270}$$

$$SS_j := .05 \cdot RR_j$$