The CGC: an effective theory of QCD at high energies

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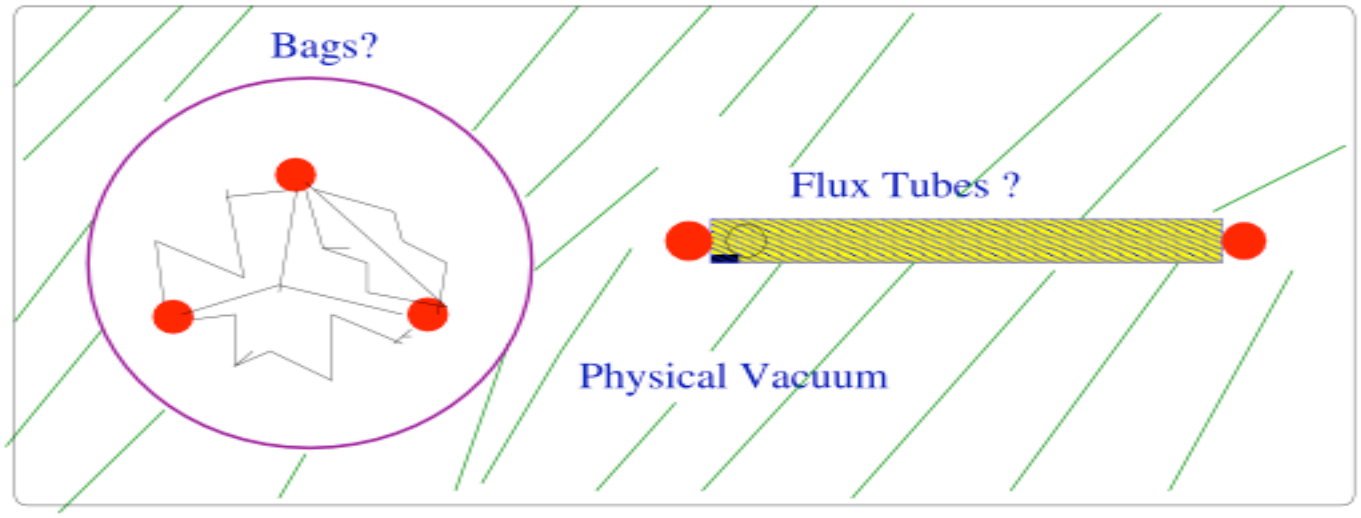
Outline of lectures

- Lecture I: *General introduction, the DIS paradigm, QCD evolution, saturation.*
- Lecture II: *The IMF wavefunction, the MV model.*
- Lecture III: *Quantum evolution in the CGC, Wilson RG, analytic and numerical solutions.*
- Lecture IV: *DIS and hadronic scattering at high energies; Heavy Ion collisions at RHIC.*

The Color Glass Condensate: An effective field theory of QCD at high energies

- Life on the Light Cone
- The MV-model
- Quantum evolution: a Wilsonian RG
- The JIMWLK equations
- Analytical approximations and numerical solutions

Unlike QED, the QCD light cone vacuum is very complicated:
--various topological objects, Instantons, Monopoles, Skyrmions, … Hadrons may be bags or flux tubes or solitons:
Complex phenomena - Chiral symmetry breaking, Confinement,…



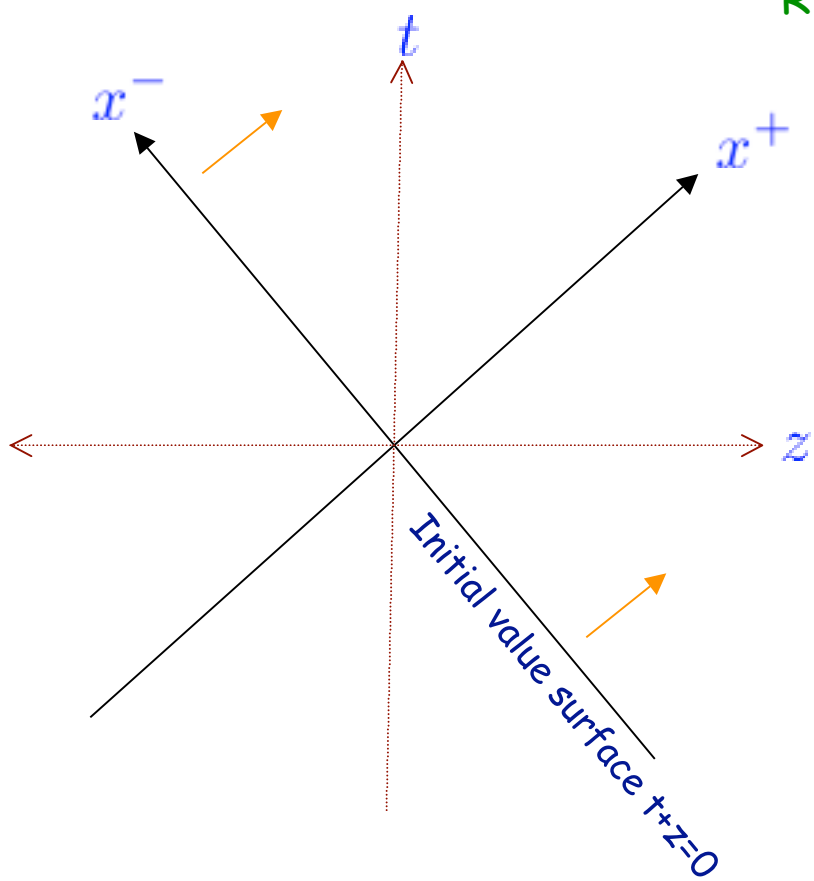
- Given this, how does one describe the structure of hadrons in high energy scattering?

How does one construct a Lorentz invariant wave fn for a hadron?

Partial answer: formulate the theory on the light cone

Life on the light cone

RV, nucl-th/9808023



Quantize theory on light like surface: $x^+ = 0$

Quantum field theories quantized on light like surfaces have remarkable properties

Light cone algebra:

$$x^\mu \equiv (x^0, x^1, x^2, x^3) = (t, \vec{x})$$

$$x^\pm = \frac{(t \pm z)}{\sqrt{2}}\,;\; \partial_\pm = \frac{1}{\sqrt{2}}(\partial_t \pm \partial_z)\,;\; A^\pm = \frac{(A^0 \pm A^z)}{\sqrt{2}}$$

$$g^{++} = g^{--} = 0\,;\; g^{-+} = g^{+-} = 1\,;\; g^{xx} = g^{yy} = -1$$

$$=> A_\pm = A^\mp \;\&\; A_x = -A^x$$

For spinors, define projection operator $\alpha^\pm = \frac{\gamma^\mp \gamma^\pm}{2}$

Project out two component spinors $\psi_\pm = \alpha^\pm \psi$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \longrightarrow \psi_+ = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \\ \psi_4 \end{pmatrix} \;\&\; \psi_- = \begin{pmatrix} 0 \\ \psi_2 \\ \psi_3 \\ 0 \end{pmatrix}$$

ψ_+ and A_{x,y} are the dynamical “good” fields in which the physical content of the theory is expressed

Light cone quantization:

$$\psi_+ = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm 1/2} \left[e^{ik\cdot x} b_s(k;x^+) + e^{-ik\cdot x} d_s^\dagger(k;x^+) \right]$$

$$\left\{ b_s(k;x^+), b_{s'}^\dagger(k';x^+) \right\} = \left\{ d_s(k;x^+), d_{s'}^\dagger(k';x^+) \right\} = (2\pi)^3 \delta^{(3)}(k-k')\delta_{ss'}$$

$$A_i^a(x) = \int \frac{d^3k}{\sqrt{2k^+}(2\pi)^3} \sum_{\lambda=1,2} \delta_{\lambda i} \left[e^{ik\cdot x} a_\lambda^a(k;x^+) + c.c \right]$$

$$[a_\lambda^a(k;x^+), a_\lambda'^{a\dagger}(k';x^+)] = (2\pi)^3 \delta^{(3)}(k-k')\delta_{\lambda\lambda'}$$

Light cone QCD Hamiltonian in light cone gauge: $A^+ = 0$

$$\boxed{P^-_{\rm QCD} = P^-_0 + V_{\rm QCD}}$$

$$P^-_{0,\rm fermi} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=1/2} \frac{(k_t^2 + M^2)}{2k^+} \left(b_s^\dagger(k) b_s(k) + d_s^\dagger(k) d_s(k)\right)$$

$$P^-_{0,\rm bose} = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \frac{(k_t^2 + M^2)}{2k^+} a_\lambda^{a\dagger}(k) a_\lambda^a(k)$$

- The QCD vacuum is ``trivial" in light cone quantization. It is an eigenstate of both $P^-_{\rm QCD}$ & P^-_0

Physical states therefore expressed in terms of Fock states of bare quanta => PARTON MODEL

In light cone quantization, the boost operator commutes with the Hamiltonian

- a) *Boosts do not generate particles as in equal time quantization.*

b) *Light cone wave-functions are boost invariant*

Weinberg, 1966
Susskind, 1968

Q.F.T on the light cone

isomorphism

Two dimensional quantum mechanics

Light cone dispersion relation:

Momenta

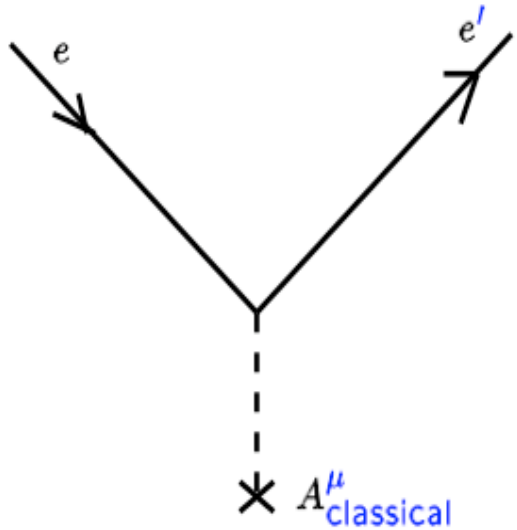
$$P^- = \frac{(P_t^2 + M^2)}{2P^+}$$

Energy

Mass

Light cone pert. theory = Rayleigh-Schrodinger pert. theory

Example: electron scattering off an external potential in QED



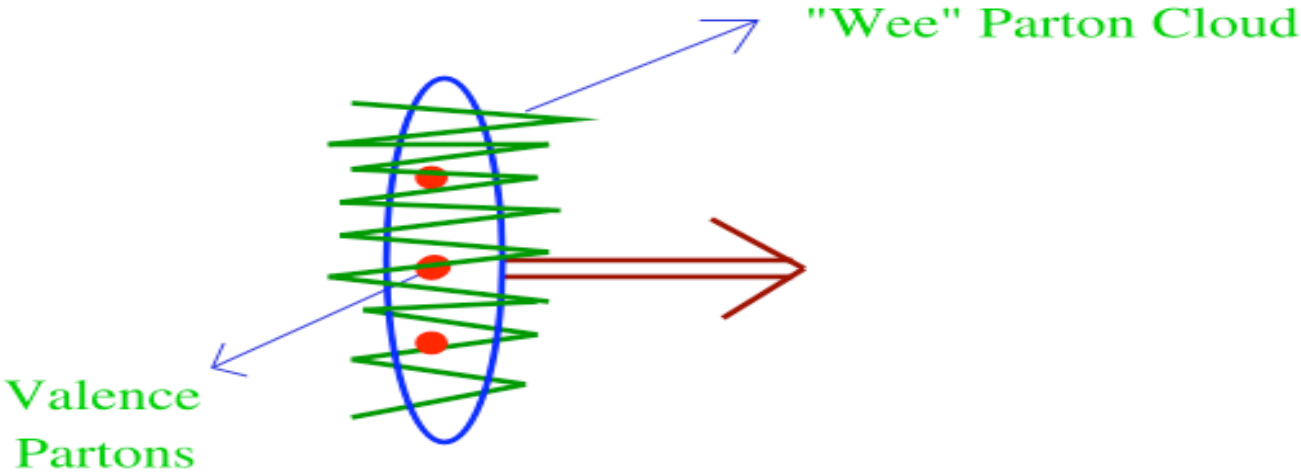
Bjorken,Kogut,Soper, 1971

$$|e^-_{\text{phys.}}> = a_1|e^-_{\text{bare}}> + a_2|e^-\gamma^*> + a_3|e^-\gamma^* e^+ e^-> + \cdots$$

- Scattering of physical state is complex at high energies due to many interacting quanta.
- Mutual interactions of the quanta ("partons") is simple-slowed down by time dilation.
- Scattering of the partons off the potential is simple-they acquire an eikonal phase

QFT basis of Bj scaling

A hadron at high energies



$$|h>=|qqq>+|qqqg>+\cdots+|qqq\cdots gggq\bar{q}>$$

Each wee parton carries only a small fraction $x^+ = k^+/P^+$ of the momentum P^+ of the hadron

What is the behavior of wee partons in a high energy hadron?

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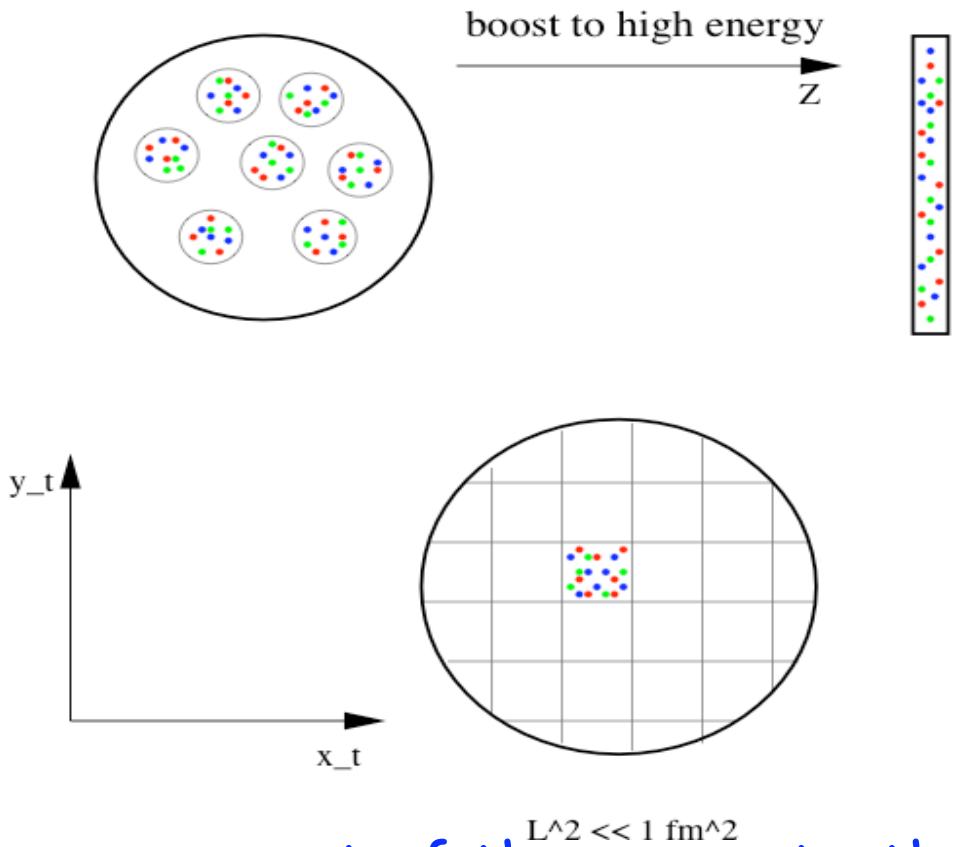
The MV model

McLerran, RV; Kovchegov
Jalilian-Marian,Kovner,McLerran,Weigert

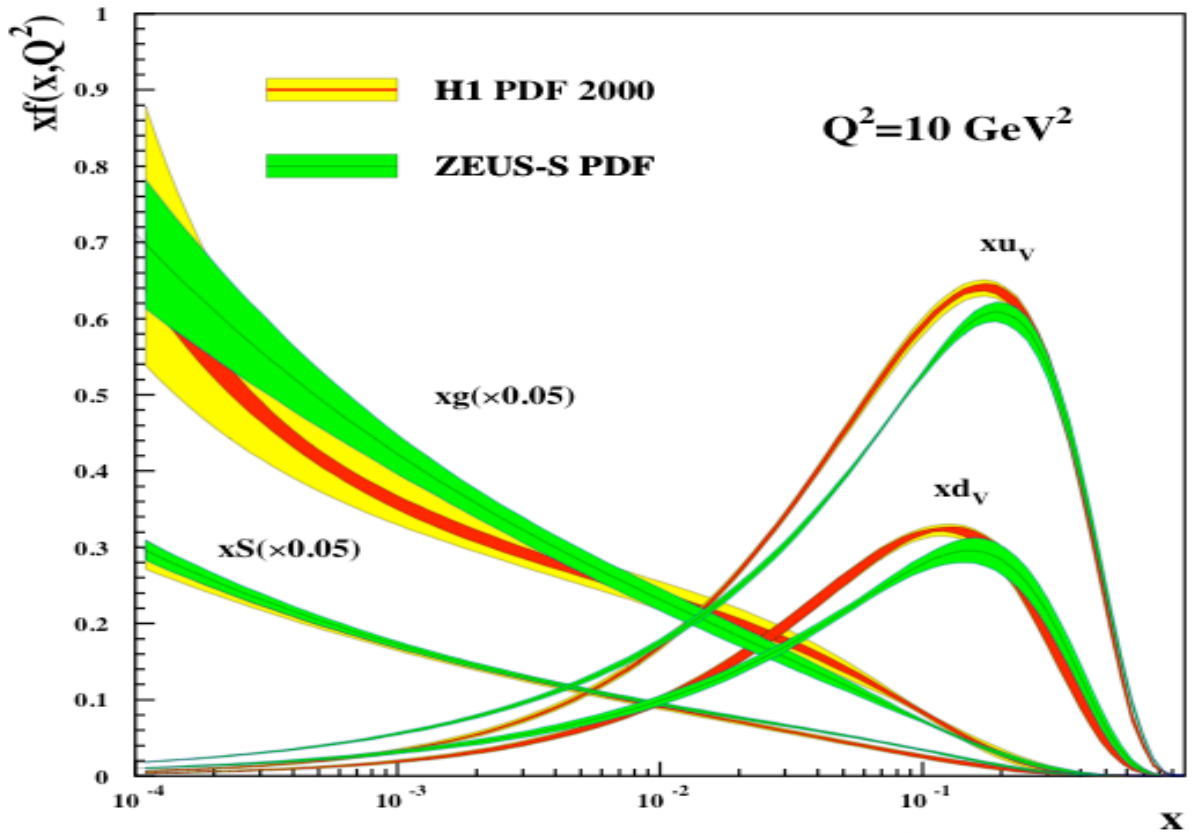
Consider large nucleus in the IMF frame: $P^+ \to \infty$



One large component of the current-others suppressed by $\frac{1}{P^+}$

Wee partons see a large density of valence color charges at small transverse resolutions.

Born-Oppenheimer: separation of large x and small x modes

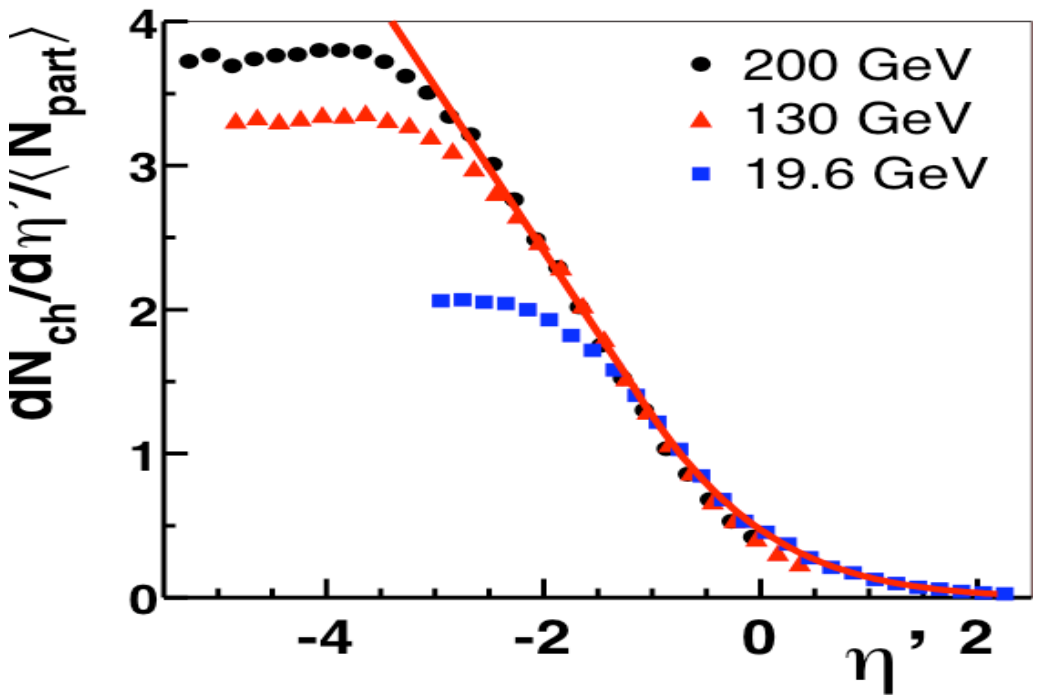


$$\tau_{\rm wee} \sim \frac{1}{k^-} = \frac{2k^+}{k_\perp^2} \equiv \frac{2x\,P^+}{k_\perp^2}$$

$$\tau_{\rm valence} = \frac{2P^+}{k_\perp^2} >> \tau_{\rm wee} \text{ for } x << 1$$

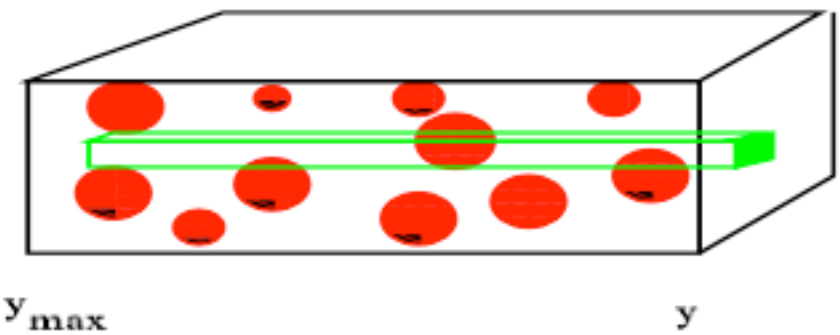
Valence partons are static over wee parton life times

Limiting fragmentation



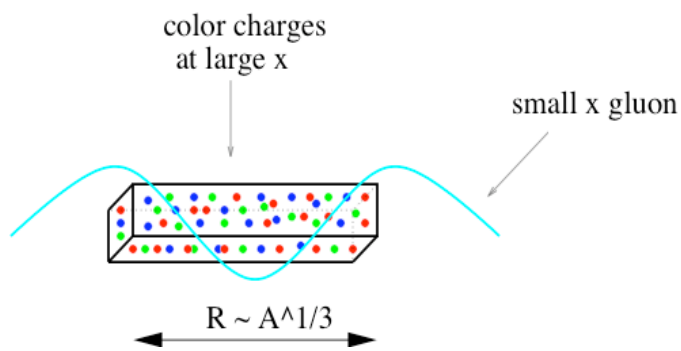
Suggestive that valence partons are recoil-less sources-unaffected by Bremsstrahlung of wee partons

Random sources



$$\lambda_{\rm wee} \sim \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\rm valence} \equiv \frac{Rm_p}{P^+}$$

$$=> x << A^{-1/3};$$



$$< \rho^a >= 0; < \rho^a(x_\perp)\rho^b(y_\perp) >= \mu_A^2 \delta^{ab} \delta^{(2)}(x_\perp - y_\perp)$$

Gaussian random sources

The effective action

Scale separating sources and fields

Generating functional:

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional describing distribution of the sources

$$S[A,\rho] = \frac{-1}{4} \int d^4x\, F^2_{\mu\nu} + \frac{i}{N_c} \int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

wher e $U_{-\infty,\infty}[A^-] = \mathcal{P} \exp\left(ig \int dx^+ A^{-,a} T^a \right)$

To lowest orde $= -J^+ A^-$ with $J^+ = g\, \rho(x_\perp)\, \delta(x^-)$

For a large nucleus,

$$W[\rho] = \exp\left(-\int d^2x_\perp \frac{\rho^a \rho^a}{2\,\mu_A^2}\right)$$

where, for valence quark sources, on $\mu_A^2 = \frac{g^2 A}{2\pi R_A^2} \propto A^{1/3}\,\mathrm{fm}^{-2}$

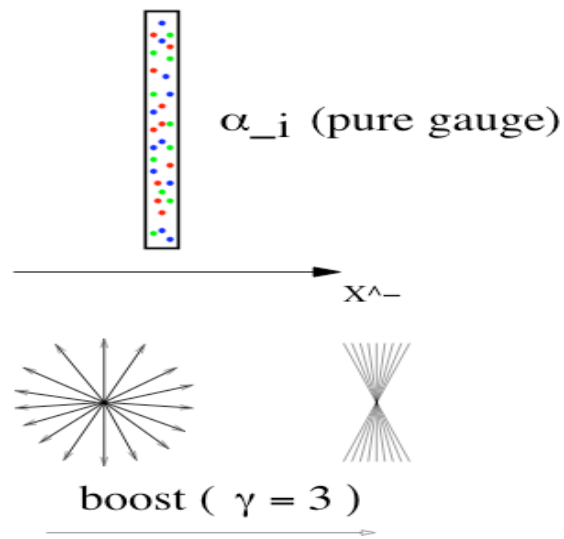
For A >>1, $\mu_A^2 >> \Lambda_{\mathrm{QCD}}^2$ and $\alpha_S(\mu_A^2) << 1$

Effective action describes a weakly coupled albeit non-perturbative system

The classical field of the nucleus at high energies

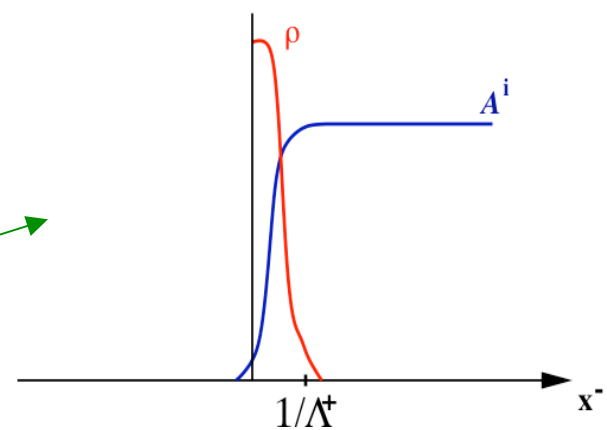
Saddle point of effective action-> Yang-Mills equations

$$D_\mu F^{\mu\nu} = \delta^{\nu +}\delta(x^-)\rho^a(x_\perp)$$



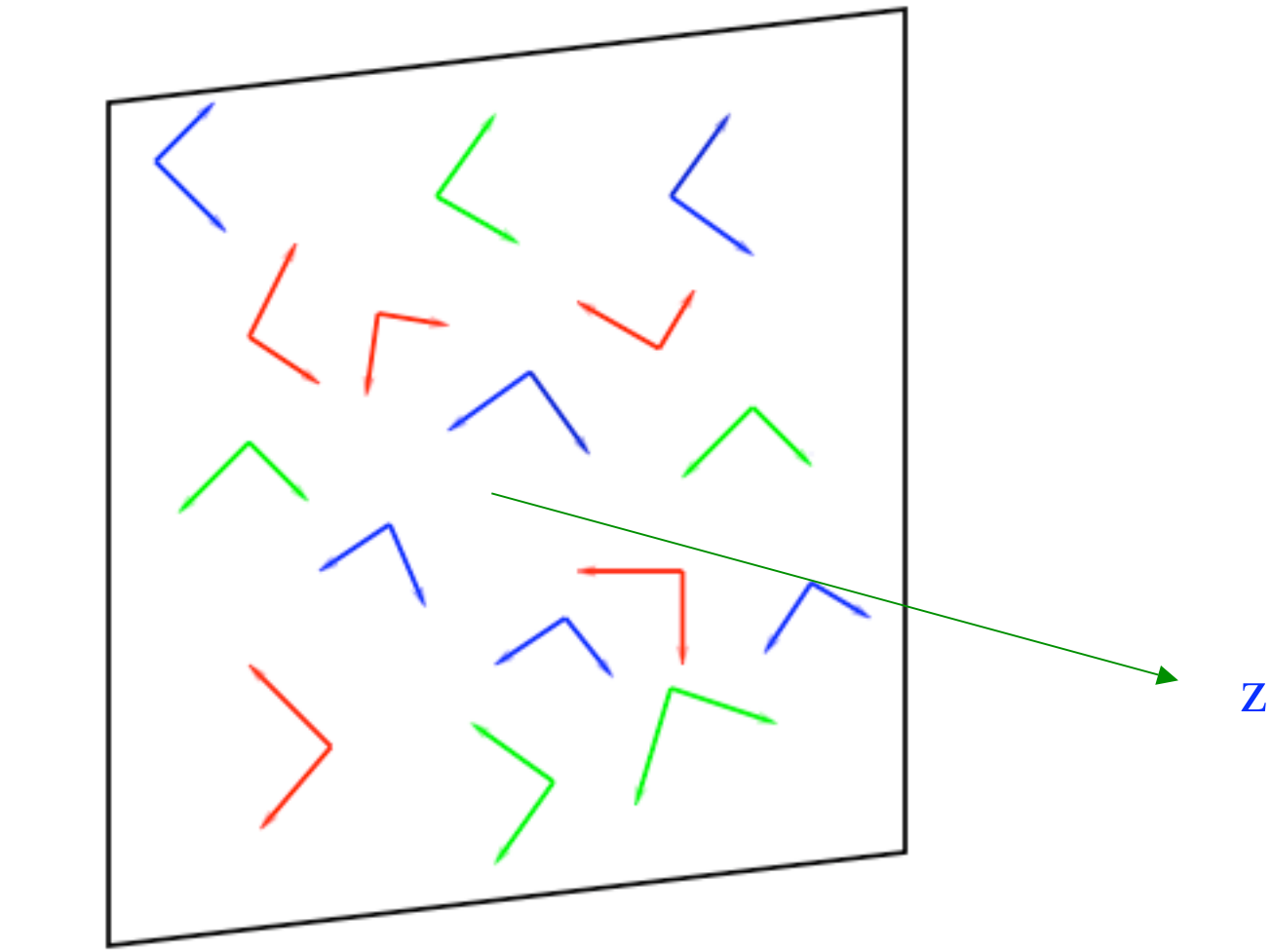
Solutions are non-Abelian Weizsäcker-Williams fields

$$A^+ = A^- = 0\ ;$$
$$F^{ij} = 0 => A^i = \theta(x^-)\alpha^i\ ,$$
$$\text{where } \alpha^i = \frac{-1}{ig}U\nabla^i U^\dagger$$
$$\text{and } \nabla\cdot\alpha = g\rho$$



Careful solution requires smearing in x^-

Random Electric & Magnetic fields in the plane of the fast moving nucleus

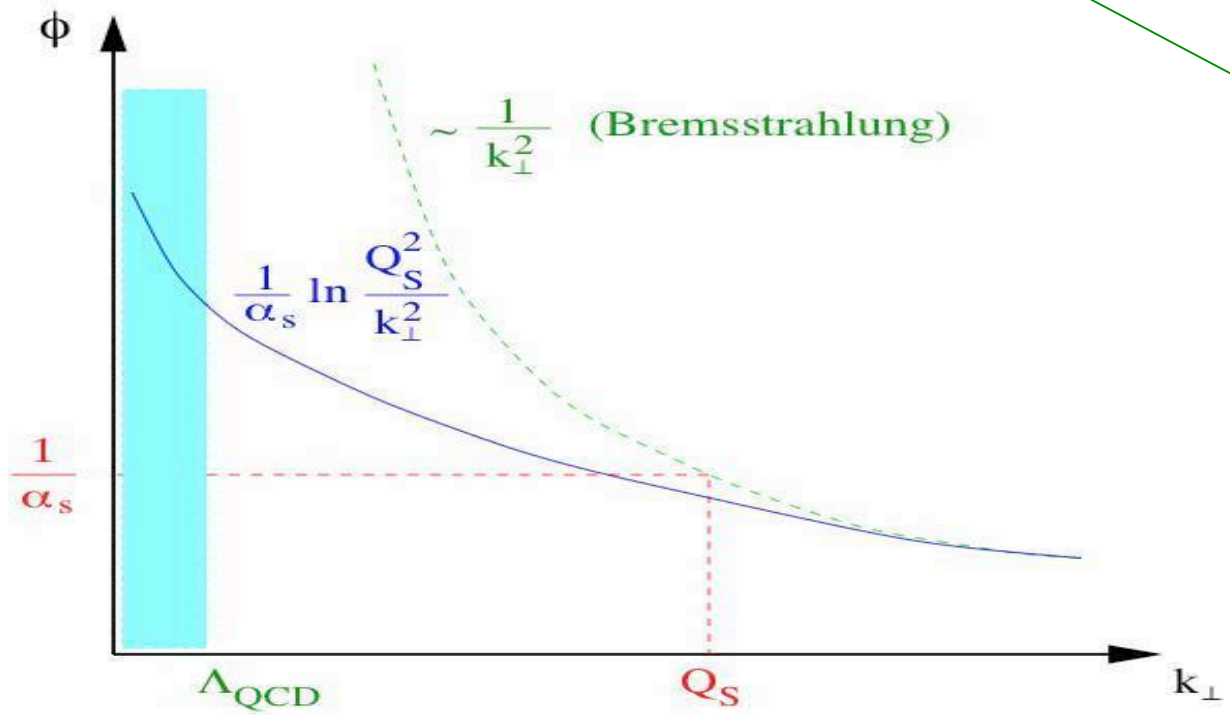


$x^- = 0$

Average over ρ^a to compute gluon distribution $< AA >_\rho$

$$< AA >_\rho = \int [d\rho] A_{\rm cl.}(\rho) A_{\rm cl.}(\rho) W_{\Lambda^+}[\rho]$$

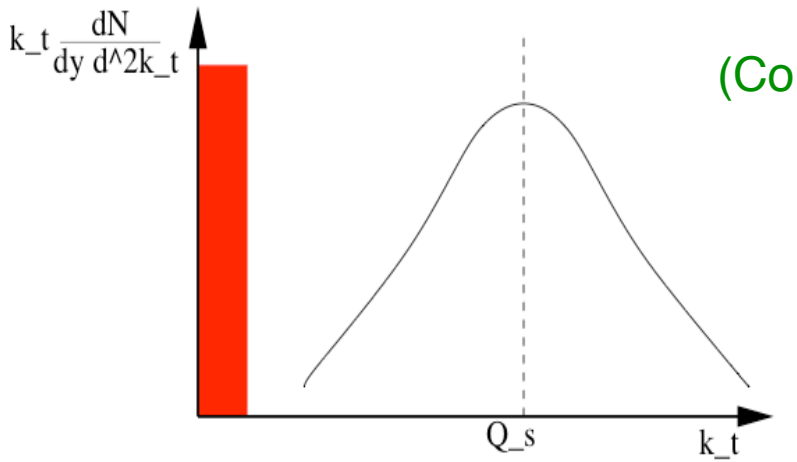
Gaussian in MV



ϕ = gluon phase space densi $\frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN}{\pi R^2 d^2k_\perp dy}$

$Q_s^2 \approx \alpha_S N_c \mu_A^2 \ln\left(\frac{Q_s^2}{\Lambda^2}\right) \sim A^{1/3} \ln A \approx A^{1/3}$ for A >> 1

The Color Glass Condensate



(Condensado de vidro de cor)

- ✓ *Typical momentum of gluons is* Q_s
- ✓ *Bosons with large occupation #* $\sim \frac{1}{\alpha_S}$ *- form a condensate*
- ✓ *Gluons are colored*
- ✓ *Random sources evolving on time scales much larger than natural time scales-very similar to spin glasses*

Hadron/nucleus at high energies is a Color Glass Condensate

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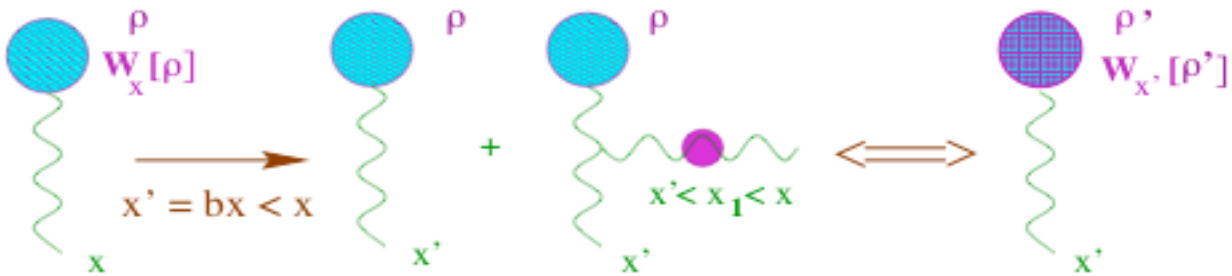
Quantum evolution in the Color Glass Condensate

- The MV-model is a classical small x effective theory for a large nucleus, with Gaussian sources

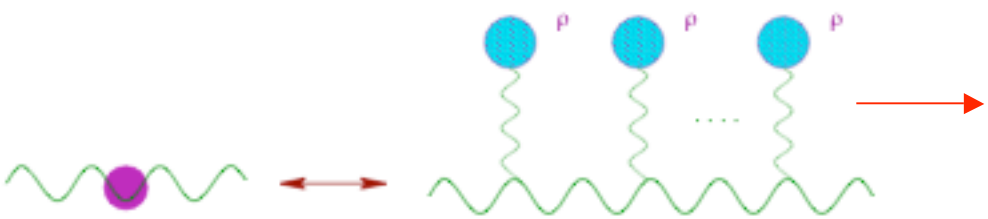
- Small x quantum corrections are large-significantly modifying this simple picture-correlations are no longer Gaussian

- The large small x corrections can be incorporated in a Wilsonian Renormalization Group procedure- fluctuations in fields at the scale x_1 are incorporated in the sources at the next scale x_2

Wilson RG at small x



Color charge grows due to inclusion of fields into hard source with decreasing x: $\rho' = \rho + \delta\rho => W_x[\rho] \to W_{x'}[\rho']$



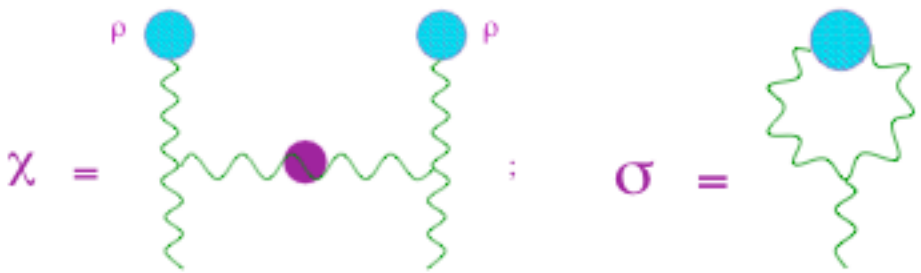
Because of strong fie $A \sim 1/g$
All insertions are O(1)

$W_x[\rho]$ *obeys a non-linear Wilson renormalization group equation-the JIMWLK equation*

(Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)

At each step in the evolution, compute 1-point and 2-point functions in the background field

$$\sigma^a(x)[\rho] =< \delta\rho^a_Y(x) >_\rho \; ; \; \chi^{ab}(x,y)[\rho] =< \delta\rho^a_Y(x)\delta\rho^b_Y(y) >_\rho$$



$$\sigma^a(x) = \frac{1}{2}\int d^2y \frac{\delta\chi^{ab}(x,y)}{\delta\rho^b_Y(y)}$$

The JIMWLK (functional RG) equation:

$$\frac{\partial W_x[\rho]}{\partial \ln(1/x)} = \frac{1}{2}\int_{x_\perp, y_\perp} \frac{\delta}{\delta\rho^a_x(x_\perp)}\chi^{ab}(x_\perp, y_\perp)[\rho]\frac{\delta}{\delta\rho^b_x(y_\perp)}W_x[\rho]$$

- An infinite hierarchy of ordinary differential equations for the correlators $< A_1 A_2 \cdots A_n >_y$

Correlation Functions

Change of variables: $\rho^a \to \alpha^a \, ; \, \nabla^2 \alpha = \rho$

$$< O[\alpha] >_Y = \int [d\alpha] O[\alpha] W_Y[\alpha]$$

Iancu, McLerran; Weigert

Brownian motion in functional space: Fokker-Planck equation!

"diffusion coefficient"

$$=> \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha^a_Y(x)} \chi^{ab}_{x,y} \frac{\delta}{\delta \alpha^b_Y(y)} O[\alpha] >_Y$$

"time"

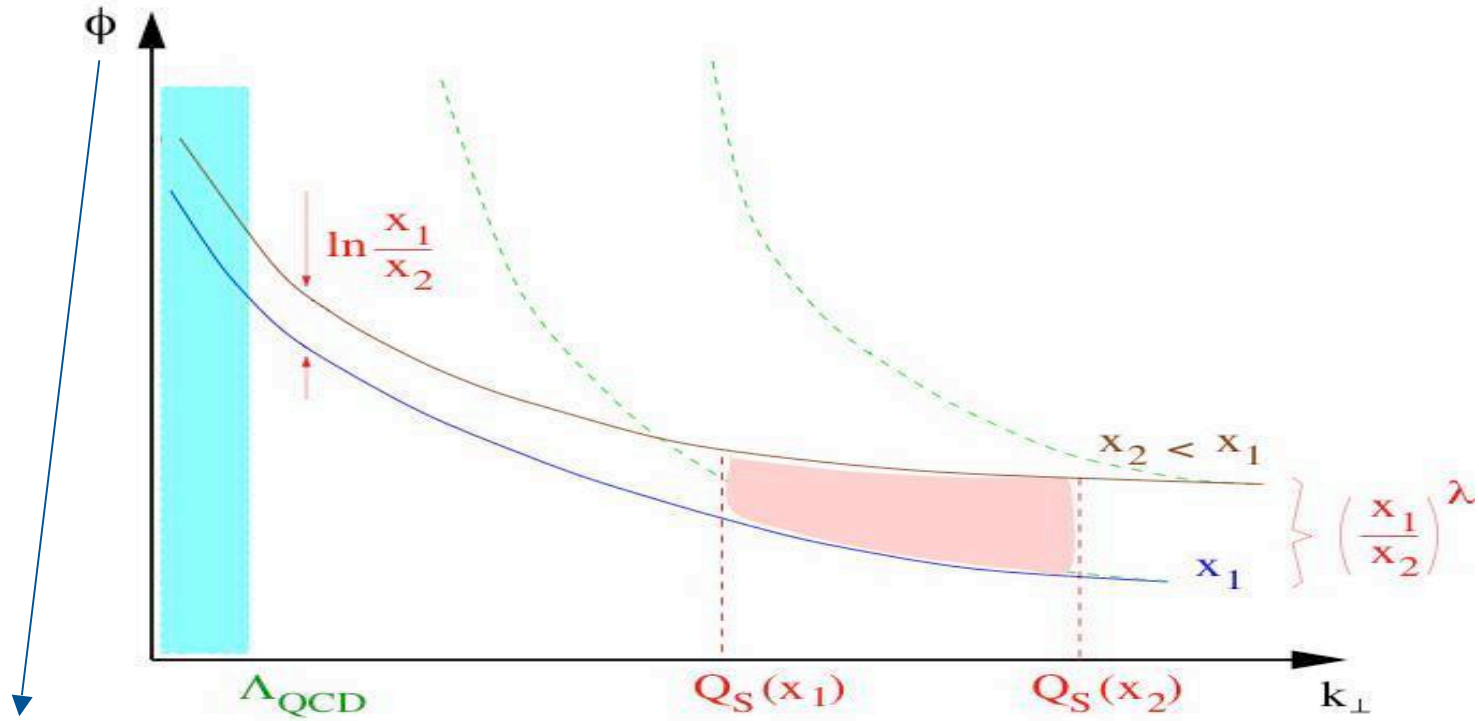
Consider the 2-point functior $< \alpha(x_\perp) \alpha(y_\perp) >_Y$

Can solve JIMWLK in the weak field li $g\,\alpha << 1$

Recover the BFKL equation in this low density limit

Can also solve JIMWLK in the strong field $g\,\alpha \sim 1$

Iancu,McLerran



Gluon phase space density

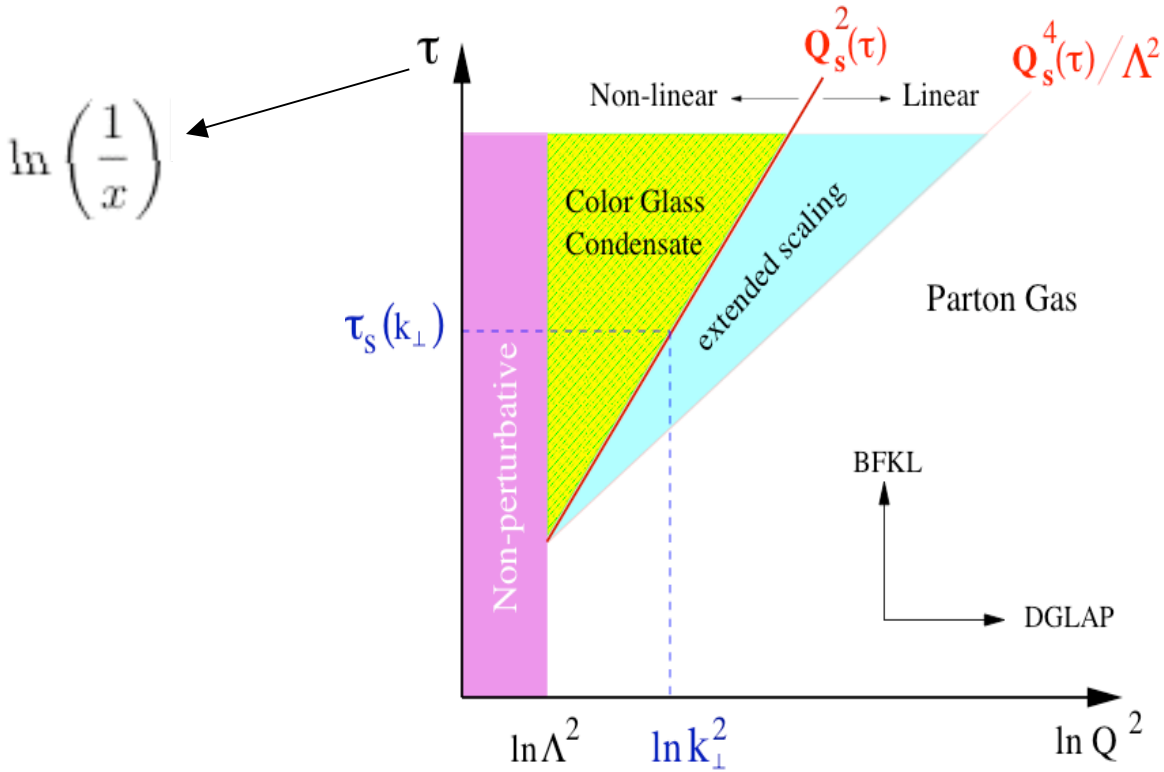
$$\ln k^2 >> \alpha_s Y(\mathrm{MV}, \mathrm{DGLAP}) : \phi \approx \frac{\mu_A^2}{k^2}$$

$$\ln k^2 \sim \alpha_s Y \text{ but } k^2 >> Q_s^2(Y)\,(\mathrm{BFKL}) : \phi \approx \left(\frac{\mu_A^2}{k^2}\right)^{1/2} e^{\omega \alpha_S Y}$$

$$k^2 << Q_s(Y) : \phi \approx \frac{1}{\alpha_S} \ln\left(\frac{Q_s^2(Y)}{k^2}\right)$$

How does one compute Q_s(Y) ?

Novel regime of QCD evolution at high energies



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