Few-Nucleon Forces and Systems in Chiral Effective Field Theory

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Lecture 1: Chiral Perturbation Theory: the basics

Lecture 2: Inclusion of nucleon(s)

Lecture 3: Chiral EFT & nuclear forces

Lecture 4: Applications

- 2 nucleons up to N^3LO
- 3 and more nucleons up to N^2LO
- Quark mass dependence of the nuclear force
- The πd system
- Isospin-breaking nuclear forces
- Summary and outlook

Chiral EFT a la Weinberg

effective chiral Lagrangian

work out nuclear forces

fix LECs from the data

make predictions

2 nucleons up to N^{3}LO

- 1π-exchange: $V_{1\pi}(q) = -\left(\frac{g_A}{2F_\pi}\right)^2 \left(1 - \frac{p^2+p'^2}{2m^2}\right) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{q^2 + M_\pi^2}$,

- 2π-exchange: $V_{2\pi} = V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)}$,

$$V_{2\pi}^{(2)}(\vec{q}) = -\frac{1}{384\pi^2 F_\pi^4} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 L^{\tilde{\Lambda}}(q) \left\{ 4M_\pi^2(5g_A^4 - 4g_A^2 - 1) + q^2(23g_A^4 - 10g_A^2 - 1) + \frac{48 g_A^4 M_\pi^4}{4M_\pi^2 + q^2} \right\}$$
$$+ \frac{3g_A^4}{64\pi^2 F_\pi^4} L^{\tilde{\Lambda}}(q) \left[q^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) - (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) \right]$$

where: $q = |\vec{p}\,' - \vec{p}|$

$$V_{2\pi}^{(3)}(\vec{q}) = -\frac{3g_A^2}{16\pi F_\pi^4} \left\{ 2M_\pi^2(2c_1 - c_3) - c_3 q^2 \right\} (2M_\pi^2 + q^2) A^{\tilde{\Lambda}}(q)$$
$$+ \frac{g_A^2}{32\pi F_\pi^4} c_4 (4M_\pi^2 + q^2) A^{\tilde{\Lambda}}(q)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[q^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) - (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) \right],$$

$$V_{2\pi}^{(4)}(q) = \ldots$$

+ $1/m$ - corrections consistent with: $\left[\left(2\sqrt{p^2 + m} - 2m \right) + V \right] \Psi = E\Psi$ and $V_{1\pi}(q)$

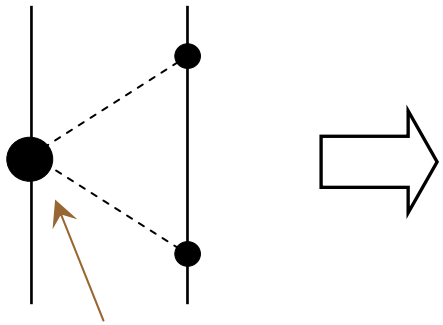
- 3π-exchange: …

- 24 contact terms (S- P- and D-waves): $V_{\rm cont} = C_S + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_1\, \vec{q}^{\,2} + C_2\, \vec{k}^2 + \ldots$

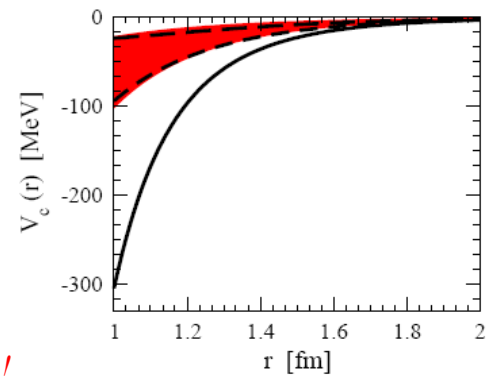
- Isospin-breaking corrections in accordance with the Nijmegen PWA '93:

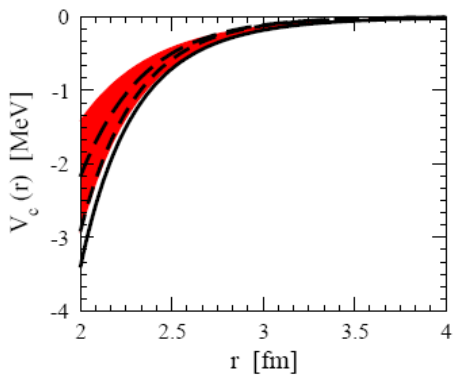
  1γ-, 2γ-, 1π-exchange + 2 contact terms with no derivatives.

- The strongest contribution to the 2π-exchange potential is given by the isoscalar central component that arises from the triangle diagram at N^2LO



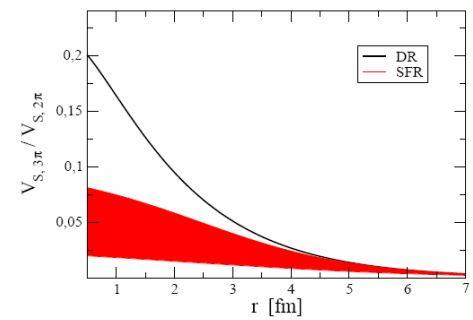
$c_{1,3,4}$ — *numerically large !!!*



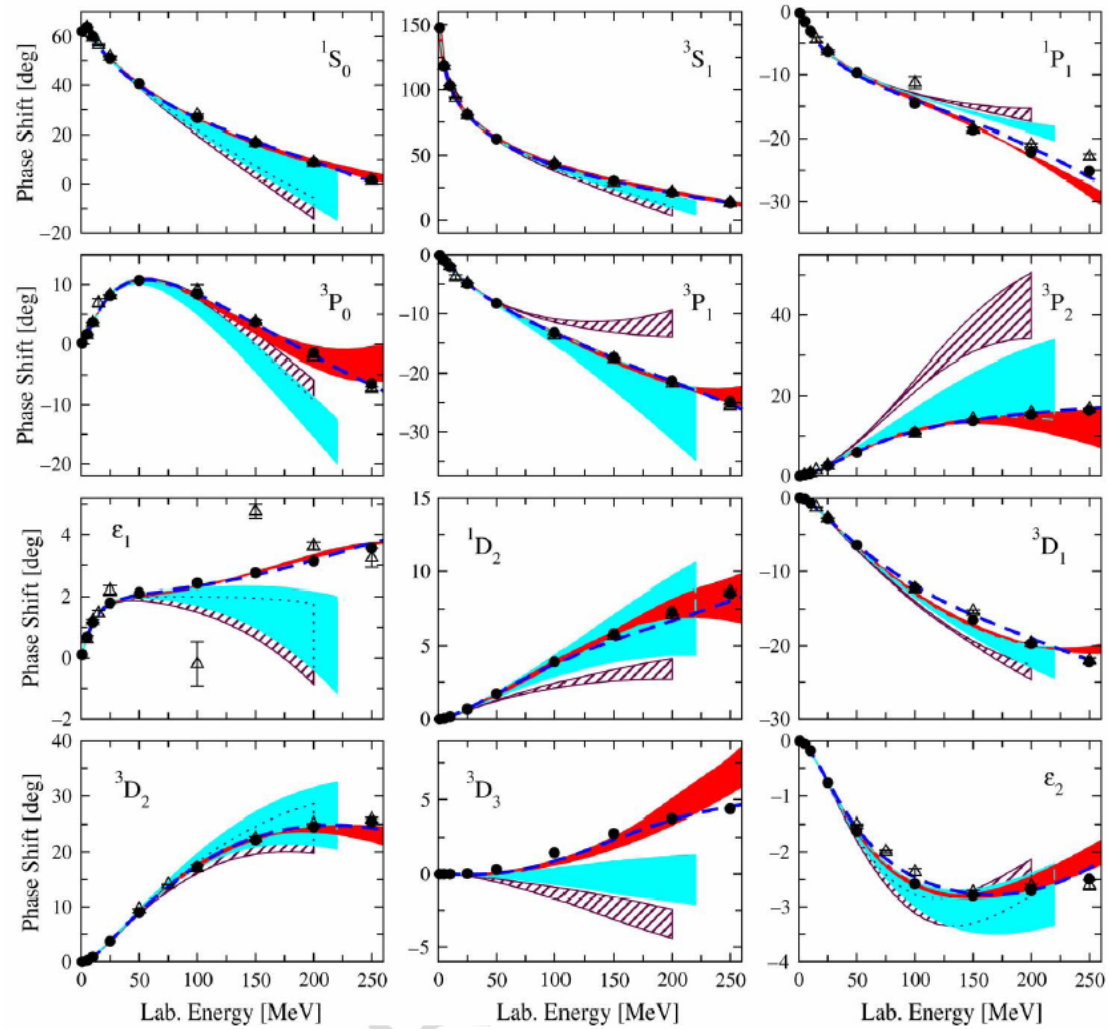


- The leading 3π-exchange is negligibly small ***(Kaiser ´00)***

Notice however: the corrections to the 3π-exchange at N^4LO are significant ***(Kaiser ´01)*** !!!

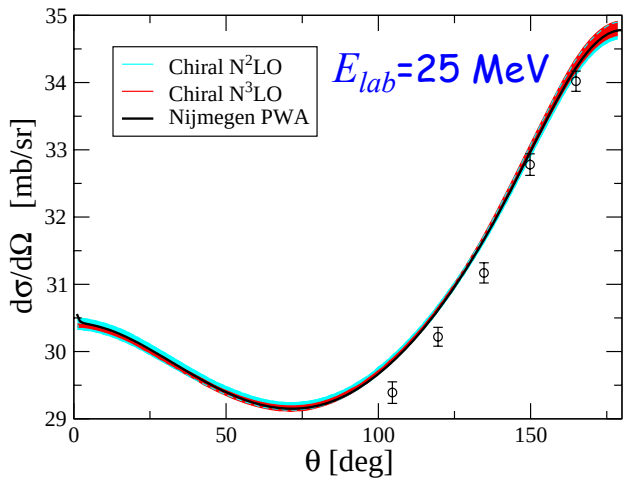


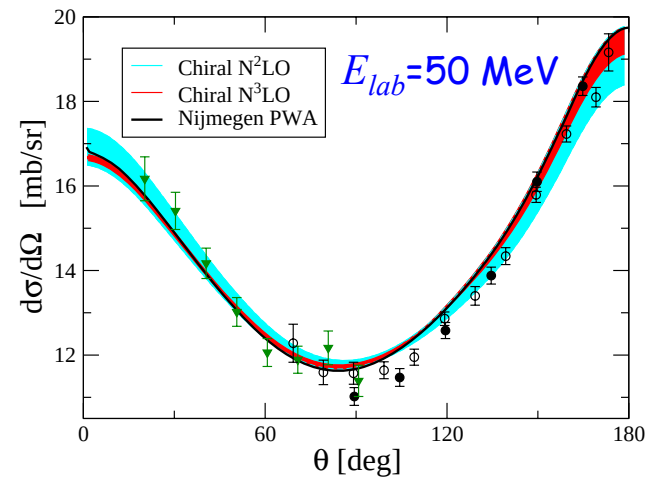
Neutron-proton phase shifts up to N^{3}LO



Results from: *Entem & Machleidt, PRC 68 (2003); E.E., Meißner & Glöckle, NPA 747 (2005)*

Differential cross section for np scattering





Deuteron observables

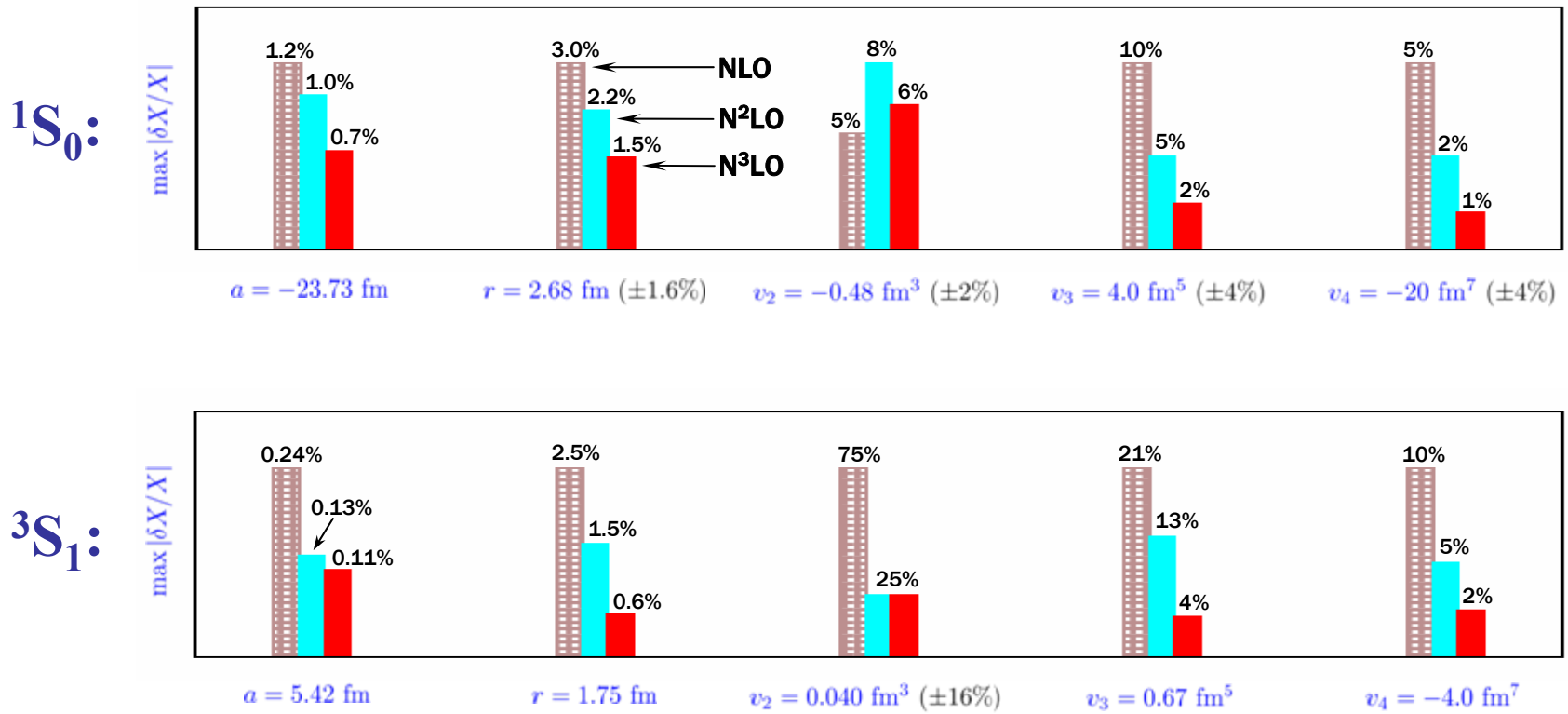
	NLO	N^2LO	N^3LO	Exp
E_d [MeV]	-2.171…-2.186	-2.189…-2.202	-2.216…-2.223	-2.225
A_S [$fm^{-1/2}$]	0.868…0.873	0.874…0.879	0.882…0.883	0.8846(9)
η	0.0256…0.0257	0.0255…0.0256	0.0254…0.0255	0.0256(4)

At large r : $u(r) \to A_S\, e^{-\gamma r}\,, \qquad w(r) \to \eta\, A_S\, e^{-\gamma r}\left(1 + \frac{3}{\gamma r} + \frac{3}{\gamma^2 r^2}\right)$

Do existing NN data show any evidence for chiral 2π-exchange?

Low energy S-wave threshold parameters

S-wave threshold (effective range) expansion: $k \cot \delta(k) = -\frac{1}{a} + \frac{1}{2} r k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \ldots$

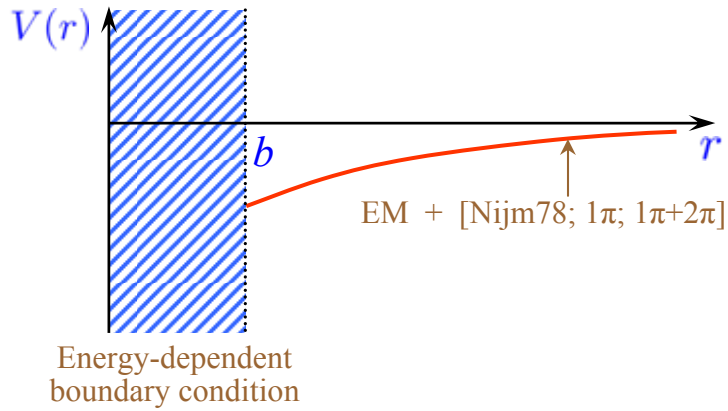


Values for a and r extracted from NPWA, de Swart, Terheggen & Stoks '95;

v_i are based on NIJM-II, see also: Pavon Valderrama & Ruiz Arriola nucl-th/0407113.

Evidence of the 2-exchange from NN phase-shift analysis

Chiral 2π-exchange potential at NLO and N^2LO has been tested in an energy-dependent proton-proton partial-wave analysis, Rentmeester et al. '99, '03



	$b = 1.4$ fm		$b = 1.8$ fm	
	#BC	$\chi^2_{\rm min}$	#BC	$\chi^2_{\rm min}$
Nijm78	19	1968.7	–	–
OPE	31	2026.2	29	1956.6
OPE + TPE(l.o.)	28	1984.7	26	1965.9
OPE + χTPE	23	1934.5	22	1937.8

Three nucleons

E.E., Nogga, Glöckle, Kamada, Meißner, Witala ´00, ´02

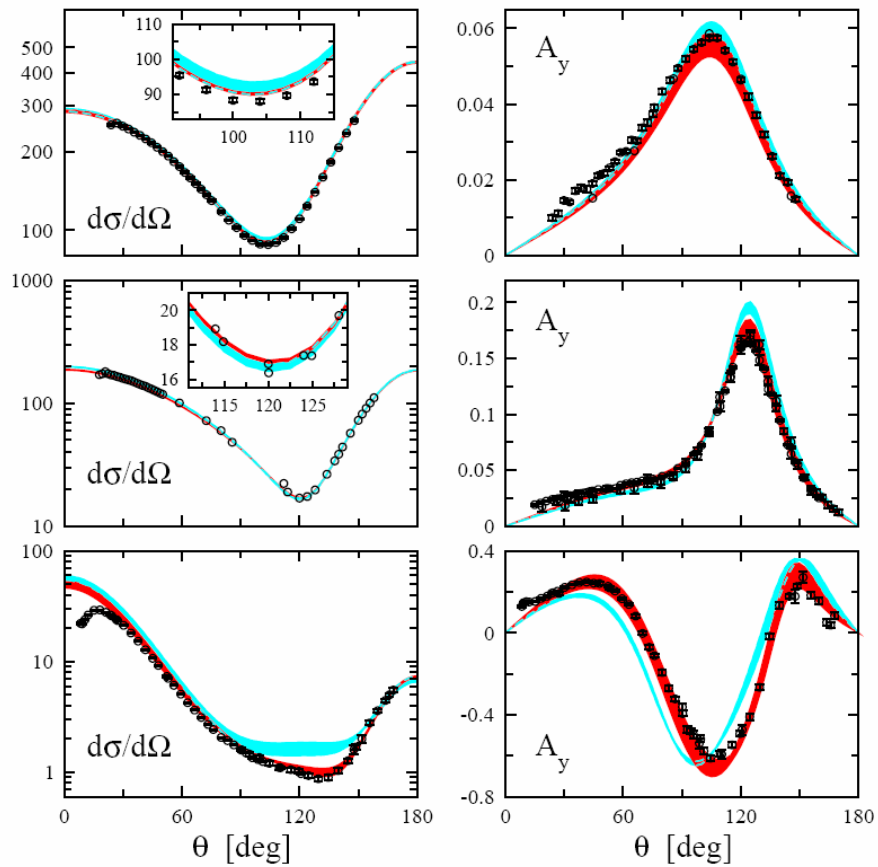
- parameter-free results at LO, NLO
- the LECs D and E entering the 3NF at NNLO are fixed from the ^{3}H BE and $^2a_{nd}$

Elastic nucleon-deuteron scattering observables

E=3 MeV

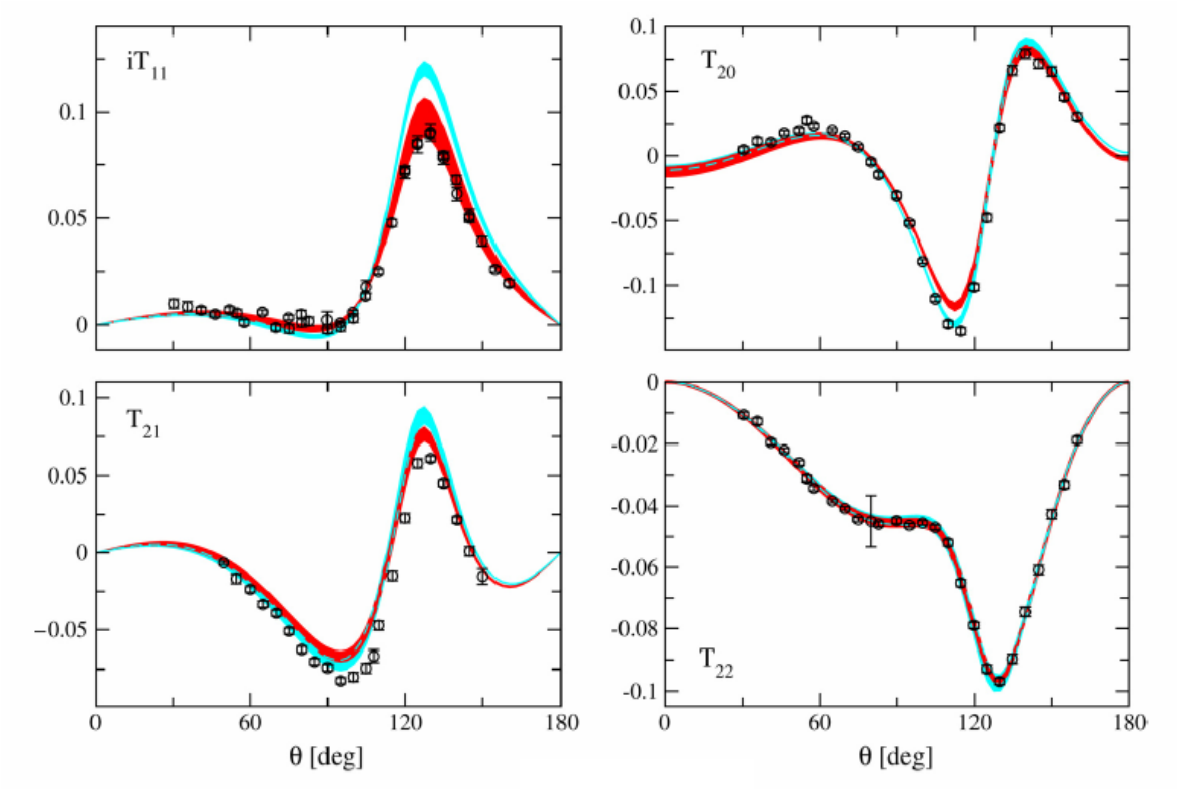
E=10 MeV

E=65 MeV

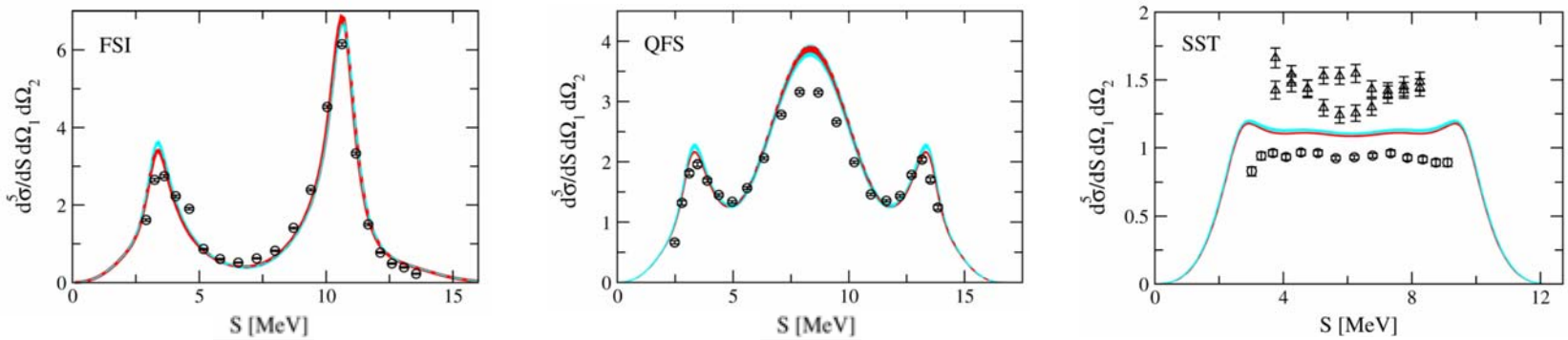


Solving Faddeev-Yakubovsky equations is numerically intensive. Calculations performed on supercomputers at the NIC, Jülich

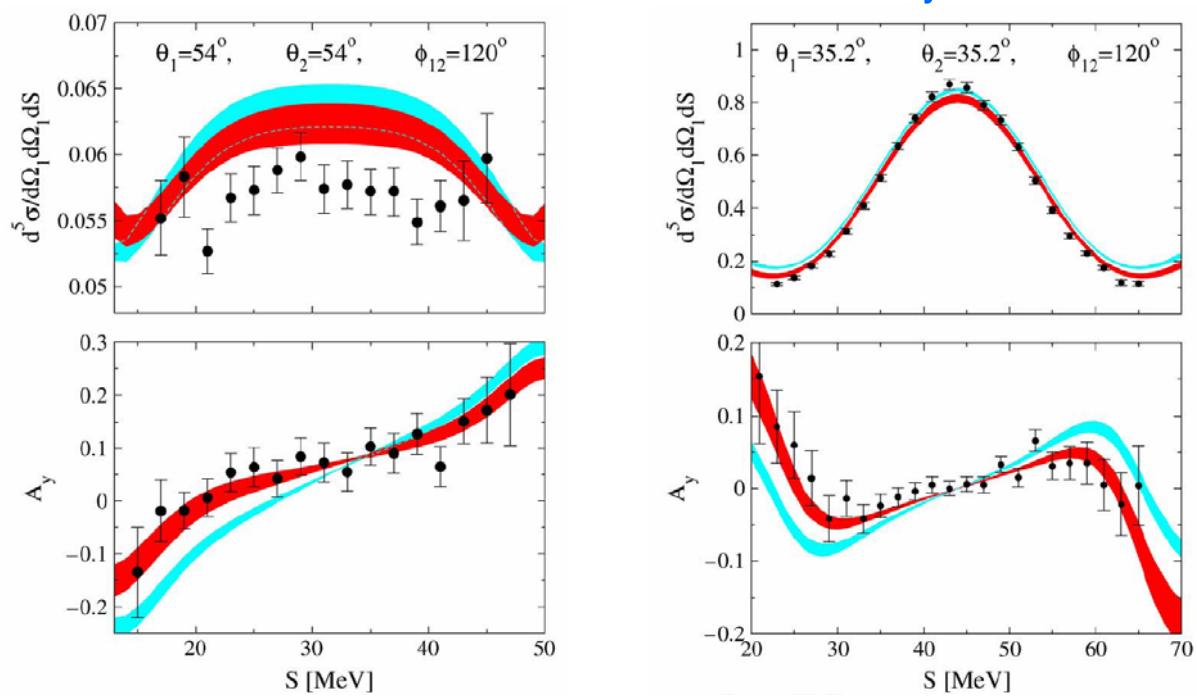
Tensor analyzing powers for elastic nucleon-deuteron scattering observables at 10 MeV



Nd break-up cross section at E_N=13 MeV

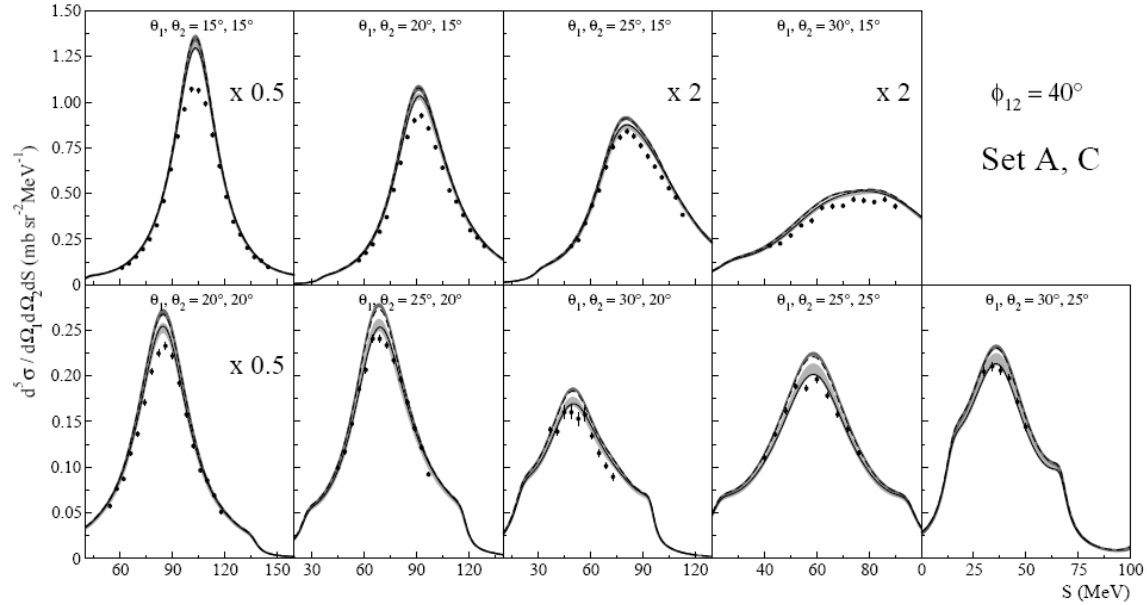


Nd break-up cross section and nucleon A_y at E_N=65 MeV

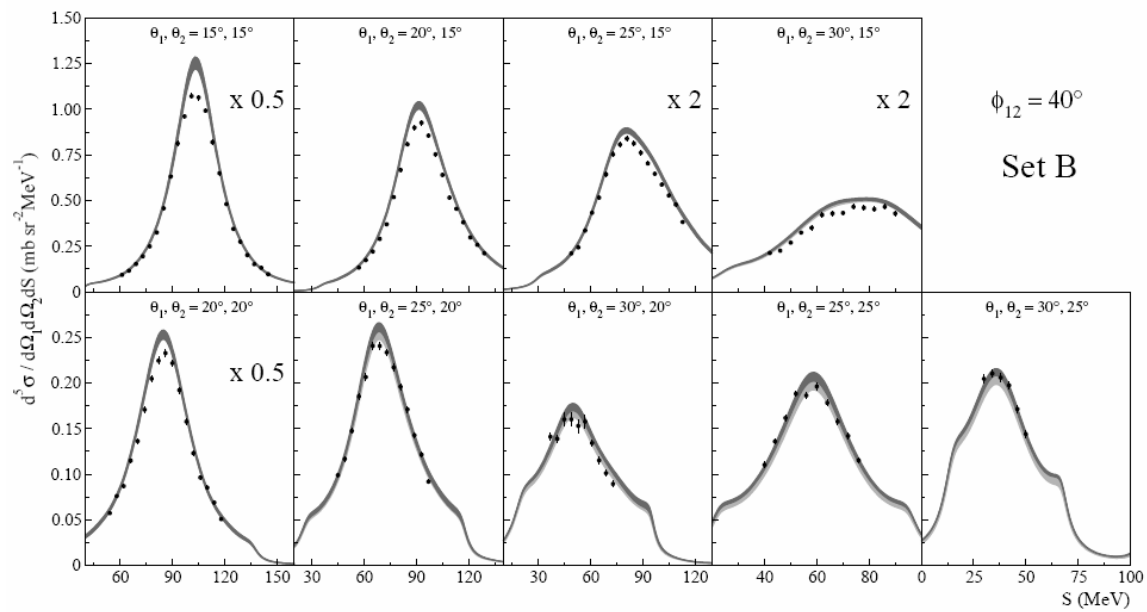


High precision cross-section data of the deuteron-proton breakup at 130 MeV for 72 kinematically complete kon-figurations compared with theory, *Kistryn et al. '05*

Modern high-precision potentials →



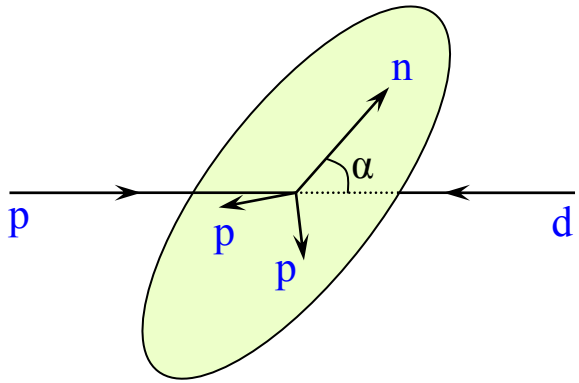
Chiral NNLO →



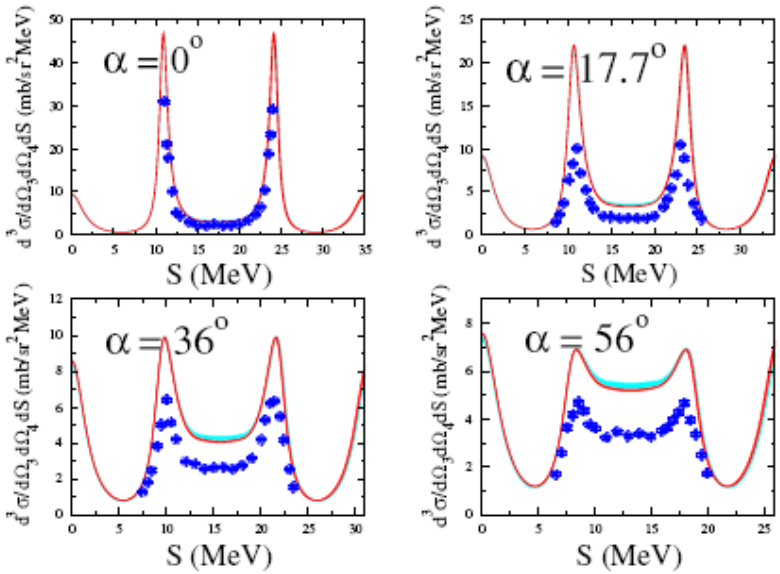
Some open problems…

Deuteron breakup in the Symmetric Constant Relative Energy (SCRE) configuration at E_d=19 MeV

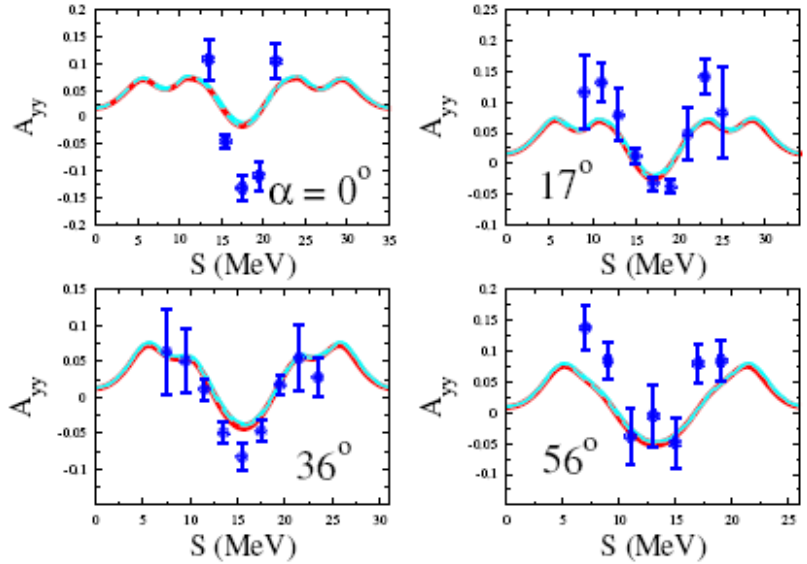
Ley et al., PRC 2006, in press



Differential cross section

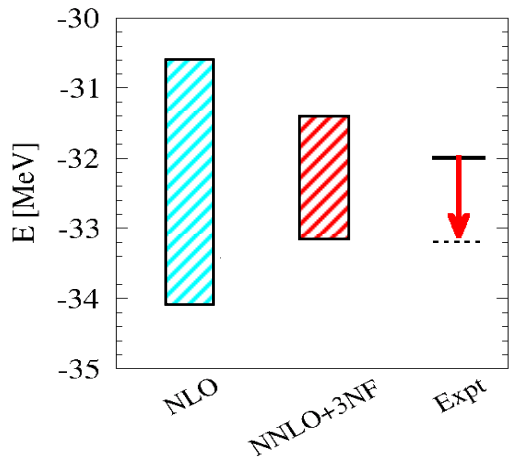


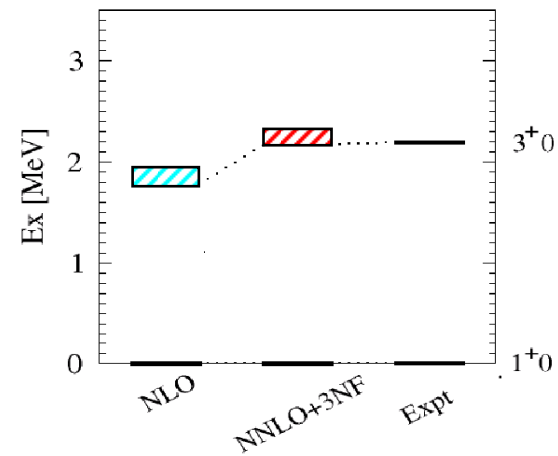
Tensor analyzing power A_{vv}



More nucleons...

Predictions for ^{6}Li ground and excited states





Calculations carried out by *Nogga et al.* within the No Core Shell Model

Even heavier nuclei studied based on the Idaho N^{3}LO chiral potential and N^{2}LO 3NF by *Navrátil et al. '05*

Quark mass dependence of the nuclear force

Beane & Savage '02, 03; E.E., Meißner & Glöckle '02, '03

- today's lattice calculations adopt large m_q (or M_π)
- chiral EFT might be used to extrapolate to physical values of M_π

Sources of the M-dependence of V_{NN} (at NNLO)

- **1π-exchange:** $V_{1\pi} = -\frac{1}{4}\left(\frac{g_{\pi N}}{m_N}\right)^2 \frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^{\,2}+M^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,,$

where $$\frac{g_{\pi N}}{m_N} = \frac{g_A}{F_\pi}\left[1-\frac{g_A^2 M^2}{4\pi^2 F_\pi^2}\ln\frac{M}{M_\pi}-\frac{2M^2}{g_A}\bar{d}_{18}+\left(\frac{g_A^2}{16\pi^2F_\pi^2}-\frac{4}{g_A}\bar{d}_{16}+\frac{1}{16\pi^2F_\pi^2}\bar{l}_4\right)(M_\pi^2-M^2)\right]$$

Input:

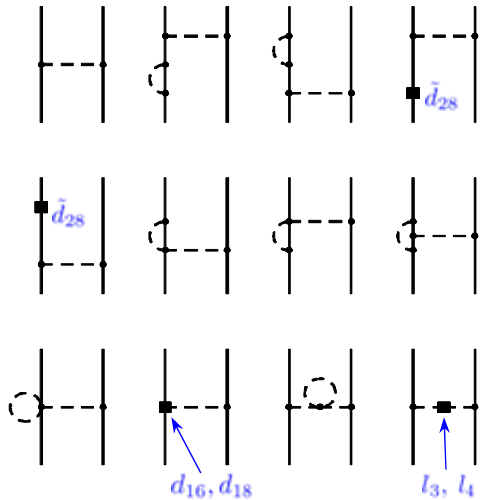
$g_A = 1.26, \qquad F_\pi = 92.4\ \text{MeV}, \qquad M_\pi = 138\ \text{MeV}$

$g_{\pi N}\Big|_{M=M_\pi} = 13.1\ldots13.4 \quad\Longrightarrow\quad \bar{d}_{18} \cong -0.97\ \text{GeV}^{-2}$

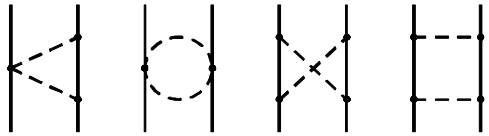
$\bar{l}_4 = 4.30$ *(Gasser & Leutwyler '84)*

$\bar{\tilde{d}}_{28} = 0, \qquad \bar{d}_{16} = -1.23^{+0.32}_{-0.53}\ \text{GeV}^{-2}$

(from $\pi N \to \pi\pi N$, *Fettes, Bernard & Meißner '00*)



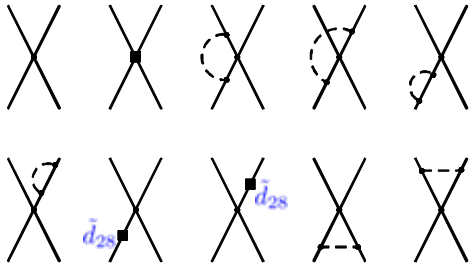
2π-exchange:



$$V_{2\pi} = -\frac{3g_A^4}{64\pi F_\pi^4}\left\{L(q) + \ln\frac{M}{M_\pi}\right\}\left(\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q} - \vec{q}^{\,2}\vec{\sigma}_1\cdot\vec{\sigma}_2\right)$$
$$-\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{384\pi^2F_\pi^2}\left\{L(q)\left[4M^2(5g_A^4-4g_A^2-1)+\frac{48g_A^4M^4}{4M^2+\vec{q}^{\,2}}+\vec{q}^{\,2}(23g_A^4-10g_A^2-1)\right]+\vec{q}^{\,2}\ln\frac{M}{M_\pi}(23g_A^4-10g_A^2-1)\right\}$$

where $L(q) = \frac{\sqrt{4M^2+\vec{q}^{\,2}}}{q}\ln\frac{\sqrt{4M^2+\vec{q}^{\,2}}+q}{2M}$, $\quad \vec{q}=\vec{p}-\vec{p}\,'$, $\quad q=|\vec{q}|$, $\quad \vec{k}=\frac{1}{2}(\vec{p}+\vec{p}\,')$.

Contact terms:



$$V_{\rm cont} = C_1\vec{q}^{\,2}+\ldots+C_7\vec{\sigma}_1\cdot\vec{k}\,\vec{\sigma}_2\cdot\vec{k}+\left(C_S+M^2\left[\bar{D}_S-\beta_S\ln\frac{M}{M_\pi}\right]\right)$$
$$+\left(C_T+M^2\left[\bar{D}_T-\beta_T\ln\frac{M}{M_\pi}\right]\right)\vec{\sigma}_1\cdot\vec{\sigma}_2$$

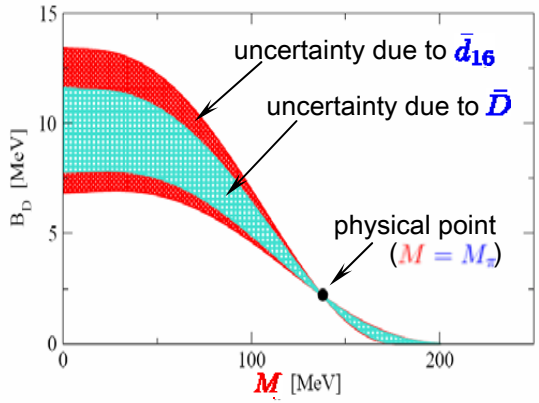
where $\beta_S = \frac{3g_A^2}{32\pi^2F_\pi^4}(8F_\pi^2C_T-5g_A^2+2)$, $\quad \beta_T = \frac{3g_A^2}{64\pi^2F_\pi^4}(16F_\pi^2C_T-5g_A^2+2)$

The combinations $C_S+\bar{D}_S\,M_\pi^2$ and $C_T+\bar{D}_T\,M_\pi^2$ as well as $C_1,\ldots,C_7$ are fixed from fit to low-energy S- and P-waves.

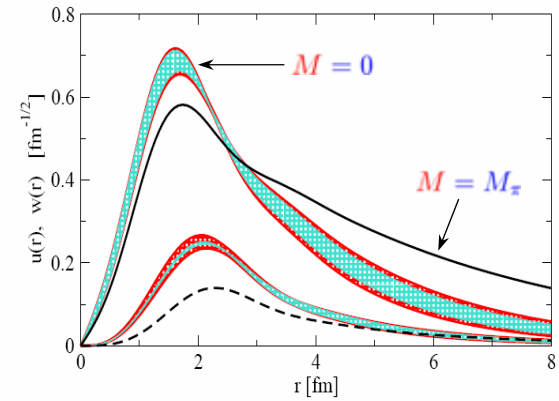
Problem: the constants $\bar{D}_S$ and $\bar{D}_T$ cannot be determined from NN scattering!

Naturalness assumption: $\bar{D}_{S,T} = \frac{\alpha_{S,T}}{F_\pi^2\Lambda_\chi^2}$, where $\alpha_{S,T}\sim 1$. We use: $-3.0<\alpha_{S,T}<3.0$.

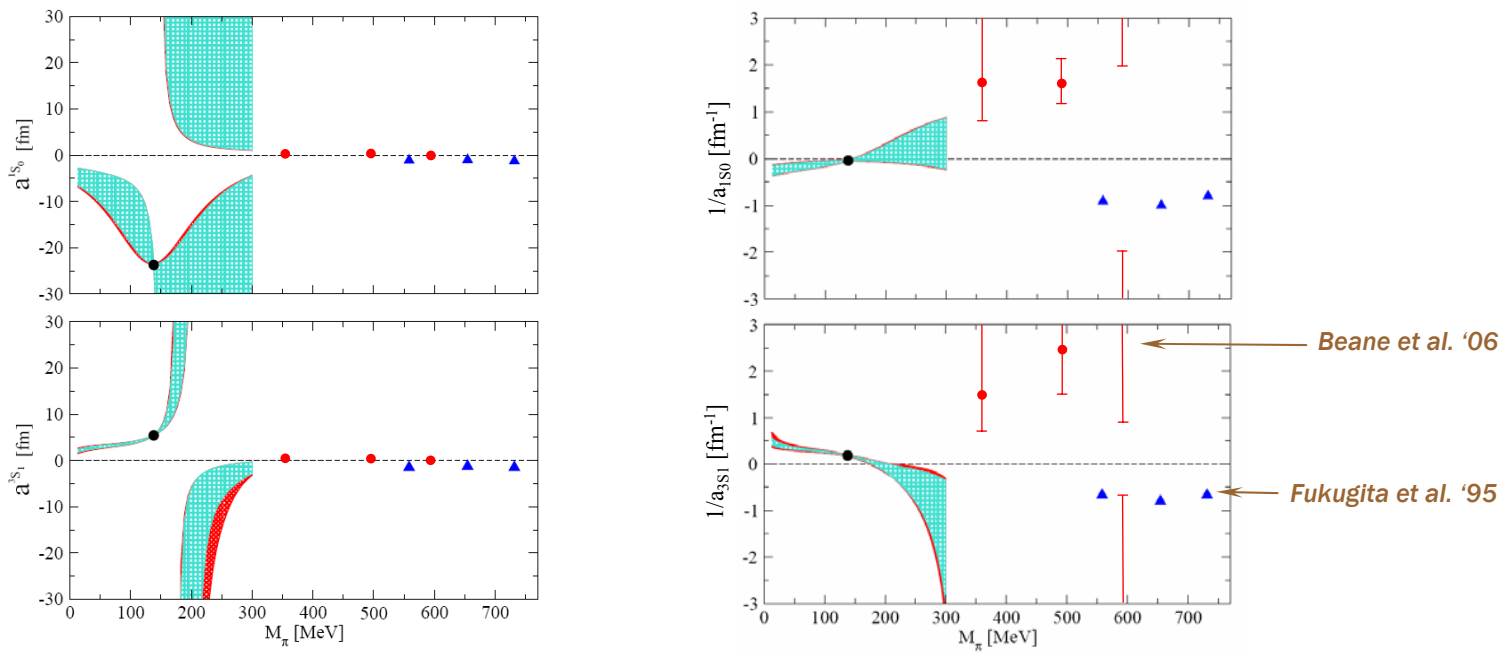
Deuteron B_D as a function of M



Deuteron WF in the chiral limit



Chiral extrapolation of the S-wave scattering lengths



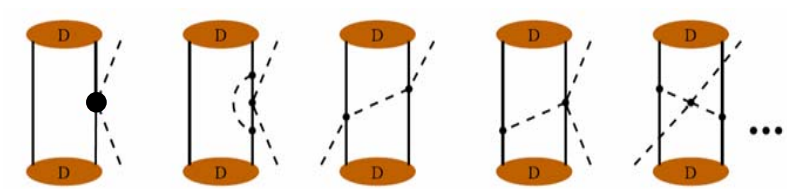
πN scattering length from πd scattering

Beane, Bernard, E.E., Meißner and Phillips, '03

In the limit of exact isospin symmetry at threshold: $T^{ba}_{\pi N} \propto \left[\delta^{ab} a^+ + i\epsilon^{bac}\tau^c a^-\right]$

- No πN data at very low energy.
- Extractions of a^+ and a^- from the level shifts and lifetime of pionic hydrogen have large error bars.
- πd scattering length $a_{\pi d}$ measured with high accuracy.

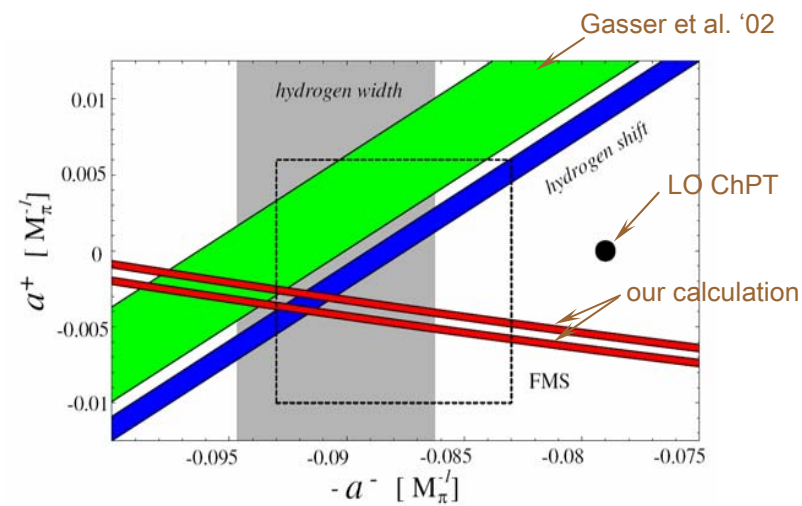
$\Longrightarrow$ use chiral EFT to extract a^+ and a^- from $a_{\pi d}$



Novel power counting: $\sqrt{m_N E_d} \sim 45 \text{ MeV} \ll M_\pi$

$$\mathrm{Re}\, a_{\pi d} = \frac{2(1+\mu)}{(1+\mu/2)}\left(a^+ + (1+\mu)\left[(a^+)^2 - 2(a^-)^2\right]\frac{1}{2\pi^2}\left\langle\frac{1}{\vec{q}^{\,2}}\right\rangle + (1+\mu)^2\left[(a^+)^3 - 2(a^-)^2(a^+ - a^-)\right]\frac{1}{4\pi}\left\langle\frac{1}{|\vec{q}|}\right\rangle\right) + a_{\text{boost}}$$

where $\mu = M_\pi/m_N$.



Further constraints from "heavy-pion" EFT, Beane & Savage '02

Update:

- new experimental data on pionic hydrogen
- Incorporation of the isospin-breaking effects

Meißner, Raha & Rusetsky ´05

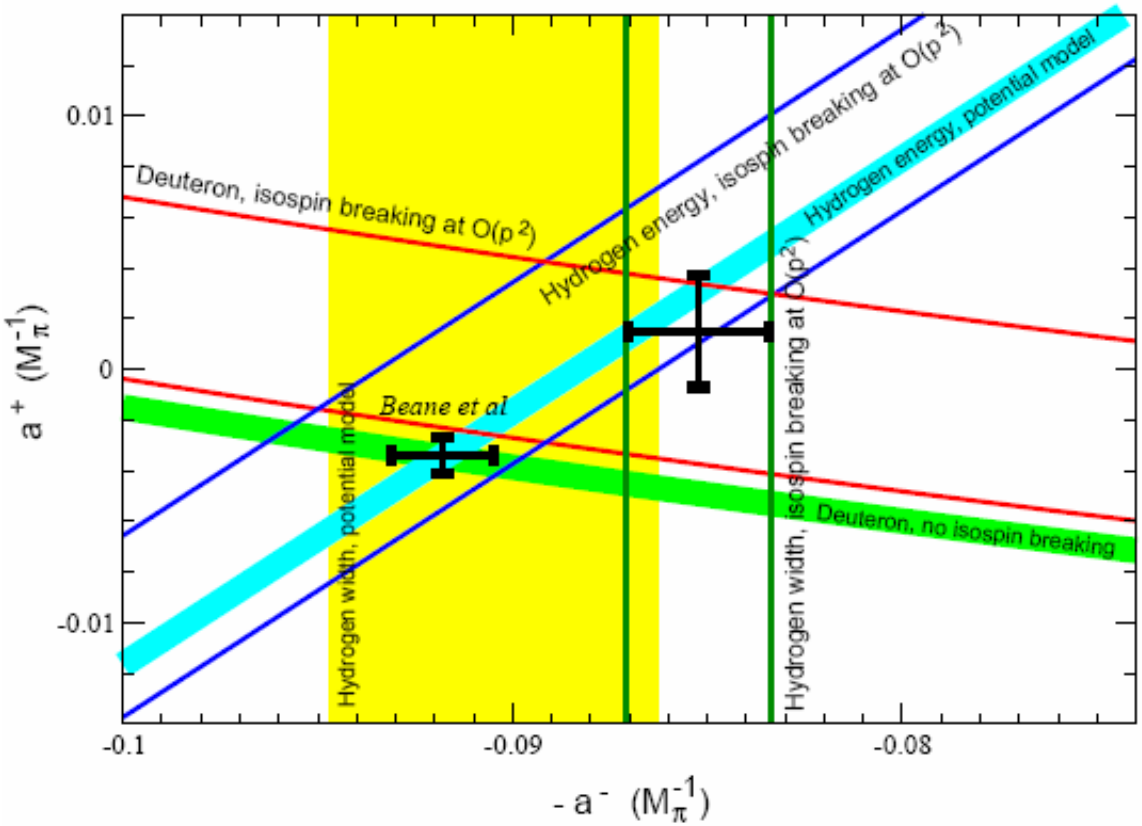


figure from: *Meißner, Raha & Rusetsky , nucl-th/0512035*

Isospin violating nuclear forces

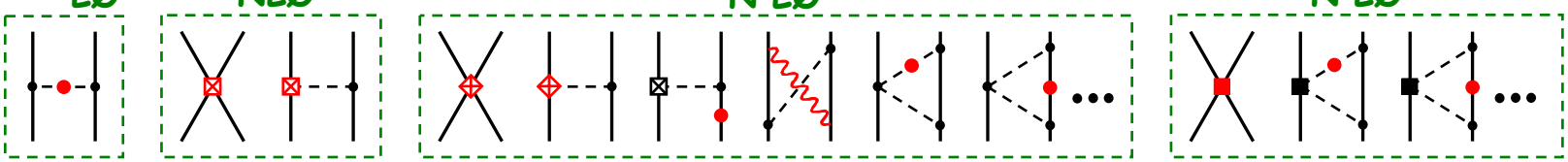
$$\mathcal{L}_{\rm QCD} = \underbrace{-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,\,a} + \bar{q}i\not{D}q}_{\text{chiral invariant}} \underbrace{- \bar{q}\mathcal{M}q - \bar{q}Q\gamma_\mu A^\mu q}_{\text{break chiral (and isospin) symm.}} \Longrightarrow \mathcal{L}_{\rm eff}$$

$\mathcal{L}_{\rm eff}$ includes:

- strong isospin breaking terms $\propto (m_u - m_d)^n$,
- electromagnetic isospin breaking terms $\propto Q_{\rm ch}^{2n}$, $\quad Q_{\rm ch} = \frac{e}{2}(1+\tau^3)$,
- coupling to (soft) photons $\propto e$.

2NF (Long-range terms up to N³LØ depend on $(\delta m)^{\rm str}$, $(\delta m)^{\rm em}$, δM_π and $\delta g_{\pi N}$.)

LØ NLØ N²LØ N³LØ

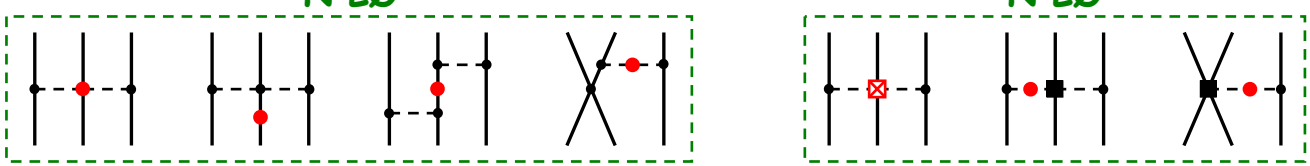


van Kolck et al., '93, '96, '98; Friar et al., '99, '03, '04; Niskanen '02; Epelbaum et al., '05

Epelbaum et al., '05

3NF (Up to N³LØ depends on $(\delta m)^{\rm str}$, $(\delta m)^{\rm em}$, δM_π and f_1.)

N²LØ N³LØ



Epelbaum et al., '04; Friar et al., '04

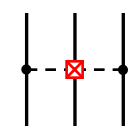
Epelbaum et al., '04

Isospin-breaking 2NF

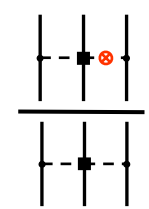
- Isospin-violating 2N force is calculated up to N^3LO.
- The long-range pieces (1γ-, 2γ-, $\pi\gamma$-, 1π-, 2π-exchange) can be used in the (Nijmegen) PWA. The charge-dependent πNN coupling constant is the only unknown LEC (provided $(\delta m)^{str}$ is taken from Gasser & Leutwyler '82) and can be determined in PWA.
- The LECs corresponding to short-range terms can, in principle, be fixed from PWA + nn scattering length.

Isospin-breaking 3NF

- Isospin-violating 3N force is calculated up to N^3LO.
- One new (and unknown) low-energy constant f_1 →



Notice: charge-symmetry conserving 3NF expected to yield large effects:

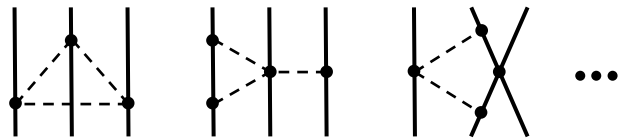


$$\propto \frac{2\delta M_\pi^2}{M_\pi^2} \sim 14\% \text{ (!)}$$

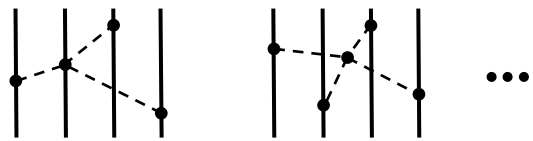
Future plans

- Few-nucleon systems at N^{3}LO.

  — work on 1-loop corrections to the 3NF is in progress. No new contact terms
  $\Longrightarrow$ expect large predictive power!



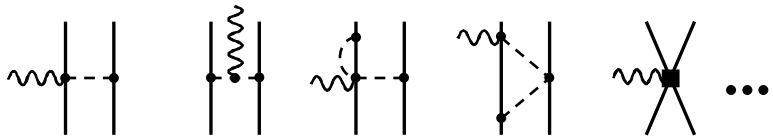
  — 4NF at this order has already been worked up (E.E. in preparation).
  No new parameters!



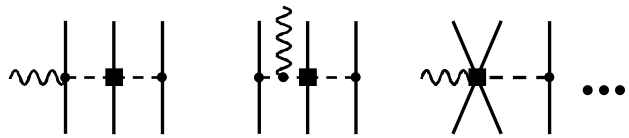
  — numerical implementation nontrivial!

- Reactions with electroweak probes.

  — 2N currents: some work already done, Park et al. '93, 96



  — 3N currents: have not yet been worked out



$\Longrightarrow$ applications to processes like $^3\vec{\mathrm{He}}\,(\vec{e},\,e)$, $^3\vec{\mathrm{He}}\,(\vec{e},\,en)$

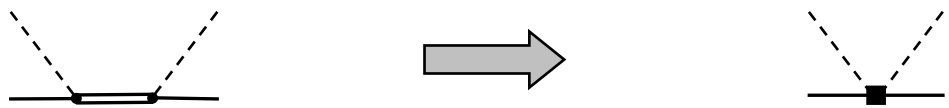
$pp \to \mathrm{D}\,e^+\,\nu_e$, $p\,^3\mathrm{He} \to {}^4\mathrm{He}\,e^+\,\nu_e$, $\ldots$

Inclusion of the Δ.

Δ-isobar is known to be important due to:

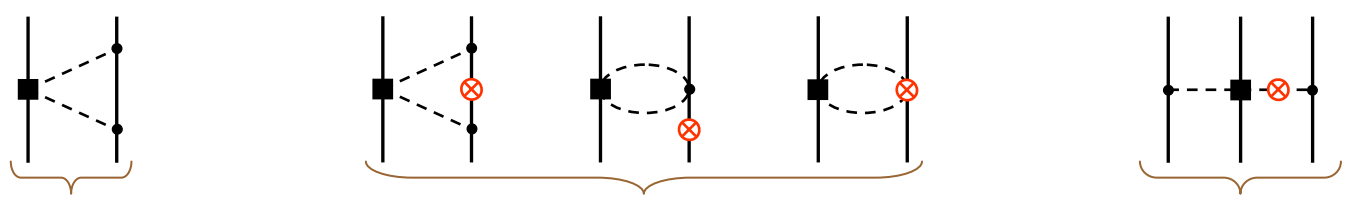
1) Low excitation energy: $\Delta m = m_\Delta - m_N = 293\ \mathrm{MeV} \sim 2M_\pi$
2) Strong coupling to the πN–system, i.e. $g_{\pi N \Delta}$ Is large

EFT with Δ $(\Delta m \sim M_\pi)$ Δ-less EFT $(\Delta m \gg M_\pi)$



$c_3 = -2c_4 = -\frac{h_A^2}{9\Delta m}$ — large!

Examples of diagrams in the Δ-less EFT which yield unnaturally large contributions:



sub-leading TPEP *sub-leading isospin-breaking TPEP* *Charge-symmetry conserving 3NF*

Including Δ's would probably lead to the nuclear force contributions of a more natural size, since the big portion of the terms $\propto c_i$ is shifted to lower orders.

Expect: better convergence, applicability at higher energies…

Summary

- The 2N system has been analyzed up to N3LO. Accurate results for deuteron and scattering observables at low energy.
- 3N, 4N and 6N systems studied up to N2LO including the chiral 3NF. The results look promising.
- Many other applications considered.

Challenges

- Increasing the energy range (including the Δ),
- Better understanding of the non-perturbative renormalization,
- Including more information from QCD (lattice, large Nc …)