QCD and Hadron Phenomenology (I)

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What are the aims of hadron physics?

Lecture plan

A. Hadron physics

– Aims
– Hard probes of hadron structure
– QCD tests in hard scattering
– Similarities between soft and hard processes

B. Dynamics of hard processes

– Resolution
– Rescattering
– Light-Front time ordering
– Fock states
– DIS as dipole scattering
– Shadowing
– Diffraction
– Coherence at large x

C. Bound states in field theory

– General features of relativistic states
– Dyson-Schwinger and Bethe-Salpeter equations
– Wave functions at equal time vs. equal Light-Front time
– Lorentz contraction

D. The confinement regime

– The QCD vacuum
– QCD sum rules
– Boundary condition in field theory
– Dressed perturbation theory
– Spontaneous breaking of chiral symmetry

The divisibility of matter

Since ancient times we have wondered whether matter can be divided into smaller parts *ad infinitum*, or whether there is a smallest constituent.

Democritus, ~ 400 BC
Vaisheshika school

Common sense suggest that these are the two possible alternatives. However, physics requires us to refine our intuition.

Quantum mechanics shows that atoms (or molecules) are the identical smallest constituents of a given substance
– yet they can be taken apart into electrons, protons and neutrons.

Hadron physics gives a new twist to this age-old puzzle: Quarks can be removed from the proton, but cannot be isolated. Relativity – the creation of matter from energy – is the new feature which makes this possible.

We are fortunate to be here to address – and hopefully develop an understanding of – this essentially novel phenomenon!

Can confinement be understood?

Hadrons are *"infinitely complicated"* structures – but so are QED atoms.
We have two powerful tools:

Experimental data. Hadronic resolution (~ 0.1 fm) is routinely achieved.
- Hadron structure is being revealed in unprecedented detail
- Data indicates similarities between short and long distance scales

Ex: What electron energy is required to probe a proton (at rest) with a resolution of 0.1 fm in $ep \rightarrow ep$ scattering?

Theory. The fundamental interaction is given by L_{QCD}:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

- Physics at short distance is understood through perturbation theory
  $\alpha_s(m_Z) = 0.1189(10)$ Quark-Gluon coupling strength, cf. $\alpha = 1/137$
- Insights into soft phenomena exist through qualitative models and quantitative numerical (lattice) calculations

The mission is to find a systematic approximation scheme based on L_{QCD}
- it may be wise to let data indicate the choice of scheme

The accuracy of measurement and theory

Many of our most accurate predictions come from QED atoms.
For example, the $2S_{1/2} - 1S_{1/2}$ splitting in Hydrogen:

$$\Delta(2S_{1/2} - 1S_{1/2})_H = 2\,466\,061\,413\,187.103(46)\ \text{kHz} \quad \text{QED}$$
$$= 2\,466\,061\,413\,187.103(46)\ \text{kHz} \quad \text{EXP}$$

U.D. Jentschura et al, PRL 95 (2005) 163003

The QED result is based on perturbation theory:
– an expansion in $\alpha = e^2/4\pi \approx 1/137.035\,999\,11(46)$

However, the series must diverge since for any $\alpha = e^2/4\pi < 0$ the electron charge e is imaginary: The Hamiltonian is not hermitian and probability not conserved.

F. Dyson

The perturbative expansion is believed to be an asymptotic series.
The good agreement with QED seems fortuituous, from a purely theoretical point of view.

For a recent discussion of the truncation effects in asymptotic expansions see Y. Meurice, hep-th/0608097

TABLE I. Transition frequencies in hydrogen ν_H and in deuterium ν_D used in the 2002 CODATA least-squares adjustment of the values of the fundamental constants and the calculated values. Hyperfine effects are not included in these values.

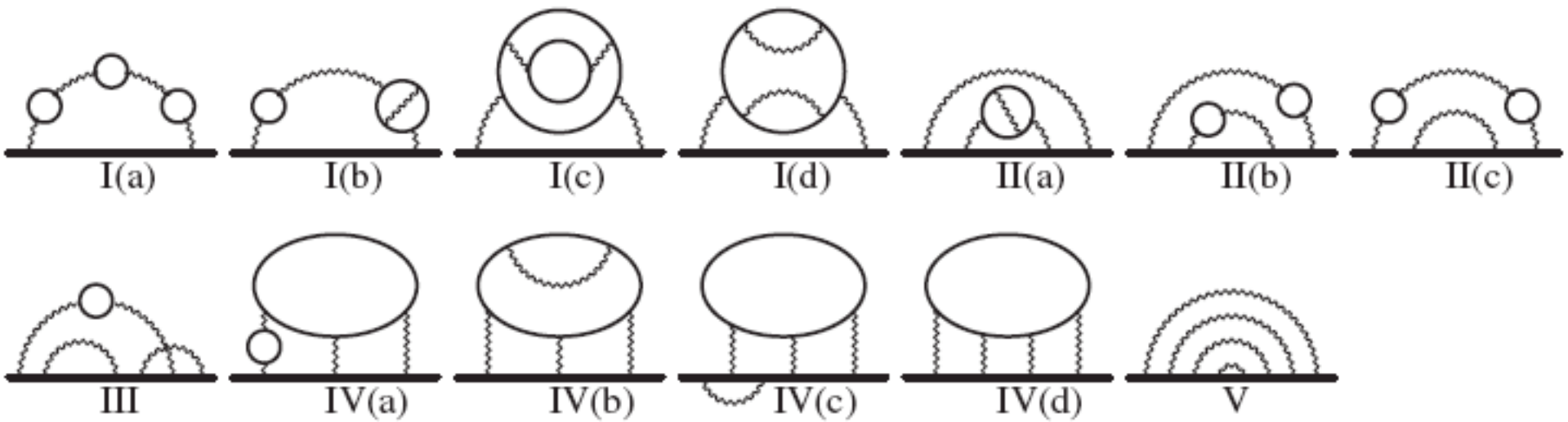
Experiment	Frequency interval(s)	Reported value ν/kHz	Calculated value ν/kHz
Niering *et al.* [1]	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.103(46)	2 466 061 413 187.103(46)
Weitz *et al.* [2]	$\nu_H(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	4 797 338(10)	4 797 331.8(2.0)
	$\nu_H(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	6 490 144(24)	6 490 129.9(1.7)
	$\nu_D(2S_{1/2} - 4S_{1/2}) - \frac{1}{4}\nu_D(1S_{1/2} - 2S_{1/2})$	4 801 693(20)	4 801 710.2(2.0)
	$\nu_D(2S_{1/2} - 4D_{5/2}) - \frac{1}{4}\nu_D(1S_{1/2} - 2S_{1/2})$	6 494 841(41)	6 494 831.5(1.7)
Huber *et al.* [3]	$\nu_D(1S_{1/2} - 2S_{1/2}) - \nu_H(1S_{1/2} - 2S_{1/2})$	670 994 334.64(15)	670 994 334.64(15)
de Beauvoir *et al.* [4]	$\nu_H(2S_{1/2} - 8S_{1/2})$	770 649 350 012.0(8.6)	770 649 350 016.1(2.8)
	$\nu_H(2S_{1/2} - 8D_{3/2})$	770 649 504 450.0(8.3)	770 649 504 449.1(2.8)
	$\nu_H(2S_{1/2} - 8D_{5/2})$	770 649 561 584.2(6.4)	770 649 561 578.2(2.8)
	$\nu_D(2S_{1/2} - 8S_{1/2})$	770 859 041 245.7(6.9)	770 859 041 242.6(2.8)
	$\nu_D(2S_{1/2} - 8D_{3/2})$	770 859 195 701.8(6.3)	770 859 195 700.3(2.8)
	$\nu_D(2S_{1/2} - 8D_{5/2})$	770 859 252 849.5(5.9)	770 859 252 845.1(2.8)
Schwob *et al.* [5]	$\nu_H(2S_{1/2} - 12D_{3/2})$	799 191 710 472.7(9.4)	799 191 710 481.9(3.0)
	$\nu_H(2S_{1/2} - 12D_{5/2})$	799 191 727 403.7(7.0)	799 191 727 409.1(3.0)
	$\nu_D(2S_{1/2} - 12D_{3/2})$	799 409 168 038.0(8.6)	799 409 168 041.7(3.0)
	$\nu_D(2S_{1/2} - 12D_{5/2})$	799 409 184 966.8(6.8)	799 409 184 973.4(3.0)
Bourzeix *et al.* [6]	$\nu_H(2S_{1/2} - 6S_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 3S_{1/2})$	4 197 604(21)	4 197 600.3(2.2)
	$\nu_H(2S_{1/2} - 6D_{5/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 3S_{1/2})$	4 699 099(10)	4 699 105.4(2.2)
Berkeland *et al.* [7]	$\nu_H(2S_{1/2} - 4P_{1/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	4 664 269(15)	4 664 254.3(1.7)
	$\nu_H(2S_{1/2} - 4P_{3/2}) - \frac{1}{4}\nu_H(1S_{1/2} - 2S_{1/2})$	6 035 373(10)	6 035 384.1(1.7)
Hagley and Pipkin [8]	$\nu_H(2S_{1/2} - 2P_{3/2})$	9 911 200(12)	9 911 197.6(2.4)
Lundeen and Pipkin [9]	$\nu_H(2P_{1/2} - 2S_{1/2})$	1 057 845.0(9.0)	1 057 844.0(2.4)
Newton *et al.* [10]	$\nu_H(2P_{1/2} - 2S_{1/2})$	1 057 862(20)	1 057 844.0(2.4)

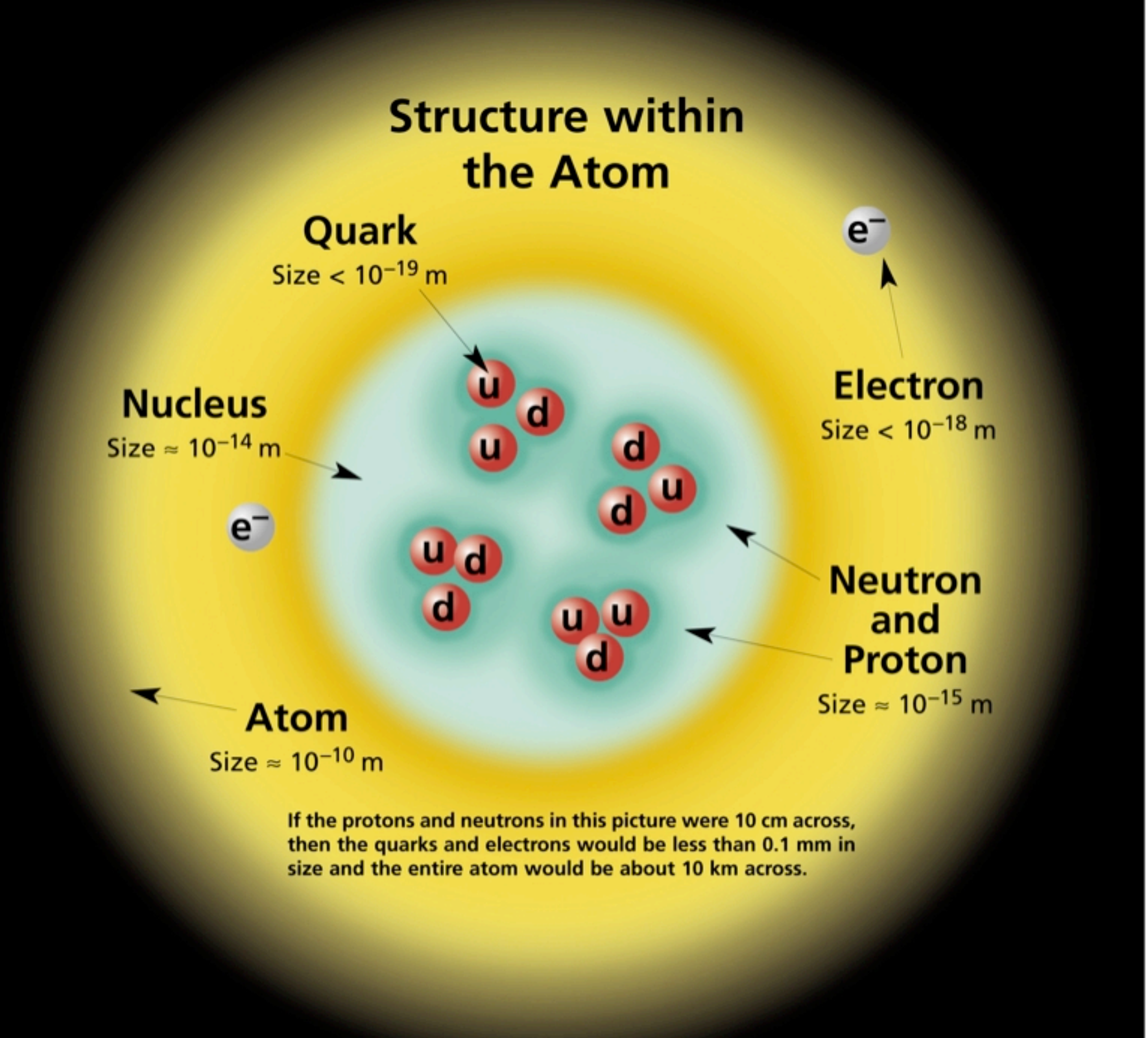
PRL 97, 030802 (2006) PHYSICAL REVIEW LETTERS week ending 21 JULY 2006

New Determination of the Fine Structure Constant from the Electron g Value and QED

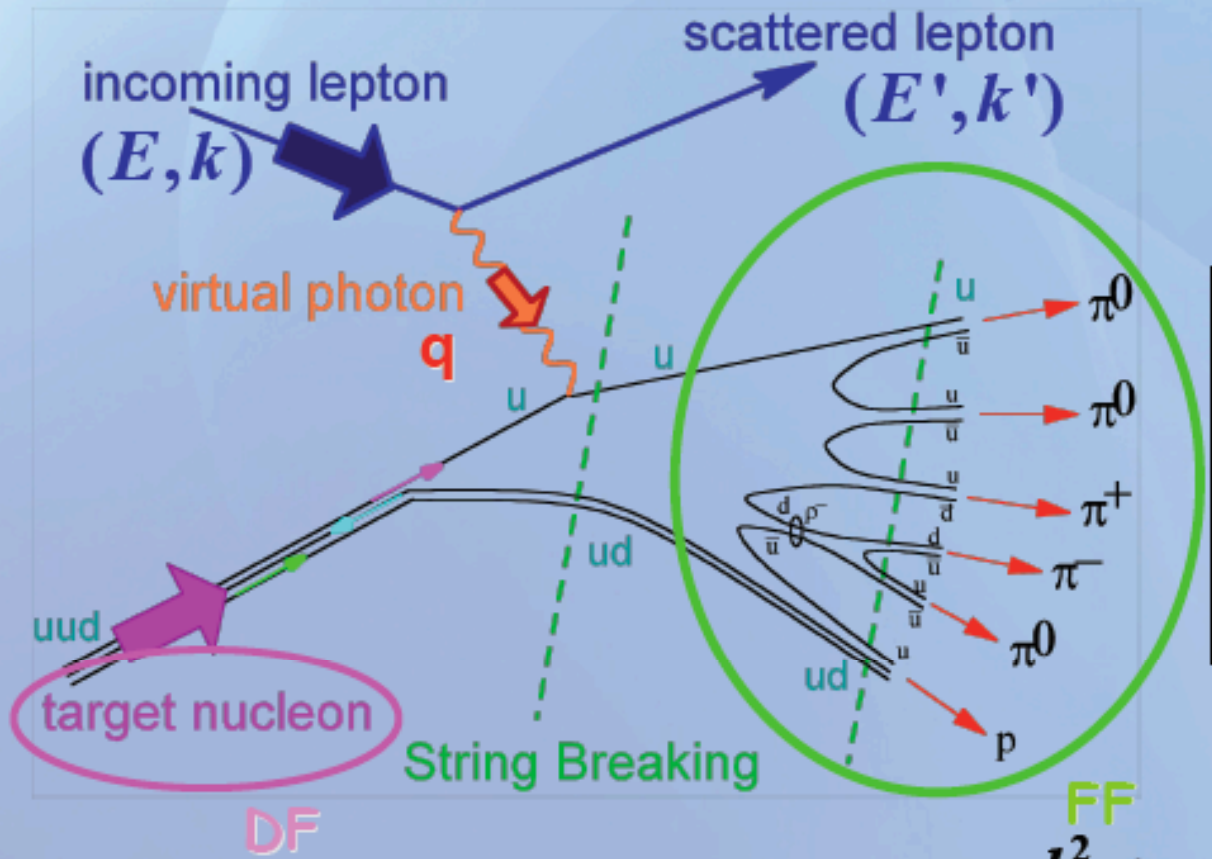
G. Gabrielse,[1] D. Hanneke,[1] T. Kinoshita,[2] M. Nio,[3] and B. Odom[1,*]

Quantum electrodynamics (QED) predicts a relationship between the dimensionless magnetic moment of the electron (g) and the fine structure constant (α). A new measurement of g using a one-electron quantum cyclotron, together with a QED calculation involving 891 eighth-order Feynman diagrams, determine $\alpha^{-1} = 137.035\,999\,710\,(96)$ [0.70 ppb]. The uncertainties are 10 times smaller than those of nearest rival methods that include atom-recoil measurements. Comparisons of measured and calculated g test QED most stringently, and set a limit on internal electron structure.




Structure within the Atom
Quark
Size < 10^{-19} m
Electron
Size < 10^{-18} m
Nucleus
Size ≈ 10^{-14} m
Neutron and Proton
Size ≈ 10^{-15} m
Atom
Size ≈ 10^{-10} m
u
d
e⁻
If the protons and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

Deep Inelastic Scattering



Important kinematic variables:

$$Q^2 = -q^2 = -(k-k')^2$$

$$\nu = E - E'$$

$$x = \frac{Q^2}{2M\nu} \qquad z = \frac{E_h}{\nu}$$

cross section: $$\frac{d^2\sigma}{d\Omega dE'} \sim L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu} = -g^{\mu\nu} F_1 - \frac{p^\mu p^\nu}{\nu} F_2 + \frac{i}{\nu}\varepsilon^{\mu\nu\lambda\sigma} q^\lambda s^\sigma g_1 + \frac{i}{\nu^2}\varepsilon^{\mu\nu\lambda\sigma} q^\lambda (p\cdot q s^\sigma - s\cdot q p^\sigma) g_2$$

$$\underbrace{-r_{\mu\nu} b_1 + \frac{1}{6}(s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) b_2 + \frac{1}{2}(s_{\mu\nu} - u_{\mu\nu}) b_3 + \frac{1}{2}(s_{\mu\nu} - t_{\mu\nu}) b_4}_{\text{Spin 1}}$$

Scaling of Deep Inelastic Cross section, e + p → e + X

$x = 1/\omega = 0.25$

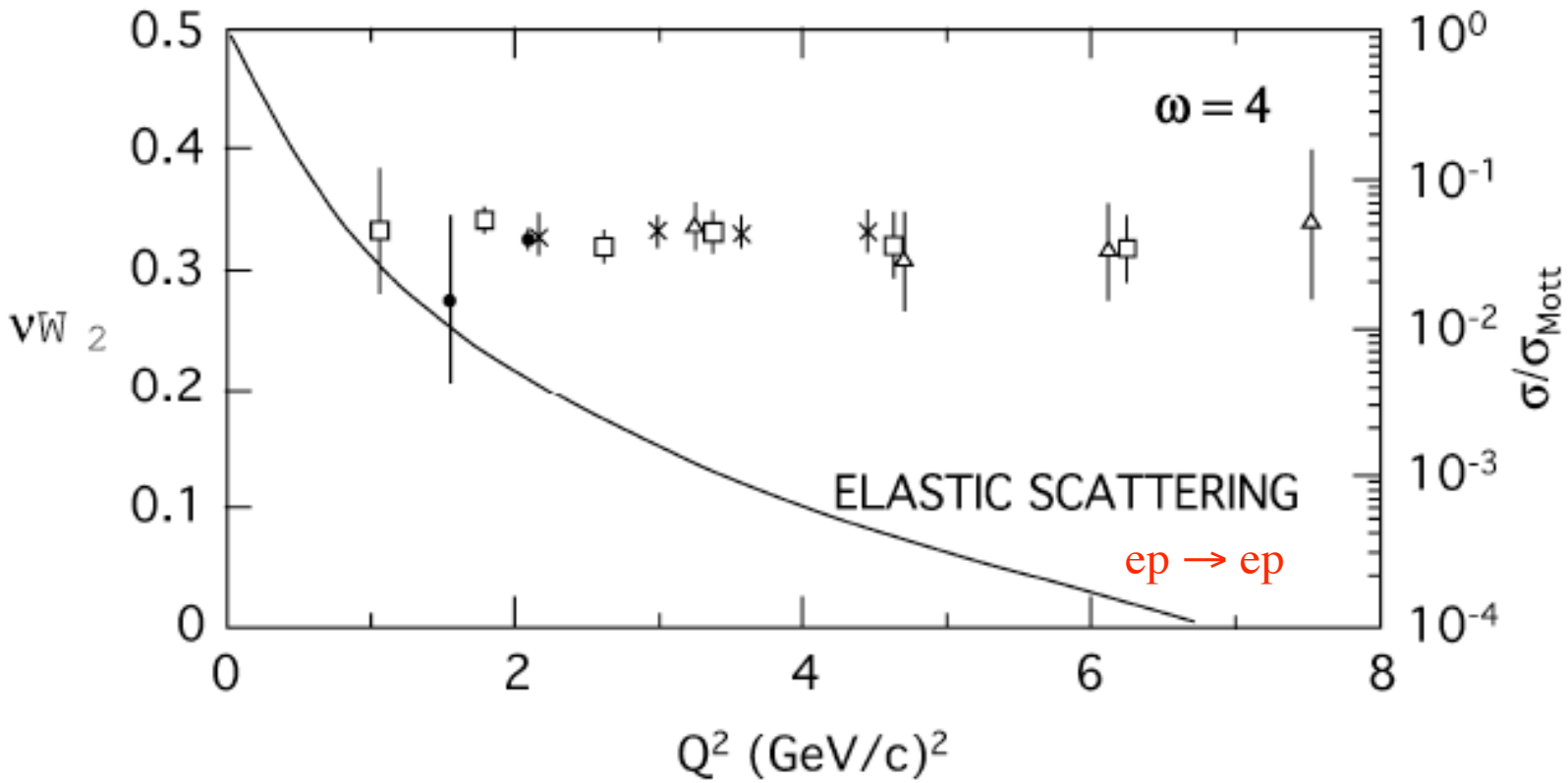
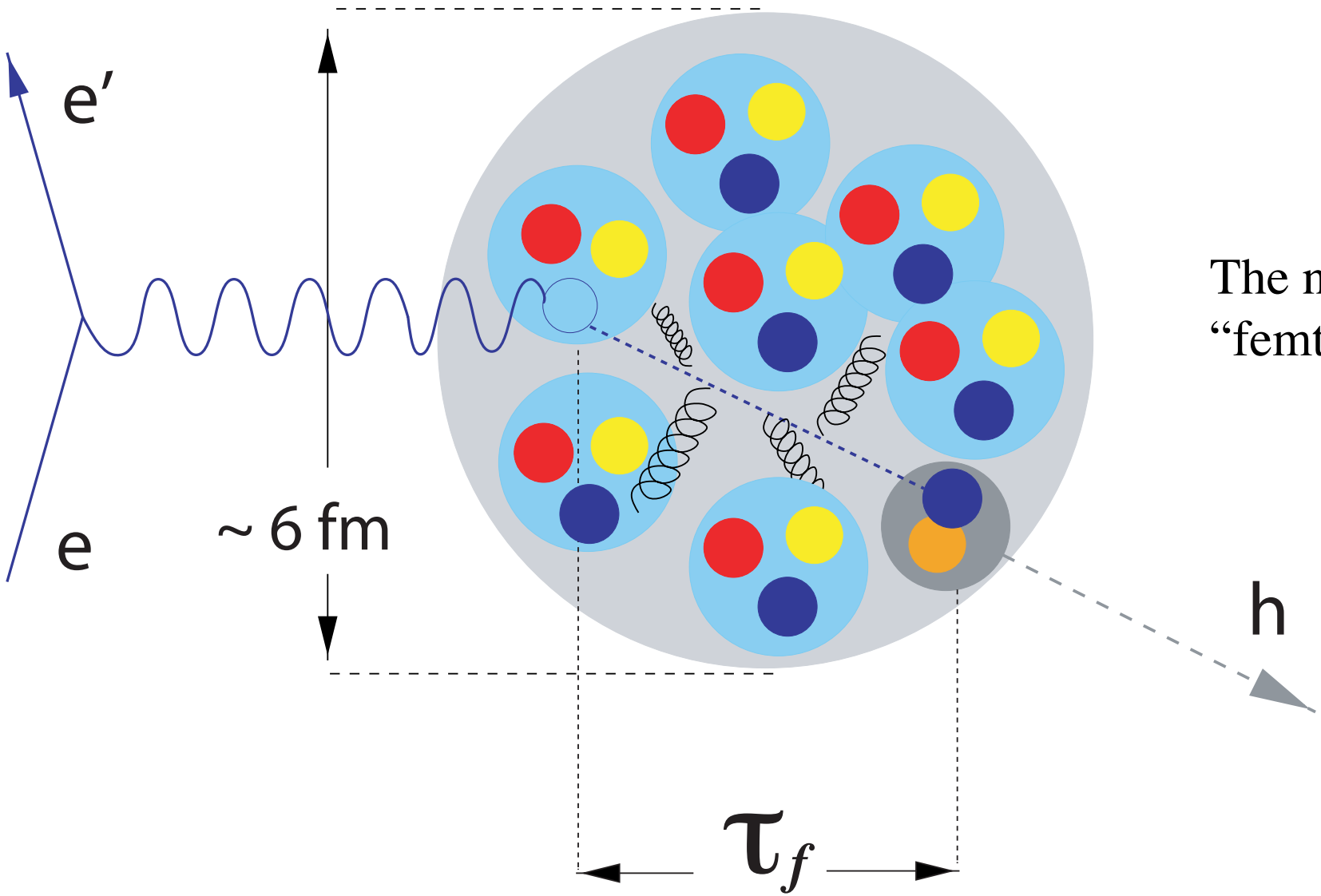


Figure 1: The structure function of the proton, describing the electron- proton scattering in units of the Mott cross section, i.e. the generalised Rutherford cross section at high energies, as measured at SLAC [7]. $\omega = 4$ corresponds to $x = 0.25$ in figure 13.

Semi-inclusive Deep Inelastic Scattering (SIDIS)

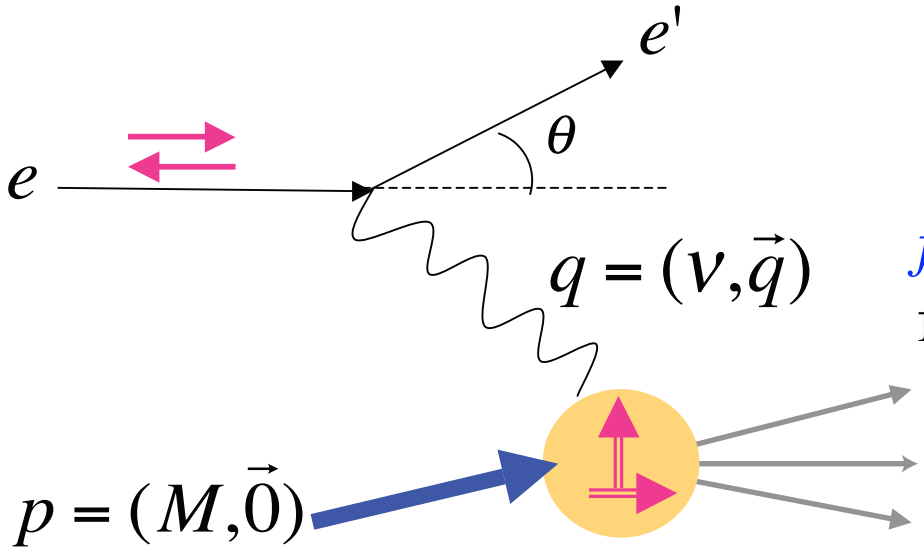
$$e + A \rightarrow e + h + X$$



The nucleus is a "femtodetector"

Hadron formation length

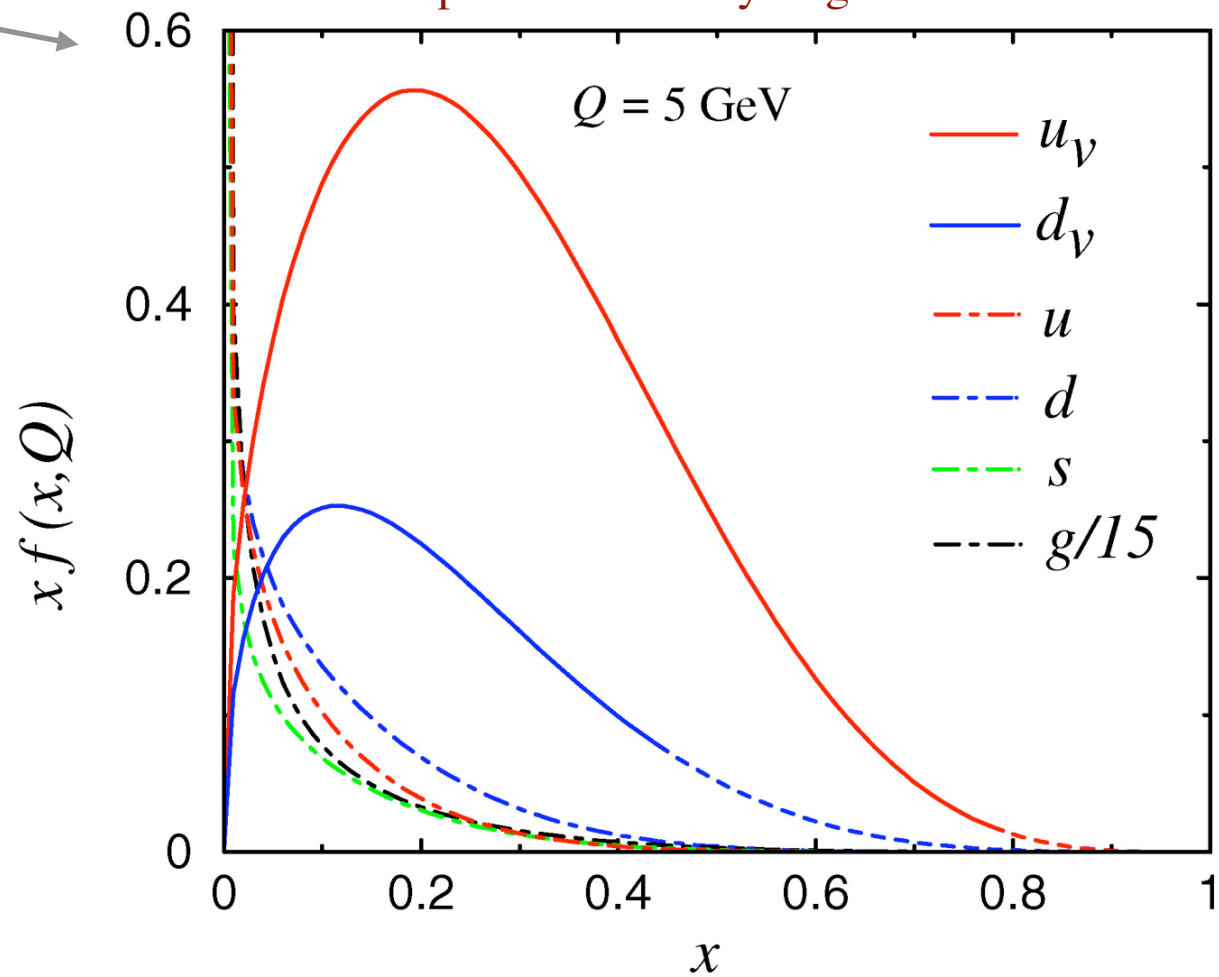
QCD at short distances: Perturbation theory



Hadron structure is probed by a variety of probes, including real and virtual photons

$f(x)$: Probability that a parton carries a fraction x of the proton momentum

Ex: Sketch the x-distribution of the electron and photon in the Hydrogen atom.



Data has revealed the pointlike nature of quarks and gluons, their electric and color charges, their momentum and spin distributions, etc.

Hadrons are highly relativistic bound states, with, 50% of the proton momentum carried by gluons.

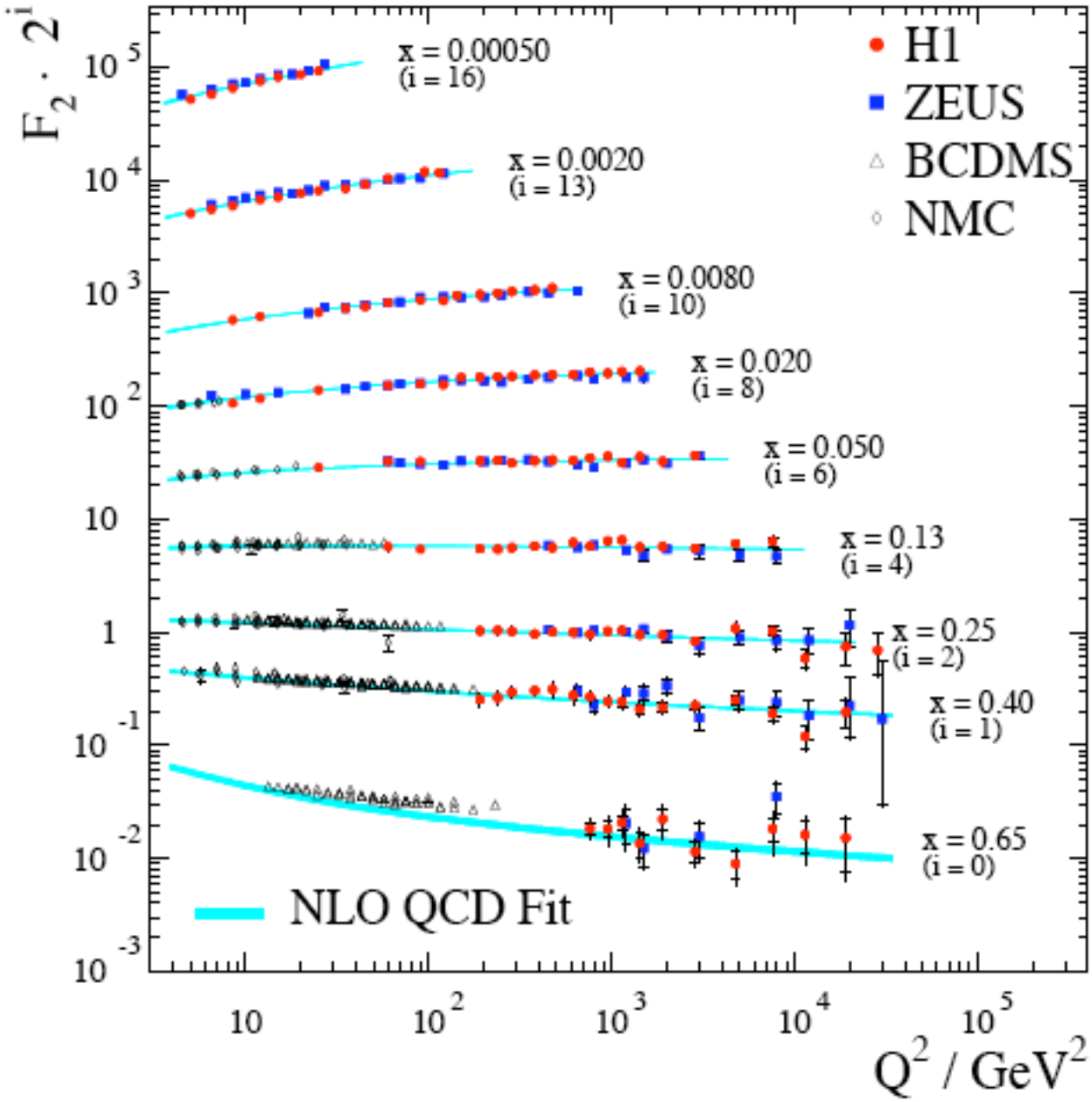
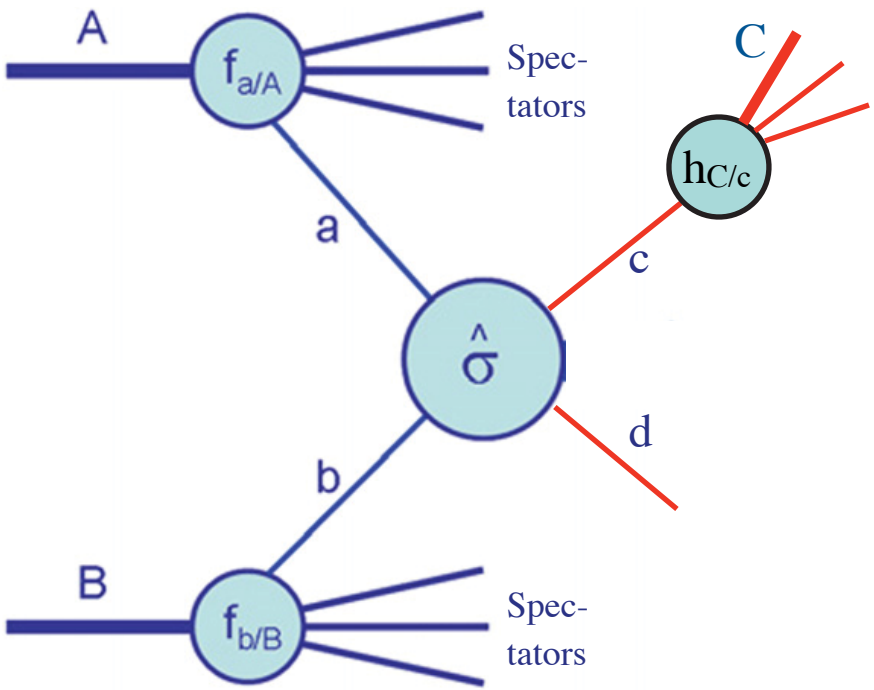


Figure 13: Summary of measurements of F_2 [66]. For better visibility, the results for different values of x were multiplied with the given factors of 2^i.

Soft Coherence in Hard Inclusive Processes

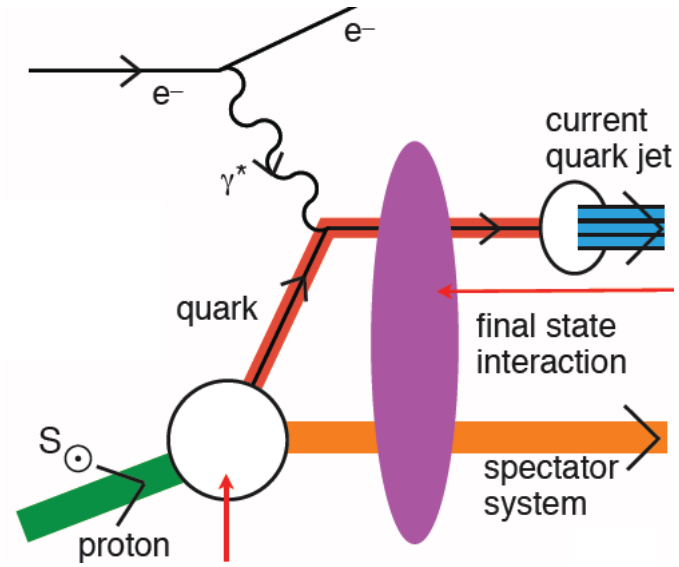
Cartoon:

- One active parton in each hadron
- No interactions with spectators
- Hard subprocess is pointlike



In fact:

Soft spectator interactions influence the hard process, even as $Q^2 \to \infty$



$\Rightarrow$

- Shadowing of parton distrib.
- Hard diffractive scattering
- SSA (SIDIS) = – SSA (DY)
- DVES?

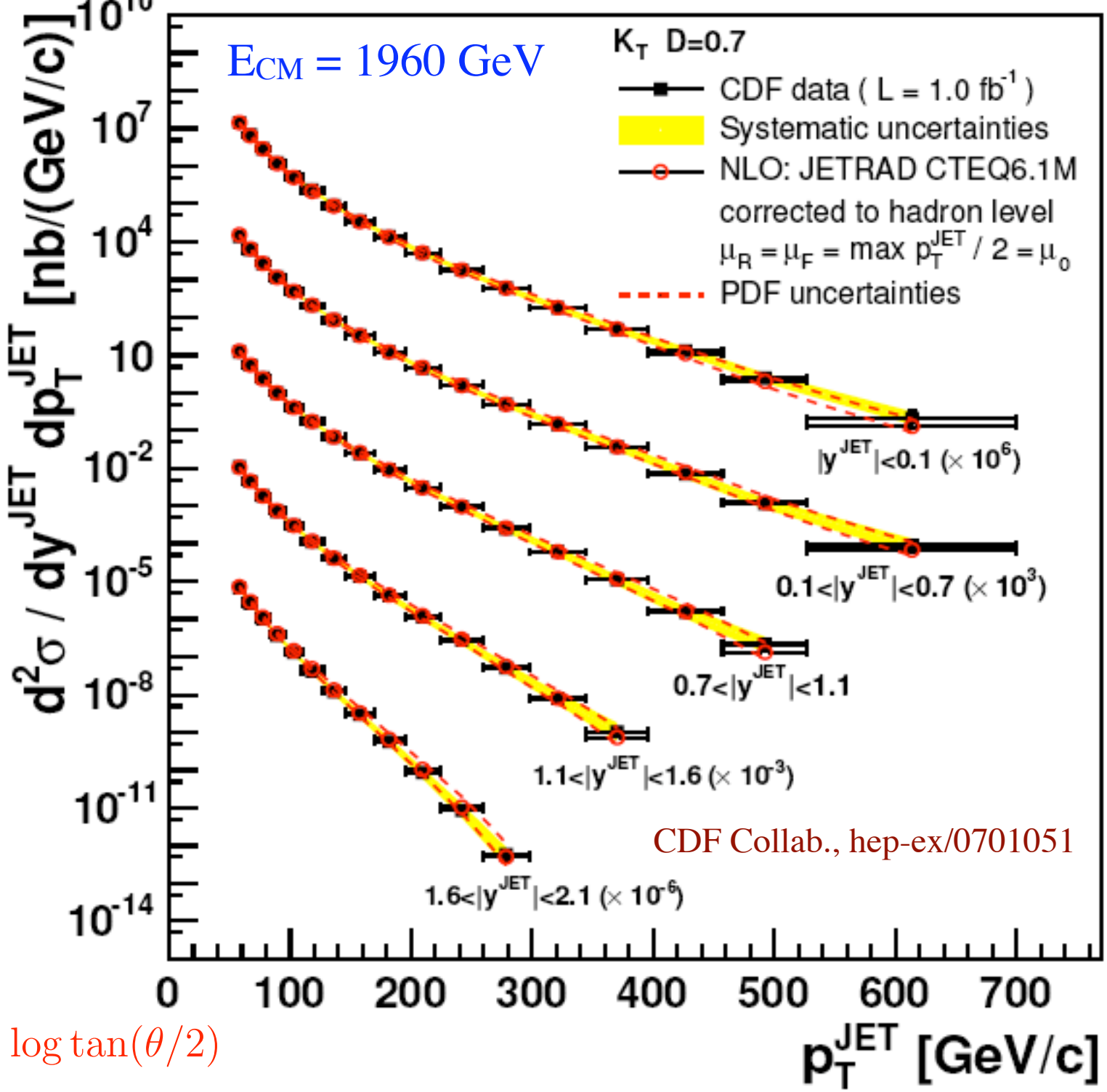
Highest energy:

$p\bar{p} \rightarrow$ jet + X

E_{CM} = 1.96 TeV

Quarks and gluons are pointlike down to the best resolution that has been reached

Ex: Estimate the maximum radius of quarks and gluons, given the agreement of QCD with the Fermilab jet data.



Rapidity:

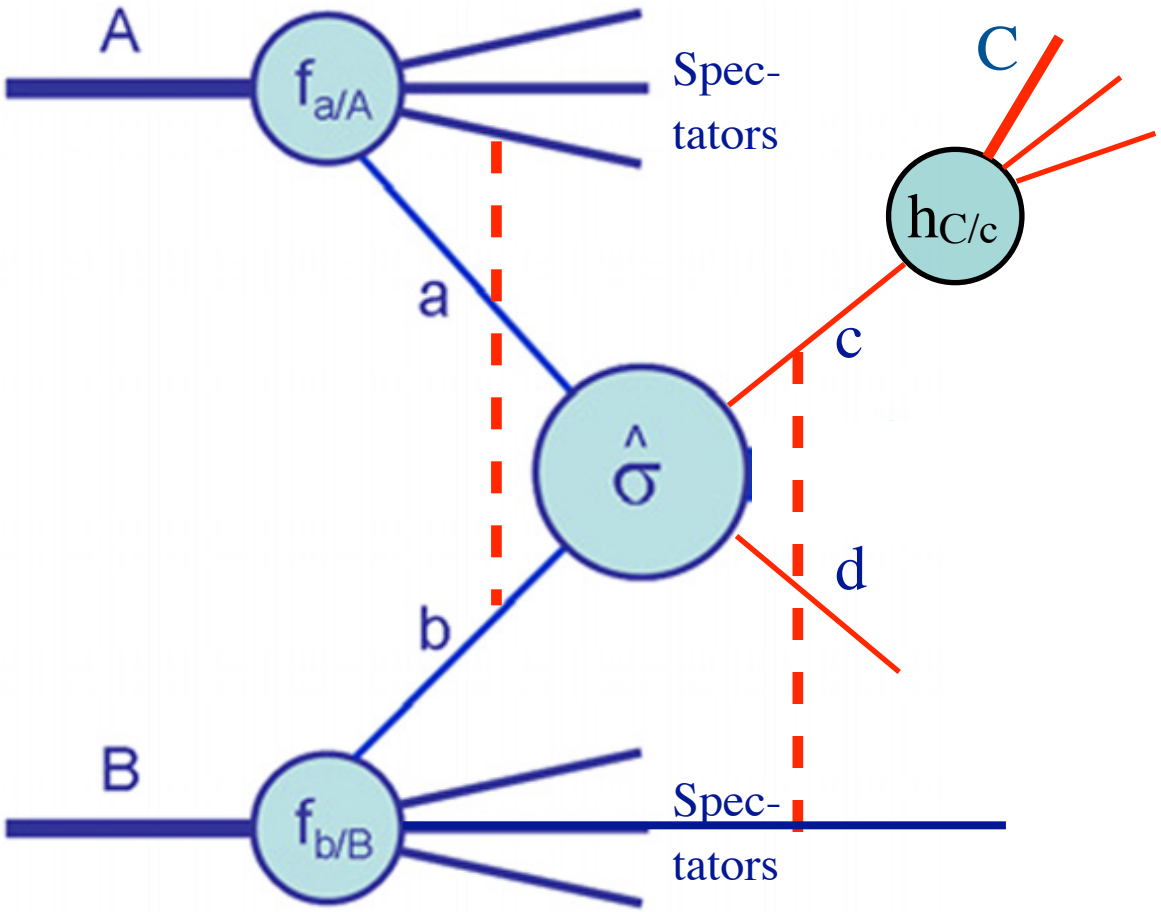
$$y = \log \frac{E + p_{\parallel}}{\sqrt{m^2 + p_{\perp}^2}} \simeq -\log \tan(\theta/2)$$

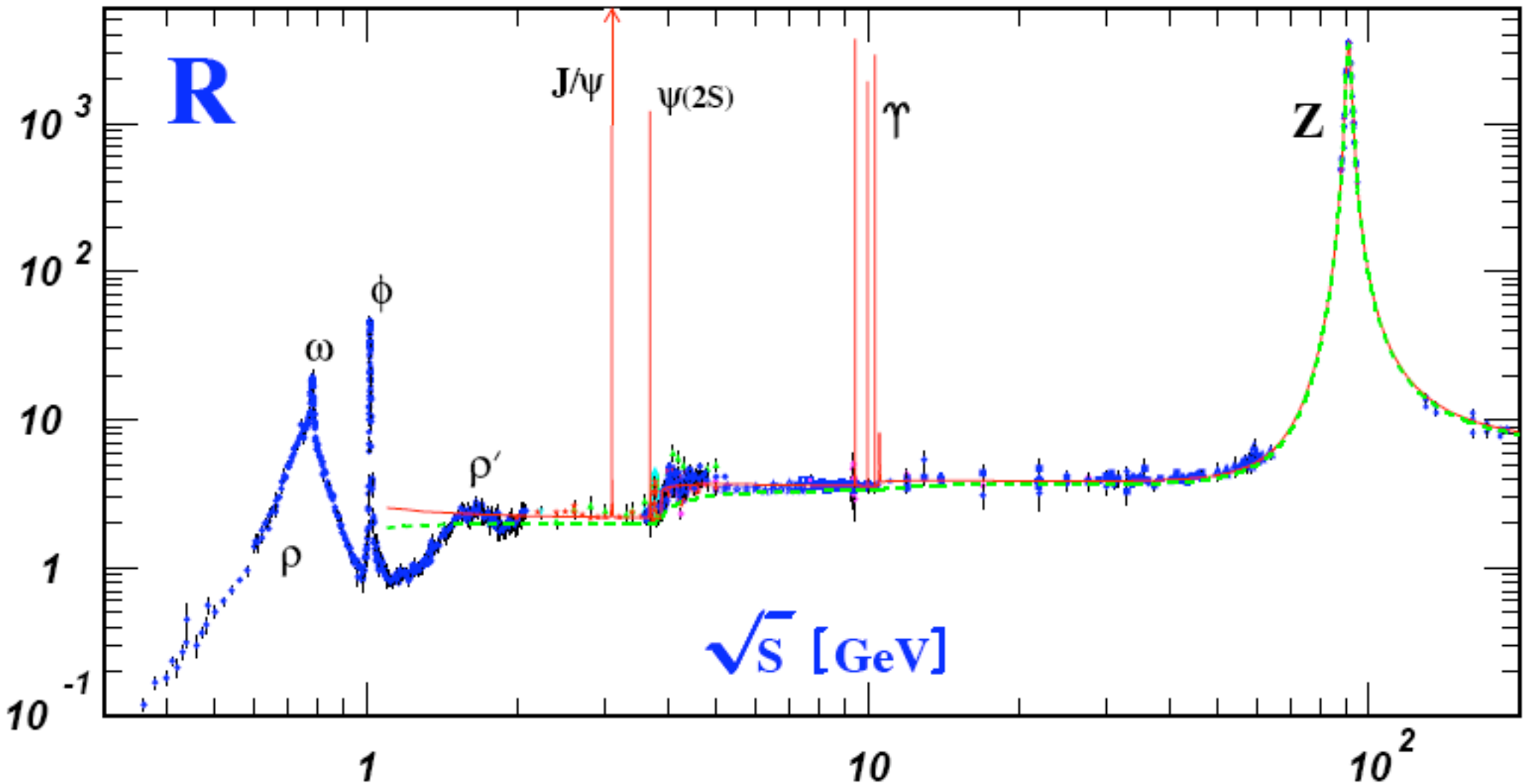
Breaking News: QCD factorization violated

No universality of ($k_\perp$-dependent) parton distributions in hh → h

J. Collins and J-W. Qiu, arXiv:0705.2141 [hep-ph]

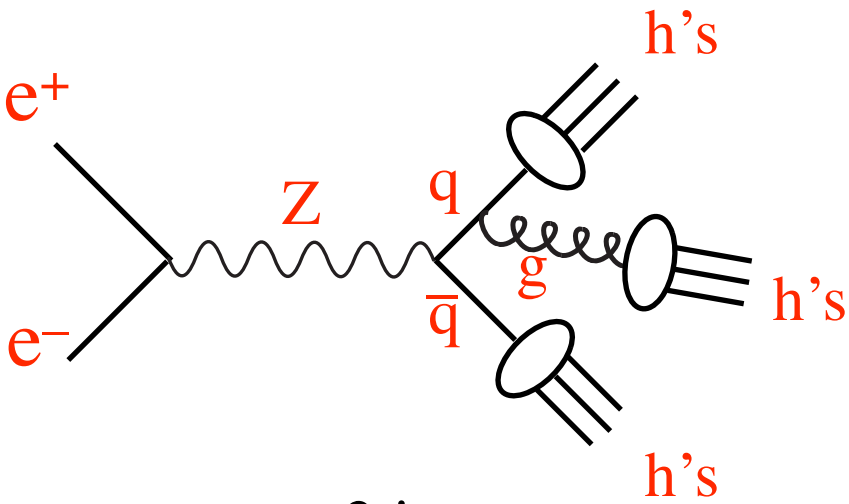
Soft rescattering of active partons on spectators in both initial and final states is not consistent with universal parton distributions.



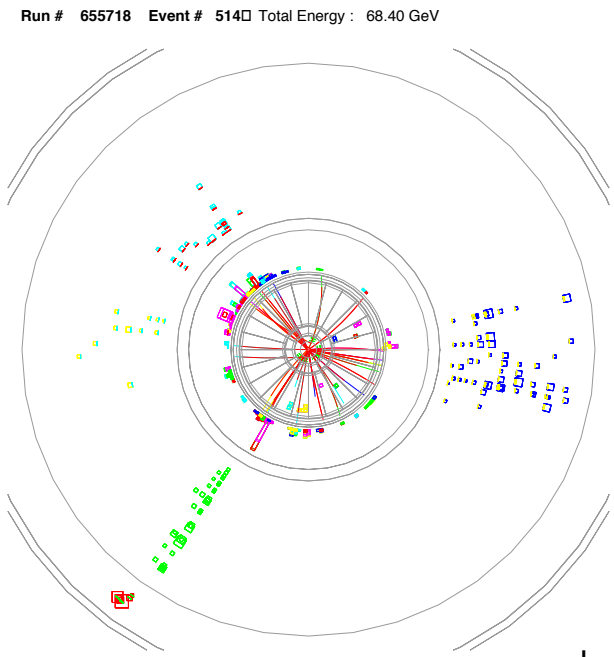


$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$e^+ e^- \rightarrow$ hadrons L3 events at $E=m_Z= 91.2$ GeV

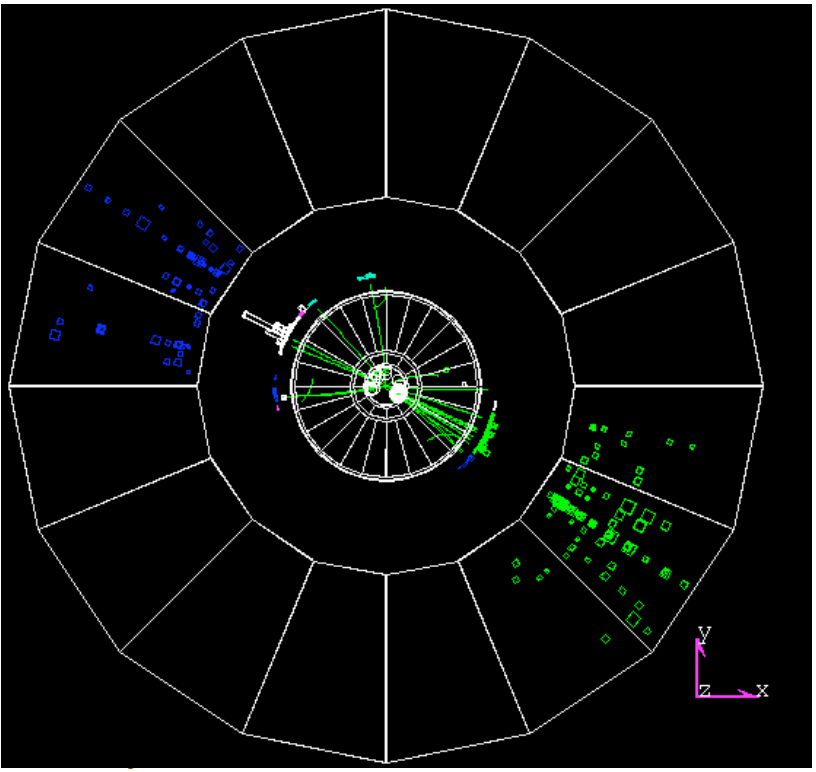


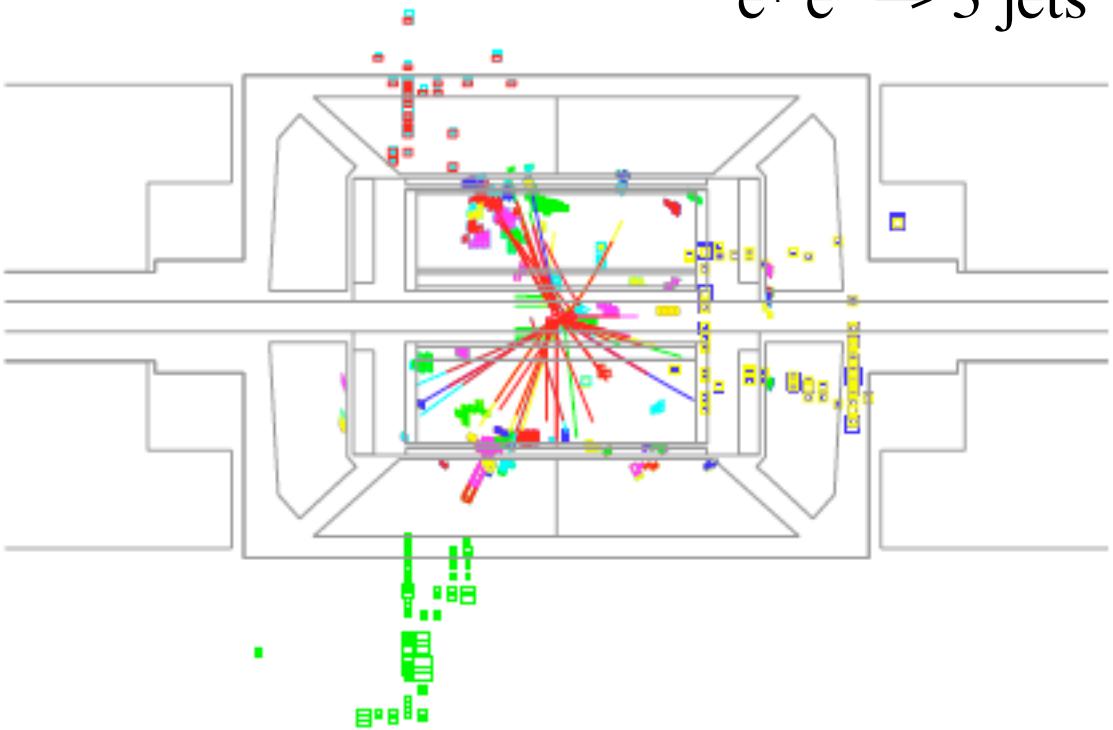
$e^+ e^- \rightarrow$ 2 jets



$e^+ e^- \rightarrow$ 3 jets

5:02 PM





QCD has been extraordinarily successful in explaining the data

Rate of 3-jet events in $e^+ e^-$ annihilations

$e^+ e^- \rightarrow q\, q\, g$
$\rightarrow$ 3 jets

Ex: Estimate the CM energy in e+ e– annihilations at which 2-jet structure emerges. In quark fragmentation, pions get an average fraction $\langle z \rangle \approx 0.1$ of the quark energy, and $\langle p_\perp \rangle \approx 350$ MeV.

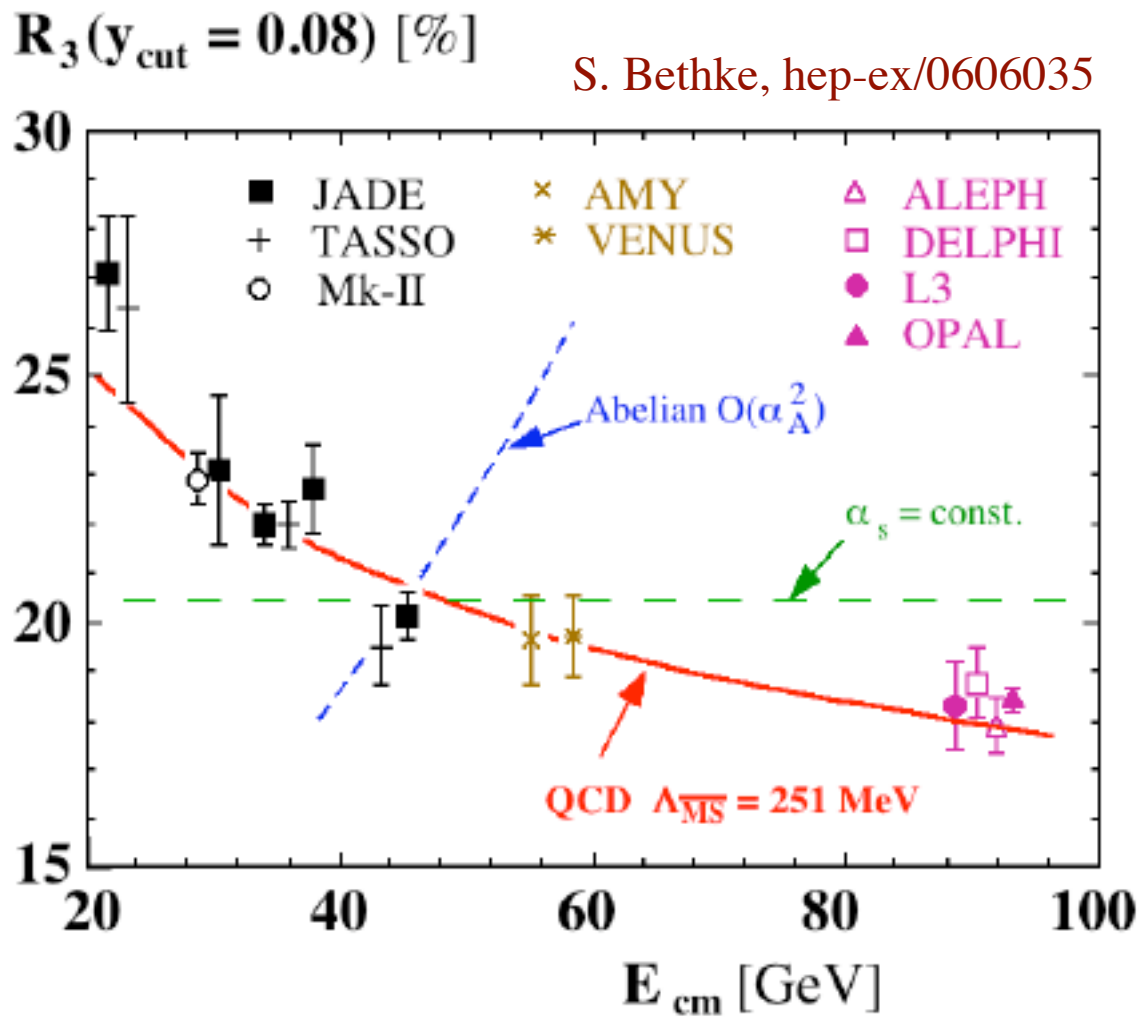


Figure 8: Energy dependence of 3-jet event production rates, measured using the JADE jet finder at a scaled jet energy resolution $y_{cut} = 0.008$. The errors are experimental. The data are not corrected for hadronisation effects. They are compared to theoretical expectations of QCD, of an abelian vector gluon model, and to the hypothesis of a constant coupling strength.

S. Bethke, hep-ex/0606035

Angular distribution of 4-jet events in $e^+ e^-$ annihilations

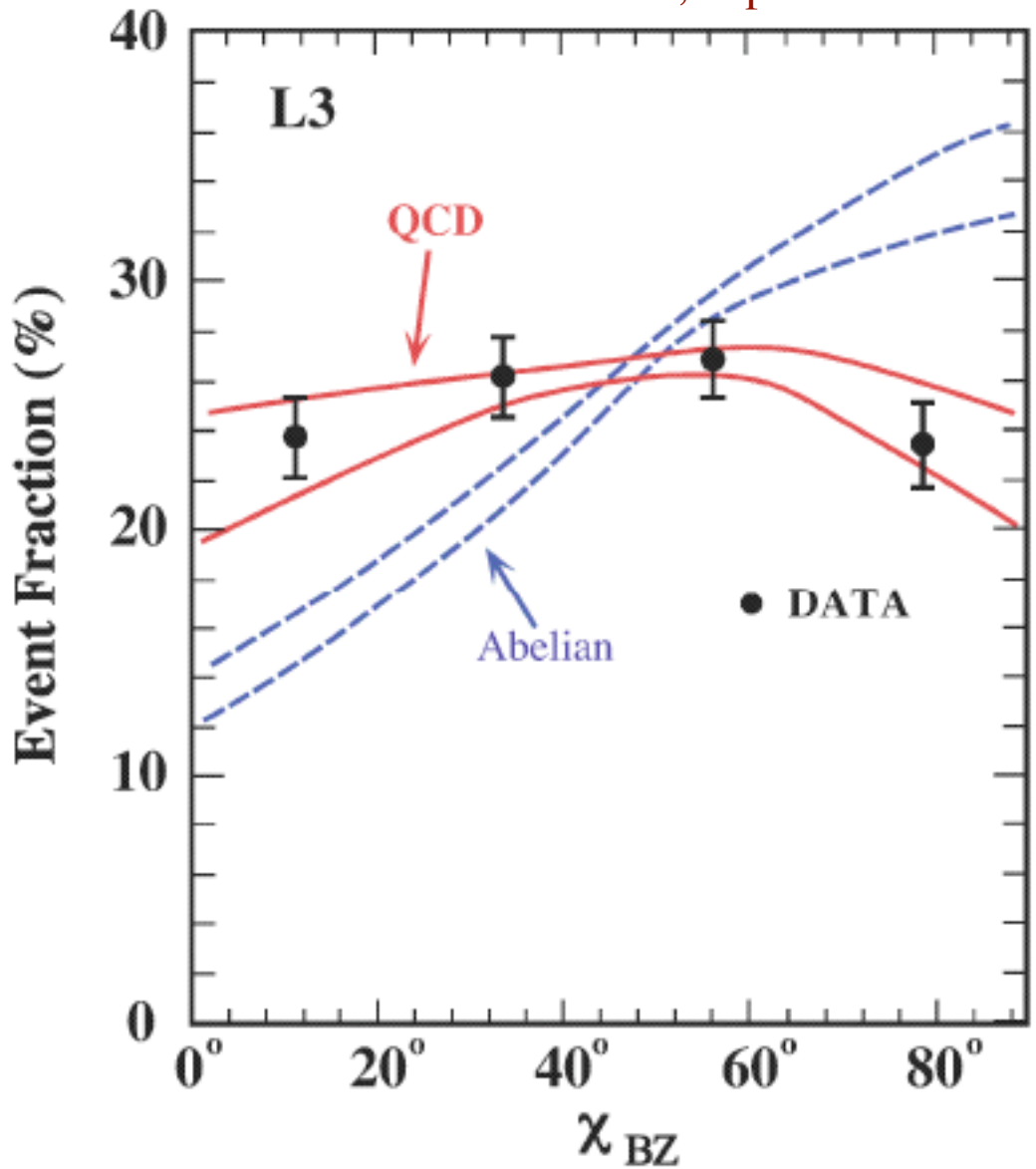
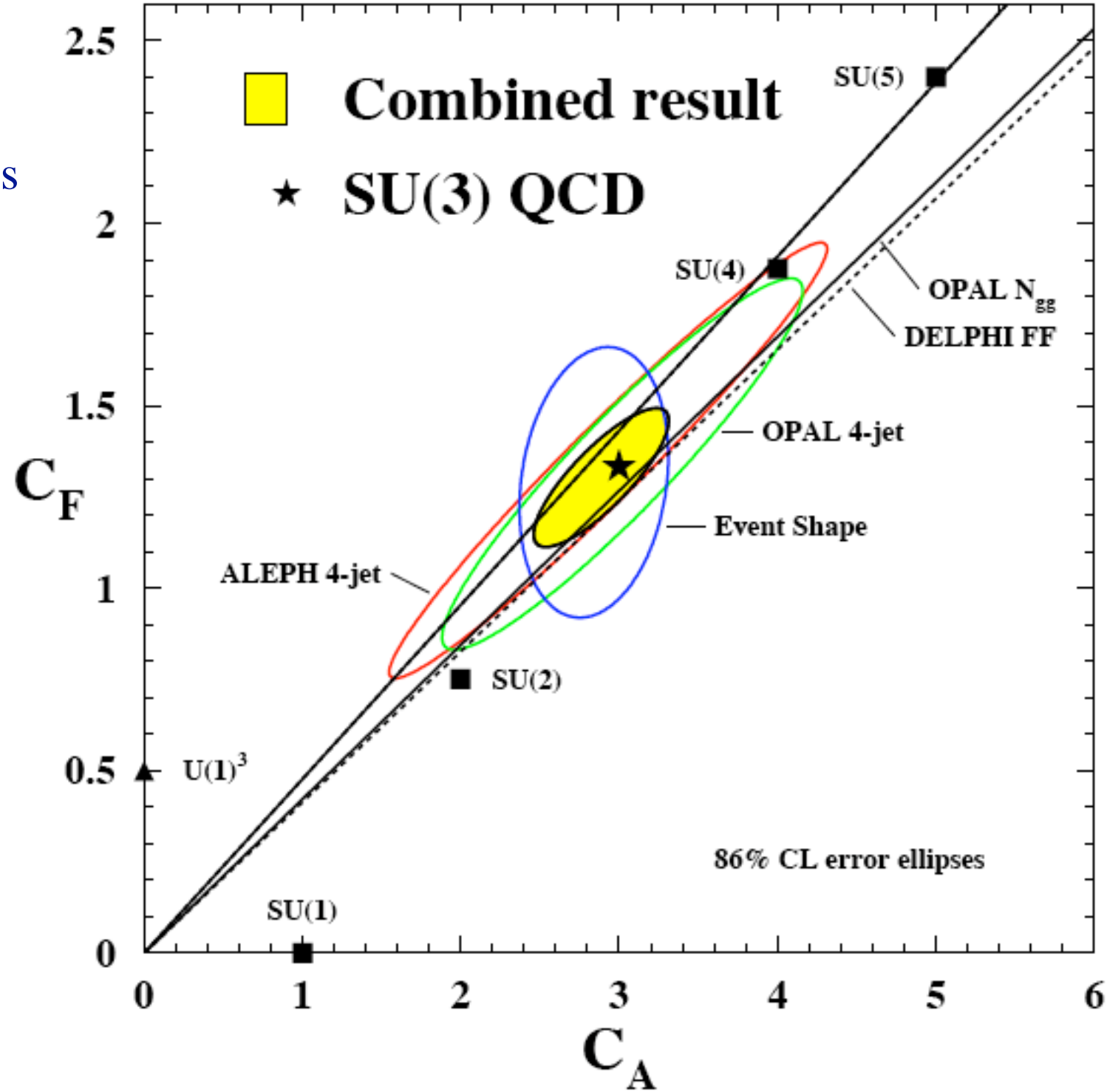


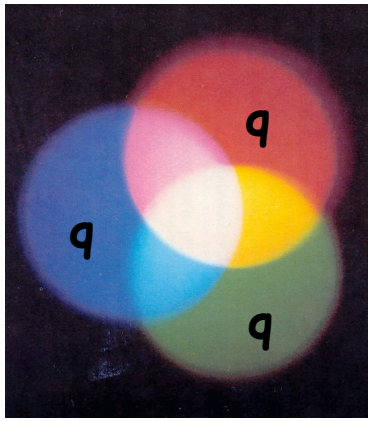
Figure 10: Distribution of the azimuthal angle between two planes spanned by the two high- and the two low-energy jets of hadronic 4-jet events measured at LEP [54], compared to the predictions of QCD and of an abelian vector gluon model where gluons carry no colour charge [27].

S. Bethke, hep-ex/0606035

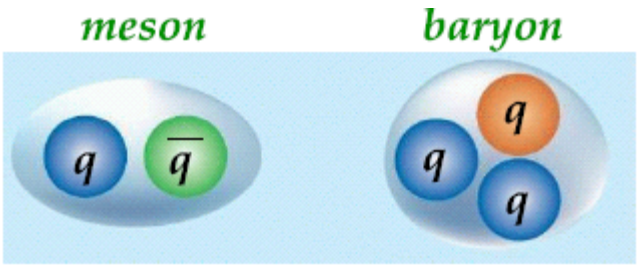
Measurement of quark and gluon color charges in $e^+ e^-$ annihilations



QCD at long distances: Color confinement



Mystery remains:
Why do quarks and gluons always form colourless hadrons, with only two configurations predominating?

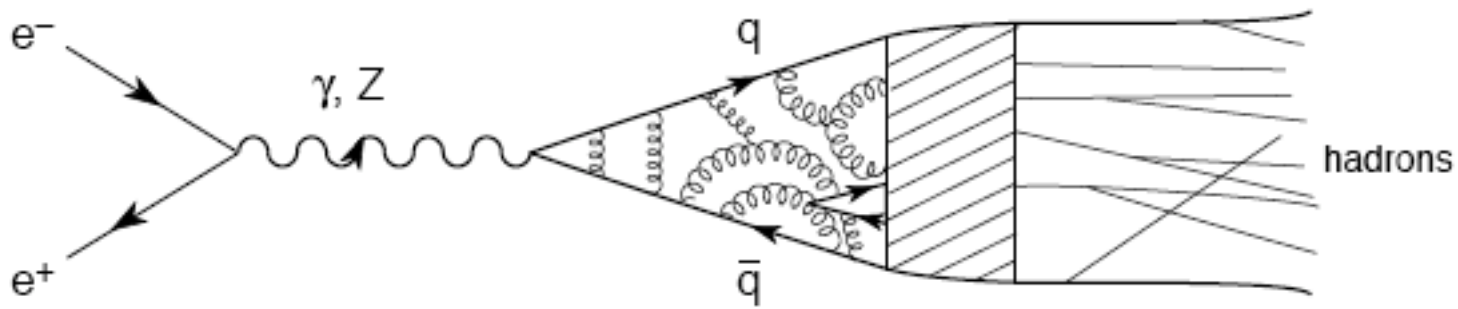


The interactions are given by $\mathcal{L}_{QCD} = \bar{\Psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$

Hadron structure is being studied experimentally using $E_{lab} \geq 1$ GeV = 0.2 fm^{-1} at Jefferson Lab, DESY, Fermilab, CERN, BNL, KEK, SLAC, Mainz, GSI, ...

How could we fail to reach an understanding of the basic features?

Bridging the quark and gluon transition to hadrons



$e^+e^- \to$ hadrons.

Data suggests:

- The momentum and multiplicity distribution of hadrons is similar to that of quarks and gluons in perturbative QCD
- Parton distributions $f_{q/N}(x, Q^2)$ extrapolate smoothly from high to low Q^2

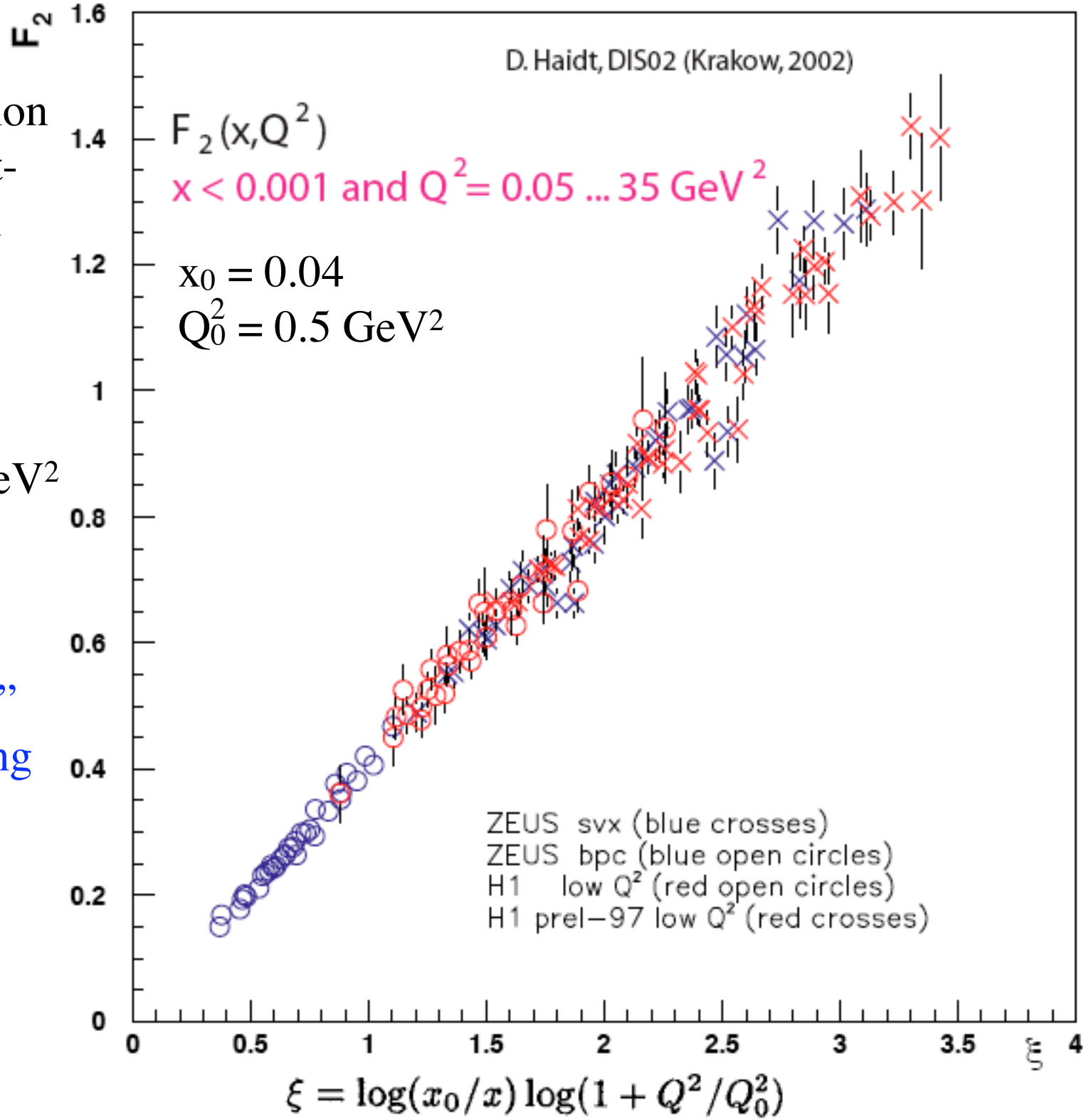
There is a similarity between hard PQCD and soft NPQCD processes

$\Rightarrow$ The quark hadron transition is soft rearrangement process

The F_2 structure function can be simply parametrized from low to high values of Q^2

The Q^2 dependence freezes at $Q_0^2 = 0.5$ GeV^2

There is no evidence of a "phase transition" between short and long distance phenomena



Local parton-hadron duality

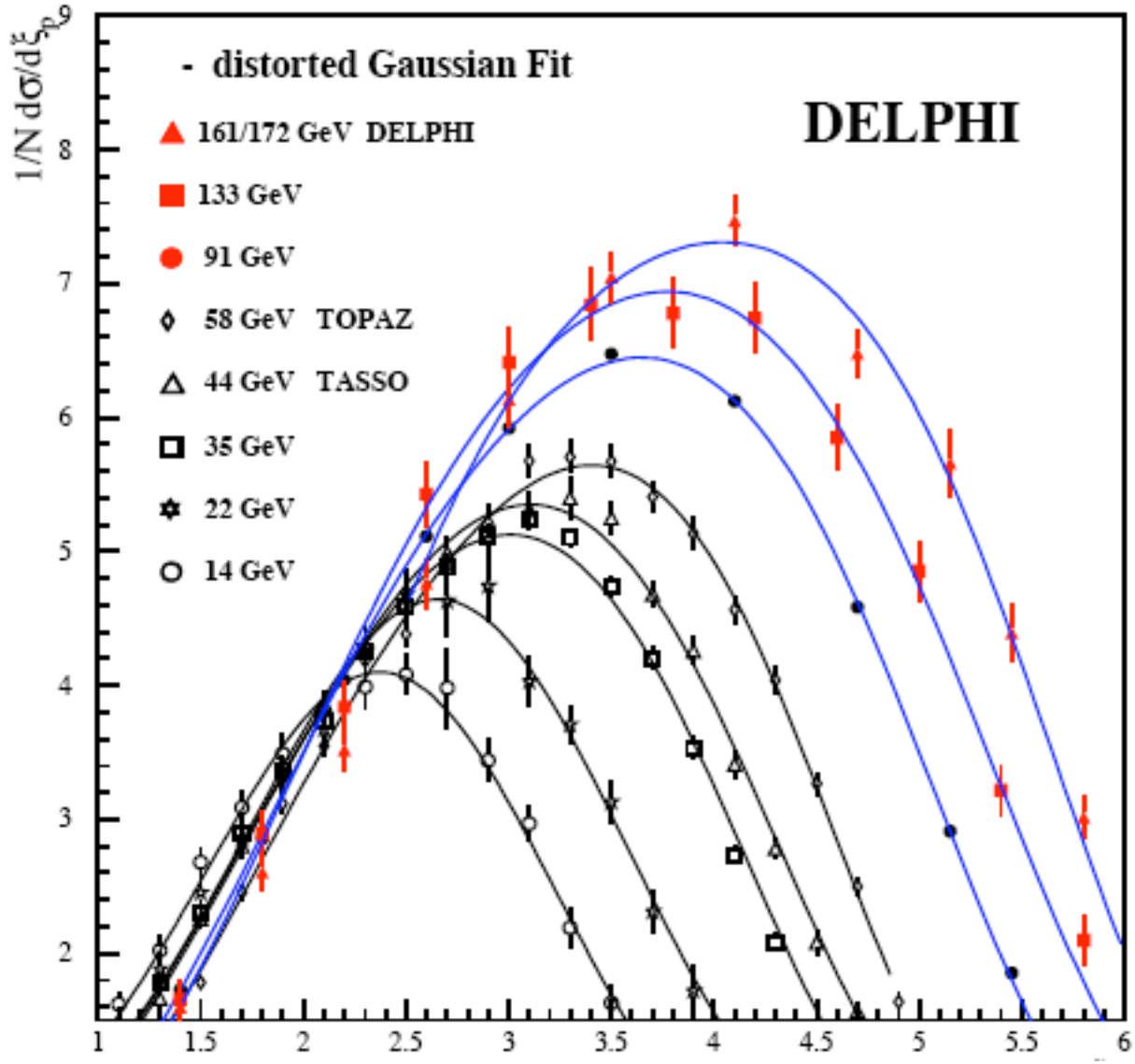
The momentum distribution of partons in QCD agrees with the measured distribution of charged hadrons (up to one normalization parameter)

Dokshitzer: The transition from partons to hadrons is local in momentum space, ie., no redistribution of momentum occurs.

Ex: Estimate the energy of the hadrons/partons at the maximum of the curves.

$x = E_h/E_e$

$e^+ e^- \rightarrow h^\pm + X$ Y. Dokshitzer, hep-ph/0306287



Duality in the proton structure function

Bloom and Gilman, PRL **25** (1970) 1140

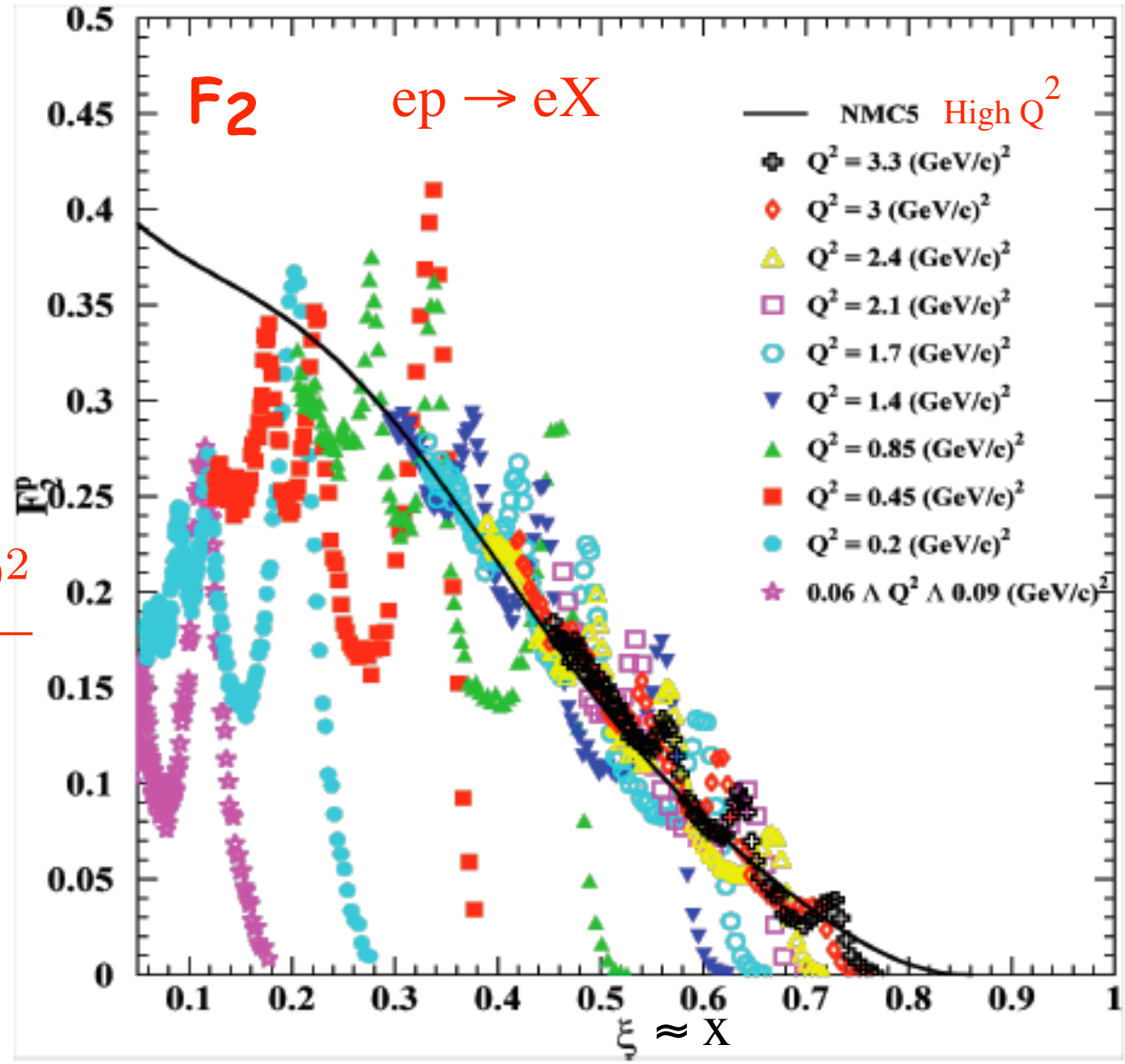
$$F_2(x) = \sum_q e_q^2 \, x f_q(x)$$

Resonance contributions at low Q^2 average the scaling distribution ($Q^2 \to \infty$ at fixed x)

$$M_{N*}^2 = m_N^2 + \frac{(1-x_B)Q^2}{x_B}$$

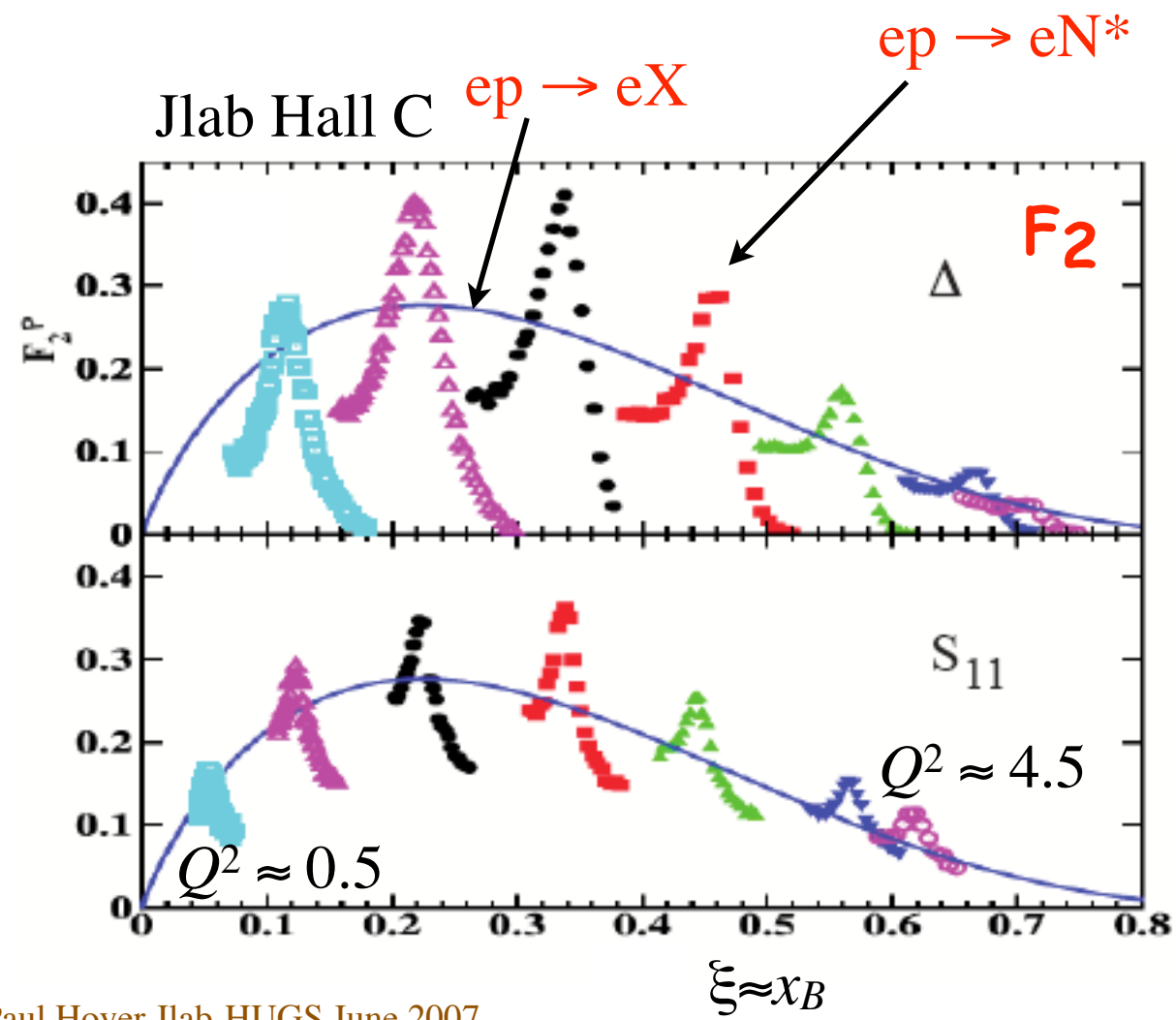
$$= M_X^2$$

I. Niculescu et al, Phys. Rev. Lett. 85, 1182 (2000)

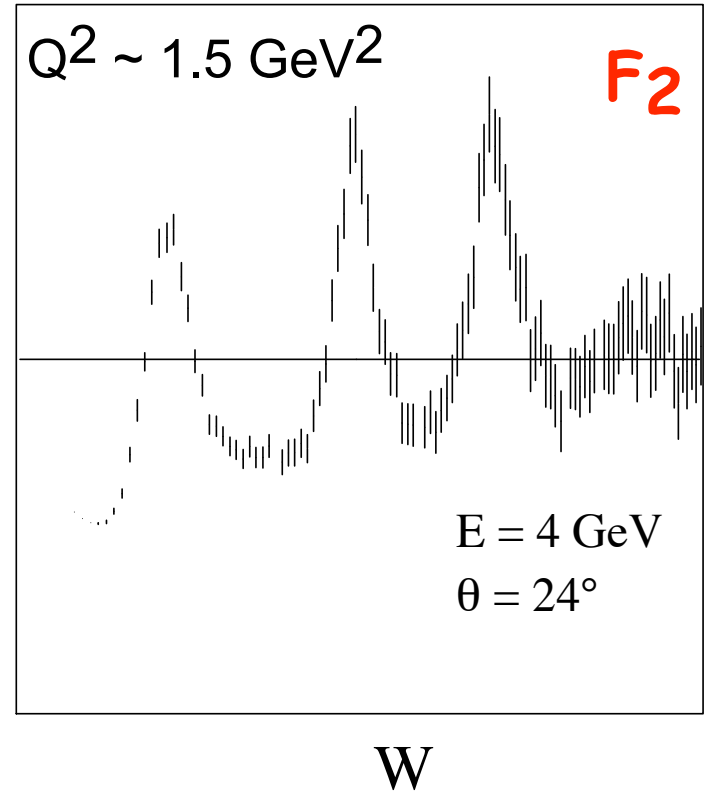


Duality between ep → eN* resonance contributions at low Q^2 and the scaling curve for ep → eX at high Q^2 works locally.

W. Melnitchouk et al,, Phys. Rep. 406 (2005) 127Workshop on Duality, Frascati 2005
http://www.lnf.infn.it/conference/duality05/



Resonance Region F_2 vs. NNLO Scaling Curve



Ex: Estimate the difference between the Nachtmann variable ξ and x_B for the relevant Q^2. Can you derive the expression for ξ ?

$$\xi = \frac{2x}{1+\sqrt{1+4m_p^2x^2/Q^2}}$$

What we learnt so far

1. PQCD works quantitatively in hard processes: $\alpha_s(M_Z) = 0.1189 \pm 0.0010$

2. There is a surprising similarity between short and long-distance physics

$\Rightarrow$ Is an expansion in α_s relevant also at low Q^2 ?

The coupling α_s may cease to run (*i.e.*, freeze) at $Q \sim \Lambda_{QCD}$

Scale dependence of the QCD coupling α_s

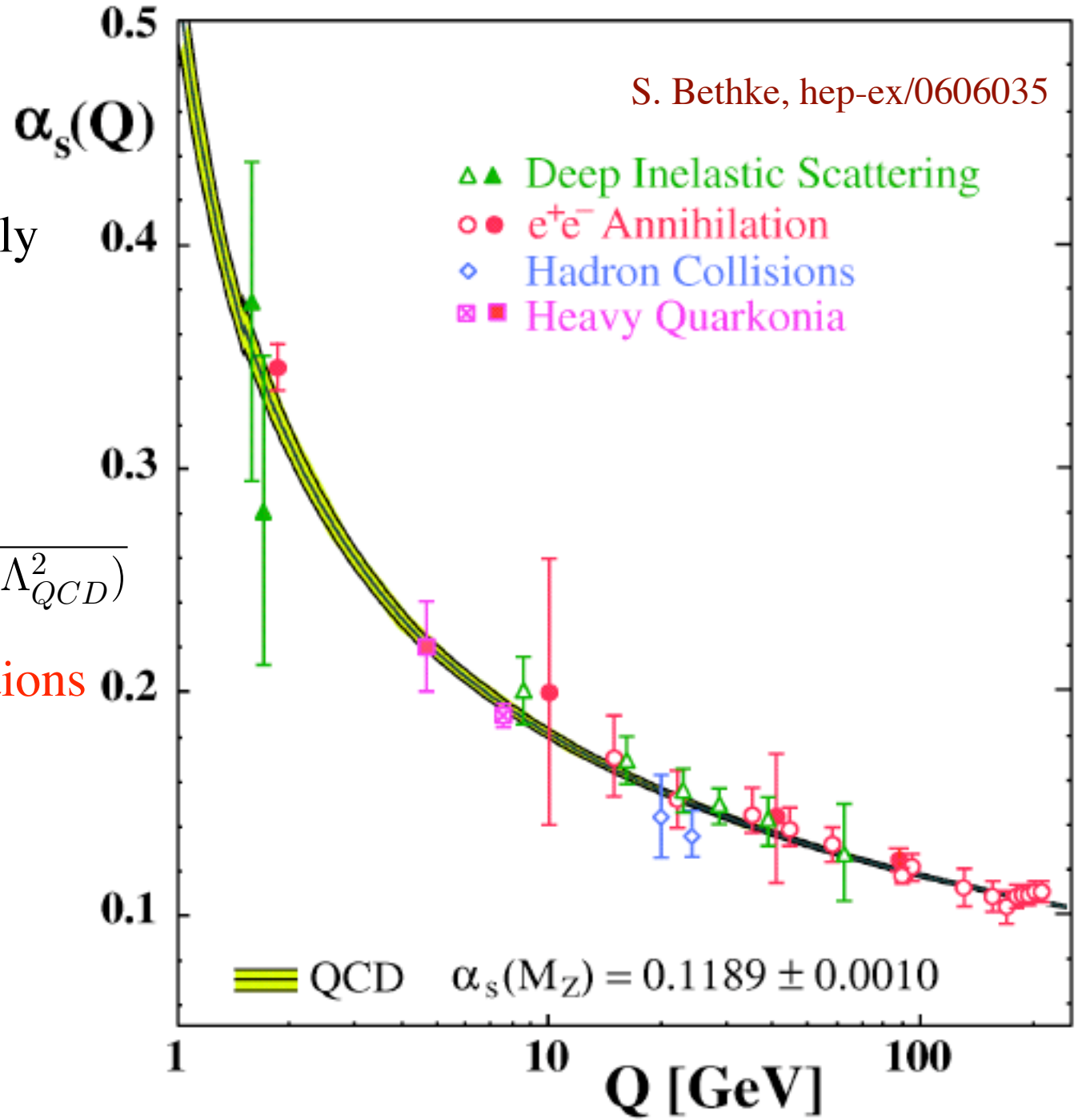
The coupling can be determined perturbatively only for Q ≥ 1 GeV

The expression

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2_{QCD})}$$

is valid only up to corrections of order $1/\log(Q^2)$.

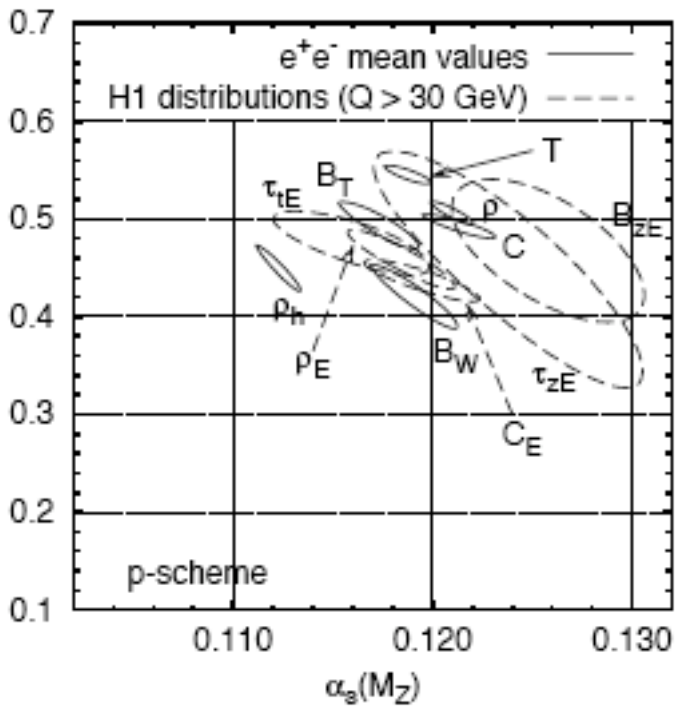
Ex: How big is $\log(Q^2/\Lambda^2_{QCD})$ at the largest value plotted for α_s?
Λ_{QCD} = 200 MeV



It is plausible that α_s freezes as Q → 0

Analysis of power corrections

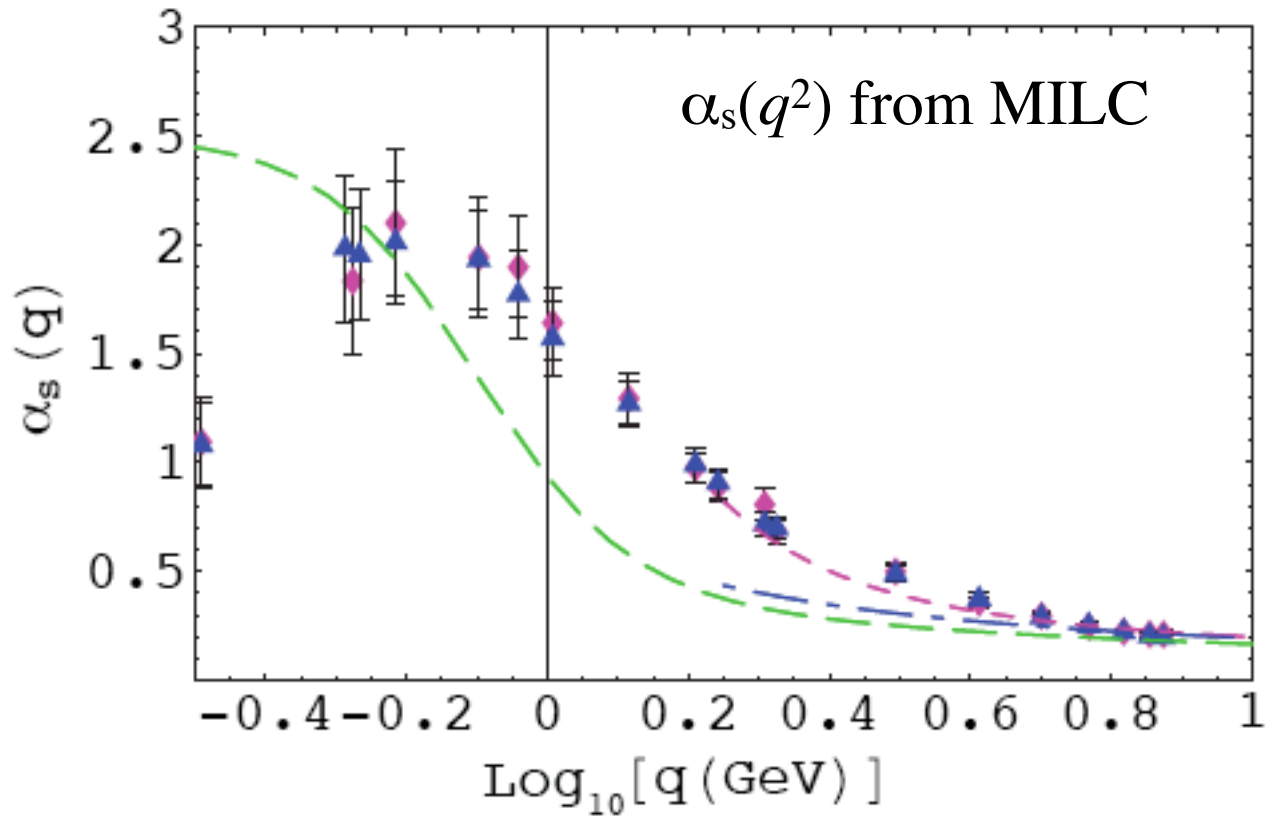
$$\alpha_0 = \frac{1}{2\ \mathrm{GeV}} \int_0^{2\ \mathrm{GeV}} dk\, \alpha_s(k^2)$$



Y. Dokshitzer, hep-ph/0510199

Lattice determination

Furui and Nakjima, hep-lat/0609024



In general: Do not expect sensitivity to Q when Q << Λ_{QCD}

Constituent Quark Model

Mesons $q\bar{q}$
Baryons qqq

http://pdg.lbl.gov/

Hadron spectrum is similar to non-relativistic QED atoms:
Another sign of perturbation theory

In strongly coupled field theory the spectrum need not reflect constituent quantum numbers

Cf. QED in 1+1 dimensions (the Schwinger model):

Theory of a free, pointlike boson as $m_e/e \to 0$. See also:

S. Coleman, Ann. Phys (NY). **101** (1976) 239

$n\,^{2s+1}\ell_J$	J^{PC}	$I = 1$ $ud,\ \bar{u}d,\ \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$
$1\,^1S_0$	0^{-+}	π
$1\,^3S_1$	1^{--}	$\rho(770)$
$1\,^1P_1$	1^{+-}	$b_1(1235)$
$1\,^3P_0$	0^{++}	$a_0(1450)$
$1\,^3P_1$	1^{++}	$a_1(1260)$
$1\,^3P_2$	2^{++}	$a_2(1320)$
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$
$1\,^3D_1$	1^{--}	$\rho(1700)$
$1\,^3D_2$	2^{--}	
$1\,^3D_3$	3^{--}	$\rho_3(1690)$
$1\,^3F_4$	4^{++}	$a_4(2040)$
$1\,^3G_5$	5^{--}	$\rho_5(2350)$
$1\,^3H_6$	6^{++}	$a_6(2450)$
$2\,^1S_0$	0^{-+}	$\pi(1300)$
$2\,^3S_1$	1^{--}	$\rho(1450)$

Baryon magnetic moments

Baryon Moment	1983 Data Ref[26]	From Naive Model[25]
$\mu(p)$	2.793±0.000	2.79
$\mu(n)$	−1.913±0.000	−1.86
$\mu(\Lambda)$	−0.613±0.005	−0.58
$\mu(\Sigma^+)$	2.38±0.02	2.68
$\mu(\Sigma^-)$	−1.11±0.04[27]	−1.05
$\mu(\Xi^0)$	−1.25±0.014	−1.40
$\mu(\Xi^-)$	−0.60±0.04	−0.47

1995

$\mu_{\Omega^-} = (-2.019 \pm 0.054)\mu_N$ $\quad -1.84\mu_N$

Charmonium spectrum: Charm-Anticharm mesons

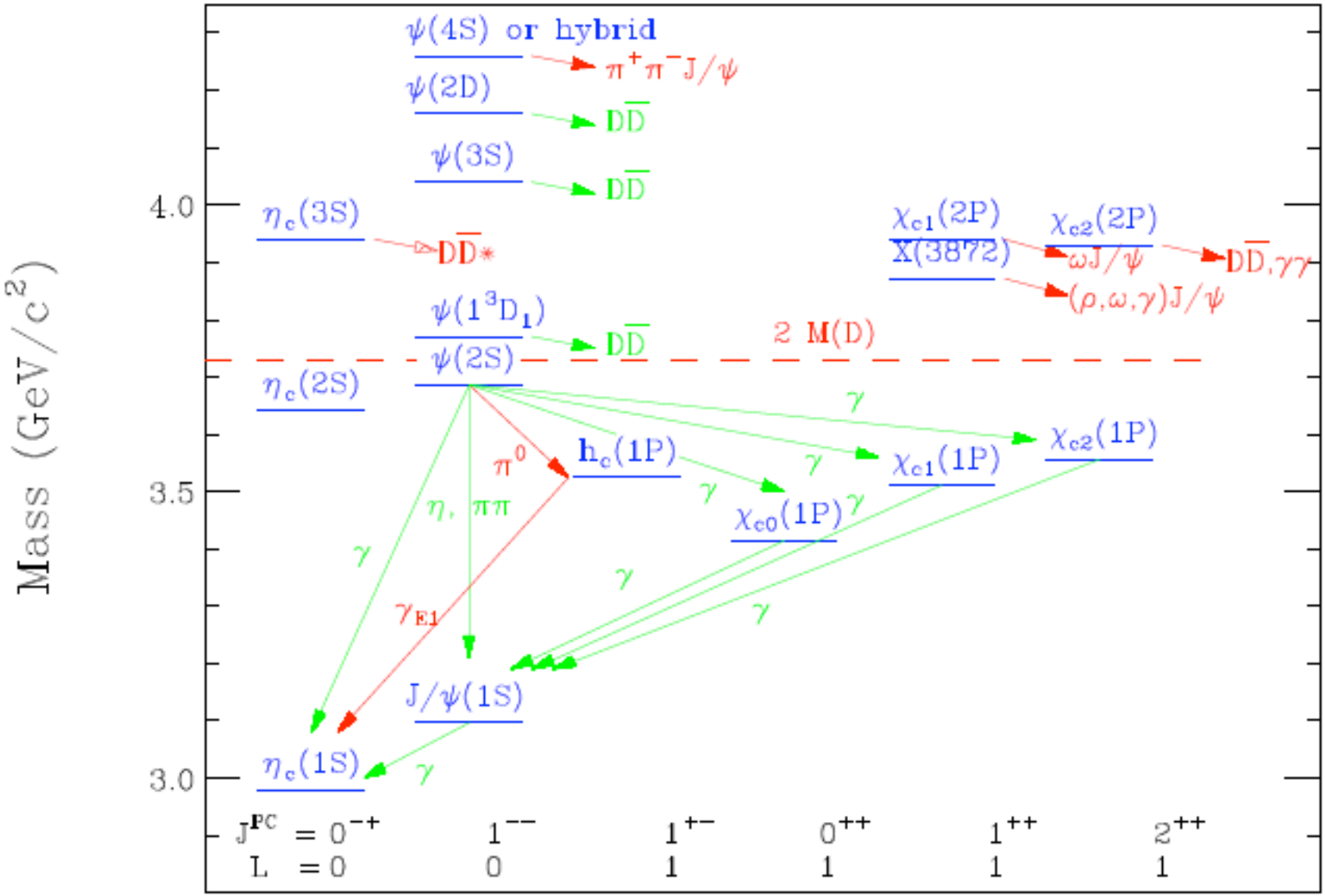


Figure 7: Charmonium states including levels above charm threshold.

J. L. Rosner, hep-ph/0609195

Importance of the perturbative expansion

The perturbative expansion has been crucial for establishing QED and QCD. It satisfies analyticity and unitarity – very non-trivial constraints!

The coupling need not be large to generate qualitatively need physics: Consider the behavior of high energy electrons in ordinary matter

The crucial new aspect of QCD appears to be its ground state: The QCD vacuum is a condensate of quarks and gluons

Shifman, Vainshtein and Zakharov (1979)

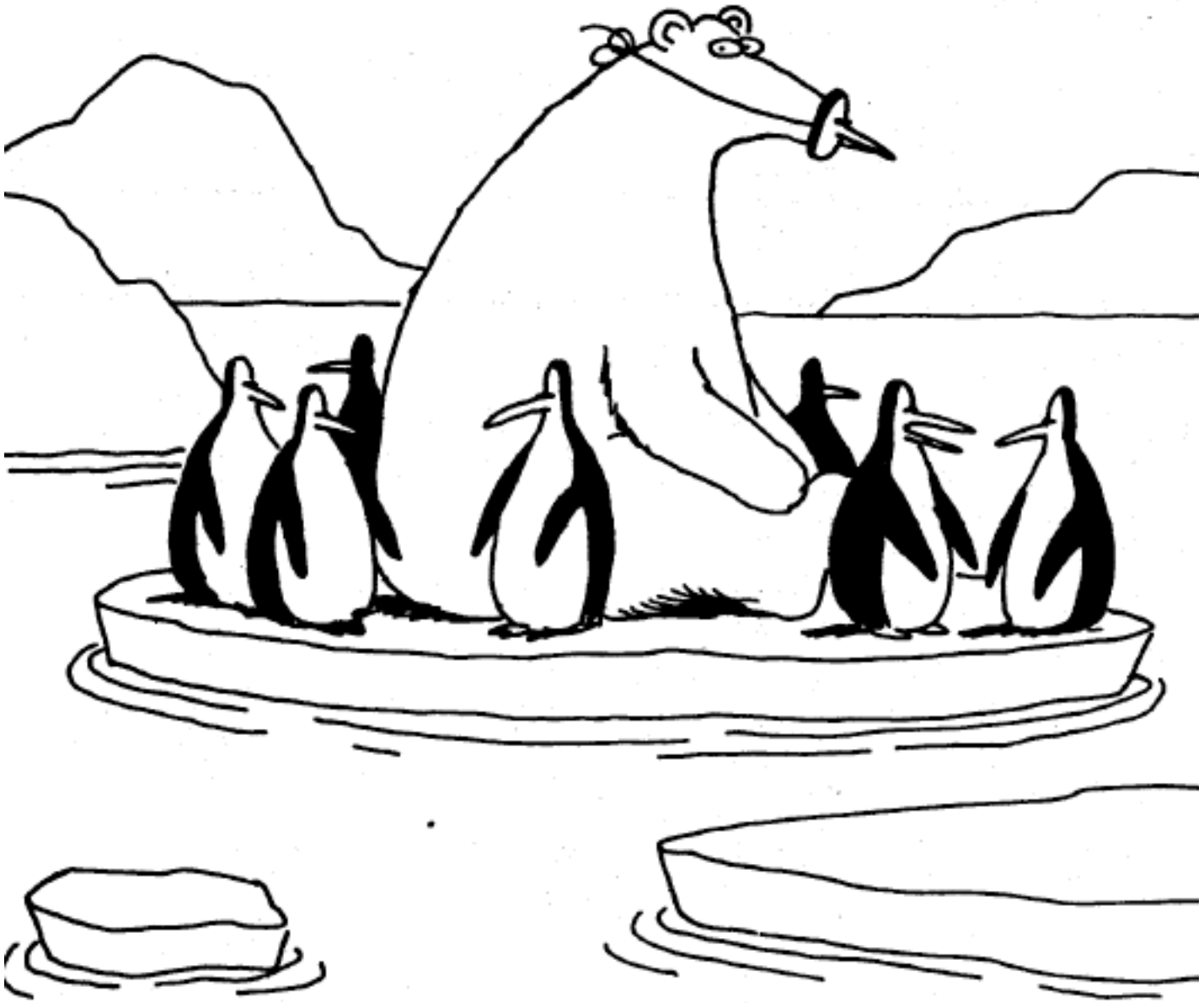
The successful "QCD sum rule" approach combines a perturbation expansion with effects of the vacuum

Colangelo and Khodjamirian, hep-ph/0010175

$\Longrightarrow$ Need to consider around which state the expansion in is made

Standard perturbation theory expands around the empty state

D. Dietrich, PH, M. Järvinen, S. Peigné, JHEP 0703 (2007) 105



"And now Edgar's gone. ... Something's going on around here."

Gary Larson, The Far Side