# QCD and Hadron Phenomenology (II) 

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## Dynamics of DIS: $e+p \rightarrow e+X$

Deep Inelastic Scattering (DIS) was the key to discovering quarks as physical, pointlike constituents of the proton (SLAC, 1969)
 Nevertheless, our understanding
of the dynamics is still developing

The QCD factorization proofs have allowed to put the analysis in terms of parton distributions on a firm basis

- but their complicated structure have to some extent stymied physical insight



## QCD Factorization in Hard Inclusive Processes

- One active parton in each hadron
- No interactions with spectators
- Hard subprocess is pointlike



## Transverse resolution

Bjorken limit:
In the target rest frame: $p=(m, \mathbf{0})$
take $p_{e} \rightarrow \infty$ and choose "deep inelastic" events where the photon has large virtuality $Q^{2}=-q^{2}$ and gets a finite fraction $y$ of the beam energy: $q^{0}=y E_{e}=v$

$$
x_{B}=\frac{Q^{2}}{2 p \cdot q}=\frac{Q^{2}}{2 m \nu} \quad \text { fixed }
$$

Then its transverse momentum

$$
q_{\perp}=\sqrt{1-y} Q
$$ implies a resolution

$r_{\perp} \sim 1 / q_{\perp} \sim 1 / Q$ in the transverse direction.

The probability to hit a single parton is $\sigma_{\text {DIS }} \sim 1 / Q^{2} \quad$ (dimensional scaling)
Probability to hit two partons is $\sigma_{\mathrm{HT}} \sim \Lambda_{\mathrm{QCD}}^{2} / Q^{4} \quad$ (higher twist contribution)

## Longitudinal resolution

Through a small rotation $\theta \sim 1 / \mathrm{Q}$
Ex: Find $\theta$. align $\boldsymbol{q}$ along the negative $z$-axis

$$
\begin{aligned}
& q=\left(q^{0}, q^{x}, q^{y}, q^{z}\right)=\left(\nu, 0,0,-\sqrt{\nu^{2}+Q^{2}}\right) \\
& q=\left(q^{+}, q^{-}, \boldsymbol{q}_{\perp}\right) \simeq\left(\frac{-Q^{2}}{2 \nu}, 2 \nu, \mathbf{0}\right)
\end{aligned}
$$



$$
q^{ \pm}=q^{0} \pm q^{3}
$$

Then the resolution in the space coordinate r , from $\exp (\mathrm{i} r \cdot q) \leq 1$, is

$$
r \cdot q=\frac{1}{2}\left[r^{+} q^{-}+r^{-} q^{+}\right] \leq 1
$$

$r^{+} \sim 1 / q^{-} \sim 1 / \nu \rightarrow 0 \quad$ Ex: How far can a photon travel in $r^{+}=1 / v$ ?
The photon probes the proton at equal Light-Front (LF) time, $r^{+}=t+z \approx 0$
$r \sim 1 / q^{+} \sim 2 v / Q^{2}=1 / m x_{B} \quad$ "Ioffe length"
The photon resolution is finite along the light-cone '-' direction
Note: Since $t \approx-z$, the resolution in $z$ is $1 / 2 m x_{B} \quad x_{B}=0.1 \Rightarrow \Delta \mathrm{z}=1 \mathrm{fm}$

## The Handbag

The scaling (leading twist) contribution to odis arises when the same quark is hit in the amplitude and (amplitude)*.

The photon vertices may be separated by the finite resolution distance $r^{-}$

According to the optical theorem, the inclusive cross section is given by the discontinuity (imaginary part, - - - -) of the handbag (forward) amplitude:

$$
\sum_{X}\left|T\left(\gamma^{*}+p \rightarrow X\right)\right|^{2}=\operatorname{Disc} T\left(\gamma^{*}+p \rightarrow \gamma^{*}+p\right)
$$

Ex: 1. Estimate the distance $r^{-}$(in fermis) at (a) Jlab, $x_{B}=0.2$ and (b) Hera, $x_{B}=0.0001$
2. With $p_{\mathrm{q}}=x p$, determine the value of $x$ for which the struck quark is on-shell: $\left(p_{\mathrm{q}}+q\right)^{2}=0$.
3. Draw a higher twist diagram. Explain why it is small at high $Q^{2}$.

All partons in the target except the quark (or gluon which interacts with the virtual photon) are called spectators. Interactions between the spectators during the coherence time of the $\gamma^{*}$ can be neglected at leading twist in Feynman gauge.
This is seen from Light-Front (LF) time ordered diagrams. The three on-shell (imaginary) parts cancel each other in the left-hand model diagram:


Spectator interactions do not affect $\sigma_{\text {DIS }}$ in Feynman gauge


Rescattering of struck quark with spectators within coherence length does affect $\sigma_{\text {DIS }}$ in Feynman gauge

## Parton distribution with rescattering



Soft rescattering of the struck parton on the color field of the spectators gives rise to the "Wilson" line in the matrix element that gives the parton distribution

$$
\begin{aligned}
f_{q / N}\left(x_{B}, Q^{2}\right) & \left.=\frac{1}{8 \pi} \int d r^{-} e^{-i m x_{B} x^{-} / 2}\langle N(p)| \bar{q}\left(r^{-}\right) \gamma^{+} W\left[r^{-}, 0\right] q(0)|N(p)\rangle \right\rvert\, \begin{array}{c}
r^{+}=0 \\
r_{\perp} \sim 1 / Q \\
\hline
\end{array} \\
& \text { where the Wilson line } \quad W\left[r^{-}, 0\right] \equiv \mathrm{P} \exp \left[\frac{i g}{2} \int_{0}^{r^{-}} d x^{-} A^{+}\left(x^{-}\right)\right]
\end{aligned}
$$

arises from rescattering of the struck quark on the color field of target spectators

- Only instantaneous Coulomb exchange $\mathrm{A}^{+}$(specific to gauge theory)
- Wilson line ensures gauge invariance of the matrix element


## Physical relevance of rescattering

The Wilson line was long thought to have no physical effects, since it is unity in LF $\left(\mathrm{A}^{+}=0\right)$ gauge. However:

- Rescattering within the coherence length is expected to occur and to affect the DIS cross section in a non-trivial way
- The argument that spectator interactions are unimportant was shown only in Feynman gauge
- An explicit perturbative model calculation showed that the rescattering effect indeed shifts from the struck quark to spectators when going from Feynman to LF gauge

Rescattering implies that the parton distribution does not directly correspond to the LF wave function of the target in isolation:

Structure functions are not parton probabilities
In fact, shadowing and diffraction, which contribute to $f_{q / N}(x)$ at leading twist, require rescattering.

## LF time ordering: $x^{+}=x^{0}+x^{3}$

In the usual "covariant" perturbation theory we describe particle propagation using 4-momenta. Thus the propagator of a scalar particle is

$$
D(p)=\frac{i}{p^{2}-m^{2}+i \varepsilon}=\frac{i}{p^{+} p^{-}-\boldsymbol{p}_{\perp}^{2}-m^{2}+i \varepsilon}
$$

The propagator goes "on the mass shell" as $p^{2} \rightarrow m^{2}$ : On-shell particles can propagate an infinite distance and register in detectors.

We can equally well describe propagation over a definite LF time $x^{+}$, by Fourier transforming the propagator over the conjugate variable $p^{-}$:
$\mathrm{D}\left(x^{+}, p^{+}, \boldsymbol{p}_{\perp}\right)=$
$\int \frac{d p^{-}}{2 \pi} \frac{i}{p^{+} p^{-}-\boldsymbol{p}_{\perp}^{2}-m^{2}+i \varepsilon} e^{-i p^{-} x^{+} / 2}=\left.\frac{\theta\left(x^{+} p^{+}\right)}{\left|p^{+}\right|} e^{-i p^{-} x^{+} / 2}\right|_{p^{-}=\frac{p_{\perp}^{2}+m^{2}}{p^{+}}}$
Note that $p^{-}$is now defined such that $p^{2}=m^{2}$ : The particle is "always on-shell"

An arbitrary Feynman diagram can be LF time ordered by F.T. $p^{-}$into $x^{+}$, allowing us to "follow" the time development of the process

Covariant Feynman diagram: $p_{13}=p_{1}-p_{3}$ (4-momenta conserved)


$$
T=g^{2} \frac{\left(p_{1}+p_{3}\right) \cdot\left(p_{2}+p_{4}\right)}{\left(p_{1}-p_{3}\right)^{2}+i \varepsilon}
$$

LF time ordered diagrams: $x^{+}$increases from left to right: $p^{-}$not conserved!


Ex: Show the equivalence of the covariant and LF time-ordered expressions for T .
Note that the external momenta $p_{1}, \ldots, p_{4}$ are on-shell.

In general, a single covariant diagram breaks into a sum of several LF-ordered ones, where each line with $p^{+}>0$ moves forward in $x^{+}$. The sum of diagrams gives equivalent results, but each LF-ordered diagram may contain spurious singularities, which cancel in their sum.

An advantage of LF-ordering is that it allows us to consider the particle content at a given instant of LF time. For example, a scalar may fluctuate into a pair:


$$
T=\frac{g}{p^{-}-p_{1}^{-}-p_{2}^{-}}
$$

The lowest order in $g$ expression for a two-scalar Fock state in a scalar particle

Note: None of the particles need to be on-shell, and LF energy is generally not conserved: $p^{-} \neq p_{1}^{-}+p_{2}^{-}$

The LF energy difference is, by the uncertainty principle, inversely related to the life-time of the Fock state
$p^{-}-p_{1}^{-}-p_{2}^{-} \sim 1 / \tau^{+}$


## Fock states of the proton

LF time ordering allows us to define a snapshot of the proton in terms of its Fock states luud>, ...., each constituent is taken at the same $x^{+}$

$$
\begin{aligned}
|N\rangle & =\int\left[\prod_{i} \frac{d x_{i} d^{2} \boldsymbol{k}_{\perp i}}{16 \pi^{3}}\right]\left[\psi_{u u d}\left(x_{i}, \boldsymbol{k}_{\perp i}, \lambda_{i}\right)|u u d\rangle\right. \\
& \left.+\psi_{u u d g}(\ldots)|u u d g\rangle+\ldots+\psi \ldots(\ldots)|u u d q \bar{q}\rangle+\ldots\right]
\end{aligned}
$$

The LF Hamiltonian (given by $L_{Q C D}$ ) species the $x^{+}$- development of each Fock state, it may propagate or turn into another Fock state through the creation and annihilation of quarks and gluons.

The weight of each Fock state is given by its Fock amplitude $\psi$, which describes the momentum, spin, etc. distributions of the partons.

A full description of the proton implies giving all the Fock amplitudes $\psi$.

## Light-Front Wavefunctions

$$
\begin{aligned}
& \text { Fixed } \tau=t+z / c \quad \text { F.T. }<0\left|\psi\left(y_{1}\right) \psi\left(y_{2}\right) \psi\left(y_{3}\right)\right| p>\left.\right|_{\tau_{i}=0} \\
& x_{i} P^{+}, x_{i} \vec{P}_{\perp}+\vec{k}_{\perp i} \\
& \sum_{i}^{n} x_{i}=1 \\
& \sum_{i}^{n} \vec{k}_{\perp i}=\overrightarrow{0}_{\perp} \\
& \xrightarrow{ } \\
& \Psi_{n}\left(x_{i}, \vec{k}_{\perp i}, \lambda_{i}\right) \\
& P^{+}=P^{0}+P^{z}
\end{aligned}
$$

Invariant under boosts! Independent of $P^{\mu}$

Jyväskylä, Finland
March 24, 2007

Novel QCD Phenomena

Stan Brodsky, SLAC

## Light-Front wave functions

LF wave functions allow to describe hadrons in arbitrary motion using the same wave function.

- They are closely related to hard scattering processes (e.g., DIS)
$f_{q / N}\left(x_{B}, Q^{2}\right)=\left.\frac{1}{8 \pi} \int d r^{-} e^{-i m x_{B} x^{-} / 2}\langle N(p)| \bar{q}\left(r^{-}\right) \gamma^{+} W\left[r^{-}, 0\right] q(0)|N(p)\rangle\right|_{\substack{r+=0 \\ r_{\perp} \sim \sim / Q}} ^{\substack{ \\\hline}}$
- For non-relativistic motion they coincide with equal-time wave functions The constituents do not move in the time a light-ray connects them
- Are usually considered in $\mathrm{A}^{+}=0(\mathrm{LF})$ gauge, to eliminate unphysical dof's Wave functions are not physically measureable: They are gauge-dependent
- Are often referred to as "infinite momentum frame" wf's, since they coincide with equal-time ( $x^{0}=0$ ) wf's in the $p^{+} \rightarrow \infty$ frame


## Vagaries of ordering in $x^{+}$

In $x^{+}$-ordered diagrams $p^{+}$and $\boldsymbol{p}_{\perp}$ are conserved, but $p^{-} \equiv\left(\boldsymbol{p}_{\perp}{ }^{2}+m^{2}\right) / p^{+}$is not

The conservation of $p^{+}$and the requirement that only $p^{+}>0$ lines move forward in $x^{+}$ implies that particles are not created from nothing: $k_{1}^{+}+k_{2}^{+}+k_{3}^{+}=0$ and $k_{i}^{+}>0$ are not consistent

Consequently: $\quad H_{L F}|0\rangle=0$


The trivial (empty) vacuum is an exact eigenstate of the full Hamiltonian!

This is too good to be true: Where did the QCD condensates go?
Apparently into zero-modes, having $k_{i}^{+}=0$
An $x^{+}=0$ surface allows causal connections using signals with the speed of light: E.g., photons with $k^{+}=0$ but $k^{-} \neq 0$

Quantization on an LF surface is not quite OK:

$$
\left[\phi\left(x^{+}, \boldsymbol{x}\right), \pi\left(x^{+}, \boldsymbol{y}\right)\right]=i \delta^{3}(\boldsymbol{x}-\boldsymbol{y})
$$

The "physical" LF gauge $\mathrm{A}^{+}=0$ has no ghosts, but a gluon propagator which is singular at $\mathrm{k}^{+}=0$ :

$$
d_{L F}^{\mu \nu}(k)=\frac{i}{k^{2}+i \epsilon}\left[-g^{\mu \nu}+\frac{n^{\mu} k^{\nu}+k^{\mu} n^{\nu}}{k^{+}}\right], \quad n \cdot A=A^{+}=0
$$

The $k^{+}=0$ pole makes $x^{+}$- ordering delicate: $\theta\left(k^{+} x^{+}\right) / k^{+}$

This is especially true for DIS, where in the Bjorken limit the rescattering propagators contribute for $k^{+} \sim 1 / v \rightarrow 0$

Using $x^{+}$-time ordering in DIS with $\mathrm{A}^{+}=0$
 gauge introduces many spurious singularities

Also the renormalization procedure is complicated by $x^{+}$-time ordering

## The two views of DIS

The LF time $\left(x^{+}\right)$development in DIS depends on the direction of the photon momentum $q$, due to the $\theta\left(x^{+} p^{+}\right)$in the quark propagators:

$$
q^{\mathrm{z}}<0: q^{+}=-m x_{B}
$$



Virtual photon scatters on a quark in the target with $k^{+}=m x_{B}$
$\sigma_{\text {DIS }}$ given $\approx$ by probability for the quark in the target wf.
"Infinite momentum frame"


Virtual photon splits into a quarkantiquark pair. The antiquark has a finite $k^{+}$in the target rest frame $\sigma_{\text {DIS }}$ given by the scattering of the asymmetric $q \bar{q}$ pair in the target
"Target rest frame"

The two views are related by a rotation of $180^{\circ}$, but rotations are not kinematic (explicit) symmetries in the LF matrix element.

$$
\gamma^{*} \sim \sim_{\sim}^{q^{+} \approx 2 v} \underbrace{k_{1}^{+} \approx 2 v}_{\substack{\text { \& } \\ k_{\perp} \approx \Lambda_{\mathrm{QCD}} \\ k_{2}^{+} \approx \Lambda_{\mathrm{QCD}}}}
$$



The virtual photon fluctuates into a similarly short-lived quark pair: $\Delta x^{+} \sim 1 / \Delta k^{-}$

$$
k_{1}^{-}+k_{2}^{-} \geq q^{-} \approx m x_{B} \quad \text { with } \quad k^{-}=\left(\boldsymbol{k}_{\perp}^{2}+\mathrm{m}^{2}\right) / k^{+}
$$

The asymmetric pair has large transverse size:

$$
r_{\perp} \sim 1 / k_{\perp} \approx 1 / \Lambda_{\mathrm{QCD}}
$$

It has a large cross section, but the splitting probability is $\sim 1 / \mathrm{Q}^{2}$

$$
\sigma\left(\gamma^{*} q \rightarrow q\right) \sim 1 / Q^{2}
$$

The symmetric pair has small transverse size: $r_{\perp} \sim 1 / k_{\perp} \approx 1 / \mathrm{Q}$
Its cross section is $\sim 1 / \mathrm{Q}^{2}$ (due to color transparency!) but the splitting probability is $O(1)$

$$
\sigma\left(\gamma^{*} \mathrm{~g} \rightarrow \mathrm{qq}\right) \sim \alpha_{\mathrm{s}} / \mathrm{Q}^{2}
$$

$$
\begin{aligned}
& \sigma(\text { rescat }) \sim \mathrm{P}_{\text {split }} \cdot \sigma^{2}(\mathrm{qq}) \propto 1 / \mathrm{Q}^{2} \\
& \gamma^{*} \sim \sim \sim q_{k_{2}^{+} \approx v \xi_{\xi^{+}} \xi_{\xi}^{\xi}}^{k_{1}^{+} \approx v} k_{\perp} \approx Q \\
& \sigma(\text { rescat }) \sim P_{\text {split }} \cdot \sigma^{2}(q q) \propto 1 / Q^{4}
\end{aligned}
$$

Rescattering probability from the large (asymmetric) pair is independent of $\mathrm{Q}^{2}$, whereas that from the compact pair is suppressed by $1 / \mathrm{Q}^{2}$

Recall: Longitudinal coherence length $L_{I} \sim 1 / m x_{B}$ remains finite in the scaling limit, hence does not depend on $Q^{2}$ at fixed $x_{B}$.

Note: Soft (re-)scattering from asymmetric state influences hard process!

## QCD factorization in hard collisions

- One active parton in each hadron
- No interactions with spectators
- Hard subprocess is pointlike


It ain't quite so!
Spectators influence the hard subprocess, even at "infinite hardness".

In certain situations the spectator effects are crucial.

## Universality of fragmentation in nuclear matter

Hadronization is independent of target size in DIS on nuclei when the struck quark has high energy $v$


Ex: Estimate the formation length of a pion with $\mathrm{z}=0.5$ at $\mathrm{v}=50 \mathrm{GeV}$.

## Shadowing in $v \mathrm{~A} \rightarrow v+\mathrm{X}$

How can the neutrino, which penetrates lightyears of matter,"know" that there are nucleons in front of the one it hits?



Shadowing requires at least two neutrino interactions in the nucleus!

Diffractive DIS: $\mathrm{e}+\mathrm{p} \rightarrow \mathrm{e}+\mathrm{X}+\mathrm{p}$

$$
W=200-245 \mathrm{GeV}
$$

Intuitive picture of DIS: A color string extending from the struck quark to the target fills the rapidity interval with hadrons:


DDIS: No hadrons emerge in an extended rapidity region.


DDIS/DIS $\approx 10 \%$, independent of $\mathrm{Q}^{2}$
DDIS requires color singlet exchange, i.e., at least two partons from target


## Shadowing in $v \mathrm{~A} \rightarrow v+\mathrm{X}$

The neutrino interacts via its
aponent (rare) $q \bar{q}$ Fock component

Hard subprocess

$$
\begin{gathered}
\text { po } \\
- \\
\hline
\end{gathered}
$$

## Shadowing dynamics



Negative interference between single and double scattering due to factor $i^{2}=-1$ from elastic scattering and on-shell intermediate state

How about the antishadowing and EMC effects?

The dip-bump structure in the A-dependence may be understood as simply arising from the Fourier-transform in the parton distribution:


$$
f_{q / N}\left(x_{B}, Q^{2}\right)=\frac{1}{8 \pi} \int d x^{-} e^{-i x_{B} p^{+} x^{-} / 2}\langle N(p)| \bar{q}\left(x^{-}\right) \gamma^{+} W\left[x^{-}, 0\right] q(0)|N(p)\rangle_{x^{+}=0}
$$

The matrix element in coordinate space, $\mathrm{x}^{-}=\mathrm{x}^{0}-\mathrm{x}^{3}$, has only shadowing

$$
\begin{aligned}
& \langle N| \bar{q}\left(x^{-}\right) \gamma^{+} W\left[x^{-}, 0\right] q(0)|N\rangle-\left(x^{-} \rightarrow-x^{-}\right) \\
& \quad=4 i m \int_{0}^{1} d x_{B}\left[f_{q / N}\left(x_{B}\right)+f_{\bar{q} / N}\left(x_{B}\right)\right] \sin \left(\frac{1}{2} m x_{B} x^{-}\right)
\end{aligned}
$$

## Nuclear Dependence of Parton Distributions

The parton distribution in momentum space $\mathrm{f}_{\mathrm{q} / \mathrm{A}}\left(\mathrm{x}_{\mathrm{B}}\right)$ has a complicated A-dependence:


Within the experimental resolution, data shows only shadowing when plotted in coordinate space, from
$z=t \approx 2.5 \mathrm{fm}$

When transformed to coordinate ( $\mathrm{x}^{-}$) space the A-dependence is much simpler:


Error estimate in coordinate space


Reconstruction of $\mathrm{R}_{\mathrm{C}}\left(\mathrm{x}_{\mathrm{B}}\right)$, assuming no nuclear effect in coordinate space for $x^{-}<\mathrm{w}$


## Compact $\mathrm{q} \overline{\mathrm{q}}$ states: Coherent $\mathrm{J} / \psi$ production

In $\gamma+\mathrm{A} \rightarrow \mathrm{J} / \psi+\mathrm{A}$ the charm quark pair is produced as a compact color singlet configuration, $r_{\perp} \sim 1 / \mathrm{m}_{\mathrm{c}}$

The compact quark pair can scatter
 coherently on all A nucleons, giving an enhanced production on nuclei:

$$
\sigma_{\mathrm{coh}} \sim \mathrm{~A}^{\alpha}, \quad \alpha=1.40 \pm 0.04
$$

Sokoloff et al, PRL 57 (1986) 3003


