Nucleon and Nuclear Structure Functions (2)

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Outline

- Experimental information on nuclear effects in structure functions and parton distributions.
- Interpretation of data. Sketch of major mechanisms of nuclear effects in DIS.
- Phenomenology of nuclear DIS and development of a quantitative model of nuclear structure functions.

Structure Functions

Inclusive lepton scattering $e^-(k) + A(p) \to e^-(k') +$ undetected states is described in leading order in one-photon-exchange approximation:

$$\begin{aligned}
\mathrm{d}\sigma =& \frac{\alpha^2}{Q^4} L^{\mu\nu} W_{\mu\nu} \frac{\mathrm{d}^3 k'}{(p\cdot k) E'} \\
L_{\mu\nu} =& 2\left(k_\mu k'_\nu + k_\nu k'_\mu\right) + q^2 g_{\mu\nu} \\
W_{\mu\nu}(p,q) =& \frac{1}{4\pi} \int \mathrm{d}^4 z \exp(iq\cdot z) \langle p| \left[J_\mu(z), J_\nu(0)\right] |p\rangle \\
=& F_1 \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)
\end{aligned}$$

$$\frac{\mathrm{d}^2\sigma^{\text{unpol}}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\,\frac{F_2}{x}\left[\left(1 - y - \frac{Mxy}{2E}\right) + \frac{y^2\gamma^2}{2(1+R)}\right],$$

$$x = Q^2/2p\cdot q,\ y = p\cdot q/p\cdot k,$$

$$R = F_L/F_T,\ \gamma^2 = 1 + 4M^2x^2/Q^2.$$

Experimental evidence of nuclear effects in PDFs

- Data on nuclear effects in DIS are available in the form of the ratio $\mathcal{R}_2(A/B) = F_2^A/F_2^B$.

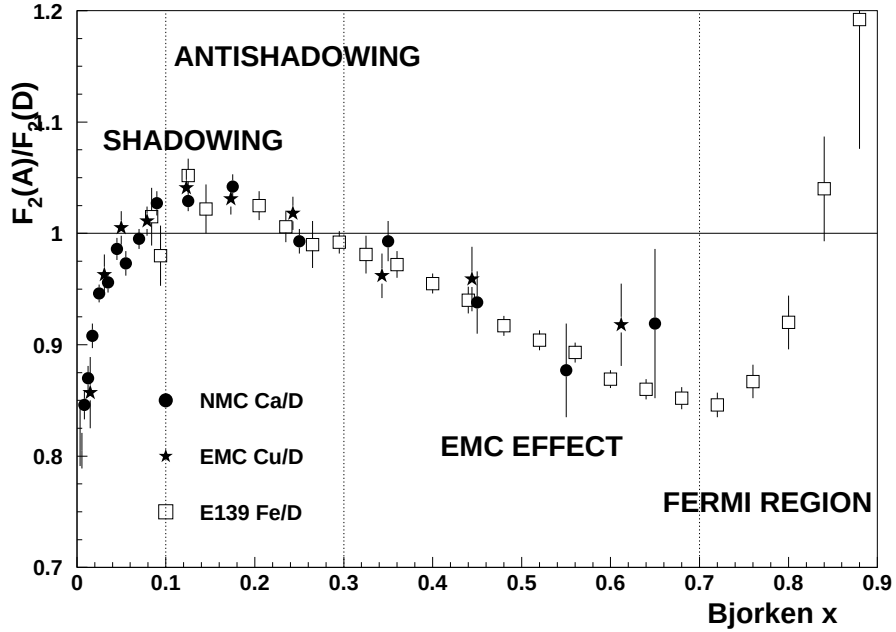
- Targets:
  Variaty of nuclear targets from the deuteron ^{2}D to lead ^{208}Pb

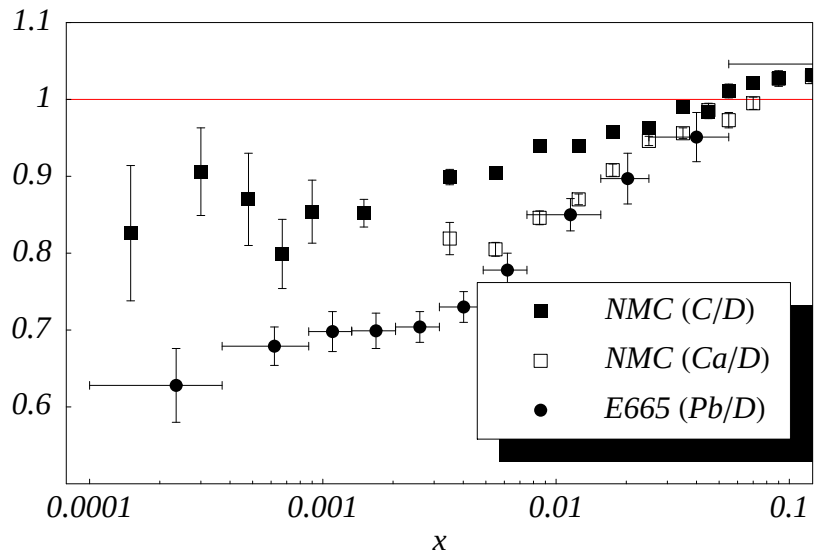
- Experiments:
  – Muon beam at CERN (EMC, BCDMS, NMC) and FNAL (E665).
  – Electron beam at SLAC (E139, E140), HERA (HERMES) and recently at JLab.

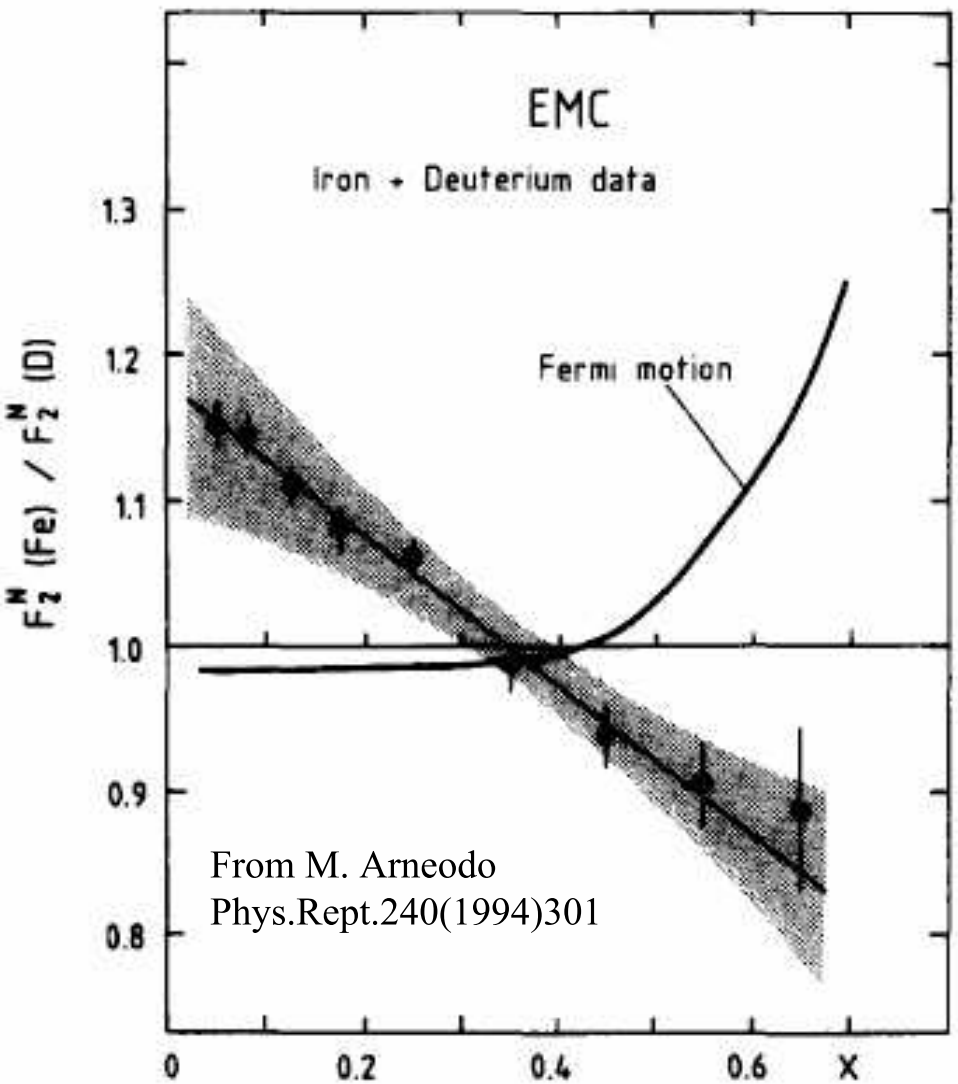
- Kinematics and statistics:
  Data covers the region $10^{-4} < x < 0.9$ and $0 < Q^2 < 150\,\mathrm{GeV}^2$. There available about 600 data points with $Q^2 > 1\ \mathrm{GeV}^2$.

Data on $\mathcal{R}$ show a weak Q^2 dependence that suggests scaling origin of (at least a part of) nuclear effects. Characteristic nuclear effects were observed for different kinematical regions of x.

- Nuclear shadowing at small values of Bjorken x ($x < 0.05$).
- Antishadowing at $0.1 < x < 0.25$.
- A well with a minimum at $x \sim 0.6 \div 0.75$ (EMC effect).
- Enhancement at large $x > 0.75 \div 0.8$ (Fermi motion region).







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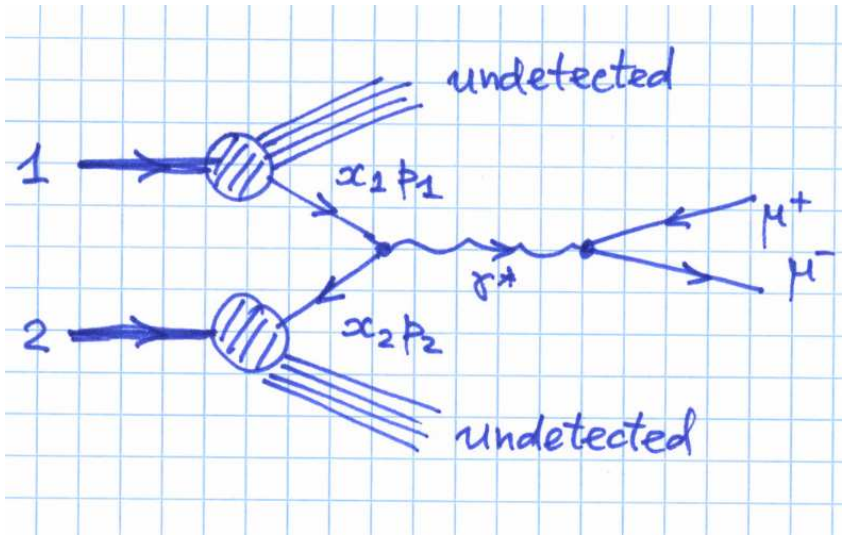
Drell-Yan reaction

Drell-Yan production of lepton pairs $B + T \to \mu^+\mu^- +$ anything is also determined by parton distributions



$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_B\mathrm{d}x_T} = \frac{4\pi\alpha^2}{9Q^2} K \sum_a e_a^2 \left[q_a^B(x_B)\bar{q}_a^T(x_T) + \bar{q}_a^B(x_B) q_a^T(x_T) \right]$$

$$x_T x_B = Q^2/s, \quad x_B - x_T = 2q_L/\sqrt{s} = x_F$$

Q^2 is invariant mass squared of the lepton pair, q_L the longitudinal momentum of the lepton pair, s the center-of-mass energy squared.

E772 data on Drell-Yan ratios

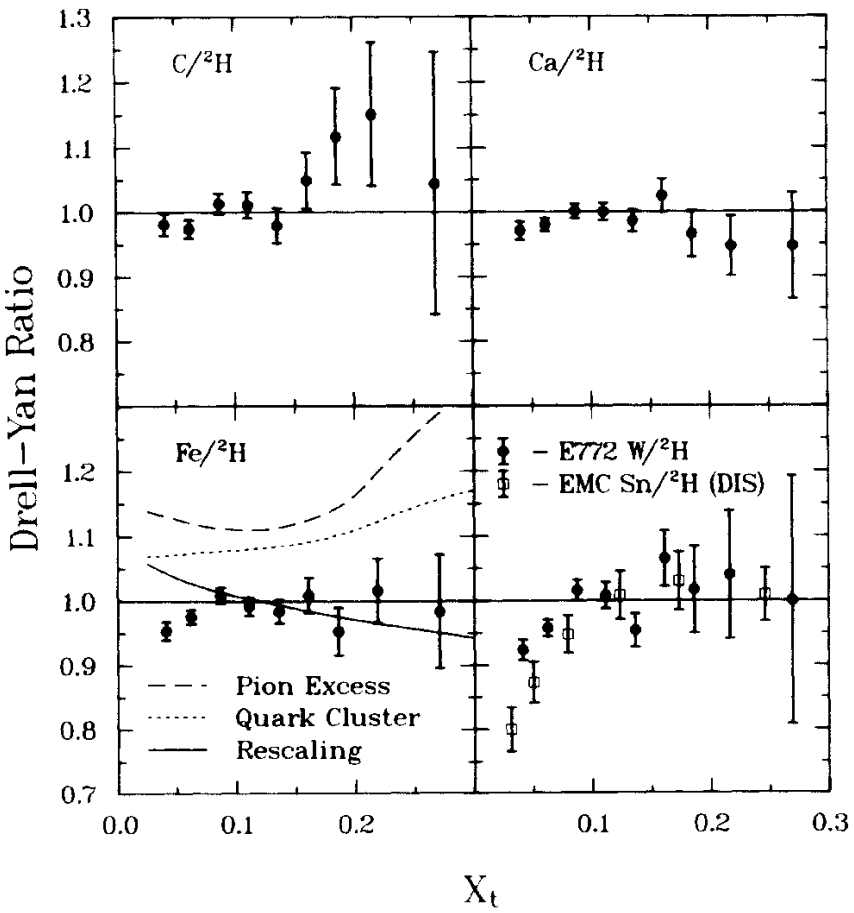


FIG. 3. Ratios of the Drell-Yan dimuon yield per nucleon, $Y_A/Y_{^2H}$, for positive x_F. The curves shown for Fe/^{2}H are predictions of various models of the EMC effect. Also shown are the DIS data for Sn/^{2}H from the EMC (Ref. 4).

In E772 experiment $s = 1600\ \mathrm{GeV}^2$. At $x_F > 0.2$ process is dominated by $q^B\bar{q}^T$ annihillation. The ratio of DY yields for different targets is proportional to the corresponding ratio of antiquark distributions.

Other data

Events with $x > 1$ (kinematically not allowed for free nucleon) were observed by BCDMS ($^{12}\mathrm{C}$) and recently in neutrino DIS by CCFR ($^{56}\mathrm{Fe}$).

Direct measurements of nuclear effects in neutrino cross sections were done by BEBC ($^{20}\mathrm{Ne}/\mathrm{D}$) and CDHSW ($^{56}\mathrm{Fe}/p$) (large stat. and syst. uncertainties).

High-energy cross section data are available from recent measurements by NOMAD ($^{12}\mathrm{C}$), CCFR–NuTeV ($^{56}\mathrm{Fe}$), CHORUS ($^{208}\mathrm{Pb}$). These data can be a good source of additional information on both the nucleon and the nuclear PDFs.

The measurements of nuclear effects in neutrino scattering are planned at MINERνA experiment (FNAL) (C, Fe, Pb targets).

Why Nuclear Effects Survive in DIS ?

The analysis of characteristic space-time scales involved into DIS helps to approach nuclear physics of the process. Typical regions in configuration space which contribute to DIS hadronic tensor:

- $t^2 - z^2 \sim Q^{-2}$ DIS proceeds near the light cone.
- $t \sim z \sim L = (Mx)^{-1}$ DIS can proceed at the distances not small in hadronic scale (in the target rest frame) $\Rightarrow$ the reason for nuclear effects to survive even at high Q^2.

L has to be compared with average distance between bound nucleons $d = (3/4\pi\rho)^{1/3} \sim 1.2\,\mathrm{Fm}$ (ρ is the nucleon number density in central region of heavy nuclei). One should distinguish two different regions:

- $L < d \Rightarrow$ Nuclear DIS $\approx$ incoherent sum of contributions from bound nucleons.
- $L \gg d \Rightarrow$ Coherent effects of interactions with a few nucleons are important.

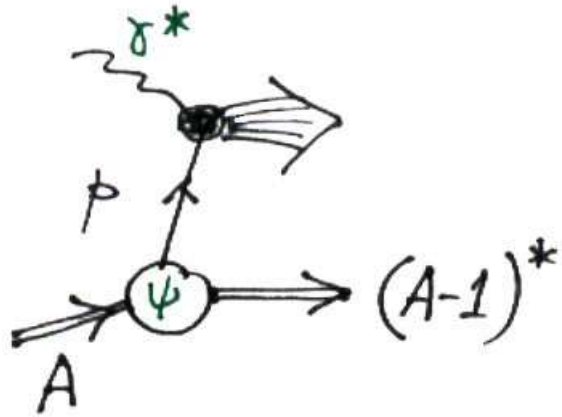
Nuclear DIS in impulse approximation (IA)

Fermi Motion and Nuclear Binding corrections

In incoherent scattering approximation the nuclear SFs can be written as a convolution of (bound) nucleon SFs and nuclear spectral function:

$$F_2^A(x,Q^2) = \int \mathrm{d}^4p\, \mathcal{P}_A(p) \left(1+\frac{p_z}{M}\right) F_2^N(x',Q^2,p^2),$$

$$x = \frac{Q^2}{2p\cdot q}, \quad x' = \frac{Q^2}{2p\cdot q} = \frac{x}{1+(\varepsilon+p_z)/M}$$



Similar equations also hold in IA for $F_T,\ F_L$. Fermi motion and binding effect is driven by nuclear spectral function ($\boldsymbol{p} = (M+\varepsilon,\ \boldsymbol{p}$)

$$\mathcal{P}_A(p) = \sum_n |\psi_n(\boldsymbol{p})|^2 \delta(\varepsilon + E_n(A-1) - E_0(A)).$$

Spectral function describes probability to find a bound nucleon with momentum $\boldsymbol{p}$ and energy $p_0 = M+\varepsilon$.

Nuclear spectral function

Mean-field picture

Nucleus in a first approximation can be viewed as a system of protons and neutrons bound to a self-consistent potential (mean field model, MF). Nucleons occupy the MF energy levels according to Fermi statistics and thus distributed over momentum (Fermi motion) and energy states. MF nuclear spectral function

$$\mathcal{P}_{\mathrm{MF}}(\varepsilon, \boldsymbol{p}) = \sum_{\lambda<\lambda_F} n_\lambda |\phi_\lambda(\boldsymbol{p})|^2 \delta(\varepsilon - \varepsilon_\lambda)$$

where sum is taken over occupied levels with ϕ_λ the wave function and n_λ the occupation number of the level λ (λ_F the Fermi level). MF model is a reasonable approximation if nucleon separation energy and momenta are not high (in nuclear ground state scale, $|\varepsilon| < 50\,\mathrm{MeV}$ and $k < 300\,\mathrm{MeV/c}$).

Fermi gas model as limit of MF spectrum (can be used for large nuclei)

$$\mathcal{P}_{\mathrm{MF}}(\varepsilon, \boldsymbol{p}) = \theta(p_F - |\boldsymbol{p}|)\delta(\varepsilon - V - \boldsymbol{p}^2/2M)$$

.

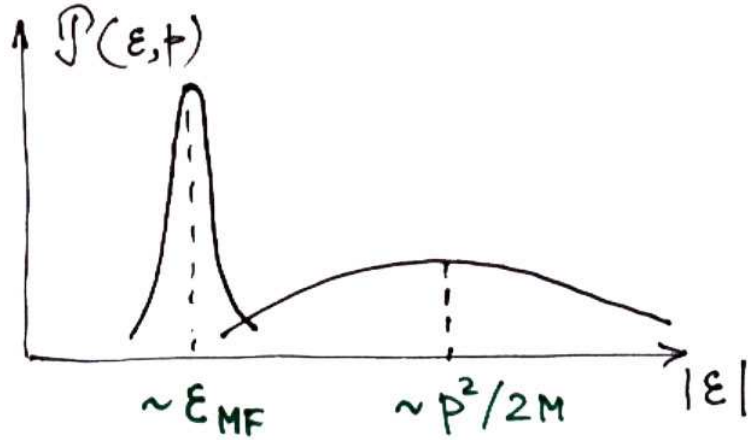
Nuclear spectral function

Nucleon short-range correlation effects

As the separation energy $|\varepsilon|$ becomes higher, the MF approximation becomes less accurate. High-energy and high-momentum component of nuclear spectrum can not be described in the MF model. These effects are driven by short-range NN correlations in nuclear ground state.

$$\mathcal{P}_{\rm cor}(\varepsilon, \boldsymbol{p}) \approx n_{\rm rel}(\boldsymbol{p}) \left\langle \delta\left(\varepsilon + \frac{(\boldsymbol{p}+\boldsymbol{p}_{A-2})^2}{2M} + E_{A-2} - E_A\right)\right\rangle_{A-2}$$

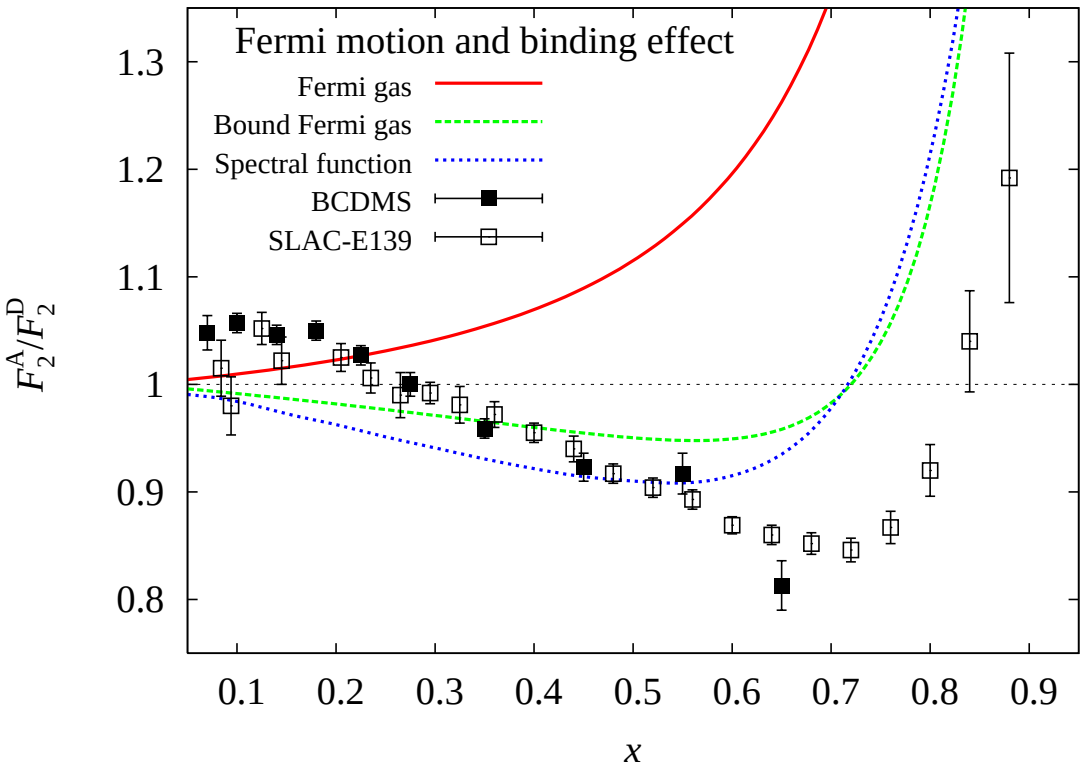
The full spectral function can be approximated by a sum of the MF and correlation parts $\mathcal{P} = \mathcal{P}_{\rm MF} + \mathcal{P}_{\rm cor}$.



Note that characteristic nuclear momentum and energy are small comapared to the nucleon mass M. Expand $F_2^N(x')$ in series in a small parameter $(\varepsilon + p_z)/M$ and integrate term by term

$$\frac{1}{A}F_2^A(x) \approx F_2^N(x) - \frac{\langle\varepsilon\rangle}{M}\, xF_2^{N'}(x) + \frac{\langle \boldsymbol{p}^2\rangle}{6M^2}\, x^2 F_2^{N''}(x)$$

Consider $\mathcal{R}(x) = \frac{1}{A}F_2^A(x)/F_2^N(x)$. The slope at $x < 0.5$ is controlled by $xF_2^{N'}(x)$ while the rise at large $x > 0.7$ is due to $x^2F_2^{N''}(x)$.



- Fermi gas momentum distribution qualitatively explains the trend of data at $x > 0.7$ but fails to quantitatively explain data.
- Binding correction is important and brings the calculation closer to data in the dip region.
- However, even more sophisticated calculations with realistic nuclear spectral function fail to quantitatively explain the slope of the ratio F_2^A/F_2^D and the dip position.

Effects beyond IA

Nucleon off-shell effect

Bound nucleons are off-mass-shell $p^2 = (M+\varepsilon)^2 - \boldsymbol{p}^2 < M^2$. In off-shell region nucleon structure functions depend on additional variable $F_2(x, Q^2) \;\Rightarrow\; F_2(x, Q^2, p^2)$.

A few model calculations exist for off-shell effect in DIS structure functions (Melnitchouk–Schreiber–Thomas, Gross–Liuti, Kulagin–Piller–Weise).

We follow phenomenological approach and extract off-shell dependence from data on the ratio of nuclear structure functions (S.K. & R.Petti, NPA765(2006)126). The virtuality parameter $v = (k^2 - M^2)/M^2$ (average virtuality $\langle v \rangle \sim -0.15$ for iron) is small. Expand in v:

$$F_2^N(x, p^2) = F_2^N(x)\big(1 + v\,\delta f(x)\big)$$

Off-shell correction $\Longleftrightarrow$ modification of nucleon parton distributions in nuclear environment.

Missing nucleon light-cone momentum in IA

In impulse approximation the nuclear PDFs are actually the convolutions of the nucleon light-cone momentum distribution $f_{N/A}(y)$ and the nucleon PDFs

$$q^A(x) = \int \mathrm{d}y\,\mathrm{d}z\; f_{N/A}(y) q^N(z)\,\delta(x - yz)$$

$$f_{N/A}(y) = \int \mathrm{d}^4p\; \mathcal{P}(\varepsilon, \boldsymbol{p}) \left(1 + \frac{p_z}{M}\right) \delta\left(y - 1 - \frac{\varepsilon + p_z}{M}\right)$$

Nucleon distribution:

$\Rightarrow$ Normalization: $\int \mathrm{d}y\; f_{N/A}(y) = A$ the number of bound nucleons.

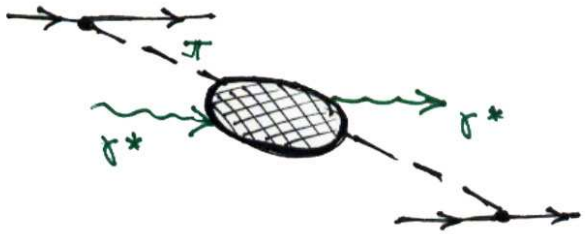
$\Rightarrow$ Average nucleon light cone momentum: $\langle y \rangle_N = 1 + \dfrac{\langle \varepsilon \rangle}{M} + \dfrac{\langle p^2 \rangle}{3M^2}$.

Because of binding $\varepsilon < 0 \;\Rightarrow\; \langle y \rangle_N \sim 0.95 \;\Rightarrow$ bound nucleons do not carry all the nuclear momentum (in a frame in which nucleus moves with a large momentum).
Who balances the missing momentum?

Nuclear pion effect

Leptons can scatter on nuclear meson field which mediate interaction between bound nucleons. This process generate a pion correction to nuclear sea quark distribution

$$\delta q^{\pi/A}(x,Q^2) = \int_x \frac{\mathrm{d}y}{y} f_{\pi/A}(y) q^{\pi}(\frac{x}{y},Q^2)$$



• Contribution from nuclear pions (mesons) is important to balance nuclear light-cone momentum $\langle y \rangle_\pi + \langle y \rangle_N = 1$.

• The nuclear pion distribution function is localized in a region of $y < p_F/M \sim 0.3$. For this reason the pion correction to nuclear (anti)quark distributions is localized at $x < 0.3$.

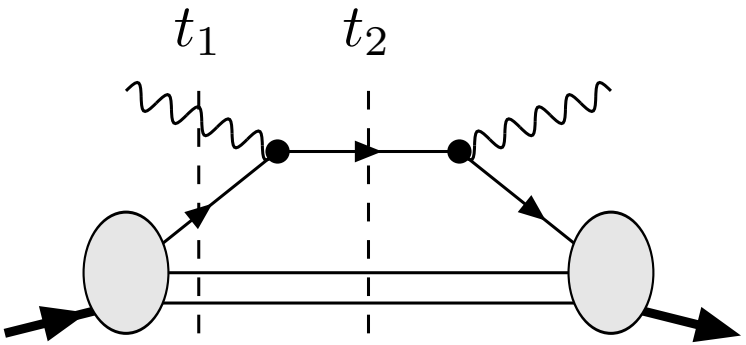
• The magnitude of the correction is driven by average number of "pions" $n_\pi = \int \mathrm{d}y\, f_{\pi/A}(y)$. By order of magnitude $n_\pi/A \sim 0.1$ for a heavy nucleus like ^{56}Fe.

• Nuclear pion correction effectively leads to enhancement of nuclear sea quark distribution and does not affect the valence quark distribution (for isoscalar nuclear target).

Coherent nuclear corrections

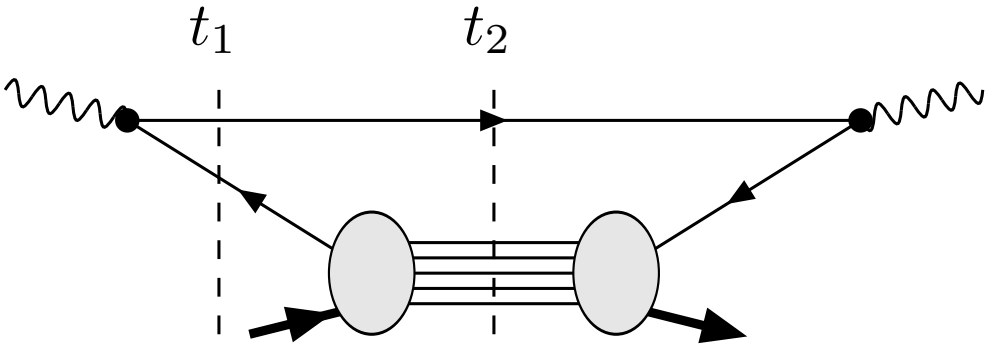
Two different mechanisms of DIS:

(I) QE scattering off bound quark. This process dominates at intermediate and large values of x and the structure functions are determined by quark wave functions.



Nuclear effects arise because of averaging with nucleon distributions in a nucleus.

(II) Conversion $\gamma^* \to q\bar{q}$. Then $q\bar{q}$ state propagates in a target. This process dominates at small x since life time of $q\bar{q}$ state grows as $1/(Mx)$. The structure functions are determined by quark scattering amplitudes.



Nuclear effects arise due to interaction of intermediate states during propagation in matter. Relative correction to a single scattering term $\delta\mathcal{R} = \delta F_T^A / F_T^N \Rightarrow \delta\sigma_T^{\text{mult.sc.}}/\sigma_T$

Phenomenology of nuclear DIS

Motivation: The development of a quantitative model providing predictions of nuclear cross sections (structure functions) and corresponding uncertainties to be used in the analyses of present and future lepton scattering data from nuclear targets.

Approach: Take into account major nuclear mechanisms such as FMB, off-shell, nuclear pion and shadowing corrections. Parameterize off-shell correction function and effective scattering amplitude responsible for shadowing and extract these quantaties from data (for more detail see S.K. & R.Petti, NPA765(2006)126).

Phenomenological parameters

Off-shell corrections to the nucleon structure functions and effective scattering amplitude of hadronic component of virtual photon off the nucleon are treated phenomenologically.

Off-shell correction function $\delta f_2(x) = C_N(x - x_1)(x - x_0)(h - x)$.

From preliminary studies we observe that h is fully correlated with x_0, i.e. $h = 1 + x_0$. The nuclear valence number normalization helps to fix $x_1 = 0.05$. C_N and x_0 are fit parameters.

Effective amplitude

$$\bar{a}_T = \bar{\sigma}_T(i + \alpha)/2$$

$$\bar{\sigma}_T = \sigma_1 + \frac{\sigma_0 - \sigma_1}{1 + Q^2/Q_0^2}$$

Parameters $\sigma_0 = 27\,\mathrm{mb}$ and $\alpha = -0.2$ were fixed in order to match the VMD model at low Q. Parameter $\sigma_1 = 0$ (preferred by preliminary fits and fixed in final fit). Q_0^2 is adjustable parameter controlling transition between low and high Q regions.

Analysis

We use the data from electron and muon DIS in the form of ratios $\mathcal{R}_2(A/B) = F_2^A/F_2^B$ for a variaty of targets. The data are available for A/D and $A/^{12}\mathsf{C}$ ratios.

We perform a fit and minimize $\chi^2 = \sum_{\mathsf{data}}(\mathcal{R}_2^{exp} - \mathcal{R}_2^{th})^2/\sigma^2(\mathcal{R}_2^{exp})$ with σ the experimental uncertainty. In the fit we use data with $Q^2 > 1\,\mathrm{GeV}^2$ (overall about 560 points). Then we validate the predictions with the data not used in the analysis (data of $Q^2 < 1\,\mathrm{GeV}^2$ and data on D/p ratio of structure functions.

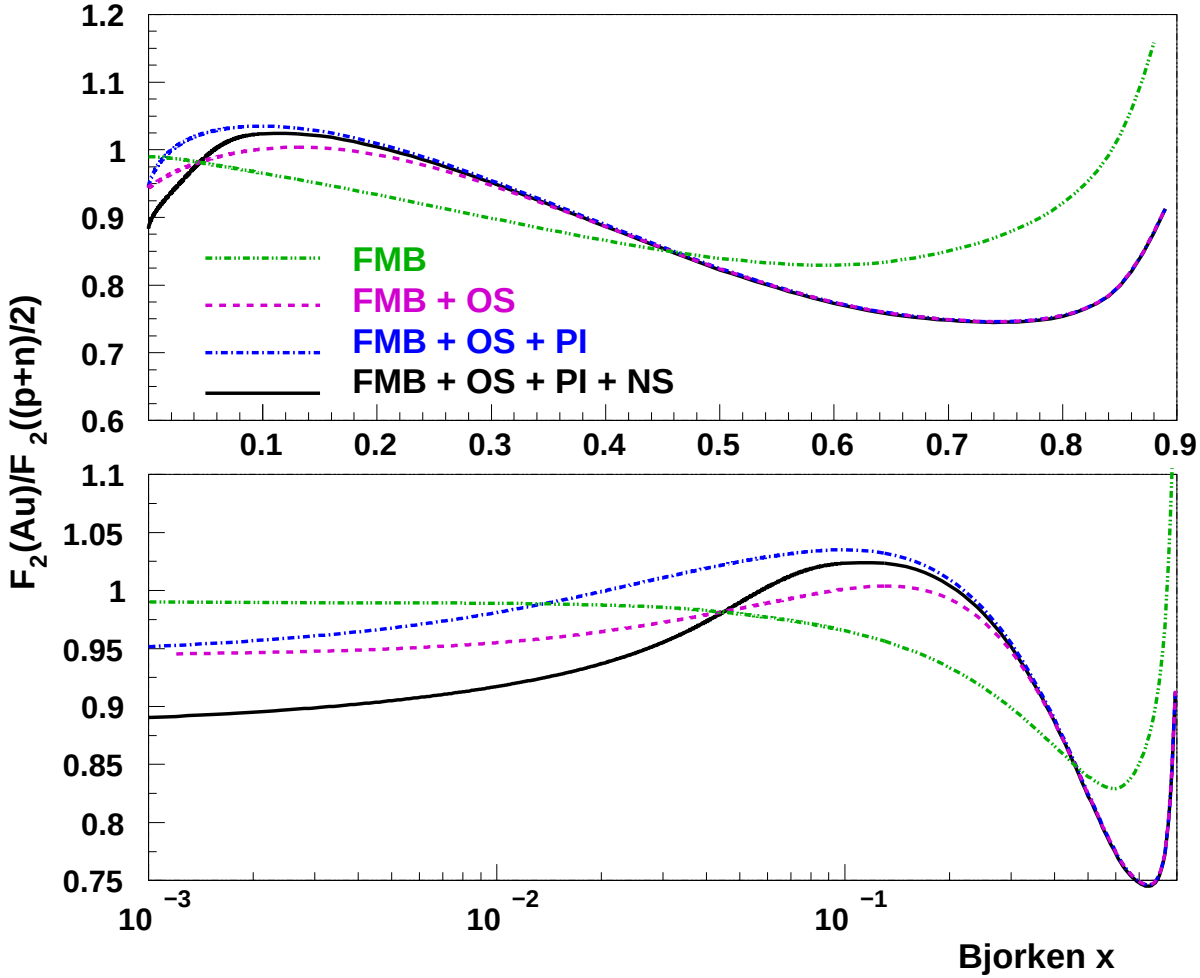
Results

- Very good agreement with data (both for x and Q^2 dependence) for entire kinematical region

$$\chi^2/\mathrm{d.o.f} = 459/556$$

- Phenomenological parameters from global fit (statistical + syst./theoretical uncertainties)

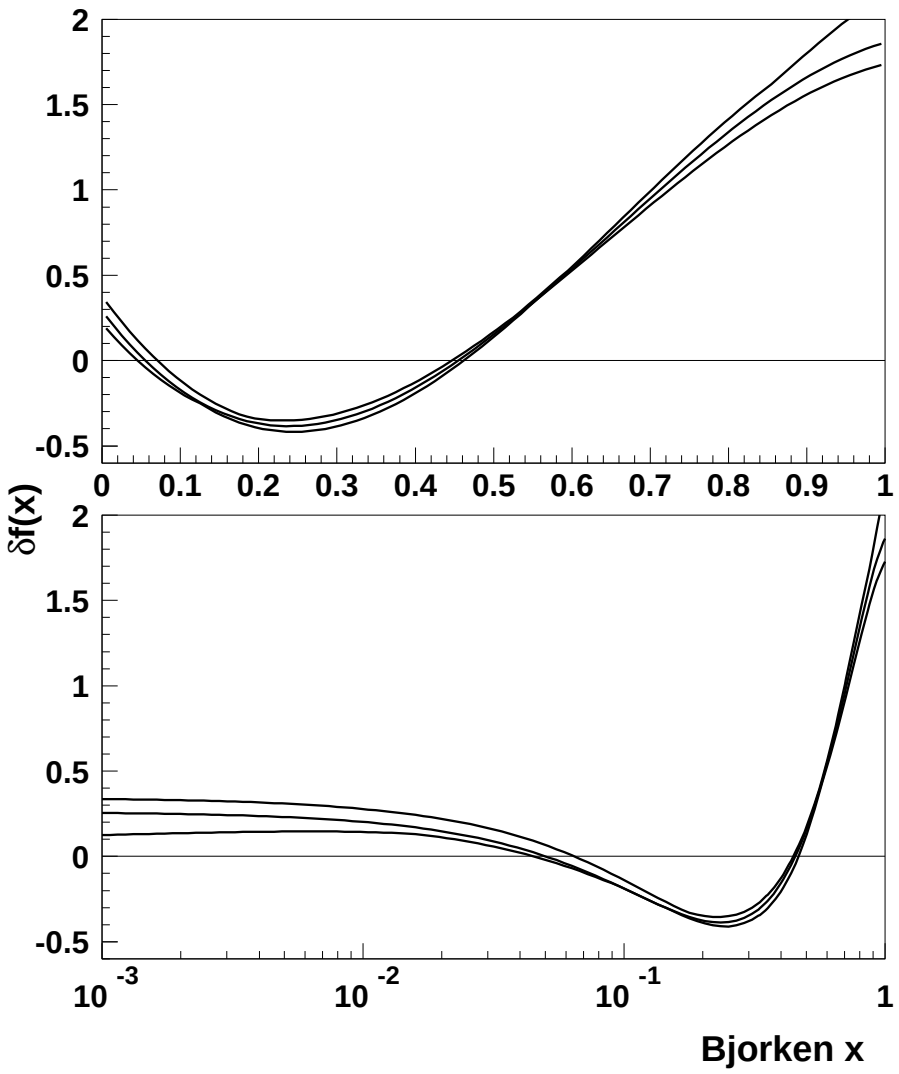
$$C_N = 8.1 \pm 0.3 \pm 0.5$$
$$x_0 = 0.448 \pm 0.005 \pm 0.007$$
$$Q_0^2 = 1.43 \pm 0.06 \pm 0.2\ \mathrm{GeV}^2$$



The EMC ratio for gold and the isoscalar nucleon calculated at $Q^2 = 10\ \mathrm{GeV}^2$. The labels on the curves mark the effects included in turn: Fermi motion and nuclear binding (FMB), off-shell correction (OS), nuclear pion excess (PI) and corrections from coherent nuclear processes (NS).

Off-shell function

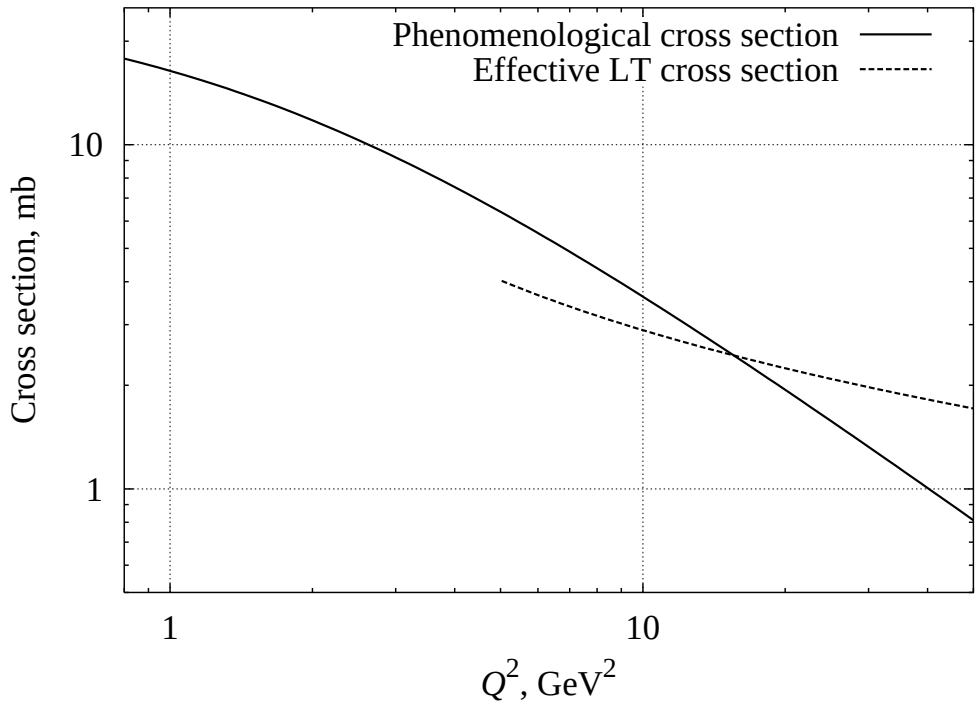
- The function $\delta f(x)$ provides a measure of modification of quark distributions in bound nucleon.
- The off-shell effect results in the enhancement of the structure function for $x_1 < x < x_0$ and depletion for $x < x_1$ and $x > x_0$.
- The phenomenological function $\delta f(x)$ suggests the increase in the radius of the bound nucleon valence region (in Fe by $\sim 10\%$).



Effective cross section

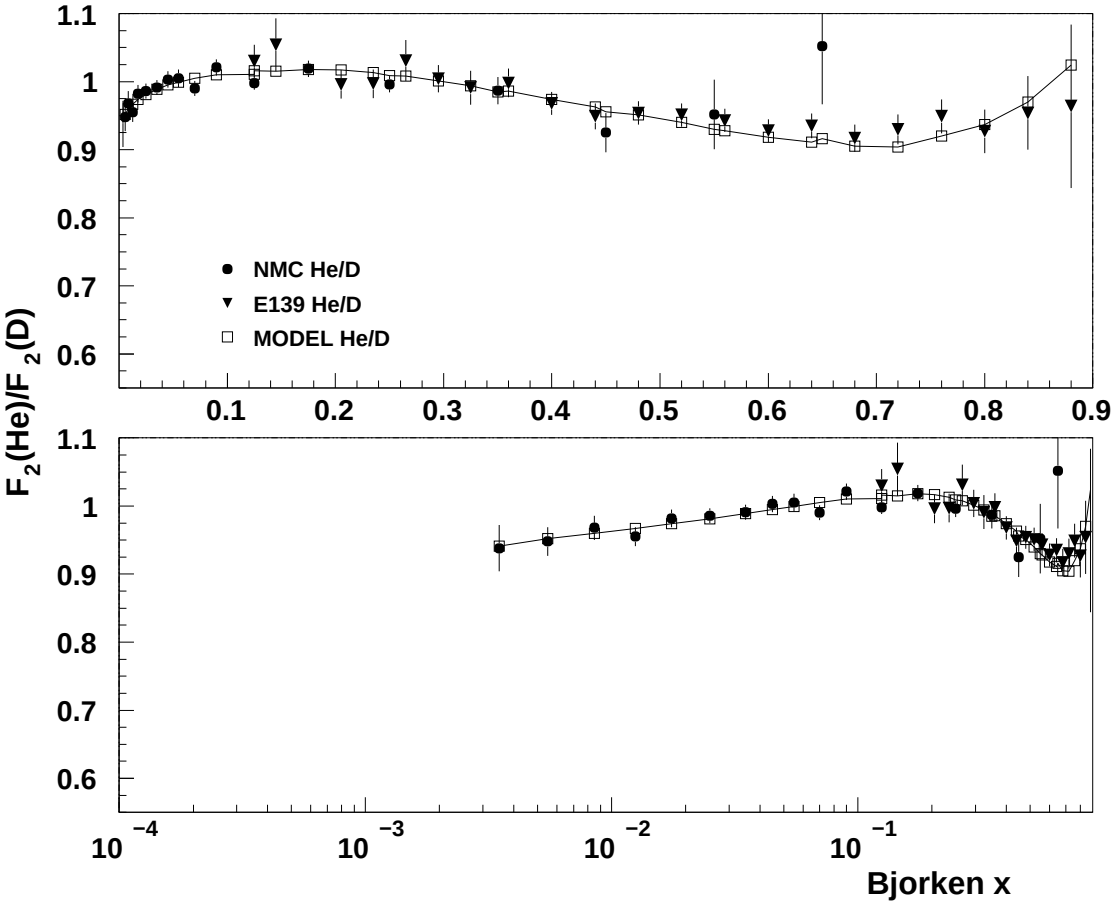
- The monopole form $\bar{\sigma} = \sigma_0/(1 + Q^2/Q_0^2)$ provides a good fit to existing data on nuclear shadowing for $Q^2 < 20\ \mathrm{GeV}^2$ (ratio $F_2(\mathrm{Sn})/F_2(\mathrm{C})$ from NMC).

- This does not necessarily mean that $\bar{\sigma}$ vanish as $1/Q^2$ at high Q. Effective cross section at high Q can be calculated from the normalization condition of the valence quark distribution $\delta N_{\mathrm{val}}^{\mathrm{off-shell}} + \delta N_{\mathrm{val}}^{\mathrm{shad}} = 0$.

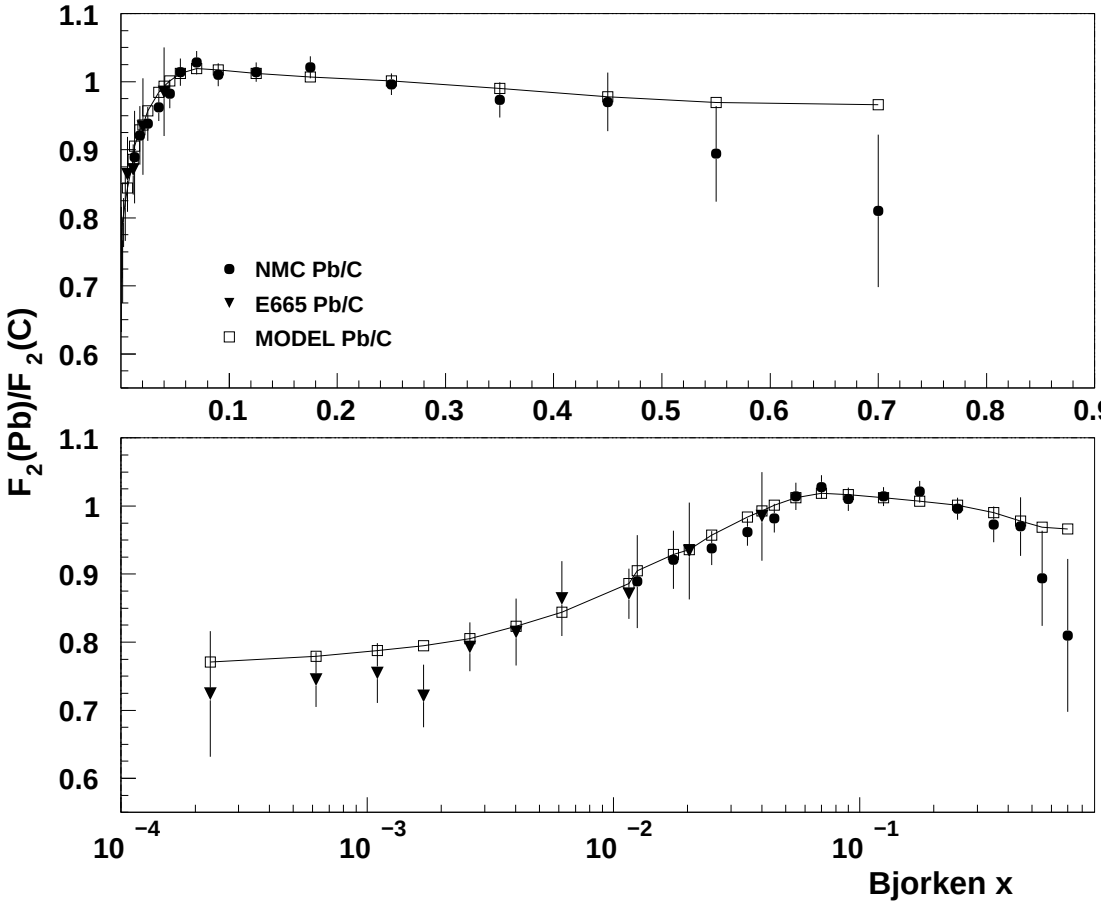


Ratios $\mathcal{R}_2(x, A/B) = F_2^A/F_2^B$

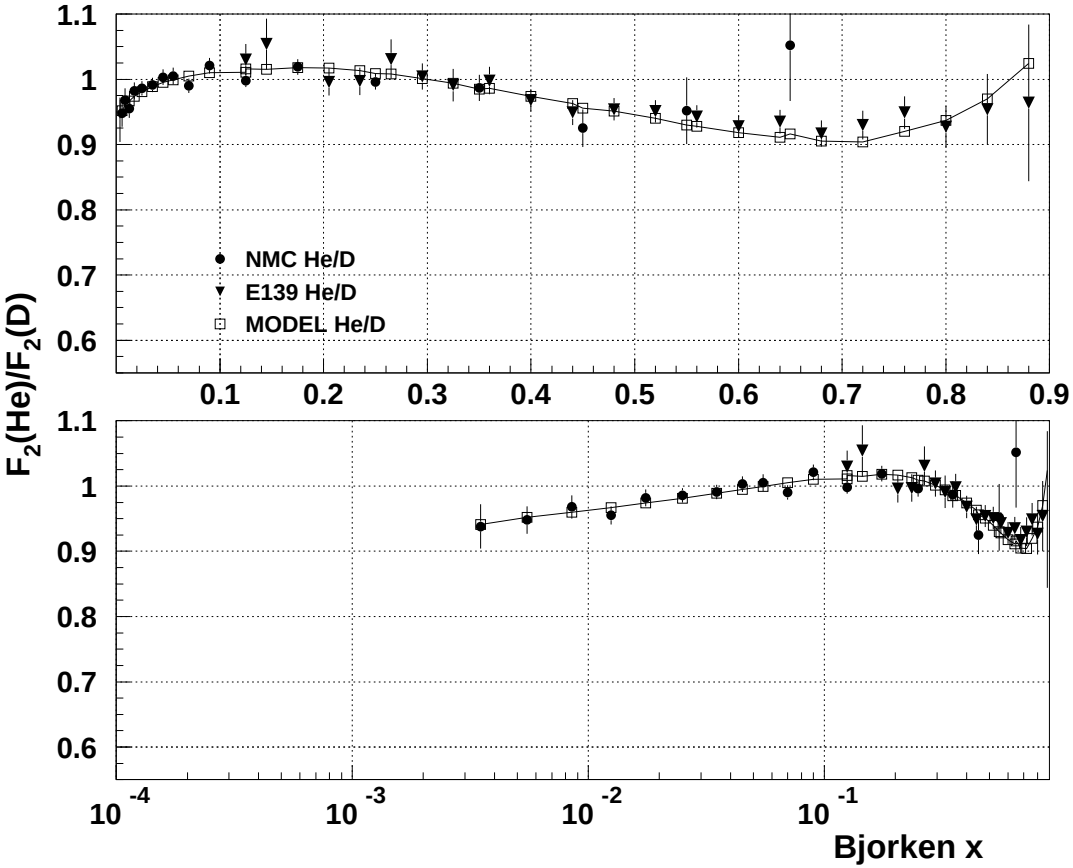
^{4}He/D



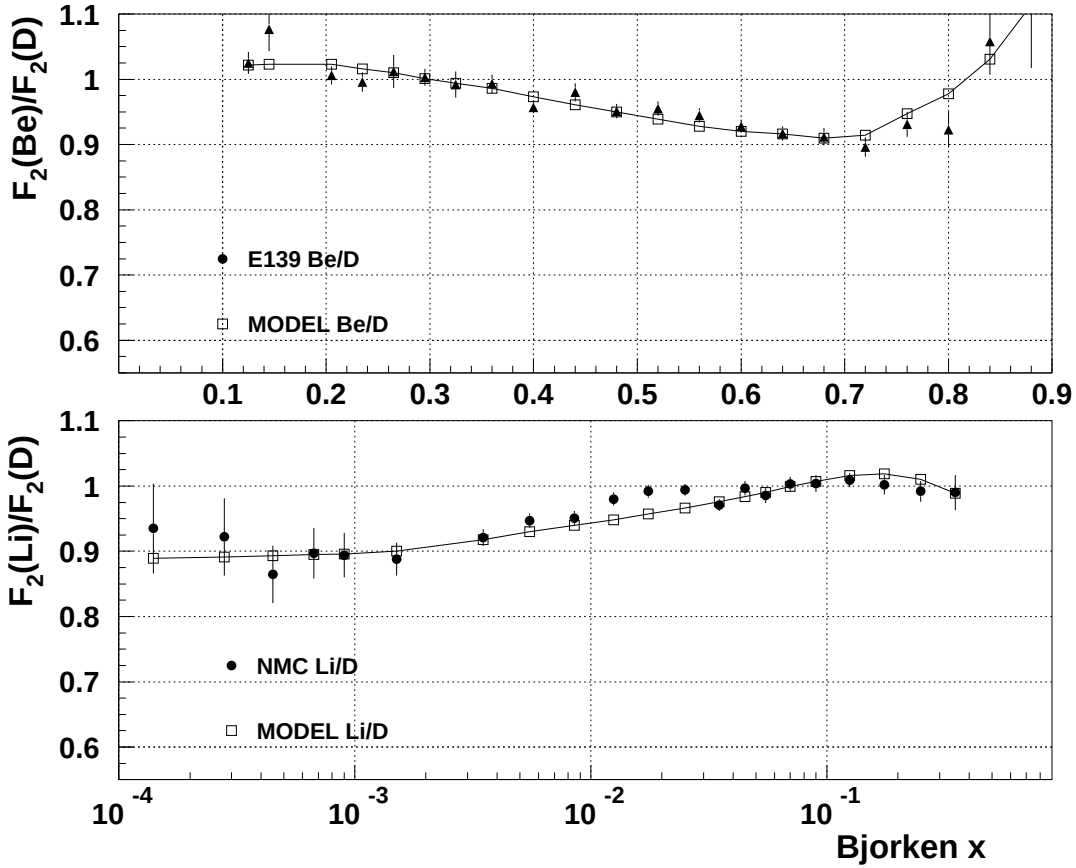
^{208}Pb/^{12}C



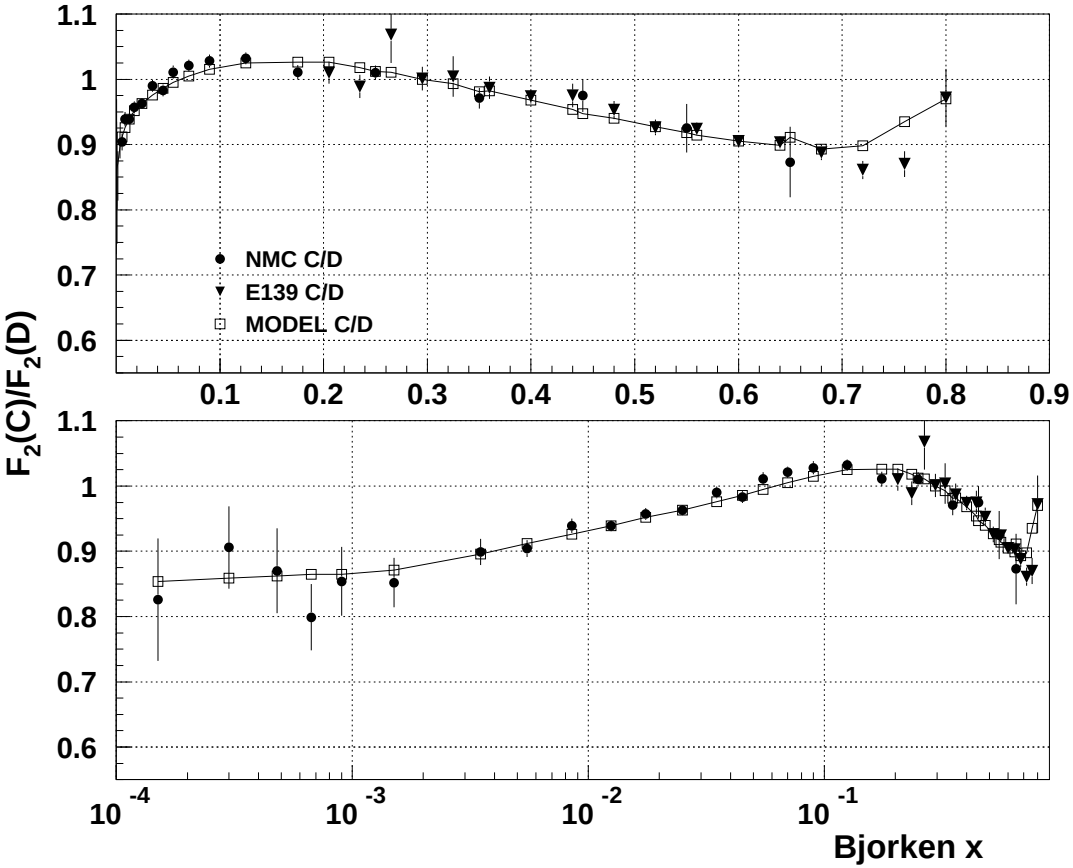
^{4}He/D



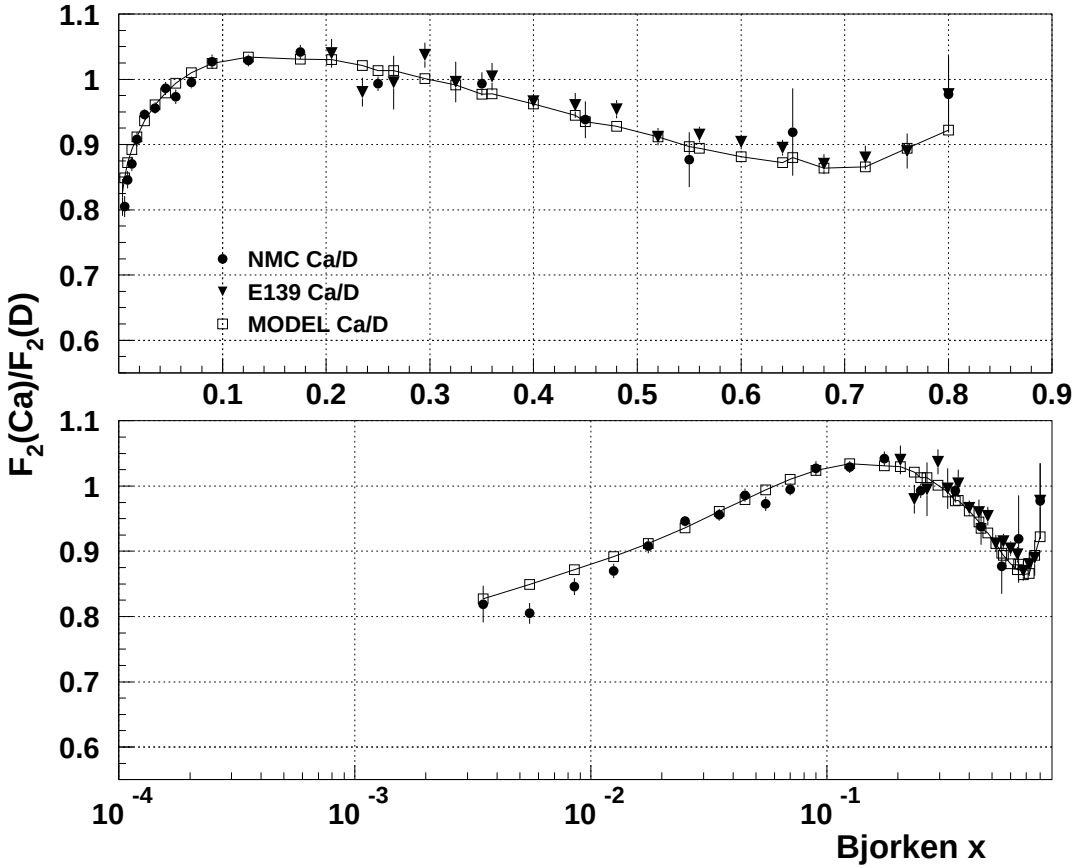
^{7}Li/D & ^{9}Be/D



$^{12}C/D$

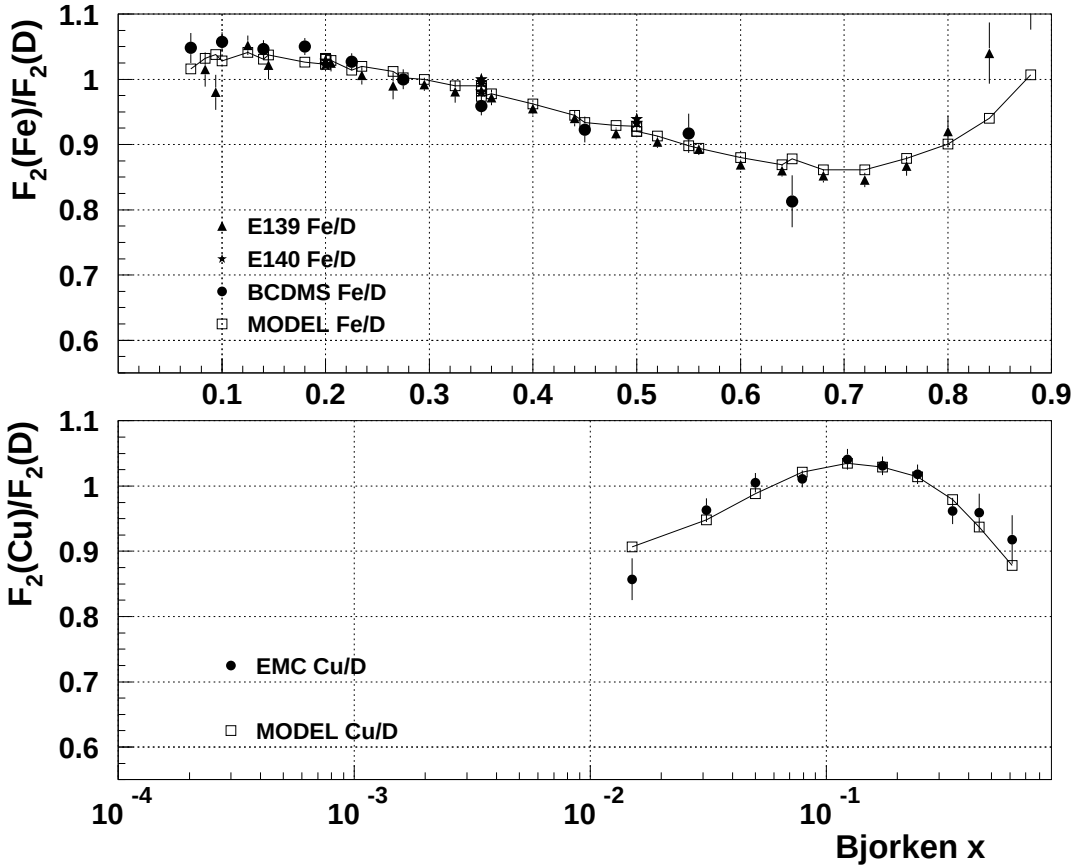

NMC C/D
E139 C/D
MODEL C/D
$F_2(C)/F_2(D)$
Bjorken x

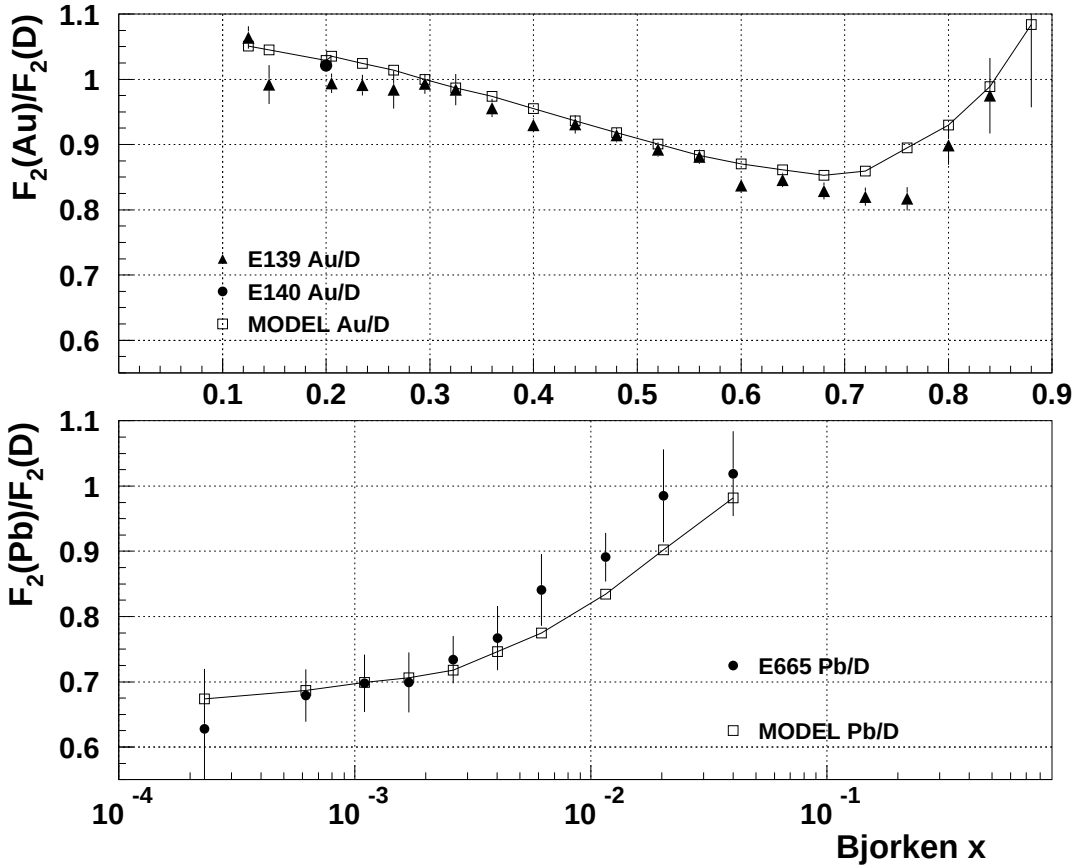
$^{40}Ca/D$


NMC Ca/D
E139 Ca/D
MODEL Ca/D
$F_2(Ca)/F_2(D)$
Bjorken x

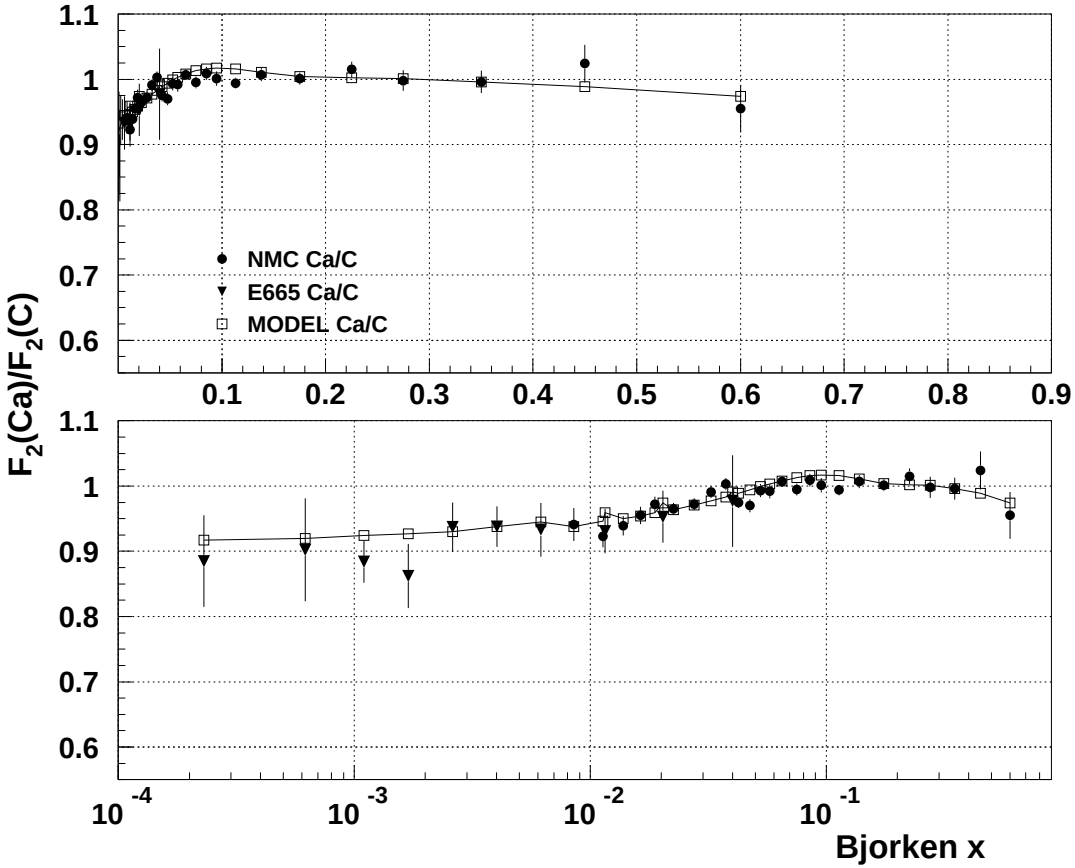
^{56}Fe/D & ^{63}Cu/D

^{197}Au/D & ^{207}Pb/D

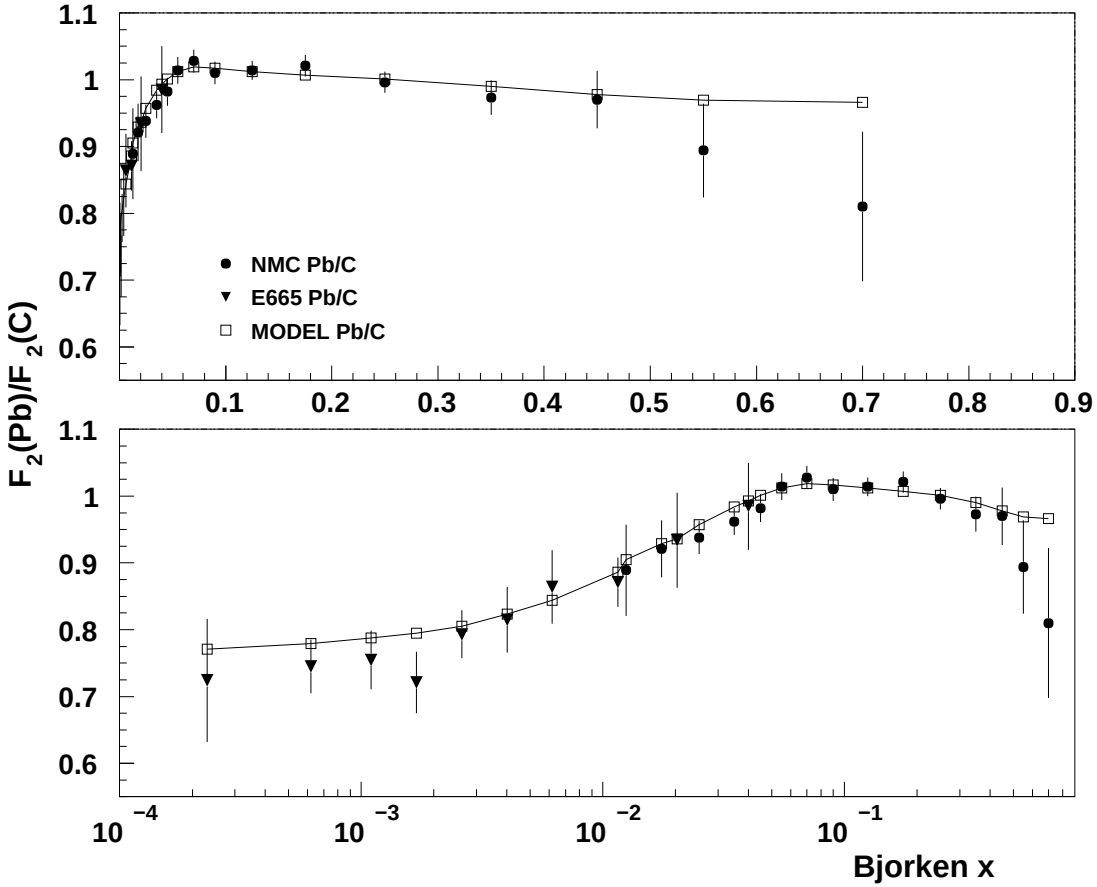




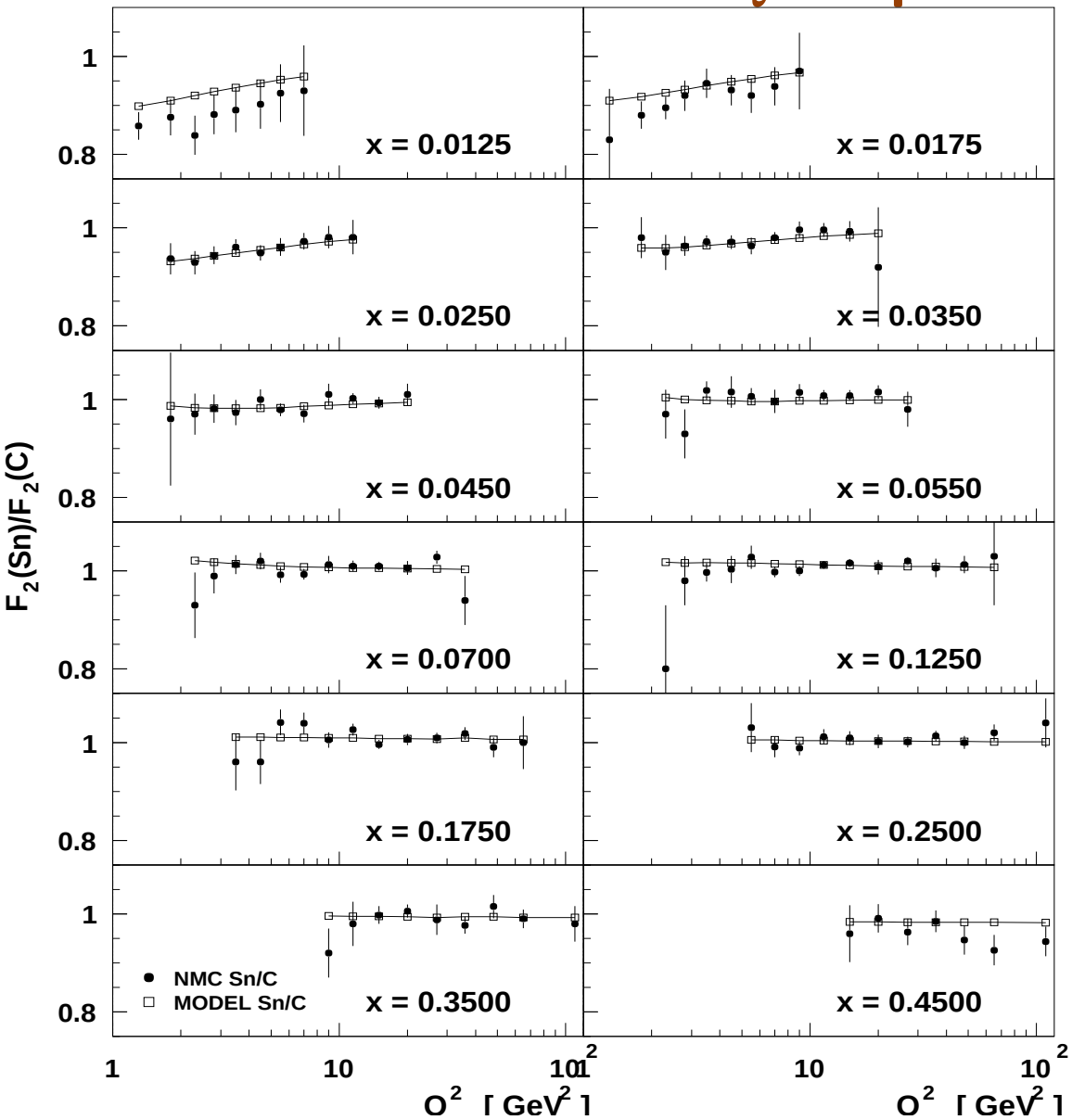
$^{40}Ca/^{12}C$

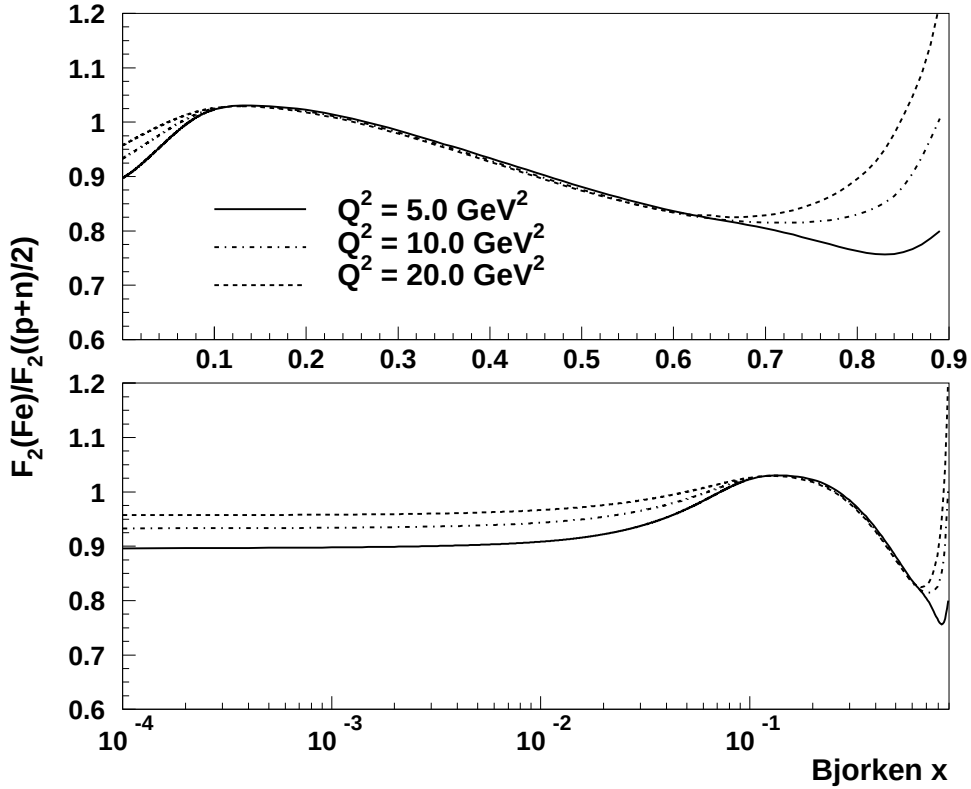


$^{207}Pb/^{12}C$



Q^2 dependence of $\mathcal{R}_2$

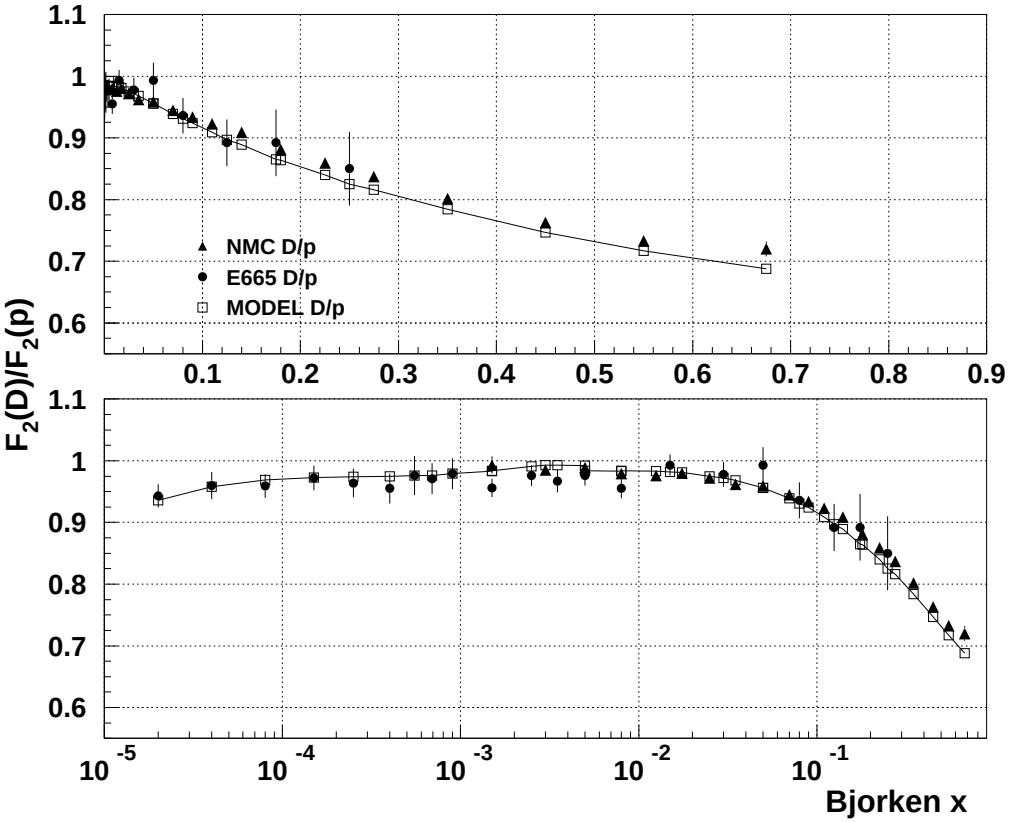


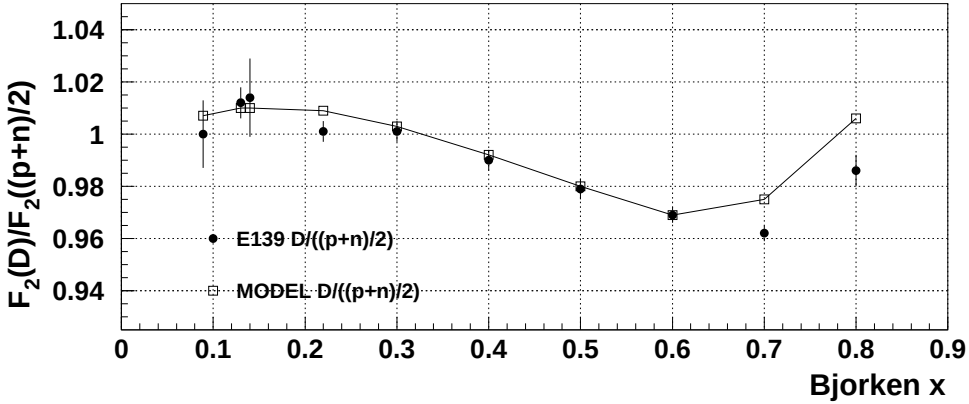


Significant Q^2 dependence observed at $x < 0.05$ (the Q^2 dependence of shadowing effect) and for $x > 0.7$ (the Q^2 dependence of target mass correction)

Deuteron structure functions

Comparison of E665 and NMC D/p data to our calculations (curve with open squares). Note that these data were not used in our fit. The data points with $x < 10^{-3}$ also have $Q^2 < 0.5\ \mathrm{GeV}^2$.





Comparison of Gomez et.al. extraction of D/$(p+n)$ ratio from E139 nuclear data which was based on extrapolation and the nuclear density model of Frankfurt & Strikman (closed circles) to our calculations (curve with open squares).

Summary

- Exciting nuclear effects were measured in electron and muon DIS experiments.

- A few nuclear mechanisms produce substantial corrections to nuclear structure functions. However, data are not explained by standard approaches.

- Phenomenological analysis of data indicates the presence of substantial modification of bound nucleon quark distribution.

- I discussed approach in which this modification is associated with off-shell effect. By performing phenomenological analysis of data this correction was extracted from data and a model was developed that quantitatively describes data with very good accuracy.

- This approach was verified by confronting calculations with data not used in the analysis and by predicting nuclear effects in neutrino scattering.