Nucleon and Nuclear Structure Functions (3)

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HUGS 2007, Jefferson Lab., June 12, 2007

Outline

- Introduction to neutrino charged-current (CC) and neutral-current (NC) inelastic scattering. Kinematics, currents, cross sections and structure functions. Similarities and differences between charged-lepton (CL) and neutrino scattering.

- Relations between CC and NC neutrino structure functions and their using in electroweak studies (measurements of $\sin^2\theta_W$).

- Problems associated with nuclear effects.

Currents in the Standard Model

$$J_\mu^{\rm em} = \sum_{q=u,d,\dots} e_q^2\ \bar\psi_q\gamma_\mu\psi_q$$

$$V_\mu^a = \sum_{\rm generations} \bar\psi\gamma_\mu\frac{\tau^\pm}{2}\psi, \qquad \psi = \begin{pmatrix} u \\ d \end{pmatrix},\ \psi = \begin{pmatrix} c \\ s \end{pmatrix}, \dots$$

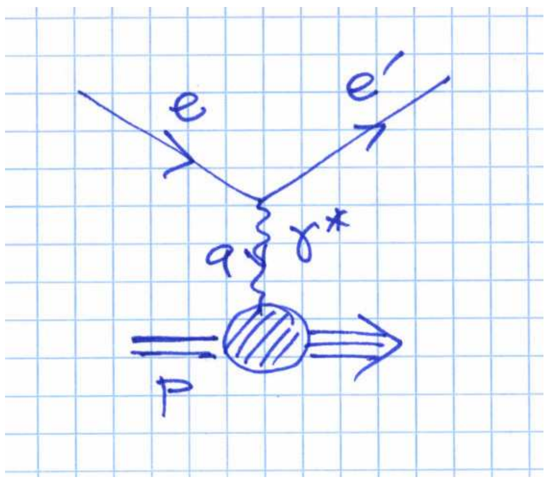
$$A_\mu^a = \sum_{\rm generations} \bar\psi\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi$$

$$J_\mu^\pm = V_\mu^\pm - A_\mu^\pm, \quad V_\mu^\pm = V_\mu^1 \pm iV_\mu^2,$$

$$J_\mu^0 = \sqrt{2}\left(V_\mu^3 - A_\mu^3 - 2\sin^2\theta_W J_\mu^{\rm em}\right) \qquad = \sum_{q=u,d,\dots} \bar\psi_q\gamma_\mu(g_V^q - g_A^q\gamma_5)\psi_q$$

charges	$u,\ c$	$d,\ s$	$\nu_e,\ \nu_\mu$	$e^-,\ \mu^-$
e	$\frac{2}{3}$	$-\frac{1}{3}$	0	-1
$\sqrt{2}\,g_V$	$1-\frac{8}{3}\sin^2\theta_W$	$\frac{4}{3}\sin^2\theta_W - 1$	1	$4\sin^2\theta_W - 1$
$\sqrt{2}\,g_A$	1	-1	1	-1

Neutrino inclusive inelastic scattering



CC neutrino scattering: $\nu + \text{target} \to \mu^- + \text{anything}$
NC neutrino scattering: $\nu + \text{target} \to \nu + \text{anything}$
Inclusive inelastic cross section:

$$\mathrm{d}\sigma^{(\nu,\bar\nu)} = \frac{G_F^2}{32\pi^2} L_{\mu\lambda}^{(\nu,\bar\nu)} H_{\mu\lambda}^{(\nu,\bar\nu)} \frac{d^3\boldsymbol{k}'}{(pk)E'}$$

Leptonic tensor (polarization of outgoing lepton is not detected)

$$L_{\mu\lambda}^{(\nu,\bar\nu)} = 8\left[k_\mu k'_\lambda + k_\lambda k'_\mu - kk' g_{\mu\lambda} \pm i\varepsilon_{\mu\lambda\alpha\beta}\, k_\alpha k'_\beta\right]$$

Hadronic tensor provides information about the target's response to the studied interaction

$$H_{\mu\nu}^{(\nu,\bar\nu)}(p,q) = \frac{1}{4\pi}\int d^4\xi \exp(iq\cdot\xi)\, \langle p|\left[J_\mu^{\pm}(\xi), J_\nu^{\mp}(0)\right]|p\rangle$$

For NC scattering $L_{\mu\lambda}(\text{NC}) = \frac{1}{2} L_{\mu\lambda}(\text{CC})$ since (anti)neutrino can only be (right)left-polarized.

Hadronic tensor

The hadronic tensor is parametrized in terms of 5 independent scalar structure functions for both CC and NC.

$$W_{\mu\nu}(p,q) = -\widetilde{g}_{\mu\nu} F_1(x,Q^2) + \frac{\widetilde{p}_\mu \widetilde{p}_\nu}{p\cdot q} F_2(x,Q^2) + i\,\frac{\varepsilon_{\mu\nu\alpha\beta}\, p^\alpha q^\beta}{2\,p\cdot q} F_3(x,Q^2) + \frac{q_\mu q_\nu}{q^2} F_4(x,Q^2) + \frac{q_\mu p_\nu + q_\nu p_\mu}{p\cdot q} F_5(x,Q^2),$$

where $Q^2 = -q^2$, $x = \frac{Q^2}{2p\cdot q}$ and $\widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$, $\quad \widetilde{p}_\mu = p_\mu - \frac{p\cdot q}{q^2} q_\mu$

- The term F_3 is because of VA interference.
- The terms F_4 and F_5 are present because the axial-vector current is not conserved however their contribution to cross section is suppressed by the ratio $m^2_{\rm lept}/E$.

CC and NC neutrino and antineutrino cross sections in terms of Bjorken x and inelasticity $y = (E - E')/E$, E is energy of incoming neutrino beam.

$$\frac{\mathrm{d}^2\sigma^{(\nu,\bar\nu)}_{\mathrm{CC}}}{\mathrm{d}x\mathrm{d}y} = \frac{\frac{1}{\pi}G_F^2 M E}{(1+\frac{Q^2}{M_W^2})^2}\left[Y_+F_2^{W^\pm} - y^2 x F_L^{W^\pm} \pm Y_- x F_3^{W^\pm}\right],$$

$$\frac{\mathrm{d}^2\sigma^{(\nu,\bar\nu)}_{\mathrm{NC}}}{\mathrm{d}x\mathrm{d}y} = \frac{\frac{1}{2\pi}G_F^2 M E}{(1+\frac{Q^2}{M_Z^2})^2}\left[Y_+F_2^{Z} - y^2 x F_L^{Z} \pm Y_- x F_3^{Z}\right],$$

$$F_L = (1+\tfrac{4x^2M^2}{Q^2})\frac{F_2}{2x} - F_1,$$

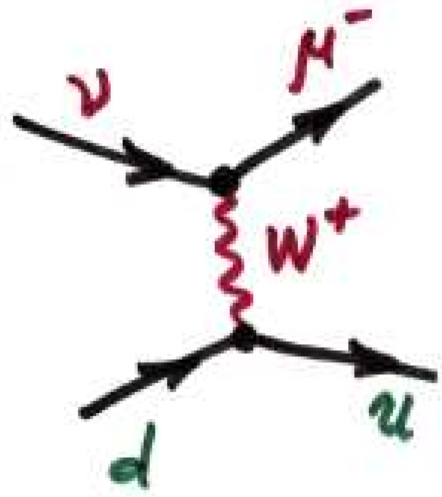
$$Y_+ = \tfrac{1}{2}\left[1+(1-y)^2\right] + \tfrac{M^2x^2y^2}{Q^2},\;\; Y_- = \tfrac{1}{2}\left[1-(1-y)^2\right].$$

If $R^\nu = \sigma^\nu_{\mathrm{NC}}/\sigma^\nu_{\mathrm{CC}}$ and/or $R^{\bar\nu} = \sigma^{\bar\nu}_{\mathrm{NC}}/\sigma^{\bar\nu}_{\mathrm{CC}}$ are measured with appropriate accuracy then we can use theory in order to derive $\sin^2\theta_W$ from data.

Basic scattering processes in neutrino DIS

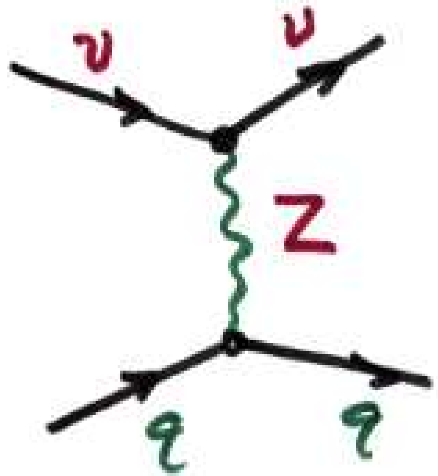
CC structure functions:

$$
\begin{aligned}
F_2^{\nu} = F_2^{W^+} &= 2x(d + \bar{u} + s + \bar{c}),\\
xF_3^{\nu} = xF_3^{W^+} &= 2x(d - \bar{u} + s - \bar{c}),\\
F_2^{\bar{\nu}} = F_2^{W^-} &= 2x(u + \bar{d} + c + \bar{s}),\\
xF_3^{\bar{\nu}} = xF_3^{W^-} &= 2x(u - \bar{d} + c - \bar{s}).
\end{aligned}
$$



NC structure functions:

$$
\begin{aligned}
F_2^Z &= x \sum_{q=u,d,\dots} (g_V^2 + g_A^2)\,(q + \bar{q})\,,\\
xF_3^Z &= 2x \sum_{q=u,d,\dots} g_V g_A\,(q - \bar{q})\,.
\end{aligned}
$$



Relations beween CC and NC neutrino scattering

Relations between NC and CC structure functions in the Parton model

$$
\begin{aligned}
2\,F_2^Z &= C_2^0\,F_2^{\nu+\bar\nu} - C_2^1\,xF_3^{\nu-\bar\nu},\\
2\,xF_3^Z &= C_3^0\,xF_3^{\nu+\bar\nu} - C_3^1\,F_2^{\nu-\bar\nu}.
\end{aligned}
$$

The coefficients $C_{2,3}^{0,1}$:

$$
\begin{aligned}
C_2^0 &= \frac{1}{2}\left(g_V^2+g_A^2\right)_{u+d} = 1-2\sin^2\theta_W+\frac{20}{9}\sin^4\theta_W,\\
C_2^1 &= \frac{1}{2}\left(g_V^2+g_A^2\right)_{u-d} = -\frac{2}{3}\sin^2\theta_W(1-2\sin^2\theta_W),\\
C_3^0 &= \left(g_V g_A\right)_{u+d} = 1-2\sin^2\theta_W,\\
C_3^1 &= \left(g_V g_A\right)_{u-d} = -\frac{2}{3}\sin^2\theta_W.
\end{aligned}
$$

Consider the ratios of NC and CC cross-sections of definite C parity

$$C-\text{odd}: R^- = \frac{\sigma^\nu_{\rm NC}-\sigma^{\bar\nu}_{\rm NC}}{\sigma^\nu_{\rm CC}-\sigma^{\bar\nu}_{\rm CC}} = \frac{Y_-\, xF_3^Z}{Y_-\, xF_3^{\nu+\bar\nu}+Y_+\, F_2^{\nu-\bar\nu}-\frac{y^2}{2}F_L^{\nu-\bar\nu}},$$

$$C-\text{even}: R^+ = \frac{\sigma^\nu_{\rm NC}+\sigma^{\bar\nu}_{\rm NC}}{\sigma^\nu_{\rm CC}+\sigma^{\bar\nu}_{\rm CC}} = \frac{Y_+\, F_2^Z+\frac{y^2}{2}F_L^Z}{Y_+\, F_2^{\nu+\bar\nu}+\frac{y^2}{2}F_L^{\nu+\bar\nu}+Y_-\, xF_3^{\nu-\bar\nu}},$$

Isospin symmetry suggests a simple relation between C-odd NC and CC cross sections and structure functions for isoscalar target. For isoscalar target $F_{2,L}^{\nu-\bar\nu}=0$ (but this does not hold for $xF_3^{\nu-\bar\nu}=2x(s+\bar s)$!)

$$R^- = \frac{F_3^Z}{F_3^{\nu+\bar\nu}} = \frac{1}{2}-s_W^2$$

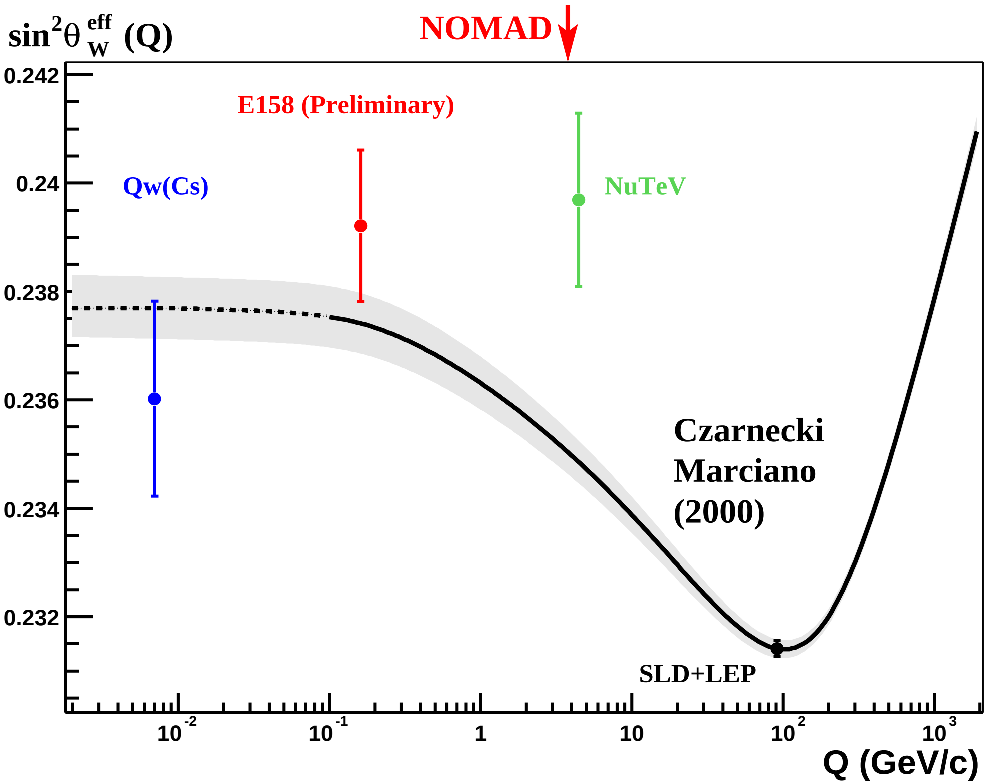
Paschos & Wolfenstein, 1973

NuTeV puzzle

NuTeV Collaboration derives $\sin^2\theta_W$ from neutrino and antineutrino DIS cross section measurement on a complex target (mainly iron).

$$\sin^2\theta_W(\mathrm{NuTeV}) =$$
$$0.2277 \pm 0.0013(\mathrm{stat}) \pm 0.0009(\mathrm{syst})$$

This is about 3σ larger than the value derived from the Standard Model fit to other electroweak measurements, $\sin^2\theta_W(\mathrm{SM}) = 0.2227 \pm 0.00004$.



Remarks on the PW relation

\+ The PW relation is based on the isospin symmetry and stable against strong interaction corrections (it holds for any isoscalar target including nuclei beyond the Parton model). It holds for both differential (structure functions) and total cross sections. It can be generalized to other (exclusive) reactions. The PW relation is a good (but expensive) tool to measure $\sin^2\theta_W$.

— The PW relation holds for isoscalar target (e.g. deuterium) and must be corrected for non-isoscalarity in heavy nuclei (note a typical complex nucleus has excess of neutrons over protons). The PW relation is violated by C-odd strange-quark contribution ($s - \bar{s}$ asymmetry in the nucleon) and/or if isospin symmetry is not exact ($u_p \neq d_n$).

The PW relation for total cross sections

The ratio R^- for the total cross sections assuming a small admixture of isovector component in a target (nucleus with a small neutron excess):

$$
\begin{aligned}
R^-_{\rm tot} &= \frac{1}{2} - \sin^2\theta_W + \delta R^-_{\rm tot}, \\
\delta R^-_{\rm tot} &= \left(\frac{x_1^- - x_s^-}{x_0^- + x_s^-}\right)\cdot\left[1 - \frac{7}{3}s_W^2 + \left(\frac{8}{9}\frac{\alpha_S}{\pi} + 5.34\frac{\alpha_S^2}{\pi^2}\right)\left(\frac{1}{2} - \sin^2\theta_W\right)\right], \\
x^-_{0,1} &= \int \mathrm{d}x\, x(u_{\rm val} \pm d_{\rm val}), \\
x^-_s &= \int \mathrm{d}x\, x(s - \bar{s}),
\end{aligned}
$$

Neutron Excess Effect in Heavy Nuclei

The *neutron excess* correction to R^- for the total cross sections:

$$\delta R^-_{\rm tot} = \left(\frac{x_1^-}{x_0^-}\right)_A \left(1 - \tfrac{7}{3}\sin^2\theta_W + \mathcal{O}(\alpha_S)\right)$$

Use isospin relations for quark distributions in proton and neutron $u_p = d_n,\ d_p = u_n$. If we neglect all possible nuclear effects then $x_{0/A} = A\, x_{0/p},\ x_{1/A} = (Z-N)\, x_{1/p}$.

$$\delta R^-_{\rm tot} \approx \beta \left(\frac{x_1^-}{x_0^-}\right)_{\text{proton}} \left(1 - \tfrac{7}{3}\sin^2\theta_W\right).$$

$\beta = (Z-N)/A$. The magnitude of the neutron excess correction in a heavy nucleus:

$$\begin{aligned} \beta(^{56}\mathrm{Fe}_{26}) &= -0.07, \\ x^-_{1/p}/x^-_{0/p} &= 0.43 \qquad \Rightarrow \qquad \delta R^- = -0.015 \end{aligned}$$

Remark:

$|\delta R^-| \approx 10\sigma$ of NuTeV measurement of $\sin^2\theta_W$. For this reason the neutron excess correction as well as nuclear corrections to PDFs and the PW relation must be treated with care in precision electroweak measurements.

Nuclear Effects

Nuclear PDFs can be written as convolution of nuclear spectral function and (bound) proton and neutron PDFs:

$$
\begin{aligned}
q_{a/A}(x) &= \left\langle q_{a/p}\right\rangle_p + \left\langle q_{a/n}\right\rangle_n + \delta q_a \\
x\langle q_{a/p}\rangle_p &= \int \mathrm{d}\varepsilon \mathrm{d}^3\boldsymbol{k}\, \mathcal{P}_p(\varepsilon, \boldsymbol{k}) \left(1 + \frac{k_z}{M}\right) x' q_{a/p}(x', Q^2, k^2).
\end{aligned}
$$

Quark distributions in bound proton and neutron, $q_{a/p}$ and $q_{a/n}$, depend on $x' = Q^2/(2k \cdot q)$, where $k = (M + \varepsilon, \boldsymbol{k})$. Since bound nucleons are off-mass-shell their quark distributions generally depend on nucleon virtuality $k^2 = (M + \varepsilon)^2 - \boldsymbol{k}^2$ leading to in-medium correction to quark distributions.

Isospin Dependence of Nuclear Effects

Use the isospin relations for quark distributions $u_p = d_n,\ d_p = u_n$ to see that the isoscalar ($q_0 = u + d$) and the isovector ($q_1 = u - d$) quark distributions are determined by the isoscalar $\mathcal{P}_0 = \mathcal{P}_p + \mathcal{P}_n$ and the isovector $\mathcal{P}_1 = \mathcal{P}_p - \mathcal{P}_n$ nucleon distributions in a nucleus

$$\mathcal{P}_p q_{a/p} + \mathcal{P}_n q_{a/n} =$$
$$\tfrac{1}{2}\left(\mathcal{P}_p + \mathcal{P}_n\right)\left(q_{a/p} + q_{a/n}\right) + \tfrac{1}{2}\left(\mathcal{P}_p - \mathcal{P}_n\right)\left(q_{a/p} - q_{a/n}\right).$$

$$\begin{aligned} q_{0/A} &= \left\langle q_{0/p}\right\rangle_0 + \delta q_0, \\ q_{1/A} &= \left\langle q_{1/p}\right\rangle_1 + \delta q_1, \end{aligned}$$

Nuclear effects on average quark light-cone momentum for isoscalar and isovector combinations of quark distributions calculated for $^{56}\mathrm{Fe}_{26}$ at $Q^2 = 20\,\mathrm{GeV}^2$

Bookkeeping of nuclear effects	$x_{0/p}$	$x_{1/p}$	$\dfrac{x_{0/A}}{x_{0/p}}$	$\dfrac{x_{1/A}}{x_{1/p}}\dfrac{1}{\beta}$	$R_{1/0}$
IA	0.367	0.163	0.965	1.013	1.05
$\mathrm{IA + OS + NS}$	0.367	0.163	0.997	1.027	1.03

$$\left(\frac{x_1}{x_0}\right)_A = \beta \left(\frac{x_1}{x_0}\right)_p R_{1/0}$$

For more details on nuclear effects in context of NuTeV effect and neutrino scattering see:
S.K., PRD **67**,091301,2003; hep-ph/0406220; hep-ph/0409057;
S.K & R. Petti, NPA **765**,126,2006; hep-ph/0703033

Evaluation of α_S correction to R^-

Using Alekhin 2002 PDF set obtained from global fit to DIS data we compute α_S correction to NNLO approx. Take $Q^2 = 20\,\mathrm{GeV}^2$.

pQCD order	$(x_1^-/x_0^-)_{\text{proton}}$	α_S
LO	0.457	0.249
NNLO	0.434	0.206

$$\delta R^-(\mathrm{NNLO}) - \delta R^-(\mathrm{LO}) = \begin{array}{ll} -0.8\cdot 10^{-3} & \text{using LO PDF and } \alpha_S \\ 0.7\cdot 10^{-4} & \text{using NNLO PDF and } \alpha_S \end{array}$$

Cancellation of pQCD corrections in R^- (as was anticipated from general arguments). LO seems to give a good approximation (error $< 1\%$). However, PDF should be consistent with α_S order of pQCD analysis.

Evaluating Corrections to NuTeV $\sin^2\theta_W$

Recall that $\Delta s_W^2(\text{NuTeV} - \text{SM}) = 5 \cdot 10^{-3}$ to be compared with the standard deviation $\sigma \approx 1.6 \cdot 10^{-3}$.

- NuTeV performed their analysis in LO QCD. The update to NLO (or better to NNLO) approximation is necessary.
- NuTeV target is (mainly) iron. The non-isoscalarity correction (neutron excess) is large ($\sim 10\sigma$). NuTeV accounts for this correction, however, nuclear effects should be properly analysed.

$$R^- = \frac{1}{2} - s_W^2 + \delta R^-,$$

$$\Delta s_W^2 = \delta R^-(\text{NNLO+FMB+neut.ex.}) - \delta R^-(\text{LO+neut.ex.}) =$$

$$-0.8 \cdot 10^{-3} \sim -\tfrac{1}{2}\sigma.$$

One finds *smaller* mixing angle if Fermi motion and binding corrections are applied for both isovector and isoscalar distributions.

Summary

- We discussed CC and NC neutrino DIS with particular emphasis on relation between CC and NC observables and heavy target effects.

- We discussed the Paschos–Wolfenstein relation for isoscalar target and also evaluated corrections to this relation for heavy nonisoscalar targets.

- Nuclear corrections to PW were found to be essential in the precision extraction of $\sin^2\theta_W$.

- In the context of NuTeV effect, the improved analysis with the account of QCD and nuclear effects is needed in order to reliably extract $\sin^2\theta_W$ and estimate its uncertainty.