HUGS Summer School
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Structure Functions at Low Q^2

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- Intriguing phenomena have been observed in low Q^2 structure functions

  → quark-hadron (Bloom-Gilman) duality

  → surprisingly small higher twist effects in (low) moments of structure functions

- Description of low Q^2 dynamics requires understanding "transition" region from resonances to scaling

  → how do resonances combine to form scaling function?

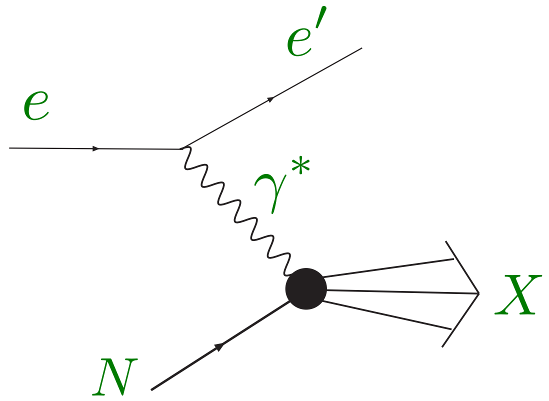
  → QCD moments of structure functions

  → target mass corrections (TMC)

- References: WM, Ent, Keppel, Phys. Rept. 406 (2005) 127

Electron scattering

Inclusive cross section for $eN \to eX$



$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2 \cos^2\frac{\theta}{2}}{Q^4}\left(2\tan^2\frac{\theta}{2}\frac{F_1}{M} + \frac{F_2}{\nu}\right)$$

$$\left.\begin{array}{l} \nu = E - E' \\ Q^2 = \vec{q}^{\,2} - \nu^2 = 4EE'\sin^2\dfrac{\theta}{2} \end{array}\right\} \quad x = \frac{Q^2}{2M\nu}$$

Bjorken scaling variable

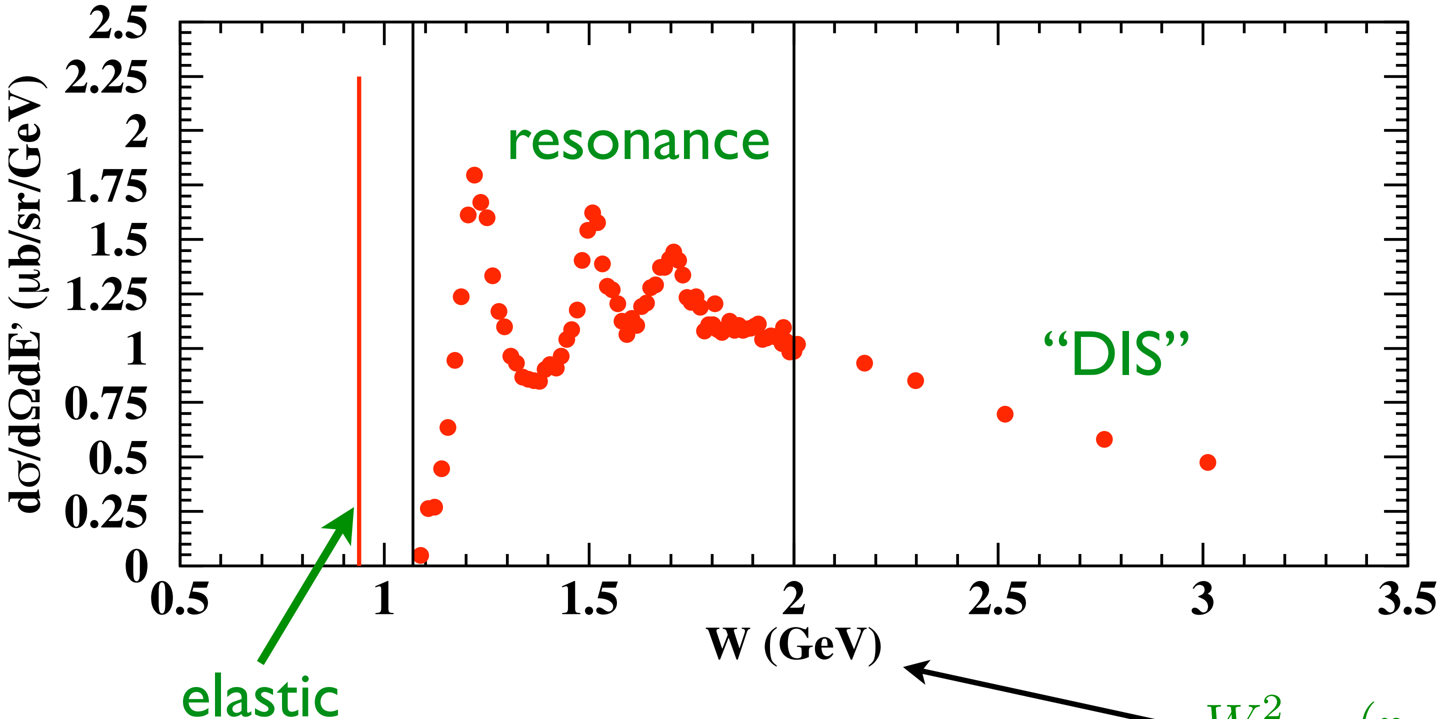
F_1 , F_2 "structure functions"

$\longrightarrow$ contain all information about structure of nucleon

$\longrightarrow$ functions of x, Q^2 in general

Resonance-DIS transition

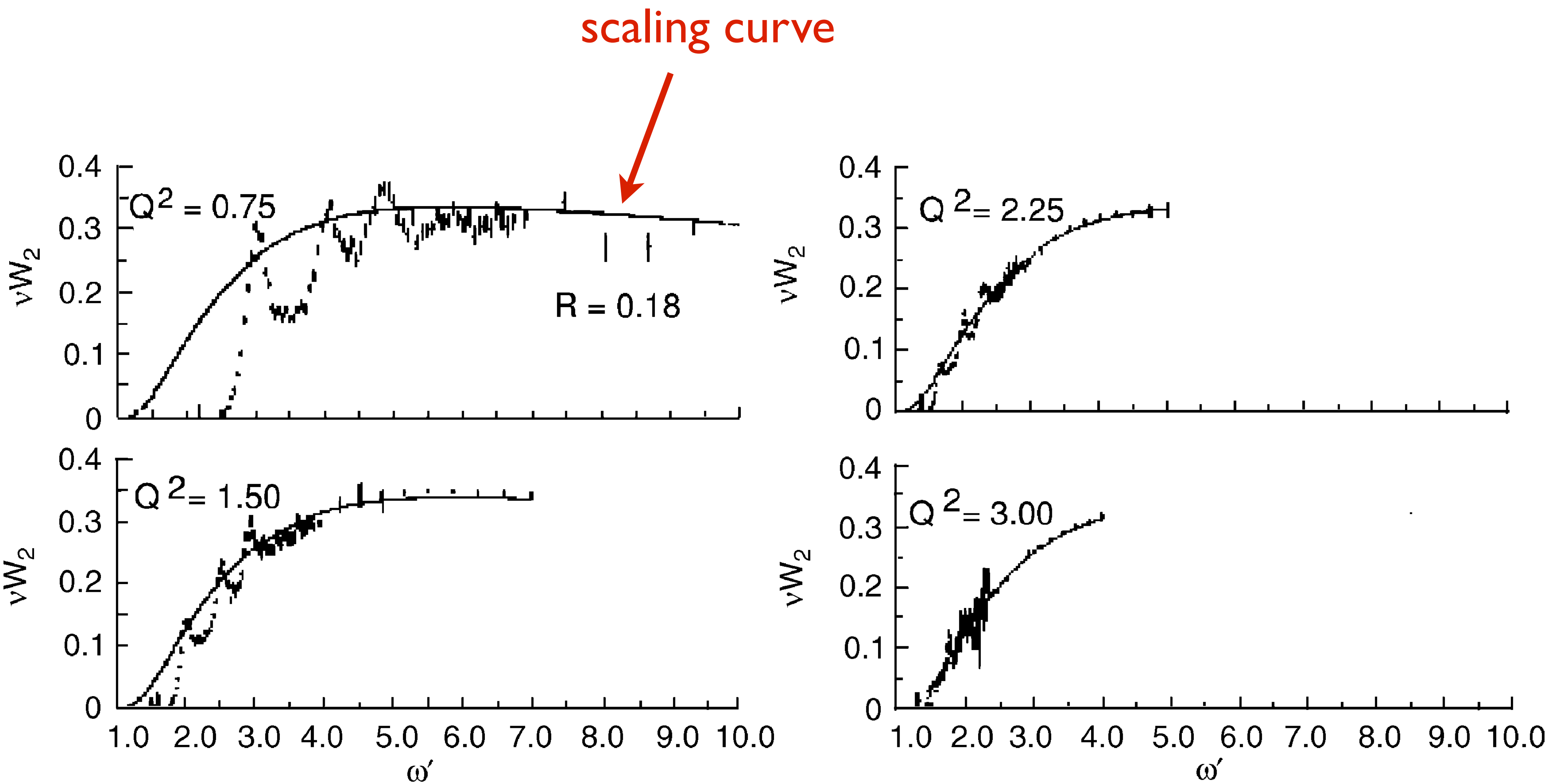
- As W decreases, DIS region gives way to region dominated by nucleon resonances



$$W^2 = (p_N + q)^2$$
$$= M^2 + 2M\nu - Q^2$$

hadronic final state mass

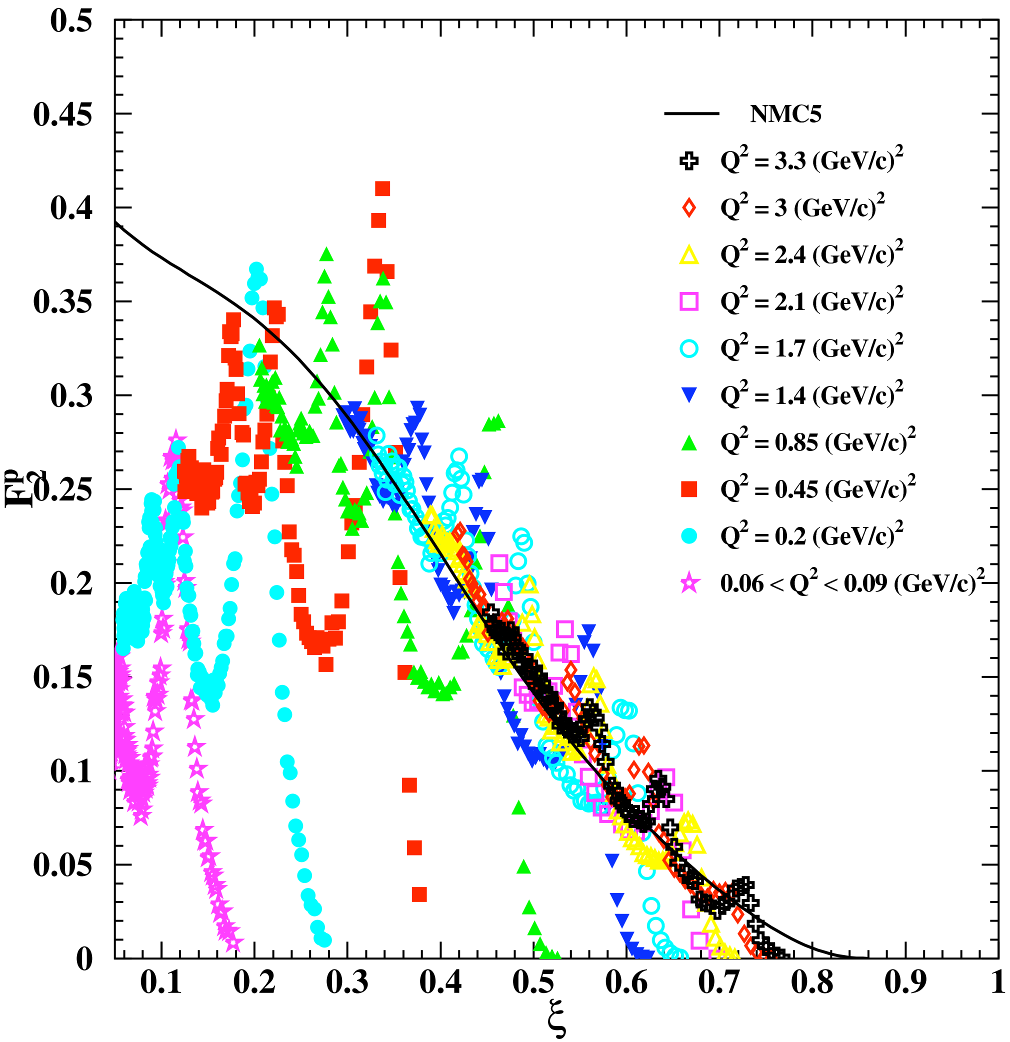
$$x = \frac{Q^2}{W^2 - M^2 + Q^2}$$



Bloom, Gilman, Phys. Rev. Lett. 85 (1970) 1185

➡ resonance – scaling duality in proton $\nu W_2 = F_2$ structure function

Resonance-DIS transition



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Niculescu et al., Phys. Rev. Lett. 85 (2000) 1182

Average over

(strongly Q^2 dependent)

resonances

$\approx$ Q^2 independent

scaling function

"Bloom-Gilman duality"

Bloom-Gilman duality

Average over (strongly Q^2 dependent) resonances
$\approx Q^2$ independent scaling function

Finite energy sum rule for eN scattering

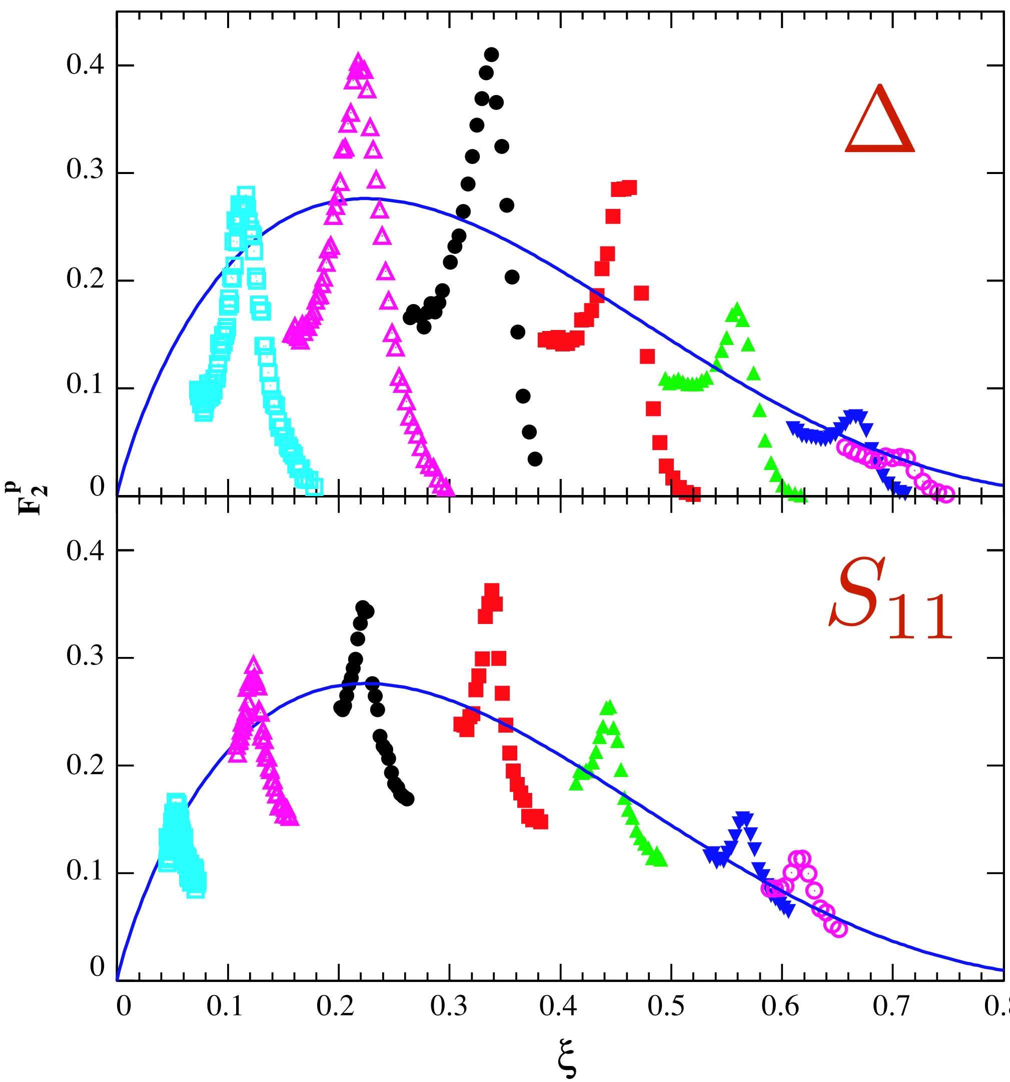
$$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$

measured structure function
(function of ν and Q^2)

scaling function
(function of ω' only)

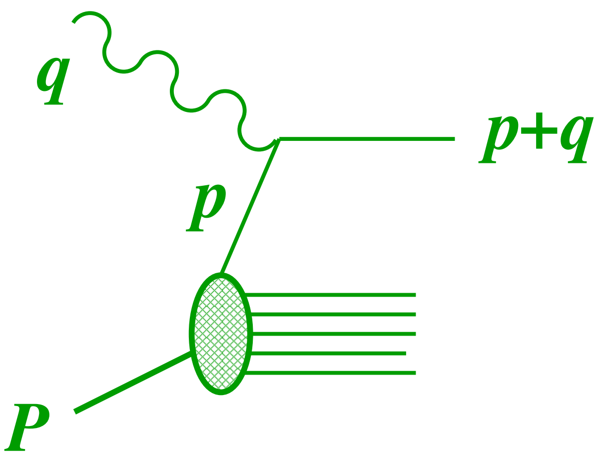
$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

Local Bloom-Gilman duality



$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$ Nachtmann scaling variable

Parton kinematics



$$(p+q)^2 = m_q^2 \quad \begin{cases} m_q = 0 \\ p_T = 0 \end{cases}$$

light-cone fraction of target's momentum carried by parton

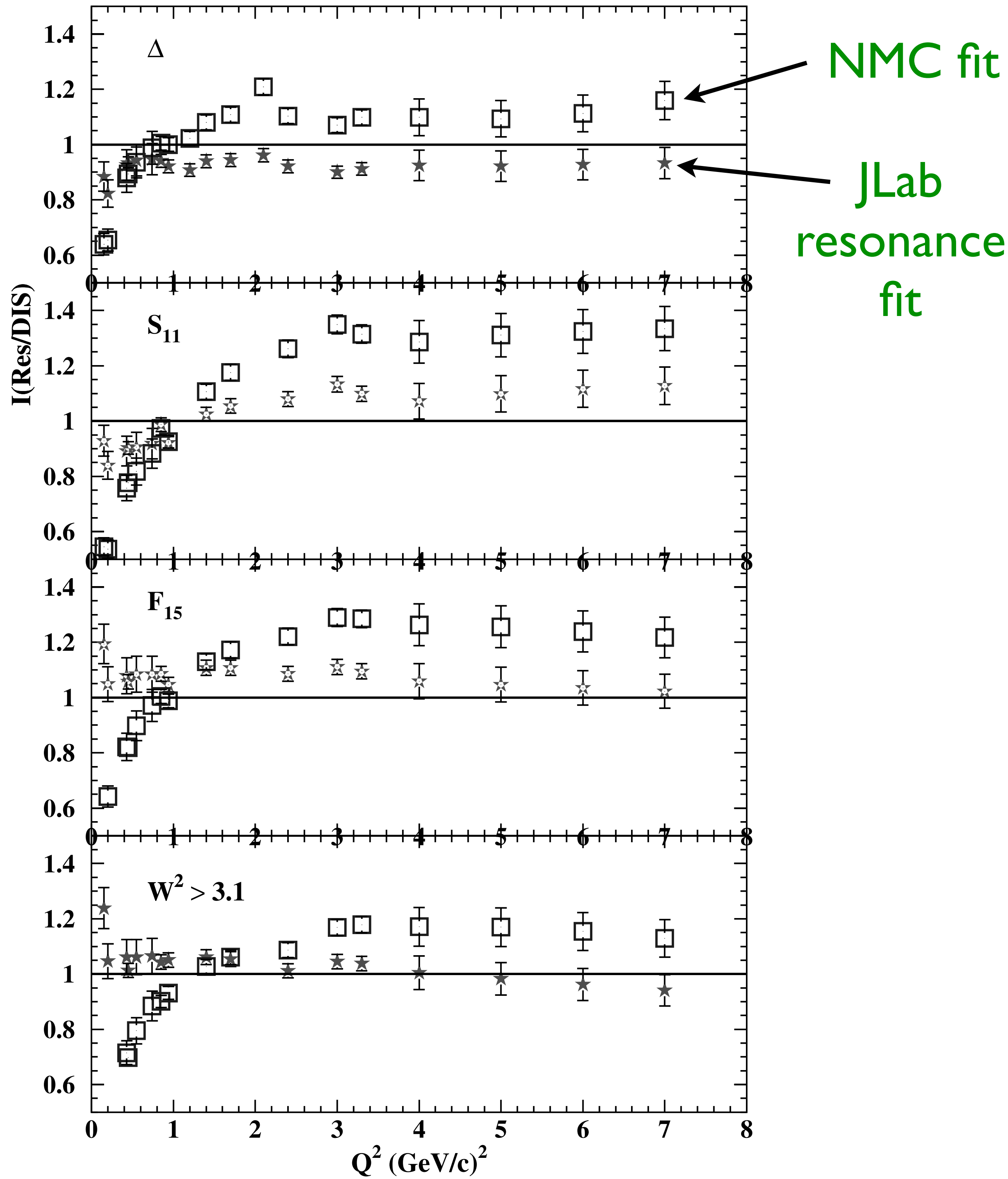
$$\xi = \frac{p^+}{P^+} = \frac{p^0 + p^z}{M}$$

Nachtmann scaling variable

$$\Rightarrow \quad \xi = \frac{2x}{1+r}\,, \quad r = \sqrt{1 + 4M^2x^2/Q^2}$$

$$\to \quad x \quad \text{as} \quad Q^2 \to \infty$$

Integrated strength



~10% agreement for $Q^2 > 1$ GeV^2

Niculescu et al., Phys. Rev. Lett. 85 (2000) 1186

Duality in QCD

- Considerable data exists in *resonance* region, $W < 2$ GeV

→ *common wisdom:* pQCD analysis not valid in resonance region

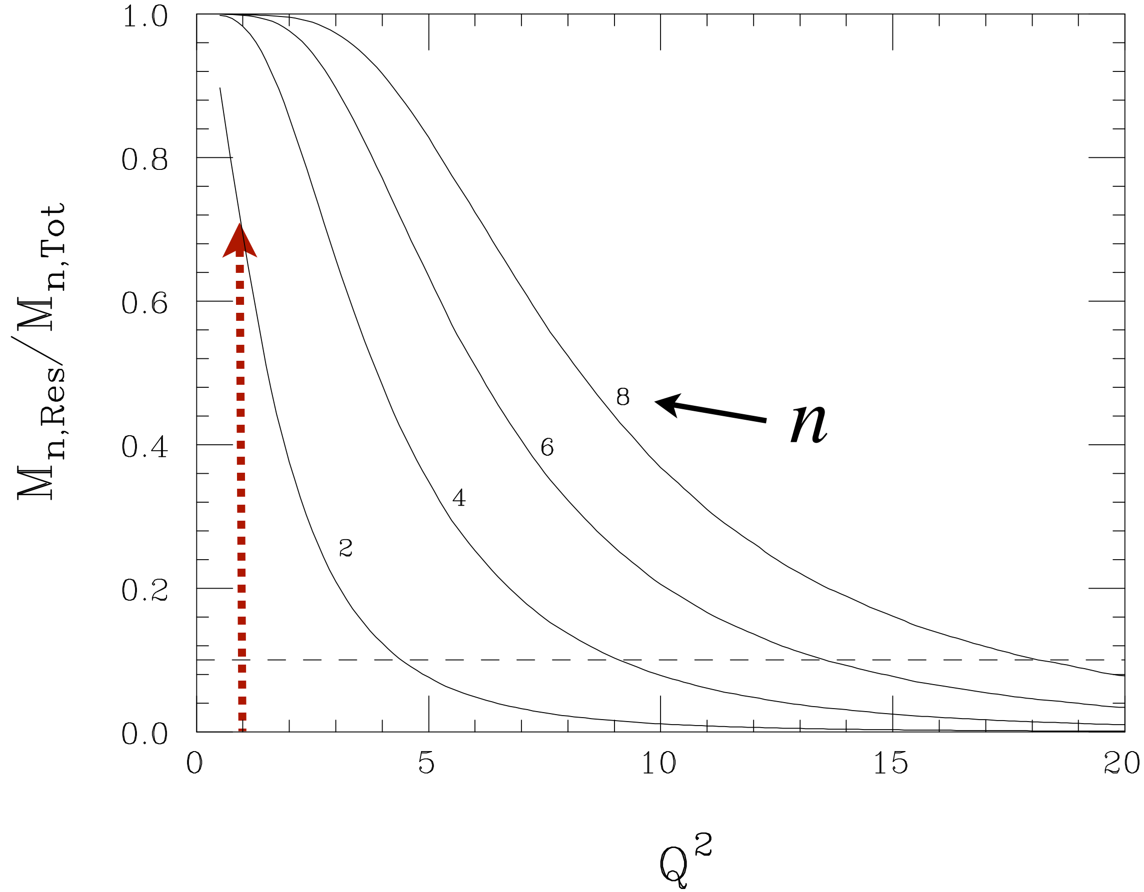
→ *in fact:* partonic interpretation of moments *does* include resonance region

- Resonances are an *integral part* of deep inelastic structure functions!

→ implicit role of quark-hadron duality

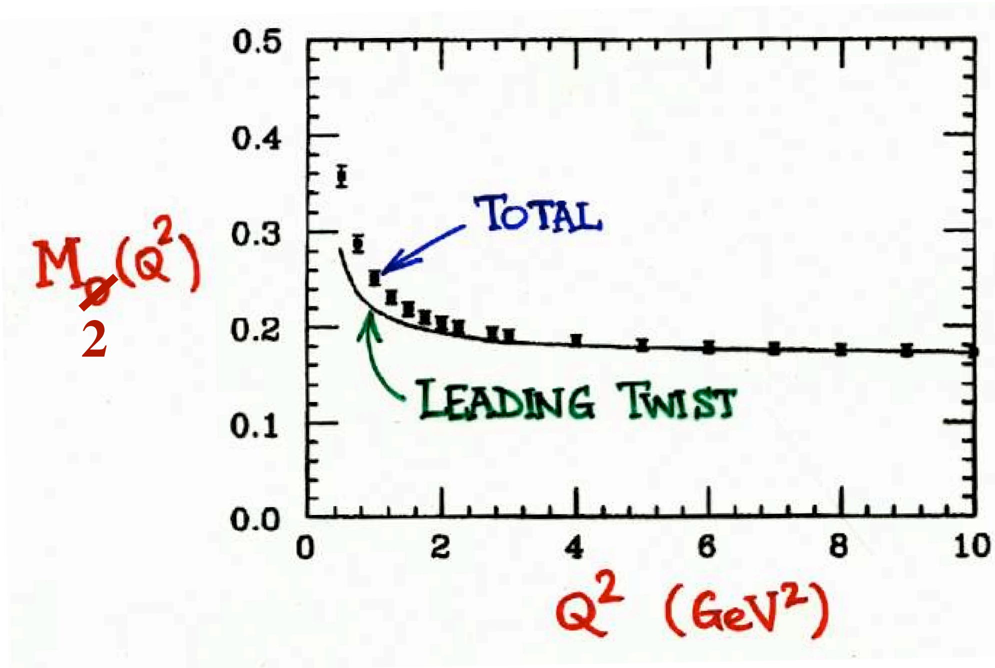
Proton F_2 moments

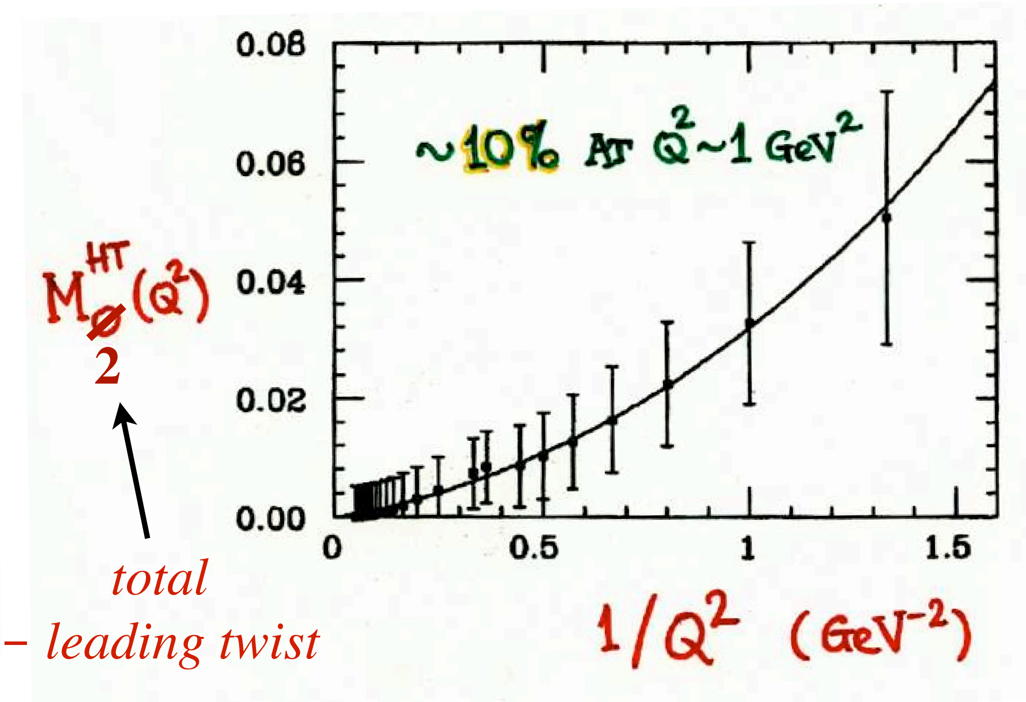
relative contribution of resonance region to n-th moment



$\Rightarrow$ At $Q^2 = 1\ \mathrm{GeV}^2$, $\sim$ <u>70%</u> of lowest moment of F_2^p comes from $W < 2\ \mathrm{GeV}$

Proton F_2 moments





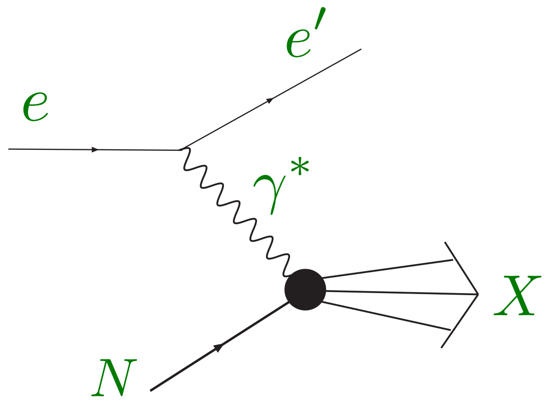
BUT resonances and DIS continuum conspire to produce only $\sim$ 10% *higher twist* contribution!

Ji, Unrau,
Phys. Rev. D 52 (1995) 72

Duality in QCD

Electron scattering in QCD

Inclusive cross section for $eN \to eX$



leptonic tensor

$$\frac{d^2\sigma}{d\Omega dE'} \sim L^{\mu\nu} W_{\mu\nu}$$

Hadronic tensor

$$W_{\mu\nu} = \sum_X \langle X|J_\mu(z)|N\rangle \langle N|J_\nu(0)|X\rangle \, \delta^4(p+q-p_X)$$

$$= \int d^4z \; e^{iq\cdot z} \langle N|J_\mu(z)J_\nu(0)|N\rangle$$

using completeness (sum over ALL states X)

$$\sum_X |X\rangle\langle X| = 1$$

"duality"

$\Longrightarrow$ in general, $N \to X$ transition matrix element very complicated

- Wilson Operator Product Expansion

  Expand product of currents $J(z)J(0)$ in a series of (nonperturbative) local operators $\widehat{\mathcal{O}}$ and (perturbative) coefficient functions C_n

$$J(z)J(0) \;\sim\; \sum_n C_n(z^2)\; z^{\mu_1} z^{\mu_2} \ldots z^{\mu_n}\; \widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}$$

- Matrix elements of $\widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}$

M^2/Q^2 corrections

$$\langle N|\widehat{\mathcal{O}}_{\mu_1\mu_2\cdots\mu_n}|N\rangle \;=\; \mathcal{A}_n(\mu^2)\; p_{\mu_1}p_{\mu_2}\cdots p_{\mu_n} \;-\; \text{traces}$$

- Moments of structure function F_2

$$M_n(Q^2) \equiv \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$
$$= \sum_i \widetilde{C}_n^i(Q^2)\; \mathcal{A}_n^i(Q^2/\mu^2)$$

where $\widetilde{C}_n(Q^2)$ is Fourier transform of $C_n(z^2)$

- Reconstruct structure function from moments via inverse Mellin transform

- **Parton model:** $$F_2(x,Q^2) = x \sum_q e_q^2\; q(x,Q^2)$$

probability to find quark type "q" in nucleon, carrying (light-cone) momentum fraction x

Duality in QCD

Operator product expansion

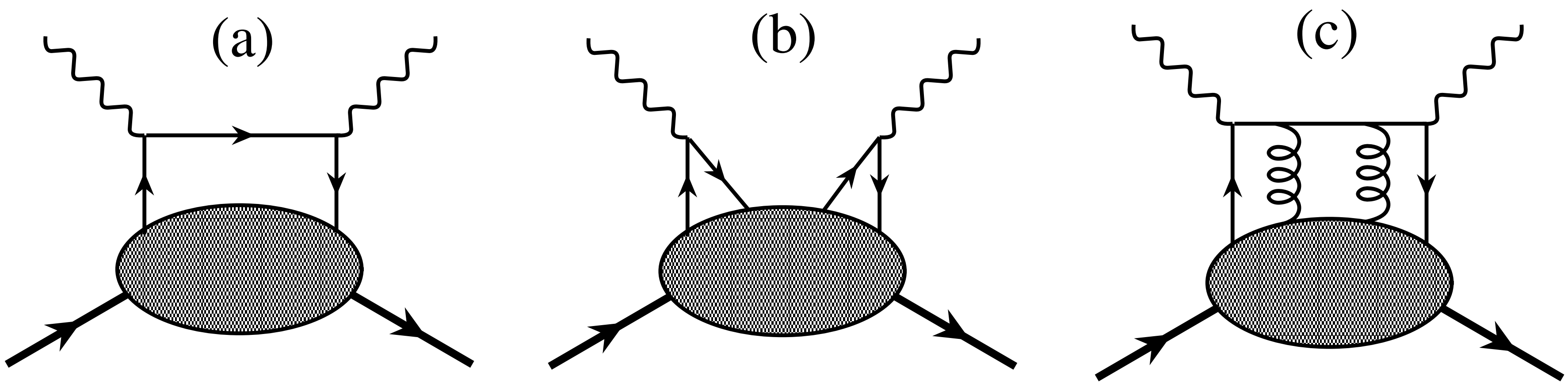
$\Longrightarrow$ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$

leading twist *higher twist*



$\tau = 2$ $\tau > 2$

single quark scattering

qq and qg correlations

Duality in QCD

Operator product expansion

$\Longrightarrow$ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of Q^2

$\Longrightarrow$ higher twist terms $A_n^{(\tau>2)}$ small

Duality in QCD

Operator product expansion

$\Longrightarrow$ expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x,Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

Duality $\Longleftrightarrow$ suppression of higher twists

de Rujula, Georgi, Politzer, Ann. Phys. 103 (1975) 315

Applications of duality

If higher twists are small (duality "works")

$\Longrightarrow$ can use single-parton approximation to describe structure functions

$\Longrightarrow$ extract *leading twist* parton distributions

If duality is violated, and if violations are small

$\Longrightarrow$ can use duality violations to extract *higher twist* matrix elements

$\Longrightarrow$ learn about nonperturbative qq or qg correlations

DIS at finite Q^2

- Formulation in terms of usual ("Cornwall-Norton") moments mixes operators of same *twist*, but different *spin*, n

  → irrelevant at large Q^2, but important at intermediate $Q^2/\nu^2 = 4M^2x^2/Q^2$

  → "target mass corrections" associated with higher spin operators (trace terms in OPE)

- Nachtmann (1973) constructed moments in which only operators with spin n contribute to the n-2 moment of structure function

  → automatically accounts for *kinematical* finite M^2/Q^2 effects

Nachtmann moments

Operator product expansion

$$\int d^4x\ e^{iq\cdot x}\langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}}\frac{2^{2k}}{Q^{4k}}\underbrace{A_{2k}\Pi_{\mu_1\cdots\mu_{2k}}}_{\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle}$$

local operators

Georgi, Politzer (1976)

$$\Pi_{\mu_1\cdots\mu_{2k}} = p_{\mu_1}\cdots p_{\mu_{2k}} - (g_{\mu_i\mu_j}\ \text{terms})$$

$$= \sum_{j=0}^{k}(-1)^j\frac{(2k-j)!}{2^j(2k)^j}g\cdots g\ p\cdots p$$

traceless, symmetric rank-$2k$ tensor

n-th Nachtmann moment of F_2 structure function

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

$\longrightarrow$ n-th moment of PDFs at finite Q^2

Relate Nachtmann and CN moments

$$\mu_2^n(Q^2) = M_2^n(Q^2) - \frac{n(n-1)}{n+2} \frac{M^2}{Q^2} M_2^{n+2}(Q^2)$$
$$+ \frac{n(n^2-1)}{2(n+3)} \frac{M^4}{Q^4} M_2^{n+4}(Q^2) - \frac{n(n^2-1)}{6} \frac{M^6}{Q^6} M_2^{n+6}(Q^2) + \cdots$$

$\longrightarrow$ mixing between lower & higher CN moments

n-th Cornwall-Norton moment of F_2 structure function

$$M_2^n(Q^2) = \int dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

"quark distribution function"

$$\Rightarrow \quad A_n = \int_0^1 dy\ y^n\ F(y)$$

$$F(y) \equiv \frac{F_2(y)}{y^2}$$

- take inverse Mellin transform (+ tedious manipulations)

  ⇒ target mass corrected structure function

$$F_2^{\mathrm{GP}}(x,Q^2) = \frac{x^2}{r^3}F(\xi) \; + \; 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 d\xi' \; F(\xi')$$
$$+ \; 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

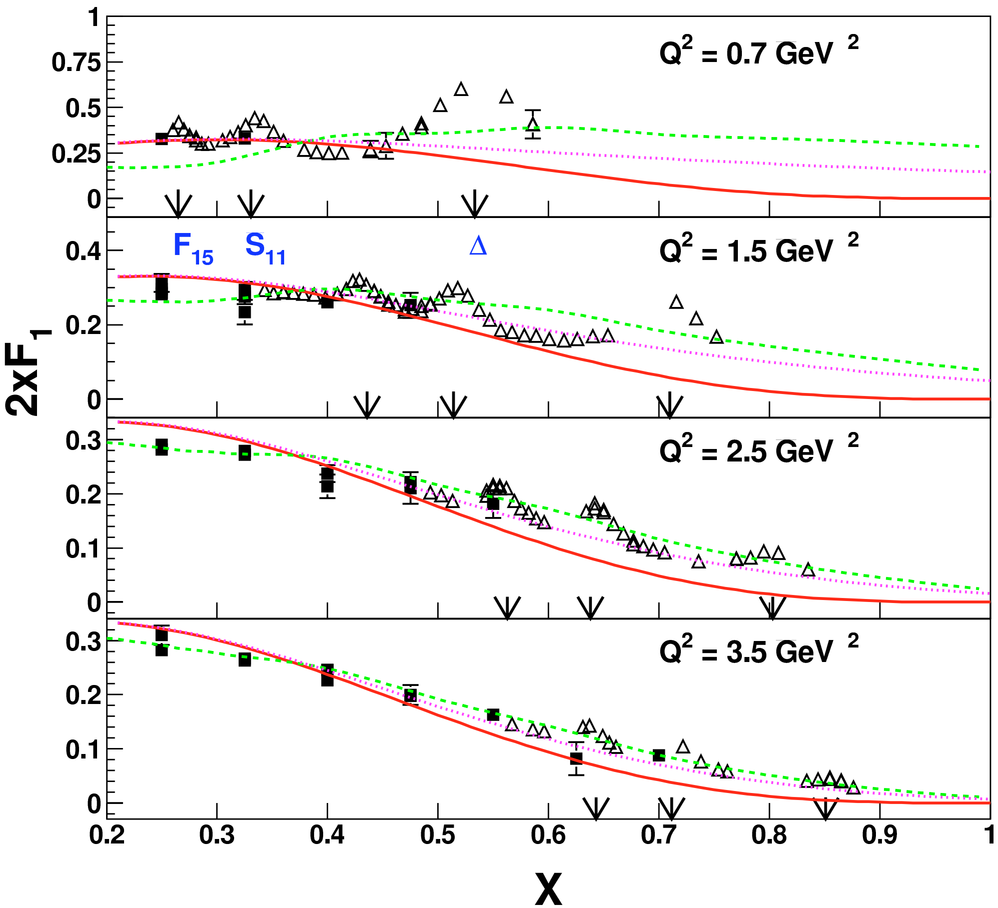
Georgi-Politzer prescription for TMCs

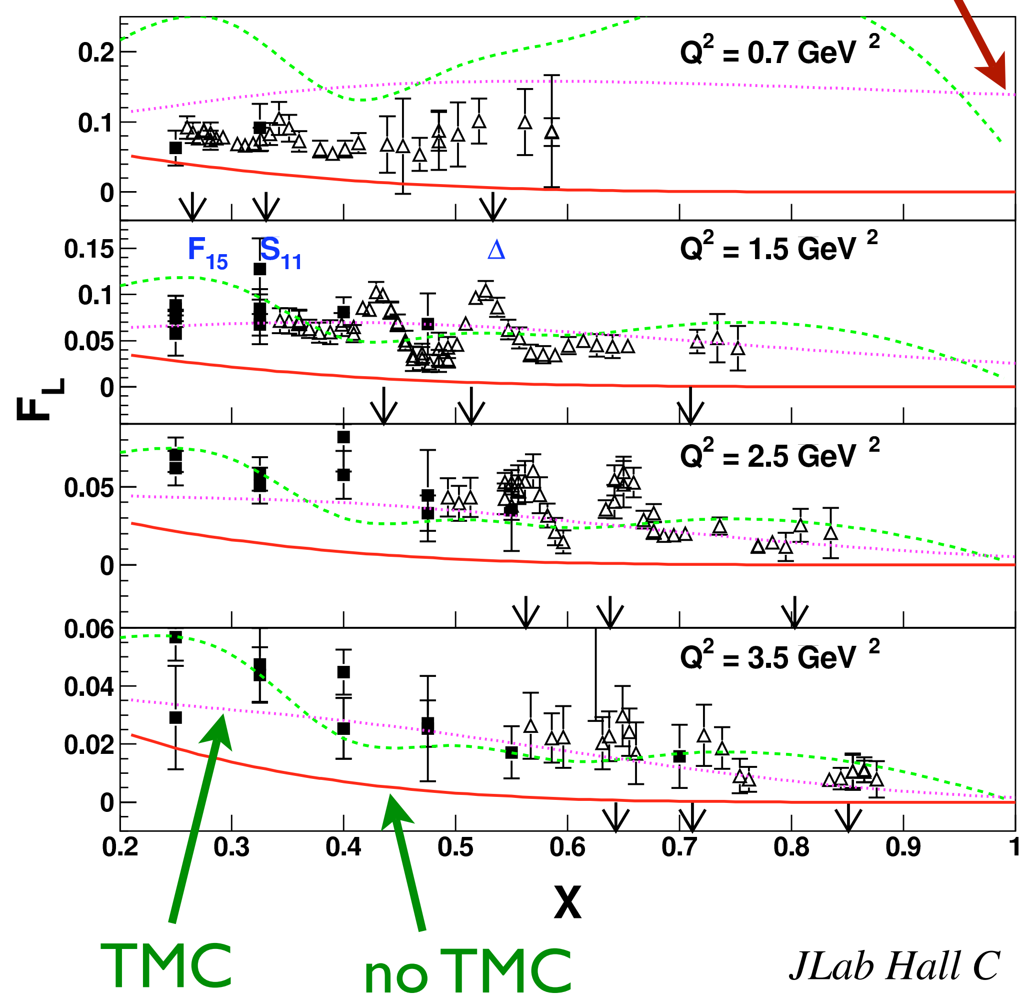
$$\xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1+4x^2M^2/Q^2}$$

... similarly for other structure functions F_1, F_L

numerically...

non-zero at x=1 !





JLab Hall C

➡ TMCs significant at large x^2/Q^2, especially for F_L

Threshold problem

- if $F(y) \sim (1-y)^{\beta}$ at large y

  then since $\xi_0 \equiv \xi(x=1) < 1$

$\Rightarrow$ $F(\xi_0) > 0$ $\longleftarrow$ recall $F_2^{\rm GP}(x,Q^2) = \frac{x^2}{r^3} F(\xi) + \cdots$

$\Rightarrow$ $F_2^{\rm GP}(x=1,Q^2) > 0$

is this physical?

$\Rightarrow$ problem with GP formulation?

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

→ n fixed, $Q^2 \to \infty$

$$\mu_2^n(Q^2) \to (\ln Q^2/\Lambda^2)^{-\lambda_n} \; A_n$$

$$A_n = \int_0^1 d\xi \; \xi^n \; F(\xi)$$

→ $n \to \infty$, Q^2 fixed

"regularized" amplitudes
(threshold-independent)

$$\mu_2^n(Q^2) \to \xi_0^n(Q^2) \; \widetilde{\mu}_2^n(Q^2)$$

Possible solutions

- Johnson/Tung - modified threshold factor

Nachtmann moment

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3} \left(\frac{3 + 3(n+1)r + n(n+2)r^2}{(n+2)(n+3)} \right) F_2(x, Q^2)$$

ansatz $\mu_2^n(Q^2) = \xi_0^n(Q^2)\ (\ln Q^2/\Lambda^2)^{-\lambda_n}\ A_n$

→ consistent with asymptotic pQCD behavior

→ not unique!

Bitar, Johnson, Tung
PLB 83B (1979) 114

Possible solutions

- Johnson/Tung - modified threshold factor

moreover, if identify A_n with $M_2^n = \int_0^1 dx\; x^{n-2}\; F_2(x)$

$$\mu_2^n(Q^2) = \xi_0^n(Q^2)\; M_2^n(Q^2)$$

$$\Longrightarrow \qquad M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{nM^2}{Q^2} M_2^n + \cdots$$

cf. exact expression

$$M_2^n(Q^2) = \mu_2^n(Q^2) + \frac{n(n-1)}{n+2}\frac{M^2}{Q^2} M_2^{n+2} + \cdots$$

$\Longrightarrow$ inconsistency at low Q^2 ?

Possible solutions

- Kulagin/Petti - expand expressions in $1/Q^2$

$$F_2^{\mathrm{TMC}}(x,Q^2) = \left(1-\frac{4x^2M^2}{Q^2}\right)F_2^{\mathrm{LT}}(x,Q^2)$$

$$+ \quad \frac{x^3M^2}{Q^2}\left(6\int_x^1\frac{\mathrm{d}z}{z^2}F_2^{\mathrm{LT}}(z,Q^2)-\frac{\partial}{\partial x}F_2^{\mathrm{LT}}(x,Q^2)\right)$$

Kulagin, Petti, NPA765 (2006) 126

→ has correct threshold behavior

Alternative solution

- work with ξ_0 dependent PDFs

→ n-th moment A_n of distribution function

$$A_n = \int_0^{\xi_{\max}} d\xi \; \xi^n \; F(\xi)$$

→ what is $\xi_{\max}$?

- GP use $\xi_{\max} = 1, \;\; \xi_0 < \xi < 1$ unphysical
- strictly, should use $\xi_{\max} = \xi_0$

Steffens, WM
PRC 73 (2006) 055202

Alternative solution

- what is effect on phenomenology?

  $\longrightarrow$ try several "toy distributions"

standard TMC ("sTMC")

$$q(\xi) = \mathcal{N}\,\xi^{-1/2}\,(1-\xi)^3\,, \qquad \xi_{\max} = 1$$

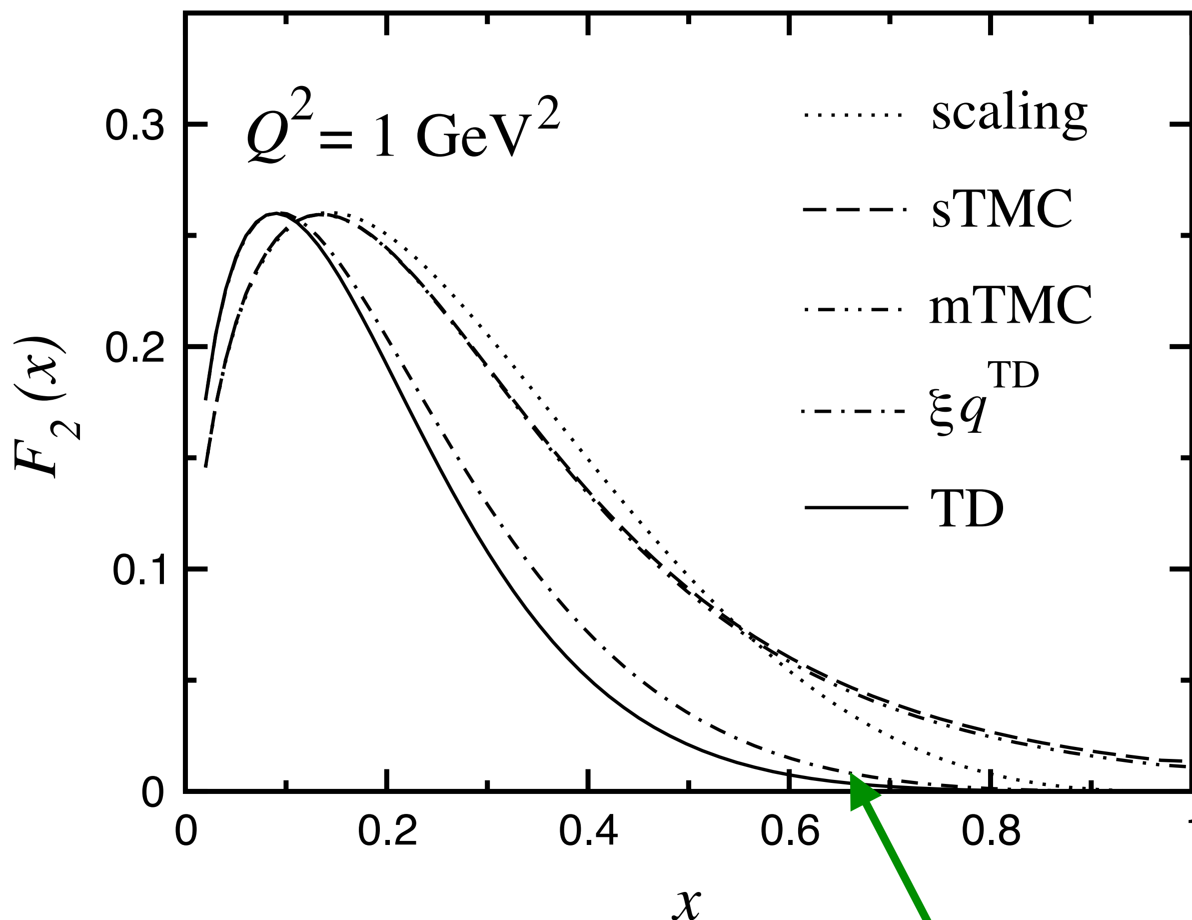
modified TMC ("mTMC")

$$q(\xi) = \mathcal{N}\,\xi^{-1/2}\,(1-\xi)^3\,\Theta(\xi-\xi_0), \qquad \xi_{\max} = \xi_0$$

threshold dependent ("TD")

$$q^{\mathrm{TD}}(\xi) = \mathcal{N}\,\xi^{-1/2}\,(\xi_0-\xi)^3\,, \qquad \xi_{\max} = \xi_0$$

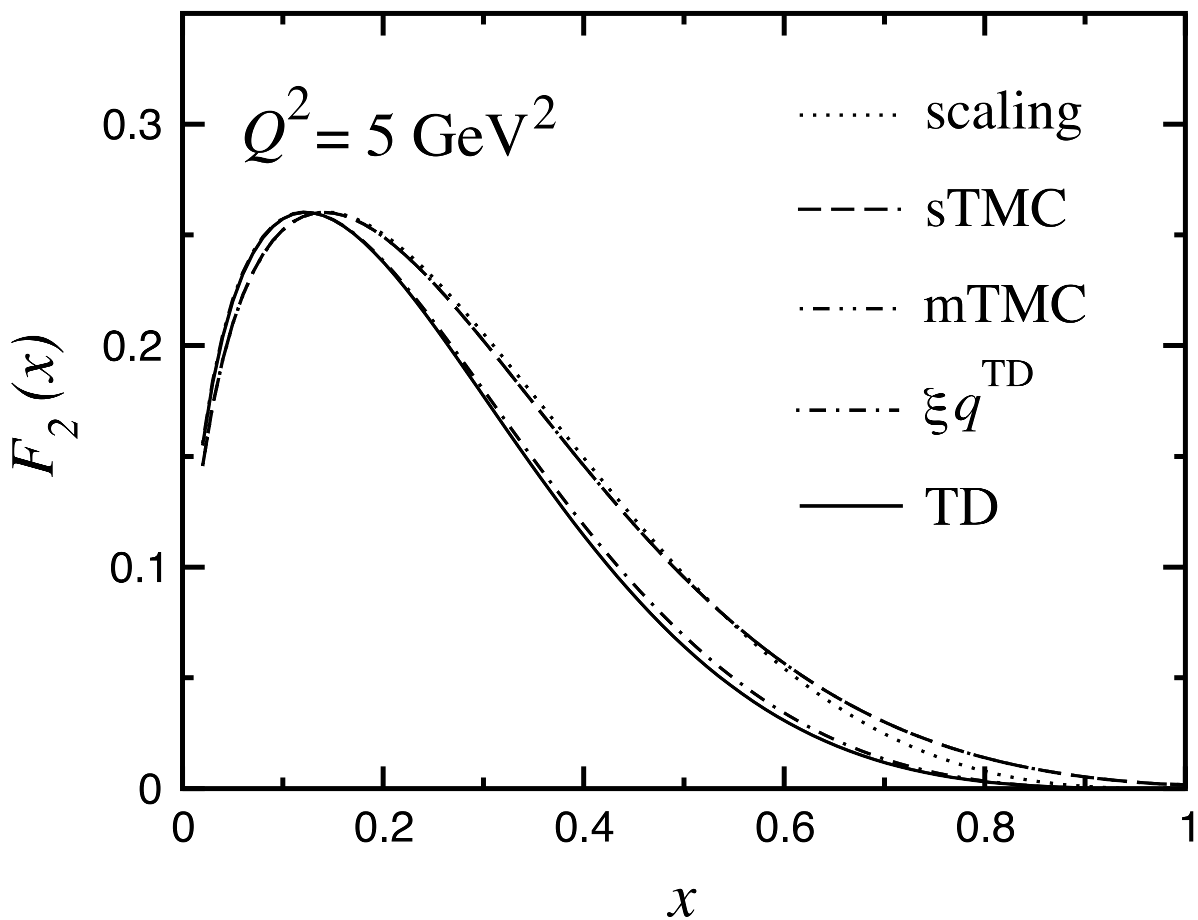
TMCs in F_2



non-zero at $x = 1$

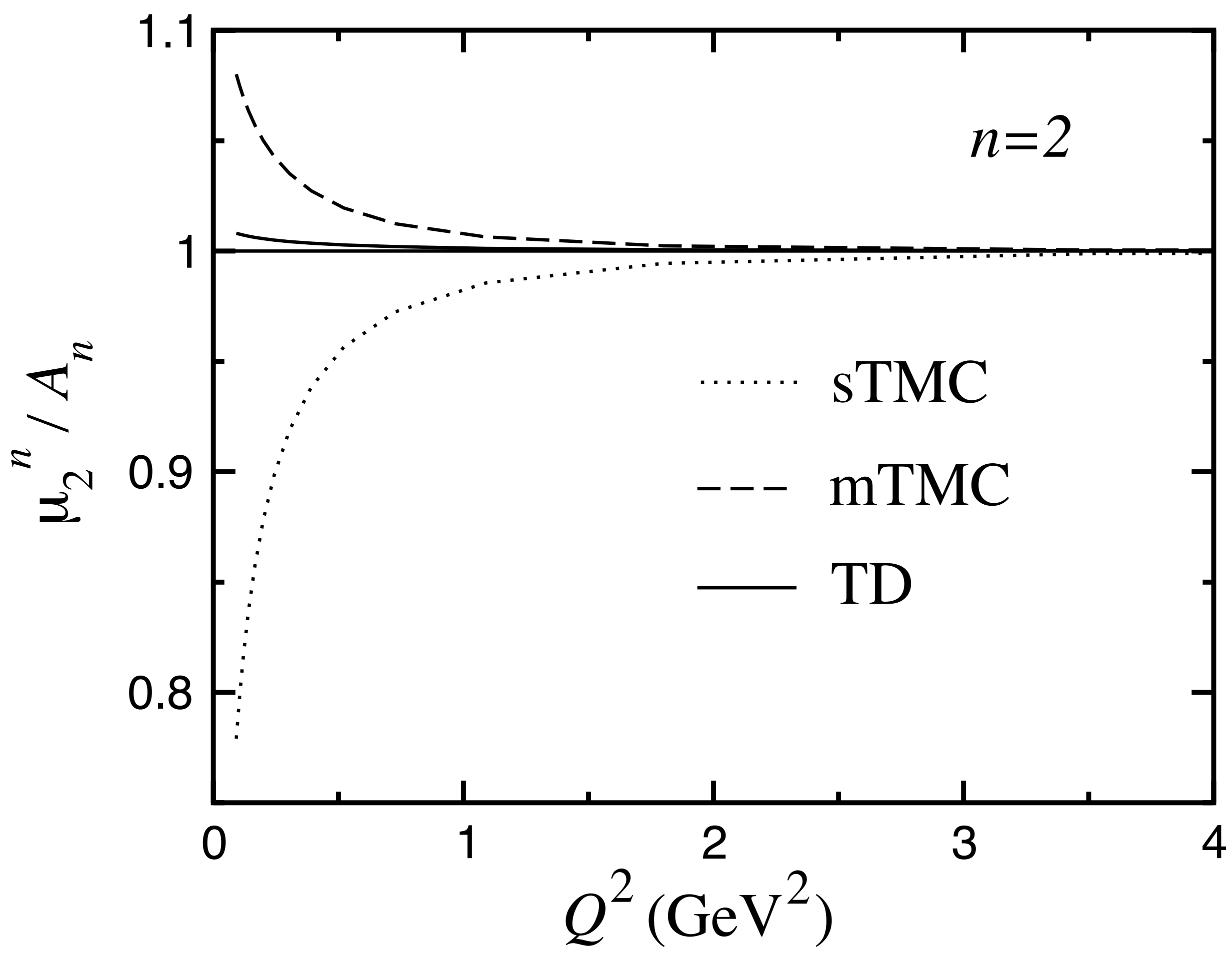
→ correct threshold behavior for "TD" correction

TMCs in F_2



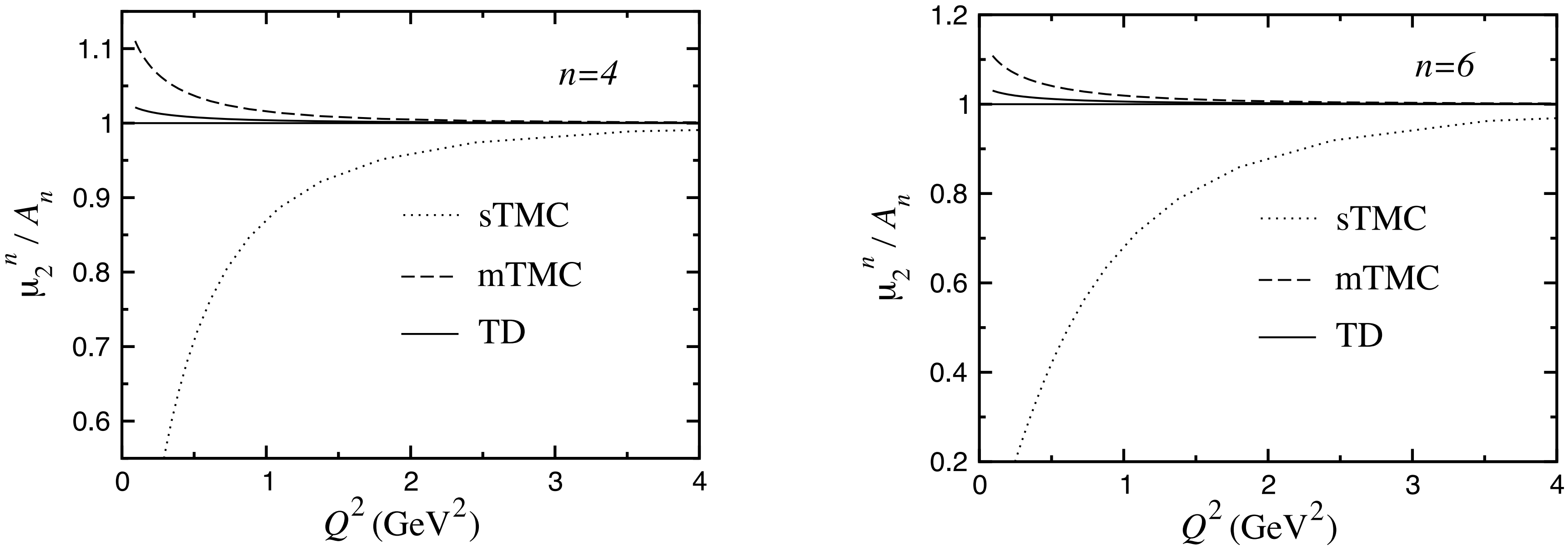
→ effect small at higher Q^2

Nachtmann F_2 moments



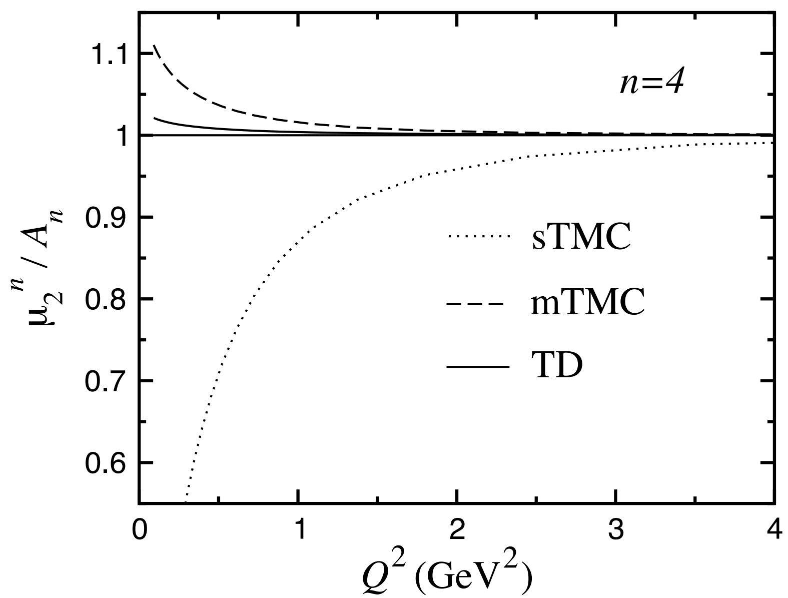
→ moment of structure function agrees with moment of PDF to 1% down to very low Q^2

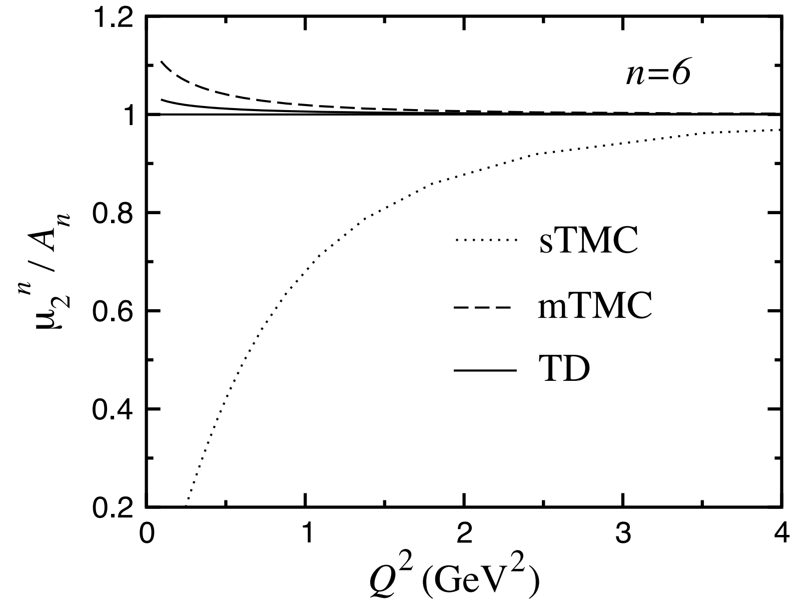
Nachtmann F_2 moments



→ higher moments show much weaker Q^2 dependence than sTMC & mTMC prescriptions

Nachtmann F_2 moments





→ $$\frac{\mu_2^n(\text{finite } Q^2)}{A_n(\text{finite } Q^2)} = \frac{\mu_2^n(Q^2 \to \infty)}{A_n(Q^2 \to \infty)}$$

→ extract PDFs from structure function data at lower Q^2

Summary

- Remarkable confirmation of quark-hadron duality in structure functions

  → higher twists "small" down to low Q^2 (~ 1 GeV^2)

- OPE "organizes" duality violations in terms of higher twists *but* need quark models to understand origin of resonance cancellations

- Target mass corrections important at low Q^2

- New TMC formulation avoids "threshold problem"

  → much weaker Q^2 dependence of moments

  → introduces ξ and ξ_0 dependent PDFs

The End