Transverse (Spin) Structure of Hadrons

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Outline

- electromagnetic form factor $\Rightarrow$ charge distribution in position space
- deep-inelastic lepton-nucleon scattering (DIS) $\Rightarrow$ momentum distribution of quarks in nucleon
- deeply virtual Compton scattering (DVCS) $\Rightarrow$ generalized parton distributions (GPDs)
  - $\hookrightarrow$ **simultaneous** determination of (longitudinal) momentum and (transverse) position of quarks in the nucleon
  - $\hookrightarrow$ 3-d images of the nucleon where $x - y$ plane in position space and z axis in momentum space
- what is orbital angular momentum?
- single-spin asymmetries (SSA)
- transverse force on quarks in polarized DIS

Physical Interpretation of Form Factors

- Nonrelativistic quantum mechanics (NRQM)
- Relativistic effects
- Breit frame interpretation
- Infinite momentum frame (IMF)

Form Factors in nonrel. QM

- Potential scattering

$$\frac{d\sigma}{d\Omega} = |f(\vec{q})|^2$$

- Born approx. ("1-photon exchange" for Coulomb interaction)

$$f(\vec{q}) = \int d^3r e^{-i\vec{q}\cdot\vec{r}} V(\vec{r})$$

- Electro-magnetic interactions: $V(\vec{r}) = eA^0(\vec{r})$ with $\vec{\nabla}^2 A^0(\vec{r}) = -4\pi\rho(\vec{r})$, i.e. $f(\vec{q}) = \frac{4\pi e}{\vec{q}^2}\int d^3r e^{-i\vec{q}\cdot\vec{r}}\rho(\vec{r})$

$$\Rightarrow \qquad \frac{d\sigma}{d\Omega} = |f(\vec{q})|^2 = \left.\frac{d\sigma}{d\Omega}\right|_{point} \times |F(\vec{q})|^2 \quad \text{with} \quad F(\vec{q}) = \int d^3r e^{-i\vec{q}\cdot\vec{r}}\rho(\vec{r})$$

$$\hookrightarrow \quad \boxed{\frac{d\sigma}{d\Omega} \longrightarrow F(\vec{q}) \longrightarrow \rho(\vec{r})}$$

Form Factor (Interpretation)

- small $\vec{q}^2$ expansion

$$F(\vec{q}) = 1 - \frac{1}{6}\langle r^2 \rangle \vec{q}^2 + \mathcal{O}(\vec{q}^4)$$

- Hofstadter (NP 1961): first measurement of $F(\vec{q})$ using electron nucleon scattering

$\hookrightarrow$ $\sqrt{\langle r_p^2 \rangle} \approx 0.86\ fm$

Issues

- Form factor of a “moving target”
- Relativistic effects
- Breit frame
- Infinite momentum frame (IMF)

Moving Target (Meson, Nucleon,..)

- Fixed target: $F(\vec{q}) \overset{FT}{\leftrightarrow} \rho(\vec{r})$ trivial

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$$\frac{d\sigma}{d\Omega} \propto \left|\langle \vec{p}' \left| H_{int} \right| \vec{p} \rangle\right|^2$$

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$(\vec{q} = \vec{p} - \vec{p}')$

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- Relation to $\rho(\vec{r})$?
- What is $\rho(\vec{r})$?

Form factor vs. charge distribution (nonrel.)

- plane wave states have uniform charge distribution

$\hookrightarrow$ meaningful definition of $\rho(\vec{r})$ requires that state is localized in position space!

$\hookrightarrow$ define localized state (center of mass frame)

$$\left|\vec{R}=\vec{0}\right\rangle \equiv \mathcal{N}\int d^3\vec{p}\,|\vec{p}\rangle$$

- define charge distribution (for this localized state)

$$\rho(\vec{r}) \equiv \left\langle \vec{R}=\vec{0}\right| j^0(\vec{r}) \left|\vec{R}=\vec{0}\right\rangle$$

Form factor vs. charge distribution (non-rel.)

- use translational invariance to relate to same matrix element that appears in def. of form factor

$$
\begin{aligned}
\rho(\vec{r}) &\equiv \left\langle \vec{R}=\vec{0} \middle| j^0(\vec{r}) \middle| \vec{R}=\vec{0} \right\rangle \\
&= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \, \langle \vec{p}' | \, j^0(\vec{r}) \, | \vec{p} \rangle \\
&= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \langle \vec{p}' | \, j^0(\vec{0}) \, | \vec{p} \rangle e^{i\vec{r}\cdot(\vec{p}-\vec{p}')}, \\
&= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}' \, F\left(-(\vec{p}'-\vec{p})^2\right) e^{i\vec{r}\cdot(\vec{p}-\vec{p}')}
\end{aligned}
$$

$\hookrightarrow$

$$
\boxed{\rho(\vec{r}) = \int \frac{d^3\vec{q}}{(2\pi)^3} F(-\vec{q}^2) e^{i\vec{q}\cdot\vec{r}}}
$$

back

Form Factors (relativistic)

- Lorentz invariance, parity, current conservation $\Rightarrow$

$$\langle p' \,|j^\mu(0)|\, p\rangle = \begin{cases} \left(p^\mu + p^{\mu\prime}\right) F(q^2) & \text{(spin } 0\text{)} \\ \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2)\right] u(p) & \text{(spin } \tfrac{1}{2}\text{)} \end{cases}$$

with $q^\mu = p^\mu - p^{\mu\prime}$.

- issues:
  - "energy factors" spoil simple interpretation of form factors as FT of charge distributions
  - q^2 depends on $\vec{p} + \vec{p}'$

Form Factor vs. Charge Distribution (rel.)

- wave packet

$$|\Psi\rangle = \int \frac{d^3p \;\; \psi(\vec{p})}{\sqrt{2E_{\vec{p}}(2\pi)^3}} \, |\vec{p}\rangle \, ,$$

- $E_{\vec{p}} = \sqrt{M^2 + \vec{p}^2}$ and covariant normalization $\langle \vec{p}' | \vec{p} \rangle = 2E_{\vec{p}} \delta(\vec{p}' - \vec{p})$
- Fourier transform of charge distribution in the wave packet

$$\begin{aligned}
\tilde{\rho}(\vec{q}) &\equiv \int d^3x e^{-i\vec{q}\cdot\vec{x}} \, \langle \Psi | \, j^0(\vec{x}) \, | \Psi \rangle \\
&= \int \frac{d^3p}{\sqrt{2E_{\vec{p}} 2E_{\vec{p}'}}} \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p}) \, \langle \vec{p}' | \, j^0(\vec{0}) \, | \vec{p} \rangle \\
&= \frac{1}{2} \int d^3p \frac{E_{\vec{p}} + E_{\vec{p}'}}{\sqrt{E_{\vec{p}} E_{\vec{p}'}}} \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p}) F(q^2).
\end{aligned}$$

Form Factor vs. Charge Distribution (rel.)

- Nonrelativistic case:
  - $$\frac{E_{\vec{p}} + E_{\vec{p}'}}{2\sqrt{E_{\vec{p}} E_{\vec{p}'}}} = 1 \qquad \text{and} \qquad q^2 = -\vec{q}^2$$

$\hookrightarrow$ Fourier transform of charge distribution in the wave packet

$$\tilde{\rho}(\vec{q}) = \int d^3p \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p}) F(\vec{q}^2)$$

- choose $\Psi(\vec{p})$ very localized in position space
  $\Psi^*(\vec{p} + \vec{q}) \approx \Psi^*(\vec{p})$

$$\hookrightarrow \qquad \tilde{\rho}(\vec{q}) = F(\vec{q}^2)$$

Form Factor vs. Charge Distribution (rel.)

- Relativistic corrections (example rms radius):

$$\tilde{\rho}(\vec{q}^2) = 1 - \frac{R^2}{6}\vec{q}^2 - \frac{R^2}{6}\int d^3p\,|\Psi(\vec{p})|^2\,\frac{(\vec{q}\cdot\vec{p})^2}{E_{\vec{p}}^2} + \int d^3p\left|\vec{q}\cdot\vec{\nabla}\Psi(\vec{p})\right|^2 - \frac{1}{8}\int d^3p\,|\Psi(\vec{p})|^2\,\frac{(\vec{q}\cdot\vec{p})^2}{E_{\vec{p}}^4},$$

[R^2 defined as usual: $F(q^2) = 1 + \frac{R^2}{6}q^2 + \mathcal{O}(q^4)$]

- If one completely localizes the wave packet, i.e. $\int d^3p\left|\vec{q}\cdot\vec{\nabla}\Psi(\vec{p})\right|^2 \to 0$, then relativistic corrections diverge ($\Delta x\Delta p \sim 1$)

$$\frac{R^2}{6}\int d^3p\,|\Psi(\vec{p})|^2\,\frac{(\vec{q}\cdot\vec{p})^2}{E_{\vec{p}}^2} \to \infty, \qquad \frac{1}{8}\int d^3p\,|\Psi(\vec{p})|^2\,\frac{(\vec{q}\cdot\vec{p})^2}{E_{\vec{p}}^4} \to \infty$$

Sachs Form Factors

- in rest frame, rel. corrections contribute $\Delta R^2 \sim \lambda_C^2 = \frac{1}{M^2}$ identification of charge distribution in rest frame with Fourier transformed form factor only unique down to scale λ_C
- standard remedy: interpret $F(\vec{q})$ as Fourier transform of charge distribution in "Breit frame" $\vec{p}' = -\vec{p}$
- $G_E(q^2) \equiv F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2)$ $\qquad$ $G_M(q^2) \equiv F_1(q^2) + F_2(q^2)$
- Physics (Breit "frame"):

$$\begin{aligned} \left\langle \vec{q}/2 \left| j^0(0) \right| -\vec{q}/2 \right\rangle &= 2MG_E(q^2)\delta_{s's} \\ \left\langle \vec{q}/2 \left| \vec{j}(0) \right| -\vec{q}/2 \right\rangle &= G_M(q^2)\chi_{s'}^\dagger i\vec{\sigma} \times \vec{q}\chi_s \end{aligned}$$

$\hookrightarrow$ $G_E \xrightarrow{FT}$ charge density; $\qquad$ $G_M \xrightarrow{FT}$ magnetization density;

- flaw of this interpretation: Breit "frame" is not one single frame but a different frame for each momentum transfer

Infinite Momentum Frame Interpretation

- rel. corrections governed by $\frac{\vec{p}\cdot\vec{q}}{E_{\vec{p}}^2}$ and $\frac{\vec{q}^2}{E_{\vec{p}}^2}$

  consider wave packet $\Psi(\vec{p}_\perp)$ in transverse direction, with
  - sharp longitudinal momentum $P_z \to \infty$
  - transverse size of wave packet $r_\perp$, with $R \gg r_\perp \gg \frac{1}{P_z}$
  - take purely transverse momentum transfer

$\hookrightarrow$ $\tilde{\rho}(\vec{q}_\perp) = F(\vec{q}_\perp^2)$

$\hookrightarrow$ form factor can be interpreted as Fourier transform of charge distribution w.r.t. impact parameter in ∞ momentum frame (without λ_C uncertainties!)

  - impact parameter measured w.r.t. $\perp$ center of momentum $\mathbf{R}_\perp = \sum_{i\in q,g} x_i \mathbf{r}_\perp^i$

Derivation in LF-Coordinates

- light-front (LF) coordinates

$$p^+ = \frac{1}{\sqrt{2}}\left(p^0 + p^3\right) \qquad p^- = \frac{1}{\sqrt{2}}\left(p^0 - p^3\right)$$

- form factor for spin $\frac{1}{2}$ target (Lorentz invariance, parity, charge conservation)

$$\langle p' \,|j^\mu(0)|\, p\rangle = \bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]u(p)$$

with $q^\mu = p^\mu - p^{\mu\prime}$.

- If $q^+ = 0$ (Drell-Yan-West frame) then

$$\left\langle p', \uparrow \left|j^+(0)\right| p, \uparrow\right\rangle = 2p^+ F_1(-\mathbf{q}_\perp^2)$$

$F(\mathbf{q}_\perp^2) \to \rho(\mathbf{r}_\perp)$ in LF-Coordinates

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define charge distribution in impact parameter space

$$\rho(\mathbf{b}_\perp) \equiv \frac{1}{2p^+} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| j^+(0^-, \mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle$$

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- use translational invariance to relate to same matrix element that appears in def. of form factor

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&= \frac{|\mathcal{N}|^2}{2p^+} \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \left\langle p^+, \mathbf{p}'_\perp \right| j^+(0^-, \mathbf{b}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle \\
&= \frac{|\mathcal{N}|^2}{2p^+} \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \left\langle p^+, \mathbf{p}'_\perp \right| j^+(0^-, \mathbf{0}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \\
&= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp F_1(-\mathbf{q}_\perp^2) e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp}
\end{aligned}
$$

$$
\hookrightarrow \quad \boxed{\rho(\mathbf{b}_\perp) = \frac{\int d^2\mathbf{q}_\perp}{(2\pi)^2} F_1(-\mathbf{q}_\perp^2) e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp}}
$$

Boosts in NRQM

$$\vec{x}' = \vec{x} + \vec{v}t \qquad\qquad t' = t$$

purely kinematical (quantization surface $t = 0$ inv.)

$\hookrightarrow$ 1. boosting wavefunctions very simple

$$\Psi_{\vec{v}}(\vec{p}_1, \vec{p}_2) = \Psi_{\vec{0}}(\vec{p}_1 - m_1\vec{v}, \vec{p}_2 - m_2\vec{v}).$$

2. dynamics of center of mass

$$\vec{R} \equiv \sum_i x_i \vec{r}_i \qquad \text{with} \qquad x_i \equiv \frac{m_i}{M}$$

decouples from the internal dynamics

Relativistic Boosts

$$t' = \gamma\left(t + \frac{v}{c^2}z\right), \qquad z' = \gamma\left(z + vt\right) \qquad \mathbf{x}'_\perp = \mathbf{x}_\perp$$

- generators satisfy Poincaré algebra:

$$\begin{aligned}
[P^\mu, P^\nu] &= 0 \\
[M^{\mu\nu}, P^\rho] &= i\left(g^{\nu\rho}P^\mu - g^{\mu\rho}P^\nu\right) \\
\left[M^{\mu\nu}, M^{\rho\lambda}\right] &= i\left(g^{\mu\lambda}M^{\nu\rho} + g^{\nu\rho}M^{\mu\lambda} - g^{\mu\rho}M^{\nu\lambda} - g^{\nu\lambda}M^{\mu\rho}\right)
\end{aligned}$$

- rotations: $M_{ij} = \varepsilon_{ijk}J_k$, boosts: $M_{i0} = K_i$.
- $[K_i, P_j] = i\delta_{ij}P^0$

$\hookrightarrow$ boost operator contains interactions!

$\hookrightarrow$ in general, no useful generalization of concept of center of mass to a relativistic theory

Galilean subgroup of $\perp$ boosts

introduce generator of $\perp$ 'boosts':

$$B_x \equiv M^{+x} = \frac{K_x + J_y}{\sqrt{2}} \qquad B_y \equiv M^{+y} = \frac{K_y - J_x}{\sqrt{2}}$$

Poincaré algebra $\Longrightarrow$ commutation relations:

$$\begin{aligned} [J_3, B_k] &= i\varepsilon_{kl} B_l & [P_k, B_l] &= -i\delta_{kl} P^+ \\ [P^-, B_k] &= -iP_k & [P^+, B_k] &= 0 \end{aligned}$$

with $k, l \in \{x, y\}$, $\varepsilon_{xy} = -\varepsilon_{yx} = 1$, and $\varepsilon_{xx} = \varepsilon_{yy} = 0$.

Galilean subgroup of $\perp$ boosts

Together with $[J_z, P_k] = i\varepsilon_{kl} P_l$, as well as

$$\begin{aligned} \left[P^-, P_k\right] &= \left[P^-, P^+\right] = \left[P^-, J_z\right] = 0 \\ \left[P^+, P_k\right] &= \left[P^+, B_k\right] = \left[P^+, J_z\right] = 0. \end{aligned}$$

Same as commutation relations among generators of nonrel. boosts, translations, and rotations in x-y plane, provided one identifies

$$\begin{aligned} P^- &\longrightarrow \text{Hamiltonian} \\ \mathbf{P}_\perp &\longrightarrow \text{momentum in the plane} \\ P^+ &\longrightarrow \text{mass} \\ L_z &\longrightarrow \text{rotations around } z\text{-axis} \\ \mathbf{B}_\perp &\longrightarrow \text{generator of boosts in the plane,} \end{aligned}$$

Consequences of Galilean Subgroup

- many results from NRQM carry over to $\perp$ boosts in IMF, e.g.
- $\perp$ boosts kinematical

$$\begin{aligned}\Psi_{\boldsymbol{\Delta}_\perp}(x,\mathbf{k}_\perp) &= \Psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\boldsymbol{\Delta}_\perp)\\ \Psi_{\boldsymbol{\Delta}_\perp}(x,\mathbf{k}_\perp,y,\mathbf{l}_\perp) &= \Psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\boldsymbol{\Delta}_\perp,y,\mathbf{l}_\perp - y\boldsymbol{\Delta}_\perp)\end{aligned}$$

- Transverse center of momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{\perp,i}$ plays role similar to NR center of mass, e.g. $\int d^2\mathbf{p}_\perp \, |p^+, \mathbf{p}_\perp\rangle$ corresponds to state with $\mathbf{R}_\perp = \mathbf{0}_\perp$.

Summary: Form Factor vs. Charge Distribution

- fixed target: FT of form factor = charge distribution in position space
- "moving" target:
  - nonrelativistically: FT of form factor = charge distribution in position space, where position is measured relative to **center of mass**
  - relativistic corrections usually make idendification $F(q^2) \overset{FT}{\leftrightarrow} \rho(\vec{r})$ ambigous at scale $\Delta R \sim \lambda_C = \frac{1}{M}$
  - Sachs form factors have interpretation as charge and magnetization density in Breit "frame"
  - Infinite momentum frame: form factors can be interpreted as transverse charge distribution in fast moving proton (without rel. corrections)

Deep Inelastic Scattering (DIS)

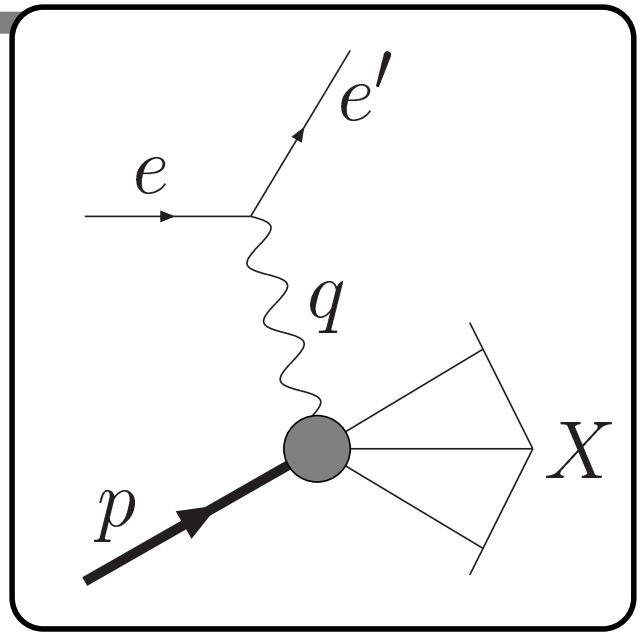
- high-energy lepton ($e^{\pm}$, $\mu^{\pm}$, ν,) nucleon scattering usually inelastic
- for $Q^2 = -q^2 = -(p-p')^2 \gg M_p^2 \sim 1$GeV^2, probe can resolve distance scales much smaller than the proton size

$\hookrightarrow$ deep inelastic scattering

- because of high Q^2, inclusive (i.e. sum over final states) cross section obtained from incoherent superposition of charged constituents

$\hookrightarrow$ DIS provided first direct evidence for the existence of quarks inside nucleons (Nobel Prize 1990: Freedman, Kendall, Taylor)

Deep Inelastic Scattering (DIS)



$$\nu = E - E'$$

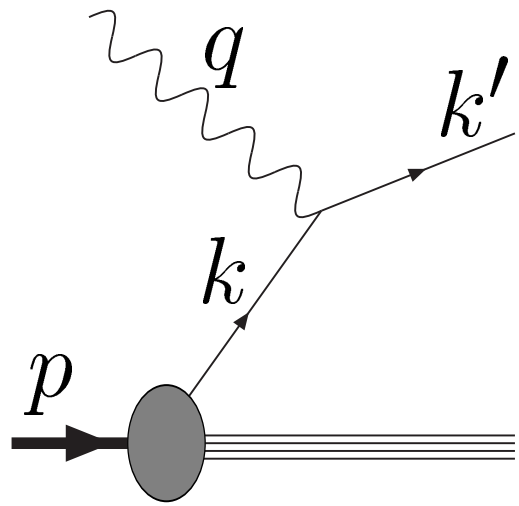
$$Q^2 \equiv -q^2 = 4EE' \sin^2 \tfrac{\theta}{2}$$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\pi\alpha^2}{MQ^4}\left\{W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2}\right\}$$

exp. result: Bjorken scaling

$$\left.\begin{array}{c} Q^2 \to \infty, \quad \nu \to \infty \\ x_{Bj} = Q^2/2M\nu \quad \text{fixed} \end{array}\right\} \Rightarrow \begin{array}{r} 2M\, W_1(Q^2,\nu) = F_1(x_{Bj}) \\ \nu\, W_2(Q^2,\nu) = F_2(x_{Bj}) \end{array}$$

Physical Meaning of $x_{Bj} = \frac{Q^2}{2p\cdot q}$



- Go to frame where $\mathbf{q}_\perp = 0$, i.e.

$$Q^2 = -q^2 = -2q^+q^- \qquad\qquad 2p\cdot q = 2q^-p^+ + 2q^+p^-$$

- Bjorken limit: $q^- \to \infty$, q^+ fixed

$\hookrightarrow$

$$x_{Bj} = -\frac{q^+q^-}{q^-p^+ + q^+p^-} \to -\frac{q^+}{p^+}$$

Physical Meaning of $x_{Bj} = \frac{Q^2}{2p\cdot q}$

- $x_{Bj} = -\frac{q^+}{p^+}$
- LC energy-momentum dispersion relation

$$k^- = \frac{m^2 + \mathbf{k}_\perp^2}{2k^+}$$

↪ struck quark with $k^{-\prime} = k^- + q^- \to \infty$ can only be on mass shell if $k^{+\prime} = k^+ + q^+ \approx 0$

↪

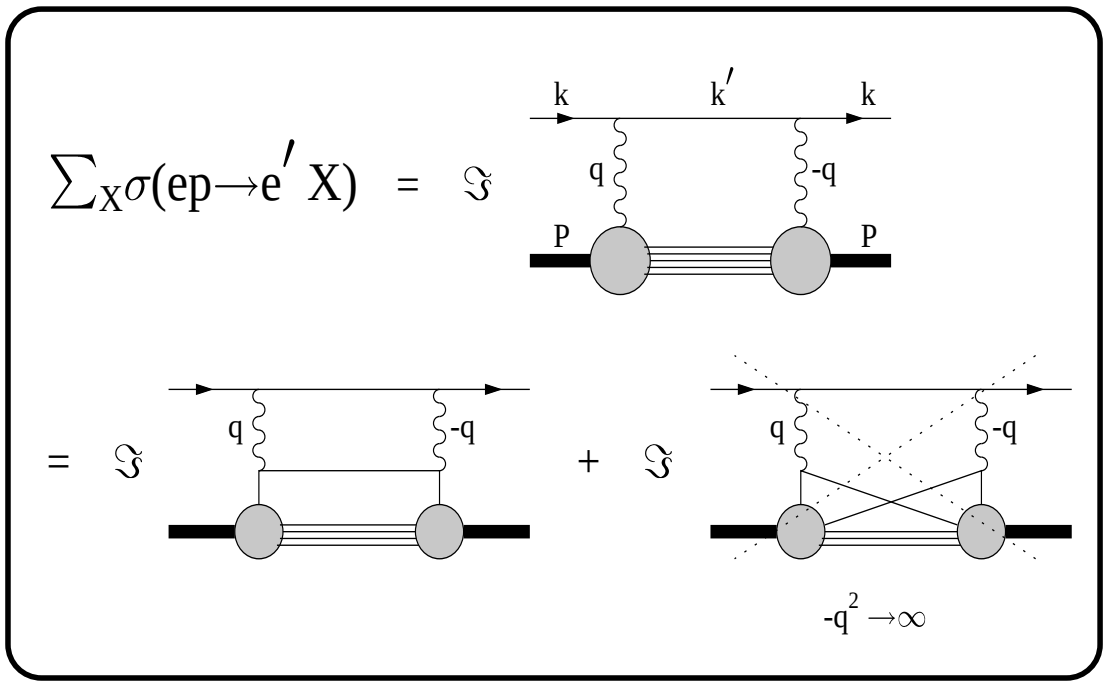
$$k^+ = -q^+ \qquad \Rightarrow \qquad x \equiv \frac{k^+}{p^+} = x_{Bj}$$

↪ x_{Bj} has physical meaning of light-cone momentum fraction carried by struck quark before it is hit by photon

DIS $\longrightarrow$ light-cone correlations

opt. theorem:

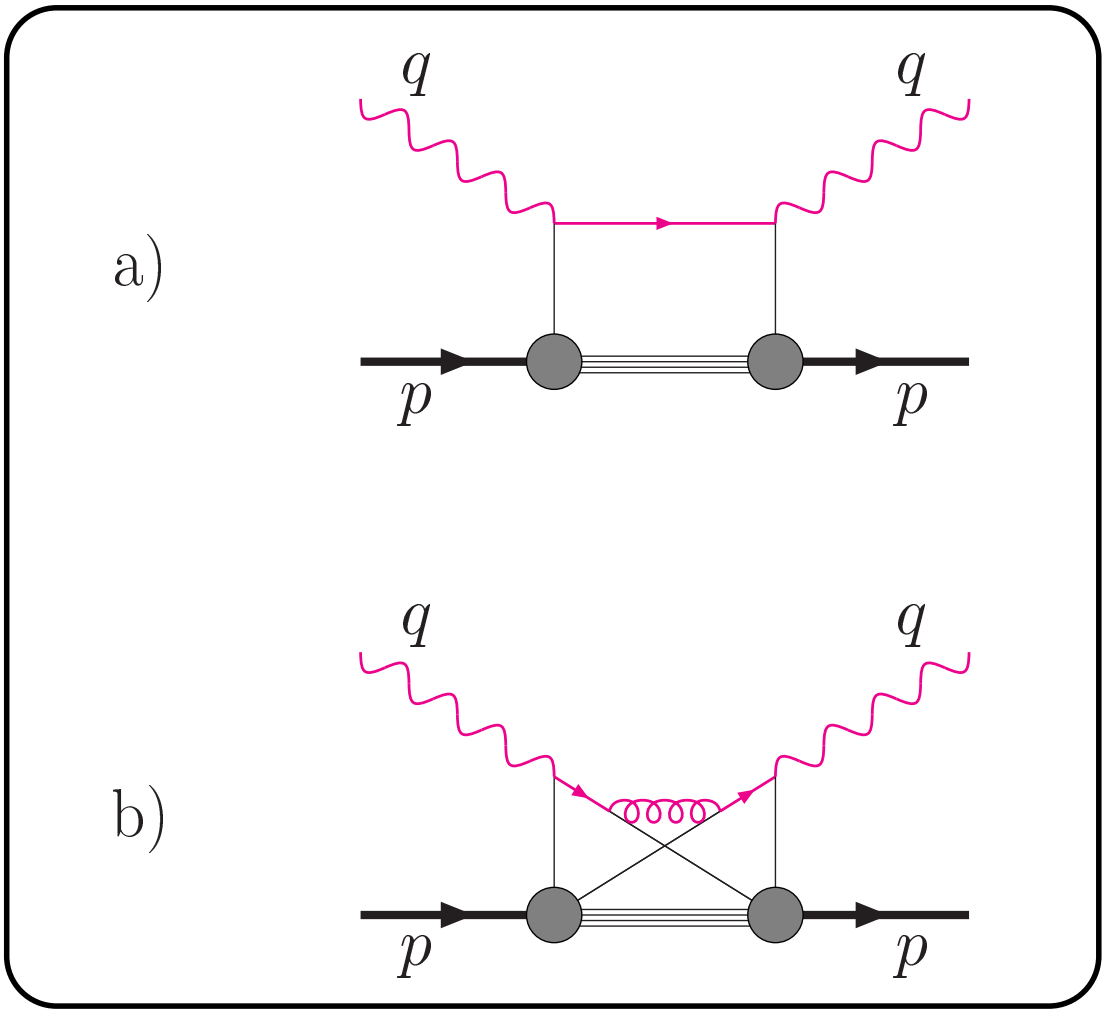
inclusive cross–section $\Leftrightarrow$ virtual, forward Compton amplitude



struck quark carries large momentum: $Q^2 \gg \Lambda^2_{\rm QCD}$

- crossed diagram suppressed (wavefunction!)
- asymptotic freedom $\Rightarrow$ neglect interactions of struck quark
- struck quark propagates along light-cone $x^2 = 0$

suppression of crossed diagrams



Flow of the large momentum q through typical diagrams contributing to the forward Compton amplitude. a) 'handbag' diagrams; b) 'cat's ears' diagram. Diagram b) is suppressed at large q due to the presence of additional propagators.

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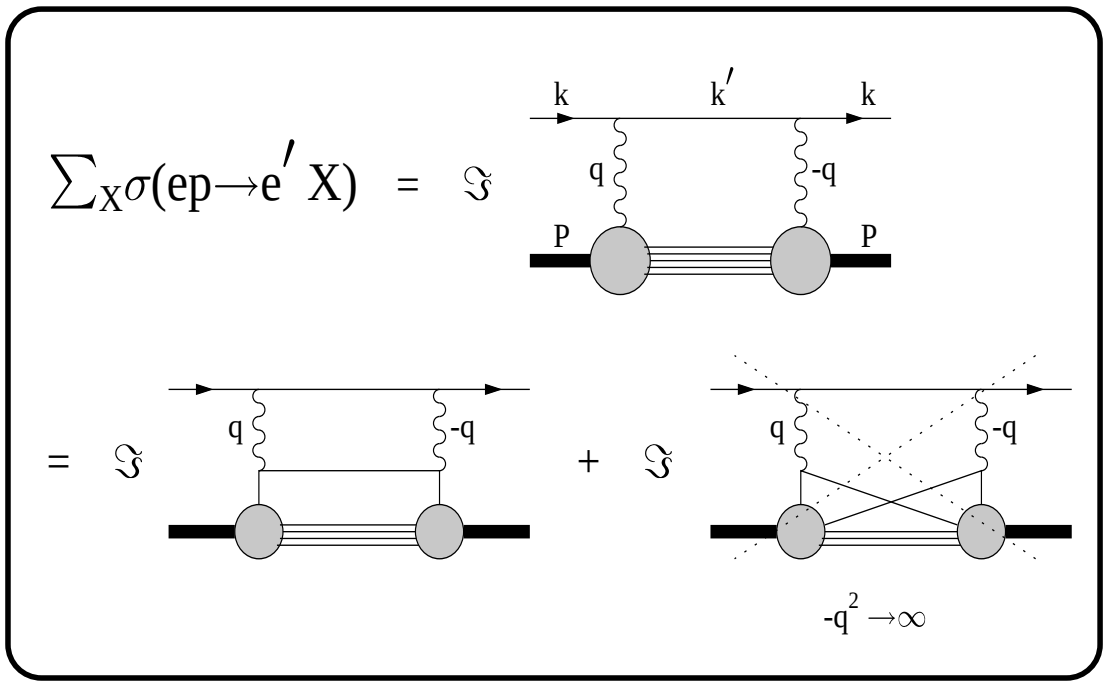

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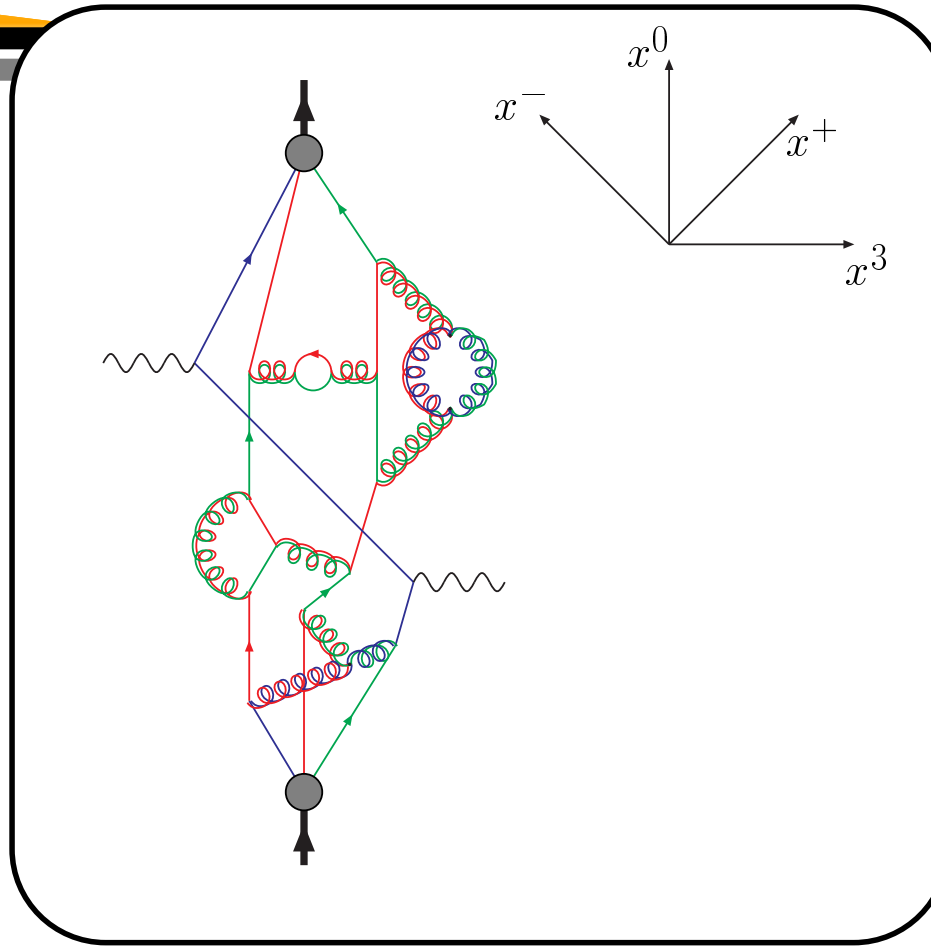


light-cone coordinates:

$$x^+ = \left(x^0 + x^3\right)/\sqrt{2}$$

$$x^- = \left(x^0 - x^3\right)/\sqrt{2}$$

DIS related to correlations along light–cone

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle P|\overline{q}(0^-, \mathbf{0}_\perp)\gamma^+ q(x^-, \mathbf{0}_\perp)|P\rangle \, e^{ix^- x_{Bj} P^+}$$

Probability interpretation!

No information about transverse position of partons!

DIS $\longrightarrow$ light-cone correlations

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle P|\overline{q}(0^-, \mathbf{0}_\perp)\gamma^+ q(x^-, \mathbf{0}_\perp)|P\rangle \, e^{ix^- x_{Bj} P^+}$$

- Fourier transform along x^- filters out quarks with light-cone momentum $k^+ = x_{Bj} P^+$
- momentum distribution = FT of equal time correlation function
- boost to IMF tilts equal time plane arbitrarily close to $x^+ = const.$ plane

$\hookrightarrow$ light-cone momentum distribution = momentum distribution in IMF

$\hookrightarrow$ $q(x) =$ light-cone momentum distribution or momentum distribution in IMF

unpolarized ep DIS

- $\sigma_{eq}^{elast} \propto Q_q^2$

$\hookrightarrow$ $e + p \to e' + X$ sensitive to:

$$\frac{4}{9}\left[u_p^{\uparrow}(x) + \bar{u}_p^{\uparrow}(x) + u_p^{\downarrow}(x) + \bar{u}_p^{\downarrow}(x)\right] +$$
$$\frac{1}{9}\left[d_p^{\uparrow}(x) + \bar{d}_p^{\uparrow}(x) + d_p^{\downarrow}(x) + \bar{d}_p^{\downarrow}(x)\right] +$$
$$\frac{1}{9}\left[s_p^{\uparrow}(x) + \bar{s}_p^{\uparrow}(x) + s_p^{\downarrow}(x) + \bar{s}_p^{\downarrow}(x)\right] + ...$$

where e.g $u_p^{\uparrow}(x)$,$u_p^{\downarrow}(x)$, $\bar{u}_p^{\uparrow}(x)$,$\bar{u}_p^{\downarrow}(x)$, ... are the distribution of u,$\bar{u}$, ... in the proton with spin parallel/antiparallel to the proton's spin

- neutron target (charge symmetry)

$\hookrightarrow$ $u_n(x) = d_p(x)$, $d_n(x) = u_p(x)$, $s_n(x) = s_p(x)$

$\hookrightarrow$ sensitive to $\frac{4}{9}d_p(x) + \frac{1}{9}u_p(x) + \frac{1}{9}s_p(x)$

$\hookrightarrow$ different linear combination of the same distribution functions!

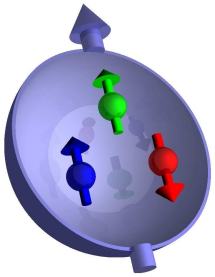
many important results from DIS:

- discovery of elementary, charged, spin $\frac{1}{2}$ constituents in the nucleon $\rightarrow$ quarks
- failure of momentum sum rule, i.e. quarks carry only about 50% of the nucleon's momentum $\rightarrow$ gluons

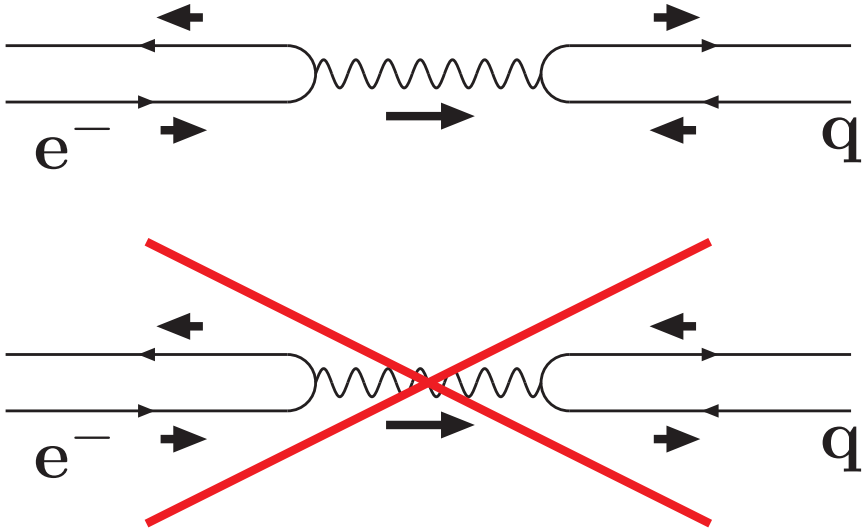
some recent puzzles:

- nuclear binding effect on structure functions (EMC collaboration): large and systematic modification of the nucleon's parton distribution in a bound nucleus
- failure of the "Ellis-Jaffe sum rule" for the spin dependent structure function $g_1(x)$ (EMC collaboration, also SMC, E142): spin fraction carried by quarks $\equiv$
  $\sum_{q=u,d,s} \int_0^1 dx \left[q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)\right] \ll 1$
  (nonrel quark model: 1) $\Rightarrow$ "spin crisis"

Longitudinally Polarized DIS



- e^-/q helicity conserved in high-energy interactions



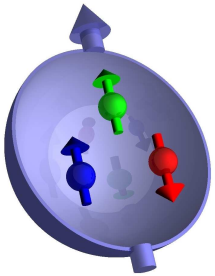
$\hookrightarrow$ longitudinally polarized e^- preferentially scatter off q with spin opposite to that of the e^-

- scattering long. pol. e^- off long. pol. nucleons $\Rightarrow$ quark/nucleon spin correlation

$$d\sigma^{\Uparrow\uparrow} - d\sigma^{\Uparrow\downarrow} \propto g_1(x) \qquad \text{with} \qquad g_1(x) = \sum_q e_q^2 \left[q_\uparrow(x) - q_\downarrow(x)\right]$$

- $q_\uparrow(x)/q_\downarrow(x) =$ probability that q has same/opposite spin as N
- $q(x) = q_\uparrow(x) + q_\downarrow(x)$
- $g_1(x)$ has been measured at CERN, SLAC, DESY, JLab, ...
- future, more precise, measurements from JLab@12GeV, EIC

Longitudinally Polarized DIS



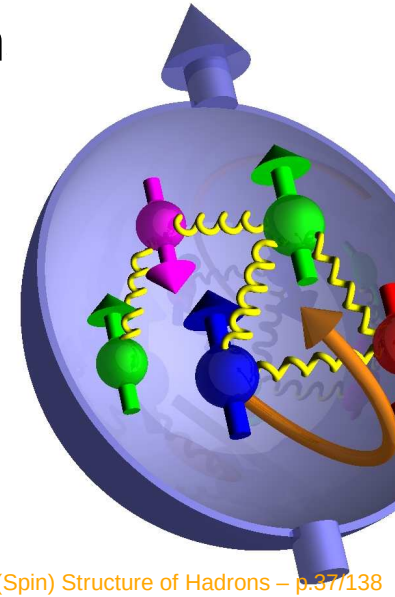
- $q_\uparrow(x)/q_\downarrow(x) =$ probability that q has same/opposite spin as N
- spin sum rule ($\rightarrow$R.Jaffe)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_{parton}$$

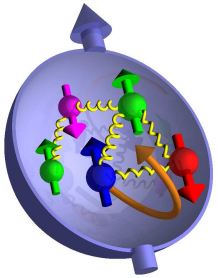
  - $\Delta\Sigma = \sum_q \Delta q \equiv \sum_q \int_0^1 dx\,[q_\uparrow(x) - q_\downarrow(x)] =$ fraction of the nucleon spin due to quark spins
  - ΔG = fraction of the nucleon spin due to gluon spins
  - $\mathcal{L}_{parton}$ = angular momentum due to quarks & gluon
- EMC collaboration (1987): only small fraction of the proton spin due to quark spins
- incl. more recent data (CERN,SLAC,DESY): $\sim 30\%$

$\hookrightarrow$ was called 'spin crisis', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

$\hookrightarrow$ quest for the remaining 70%



ΔG



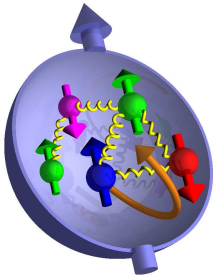
- gluons, like photons, descibed by a massless vector field and carry intrinsic angular momentum (spin) ± 1
- gluon contribution to nucleon momentum known to be large

$$\langle x_g \rangle \equiv \int_0^1 dx\, x g(x) = 1 - \sum_q \int_0^1 dx\, x q(x) \approx 0.5$$

(physics of $\langle x_g \rangle$: think of $\vec{E} \times \vec{B}$)

- conceivable that ΔG is of the same order of magnitude as $\langle x_g \rangle$ (or larger)
- several 'explanations' of the 'spin crisis' even suggested $\Delta G \sim 4 - 6$
- ΔG accessible e.g. through
  - 'QCD-evolution' of $\Delta q(x)$
  - A_{LL} in $\overrightarrow{p}\overleftarrow{p} \longrightarrow \gamma +$ jet

ΔG from QCD-evolution



- sometimes a gluon is not just a gluon (quantum fluctuations):
  - $g \longrightarrow \bar{q}q$
  - $g \longrightarrow gg, ggg$
- similar for quarks: $q \longrightarrow qg$
- quantum fluctuations short distance effects

$\hookrightarrow$ become more 'visible' as Q^2 of the probe increases

- resulting Q^2 dependence of PDFs descibed by QCD evolution equations (DGLAP evolution): coupled integro-differential equations with perturbatively calculable kernel
- $\bar{q}q$ pair 'inherits' gluon spin in $g \longrightarrow \bar{q}q$

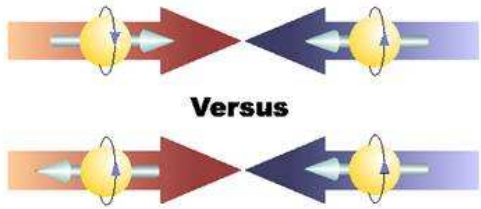
$\hookrightarrow$ infer ΔG from Q^2 dependence of Δq

- need coverage down to small x ($g(x)$ concentrated at very small x) and wide Q^2 range ('QCD evolution' slow)

$\hookrightarrow$ planned EIC (electron ion collider)

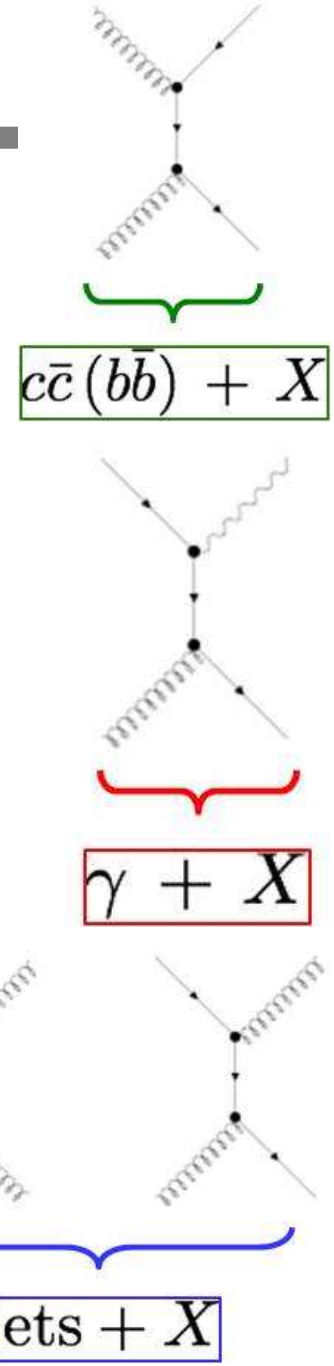
ΔG from $\vec{p} + \overleftarrow{p}$ (RHIC-spin)

- Double longitudinal-spin asymmetry: A_{LL}



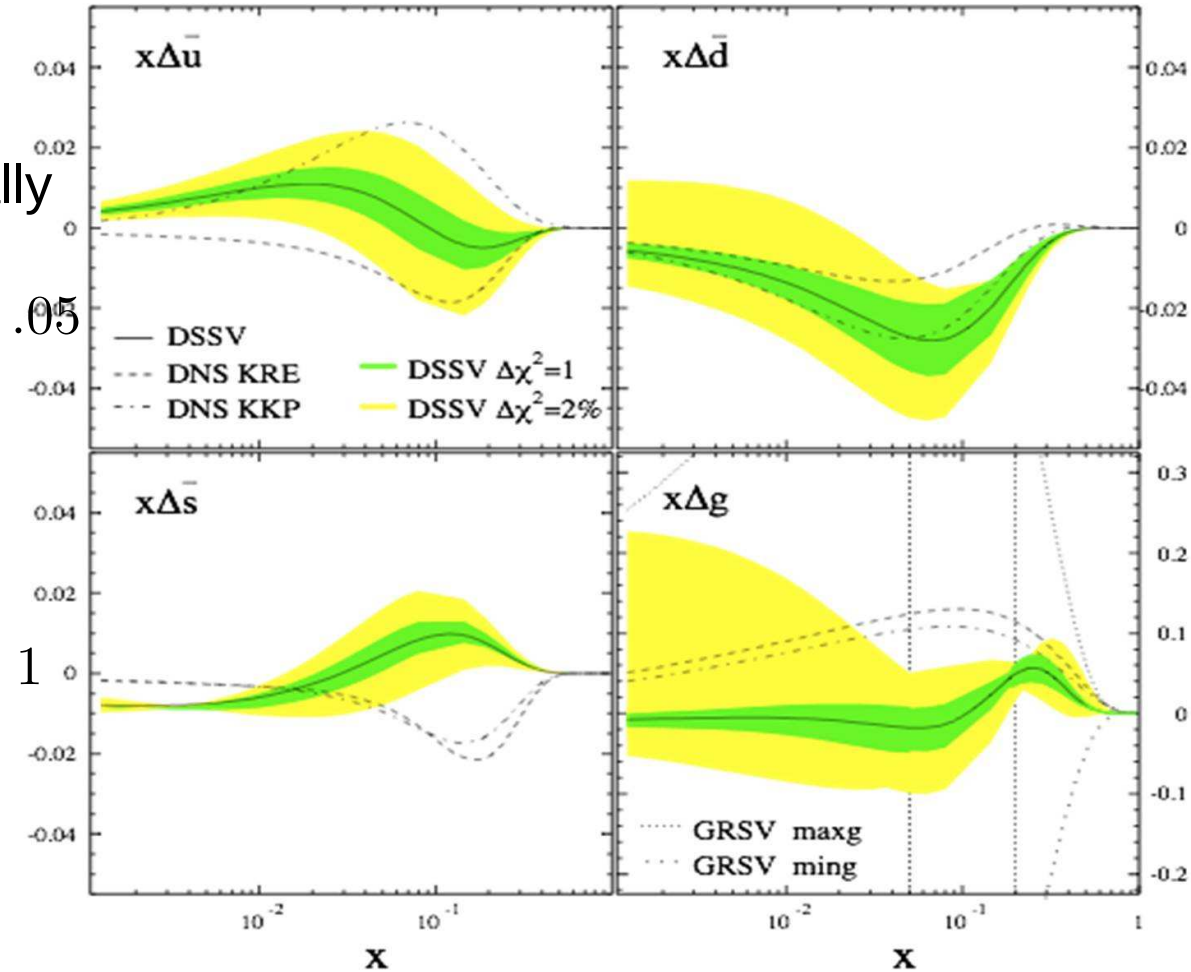
- Study helicity dependent structure functions (Gluon polarization)!

- gg and gq scattering sensitive to (relative) helicity
- use double-spin asymmetry $A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}}$ in inelastic pp-scattering at RHIC to infer ΔG directly



ΔG from $\vec{p} + \overleftarrow{p}$ (RHIC-spin)

- 'global analysis' (DIS & RHIC data):
- RHIC-spin substantially reduced error band (yellow) between $x = .05$ and $x = .2$
- despite remaining uncertainties, now evident that $|\Delta G|$ significantly less than 1
- big deal: rules out a significant role of gluonic corrections to the quark spin as explanation for spin puzzle



D. de Florian et al., hep-ph/0804.0422

- 500GeV at RHIC run with improved forward acceptance will reduce error band down to $x \sim .002$

Theoretical Status

- perturbative effects in general well understood, e.g. Q^2 dependence (="evolution") calculable in QCD (Altarelli, Parisi, Gribov, Lipatov eqs.): given $q(x, Q_0^2)$ one can calculate $q(x, Q_1^2)$ for $Q_1^2 > Q_0^2 >$ a few GeV^2

$\hookrightarrow$ important applications:

  - compare two experiments at two different Q^2
  - compare low Q^2 models or sum rules with experiments at high Q^2

- nonperturbative effects difficult!
  - power law behavior for $x \to 0$ from Regge phenomenology
  - typically, $q(x, Q_0^2)$ from some QCD-inspired models
  - lowest moments from lattice QCD

Calculating PDFs in lattice QCD

- direct evaluation: NO:
  On a Euclidean lattice, all distances are **spacelike** ($x^0 \to ix_E^0$).
  Therefore, a direct calculation of **lightlike** correlation functions on a Euclidean lattice is not possible!

- indirect evaluation: yes! Using analyticity, one can show that moments of parton distributions for a hadron h are related to expectation values of certain local operators in that hadron state

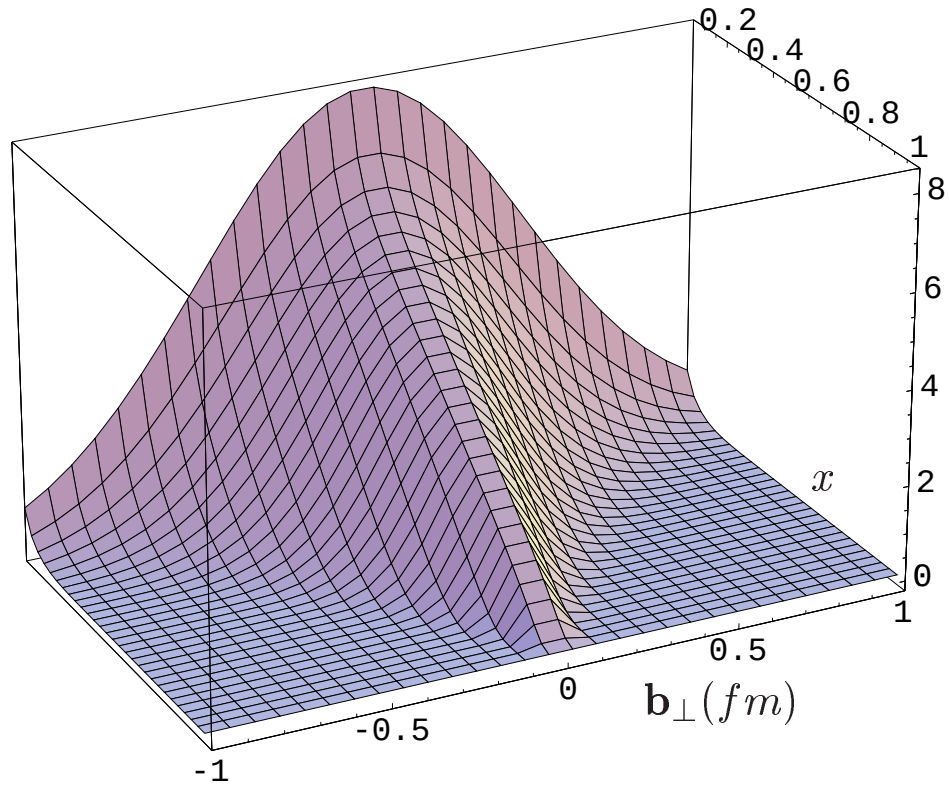
$$\int_0^1 dx f(x) x^n \leftrightarrow \langle h|\bar{\psi} D^n \psi|h\rangle$$

$\hookrightarrow$ r.h.s. of this equation can be calculated in Euclidean space (and then one could reconstruct $f(x)$ from its moments)! In practice: replace n-th derivative on the r.h.s. by appropriate finite differences. Problem: statistical noise (Euclidean lattice calculations are done using Monte Carlo techniques) makes it very hard to calculate any moment $n \gg 1$.

Summary: DIS

- DIS $\xrightarrow{Bj}$ PDF $q(x)$
- $q(x)$ is probability to find quark carrying fraction x of light-cone momentum (total momentum in IMF)
- no information about position of partons
- major results:
  - 50% of nucleon momentum carried by glue
  - significant [$\mathcal{O}(10\%)$] modification of quark distributions in nuclei ("EMC effect")
  - only 30% of nucleon spin carried by quark spin ("spin crisis")
  - more $\bar{d}$ than $\bar{u}$ in proton ("violation of Gottfried sum rule")
  - ΔG not very large
- theory:
  - perturbative "Q^2 evolution"
  - lattice calculations: lowest moments of PDFs
  - and many "QCD-inspired" models ...

3D imaging of the nucleon



Motivation (GPDs)

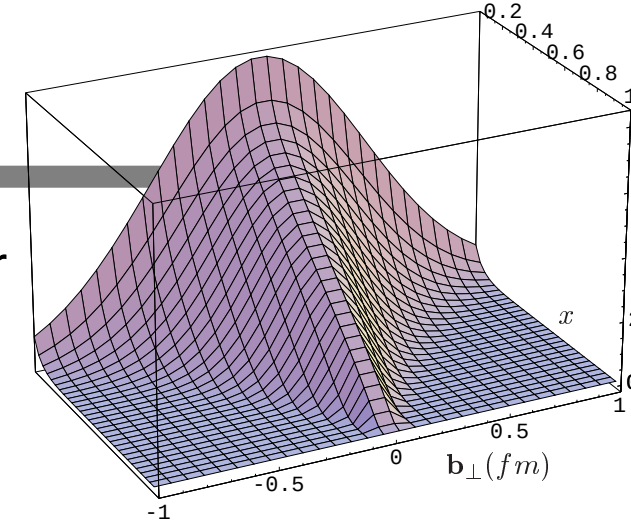
- X.Ji, PRL **78**, 610 (1997):

$$\boxed{\text{DVCS}} \quad \Leftrightarrow \quad \boxed{\text{GPDs}} \quad \Leftrightarrow \quad \boxed{\vec{J}_q}$$

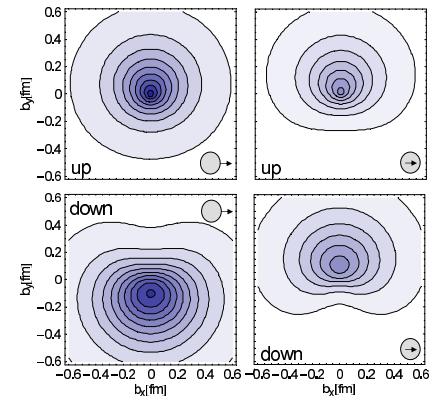
$\hookrightarrow$ GPDs are interesting physical observable!

- But:
  - do GPDs have a simple physical interpretation?
  - what more can we learn from GPDs about the structure of the nucleon?

Outline

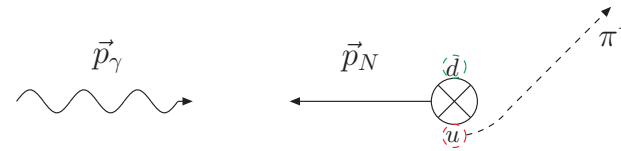


- Probabilistic interpretation of GPDs as Fourier trafos of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized



- Chromodynamik lensing and $\perp$ SSAs

$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\ \text{final state interactions}\end{array}\right\} \Rightarrow \perp \text{SSA in } \gamma N \longrightarrow \pi + X$$



- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$\int dx H_q(x,t) = F_1^q(t) \qquad \int dx \tilde{H}_q(x,t) = G_A^q(t)$$

$$\int dx E_q(x,t) = F_2^q(t) \qquad \int dx \tilde{E}_q(x,t) = G_P^q(t),$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $F_1^q(t)$, $F_2^q(t)$, $G_A^q(t)$, and $G_P^q(t)$ are the Dirac, Pauli, axial, and pseudoscalar formfactors, respectively ($t \equiv q^2 = (P' - P)^2$)

$$\langle P', S' | j^\mu(0) | P, S \rangle = \bar{u}(P', S') \left[\gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2M} F_2(q^2) \right] u(P, S)$$

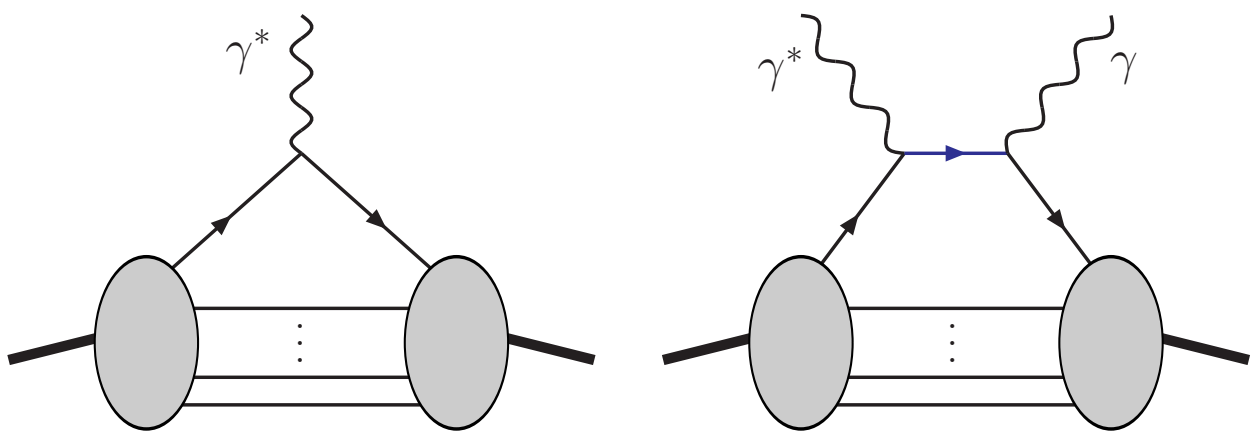
- GPDs can be probed in Deeply Virtual Compton Scattering (DVCS)

Deeply Virtual Compton Scattering (DVCS)

- virtual Compton scattering: $\gamma^* p \longrightarrow \gamma p$ (actually: $e^- p \longrightarrow e^- \gamma p$)
- 'deeply': $-q_\gamma^2 \gg M_p^2, |t| \longrightarrow$ Compton amplitude dominated by (coherent superposition of) Compton scattering off single quarks

$\hookrightarrow$ only difference between form factor (a) and DVCS amplitude (b) is replacement of photon vertex by two photon vertices connected by quark propagator (depends on quark momentum fraction x)

$\hookrightarrow$ DVCS amplitude provides access to momentum-decomposition of form factor (GPDs).



(a) (b)

Deeply Virtual Compton Scattering (DVCS)

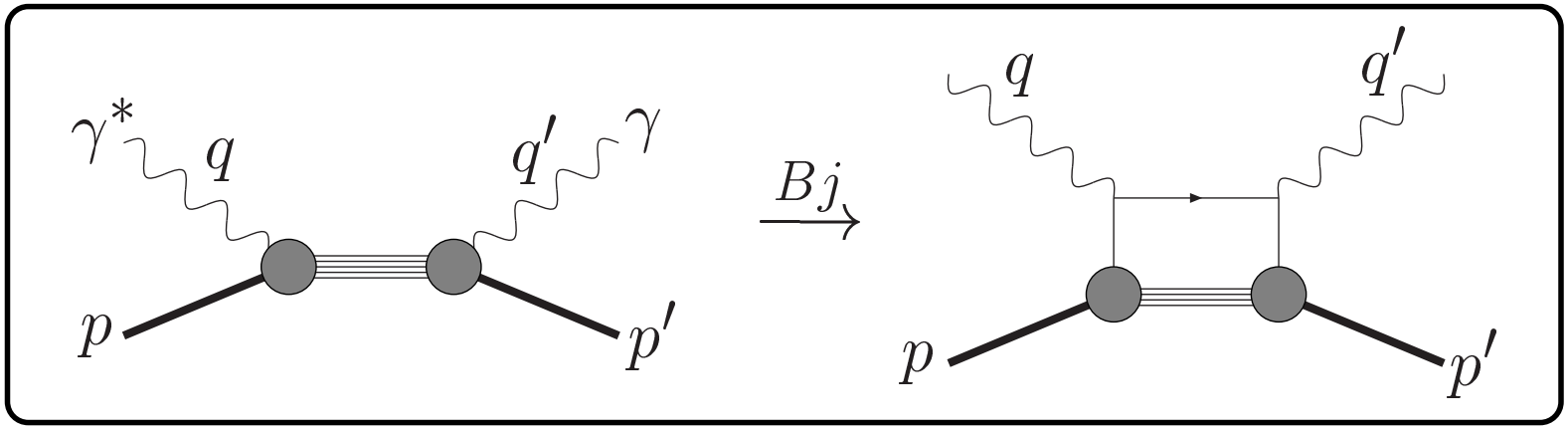
- need γ^* with several GeV^2 for Bjorken scaling
- DVCS X-section factor $\alpha = \frac{1}{137}$ smaller than elastic X-section

$\hookrightarrow$ need high luminosity e^- beam with > 10 GeV

$\hookrightarrow$ facilities suitable for detailed GPD studies:

  - 12 GeV upgrade at Jefferson Lab (higher x)
  - e^-Ion Collider (EIC): lower x, higher Q^2

Deeply Virtual Compton Scattering (DVCS)



$$T^{\mu\nu} = i\int d^4z\, e^{i\bar{q}\cdot z} \left\langle p' \left| T J^\mu\left(-\frac{z}{2}\right) J^\nu\left(\frac{z}{2}\right)\right| p \right\rangle$$

$$\overset{Bj}{\hookrightarrow} \quad \frac{g_\perp^{\mu\nu}}{2}\int_{-1}^{1} dx \left(\frac{1}{x-\xi+i\varepsilon} + \frac{1}{x+\xi-i\varepsilon}\right) H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) \quad + \quad \ldots$$

$$\bar{q} = (q+q')/2 \qquad \Delta = p' - p \qquad x_{Bj} \equiv -q^2/2p\cdot q = 2\xi(1+\xi)$$

Generalized Parton Distributions (GPDs)

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+\gamma_5 q\left(\frac{x^-}{2}\right)\right| p \right\rangle = \tilde{H}(x,\xi,\Delta^2)\bar{u}(p')\gamma^+\gamma_5u(p) + \tilde{E}(x,\xi,\Delta^2)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)$$

where $\Delta = p' - p$ is the momentum transfer and ξ measures the longitudinal momentum transfer on the target $\Delta^+ = \xi(p^+ + p^{+\prime})$.

$$\left\langle p' \middle| \hat{O} \middle| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

What is Physics of GPDs ?

with $\hat{O} = \int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x^-} \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$

- ↪ relation between PDFs and GPDs similar to relation between a charge and a form factor
- ↪ If form factors can be interpreted as Fourier transforms of charge distributions in position space, what is the analogous physical interpretation for GPDs ?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{\psi}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp | \bar{\psi}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \langle p^+, \mathbf{p}'_\perp | \bar{\psi}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp) | p^+, \mathbf{p}_\perp \rangle e^{ixp^+x^-}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar\psi(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar\psi(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar\psi(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{0}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$\times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

Impact parameter dependent PDFs

- use translational invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar\psi(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar\psi(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{b}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar\psi(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+\psi(\frac{x^-}{2},\mathbf{0}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-} \times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp H\left(x, 0, -(\mathbf{p}'_\perp - \mathbf{p}_\perp)^2\right) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

$$\hookrightarrow \quad \boxed{q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}}$$

Impact parameter dependent PDFs

- GPDs allow simultaneous determination of longitudinal momentum and transverse position of partons

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$ has interpretation as density (positivity constraints!)

$$\begin{aligned} q(x, \mathbf{b}_\perp) &\sim \left\langle p^+, \mathbf{0}_\perp \right| b^\dagger(xp^+, \mathbf{b}_\perp) b(xp^+, \mathbf{b}_\perp) \left| p^+, \mathbf{0}_\perp \right\rangle \\ &= \left| b(xp^+, \mathbf{b}_\perp) | p^+, \mathbf{0}_\perp \right\rangle \Big|^2 \geq 0 \end{aligned}$$

$$\boxed{\begin{aligned} q(x, \mathbf{b}_\perp) \geq 0 \quad &\text{for} \quad x > 0 \\ q(x, \mathbf{b}_\perp) \leq 0 \quad &\text{for} \quad x < 0 \end{aligned}}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

↪ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, \mathbf{b}_\perp)$ as difference of number densities)
- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

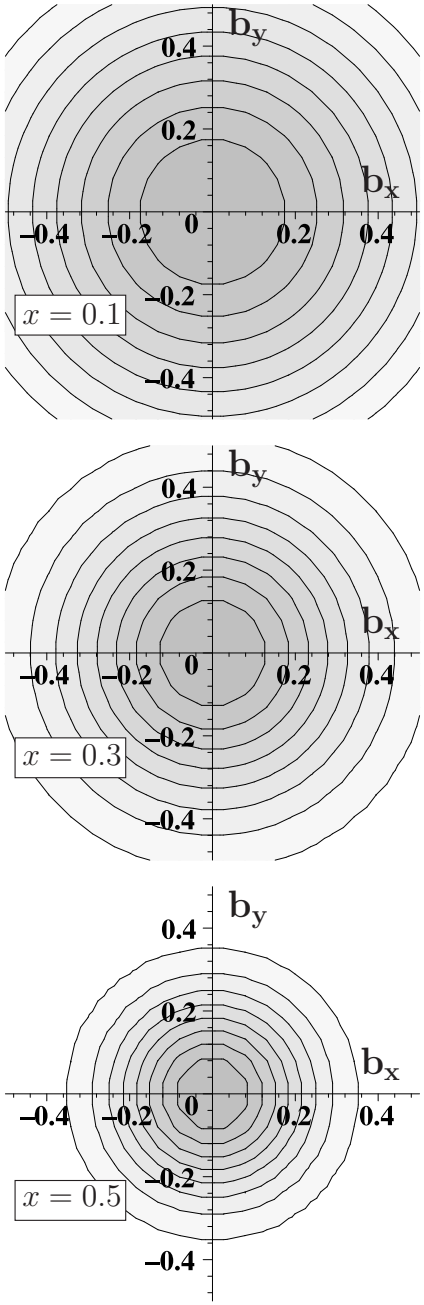
↪ for $x \to 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

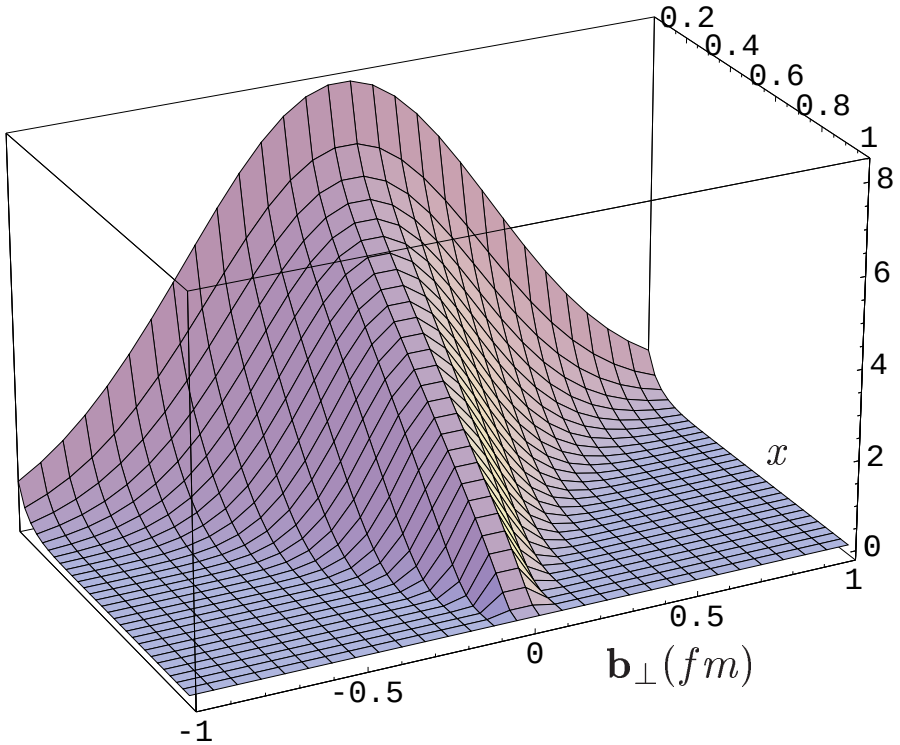
↪ $H(x, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \to 1$ (MB, 2000)

↪ consistent with lattice results for first few moments (→J.Negele)

- Note that this does not necessarily imply that 'hadron size' goes to zero as $x \to 1$, as separation $\mathbf{r}_\perp$ between active quark and COM of spectators is related to impact parameter $\mathbf{b}_\perp$ via $\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$.

$q(x, \mathbf{b}_\perp)$ for unpol. p





$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

Summary

- form factor $\overset{FT}{\leftrightarrow} \rho(\vec{r})$
- relativistic corrections! Can be avoided in 'infinite momentum frame' interpretation of 2D FT of form factors
- DIS $\longrightarrow q(x)$ light-cone momentum distribution of quarks in nucleon
- DVCS $\longrightarrow$ GPDs $\overset{FT}{\leftrightarrow} q(x, \mathbf{b}_\perp)$ 'impact parameter dependent PDFs

Transversely Deformed Distributions and $E(x, -\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- distribution of unpol. quarks in unpol (or long. pol.) nucleon:

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \equiv \mathcal{H}(x, \mathbf{b}_\perp)$$

- unpol. quark distribution for nucleon polarized in x direction:

$$q(x, \mathbf{b}_\perp) = \mathcal{H}(x, \mathbf{b}_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

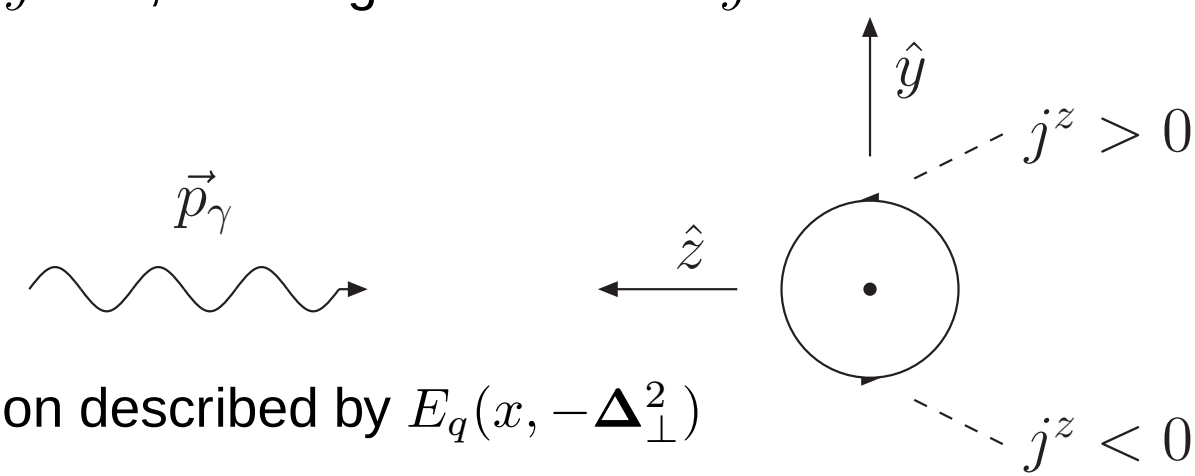
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3

Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

↪ j^+ larger than j^0 when quark current towards the γ^*; suppressed when away from γ^*

↪ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, -\mathbf{\Delta}_\perp^2)$

↪ not surprising that $E_q(x, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \, [H_q(x,0) + E_q(x,0)] \, x.$$

Transversely Deformed PDFs and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_{q/p}}{2M}$$

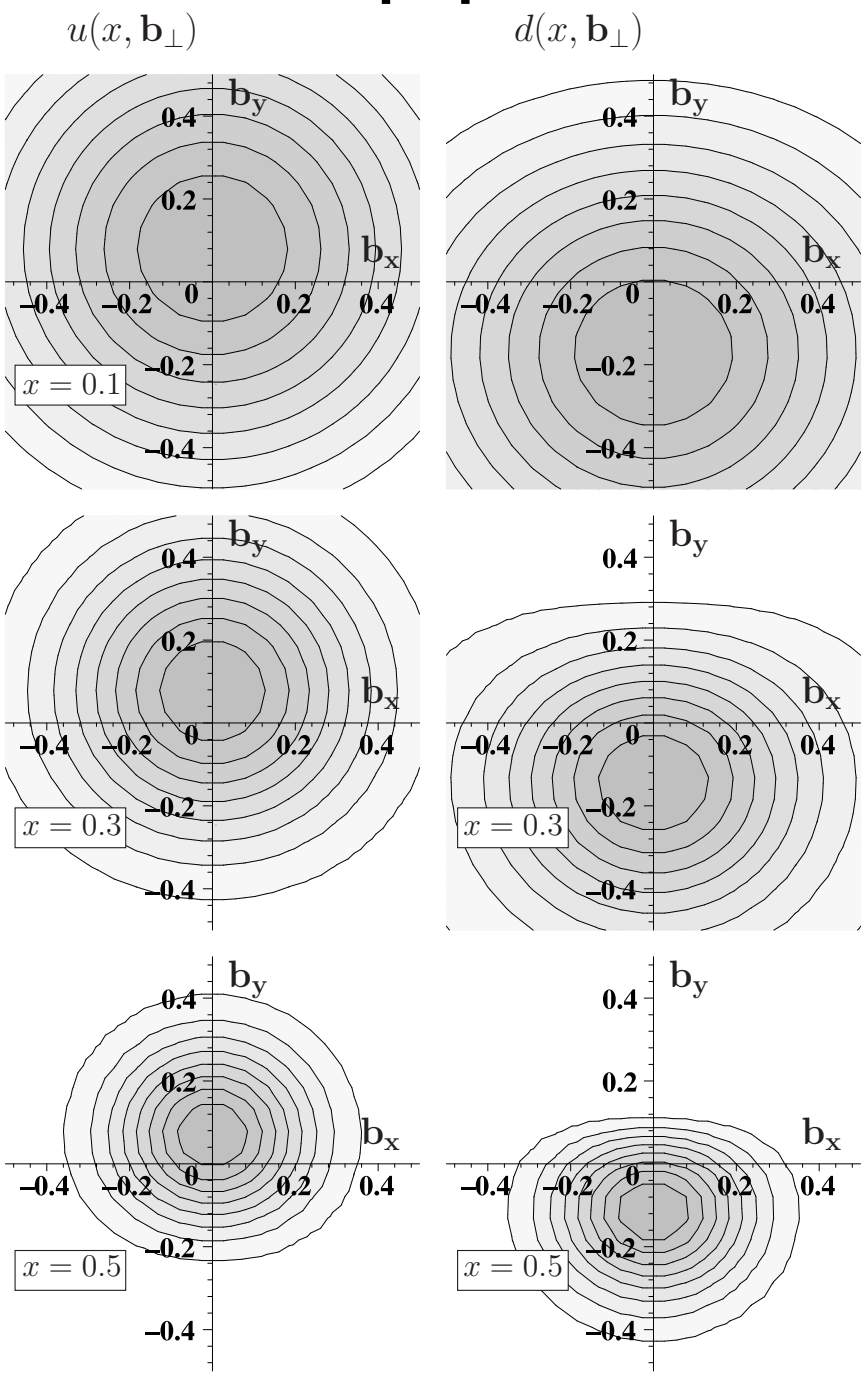
with $\kappa_{q/p} \equiv F_2^{u/d}(0)$ contribution from quark flavor q to the proton anomalous magnetic moment

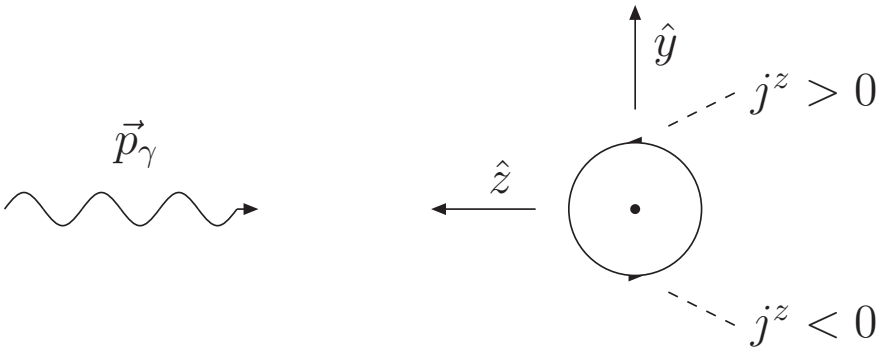
- $\kappa_p = 1.793 = \frac{2}{3}\kappa_{u/p} - \frac{1}{3}\kappa_{d/p} \quad \kappa_n = -2.033 = \frac{2}{3}\kappa_{d/p} - \frac{1}{3}\kappa_{u/p}$

$\hookrightarrow$ $\kappa_{u/p} = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_{d/p} = 2\kappa_n + \kappa_p = -2.033.$

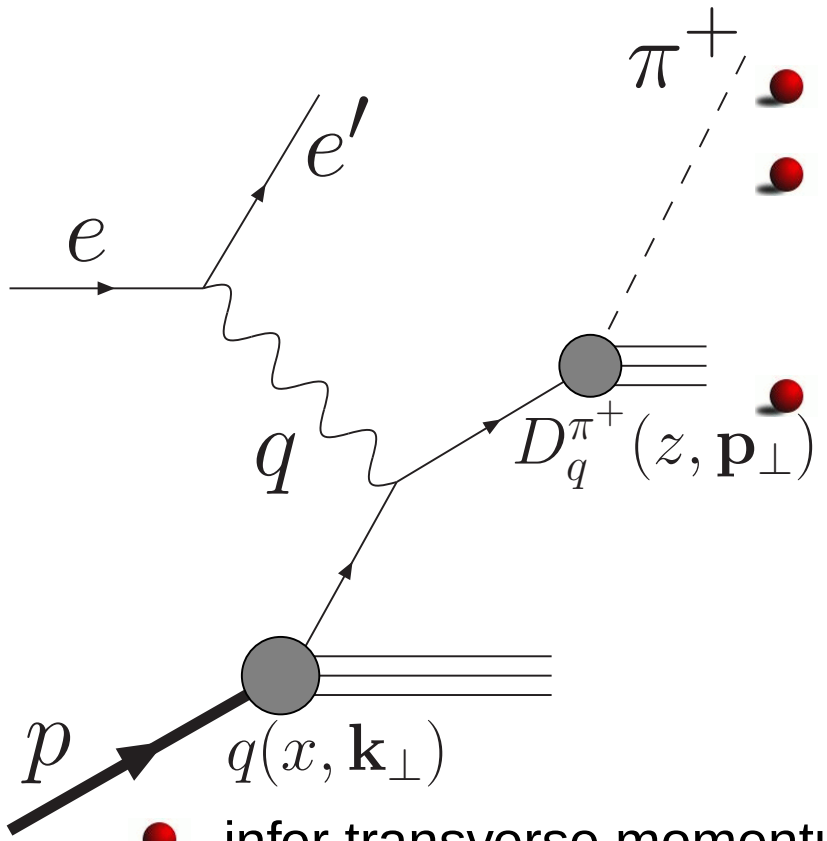
$\hookrightarrow$ $d_y^q = \mathcal{O}(0.2fm)$

p polarized in $+\hat{x}$ direction





SSAs in SIDIS ($\gamma + p\uparrow \longrightarrow \pi^+ + X$)



- SIDIS = semi-inclusive DIS
- Single-Spin-Asymmetry (SSA) = left-right asymmery in the X-section when only one spin is measured (e.g. target spin)
- example: nucleon transversely (relative to e^- beam) polarized $\longrightarrow$ left-right asymmetry of produced π-mesons relative to target pol.
- infer transverse momentum distribution $q(x, \mathbf{k}_\perp)$ of quarks in target from transverse momentum distribution of produced π (note: left-right asymmetry can also arise in 'fragmentation' process (Collins effect), but resulting asymmetry has different angular dependence...)

GPD $\longleftrightarrow$ SSA (Sivers)

- Sivers: distribution of **unpol.** quarks in $\perp$ pol. proton

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S}{M}$$

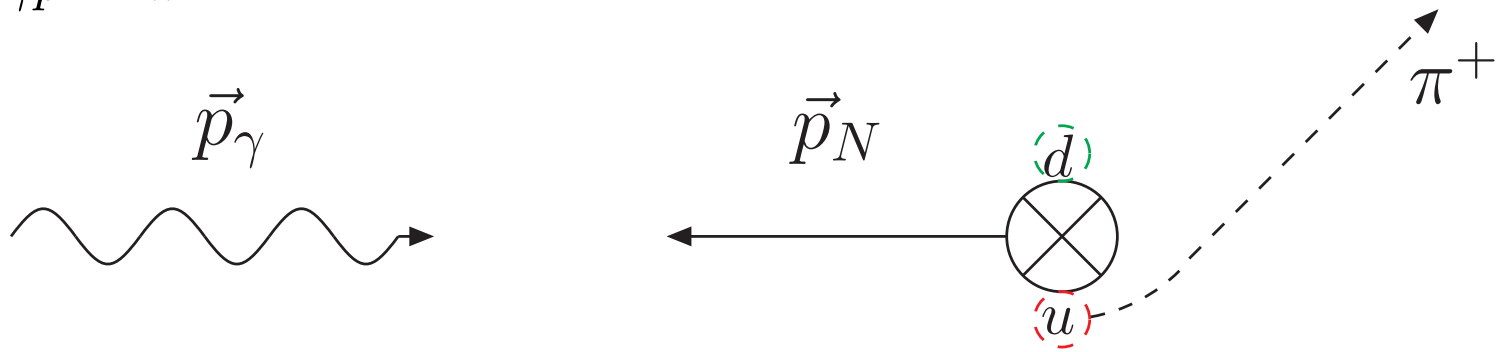
- without FSI, $f(x, \mathbf{k}_\perp) = f(x, -\mathbf{k}_\perp) \Rightarrow f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) = 0$
- with FSI, $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2) \neq 0$ (Brodsky, Hwang, Schmidt)
- Why interesting?
  - (like κ), Sivers requires matrix elements between **wave function components that differ by one unit of OAM** (Brodsky, Diehl, ..)

  $\hookrightarrow$ probe for orbital angular momentum
  - Sivers requires nontrivial final state interaction phases

  $\hookrightarrow$ **learn about FSI**

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

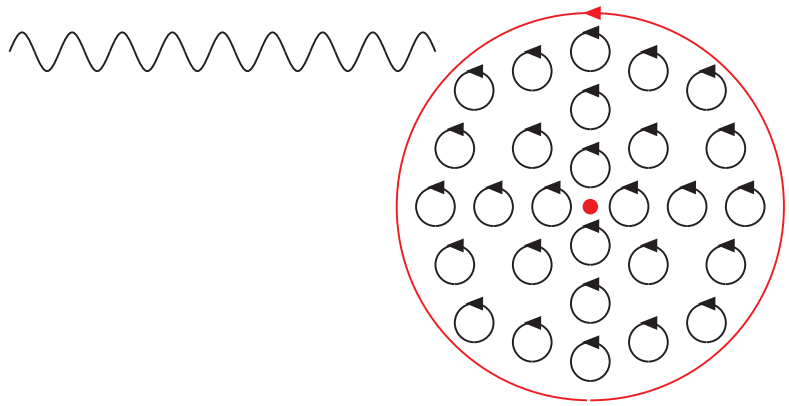
$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

Transversity Distribution in Unpolarized Target (sign)

- Consider quark in ground state hadron polarized out of the plane

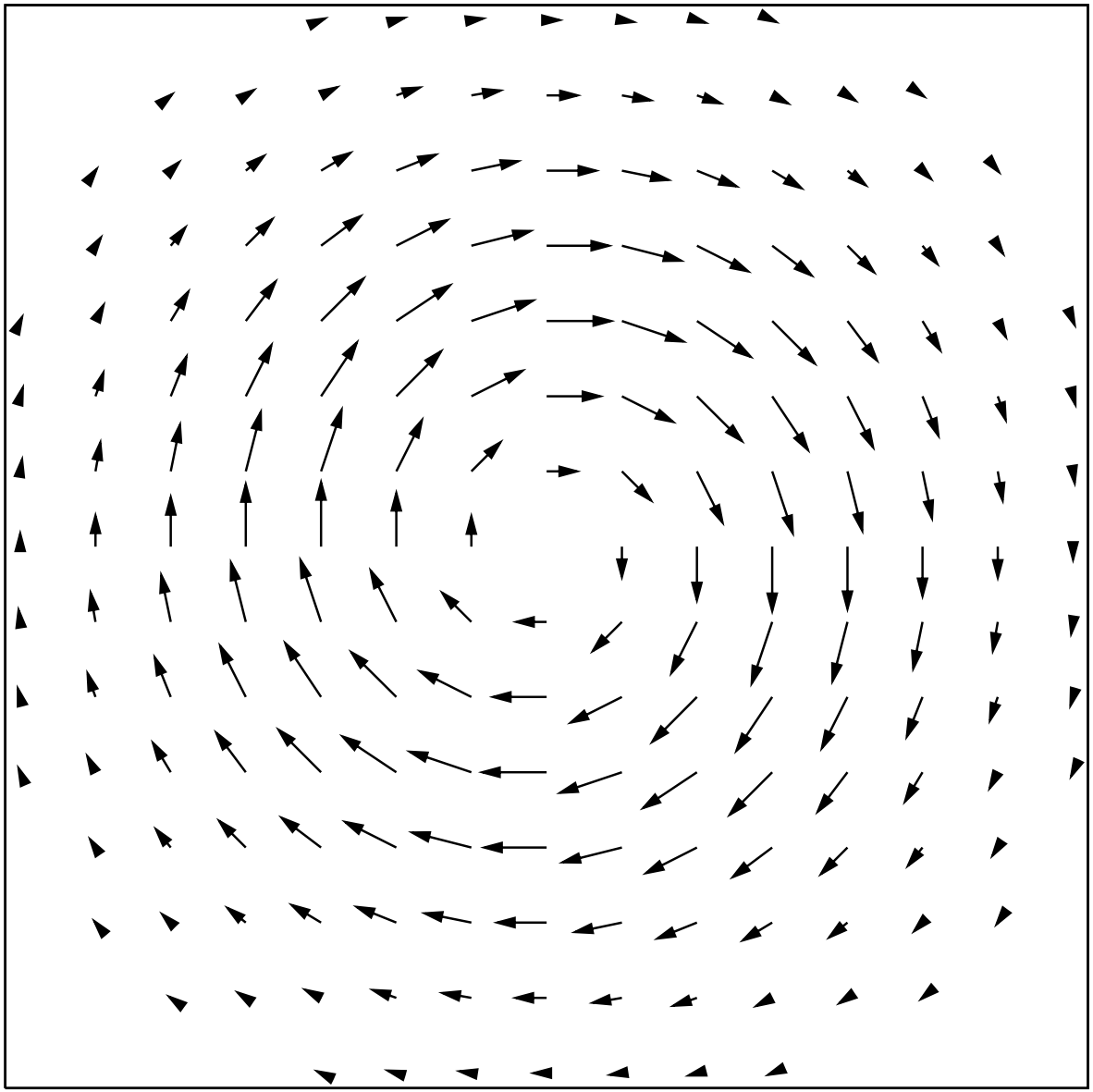
$\hookrightarrow$ expect counterclockwise net current $\vec{j}$ associated with the magnetization density in this state



- virtual photon 'sees' enhancement of quarks (polarized out of plane) at the top, i.e.

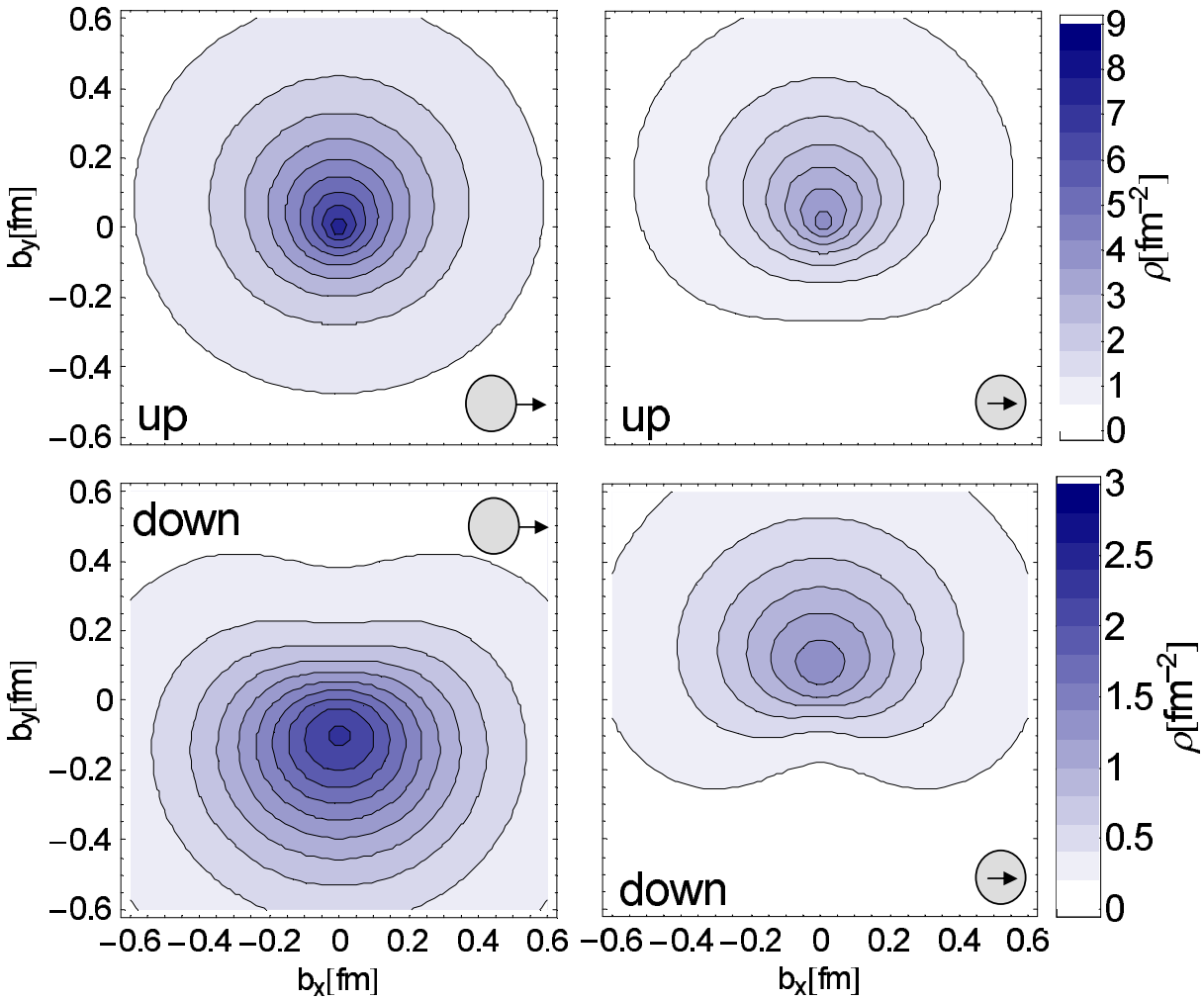
$\hookrightarrow$ virtual photon 'sees' enhancement of quarks with polarization up (down) on the left (right) side of the hadron

Transversity Distribution in Unpolarized Target



IPDs on the lattice (Hägler et al.)

- lowest moment of distribution $q(x, \mathbf{b}_\perp)$ for unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):



Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ (qualitative) connection between Boer-Mulders function $h_1^\perp(x, \mathbf{k}_\perp)$ and the chirally odd GPD $\bar{E}_T$ that is similar to (qualitative) connection between Sivers function $f_{1T}^\perp(x, \mathbf{k}_\perp)$ and the GPD E.

- Boer-Mulders: distribution of $\perp$ **pol.** quarks in **unpol.** proton

$$f_{q^\uparrow/p}(x, \mathbf{k}_\perp) = \frac{1}{2}\left[f_1^q(x, \mathbf{k}_\perp^2) - h_1^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot S_q}{M}\right]$$

- $h_1^{\perp q}(x, \mathbf{k}_\perp^2)$ can be probed in Drell-Yan (RHIC, J-PARC, GSI) and tagged SIDIS (JLab, eRHIC), using Collins-fragmentation

probing BM function in tagged SIDIS

- how do you measure the transversity distribution of quarks without measuring the transversity of a quark?
- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter

$\hookrightarrow$ (attractive) FSI provides correlation between quark spin and $\perp$ quark momentum $\Rightarrow$ BM function

- Collins effect: left-right asymmetry of π distribution in fragmentation of $\perp$ polarized quark $\Rightarrow$ 'tag' quark spin

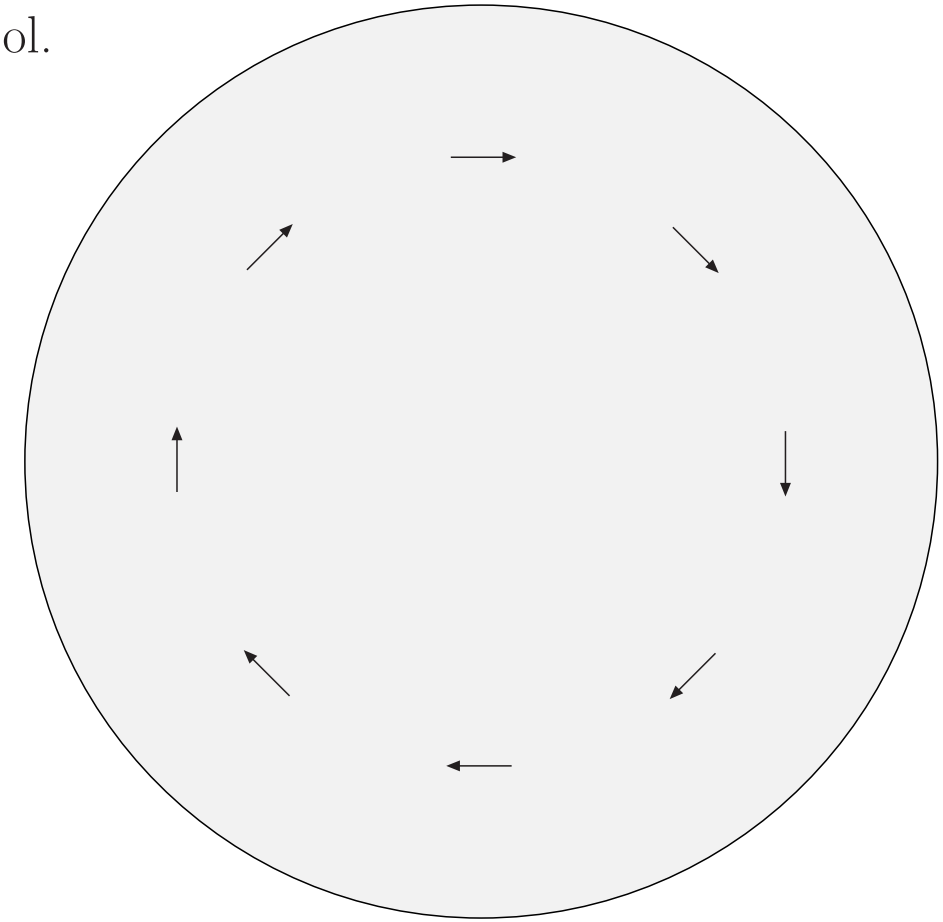
$\hookrightarrow$ $\cos(2\phi)$ modulation of π distribution relative to lepton scattering plane

$\hookrightarrow$ $\cos(2\phi)$ asymmetry proportional to: Collins $\times$ BM

probing BM function in tagged SIDIS

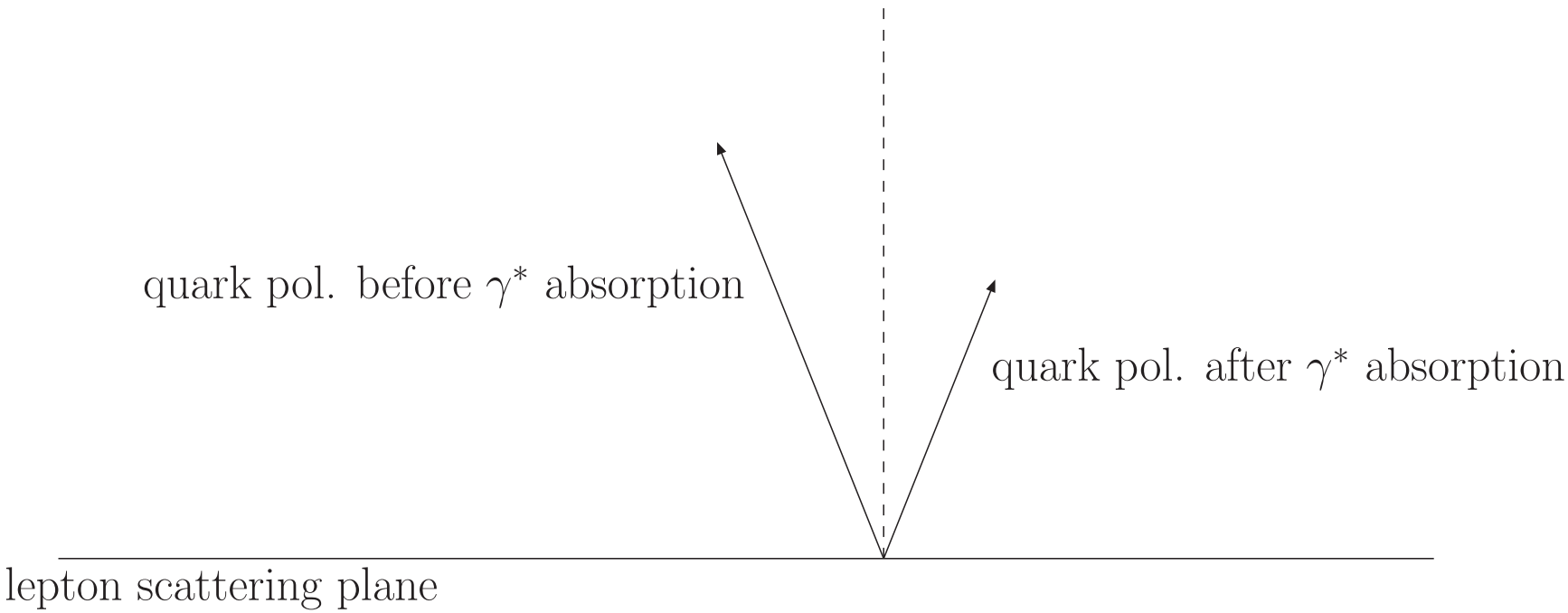
Primordial Quark Transversity Distribution

$\longrightarrow$ $\perp$ quark pol.



$\perp$ polarization and γ^* absorption

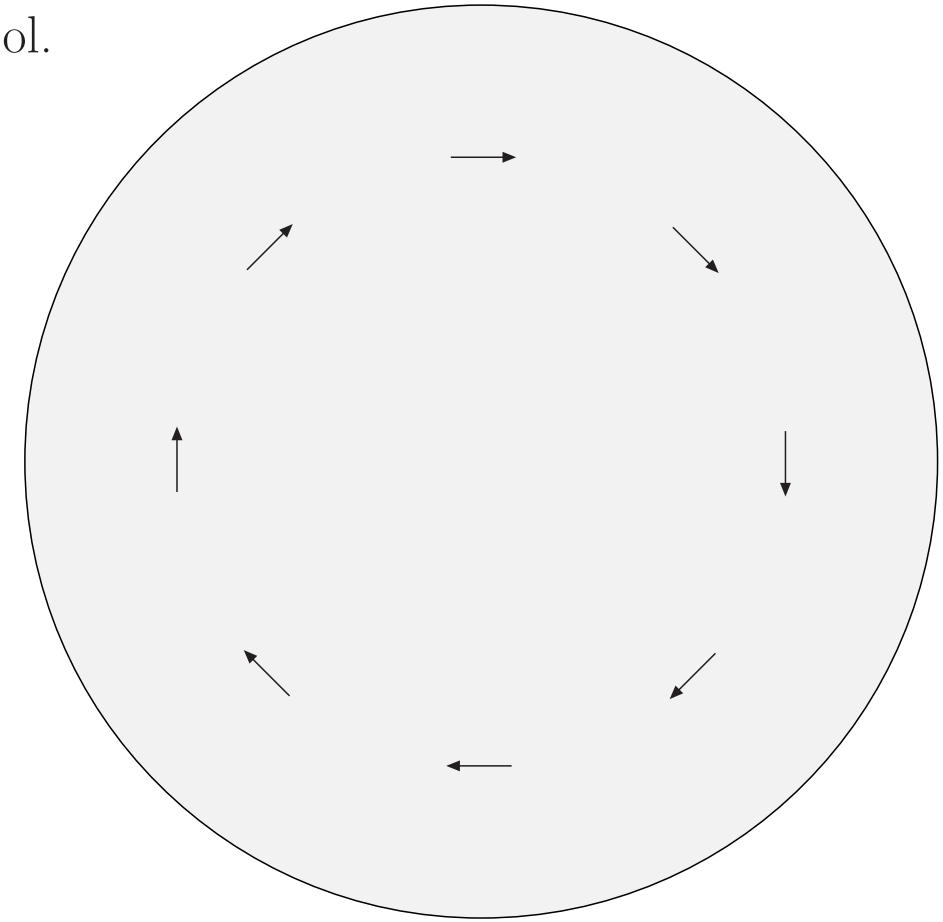
- QED: when the γ^* scatters off $\perp$ polarized quark, the $\perp$ polarization gets modified
  - gets reduced in size
  - gets tilted symmetrically w.r.t. normal of the scattering plane



probing BM function in tagged SIDIS

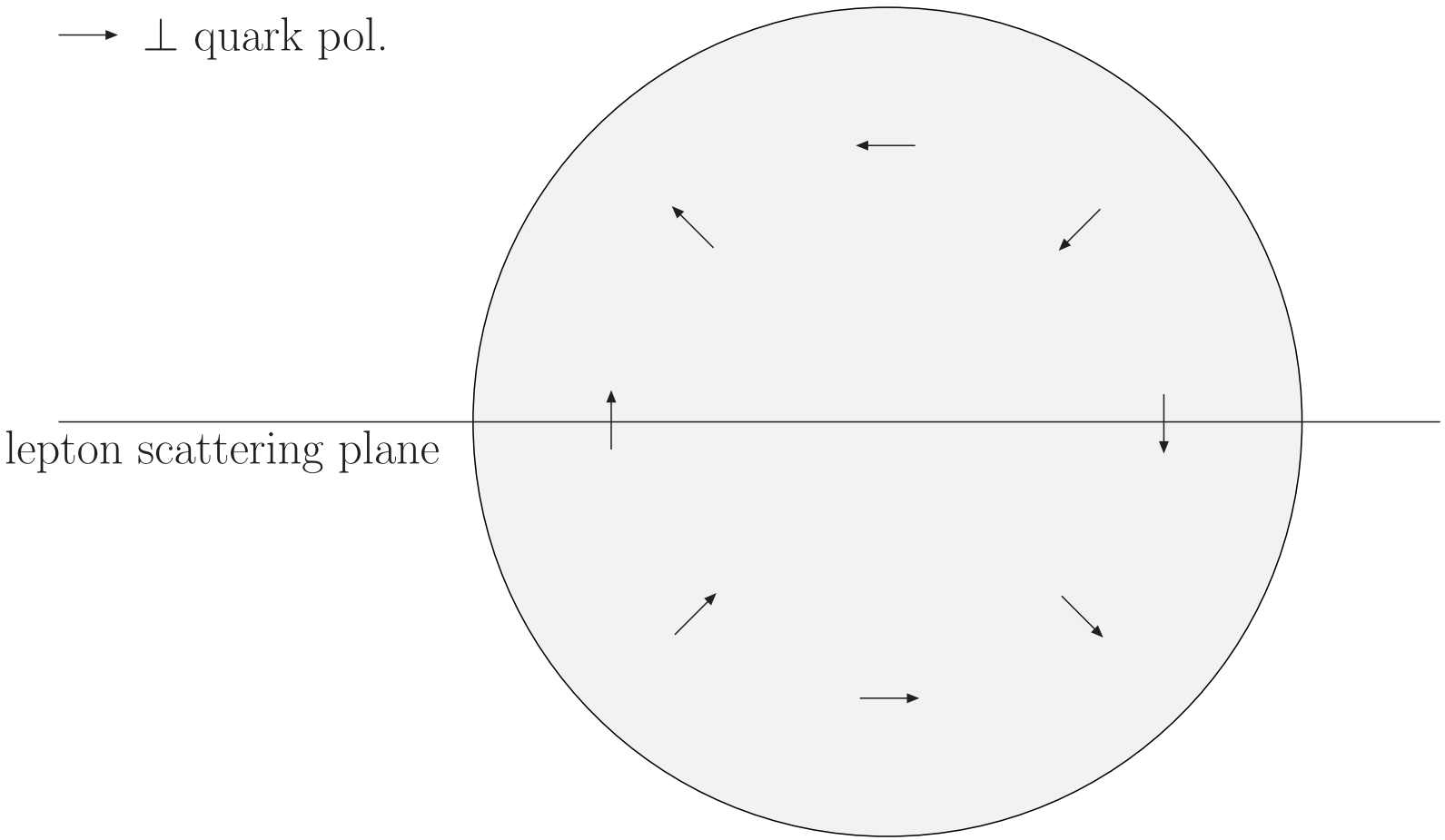
Primordial Quark Transversity Distribution

$\longrightarrow$ $\perp$ quark pol.



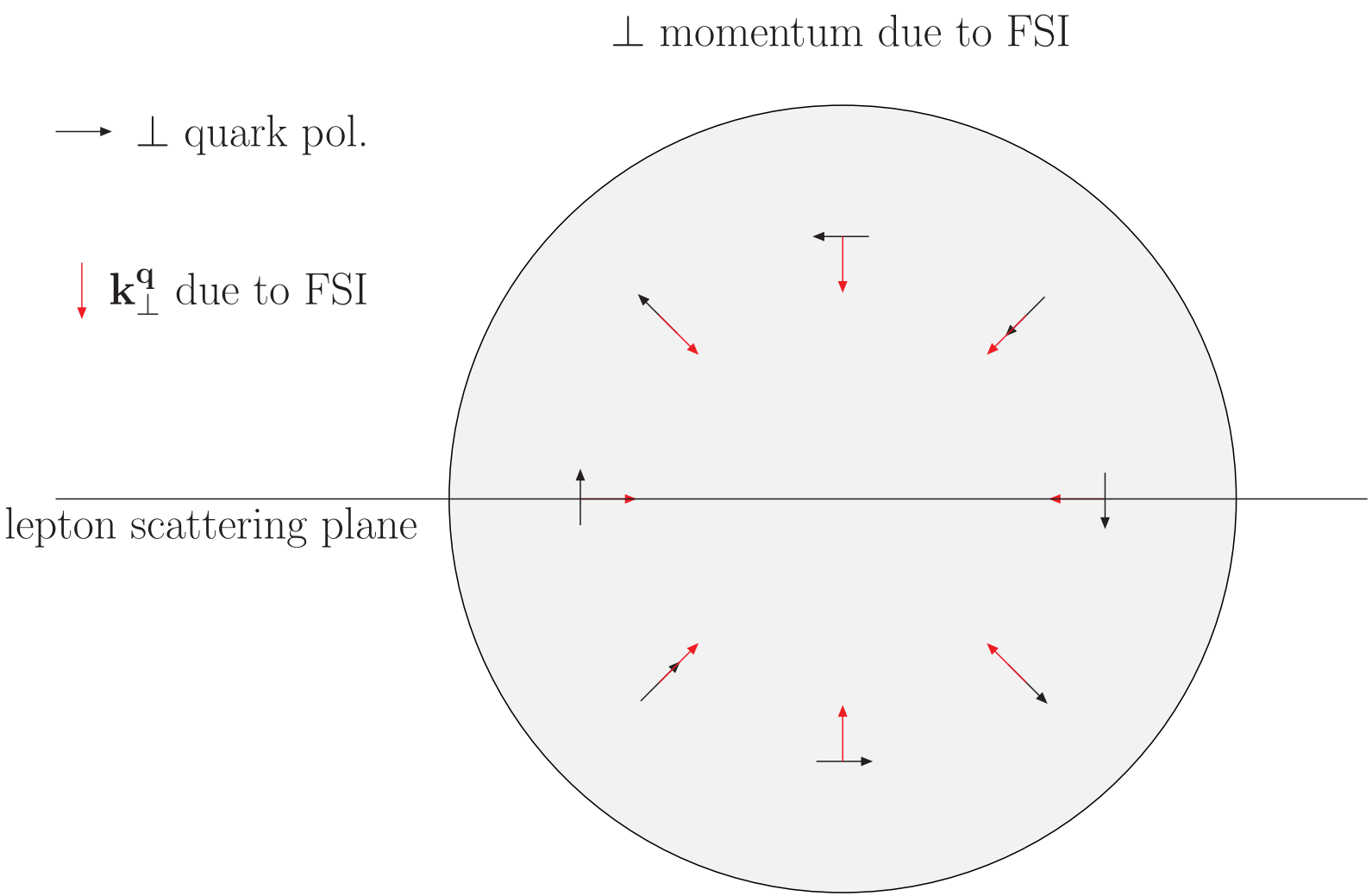
probing BM function in tagged SIDIS

Quark Transversity Distribution after γ^* absorption



quark transversity component in lepton scattering plane flips

probing BM function in tagged SIDIS

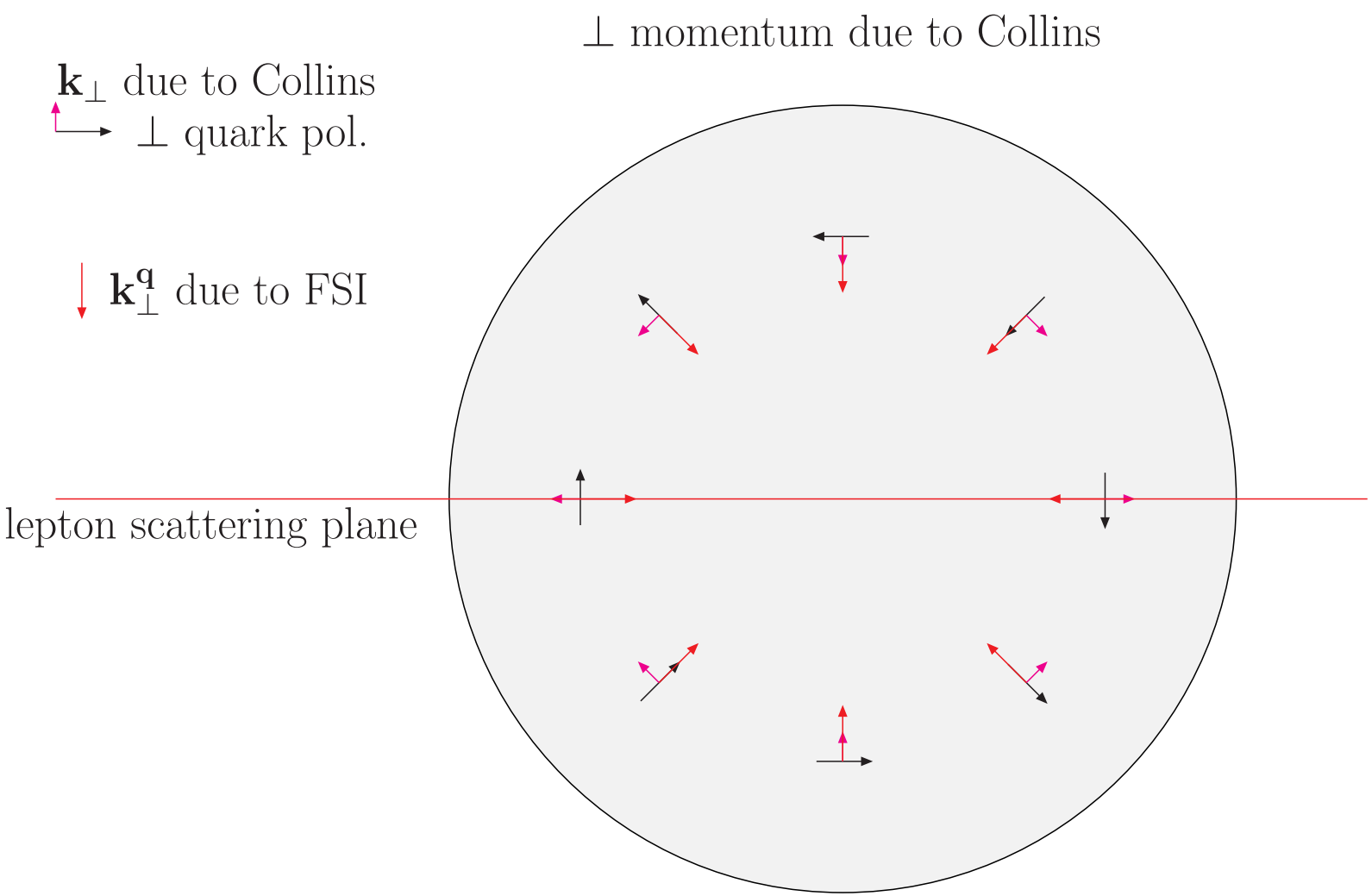


on average, FSI deflects quarks towards the center

Collins effect

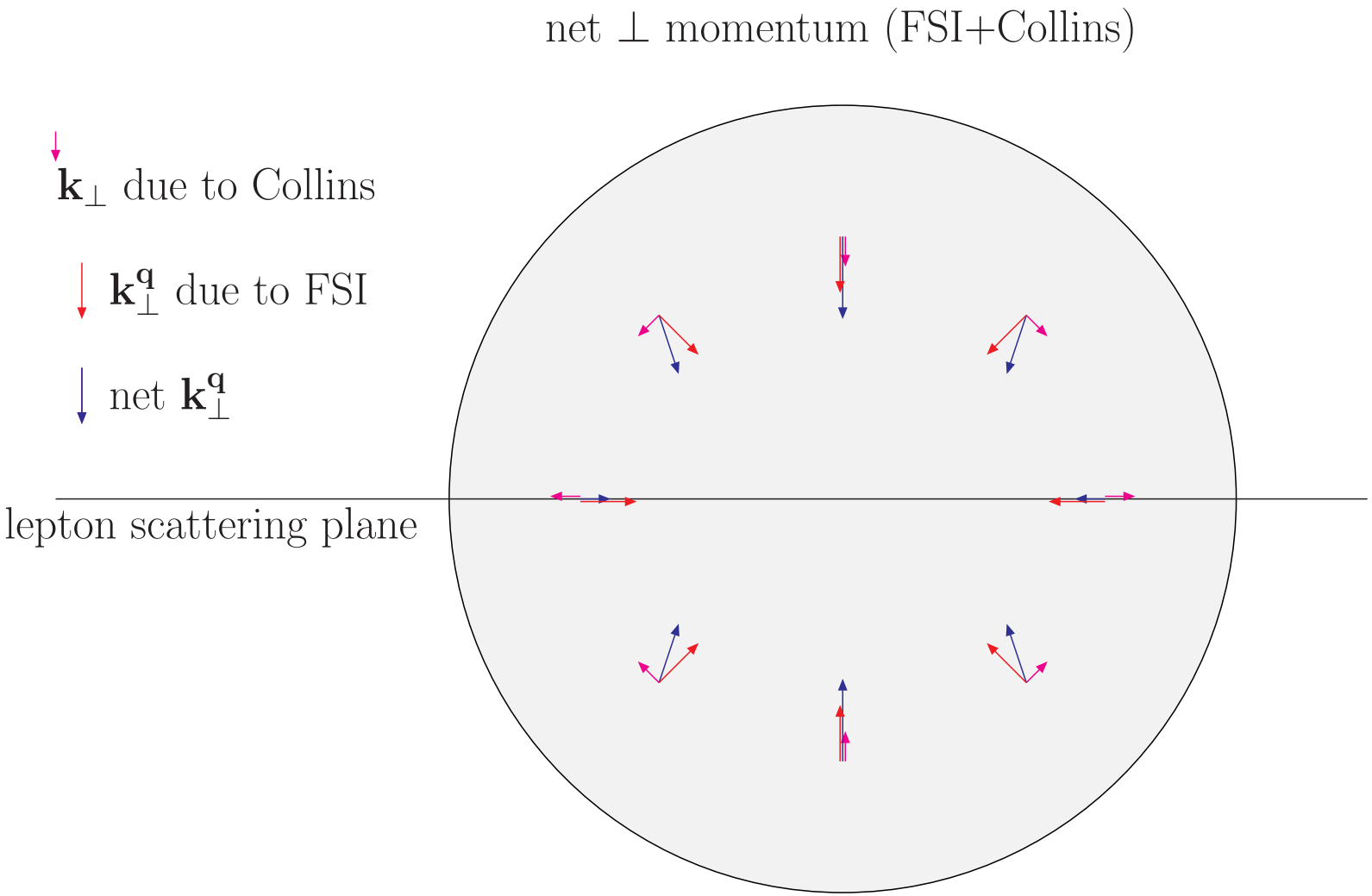
- When a $\perp$ polarized struck quark fragments, the strucure of jet is sensitive to polarization of quark
- distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric
- asymmetry parameterized by **Collins fragmentation function**
- Artru model:
  - struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with $^3P_0$ 'vacuum' quantum numbers
  - $\hookrightarrow$ pion 'inherits' OAM in direction of $\perp$ spin of struck quark
  - $\hookrightarrow$ produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up
- Artru model confirmed by HERMES experiment
- more precise determination of Collins function under way (KEK)

probing BM function in tagged SIDIS



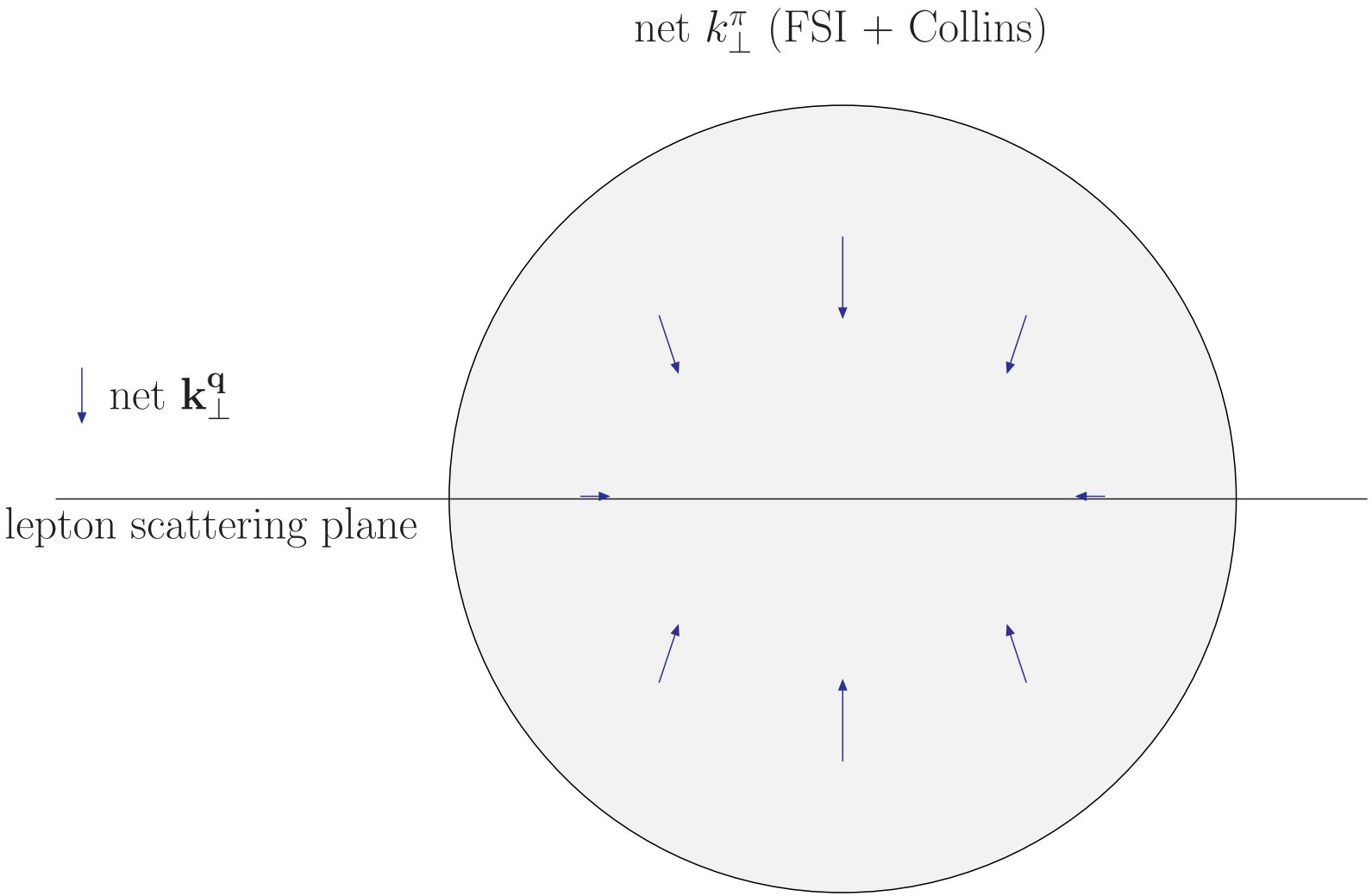
SSA of π in jet emanating from ⊥ pol. q

probing BM function in tagged SIDIS



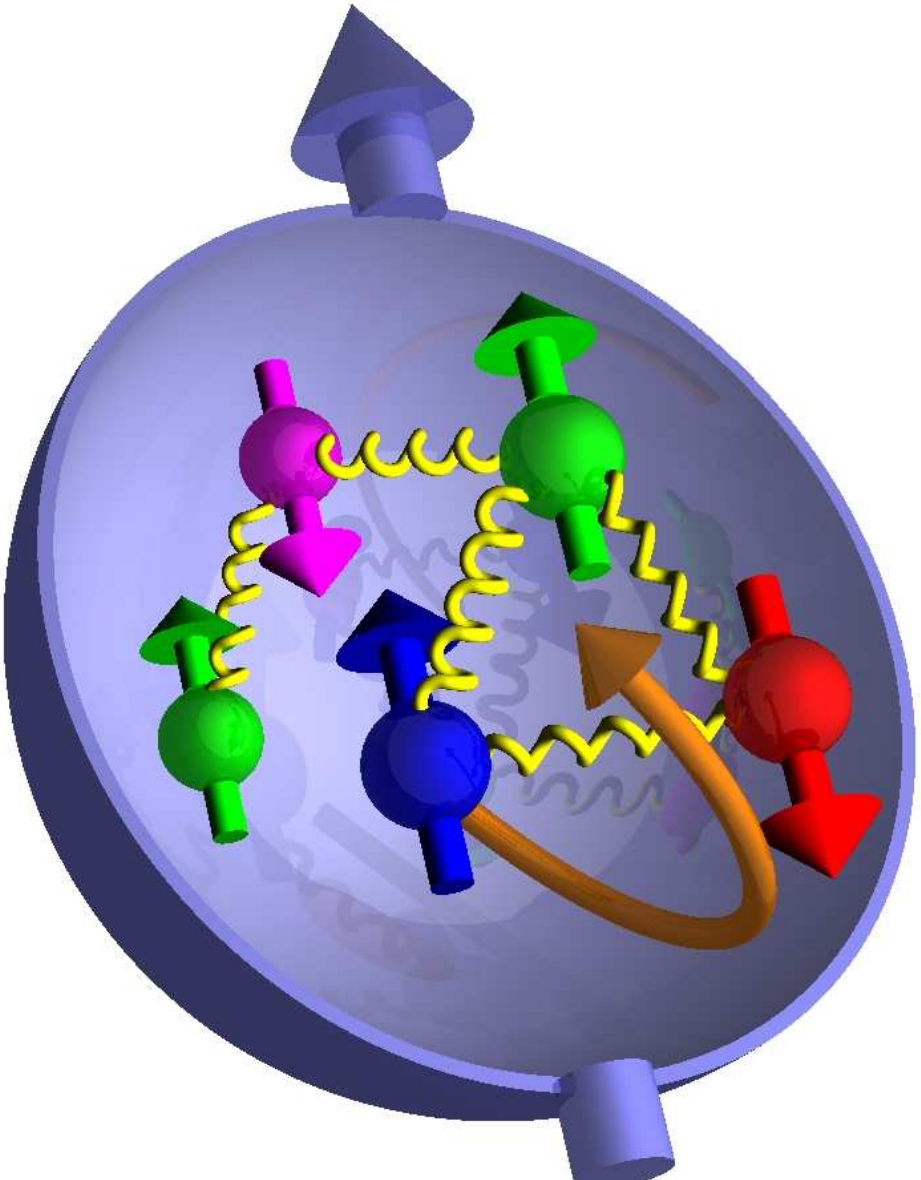
$\hookrightarrow$ in this example, enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

probing BM function in tagged SIDIS



$\hookrightarrow$ expect enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

What is Orbital Angular Momentum



Motivation

- polarized DIS: only $\sim 30\%$ of the proton spin due to quark spins

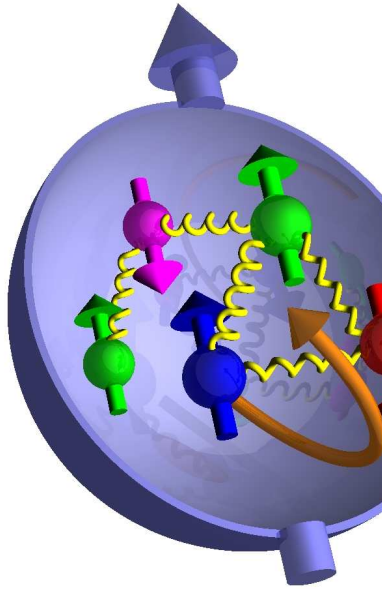
$\hookrightarrow$ 'spin crisis' $\longrightarrow$ 'spin puzzle', because $\Delta\Sigma$ much smaller than the quark model result $\Delta\Sigma = 1$

$\hookrightarrow$ quest for the remaining 70%

- quark orbital angular momentum (OAM)
- gluon spin
- gluon OAM

$\hookrightarrow$ How are the above quantities defined?

$\hookrightarrow$ How can the above quantities be measured



example: angular momentum in QED

- consider, for simplicity, QED without electrons:

$$\vec{J} = \int d^3r\, \vec{x} \times \left(\vec{E} \times \vec{B}\right) = \int d^3r\, \vec{x} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A}\right)\right]$$

- integrate by parts

$$\vec{J} = \int d^3r\, \left[E^j \left(\vec{x} \times \vec{\nabla}\right) A^j + \left(\vec{x} \times \vec{A}\right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}\right]$$

- drop 2nd term (eq. of motion $\vec{\nabla} \cdot \vec{E} = 0$), yielding $\vec{J} = \vec{L} + \vec{S}$ with

$$\vec{L} = \int d^3r\, E^j \left(\vec{x} \times \vec{\nabla}\right) A^j \qquad \vec{S} = \int d^3r\, \vec{E} \times \vec{A}$$

- note: $\vec{L}$ and $\vec{S}$ not separately gauge invariant

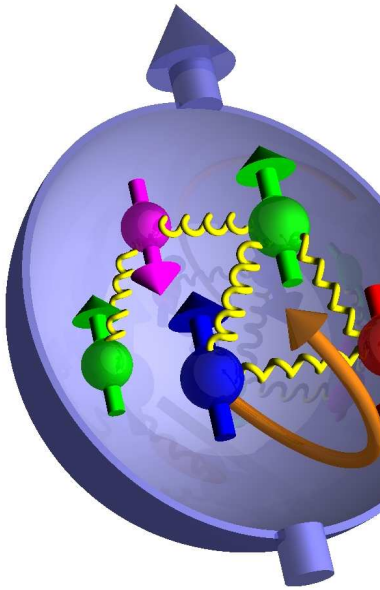
example (cont.)

- total angular momentum of isolated system uniquely defined
- ambiguities arise when decomposing $\vec{J}$ into contributions from different constituents
- gauge theories: changing gauge may also shift angular momentum between various degrees of freedom

$\hookrightarrow$ decomposition of angular momentum in general depends on ‘scheme’ (gauge & quantization scheme)

- does <u>not</u> mean that angular momentum decomposition is meaningless, but
- one needs to be aware of this ‘scheme’-dependence in the physical interpretation of exp/lattice/model results in terms of spin vs. OAM
- and, for example, not mix ‘schemes’, e.t.c.

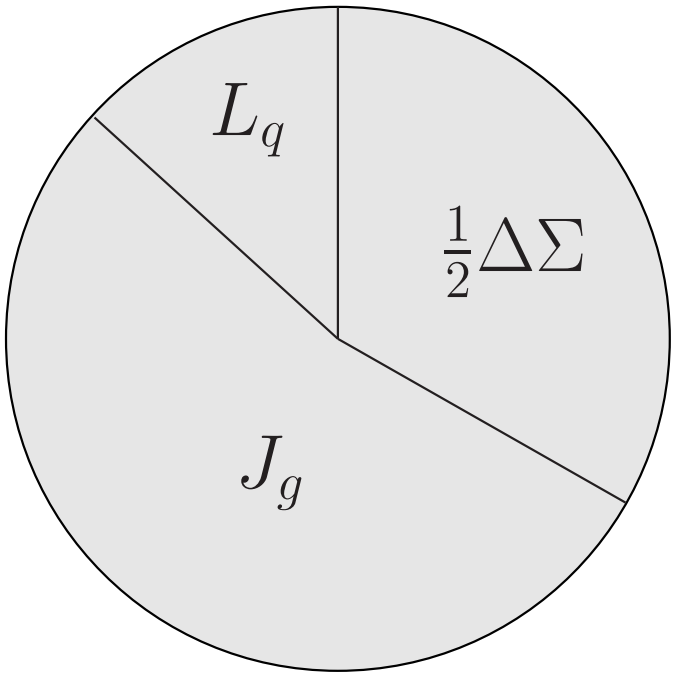
What is Orbital Angular Momentum?



- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe

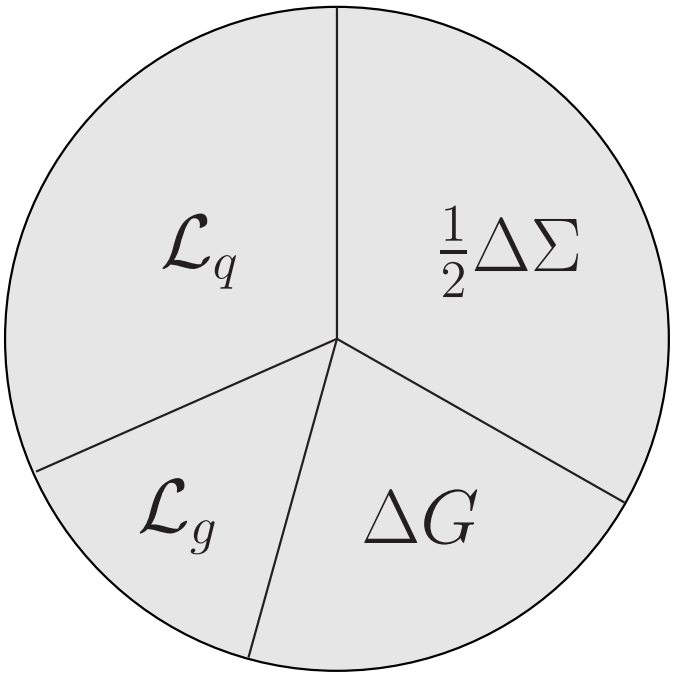
The nucleon spin pizza(s)

Ji



'pizza tre stagioni'

Jaffe & Manohar



'pizza quattro stagioni'

- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

$T^{\mu\nu}(x) \longrightarrow$ Momentum Operator

- energy momentum tensor $T^{\mu\nu} = T^{\nu\mu}$; $\quad \partial_\mu T^{\mu\nu} = 0$
- T^{00} energy density; $\quad T^{0i}$ momentum density
- $\tilde{P}^\mu \equiv \int d^3x T^{\mu 0}$ conserved

$$\frac{d}{dt}\tilde{P}^\mu = \int d^3x \frac{\partial}{\partial x^0} T^{\mu 0} \overset{\partial_\mu T^{\mu\nu}=0}{=} \int d^3x \frac{\partial}{\partial x^i} T^{\mu i} = 0$$

- $T^{\mu\nu}$ contains interactions, e.g. $T_q^{\mu\nu} = \frac{i}{2}\bar{\psi}\left(\gamma^\mu D^\nu + \gamma^\nu D^\mu\right)\psi$
- $T^{\mu 0}$ contains time derivative (don't want a Hamiltonian/momentum operator that contains time derivative!)

$\hookrightarrow$ replace by space derivative, using equation of motion, e.g. $\left(i\gamma^\mu D_\mu - m\right)\psi = 0$ to replace $iD^0\psi \to \gamma^0\left(i\gamma^k D_k - m\right)\psi$

- some of the resulting space derivatives add up to total derivative terms which do not contribute to volume integral

$$P^\mu \equiv \int d^3x \left[T^{\mu 0} + \text{`eq. of motion terms'} + \text{`surface terms'}\right]$$

Angular Momentum Operator

- angular momentum tensor $M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho}$
- $\partial_\rho M^{\mu\nu\rho} = 0$

$\hookrightarrow$ $\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk} \int d^3r M^{jk0}$ conserved

$$\frac{d}{dt}\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk} \int d^3x \partial_0 M^{jk0} = \frac{1}{2}\varepsilon^{ijk} \int d^3x \partial_l M^{jkl} = 0$$

- $M^{\mu\nu\rho}$ contains time derivatives (since $T^{\mu\nu}$ does)
  - use eq. of motion to get rid of these (as in T^{0i})
  - integrate total derivatives appearing in T^{0i} by parts
  - yields terms where derivative acts on x^i which then 'disappears'

  $\hookrightarrow$ J^i usally contains both
    - 'Extrinsic' terms, which have the structure '$\vec{x}\times$ Operator', and can be identified with 'OAM'
    - 'Intrinsic' terms, where the factor $\vec{x}\times$ does not appear, and can be identified with 'spin'

Angular Momentum in QCD (Ji)

- following this general procedure, one finds in QCD

$$\vec{J} = \int d^3x \left[\psi^\dagger \vec{\Sigma}\psi + \psi^\dagger \vec{x} \times \left(i\vec{\partial} - g\vec{A}\right)\psi + \vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]$$

with $\Sigma^i = \frac{i}{2}\varepsilon^{ijk}\gamma^j\gamma^k$

- Ji does <u>not</u> integrate gluon term by parts, <u>nor</u> identify gluon spin/OAM separately
- Ji-decomposition valid for all three components of $\vec{J}$, but usually only applied to $\hat{z}$ component, where the quark spin term has a partonic interpretation

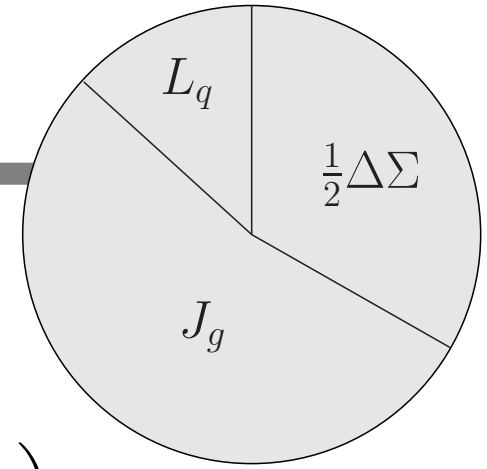
(+) all three terms manifestly gauge invariant

(+) DVCS can be used to probe $\vec{J}_q = \vec{S}_q + \vec{L}_q$

(-) quark OAM contains interactions

(-) only quark spin has partonic interpretation as a single particle density

Ji-decomposition



- Ji (1997)

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q \right) + J_g$$

with ($P^\mu = (M, 0, 0, 1)$, $S^\mu = (0, 0, 0, 1)$)

$$\begin{aligned}
\frac{1}{2}\Delta q &= \frac{1}{2} \int d^3x \, \langle P, S | \, q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) \, | P, S \rangle \qquad \Sigma^3 = i\gamma^1\gamma^2 \\
L_q &= \int d^3x \, \langle P, S | \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D} \right)^3 q(\vec{x}) \, | P, S \rangle \\
J_g &= \int d^3x \, \langle P, S | \left[\vec{x} \times \left(\vec{E} \times \vec{B} \right) \right]^3 | P, S \rangle
\end{aligned}$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

The Ji-relation (poor man's derivation)

- What distinguishes the Ji-decomposition from other decompositions is the fact that L_q can be constrained by experiment:

$$\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^{1} dx\, x \left[H_q(x,\xi,0) + E_q(x,\xi,0) \right]$$

(nucleon at rest; $\vec{S}$ is nucleon spin)

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- derivation (MB-version):
  - consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. nucleon polarized in $\hat{x}$ direction with $\vec{p} = 0$ (wave packet if necessary)
  - for such a state, $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$ and $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$

$\hookrightarrow$ $\langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$

$\hookrightarrow$ relate 2nd moment of $\perp$ flavor dipole moment to J_q^x

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  - effect sum of two effects:
    - $\langle T^{++} y \rangle$ for a point-like transversely polarized xnucleon
    - $\langle T_q^{++} y \rangle$ for a quark relative to the center of momentum of a transversely polarized nucleon
  - 2nd moment of $\perp$ flavor dipole moment for point-like nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The Ji-relation (poor man's derivation)

- derivation (MB-version):
  - $T_q^{0z} = i\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right)q$
  - since $\psi^\dagger\partial_z\psi$ is even under $y \to -y$, $i\bar{q}\gamma^0\partial^z q$ does not contribute to $\langle T^{0z}y$
  - $\hookrightarrow$ using $i\partial_0\psi = E\psi$, one finds

$$\begin{aligned}\langle T^{0z}b_y\rangle &= E\int d^3r\psi^\dagger\gamma^0\gamma^z\psi y = E\int d^3r\psi^\dagger\begin{pmatrix}0 & \sigma^z\\ \sigma^z & 0\end{pmatrix}\psi y\\ &= \frac{2E}{E+M}\int d^3r\chi^\dagger\sigma^z\sigma^y\chi f(r)(-i)\partial^y f(r)y = \frac{E}{E+M}\int d^3\end{aligned}$$

  - consider nucleon state with $\vec{p} = 0$, i.e. $E = m$ & $\int d^3r f^2(r) = 1$
  - $\hookrightarrow$ 2nd moment of $\perp$ flavor dipole moment is $\frac{1}{2M}$
  - $\hookrightarrow$ 'overall shift' of nucleon COM yields contribution $\frac{1}{2}\int dx\, xH_q(x,0,0)$ to $\langle T_q^{++}y\rangle$

The Ji-relation (poor man's derivation)

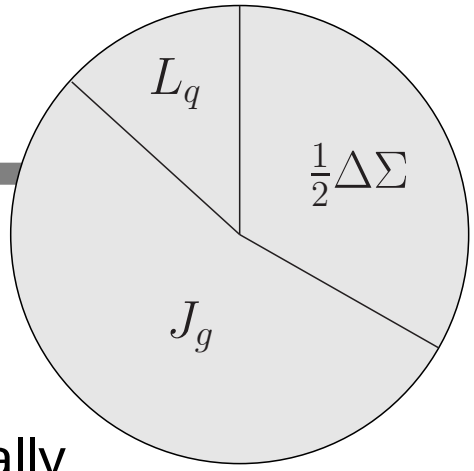
- derivation (MB-version):
  - intrinsic distortion adds $\frac{1}{2}\int dx\, x E_q(x,0,0)$ to that

$\hookrightarrow$ $J_q^x = \frac{1}{2}\int dx\, x\, [H_q(x,0,0) + E_q(x,0,0)]$

  - rotational invariance: should apply to each vector component

$\hookrightarrow$ Ji relation

Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D}\right) q + \vec{r} \times \left(\vec{E} \times \vec{B}\right)$ applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities
- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix elements of $q^\dagger \left(\vec{r} \times i\vec{D}\right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) that is local and manifestly gauge invariant has not been found

L_q for proton from Ji-relation (lattice)

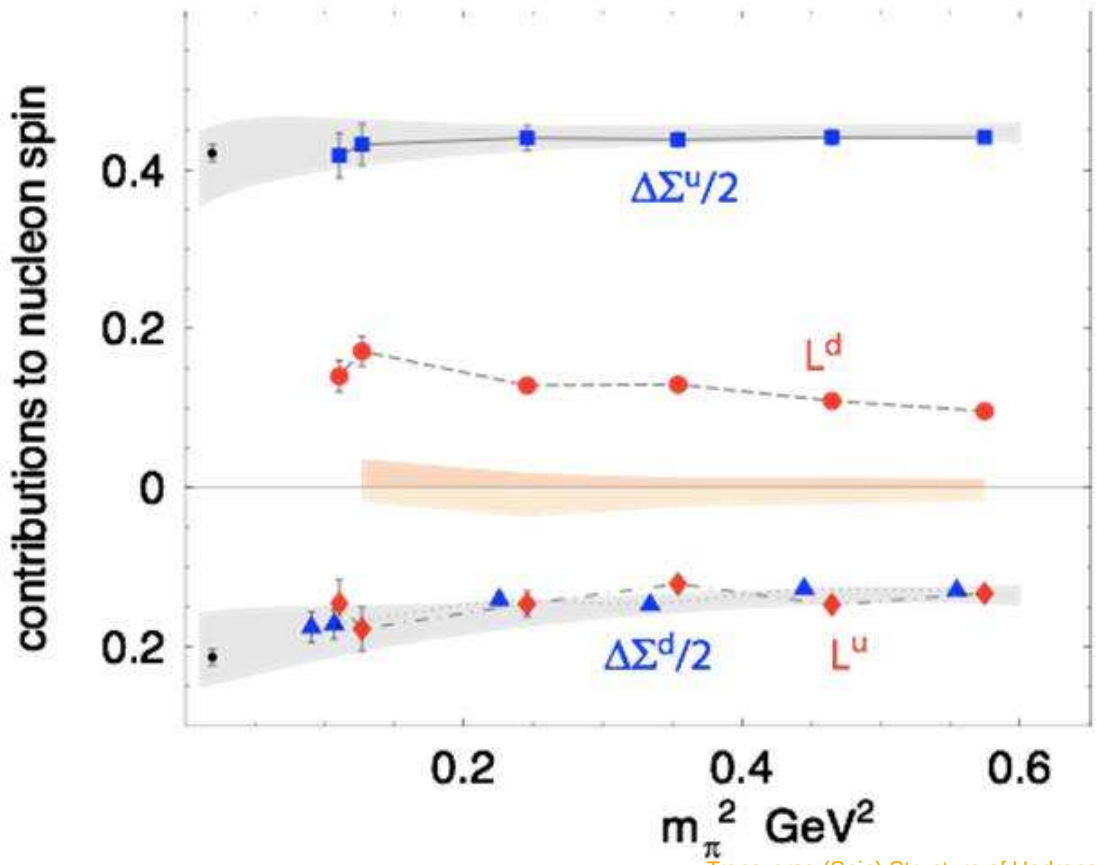
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx \, [H_q(x,0) + E_q(x,0)] \, x.$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypesurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2x_\perp \int dx^- M^{12+}$$

where $x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^-\right)$ and $M^{12+} = \frac{1}{\sqrt{2}}\left(M^{120} + M^{123}\right)$

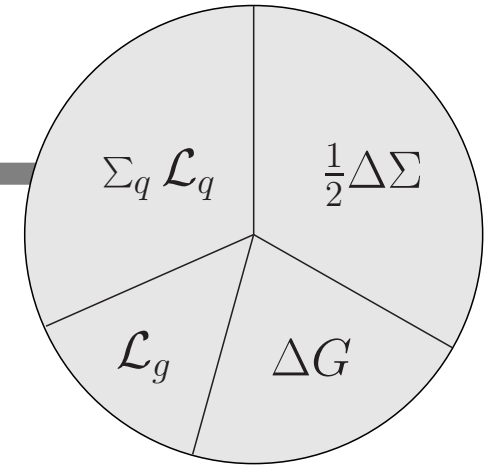
- Since $\partial_\mu M^{12\mu} = 0$

$$\int d^2\mathbf{x}_\perp \int dx^- M^{12+} = \int d^2\mathbf{x}_\perp \int dx^3 M^{120}$$

(compare electrodynamics: $\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow \quad$ flux in = flux out)

- use eqs. of motion to get rid of 'time' (∂_+ derivatives) & integrate by parts whenever a total derivative appears in the T^{i+} part of M^{12+}

Jaffe/Manohar decomposition



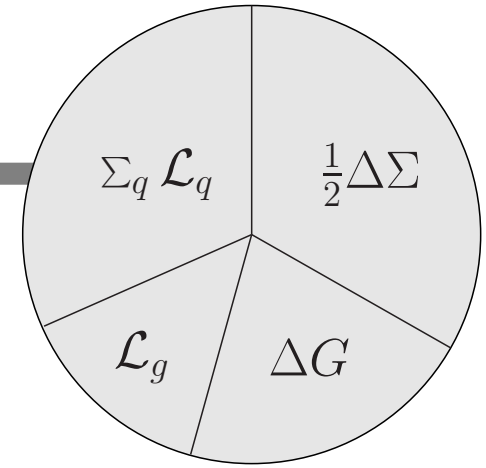
- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

where ($\gamma^+ = \gamma^0 + \gamma^z$)

$$\begin{aligned}
\mathcal{L}_q &= \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle \\
\Delta G &= \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle \\
\mathcal{L}_g &= 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle
\end{aligned}$$

Jaffe/Manohar decomposition



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\overrightarrow{p}\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to 'mix' Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- For nucleon at rest, matrix element of L_q same as that of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(xgA^y - ygA^x\right) q\Big|_{A^+=0}$

Summary part 1:

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\overrightarrow{p}\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[M.B. + Hikmat Budhathoki Chhetri (BC), PRD **79**, 071501 (2009)]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

$\hookrightarrow$ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \left(M + \frac{m}{x}\right)\phi \qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1 + ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\,[q(x) + E(x,0,0)] - \frac{1}{2}\Delta q$

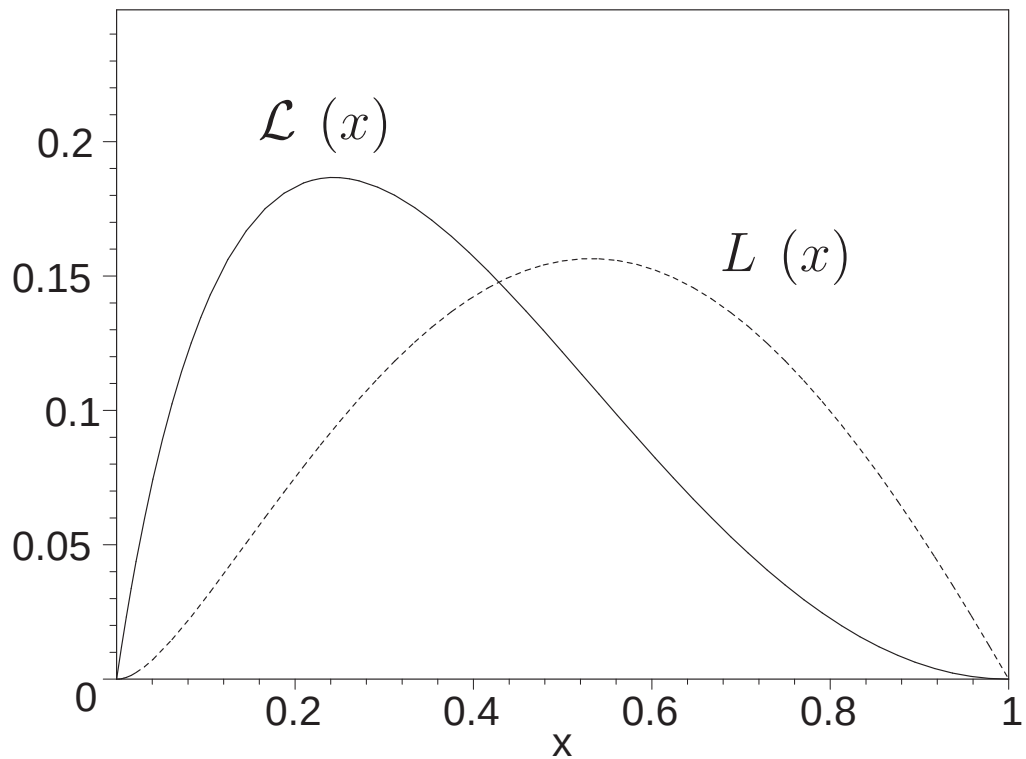
$\rightsquigarrow$ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is <u>not</u> a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ 'unintegrated Ji-relation' does <u>not</u> yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1 + ik^2}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$$

- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\,[q(x) + E(x,0,0)] - \frac{1}{2}\Delta q$

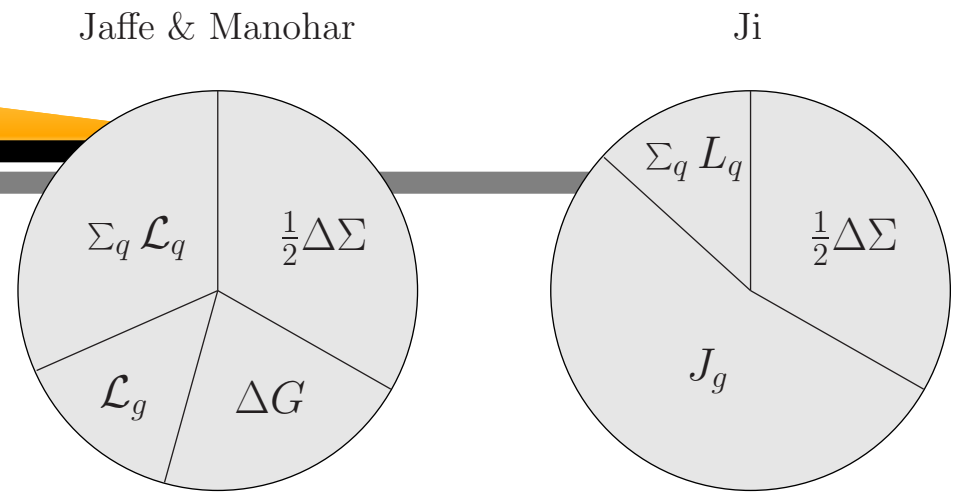
$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and <u>defining</u> $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields $\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$
- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- ↪ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- ↪ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d > L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- ↪ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary



- inclusive $\overrightarrow{e}\overleftarrow{p}/\overrightarrow{p}\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does <u>not</u> yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for O (α_s) dressed quark

Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist $\longrightarrow$ 'polarized quark distribution' $g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)$
- $\frac{1}{Q^2}$-corrections to X-section involve 'higher-twist' distribution functions, such as $g_2(x)$

$$\sigma_{TT} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- $g_2(x)$ involves quark-gluon correlations and does not have a parton interpretation as difference between number densities
- for $\perp$ polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

- $\sqrt{2} G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- matrix elements of $\bar{q} B^x \gamma^+ q$ and $\bar{q} E^y \gamma^+ q$ are sometimes called color-electric and magnetic polarizabilities

  $2M^2 \vec{S} \chi_E = \left\langle P, S \left| \vec{j}_a \times \vec{E}_a \right| P, S \right\rangle$ & $2M^2 \vec{S} \chi_B = \left\langle P, S \left| j_a^0 \vec{B}_a \right| P, S \right\rangle$

  with $d_2 = \frac{1}{4} (\chi_E + 2\chi_M)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

- QED: $\bar{q}(0) e F^{+y}(0) \gamma^+ q(0)$ correlator between quark density $\bar{q}\gamma^+ q$ and ($\hat{y}$-component of the) Lorentz-force

$$F^y = e \left[\vec{E} + \vec{v} \times \vec{B} \right]^y = e \left(E^y - B^x \right) = -e \left(F^{0y} + F^{zy} \right) = -e\sqrt{2} F^{+y}.$$

for charged paricle moving with $\vec{v} = (0, 0, -1)$ in the $-\hat{z}$ direction

$\hookrightarrow$ matrix element of $\bar{q}(0) e F^{+y}(0) \gamma^+ q(0)$ yields γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0, 0, -1)$ would experience at that point

$\hookrightarrow$ d_2 a measure for the **color Lorentz force** acting on the struck quark in SIDIS in the instant **after being hit by the virtual photon**

$$\langle F^y(0) \rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

Quark-Gluon Correlations (Interpretation)

- Interpretation of d_2 with the transverse FSI force in DIS also consistent with $\langle k_\perp^y \rangle \equiv \int_0^1 dx \int \mathrm{d}^2 k_\perp \, k_\perp^2 f_{1T}^\perp(x, k_\perp^2)$ in SIDIS (Qiu, Sterman)

$$\langle k_\perp^y \rangle = -\frac{1}{2p^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dx^- g G^{+y}(x^-) \gamma^+ q(0) \right| P, S \right\rangle$$

  semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

- matrix element defining d_2 same as the integrand (for $x^- = 0$) in the QS-integral:
  - $\langle k_\perp^y \rangle = \int_0^\infty dt F^y(t)$ (use $\mathrm{d}x^- = \sqrt{2}\mathrm{d}t$)

  $\hookrightarrow$ first integration point $\longrightarrow F^y(0)$

  $\hookrightarrow$ (transverse) force at the begin of the trajectory, i.e. at the moment after absorbing the virtual photon

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$
  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S \left| \bar{q}(0) g \tilde{G}^{\mu\nu}(0) \gamma_\nu q(0) \right| P,S \right\rangle \sim f_2$

$\hookrightarrow$ different linear combination $f_2 = \chi_E - \chi_B$ of χ_E and χ_M

$\hookrightarrow$ combine with $d_2 \Rightarrow$ disentangle electric and magnetic force

- What should one expect (sign)?
  - $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$

  $\hookrightarrow$ d_2 positive/negative for u/d quarks in proton
  - large N_C: $d_2^{u/p} = -d_2^{d/p}$
  - consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$
- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$

$\hookrightarrow$ ($M^2 \approx 5 \frac{\mathrm{GeV}}{fm}$ $\quad \langle F_u^y(0) \rangle \approx -50 \frac{\mathrm{MeV}}{fm}$ $\quad \langle F_d^y(0) \rangle \approx 28 \frac{\mathrm{MeV}}{fm}$

- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- GPDs provide decomposition of form factors w.r.t. the momentum of the active quark

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlator, but $\Delta \equiv p' - p \neq 0$.
- t-dependence of GPDs at $\xi = 0$ (purely $\perp$ momentum transfer) $\Rightarrow$ Fourier transform of impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ knowledge of GPDs for $\xi = 0$ provides novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation, e.g. $q(x, \mathbf{b}_\perp) > 0$ for $x > 0$

Summary

- $\frac{\Delta_\perp}{2M} E(x, 0, -\mathbf{\Delta}_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely distorted when is nucleon polarized in $\perp$ direction.
- (attractive) final state interaction in semi-inclusive DIS converts $\perp$ position space asymmetry into $\perp$ momentum space asymmetry

$\hookrightarrow$ simple physical explanation for observed Sivers effect in $\gamma^* p \to \pi X$

- physical explanation for Boer-Mulders effect: correlation between quark spins (transversity) and currents
- OAM in QED: $L_e \neq \mathcal{L}_e$ and $\Delta\gamma \neq J_\gamma - \mathcal{L}_\gamma$
- $d_2 \equiv 3 \int dx x^2 \bar{g}_2(x)$: (average) transverse force on quarks in DIS $\langle F_y \rangle = -M^2 d_2$ (nucleon polarized in $+\hat{x}$-direction)
- recommended reading:
  M.B., A.Miller, and W.-D.Nowak, 'Spin-Polarized High-Energy Scattering of Charged Leptons on Nucleons', hep-ph/0812.2208