

# Overview of nucleon form factor measurements

Focus on Generalized Parton Distribution and form factors

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# Spin content of the nucleon

$$\frac{1}{2} = \left( \frac{1}{2} \Delta\Sigma + L_q \right) + (\Delta G + L_g)$$

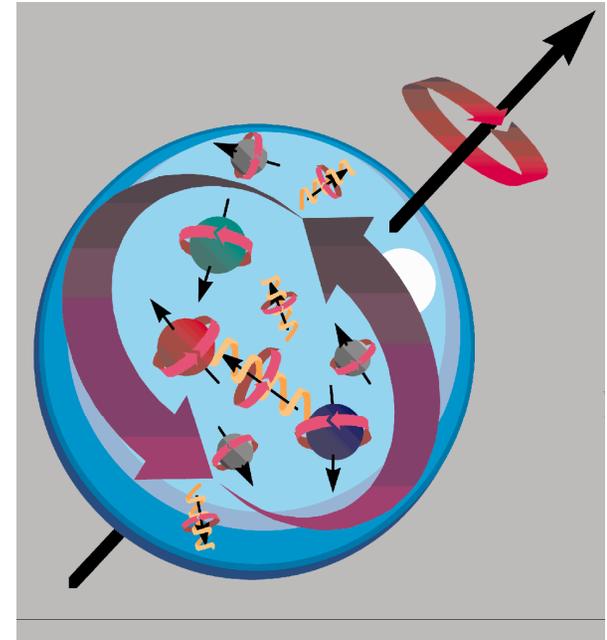
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

$$2J^q = \Delta q + 2L^q$$

Ji's sum rule

$$2J_q = \int x [H^q(x, 0, 0) + E^q(x, 0, 0)] dx$$

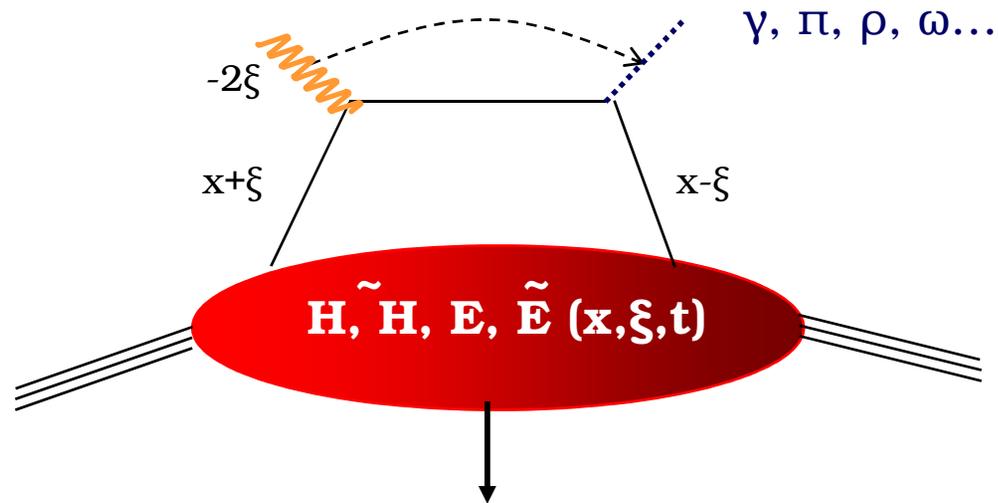
X. D. Ji, Phys. Rev. Lett. **78**, 610 (1997), Phys. Rev. D **55**, 7114 (1997).



# Generalized Parton Distributions

$H(x, \xi, t)$  and  $E(x, \xi, t)$  are general non-perturbative objects where  $x$  is the longitudinal momentum fraction  
 $\xi$  is the skewness or longitudinal momentum asymmetry  
and  $t$  is the four momentum transfer squared or  $-Q^2$

Considerable effort presently at Jlab and in future at Jlab@12 GeV to measure through deeply virtual Compton scattering and deeply virtual meson production.



# GPDs and Deep Inelastic Scattering

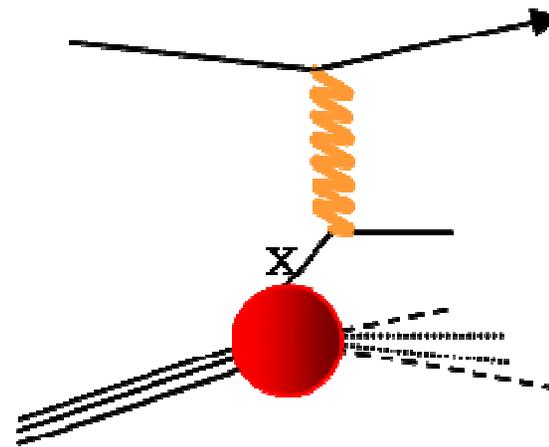
Unpolarized parton distribution

$$H(x, 0, 0) = q(x)$$

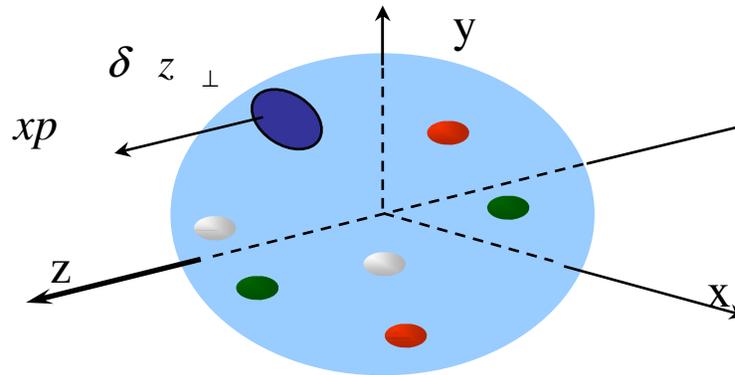
Polarized parton distributions

$$\tilde{H}(x, 0, 0) = \Delta q(x)$$

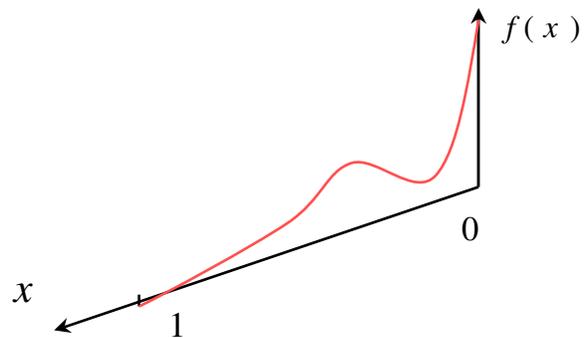
## Standard Parton Distributions



# *Parton Distributions*



**Longitudinal** momentum distribution  
(no information on the  
transverse localisation)



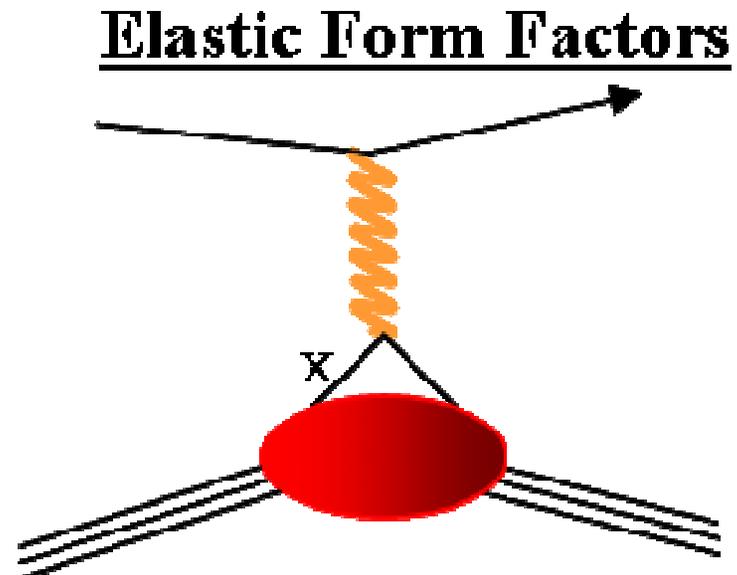
# GPD and Elastic form factors

Dirac form factor

$$F_1^q(t) = \int_{-1}^{+1} dx H^q(x, \xi, t)$$

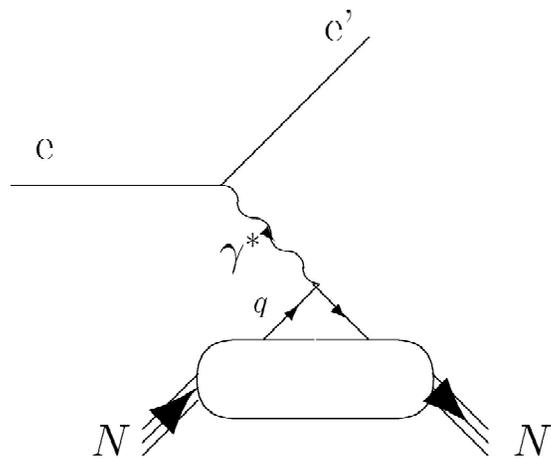
Pauli Form Factor

$$F_2^q(t) = \int_{-1}^{+1} dx E^q(x, \xi, t)$$



$E^q$  is new information about the nucleon structure that can not be accessed in DIS scattering

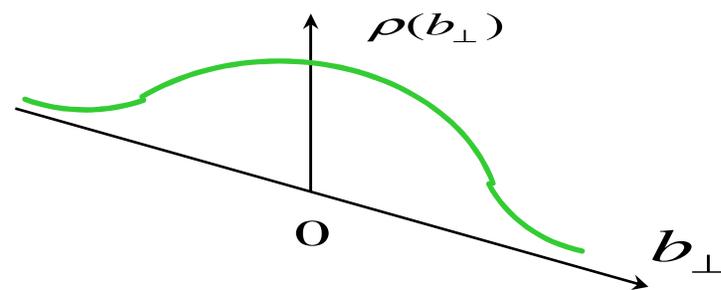
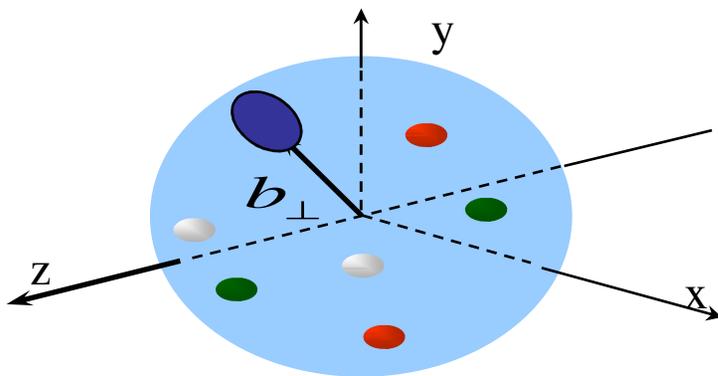
# ELASTIC SCATTERING : ep $\Rightarrow$ ep



$$t = (p' - p)^2$$

$$\frac{d\sigma}{dt} \propto A F_1(t) + B F_2(t)$$

**Transverse** localisation of partons in the nucleon  
(independently of their **longitudinal** momentum)



# GPDs and form factors

Define the nucleon  $F_1$  and  $F_2$  in terms of sum of quark form factor weighted by charge

$$F_i(t) = \sum_a e_q F_i^q(t)$$

Relate  $F_1$  and  $F_2$  to valence quark GPDs through sum rules

$$F_1^q(t) = \int_{-1}^{+1} dx H^q(x, \xi, t) \quad F_2^q(t) = \int_{-1}^{+1} dx E^q(x, \xi, t)$$

Choose  $\xi = 0$  and define nonforward parton densities

$$\mathcal{H}^q(x, t) = H^q(x, 0, t) + H^q(-x, 0, t)$$

$$\mathcal{E}^q(x, t) = E^q(x, 0, t) + E^q(-x, 0, t)$$

# Non Forward parton densities

In the  $t = 0$  limit

the nonforward parton densities equal the valence quark parton densities

$$\mathcal{H}^u(x, t = 0) = u_v(x) \quad \mathcal{H}^d(x, t = 0) = d_v(x)$$

The  $t=0$  limit of  $\mathcal{E}^q(x, 0)$  does not have a partonic distribution.

$\mathcal{E}^q(x, t)$  has new information about the structure of the nucleon

$$\kappa_q \equiv \int_0^1 dx \mathcal{E}^q(x) \quad \begin{aligned} \kappa_u &= 2\kappa_p + \kappa_n \approx +1.673 \\ \kappa_d &= \kappa_p + 2\kappa_n \approx -2.033 \end{aligned}$$

# Ansatz for the non forward parton distribution

What  $x$  and  $t$  dependent form should be chosen for  $H$  and  $E$ ?

Formulate  $H$  in terms of the valence quark dependence and term that mixes the  $x$  and  $t$  dependence

$$\mathcal{H}_G^q(x, t) = q_v(x) e^{(1-x)t/4x\lambda^2}$$

$\lambda^2$  characterizes the average transverse momentum of the quarks in the nucleon

Fit was done to proton  $F_1$  giving  $\lambda^2 = 0.7 \text{ GeV}^2$  or average transverse momentum of 300 MeV

Main problem with Gaussian form is that it is divergent in predicting the proton rms radius. This limits in usefulness below  $Q^2 = 1 \text{ GeV}^2$

# Regge Ansatz for the non forward parton distribution

$$\mathcal{H}_{R1}^q(x, t) = q_v(x) x^{-\alpha_1' t}$$

$$F_1^u(t) = \int_0^1 dx u_v(x) e^{-\alpha_1' t \ln x} \quad F_1^d(t) = \int_0^1 dx d_v(x) e^{-\alpha_1' t \ln x}$$

$$F_1^p(t) = e_u F_1^u(t) + e_d F_1^d(t) \quad F_1^n(t) = e_u F_1^d(t) + e_d F_1^u(t)$$

RMS radius  
for proton  
and neutron

$$r_{1,p}^2 = -6 \alpha_1' \int_0^1 dx \left\{ e_u u_v(x) + e_d d_v(x) \right\} \ln x$$

$$r_{1,n}^2 = -6 \alpha_1' \int_0^1 dx \left\{ e_u d_v(x) + e_d u_v(x) \right\} \ln x$$

$$\mathcal{E}_{R1}^q(x, t) = \mathcal{E}^q(x) x^{-\alpha_2' t}$$

$$\mathcal{E}^u(x) = \frac{\kappa_u}{2} u_v(x)$$

$$\mathcal{E}^d(x) = \kappa_d d_v(x)$$

## 2<sup>nd</sup> Regge Ansatz for the non forward parton distribution

$$\mathcal{H}_{R2}^q(x, t) = q_v(x) x^{-\alpha'_1} (1-x)^t$$

$$\mathcal{E}^q(x, t) = \mathcal{E}^q(x) x^{-\alpha'_2} (1-x)^t$$

Experimental find that  $F_2$  falls off faster than  $F_1$

$$\mathcal{E}^u(x) = \frac{\kappa_u}{N_u} (1-x)^{\eta_u} u_v(x) \quad \mathcal{E}^d(x) = \frac{\kappa_d}{N_d} (1-x)^{\eta_d} d_v(x)$$

$$N_u = \int_0^1 dx (1-x)^{\eta_u} u_v(x) \quad N_d = \int_0^1 dx (1-x)^{\eta_d} d_v(x)$$

$$F_2^u(t) = \int_0^1 dx \frac{\kappa_u}{N_u} (1-x)^{\eta_u} u_v(x) x^{-\alpha'_2} (1-x)^t$$

$$F_2^d(t) = \int_0^1 dx \frac{\kappa_d}{N_d} (1-x)^{\eta_d} d_v(x) x^{-\alpha'_2} (1-x)^t$$

Fit parameters  $\eta_u, \eta_d, \alpha'_1$  and  $\alpha'_2$

# Use MSRT valence parton distributions

$$\mathcal{H}_{R2}^q(x, t) = q_v(x) x^{-\alpha_1'} (1-x)^t$$

$$u_v = 0.262 x^{-0.69} (1-x)^{3.50} \left( 1 + 3.83 x^{0.5} + 37.65 x \right)$$

$$d_v = 0.061 x^{-0.65} (1-x)^{4.03} \left( 1 + 49.05 x^{0.5} + 8.65 x \right)$$

A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Phys. Lett. B **531**, 216 (2002)

# Proton and neutron RMS radius

In the 2<sup>nd</sup> Regge type model

$$r_{1,p}^2 = -6 \alpha_1' \int_0^1 dx \left\{ e_u u_v(x) + e_d d_v(x) \right\} (1-x) \ln x$$

$$r_{1,n}^2 = -6 \alpha_1' \int_0^1 dx \left\{ e_u d_v(x) + e_d u_v(x) \right\} (1-x) \ln x$$

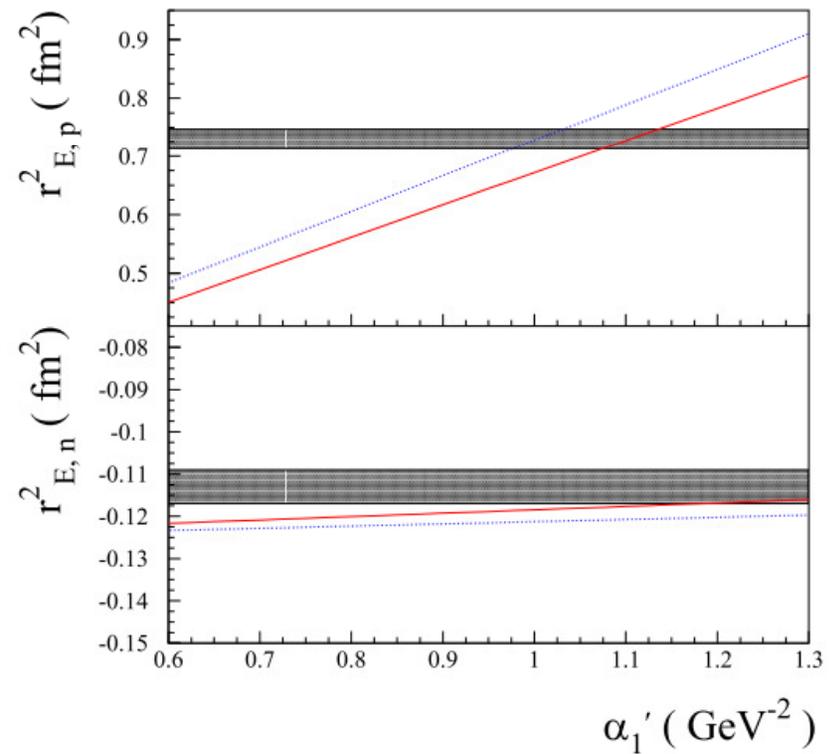
$$r_{E,p}^2 = r_{1,p}^2 + \frac{3}{2} \frac{\kappa_p}{M_N^2}$$

$$r_{E,n}^2 = r_{1,n}^2 + \frac{3}{2} \frac{\kappa_n}{M_N^2}$$

In plot

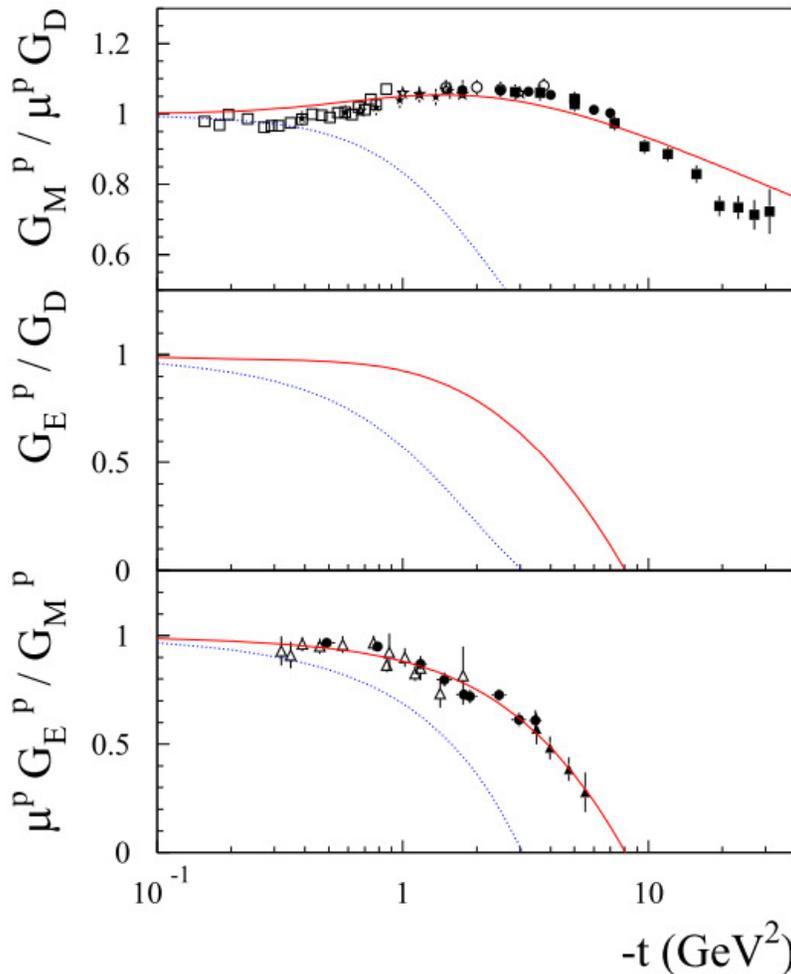
Blue line is 1<sup>st</sup> Regge type model

Red line is 2<sup>nd</sup> Regge type model



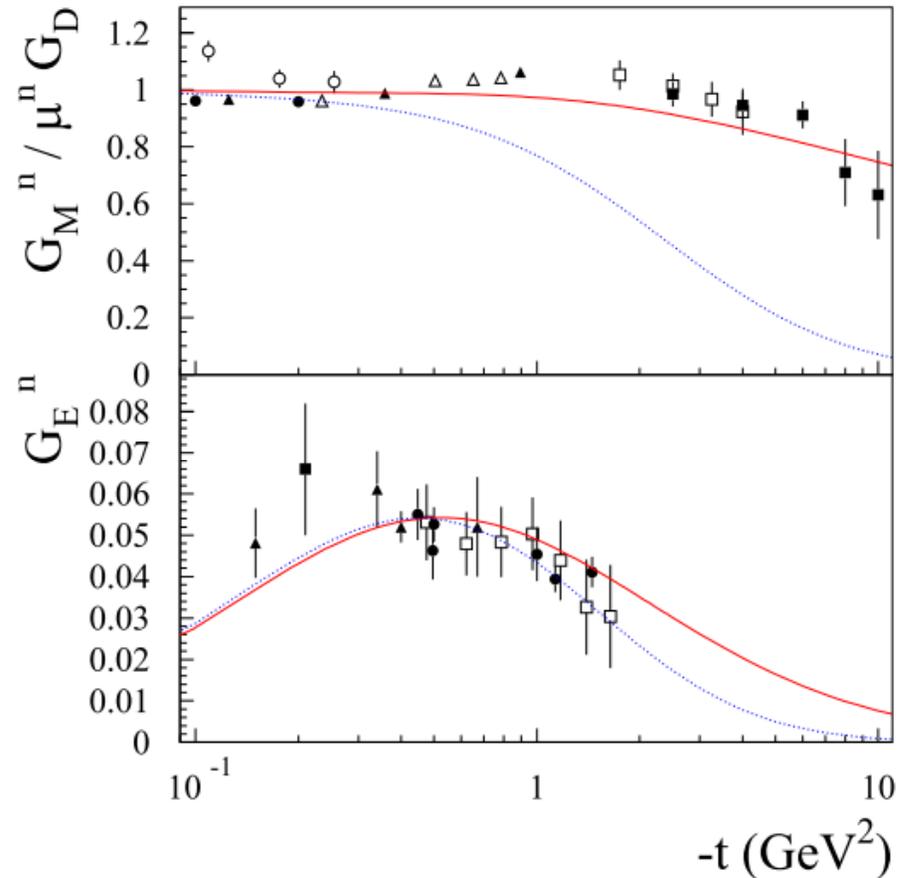
# Fit to the nucleon form factors

Proton form factors



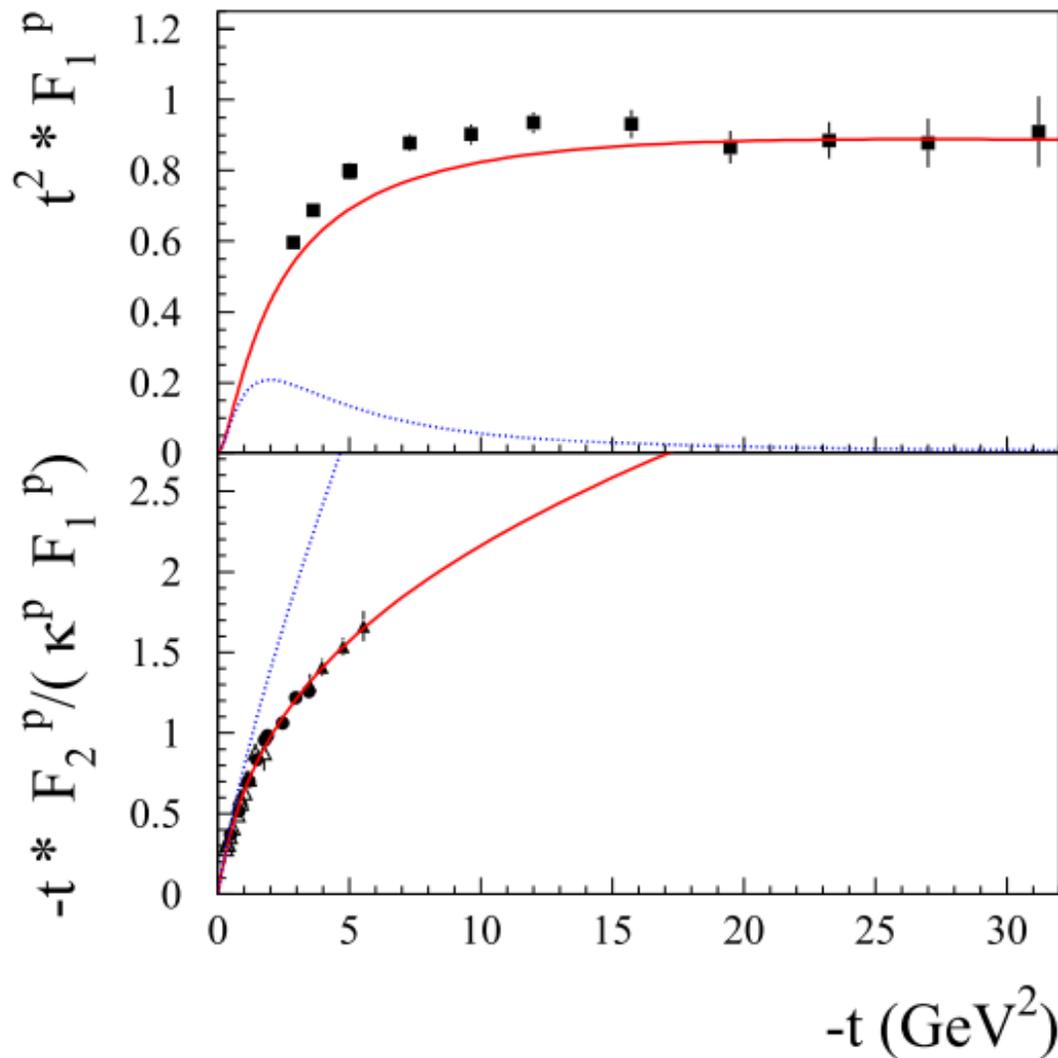
R1 (blue line) using R2 fit  
 $\alpha'_1 = 1.098$   $\alpha'_2 = 1.158$

Neutron form factors



R2 fit (red line)  
 $\eta_u = 1.52$   $\eta_d = 0.31$   
 $\alpha'_1 = 1.098$   $\alpha'_2 = 1.158$

# Large $Q^2$ behavior of the form factors



Find  $Q^4 F_1 = \text{constant}$

But calculation not assuming  
Two hard gluon exchange as  
in pQCD

Rather an overlap of soft  
wavefunctions

# Total quark angular momentum

$$2J^q = \Delta q + 2L^q$$

$$2J^q = \int_{-1}^1 dx x \{H^q(x, 0, 0) + E^q(x, 0, 0)\}$$

$$M_2^q \equiv \int_{-1}^1 dx x H^q(x, 0, 0) = \int_0^1 dx x [q_v(x) + 2\bar{q}(x)]$$

$$2J^u = M_2^u + \frac{\kappa^u}{N_u} \int_0^1 dx x (1-x)^{\eta_u} u_v(x)$$

$$2J^d = M_2^d + \frac{\kappa^d}{N_d} \int_0^1 dx x (1-x)^{\eta_d} d_v(x)$$

$$2J^s = M_2^s$$

Sea quark  $E_q$  is unknown. Need hard exclusive process like deeply virtual compton scattering

# Quark contribution to nucleon angular momentum

$$2J^q = \Delta q + 2L^q$$

	$M_2^q$ (MRST2002)	$2J^q$ (R2 model)	$2J^q$ (lattice [59])
$u$	0.40	0.63	$0.734 \pm 0.135$
$d$	0.22	-0.06	$-0.085 \pm 0.088$
$s$	0.03	0.03	
$u + d + s$	0.65	0.60	$0.65 \pm 0.16$

$$\Delta d_v \simeq -0.25 \quad 2L^d \simeq 0.2$$

$$\Delta u_v \simeq 0.6$$

[59] M. Gockeler, R. Horsley, D. Pleiter, P. E. L. Rakow, A. Schafer, G. Schierholz and W. Schroers [QCDSF Collaboration], Phys. Rev. Lett. **92**, 042002 (2004) [[arXiv:hep-ph/0304249](https://arxiv.org/abs/hep-ph/0304249)].

# Summary

- This is first glimpse into the structure of GPDs
- Form factor sensitive to  $E$ , a new information on the nucleon structure.
- What is the sea contribution to  $E$  ?
  - need hard exclusive process to access the sea contribution to GPDs

# Drell-Yan-West relation

If the parton density behaves as  $(1 - x)^\nu$  then  
the large  $Q^2$  behavior of the form factor behaves as  $t^{-\frac{(\nu+1)}{2}}$