Hadrons in the Quark Model[a]

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The Cork Model

Up Charm Top

Down Strange Bottom

- Introduction, Classification Scheme, Simple Predictions
- Meson Spectrum
- Baryons Spectrum
- Meson Transitions
- Baryons Transitions & Sundries

What is a quark?

In *Finnegan's Wake* (J. Joyce), '*three quarks for Muster Mark*'

Six flavors of quarks,

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$$

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The only hadrons we know (those that have been confirmed non-controversially) are mesons (quark and antiquark) and baryons (three quarks).

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For ground state baryons, the parity is positive; more on quantum numbers later

For mesons (pseudoscalar, $L = 0,\ S = 0,\ J^P = 0^-$):

$$\mathbf{3} \otimes \bar{\mathbf{3}} = 1 \oplus 8$$

$$K^0\,[d\bar{s}] \qquad K^+\,[u\bar{s}]$$

$$\pi^-\,[d\bar{u}] \qquad \pi^0\left[\frac{1}{\sqrt{2}}\left(u\bar{u}-d\bar{d}\right)\right] \quad \pi^+\,[-u\bar{d}]$$

$$\eta_8\left[\frac{1}{\sqrt{6}}\left(u\bar{u}+d\bar{d}-2s\bar{s}\right)\right]$$

$$K^-\,[s\bar{u}] \qquad \bar{K}^0\,[-s\bar{d}]$$

$$\eta_1\left[\frac{1}{\sqrt{3}}\left(u\bar{u}+d\bar{d}+s\bar{s}\right)\right]$$

All (light) mesons fall into these multiplets. Thus, for the vector mesons ($L = 0,\ S = 1,\ J^P = 1^-$)

$$K^{0*}\,[d\bar{s}] \qquad K^{+*}\,[u\bar{s}]$$

$$\rho^{-}\,[d\bar{u}] \qquad \rho^{0}\left[\frac{1}{\sqrt{2}}\left(u\bar{u}-d\bar{d}\right)\right] \quad \rho^{+}\left[-u\bar{d}\right]$$

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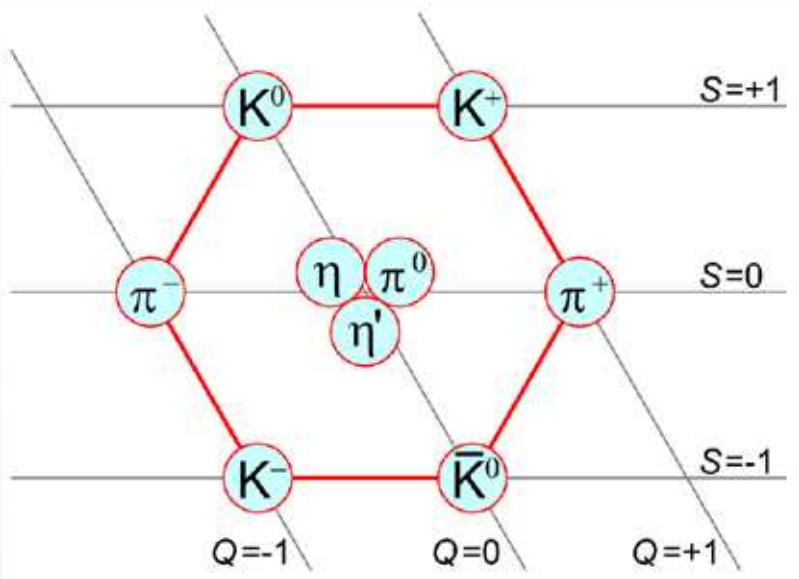
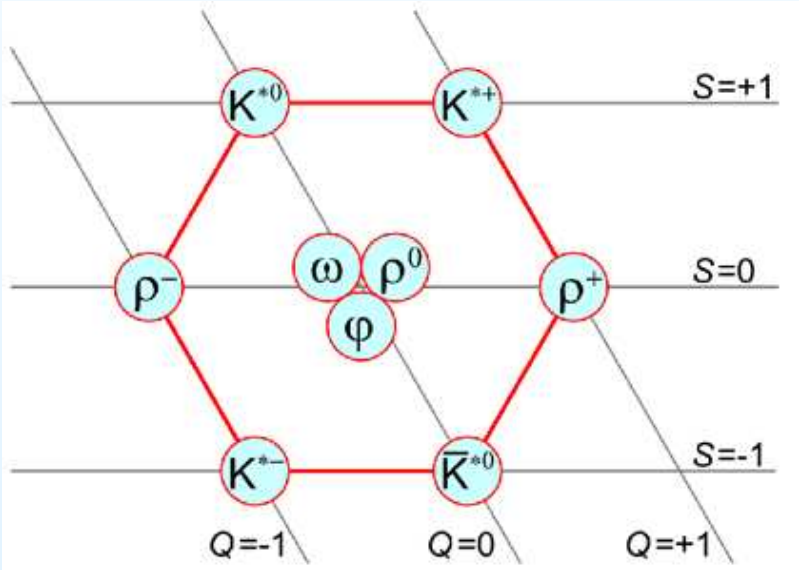

K0
K+
S=+1
π−
η
π0
η'
π+
S=0
K−
K̄0
S=-1
Q=-1
Q=0
Q=+1


K*0
K*+
S=+1
ρ−
ω
ρ0
φ
ρ+
S=0
K*−
K̄*0
S=-1
Q=-1
Q=0
Q=+1

For baryons:

$$3 \otimes 3 \otimes 3 = (6 \oplus \bar{3}) \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

$n\,[udd]$ $\quad$ $p\,[uud]$

$\Sigma^-\,[dds]$ $\quad$ $\Sigma^0\,[uds]$ $\quad$ $\Sigma^+\,[uus]$

$\Lambda\,[uds]$

$\Xi^-\,[dss]$ $\quad$ $\Xi^0\,[uss]$

$\Delta^{-}\,[ddd]$ $\Delta^{0}\,[udd]$ $\Delta^{+}\,[uud]$ $\Delta^{++}\,[uuu]$

$\Sigma^{*-}\,[dds]$ $\Sigma^{*0}\,[uds]$ $\Sigma^{*+}\,[uus]$

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Comparing equations, we find $m_{K^*} = \frac{m_\phi + m_\rho}{2}$. m_ϕ = 1.020 GeV; m_ρ = 0.77 GeV $\Longrightarrow m_{K^*} = \frac{1.79}{2} = 0.895$ GeV. Experimental masses are 0.892 GeV (charged) and 0.895 GeV (neutral).

In addition: $m_\phi - m_{K^*} = m_{K^*} - m_\rho$=120 MeV.

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We can do the same for baryons: find similar results.

Note: since ω and ρ are roughly degenerate, ω must be predominantly u and d quarks.

$\omega\rho \longrightarrow \omega$ contains a small component of $s\overline{s}$.

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For η_8, wave function is $\frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right)$, and $\langle\eta_8|H_s|\eta_8\rangle = m_0 + \frac{1}{6}(4u + 8s)$

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Note: this relation works better if the squares of the masses are (justified by chiral symmetry?) substituted:$4K^2 - \pi^2 = 3\eta_8^2$, giving $\eta_8 =$ 0.562 GeV

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Solution can be found: $\tan^2\theta = \frac{4K - \pi - 3\eta}{3\eta' - 4K + \pi} \approx 0.18$
Note that if the squares of the masses are used, this value is much smaller, ≈ 0.03

Recall that $\pi^+ = m_0 + u + d \equiv m_0 + 2u$ and $\rho^+ = m_1 + 2u$

What's the difference between m_0 and m_1? It's not isospin, since both ρ and π are isovectors.

ρ has $S = 1$, π has $S = 0$. In this simple framework, this is the only feature that can give rise to the difference between m_0 and m_1, so we might guess that the mass difference is due to a spin-spin interaction, $\vec{s}_1 \cdot \vec{s}_2$. This is usually called the contact hyperfine interaction.

In positronium, the hyperfine splitting is $\frac{2\pi\alpha}{3}\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{m_1 m_2}\,|\psi(0)|^2$

Let's assume that something similar will work here:

$$m(q_1\overline{q}_2) = M + m_1 + m_2 + a\frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{m_1 m_2}$$

$$m(q_1 q_2 q_3) = M' + m_1 + m_2 + m_2 + a' \sum_{i>j}^{3} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j}$$

For the meson case, $\vec{S} = \frac{1}{2}\left(\vec{\sigma}_1 + \vec{\sigma}_2\right) \Longrightarrow \langle \vec{\sigma}_1 \cdot \vec{\sigma}_2 \rangle = 2S(S+1) - 3$

$$\rho = M + u + d + \frac{a}{ud};\ \pi = M + u + d - \frac{3a}{ud};\ \pi - \rho = \frac{4a}{ud}$$

Similarly, $K^* - K = \frac{4a}{us}$, and assuming that this can be applied to all mesons, $D^* - D = \frac{4a}{cu}$, $B^* - B = \frac{4a}{bu}$, etc.

Does it work? Choosing $u \approx$ 330 Mev, $s \approx$ 550 MeV, $c \approx$ 1.5 GeV, $b \approx$ 5 GeV and $\rho - \pi = 630$ MeV $\Longrightarrow$

$K^* - K \approx$ 378 (396) MeV;

$D^* - D \approx$ 139 (141) MeV;

$B^* - B \approx$ 42 (46) MeV.

However, note $D_s^* - D_s \approx$ 83 (144) MeV.

For baryons, the hyperfine Hamiltonian becomes $H_{\rm hyp} = \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{m_1 m_2} + \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_3}{m_1 m_3} + \frac{\vec{\sigma}_2 \cdot \vec{\sigma}_3}{m_2 m_3}$

Let's choose $m_1 = m_2 = u$, say. Then $H_{\rm hyp} = \frac{\vec{\sigma}_1 \cdot \vec{\sigma}_2}{u^2} + \frac{(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{\sigma}_3}{u m_3}$

Black/whiteboard: color and baryon wave functions

Full set of symmetric flavor wave functions are: $\Delta^{++} = uuu,\ \Delta^{--} = ddd,\ \Omega^{-} = sss$

$$\Delta^{+} = \frac{1}{\sqrt{3}}(uud + udu + duu)\ \Delta^{0} = \frac{1}{\sqrt{3}}(ddu + dud + udd),$$

$$\Xi^{*0} = \frac{1}{\sqrt{3}}(ssu + sus + uss)\ \Xi^{*-} = \frac{1}{\sqrt{3}}(ssd + sds + dss)$$

$$\Sigma^{*+} = \frac{1}{\sqrt{3}}(uus + usu + suu)\ \Sigma^{*-} = \frac{1}{\sqrt{3}}(dds + dsd + sdd)$$

$$\Sigma^{*0} = \frac{1}{\sqrt{6}}(uds + dus + usd + sud + dsu + sdu)$$

For the two mixed symmetric representations:

p	$-\frac{1}{\sqrt{6}}(udu + duu - 2uud)$	$\frac{1}{\sqrt{2}}(udu - duu)$
n	$\frac{1}{\sqrt{6}}(udd + dud - 2ddu)$	$\frac{1}{\sqrt{2}}(udd - dud)$
Σ^+	$\frac{1}{\sqrt{6}}(usu + suu - 2uus)$	$\frac{1}{\sqrt{2}}(usu - suu)$
Σ^-	$\frac{1}{\sqrt{6}}(dsd + sdd - 2dds)$	$\frac{1}{\sqrt{2}}(dsd - sdd)$
Ξ^0	$\frac{1}{\sqrt{6}}(uss + sus - 2ssu)$	$\frac{1}{\sqrt{2}}(uss - sus)$
Ξ^-	$\frac{1}{\sqrt{6}}(dss + sds - 2ssd)$	$\frac{1}{\sqrt{2}}(dss - sds)$
Σ^0	$\frac{1}{\sqrt{6}}\left[s\frac{ud+du}{\sqrt{2}} + \frac{usd+dsu}{\sqrt{2}} - 2\,\frac{ud+du}{\sqrt{2}}s\right]$	$\frac{1}{\sqrt{2}}\left[-s\frac{ud+du}{\sqrt{2}} + \frac{usd+dsu}{\sqrt{2}}\right]$
Λ	$\frac{1}{\sqrt{2}}\left[\frac{dsu-usd}{\sqrt{2}} + s\frac{du-ud}{\sqrt{2}}\right]$	$\frac{1}{\sqrt{6}}\left[s\frac{du-ud}{\sqrt{2}} + \frac{usd-dsu}{\sqrt{2}} - 2\,\frac{du-ud}{\sqrt{2}}s\right]$

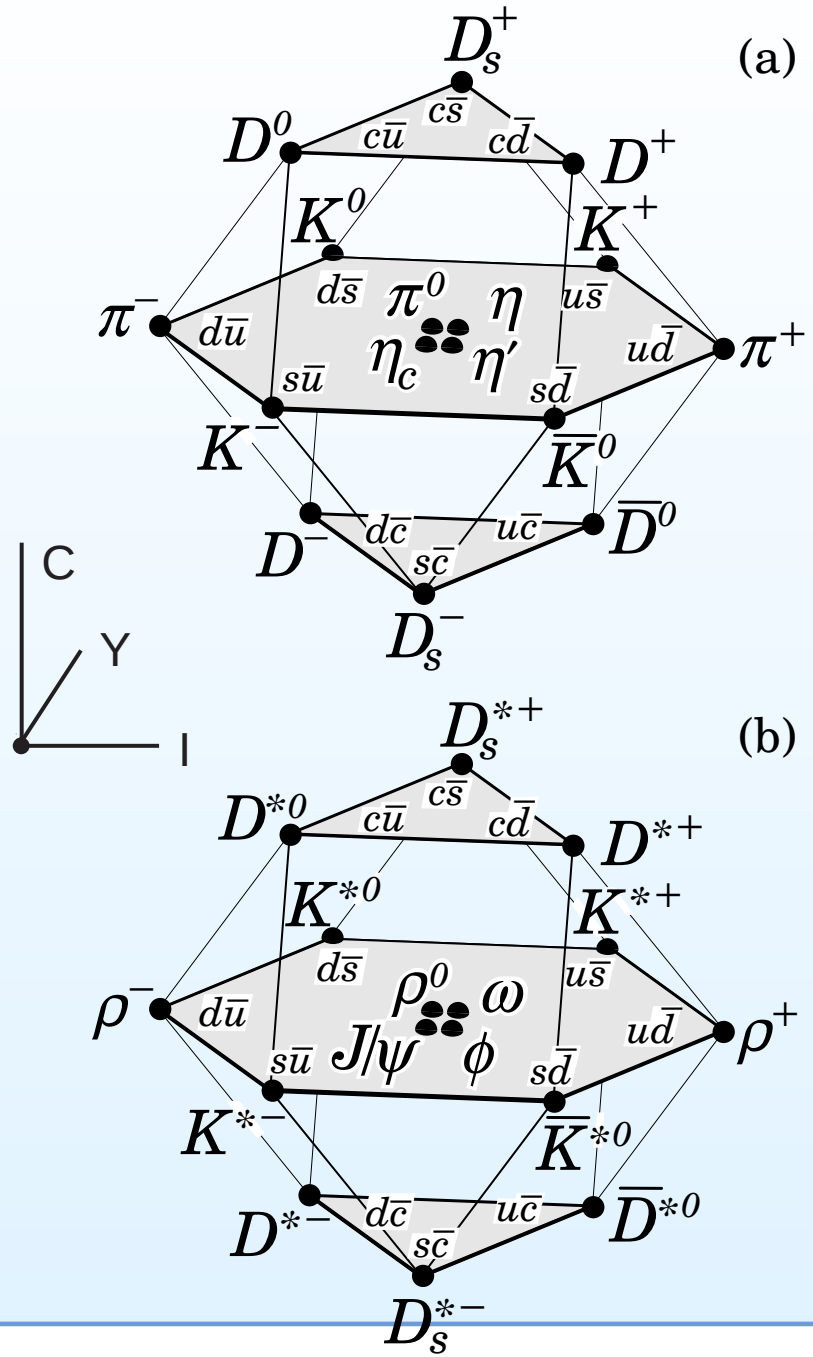
$$\Lambda_1(\text{antisymmetric}) = \frac{1}{\sqrt{6}}\left[(s(ud - du) + (usd - dsu) + (du - ud)s\right]$$

What about other quarks? Would an SU(4) classification scheme work? What about SU(5)

SU(4) multiplets would have to have the SU(3) multiplets as ‘submultiplets’

For mesons: $4 \bigotimes \overline{4} = 1 \bigoplus 15$ ($N \bigotimes \overline{N} = 1 \bigoplus N^2 - 1$)

For baryons: $4 \bigotimes 4 \bigotimes 4 = 20 \bigoplus 20 \bigoplus 20 \bigoplus 4$
($N \bigotimes N \bigotimes N = \frac{N(N+1)(N+2)}{6} \bigoplus \frac{N(N^2-1)}{3} \bigoplus \frac{N(N^2-1)}{3} \bigoplus \frac{N(N-1)(N-2)}{6}$)


(a)
D_s^+
D^0
D^+
$c\bar{s}$
$c\bar{u}$
$c\bar{d}$
K^0
K^+
$d\bar{s}$
$u\bar{s}$
π^-
π^+
$d\bar{u}$
$u\bar{d}$
π^0 η
η_c η'
$s\bar{u}$
$s\bar{d}$
K^-
$\bar{K}^0$
D^-
$\bar{D}^0$
$d\bar{c}$
$u\bar{c}$
$s\bar{c}$
D_s^-
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Y
I
(b)
D_s^{*+}
D^{*0}
D^{*+}
K^{*0}
K^{*+}
ρ^-
ρ^+
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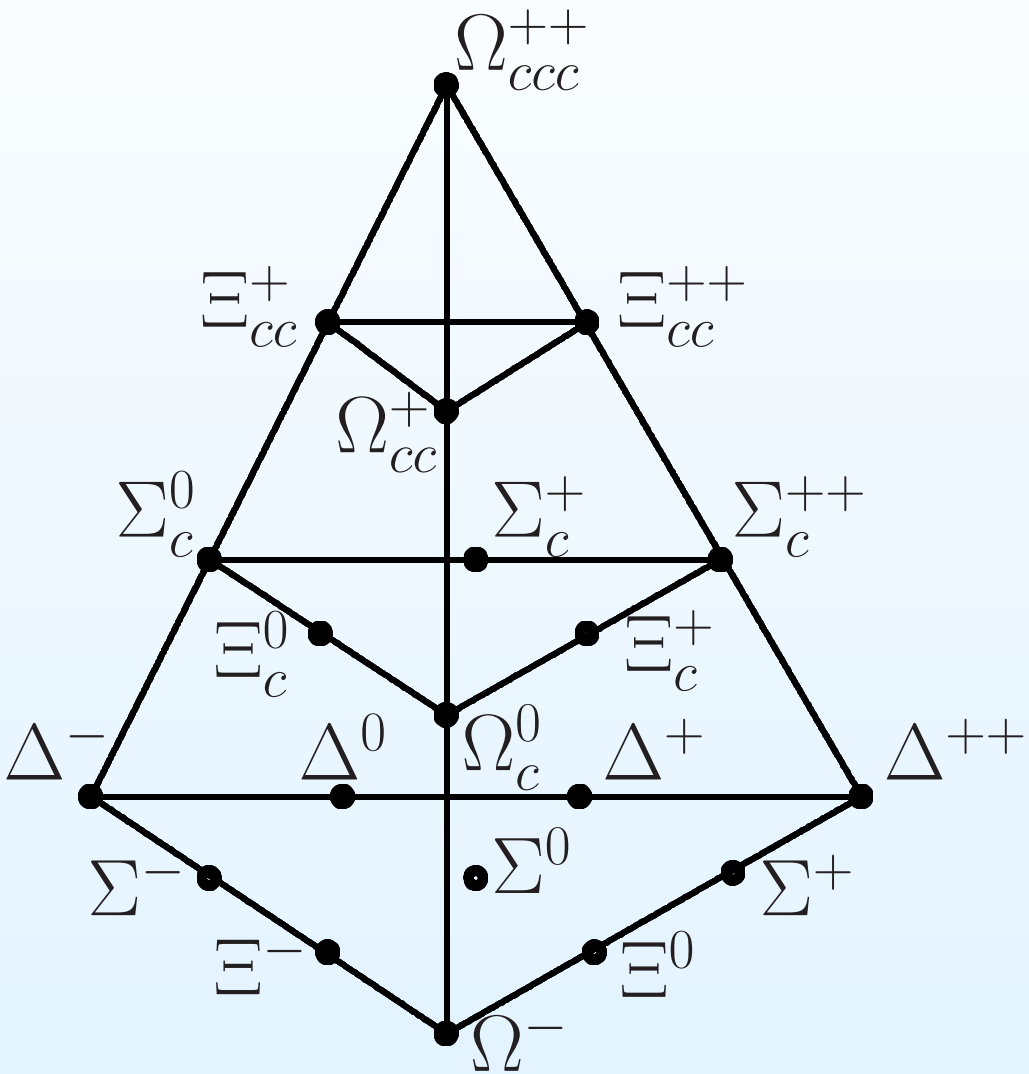
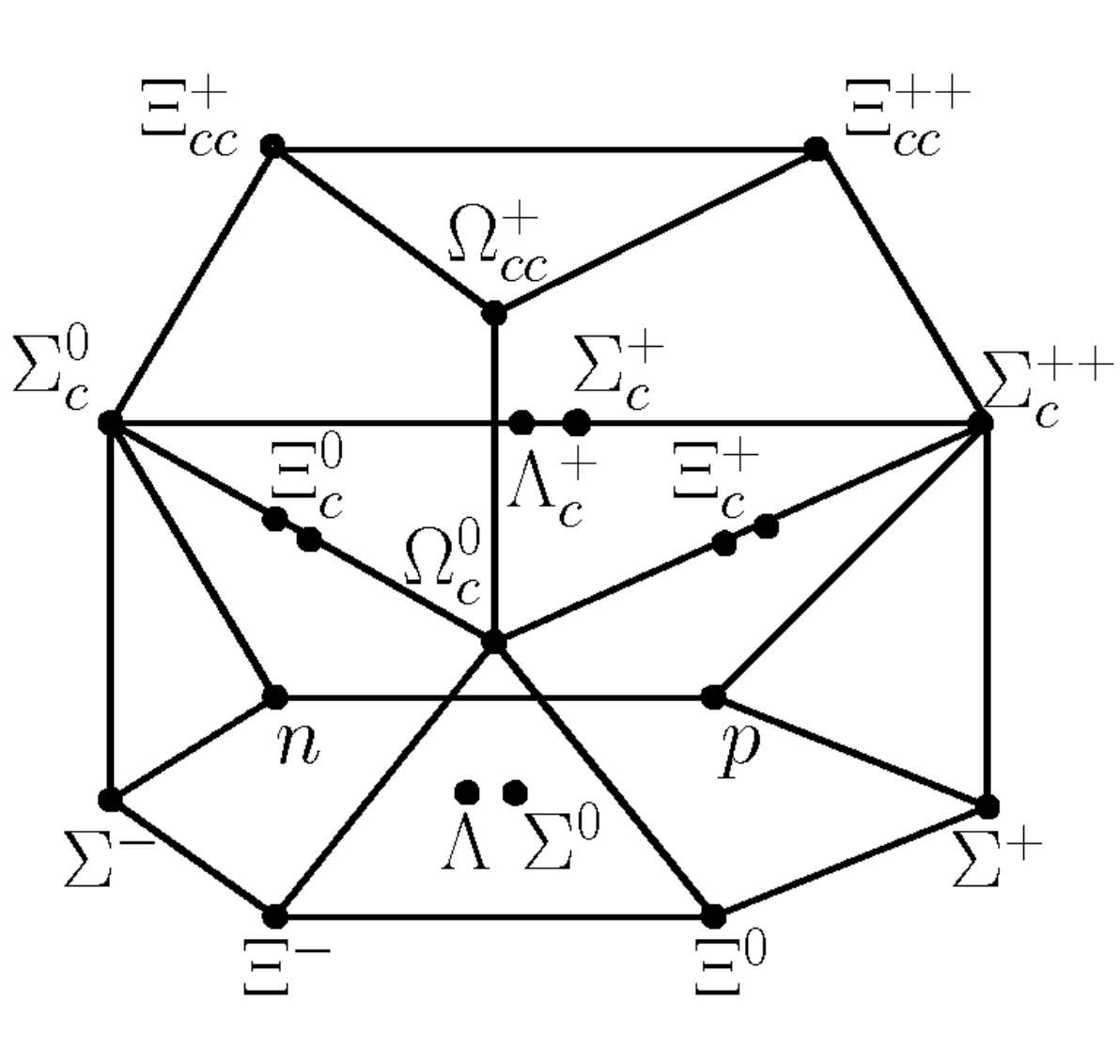
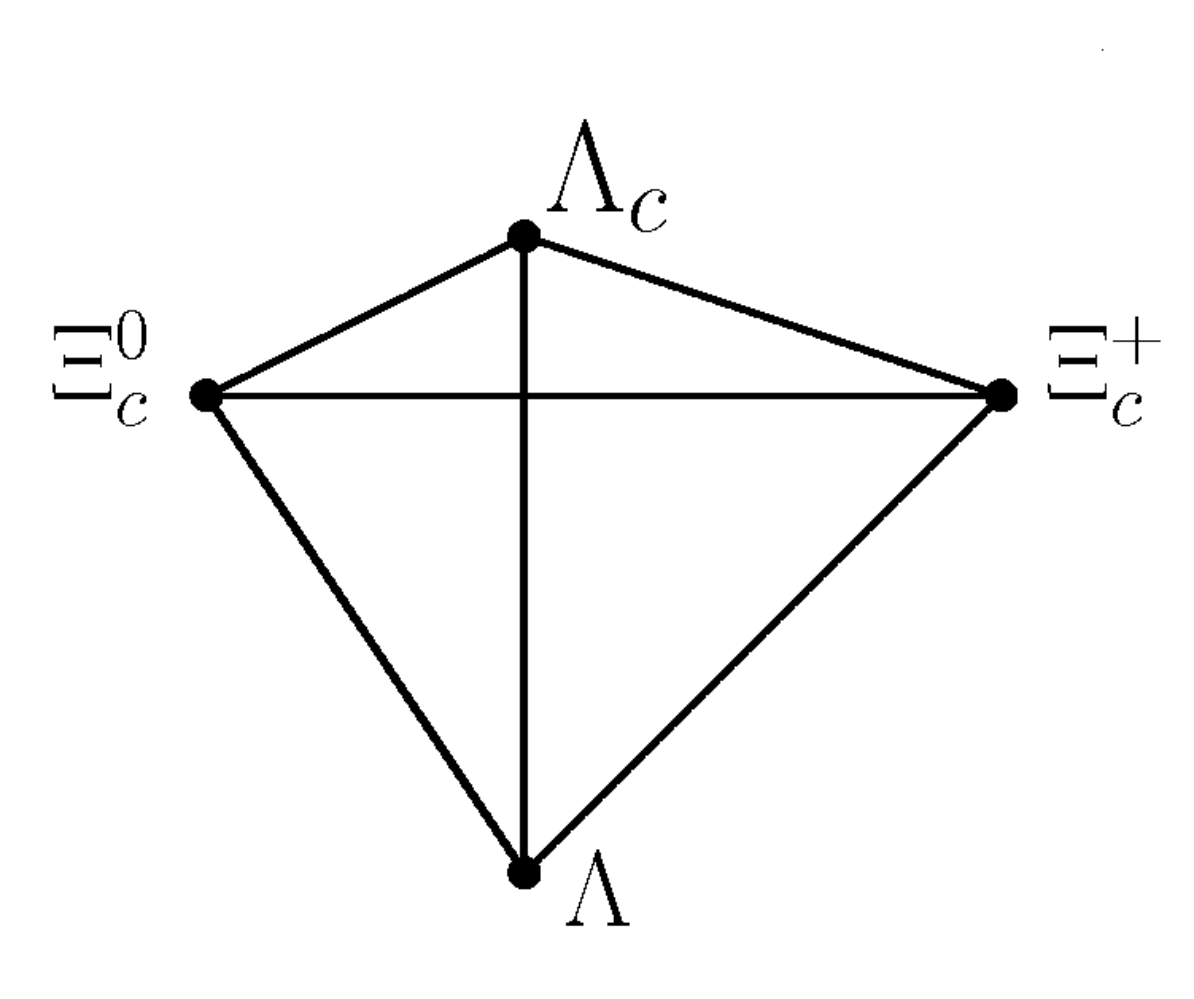
K^{*-}
$\bar{K}^{*0}$
D^{*-}
$\bar{D}^{*0}$
D_s^{*-}


Ω_{ccc}^{++}
Ξ_{cc}^{+}
Ξ_{cc}^{++}
Ω_{cc}^{+}
Σ_c^0
Σ_c^+
Σ_c^{++}
Ξ_c^0
Ξ_c^+
Ω_c^0
Δ^-
Δ^0
Δ^+
Δ^{++}
Σ^0
Σ^-
Σ^+
Ξ^-
Ξ^0
Ω^-


Ξ_{cc}^{+}
Ξ_{cc}^{++}
Ω_{cc}^{+}
Σ_c^0
Σ_c^+
Σ_c^{++}
Ξ_c^0
Λ_c^+
Ξ_c^+
Ω_c^0
n
p
Λ
Σ^0
Σ^-
Σ^+
Ξ^-
Ξ^0


Λ_c
Ξ_c^0
Ξ_c^+
Λ

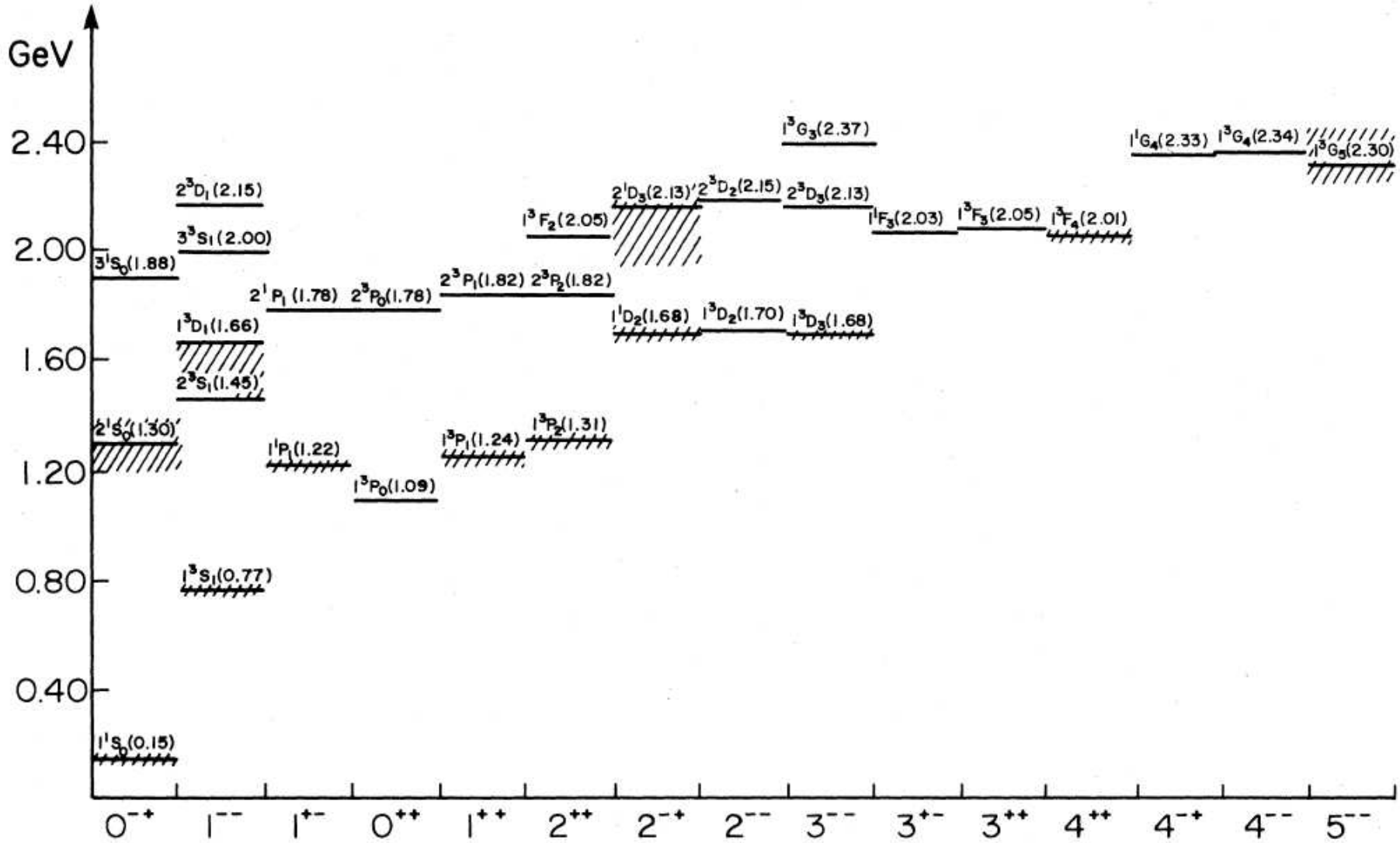
$$H^{ij}_{\rm conf} = br_{ij} - \frac{4\alpha_s}{3r_{ij}} + c$$

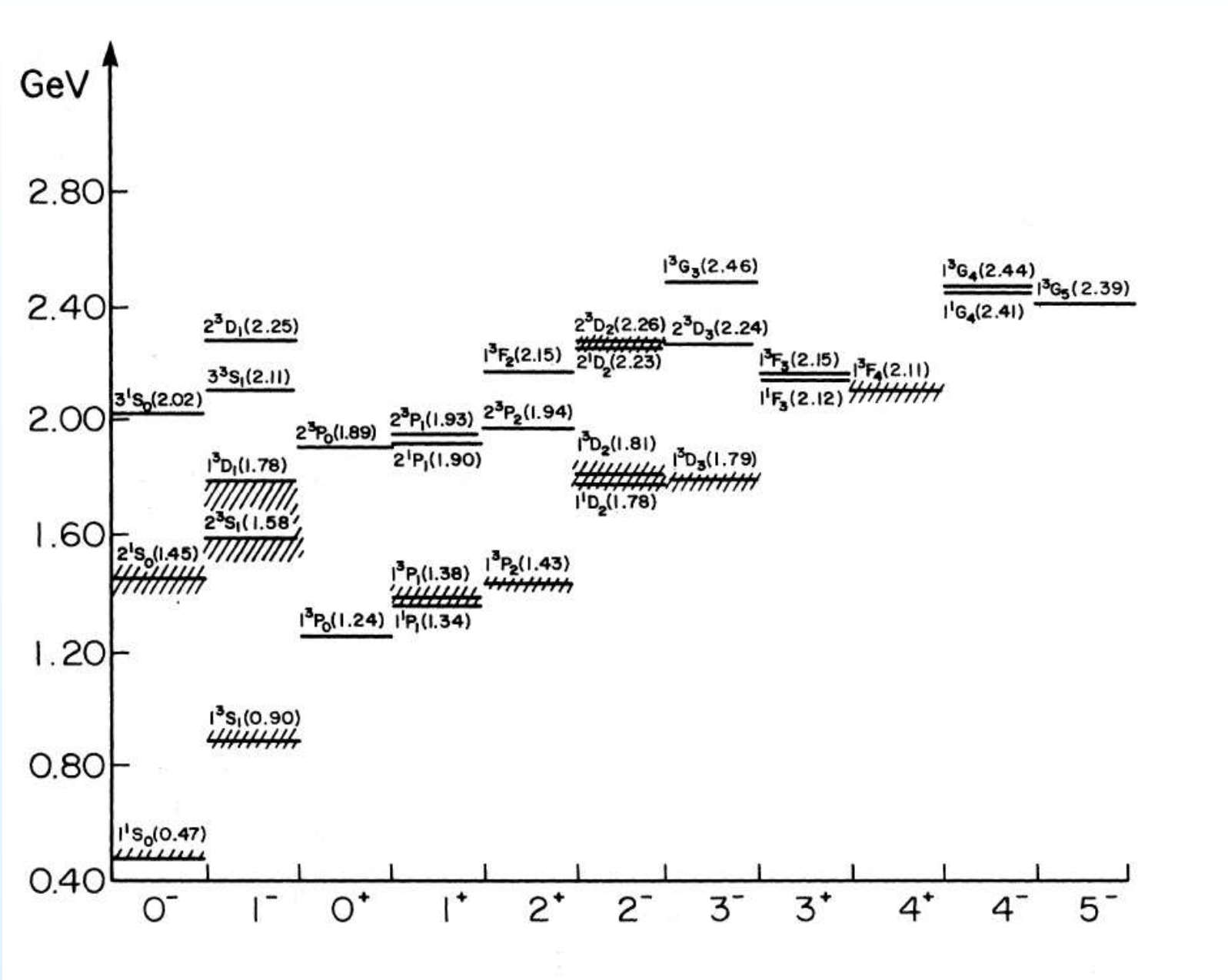
$$H^{ij}_{\rm hyp} = \left[\frac{4\alpha_{\rm con}}{3m_i m_j} \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{4\alpha_{\rm ten}}{3m_i m_j} \frac{1}{r^3_{ij}} \left(\frac{3\mathbf{S}_i \cdot \mathbf{r}_{ij} \mathbf{S}_j \cdot \mathbf{r}_{ij}}{r^2_{ij}} - \mathbf{S}_i \cdot \mathbf{S}_j \right) \right],$$

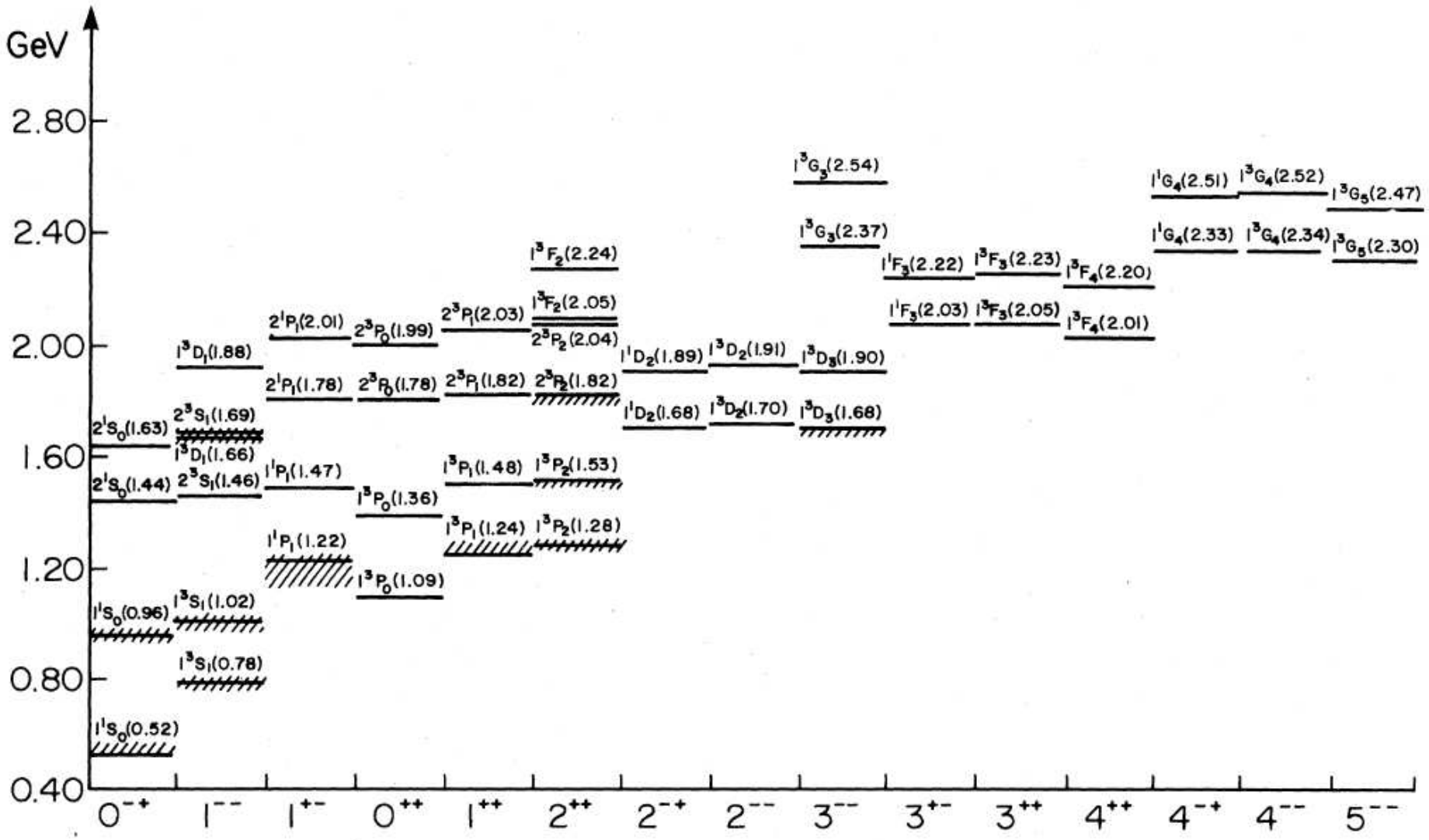
$$H^{ij}_{\rm SO} = H^{ij}_{\rm SO(cm)} + H^{ij}_{\rm SO(TP)}$$

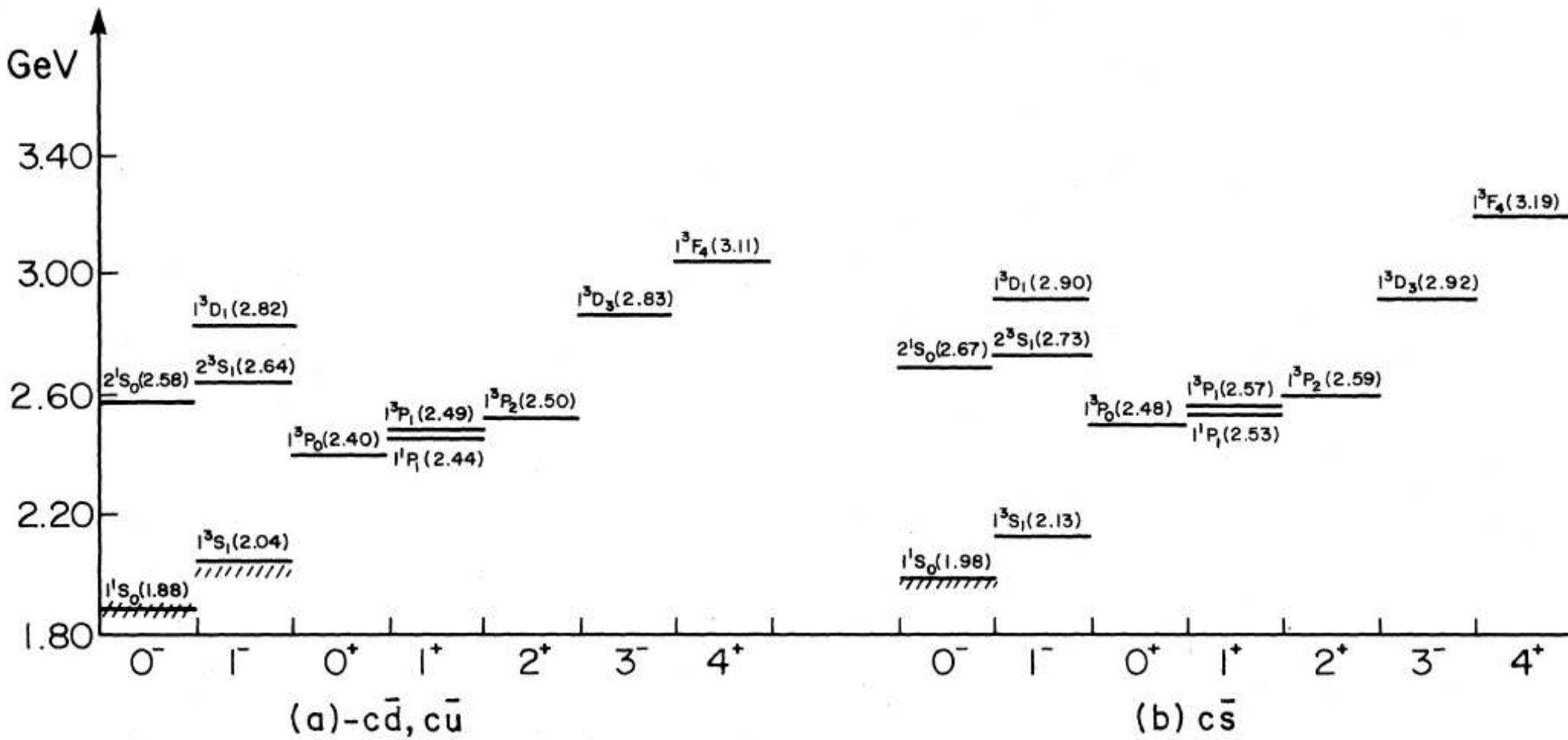
$$H^{ij}_{\rm SO(cm)} = \frac{4\alpha_s}{3r^3} \left[\frac{1}{m_i} + \frac{1}{m_j} \right] \left[\frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right] \cdot \vec{L}$$

$$H^{ij}_{\rm SO(TP)} = \frac{2}{3r} \frac{\partial H^{ij}_{\rm conf}}{\partial r} \left[\frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right] \cdot \vec{L}$$


GeV
2.40
2.00
1.60
1.20
0.80
0.40
$1^1S_0(0.15)$
$1^3S_1(0.77)$
$2^1S_0(1.30)$
$1^3D_1(1.66)$
$2^3S_1(1.45)$
$3^1S_0(1.88)$
$2^3D_1(2.15)$
$3^3S_1(2.00)$
$1^1P_1(1.22)$
$1^3P_0(1.09)$
$2^1P_1(1.78)$
$2^3P_0(1.78)$
$1^3P_1(1.24)$
$1^3P_2(1.31)$
$2^3P_1(1.82)$
$2^3P_2(1.82)$
$1^3F_2(2.05)$
$2^1D_3(2.13)$
$2^3D_2(2.15)$
$1^1D_2(1.68)$
$1^3D_2(1.70)$
$1^3D_3(1.68)$
$1^3G_3(2.37)$
$2^3D_3(2.13)$
$1^1F_3(2.03)$
$1^3F_3(2.05)$
$1^3F_4(2.01)$
$1^1G_4(2.33)$
$1^3G_4(2.34)$
$1^3G_5(2.30)$
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{-+}
2^{--}
3^{--}
3^{+-}
3^{++}
4^{++}
4^{-+}
4^{--}
5^{--}


GeV
2.80
2.40
2.00
1.60
1.20
0.80
0.40
$1^1S_0(0.47)$
$1^3S_1(0.90)$
$2^1S_0(1.45)$
$2^3S_1(1.58)$
$1^3D_1(1.78)$
$3^1S_0(2.02)$
$3^3S_1(2.11)$
$2^3D_1(2.25)$
$1^3P_0(1.24)$
$2^3P_0(1.89)$
$1^3P_1(1.38)$
$1^1P_1(1.34)$
$2^3P_1(1.93)$
$2^1P_1(1.90)$
$1^3P_2(1.43)$
$2^3P_2(1.94)$
$1^3F_2(2.15)$
$1^3D_2(1.81)$
$1^1D_2(1.78)$
$1^3D_3(1.79)$
$2^3D_2(2.26)$
$2^1D_2(2.23)$
$2^3D_3(2.24)$
$1^3G_3(2.46)$
$1^3F_3(2.15)$
$1^1F_3(2.12)$
$1^3F_4(2.11)$
$1^3G_4(2.44)$
$1^1G_4(2.41)$
$1^3G_5(2.39)$
0^-
1^-
0^+
1^+
2^+
2^-
3^-
3^+
4^+
4^-
5^-


GeV
2.80
2.40
2.00
1.60
1.20
0.80
0.40
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{-+}
2^{--}
3^{--}
3^{+-}
3^{++}
4^{++}
4^{-+}
4^{--}
5^{--}
$2^1S_0(1.63)$
$2^1S_0(1.44)$
$1^1S_0(0.96)$
$1^1S_0(0.52)$
$1^3D_1(1.88)$
$2^3S_1(1.69)$
$1^3D_1(1.66)$
$2^3S_1(1.46)$
$1^3S_1(1.02)$
$1^3S_1(0.78)$
$2^1P_1(2.01)$
$2^1P_1(1.78)$
$1^1P_1(1.47)$
$1^1P_1(1.22)$
$2^3P_0(1.99)$
$2^3P_0(1.78)$
$1^3P_0(1.36)$
$1^3P_0(1.09)$
$2^3P_1(2.03)$
$2^3P_1(1.82)$
$1^3P_1(1.48)$
$1^3P_1(1.24)$
$1^3F_2(2.24)$
$1^3F_2(2.05)$
$2^3P_2(2.04)$
$2^3P_2(1.82)$
$1^3P_2(1.53)$
$1^3P_2(1.28)$
$1^1D_2(1.89)$
$1^1D_2(1.68)$
$1^3D_2(1.91)$
$1^3D_2(1.70)$
$1^3G_3(2.54)$
$1^3G_3(2.37)$
$1^3D_3(1.90)$
$1^3D_3(1.68)$
$1^1F_3(2.22)$
$1^1F_3(2.03)$
$1^3F_3(2.23)$
$1^3F_3(2.05)$
$1^3F_4(2.20)$
$1^3F_4(2.01)$
$1^1G_4(2.51)$
$1^1G_4(2.33)$
$1^3G_4(2.52)$
$1^3G_4(2.34)$
$1^3G_5(2.47)$
$1^3G_5(2.30)$


GeV
3.40
3.00
2.60
2.20
1.80
$1^3D_1(2.82)$
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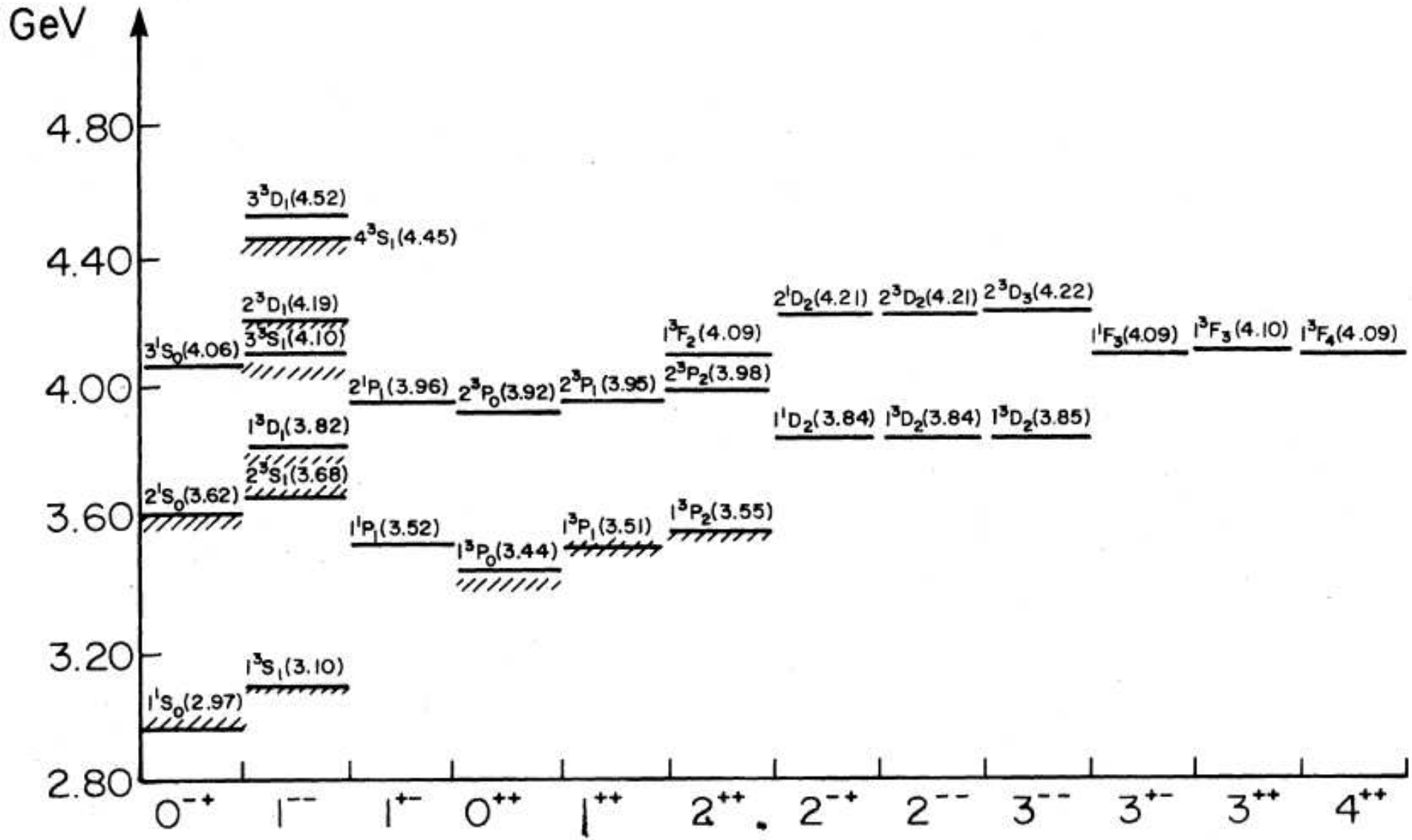
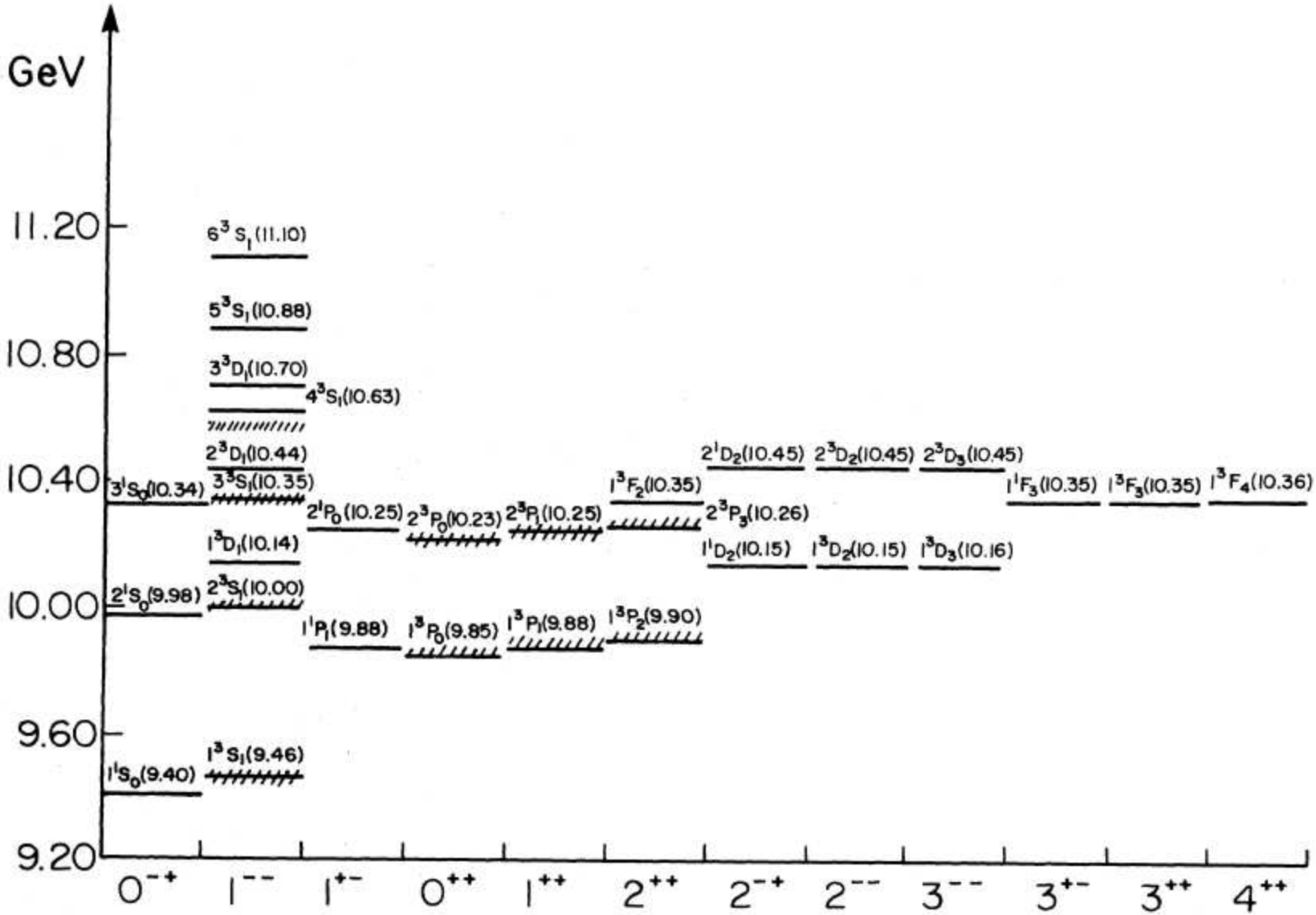
$1^3P_0(2.40)$
$1^3P_1(2.49)$
$1^1P_1(2.44)$
$1^3P_2(2.50)$
$1^3D_3(2.83)$
$1^3F_4(3.11)$
$1^3S_1(2.04)$
$1^1S_0(1.88)$
0^- 1^- 0^+ 1^+ 2^+ 3^- 4^+
(a)-$c\bar{d}$, $c\bar{u}$
$1^3F_4(3.19)$
$1^3D_1(2.90)$
$1^3D_3(2.92)$
$2^1S_0(2.67)$
$2^3S_1(2.73)$
$1^3P_0(2.48)$
$1^3P_1(2.57)$
$1^1P_1(2.53)$
$1^3P_2(2.59)$
$1^3S_1(2.13)$
$1^1S_0(1.98)$
0^- 1^- 0^+ 1^+ 2^+ 3^- 4^+
(b) $c\bar{s}$


GeV
4.80
4.40
4.00
3.60
3.20
2.80
$3^3D_1(4.52)$
$4^3S_1(4.45)$
$2^3D_1(4.19)$
$3^3S_1(4.10)$
$3^1S_0(4.06)$
$1^3D_1(3.82)$
$2^3S_1(3.68)$
$2^1S_0(3.62)$
$1^3S_1(3.10)$
$1^1S_0(2.97)$
$2^1P_1(3.96)$
$1^1P_1(3.52)$
$2^3P_0(3.92)$
$1^3P_0(3.44)$
$2^3P_1(3.95)$
$1^3P_1(3.51)$
$1^3F_2(4.09)$
$2^3P_2(3.98)$
$1^3P_2(3.55)$
$2^1D_2(4.21)$
$2^3D_2(4.21)$
$2^3D_3(4.22)$
$1^1D_2(3.84)$
$1^3D_2(3.84)$
$1^3D_2(3.85)$
$1^1F_3(4.09)$
$1^3F_3(4.10)$
$1^3F_4(4.09)$
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{-+}
2^{--}
3^{--}
3^{+-}
3^{++}
4^{++}


GeV
11.20
10.80
10.40
10.00
9.60
9.20
$6^3S_1(11.10)$
$5^3S_1(10.88)$
$3^3D_1(10.70)$
$4^3S_1(10.63)$
$2^3D_1(10.44)$
$3^3S_1(10.35)$
$3^1S_0(10.34)$
$1^3D_1(10.14)$
$2^1S_0(9.98)$
$2^3S_1(10.00)$
$1^3S_1(9.46)$
$1^1S_0(9.40)$
$2^1P_0(10.25)$
$2^3P_0(10.23)$
$2^3P_1(10.25)$
$1^1P_1(9.88)$
$1^3P_0(9.85)$
$1^3P_1(9.88)$
$1^3P_2(9.90)$
$1^3F_2(10.35)$
$2^1D_2(10.45)$
$2^3D_2(10.45)$
$2^3D_3(10.45)$
$2^3P_3(10.26)$
$1^1D_2(10.15)$
$1^3D_2(10.15)$
$1^3D_3(10.16)$
$1^1F_3(10.35)$
$1^3F_3(10.35)$
$1^3F_4(10.36)$
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{-+}
2^{--}
3^{--}
3^{+-}
3^{++}
4^{++}

λ-type: $\phi_s \chi_\lambda =^2 10;\ \Phi_\lambda^{(1)}$

$\phi_\lambda \chi_s =^4 8;\ \Phi_\lambda^{(2)}$

$\frac{1}{\sqrt{2}} (\phi_\rho \chi_\rho - \phi_\lambda \chi_\lambda) =^2 8;\ \Phi_\lambda^{(3)}$

$\phi_A \chi_\rho =^2 1;\ \Phi_\lambda^{(4)}$

ρ-type: $\phi_s \chi_\rho =^2 10;\ \Phi_\rho^{(1)}$

$\phi_\rho \chi_s =^4 8;\ \Phi_\rho^{(2)}$

$\frac{1}{\sqrt{2}} (\phi_\lambda \chi_\rho + \phi_\rho \chi_\lambda) =^2 8;\ \Phi_\rho^{(3)}$

$\phi_A \chi_\lambda =^2 1;\ \Phi_\rho^{(4)}$

A-type: $\phi_A \chi_s =^4 1;\ \Phi_A^{(1)}$

$\frac{1}{\sqrt{2}} (\phi_\lambda \chi_\rho - \phi_\rho \chi_\lambda) =^2 8;\ \Phi_A^{(2)}$

I can write the ground state wave function in a notation $(1S)^3$. With this notation, the first orbital excitation can be written $(1S)^2(1P)$.

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I can create states with particular symmetries by taking linear combinations: assuming all masses equal,
$\psi_\rho = \frac{1}{\sqrt{2}}\left(\vec{r}_1 - \vec{r}_2\right)\psi,\ \psi_\lambda = \frac{1}{\sqrt{6}}\left(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3\right)\psi,\ \psi_S = \frac{1}{\sqrt{3}}\left(\vec{r}_1 + \vec{r}_2 + \vec{r}_3\right)\psi$

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We can set the center of mass to be at the origin: $\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0$, and ψ_S becomes trivial: only two, mixed-symmetric excitations possible with $L = 1$. If $R_{\rm cm} \neq 0$, the ψ_S state corresponds to the 3 quarks in the $(1S)^3$, all moving with one unit of anguar momentum relative to some origin.

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The fully symmetric wave functions possible are: $\frac{1}{\sqrt{2}}\left(\Phi_\lambda^{(1)}\psi_\lambda + \Phi_\rho^{(1)} + \psi_\rho\right) :\ {}^2 10$
$\frac{1}{\sqrt{2}}\left(\Phi_\lambda^{(2)}\psi_\lambda + \Phi_\rho^{(2)} + \psi_\rho\right) :\ {}^4 8$
$\frac{1}{\sqrt{2}}\left(\Phi_\lambda^{(3)}\psi_\lambda + \Phi_\rho^{(3)} + \psi_\rho\right) :\ {}^2 8$
$\frac{1}{\sqrt{2}}\left(\Phi_\lambda^{(4)}\psi_\lambda + \Phi_\rho^{(4)} + \psi_\rho\right) :\ {}^2 1$

Possible angular momentum values:

$^2 10 : 1 + \frac{1}{2} \rightarrow J^P = \frac{1}{2}^-,\ \frac{3}{2}^-$ ($\Delta,\ \Sigma$, no Λ)

$^4 8 : 1 + \frac{3}{2} \rightarrow J^P = \frac{1}{2}^-,\ \frac{3}{2}^-,\ \frac{5}{2}^-$ (nucleon, Λ, Σ, no Δ)

$^2 8 : 1 + \frac{1}{2} \rightarrow J^P = \frac{1}{2}^-,\ \frac{3}{2}^-$ (same as 4^8)

$^2 1 : 1 + \frac{1}{2} \rightarrow J^P = \frac{1}{2}^-,\ \frac{3}{2}^-$ (Λ only)

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Number of states in the multiplet is 2 x 10 + 4 x 8 + 2 x 8 + 2 x 1=70. Multiplet is often referred to as $70, 1^-$.

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Examples of states in this mutliplet are

$N(1520),\ N(1535),\ \Delta(1620),\ \Delta(1700), \Lambda(1405),\ \Lambda(1520)$

What about higher excitations? Easier to illustrate for the general case (not SU(3)), separately for Λ-type and Σ-type states.

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We build components of the wave function as Clebsch-Gordan sums:

$$|J,M\rangle = \sum_{m_S} \Psi_{L,m_L}\left(\vec{\rho},\vec{\lambda}\right)\chi(S,m_S)\langle L,m_L,S,m_S|J,M\rangle$$

$$\Psi_{L,m_L}\left(\vec{\rho},\vec{\lambda}\right) = \sum_{m_\rho} \psi_{n_\rho,\ell_\rho,m_\rho}\left(\vec{\rho}\right)\psi_{n_\lambda,\ell_\lambda,m_\lambda}\left(\vec{\lambda}\right)\langle \ell_\rho,m_\rho,\ell_\lambda,m_\lambda|L,m_L\rangle$$

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Wave functions must still be fully symmetric in identical quarks. Excitations in λ are (12) symmetry (λ symmetry). Radial excitations in ρ also have λ symmetry. Orbital excitations in ρ have ρ symmetry.

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Simplified flavor wave functions: $\Sigma_Q = uuq,\ \frac{1}{\sqrt{2}}(ud+du)Q,\ ddQ$
$\Lambda_Q = \frac{1}{\sqrt{2}}(ud-du)Q,\ \Omega_Q = ssQ$
$\Xi_Q = \frac{1}{\sqrt{2}}(us-su)Q,\ \frac{1}{\sqrt{2}}(ds-sd)Q,\ \Xi'_Q = \frac{1}{\sqrt{2}}(us+su)Q,\ \frac{1}{\sqrt{2}}(ds+sd)Q$

What about higher excitations? Easier to illustrate for the general case (not SU(3)), separately for Λ-type and Σ-type states.

We build components of the wave function as Clebsch-Gordan sums:

$$|J,M\rangle = \sum_{m_S} \Psi_{L,m_L}\left(\vec{\rho},\vec{\lambda}\right)\chi(S,m_S)\langle L,m_L,S,m_S|J,M\rangle$$

$$\Psi_{L,m_L}\left(\vec{\rho},\vec{\lambda}\right) = \sum_{m_\rho} \psi_{n_\rho,\ell_\rho,m_\rho}\left(\vec{\rho}\right)\psi_{n_\lambda,\ell_\lambda,m_\lambda}\left(\vec{\lambda}\right)\langle \ell_\rho,m_\rho,\ell_\lambda,m_\lambda|L,m_L\rangle$$

Wave functions must still be fully symmetric in identical quarks. Excitations in λ are (12) symmetry (λ symmetry). Radial excitations in ρ also have λ symmetry. Orbital excitations in ρ have ρ symmetry.

Simplified flavor wave functions: $\Sigma_Q = uuq,\ \frac{1}{\sqrt{2}}(ud+du)Q,\ ddQ$
$\Lambda_Q = \frac{1}{\sqrt{2}}(ud-du)Q,\ \Omega_Q = ssQ$
$\Xi_Q = \frac{1}{\sqrt{2}}(us-su)Q,\ \frac{1}{\sqrt{2}}(ds-sd)Q,\ \Xi'_Q = \frac{1}{\sqrt{2}}(us+su)Q,\ \frac{1}{\sqrt{2}}(ds+sd)Q$
Space-spin wave function must have the same symmetry as flavor wave function.

Let's denote wave function components by $S, L, n_\rho, \ell_\rho, n_\lambda, \ell_\lambda$

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For Σ_Q with $J^P = \frac{1}{2}^+$, first component of wave function is $\frac{1}{2}_\lambda, 0, 0, 0, 0, 0$. Next easiest components to see are $\frac{1}{2}_\lambda, 0, 1, 0, 0, 0$ $\frac{1}{2}_\lambda, 0, 0, 0, 1, 0$. Other components are $\frac{3}{2}, 2, 0, 2, 0, 0$; $\frac{3}{2}, 2, 0, 0, 0, 2$; $\frac{1}{2}_\rho, 0, 0, 1, 0, 1$ and $\frac{1}{2}_\rho, 1, 0, 1, 0, 1$

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J^P	Σ_Q	Λ_Q
$\frac{1}{2}^+_1$	$\frac{1}{2}_\lambda, 0, 0, 0, 0, 0$	$\frac{1}{2}_\rho, 0, 0, 0, 0, 0$
$\frac{1}{2}^+_2$	$\frac{1}{2}_\lambda, 0, 1, 0, 0, 0$	$\frac{1}{2}_\rho, 0, 1, 0, 0, 0$
$\frac{1}{2}^+_3$	$\frac{1}{2}_\lambda, 0, 0, 0, 1, 0$	$\frac{1}{2}_\rho, 0, 0, 0, 1, 0$
$\frac{1}{2}^+_4$	$\frac{3}{2}, 2, 0, 2, 0, 0$	$\frac{1}{2}_\lambda, 0, 0, 1, 0, 1$
$\frac{1}{2}^+_5$	$\frac{3}{2}, 2, 0, 0, 0, 2$	$\frac{1}{2}_\lambda, 1, 0, 1, 0, 1$
$\frac{1}{2}^+_6$	$\frac{1}{2}_\rho, 0, 0, 1, 0, 1$	$\frac{3}{2}, 1, 0, 1, 0, 1$
$\frac{1}{2}^+_7$	$\frac{1}{2}_\rho, 1, 0, 1, 0, 1$	$\frac{3}{2}, 2, 0, 1, 0, 1$
$\frac{1}{2}^-_1$	$\frac{1}{2}_\rho, 1, 0, 1, 0, 0$	$\frac{1}{2}_\lambda, 1, 0, 1, 0, 0$
$\frac{1}{2}^-_2$	$\frac{1}{2}_\lambda, 1, 0, 0, 0, 1$	$\frac{3}{2}, 1, 0, 1, 0, 0$
$\frac{1}{2}^-_3$	$\frac{3}{2}, 1, 0, 0, 0, 1$	$\frac{1}{2}_\rho, 1, 0, 0, 0, 1$

$$H^{ij}_{\rm conf} = \sum_{i<j=1}^{3} \left(\frac{b r_{ij}}{2} - \frac{2\alpha_{\rm Coul}}{3 r_{ij}} \right).$$

$$H^{ij}_{\rm hyp} = \sum_{i<j=1}^{3} \left[\frac{2\alpha_{\rm con}}{3 m_i m_j} \frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{2\alpha_{\rm ten}}{3 m_i m_j} \frac{1}{r^3_{ij}} \left(\frac{3 \mathbf{S}_i \cdot \mathbf{r}_{ij} \mathbf{S}_j \cdot \mathbf{r}_{ij}}{r^2_{ij}} - \mathbf{S}_i \cdot \mathbf{S}_j \right) \right],$$

$$H^{ij}_{\rm SO} = H^{ij}_{\rm SO(cm)} + H^{ij}_{\rm SO(TP)}$$

$$H^{ij}_{\rm SO(cm)} = \frac{2\alpha_s}{3 r^3_{ij}} \left[\frac{\vec{r}_{ij} \times \vec{p}_i \cdot \vec{S}_i}{m_i^2} - \frac{\vec{r}_{ij} \times \vec{p}_j \cdot \vec{S}_j}{m_j^2} - \frac{\vec{r}_{ij} \times \vec{p}_j \cdot \vec{S}_i - \vec{r}_{ij} \times \vec{p}_i \cdot \vec{S}_j}{m_i m_j} \right]$$

$$H^{ij}_{\rm SO(TP)} = -\frac{1}{2 r_{ij}} \frac{\partial H^{ij}_{\rm conf}}{\partial r_{ij}} \left[\frac{\vec{r}_{ij} \times \vec{p}_i \cdot \vec{S}_i}{m_i^2} - \frac{\vec{r}_{ij} \times \vec{p}_j \cdot \vec{S}_j}{m_j^2} \right]$$

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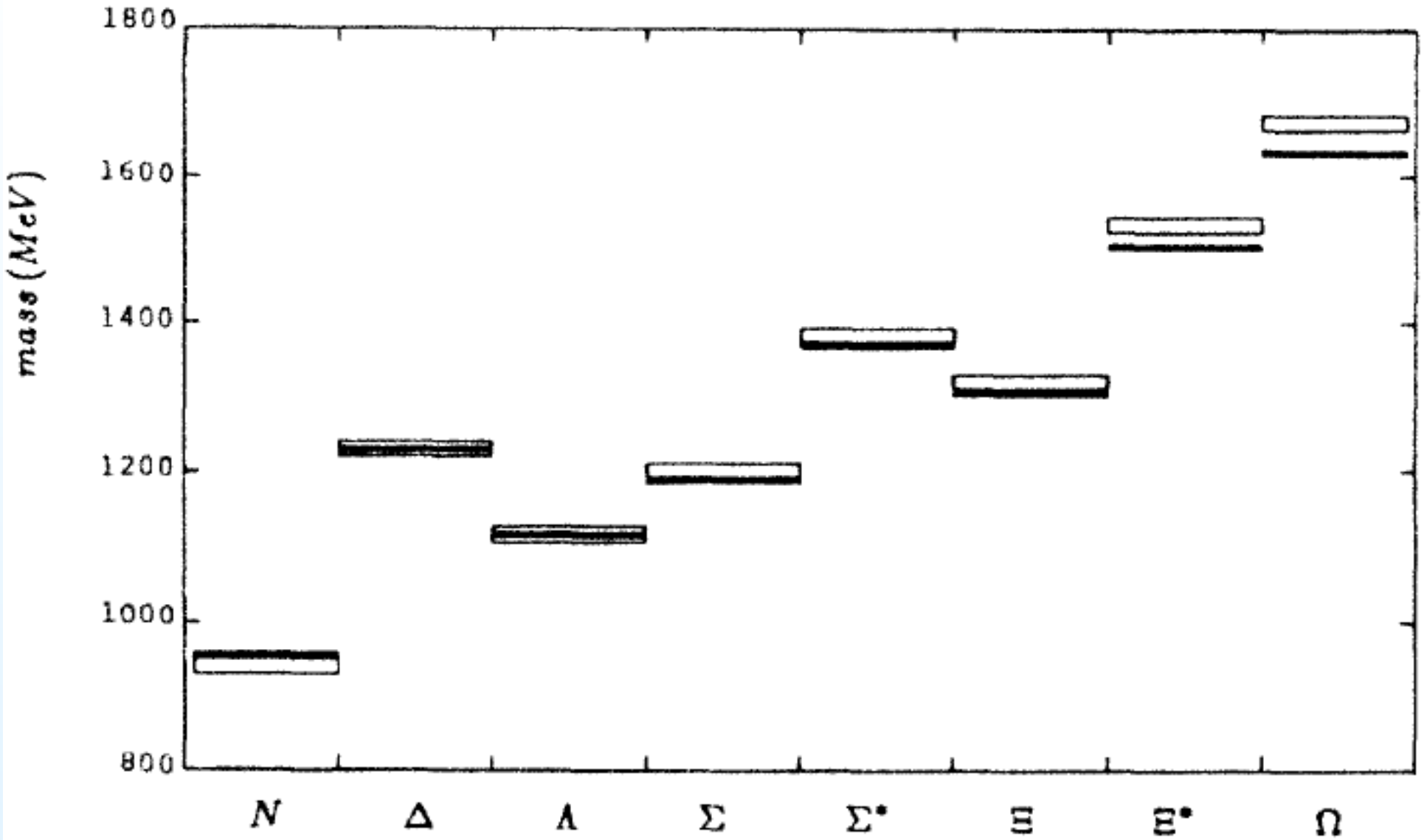
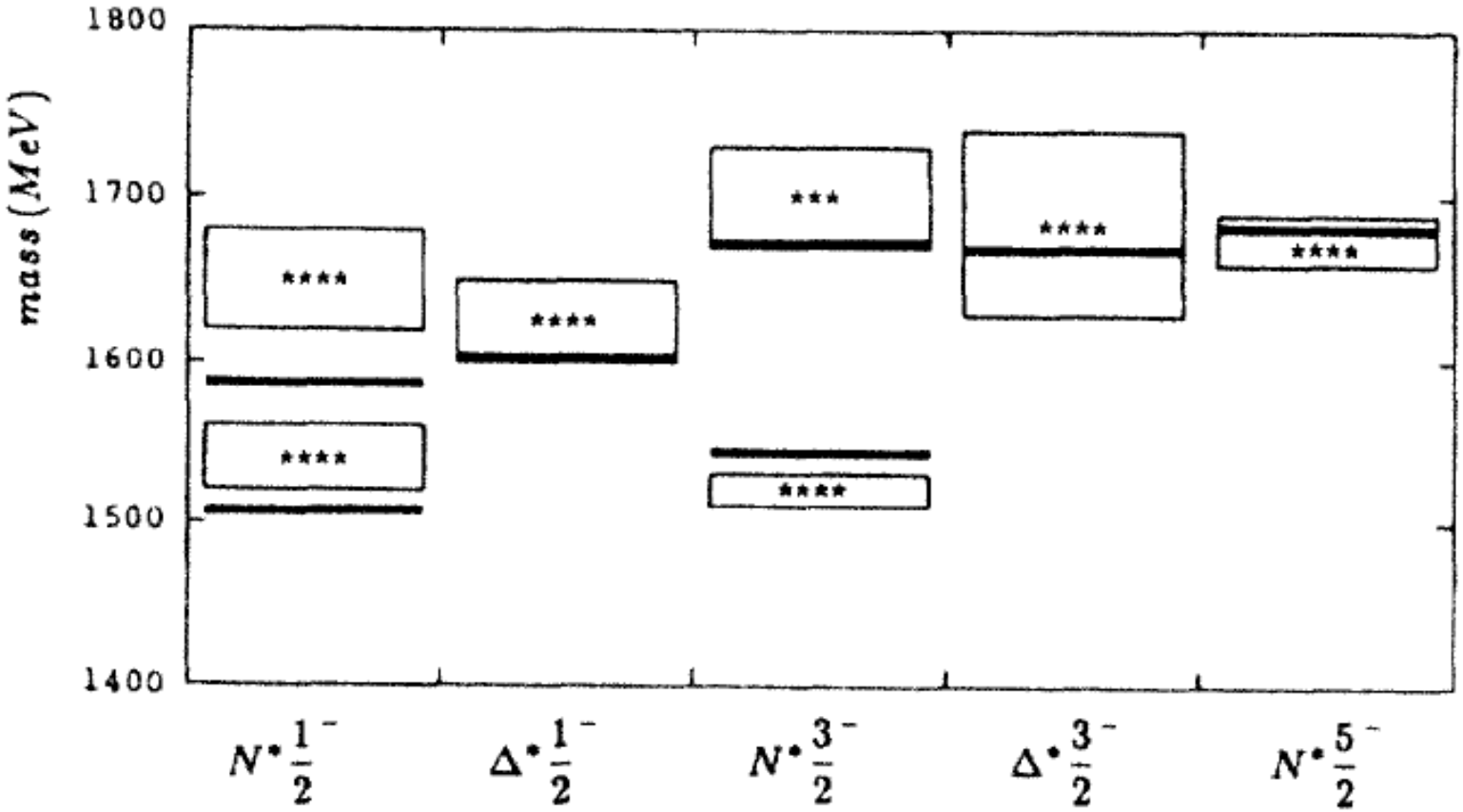
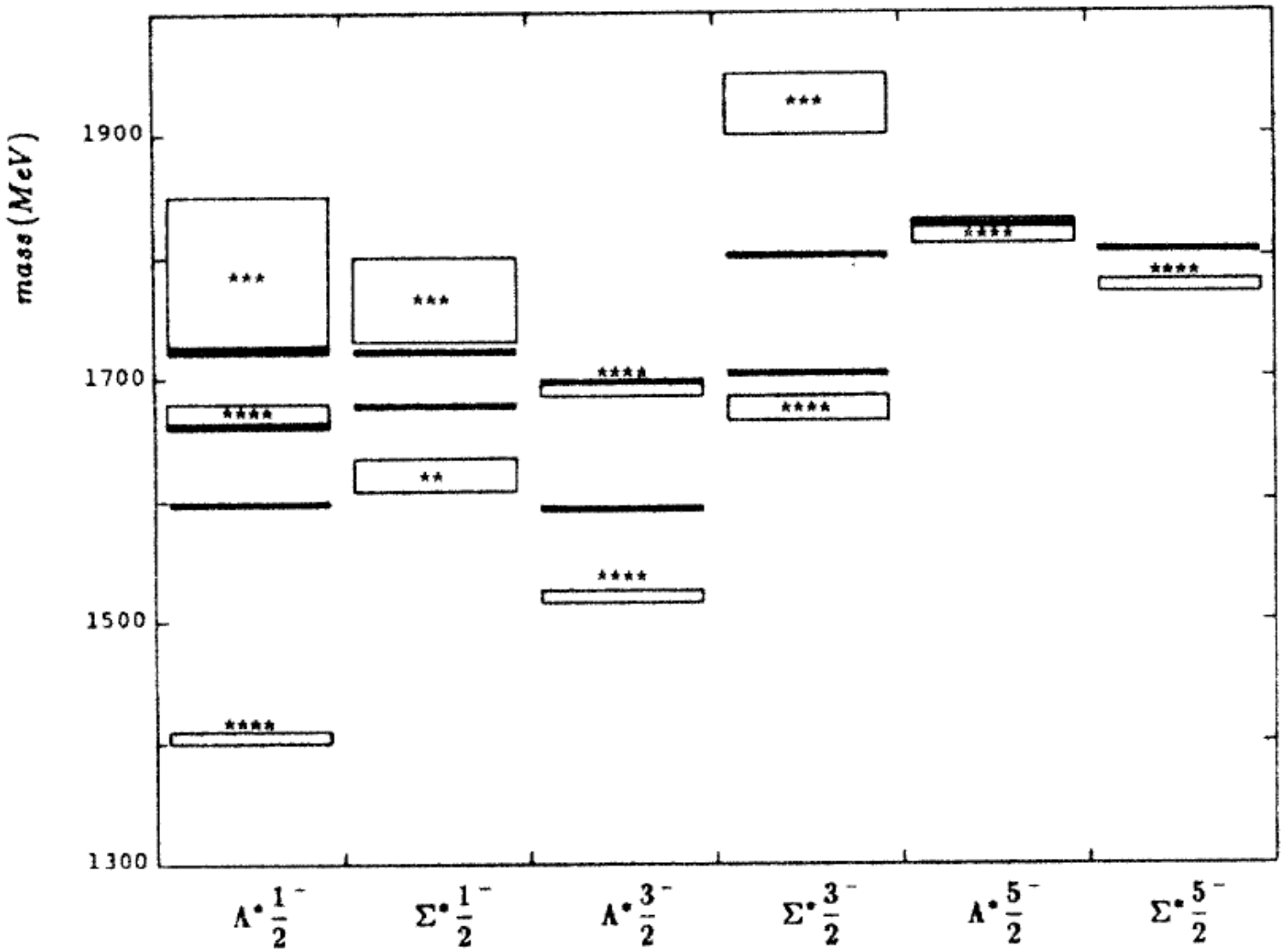
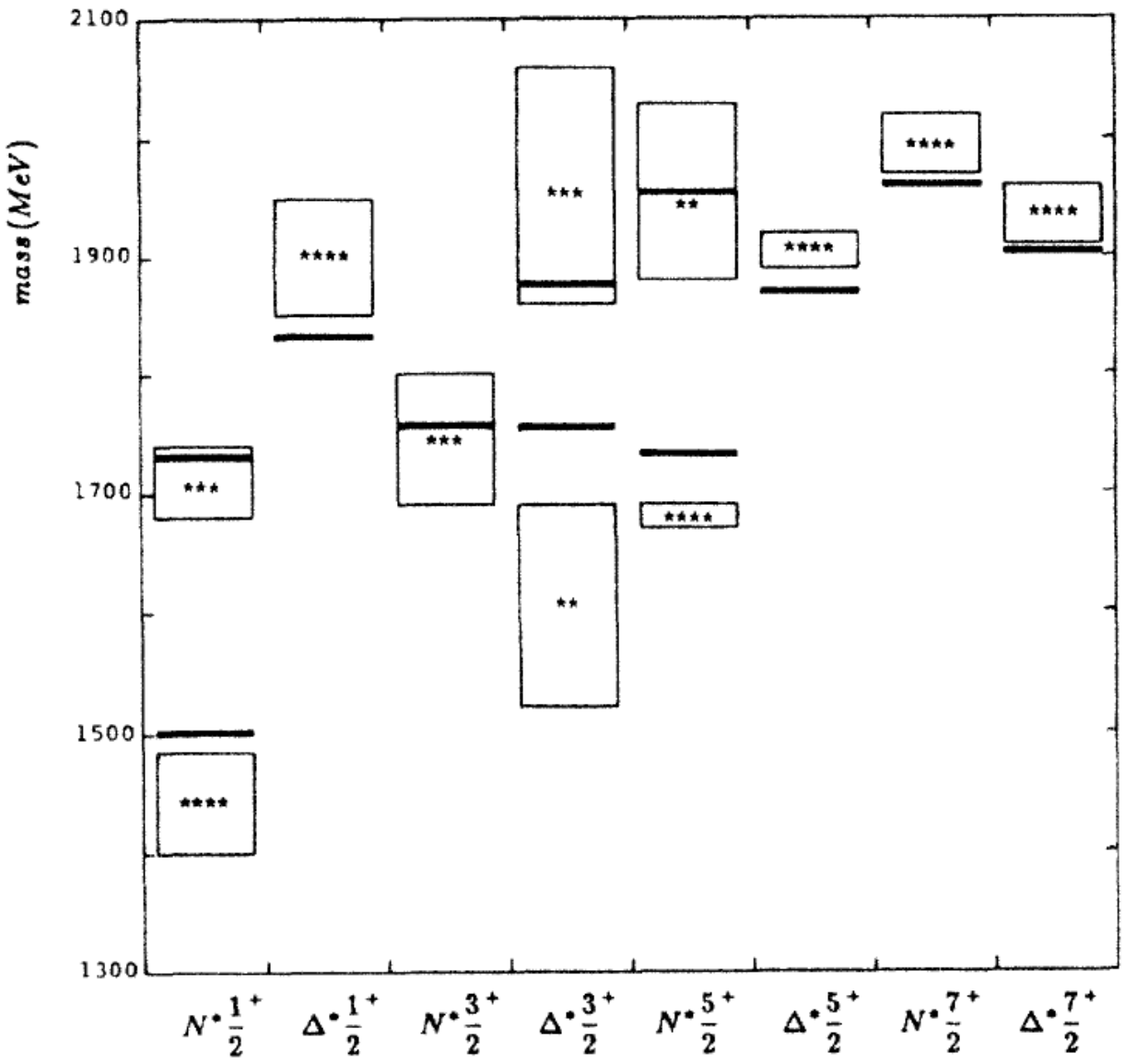
Should work with any choice of functions used to expand wave function


1800
1600
1400
1200
1000
800
mass (MeV)
N
Δ
Λ
Σ
Σ*
Ξ
Ξ*
Ω


1800
1700
1600
1500
1400
mass (MeV)
****
****
****
****
***
****
****
****
$N^* \frac{1}{2}^-$
$\Delta^* \frac{1}{2}^-$
$N^* \frac{3}{2}^-$
$\Delta^* \frac{3}{2}^-$
$N^* \frac{5}{2}^-$


mass (MeV)
1900
1700
1500
1300
***
****
**
$\Lambda^* \frac{1}{2}^-$
$\Sigma^* \frac{1}{2}^-$
$\Lambda^* \frac{3}{2}^-$
$\Sigma^* \frac{3}{2}^-$
$\Lambda^* \frac{5}{2}^-$
$\Sigma^* \frac{5}{2}^-$

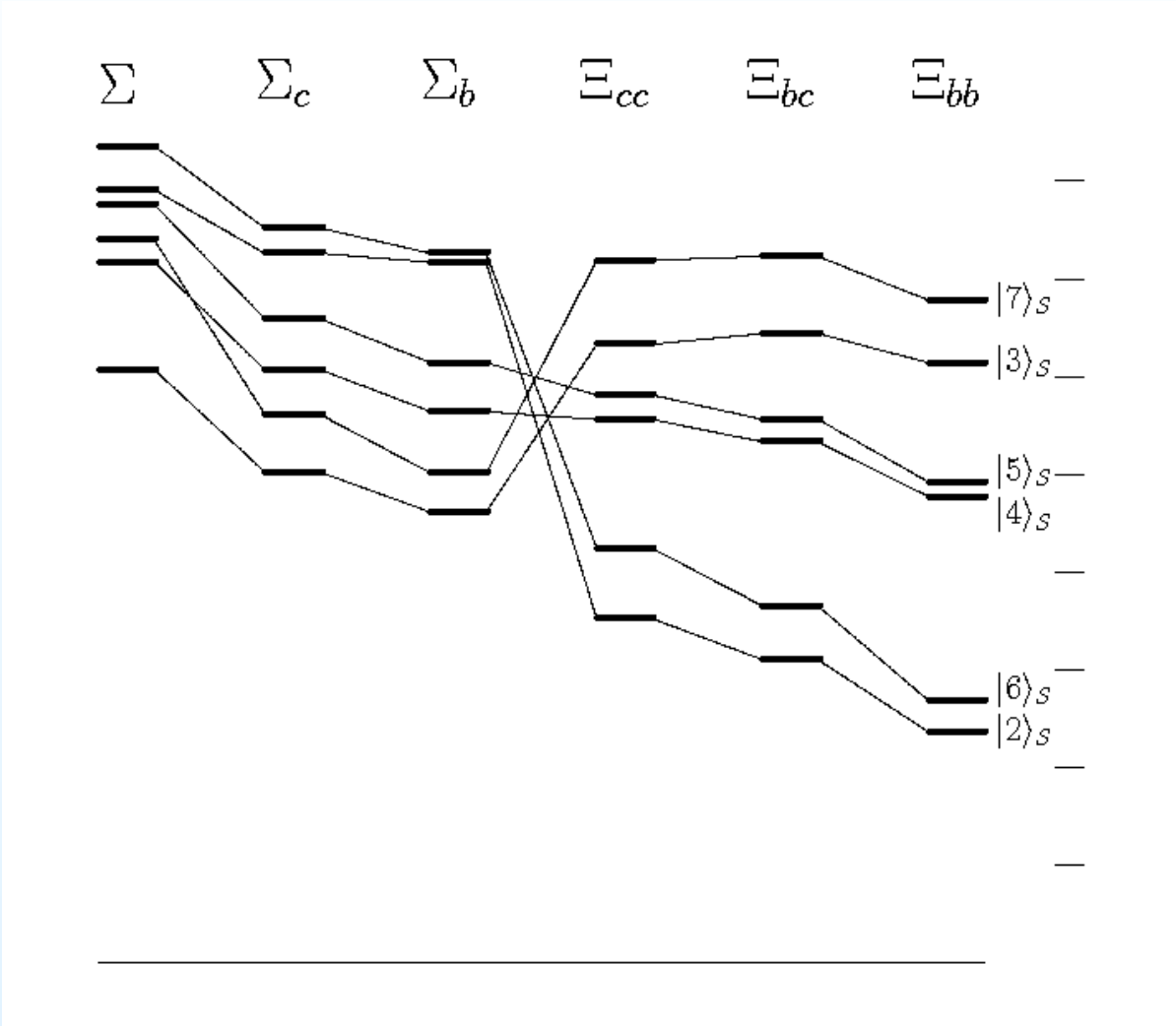

mass (MeV)
2100
1900
1700
1500
1300
***
****
***
***
**
****
**
****
****
****
$N^*\frac{1}{2}^+$
$\Delta^*\frac{1}{2}^+$
$N^*\frac{3}{2}^+$
$\Delta^*\frac{3}{2}^+$
$N^*\frac{5}{2}^+$
$\Delta^*\frac{5}{2}^+$
$N^*\frac{7}{2}^+$
$\Delta^*\frac{7}{2}^+$

State, J^P	Predicted mass (MeV)	Experiment (MeV)	Status
$N^* \frac{1}{2}^+$	1540←	1400–1480	* * * *
	1770←	1680–1740	* * *
	1880		
	1975		
$\Delta^* \frac{1}{2}^+$	1835		
	1875←	1850–1950	* * * *
$N^* \frac{3}{2}^+$	1795←	1690–1800	* * *
	1870		
	1910		
	1950		
	2030		
$\Delta^* \frac{3}{2}^+$	1795←	1520–1690	* *
	1915←	1860–2060	* * *
	1985		
$N^* \frac{5}{2}^+$	1770←	1670–1690	* * * *
	1980		
	1995←	1880–2030	* *
$\Delta^* \frac{5}{2}^+$	1910←	1890–1920	* * * *
	1990		
$N^* \frac{7}{2}^+$	2000←	1970–2020	* *
$\Delta^* \frac{7}{2}^+$	1940←	1910–1960	* * * *

Flavor	J^P	Expt. Mass	This work	[41]	[14]	[28]	[65]	[53–55]
Λ_c	$\frac{1}{2}^+$	2.285	2.268	2.265	2.297	2.272	2.292	2.284
			2.791	2.775	2.772	2.769	2.669	
	$\frac{3}{2}^+$		2.887	2.910	2.874	2.848	2.906	
			3.073	3.035	3.262	3.100	3.061	
	$\frac{5}{2}^+$		2.887	2.910				
	$\frac{1}{2}^-$	2.595	2.625	2.630	2.598	2.594	2.559	2.400[a]
			2.816	2.780	3.017	2.853	2.779	2.635[a]
	$\frac{3}{2}^-$	2.628	2.636	2.640	2.628	2.586	2.559	2.625
			2.830	2.840	3.034	2.874	2.779	2.630
	$\frac{5}{2}^-$		2.872	2.900				2.765
Σ_c	$\frac{1}{2}^+$	2.455	2.455	2.440	2.439	2.459	2.448	2.458
			2.958	2.890	2.864	2.947	2.793	
	$\frac{3}{2}^+$	2.518	2.519	2.495	2.518	2.539	2.505	2.516
			2.995	2.985	2.912	3.010	2.825	
	$\frac{5}{2}^+$		3.003	3.065				
	$\frac{1}{2}^-$		2.748	2.765	2.795	2.769	2.706	2.700
			2.768	2.770	2.805	2.817	2.791	2.915
	$\frac{3}{2}^-$		2.763	2.770	2.761	2.799	2.706	2.570
			2.776	2.805	2.799	2.815	2.791	2.570
	$\frac{5}{2}^-$		2.790	2.815				2.740

Possible Model States (flavor, mass (GeV), J^P)	Experimental State			
	$\Lambda_c(2765)$	$\Lambda_c(2880)$	$\Lambda_c(2940)$	$\Sigma_c(2800)$
	$\Lambda_c,\ 2.791,\ \frac{1}{2}^+$	$\Lambda_c,\ 2.887,\ \frac{5}{2}^+$	$\Lambda_c,\ 2.887,\ \frac{3}{2}^+$	$\Sigma_c,\ 2.768,\ \frac{1}{2}^-$
	$\Lambda_c,\ 2.816,\ \frac{1}{2}^-$	$\Lambda_c,\ 2.872,\ \frac{5}{2}^-$	$\Lambda_c,\ 2.872,\ \frac{5}{2}^-$	$\Sigma_c,\ 2.776,\ \frac{3}{2}^-$
	$\Sigma_c,\ 2.768,\ \frac{1}{2}^-$	-	$\Lambda_c,\ 2.983,\ \frac{1}{2}^+$	$\Sigma_c,\ 2.790,\ \frac{5}{2}^-$

J^P	Model		Dominant Wave Function Components		$\lvert\tan\phi\rvert$
	Unmixed	Mixed	Unmixed	Mixed	(Eq. 21)
$\frac{1}{2}^+$	7.020	7.011	$0.9881\lvert 1\rangle_3 + 0.1486\lvert 2\rangle_3$ $+0.0314\lvert 4\rangle_3 + 0.0242\lvert 3\rangle_3$	$0.8976\lvert 1\rangle_3 + 0.3839\lvert 1\rangle_1$ $+0.1526\lvert 2\rangle_3 + 0.1047\lvert 4\rangle_1$	0.4375
	7.044	7.047	$0.9885\lvert 1\rangle_1 + 0.1156\lvert 2\rangle_1$ $+0.0937\lvert 3\rangle_1 + 0.0258\lvert 4\rangle_1$	$0.9013\lvert 1\rangle_1 - 0.3908\lvert 1\rangle_3$ $+0.1636\lvert 2\rangle_1 + 0.0649\lvert 4\rangle_3$	0.4398
$\frac{3}{2}^+$	7.078	7.074	$0.9602\lvert 1\rangle_3 + 0.2789\lvert 2\rangle_3$ $-0.0102\lvert 3\rangle_3$	$0.9633\lvert 1\rangle_3 + 0.2264\lvert 2\rangle_3$ $-0.1249\lvert 3\rangle_3 + 0.0719\lvert 1\rangle_1$	0.0721
	7.386	7.371	$0.9974\lvert 6\rangle_3 - 0.0462\lvert 7\rangle_3$ $-0.0392\lvert 5\rangle_3$	$0.9643\lvert 6\rangle_3 - 0.1990\lvert 6\rangle_1$ $+0.1379\lvert 4\rangle_1 - 0.0812\lvert 7\rangle_3$	0.2607
	7.369	7.397	$0.9984\lvert 4\rangle_1 - 0.0544\lvert 6\rangle_1$ $+0.0136\lvert 7\rangle_1$	$0.9661\lvert 4\rangle_1 - 0.1612\lvert 7\rangle_3$ $-0.1548\lvert 6\rangle_3 + 0.1030\lvert 2\rangle_3$	0.2506
$\frac{5}{2}^+$	7.356	7.368	$0.9990\lvert 3\rangle_3 - 0.0368\lvert 1\rangle_3$ $+0.0186\lvert 2\rangle_3 + 0.0180\lvert 5\rangle_3$	$0.9683\lvert 3\rangle_3 - 0.1883\lvert 3\rangle_1$ $+0.1326\lvert 4\rangle_1 - 0.0806\lvert 1\rangle_3$	0.2376
	7.374	7.396	$0.9991\lvert 4\rangle_1 - 0.0415\lvert 3\rangle_1$ $+0.0120\lvert 5\rangle_1$	$0.9744\lvert 4\rangle_1 - 0.1538\lvert 3\rangle_3$ $-0.1492\lvert 1\rangle_3 - 0.0457\lvert 2\rangle_3$	0.2249
$\frac{7}{2}^+$	7.415	7.375	$\lvert 1\rangle_3$	$0.9845\lvert 1\rangle_3 - 0.1736\lvert 1\rangle_1$ $-0.0233\lvert 2\rangle_3$	0.1763
	7.564	7.562	$\lvert 1\rangle_1$	$0.9637\lvert 1\rangle_1 - 0.2022\lvert 2\rangle_3$ $+0.1747\lvert 1\rangle_3$	0.2074
$\frac{1}{2}^-$	7.206	7.227	$0.9964\lvert 2\rangle_1 - 0.0785\lvert 1\rangle_1$ $-0.0327\lvert 3\rangle_1$	$0.9723\lvert 2\rangle_1 + 0.1533\lvert 1\rangle_3$ $-0.1473\lvert 3\rangle_3 - 0.0772\lvert 3\rangle_1$	0.2177
	7.231	7.267	$0.9994\lvert 1\rangle_3 - 0.0332\lvert 3\rangle_3$	$0.9750\lvert 1\rangle_3 - 0.1633\lvert 2\rangle_1$ $-0.1054\lvert 3\rangle_1 + 0.0968\lvert 1\rangle_1$	0.2224
	7.208	7.217	$0.9991\lvert 2\rangle_1 - 0.0329\lvert 3\rangle_1$	$0.9665\lvert 2\rangle_1 + 0.1846\lvert 1\rangle_3$	0.2514


Σ
Σ_c
Σ_b
Ξ_{cc}
Ξ_{bc}
Ξ_{bb}
$|7\rangle_S$
$|3\rangle_S$
$|5\rangle_S$
$|4\rangle_S$
$|6\rangle_S$
$|2\rangle_S$

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Magnetic moments; meson radiative transitions; meson decay constants; meson semileptonic decays; meson rare decays; meson strong decays; baryon radiative decays; baryon semileptonic decays; baryon rare decays; baryon strong decays...

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Each of these represents a wealth of data. For instance, in the case of baryon semileptonic decays: form factors, total and differential decay rates, polarization asymmetries, decays to excited states, etc.

Sketch of treatment of semileptonic decays (mesons), strong decays (baryons)

Semileptonic decay: decay of a hadron to a final state containing a hadron and a pair of leptons. Mediated by weak interaction: understanding these requires understanding the interplay between the dynamics of the weak interaction leading to the decay, and the strong interaction that provides the binding in the parent and daughter hadron

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Contrasted with leptonic decays (meson decay to a pair of leptons), or weak non-leptonic decay (weak hadron decay to a final state that has no leptons: eg $\Lambda \to p\pi^-$)