

High-energy nuclear physics and QCD

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Introduction to basic concepts and phenomena of high-energy photon/electron/hadron-hadron scattering

$E_{CM} \gg 1 \text{ GeV}$ typical hadronic mass scale

$E_{CM} \gg Q$ momentum transfer in process

- Space-time picture, size of interaction region
- Hadronic structure of photon, vector dominance model JLab to HERA
- QCD dipole model, gluon field, QCD radiation HERA, EIC
- Unitarity, new regime of strong gluon fields, “saturation” LHC

Why this focus

- Simple: QM + relativity enough to understand basic concepts!
- Universal: Concepts common to photon/electron/hadron–hadron scattering
- Old and new: Same concepts apply to low- Q^2 (hadronic) and high- Q^2 (QCD) processes at high energies if $Q^2 \ll E_{CM}^2$

QCD people usually refer to high–energy scattering as “small- x physics” because $x = Q^2/E_{CM}^2(\gamma^*p) \ll 1$ in this regime

- Beautiful: Correspondence between space–time picture and mathematical formalism
- Relevant: ep at HERA, EIC, $\bar{p}p/pp/\gamma p$ at Tevatron, LHC
LHC with $E_{CM}(pp) = 14000\text{GeV}$ will expand the frontier of high–energy scattering by one order of magnitude!
Precision studies of high–energy scattering in ep with a future Electron–Ion Collider (EIC)

Outline

- 1) Introduction to high-energy scattering: Space-time picture, size of interaction region, basics of relativistic scattering theory
 - 2) Hadronic component of photon: General formulation, vector dominance model, experimental tests, extension to $Q^2 > 0$
 - 3) High-energy scattering at large Q^2 : QCD dipole model, gluon fields in nucleon, QCD radiation, HERA results
 - 4) Unitarity in high-energy scattering: Concept, black-disk regime at high Q^2 , observables at LHC and EIC
- + Compact reviews of basic topics, as needed:
Relativistic units, elementary scattering theory, . . .

Units

- Use relativistic units. Write physical quantity as quantity = $\hbar^a c^b (\text{length})^d$ and measure it in units of length:

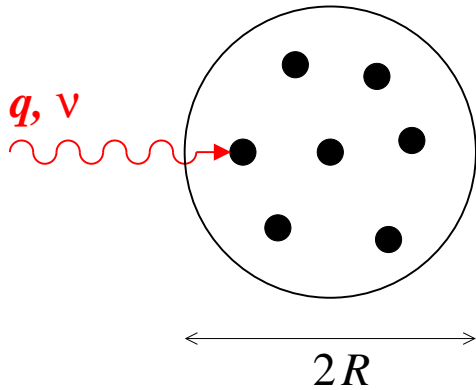
$$\text{time} = c^{-1} (\text{length}), \quad \text{energy} = \hbar c (\text{length})^{-1}, \dots$$

- Jargon: “Set \hbar and c equal to one”
- Only one basic dimension! Could equivalently use “energy” instead of “length” as basic dimension
- Conversion using magic number $\hbar c = 0.197 \text{ GeV fm} \approx 0.2 \text{ GeV fm}$

$$1 \text{ GeV} = 0.2 \text{ fm}^{-1} = \frac{1}{5 \text{ fm}}, \quad 1 \text{ fm} = 5 \text{ GeV}^{-1} = \frac{1}{0.2 \text{ GeV}}$$

... You will get used to it very quickly!

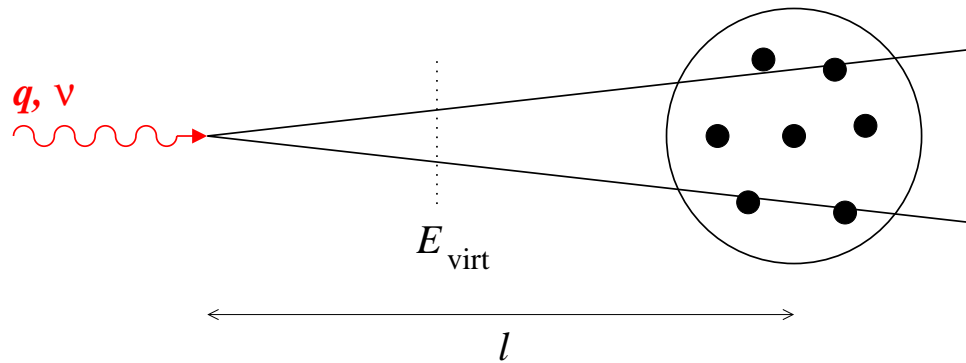
Introduction: Low vs. high energy scattering



- Example: Photon absorbed by nucleus. Photon momentum q , energy $\nu = |q|$

(A) “Low energy” $\nu < M_\pi$ (pion mass):
no particle production

- Photon hits constituent of nucleus (nucleon)
- Interaction extends over longitudinal distances $2R$ (radius of bound state), independent of energy ν !
- Information needed to describe process is contained in N -particle bound-state wave function $\Psi_P(\mathbf{r}_1, \dots, \mathbf{r}_N)$
→ inelastic cross section, form factor for elastic scattering of virtual photon, . . .



(B) “High energies:” $\nu \gg M_\pi$

- New mechanism for photon absorption: Photon converts to virtual hadronic state (energy $E_{\text{virt}} > \nu$) which interacts with nucleus
Example: $\pi^+\pi^-$ pair

- Length of interaction region l no longer determined by “size” of target R .
We will show: $l \sim \nu/M_\pi^2$, grows with energy!
- Relevant information no longer just in N -particle bound-state wave function. . . need virtual states with more particles!
- Alternative description possible: Photon has wave function with hadronic component

- Estimate interaction region length from uncertainty principle

- Virtual state can “exist” for time $\Delta t = \frac{1}{\Delta E} = \frac{1}{E_{\text{virt}} - \nu}$

- Parametrize π^\pm momenta as

$$\begin{aligned} \mathbf{k}_+ &= \alpha \mathbf{q} + \mathbf{k}_\perp, & \mathbf{q} \text{ “longitudinal” direction} \\ \mathbf{k}_- &= (1 - \alpha) \mathbf{q} - \mathbf{k}_\perp, & \mathbf{k}_\perp \text{ “transverse”, } \mathbf{q} \cdot \mathbf{k}_\perp = 0 \\ \mathbf{k}_+ + \mathbf{k}_- &= \mathbf{q} & \text{3-momentum conservation} \end{aligned}$$

- The pion energies are $E_\pm = \sqrt{\mathbf{k}_\pm^2 + M_\pi^2}$

- Calculate energy difference $\Delta E = E - \nu$ in limit $|\mathbf{q}| \gg M_\pi, |\mathbf{k}_\perp|$:

$$\begin{aligned} \Delta E &= \sqrt{\alpha^2 |\mathbf{q}|^2 + \mathbf{k}_\perp^2 + M_\pi^2} + \sqrt{(1 - \alpha)^2 |\mathbf{q}|^2 + \mathbf{k}_\perp^2 + M_\pi^2} - \nu \\ &= \alpha |\mathbf{q}| + \frac{\mathbf{k}_\perp^2 + M_\pi^2}{2\alpha |\mathbf{q}|} + (1 - \alpha) |\mathbf{q}| + \frac{\mathbf{k}_\perp^2 + M_\pi^2}{2(1 - \alpha) |\mathbf{q}|} - \nu \\ &= \frac{\mathbf{k}_\perp^2 + M_\pi^2}{2\alpha(1 - \alpha) |\mathbf{q}|} = \frac{\mathbf{k}_\perp^2 + M_\pi^2}{2\alpha(1 - \alpha) \nu} \end{aligned}$$

- Energy difference reaches minimum value for configurations with $\alpha = 1/2$, $\mathbf{k}_\perp = 0$:

$$\Delta E(\text{min}) = \frac{2M_\pi^2}{\nu} \quad \Delta t(\text{max}) = \frac{\nu}{2M_\pi^2}$$

- Since π^\pm pair moves with speed of light ($|\mathbf{q}| \gg M_\pi$), the distance traveled during its lifetime is

$$l = \frac{\nu}{2M_\pi^2}$$

- Length of interaction region (“coherence length”) grows proportionally to energy ν !

- Same conclusion can be reached more formally using time-ordered perturbation theory: “Energy denominator”
- The other mechanism (photon directly hits a nucleon, $l \sim 2R$) is also allowed at high energies, but its contribution to the cross section is suppressed by a power of $1/\nu$.

Basic idea: Do not calculate at all possible contributions, but try to identify dominant ones in the limit $\nu \rightarrow \infty$. . . simple, analytic approximations!

- Our simple example illustrates all essential features of high-energy scattering. We will only need to replace the π^\pm system by other types of hadronic fluctuations of the photon:

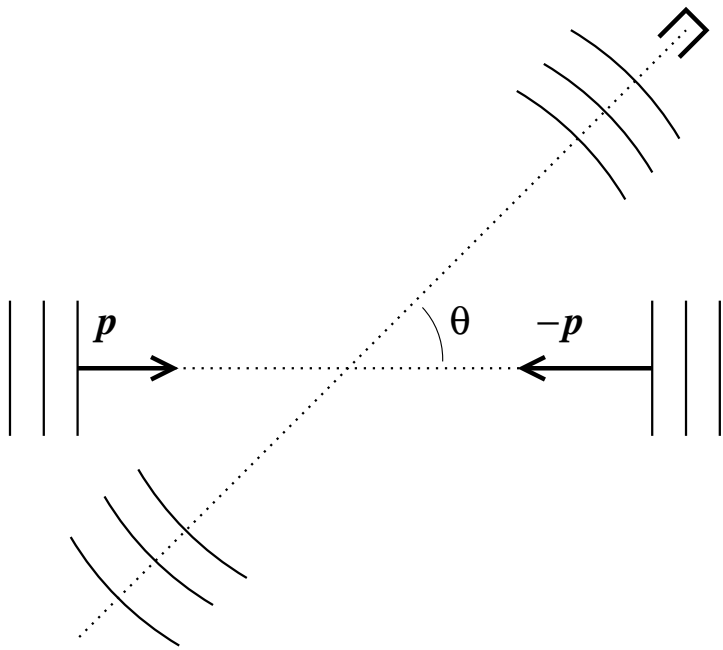
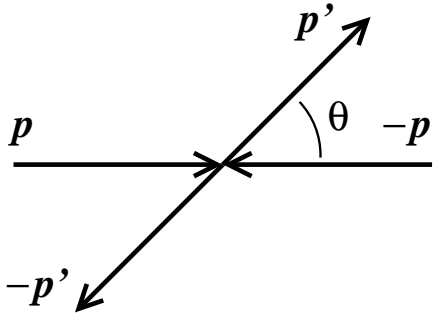
real photons, virtual w. $Q^2 < 1 \text{ GeV}^2$

vector mesons

virtual w. $Q^2 \gg 1 \text{ GeV}^2$

quark-antiquark pair w.
transverse size $\sim 1/Q$

Introduction: Scattering theory



- Non-relativistic scattering in CM frame

$$\mathbf{p}_1 = -\mathbf{p}_2 \equiv \mathbf{p} \quad \text{opposite momenta}$$

$$\mathbf{p}'_1 = -\mathbf{p}'_2 \equiv \mathbf{p}'$$

- Elastic scattering $E = E' \rightarrow |\mathbf{p}| = |\mathbf{p}'|$.

Two kinematic variables:

$$E \text{ or } |\mathbf{p}| \quad \text{CM energy (momentum)}$$

$$\Theta = \text{angle}(\mathbf{p}, \mathbf{p}') \quad \text{scattering angle}$$

- QM description: Solution of Schrödinger equation with asymptotic form

$$f(\mathbf{r}) = e^{i\mathbf{p}\cdot\mathbf{r}} + f(E, \Theta) \frac{e^{ipr}}{r}$$

$$\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2 \quad \text{relative coordinate}$$

$$f(E, \Theta) \quad \text{scattering amplitude, complex!}$$

- Basic observable: Differential cross section

$$d\sigma = \frac{\text{Number of particles scattered in } d\Omega \text{ per time}}{\text{Current density of incident wave}} \quad [\text{Dimension: Area}]$$

$$\frac{d\sigma}{d\Omega} = |f(E, \Theta)|^2$$

- Low-energy scattering: Expand incident plane wave in angular momentum eigenfunctions (partial waves), use angular momentum conservation

- Alternative formulation: Scattering amplitude as transition matrix element of operator between momentum eigenstates (free states at $t \rightarrow \pm\infty$)

$$f(E, \Theta) = (4\pi M_{\text{red}}) \langle \mathbf{p}' | \hat{S} | \mathbf{p} \rangle \quad \text{“S–Matrix”}$$

$$\hat{S}^\dagger \hat{S} = 1 \quad \text{Unitary operator if } \hat{H} \text{ hermitean}$$

$$\hat{S} = \lim_{T \rightarrow \infty} \hat{U}(T, -T) \quad \text{Time–evolution operator of system}$$

- Operator formulation useful for

→ Invariance properties: $\hat{U}^{-1} \hat{S} \hat{U} = \hat{S}$ unitary transformation

→ Conservation of probability: encoded in unitarity of \hat{S}

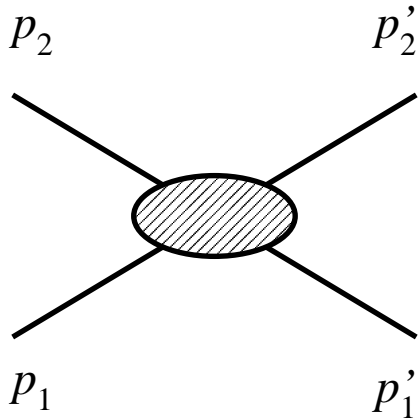
→ Approximation methods:

$$\hat{S} \rightarrow \text{Green function } \frac{1}{\hat{H} - E} \rightarrow \text{pert. expansion } \hat{H} = \hat{H}_0 - \hat{V}$$

Important for overall context, but we will not need the details. . .

- For high-energy scattering we need
 - Relativistically invariant formulation:
kinematic variables, amplitude, cross section
 - Inelastic processes: Conversion of energy into particles!

Introduction: Relativistic scattering theory



- Relativistic kinematics of two-body scattering:
Use 4-momenta $p_1^\mu = (E_1, \mathbf{p}_1)$, etc.

$$ab = a^0 b^0 - \mathbf{a} \cdot \mathbf{b}, \quad a^2 = (a^0)^2 - \mathbf{a}^2$$

- Energy-momentum relation $E_1 = \sqrt{\mathbf{p}_1^2 + M_1^2}$, etc.
 $\rightarrow p_1^2 = E_1^2 - \mathbf{p}_1^2 = M_1^2$ “mass shell condition”

- Energy-momentum conservation: $p_1^\mu + p_2^\mu = p_1'^\mu + p_2'^\mu$
- Use Lorentz invariants (= squares, scalar products of 4-vectors)
as kinematic variables

$$s \equiv (p_1 + p_2)^2 = (p_1' + p_2')^2$$

$$t \equiv (p_1' - p_1)^2 = (p_2 - p_2')^2$$

- Relation to “usual” kinematic variables in CM frame

$$p_1^\mu = (E_1, \mathbf{p}), \quad p_2^\mu = (E_2, -\mathbf{p}), \quad (p_1 + p_2)^\mu = (E_1 + E_2, \mathbf{0})$$

$$\rightarrow s = (p_1 + p_2)^2 = (E_1 + E_2)^2 \quad \text{squared total CM energy}$$

$$p_1^\mu = (E_1, \mathbf{p}), \quad p_1'^\mu = (E_1', \mathbf{p}')$$

$$\rightarrow t = (p_1' - p_1)^2 = p_1'^2 + p_1^2 - 2p_1 p_1'$$

$$= M_1^2 + M_1'^2 - 2E_1 E_1' + 2|\mathbf{p}||\mathbf{p}'| \cos \theta \quad \text{related to CM scattering angle}$$

- High-energy limit: $|\mathbf{p}|, |\mathbf{p}'| \gg \text{masses}, \quad |\mathbf{p}| \approx |\mathbf{p}'| \approx \sqrt{s}/2$

$$t = 2|\mathbf{p}|^2(-1 + \cos \theta) = -4|\mathbf{p}|^2 \sin^2(\theta/2) = -s \sin^2(\theta/2) < 0$$

- We will be interested in small-angle scattering: $s \gg 1 \text{ GeV}^2, \quad |t| \sim 1 \text{ GeV}^2$

$$\frac{|t|}{s} \approx \frac{\theta^2}{4}$$

- Like to have: Scattering amplitude, cross section as function of invariants!

- Define invariant amplitude $T(s, t)$ as that function of s and t which coincides with the “usual” amplitude in the CM frame:

$$T(s, t) \equiv \sqrt{s} f(E, \theta) \quad \text{in CM frame!}$$

The factor \sqrt{s} is conventional; the high-energy limit is assumed

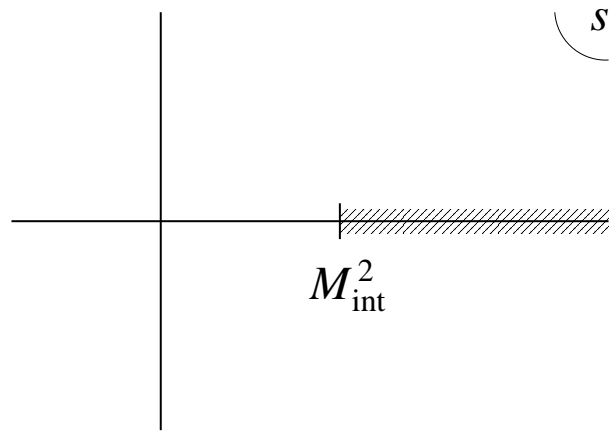
- Relation to elastic scattering cross section

$$\frac{d\sigma}{d|t|} = \frac{4\pi}{s^2} |T(s, t)|^2$$

Correspondence with “old” CM frame expression:

$$d|t| = (s/2) d(\cos \theta), \quad d\Omega = 2\pi d(\cos \theta) = (4\pi/s) d|t|$$

Invariant amplitude: Singularities



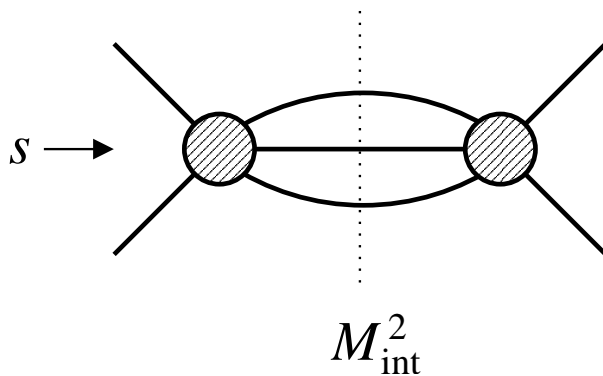
- Invariant amplitude is a complex function
... Analytic structure? Singularities?
- $T(s, t)$ has singularities (poles, cuts) at $s > 0$!

Reason: Virtual intermediate states can become real if CM energy exceeds sum of masses of particles M_{int}

virtual state $s < M_{\text{int}}^2$

real state $s > M_{\text{int}}^2$

... discontinuous change, “threshold”!



- Examples

NN scattering: $s = (M_N + M_{\Delta})^2$

$N\Delta$ interm. state

γN scattering: $s = (M_N + M_{\pi})^2$

πN interm. state

- Optical theorem for “diagonal” amplitude $ab \rightarrow ab$:
Imaginary part = inclusive cross section $ab \rightarrow X$

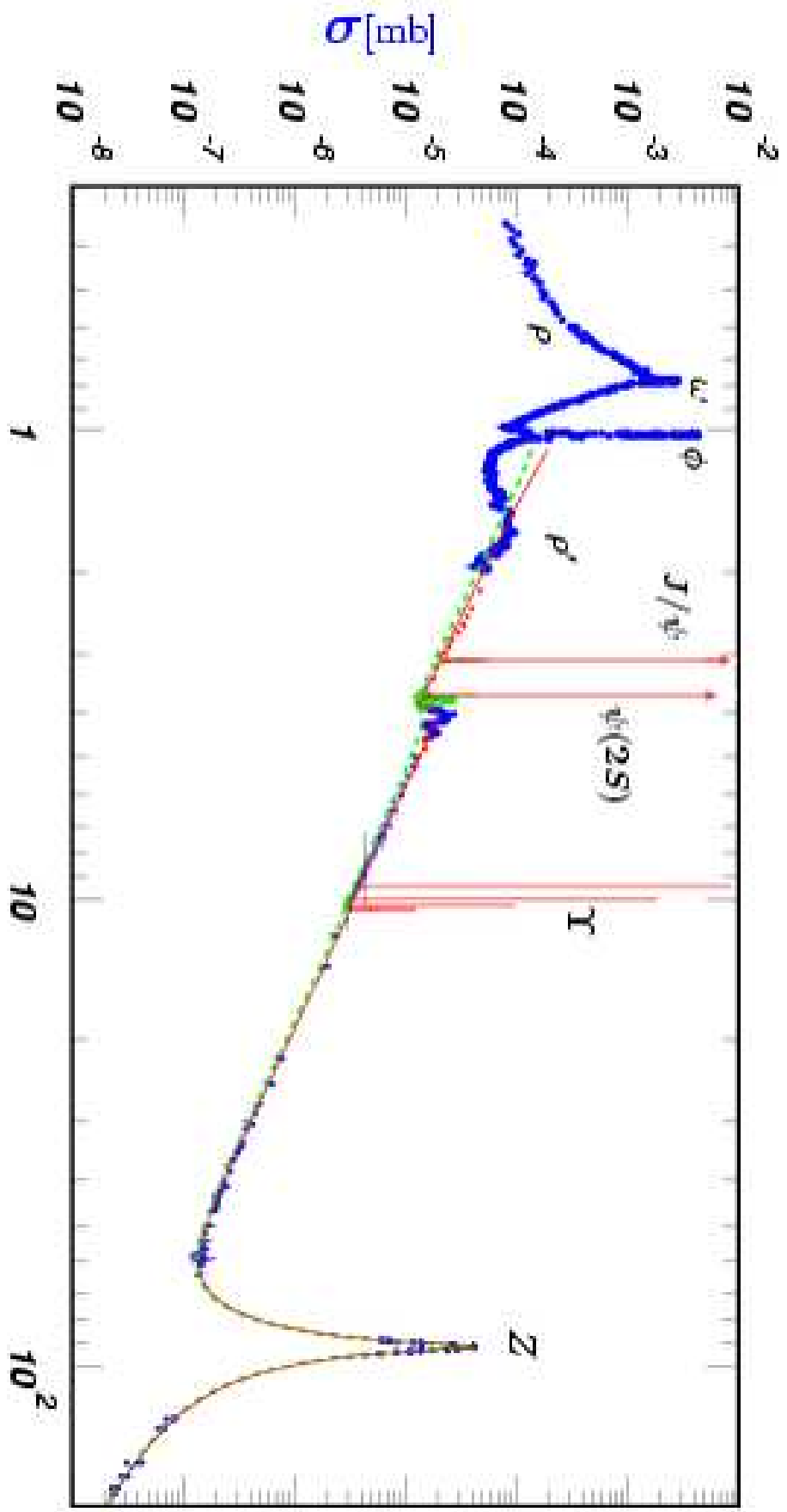
$$\text{Im } T_{ab \rightarrow ab}(s + i0, t = 0) = \frac{8\pi}{s} \sum_X \sigma_{ab \rightarrow X}(s)$$

... important, many applications!

- Example: We are interested in the total photon absorption cross section $\sigma_{\gamma p} \equiv \sum_X \sigma_{\gamma p \rightarrow X}$. Instead of calculating the individual cross sections for $\gamma p \rightarrow X$ and summing over the final states X , we simply calculate

$$T_{\gamma p \rightarrow \gamma p}(s, t = 0) \quad \text{Compton scattering amplitude at zero angle } (t = 0)$$

and take its imaginary part!

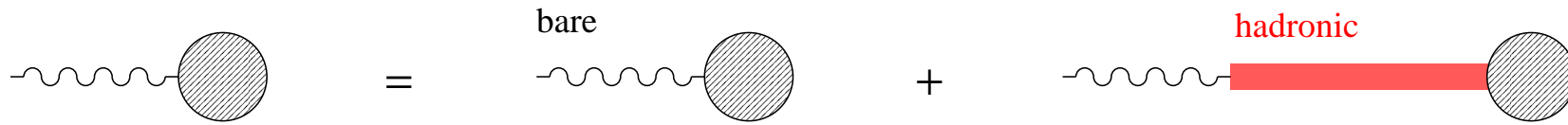


Hadronic component of photon

- Physical photon state contains hadronic component (amplitude proportional to EM coupling e)

$$|\gamma\rangle = \sqrt{Z} |\gamma_{\text{bare}}\rangle + e|h\rangle$$

- Fundamental assumption: Photon in high-energy scattering ($s \gg 1 \text{ GeV}^2$, $Q^2 < 1 \text{ GeV}^2$) couples predominantly through hadronic component



Vector dominance model

- Vector dominance model: Hadronic component saturated by vector meson states $V = \rho^0, \omega, \phi$

$$e|h\rangle = \sum_{V=\rho,\omega,\phi} \frac{e}{f_V} \frac{M_V^2}{M_V^2 + Q^2} |V\rangle$$

- “Master formula” for invariant amplitudes

$$T_{\gamma A \rightarrow B}(s, t, \dots) = \sum_V \frac{e}{f_V} \frac{M_V^2}{M_V^2 + Q^2} T_{VA \rightarrow B}(s, t, \dots) \quad T \text{ polarization}$$
$$\times \frac{Q}{M_V} \quad L \text{ polarization}$$

- Simple, high predictive power!
- L polarization in virtual photon: Amplitude must vanish for $Q^2 \rightarrow 0$. Implementation model-dependent, requires separate discussion. . .

Vector dominance model: Predictions

- Total cross section $\gamma p \rightarrow X$ proportional to hadronic cross section $V p \rightarrow X$

$$T_{\gamma p \rightarrow \gamma p}(s, t = 0) = \sum_V \frac{e^2}{f_V^2} T_{V p \rightarrow V p}(s, t = 0) \quad \text{Use optical theorem!}$$

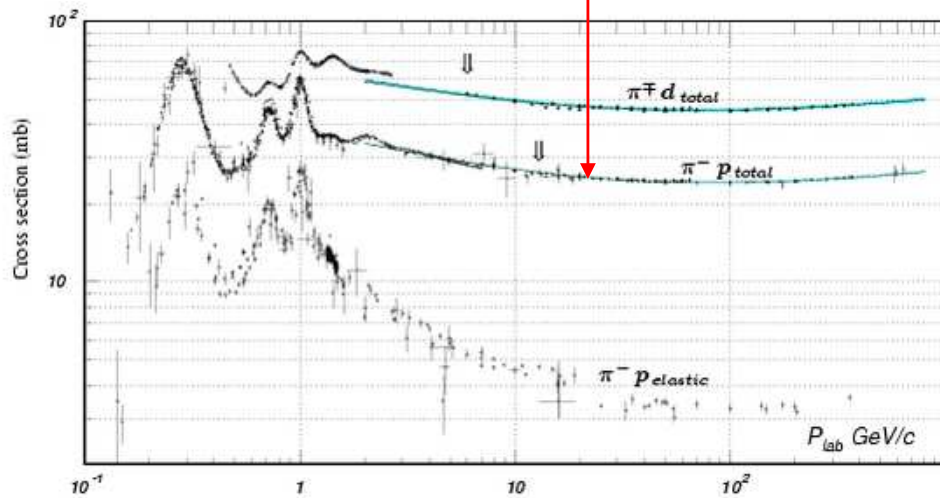
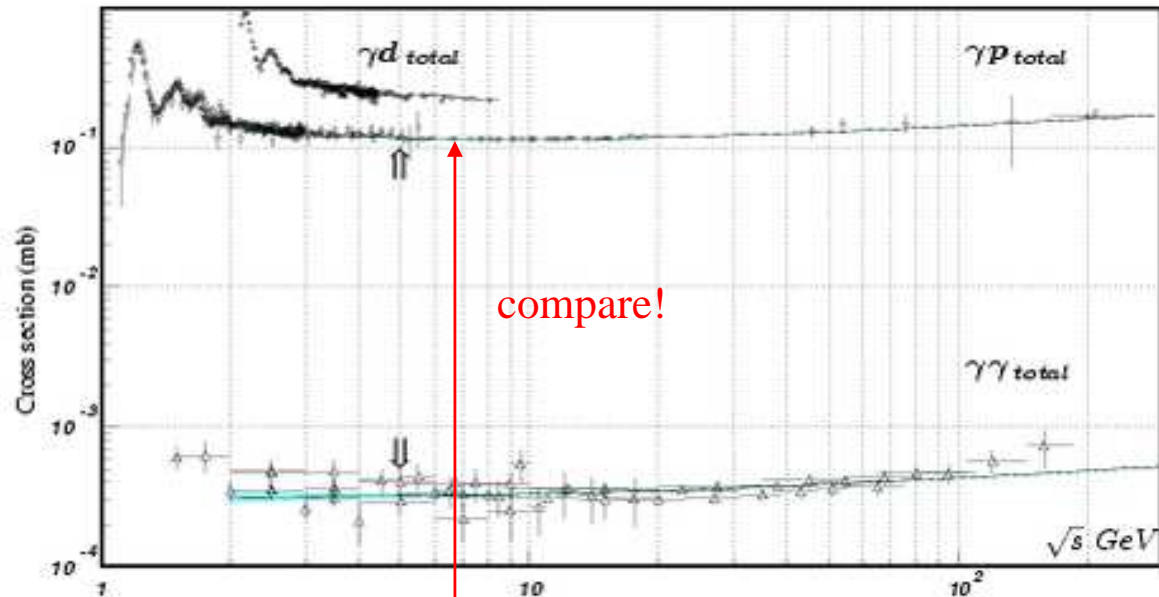
$$\sigma_{\gamma p \rightarrow X}(s) = \sum_V \frac{e^2}{f_V^2} \sigma_{V p \rightarrow X}(s) \quad \text{not measurable directly, consider } \sigma_{\pi \pm p}!$$

- Differential cross section for Compton scattering $\gamma p \rightarrow \gamma p$ proportional to that of vector meson photoproduction $\gamma p \rightarrow V p$

$$T_{\gamma p \rightarrow \gamma p}(s, t) = \sum_V \frac{e}{f_V} T_{\gamma p \rightarrow V p}(s, t) \quad \text{invariant amplitudes}$$

$$d\sigma/dt \propto |T(s, t)|^2 \quad \text{differential cross sections}$$

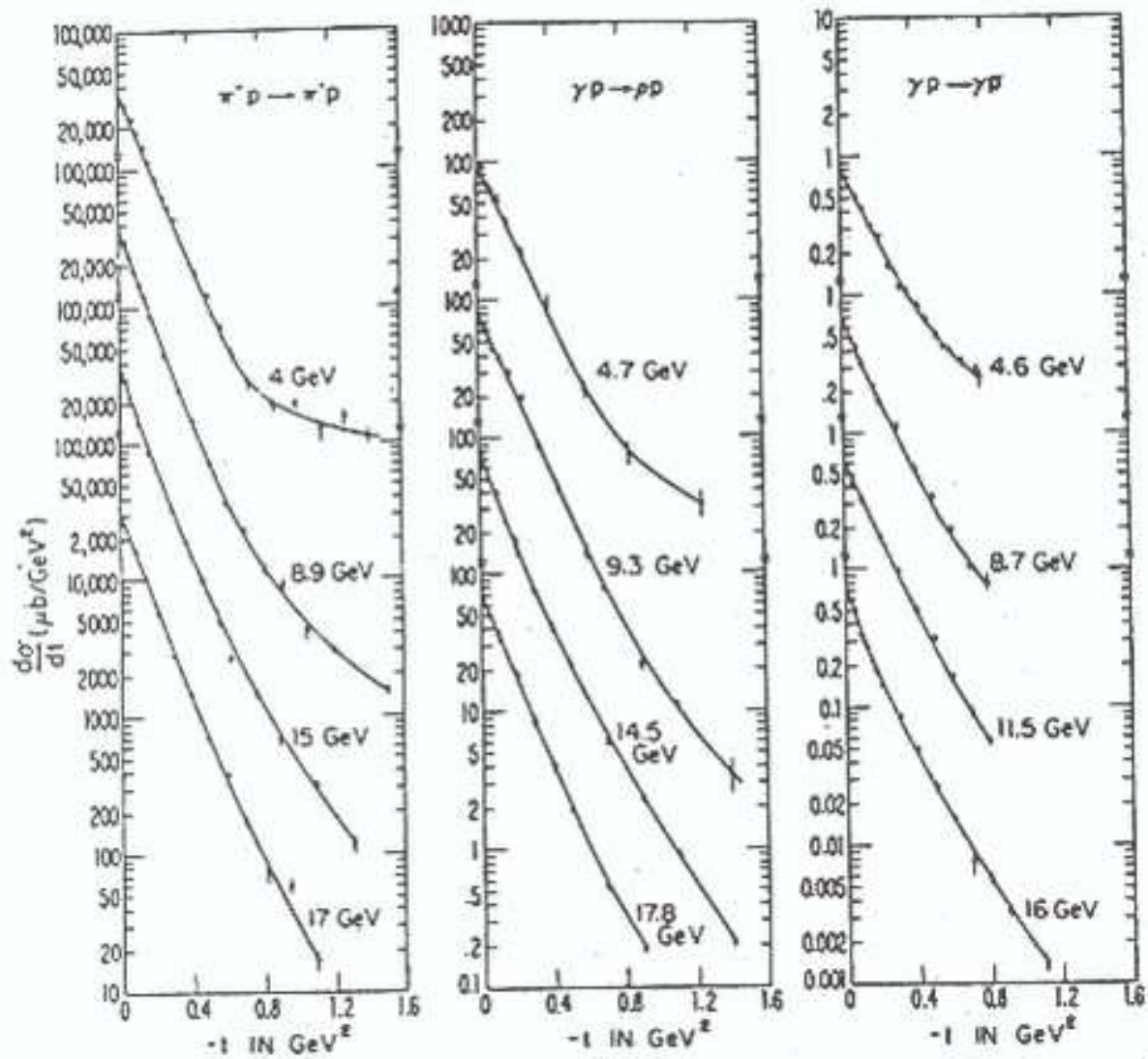
Large s : Expect ρ^0, ω, ϕ cross sections to be proportional; same s and t -dependence.



$$s = 2p_{\text{lab}} M_N$$

[Particle Data Group:
Amsler et al.]

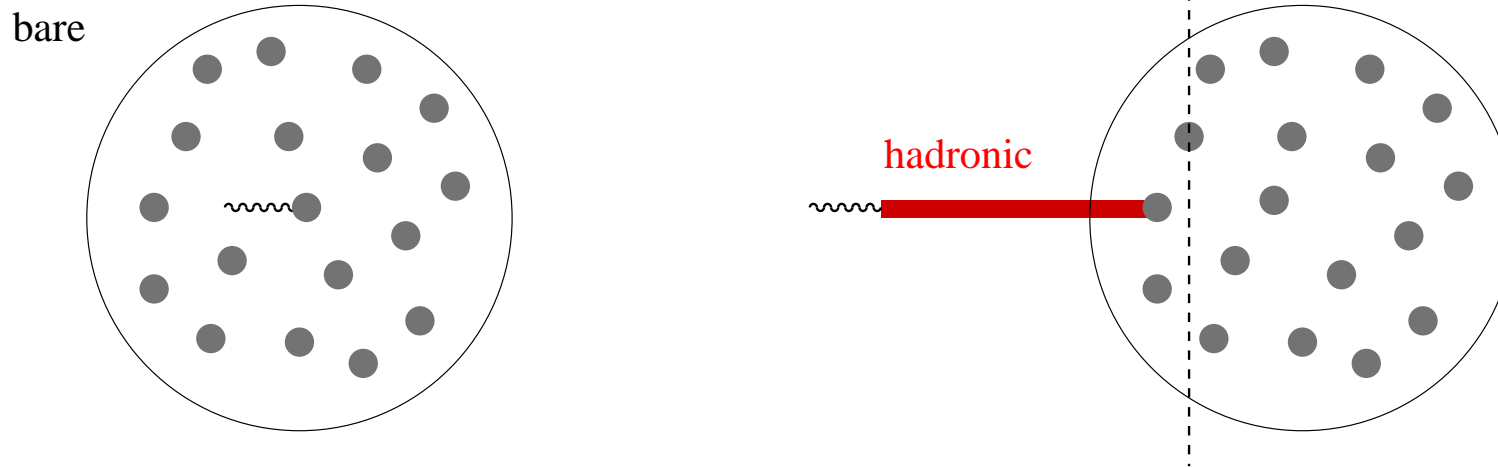
- Striking similarity of high-energy γp and $\pi^\pm p$ cross sections. Ratio $\sigma(\pi^+ p + \pi^- p) / \sigma(\gamma p) \approx 440$ at $\sqrt{s} > 10$ GeV explained by VMD factors $(e/f_V)^2$



- Same t -dependence of Compton scattering and vector meson production!

[Review Bauer et al. 1978]

Vector dominance model: Nuclear targets



- Bare component interacts weakly, penetrates entire depth of nucleus:
 $\sigma(A) = A \sigma(1)$

- Hadronic component interacts strongly, fully absorbed by surface:
 $\sigma(A) \propto (\text{surface area}) \sigma(1) \propto A^{2/3} \sigma(1)$

Downstream nucleons do not contribute to cross section: “Shadowing.”
Nucleus acts as “filter” for bare component of photon.

. . . seen in data!

Vector dominance: Higher Q^2

- Q^2 dependence of $\sigma_T(\gamma^*N)$ given by

$$\sigma_T = \sum_V \frac{e^2}{f_V^2} \left(\frac{M_V^2}{M_V^2 + Q^2} \right)^2 \sigma_V \sim \frac{1}{Q^4} \quad \text{for } Q^2 \gg M_V^2$$

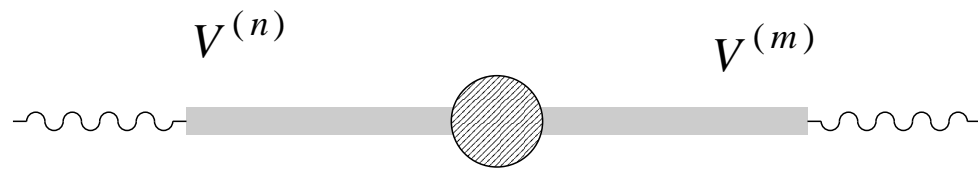
- Experiment and QCD's asymptotic freedom tell us:
Bjorken scaling for $Q^2 \propto s$:

$$\sigma_T = \pi e^2 \frac{F(x)}{Q^2}, \quad x \approx \frac{Q^2}{s} \quad \text{Bjorken variable, } F \text{ indep. of } Q^2$$

→ VM contribution drops faster with Q^2 (assuming $\sigma_V \sim \text{const.}$),
negligible in Bjorken regime

- Description in terms of hadronic components of photon can formally be extended to higher Q^2 by including higher-mass states $V, V', V'' \dots$:
“Generalized vector meson dominance,” GVMD

- However, Bjorken scaling requires off-diagonal transitions between higher-mass meson states $V^{(n)} \rightarrow V^{(m)}$: Coherent sum!



What is going on?

- QCD tells us: Relevant component of virtual photon at $Q^2 \gg 1 \text{ GeV}^2$ is really a quark-antiquark pair of size $\sim 1/Q$
 - Coherent sum of vector mesons $V^{(n)}$ tries to “mimic” small-size $q\bar{q}$ pair
 - More appropriate at high Q^2 to work directly with QCD degrees of freedom: QCD dipole model!

Use exactly same logic for $q\bar{q}$ component of virtual photon as previously for hadronic states!

New element: $q\bar{q}$ -nucleon interaction calculable in QCD, in terms of gluon field in nucleon!

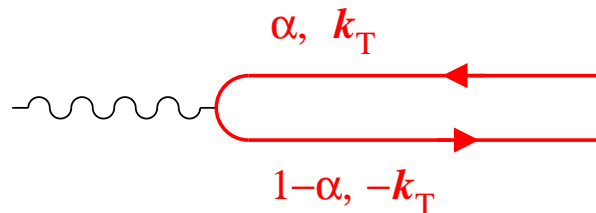
QCD dipole model: Basics

- Relevant strongly interacting component of photon at $Q^2 \gg 1 \text{ GeV}^2$ is QCD quark–antiquark pair:

$$|\gamma^*\rangle = |\gamma_{\text{bare}}^*\rangle + e|\text{pair}\rangle$$

γ^* not physical state, but reasoning valid by analogy!

- Configuration of $q\bar{q}$ pair characterized by



α longit. momentum fraction of q
 k_T transverse momentum

- Use first–order perturbation theory in EM interaction to calculate $e|\text{pair}\rangle$ state, in analogy to hadronic case. Hamiltonian density of EM interaction now given by pointlike coupling of photon to quarks:

$$\mathcal{H}_{\text{em}}(x) = e \sum_f Q_f \bar{\psi}_f(x) \gamma^\mu \psi_f(x) A_\mu(x) \quad f = u, d, s \quad \text{quark flavor}$$

- First-order perturbation theory:

$$e|\text{pair}\rangle = |q\bar{q}; \alpha, \mathbf{k}_T\rangle \frac{\langle q\bar{q}; \alpha, \mathbf{k}_T | \mathcal{H}_{\text{em}}(0) | \gamma_{\text{bare}}^* \rangle}{\nu - E_{q\bar{q}}(\alpha, \mathbf{k}_T)}$$

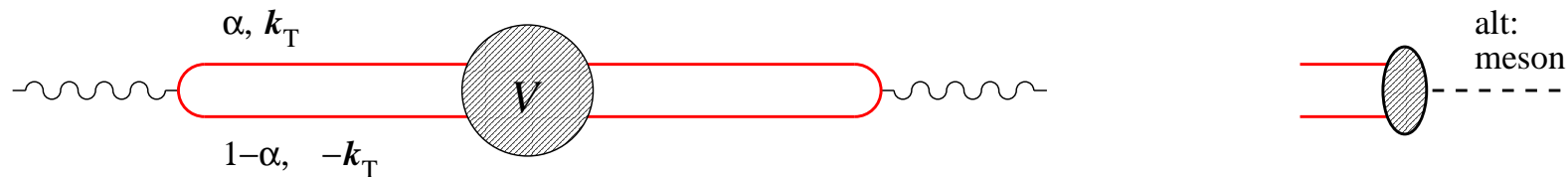
$$\equiv |q\bar{q}; \alpha, \mathbf{k}_T\rangle \psi_{\gamma^*}(\alpha, \mathbf{k}_T) \quad \dots \text{depends also on } Q^2 \text{ via } \nu!$$

- Result for $\gamma^* N \rightarrow \gamma^* N$ invariant amplitude ($t = 0$)

$$T_{\gamma^* N \rightarrow \gamma^* N}(s, t = 0) = \int \frac{d\alpha d^2 k_T}{(2\pi)^2} \int \frac{d\alpha' d^2 k'_T}{(2\pi)^2}$$

$$\times \psi_{\gamma^*}^*(\alpha', \mathbf{k}'_T) V_{q\bar{q}N \rightarrow q\bar{q}N}(\alpha', \mathbf{k}'_T; \alpha, \mathbf{k}_T) \psi_{\gamma^*}(\alpha, \mathbf{k}_T)$$

$V = (\text{kinem. factors}) \times \text{invariant amplitude of } q\bar{q} \text{ pair scattering from nucleon}$

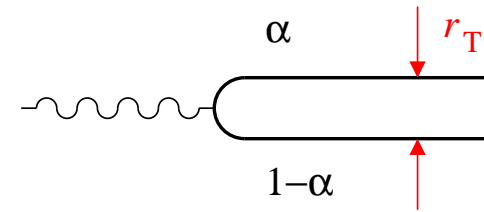


- Can be generalized to $t \neq 0$, meson production amplitude, etc.

- Convenient: Transverse coordinate representation

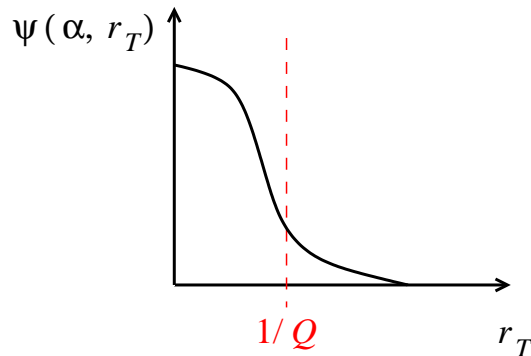
$\mathbf{k}_T \rightarrow \mathbf{r}_T$ separation of $q\bar{q}$ pair, "size"

$$\psi(\alpha, \mathbf{k}_T) = \int d^2r_T e^{i\mathbf{k}_T \cdot \mathbf{r}_T} \psi(\alpha, \mathbf{r}_T) \quad \text{etc.}$$



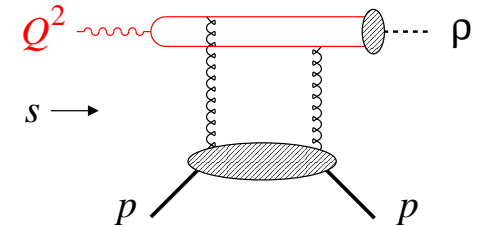
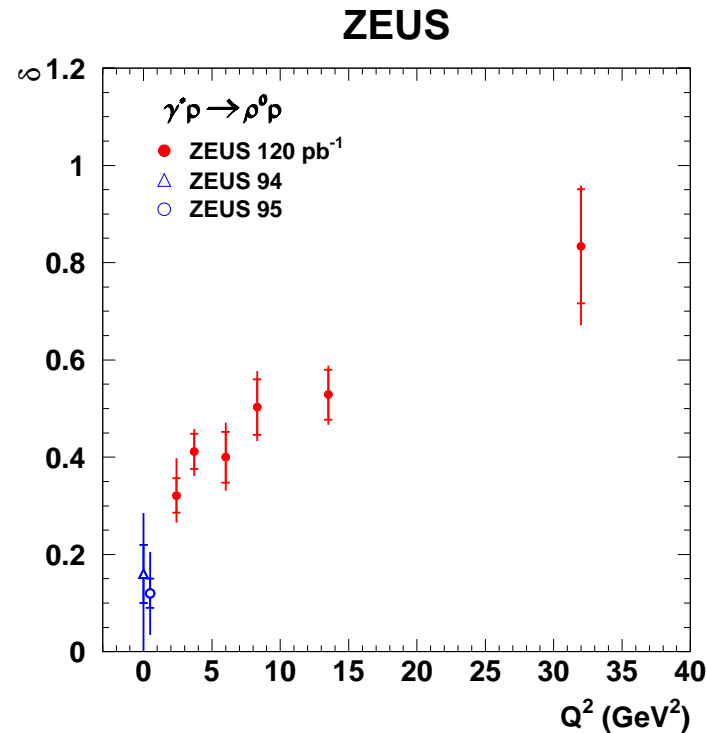
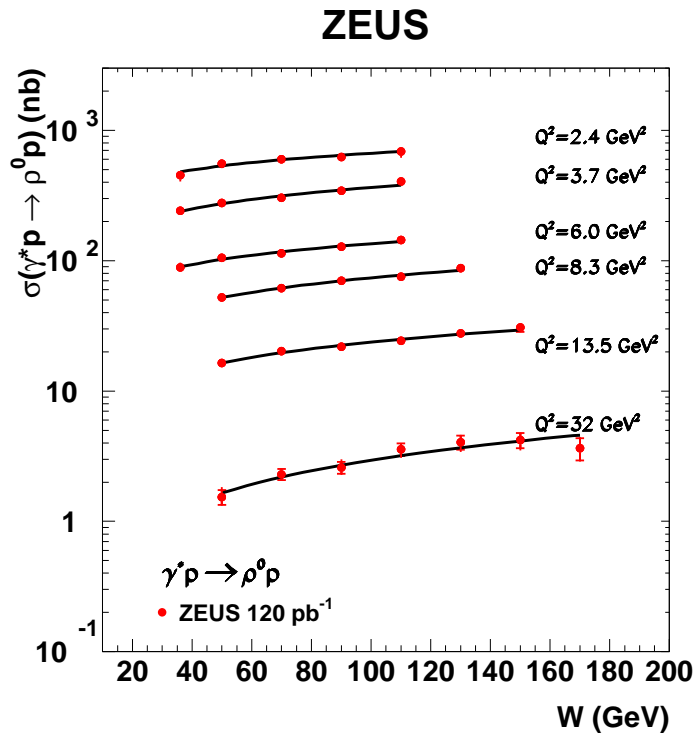
$$T_{\gamma^* N \rightarrow \gamma^* N}(s, t=0) = \int d\alpha d^2r_T \int d\alpha' d^2r'_T \\ \times \psi_{\gamma^*}^*(\alpha', \mathbf{r}'_T) V_{q\bar{q}N \rightarrow q\bar{q}N}(\alpha', \mathbf{r}'_T; \alpha, \mathbf{r}_T) \psi_{\gamma^*}(\alpha, \mathbf{r}_T)$$

- Typical r_T, r'_T in wave function of L virtual photon are of order $\sim 1/Q$



→ V calculable in QCD in well-controlled approximation!

QCD dipole model: Experimental tests



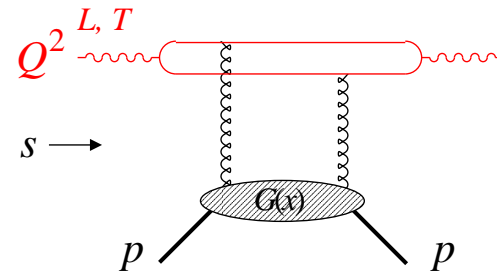
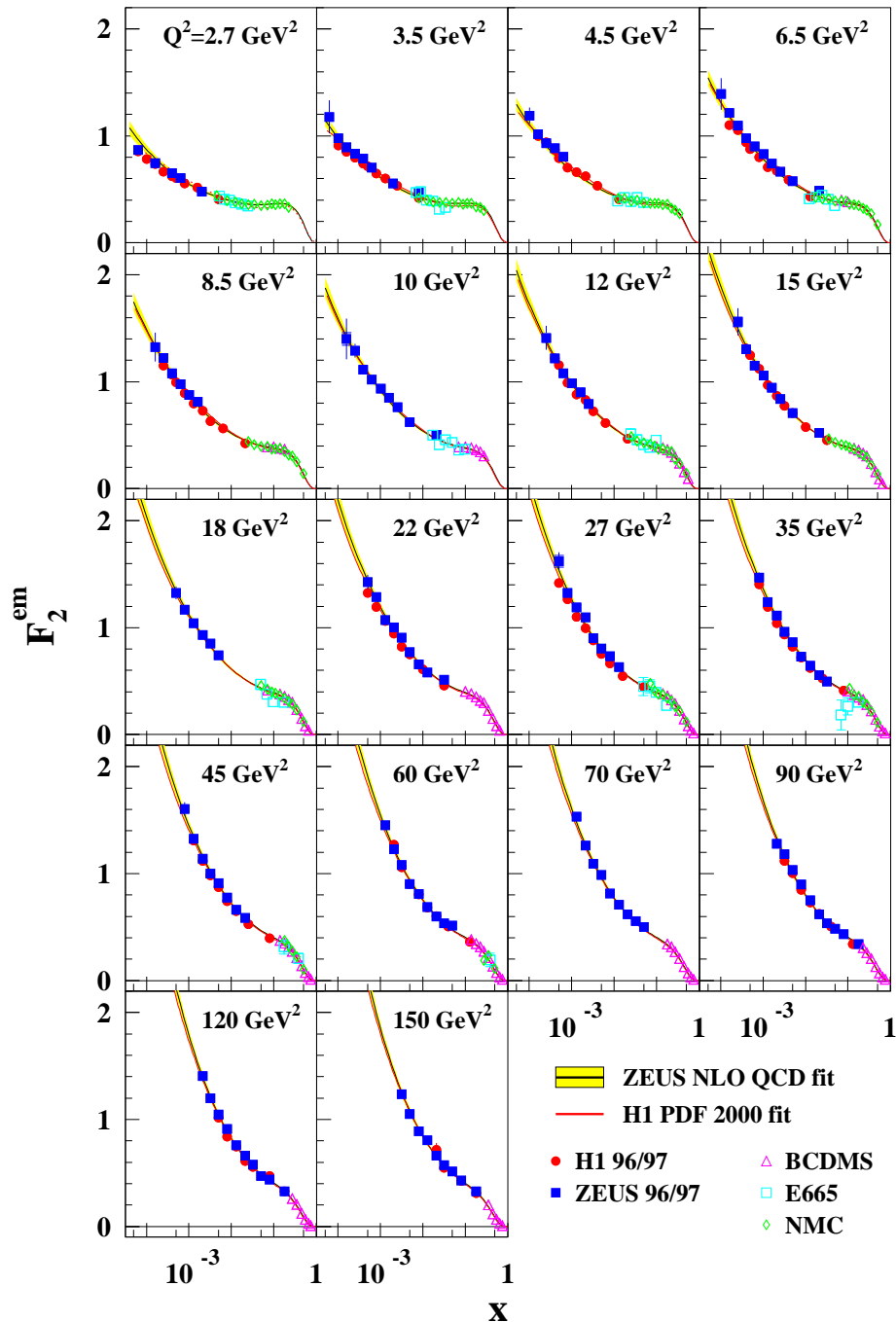
$$s = W^2$$

$$\text{Fit: } \sigma \propto W^\delta$$

[Review Levy 2009]

- s -dependence of cross section for vector meson production $\sigma(\gamma^*p \rightarrow \rho^0 p)$
 - Cross sections grows with s
 - s -dependence is steeper at higher Q^2 : Effect of QCD radiation!

HERA F_2



- s -dependence of total virtual photoabsorption cross section

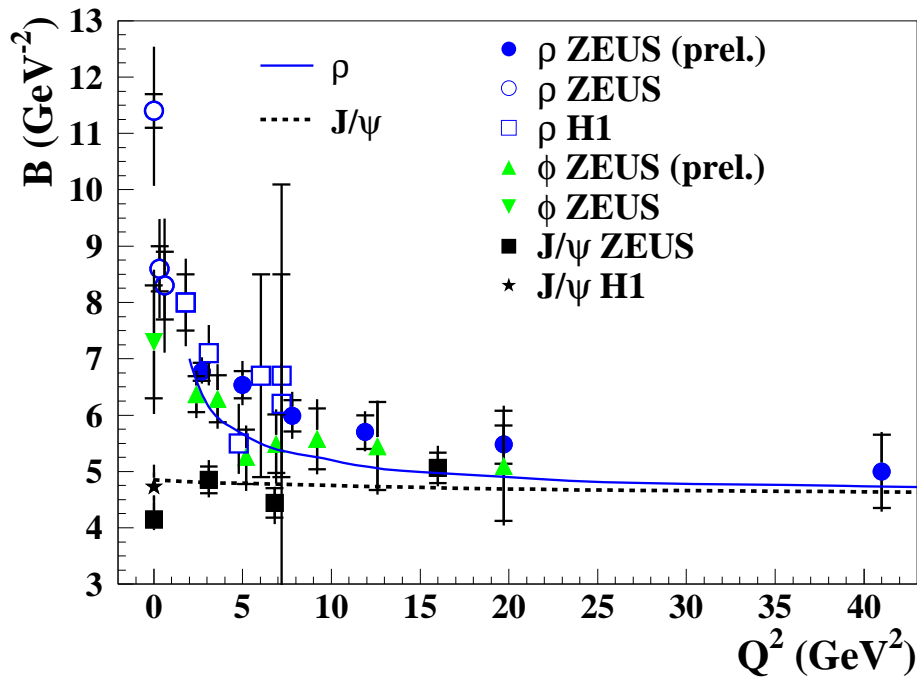
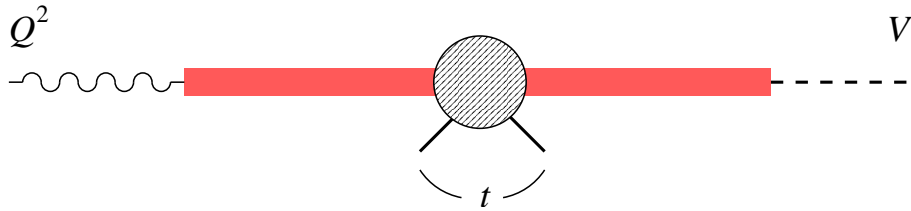
$$\sigma_T^{\gamma^*p}(s, Q^2) \propto \frac{F_2(x)}{Q^2}$$

$$x \equiv Q^2/s$$

- s -dependence becomes steeper at higher Q^2 : QCD radiation

σ_T requires cutoff at large dipole sizes.
 σ_L would be better to test the model,
 but is more difficult to measure (in progress).

Transition VMD \rightarrow dipole picture



Vector meson production $\gamma^* p \rightarrow V p$
 $d\sigma/dt \sim \exp Bt$

- Can we “measure” the transverse size of the hadronic component of γ^* ?

\rightarrow t -slope B in $\gamma^* p \rightarrow V p$
 proportional to transverse area of interaction region!

\rightarrow Observe transition from VMD regime ($Q^2 = 0$) to dipole regime ($Q^2 \gg 1 \text{ GeV}^2$)!

