Set of problems on “Large Nc QCD and the Skyrme Model”

Problem 1

Show that

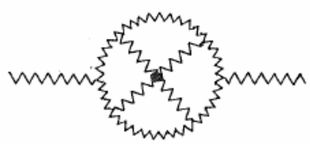
$$\left(T^A\right)^a{}_b \left(T^A\right)^c{}_d = \left(\frac{1}{2}\delta^a{}_d\,\delta^c{}_b - \frac{1}{2N_c}\delta^a{}_b\,\delta^c{}_d\right)$$

where the N_c x N_c traceless matrices T^A represent the generators of SU(N_c) in the fundamental irrep, and they are normalized as

$$Tr(T^A T^B) = \frac{\delta_{AB}}{2}$$

Problem 2

a) Using t'Hooft's double line rule find the N_c order of the following QCD diagram



b) Repeat a) for the case in which the two crossing gluons do not interact.

Problem 3

Draw three diagrams that contribute to the meson-meson scattering to leading order in the $1/N_c$ expansion. Determine explicitly their order in $1/N_c$

Problem 4

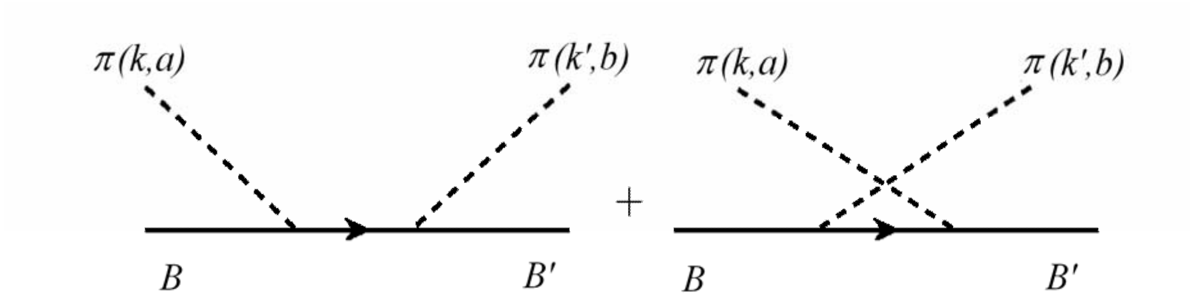
Repeat problem 3) for the case of the linked contributions to the pion-nucleon scattering amplitude.

Problem 5

Consider the operators J^i, T^a and G^{ia} for a single quark state in the case of two flavors. Show explicitly that they fulfill the SU(4) algebra.

Problem 6

Calculate the contribution to the pion-nucleon scattering amplitude of the diagrams



without assuming that the mass of the intermediate baryon state M_{int} is degenerate to the one of the initial state M_B. Assume, however, that $M_B >> \Delta M = M_{int} - M_B$. Show that the corresponding large N_c consistency relation implies

$$[X_0^{ia},[X_0^{jb},M_1]]=0 \qquad [X_0^{ia},[X_0^{jb},M_0]]=0$$

Here we have used

$$M = N_c M_1 + M_0 + \vartheta(1/N_c)$$
$$X^{ia} = X_0^{ia} + \vartheta(1/N_c)$$

Problem 7

Demostrate the identity

$$4J^i G^{ia} = (N_c + 2)I^a$$

when the operators are understood as acting on a completely symmetric N_c quark spin-flavor wavefunction.

Problem 8

a) Write the Skyrme model lagrangian

$$\mathcal{L}_{SK} = \frac{f_\pi^2}{4} Tr\left[\partial_\mu U^\dagger \partial^\mu U\right] + \frac{1}{16e^2} Tr\left[\partial_\mu U^\dagger \partial_\nu U \partial^\mu U^\dagger \partial^\nu U - \partial_\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger \partial^\mu U\right]$$

In terms of the “left” Maurer-Cartan operator $L_\mu = U^\dagger \partial_\mu U$

b) Find the way in which L_μ transforms under “left” and “right” (global) transformations defined by

$U \to T_L\, U$ donde $T_L = \exp\left(-i\ \vec{\varepsilon}_L \bullet \frac{\vec{\tau}}{2} \right)$

$U \to U\, T_R^\dagger$ donde $T_R = \exp\left(-i\, \vec{\varepsilon}_R \bullet \frac{\vec{\tau}}{2} \right)$

c) Using the results of items a) and b) show that $\mathcal{L}_{SK}$ is invariant under chiral transformations and find the associated Noether currents (in the case of the "right" transformations it might be convenient to use the "right" Maurer-Cartan operator $R_\mu = U \partial_\mu U^\dagger$)

Problem 9

Consider the quadratic contribution to $\mathcal{L}_{SK}$

$$\mathcal{L}_{SK}^{(2)} = \frac{f_\pi^2}{4} Tr\left[\partial_\mu U^\dagger \partial^\mu U \right]$$

a) Using *hedgehog* ansatz

$$U_h(\vec{r}) = \exp\left(i\, \vec{\tau} \bullet \breve{r}\, F(r) \right)$$

Find the explicit form of the corresponding contribution to the skyrmion mass.

b) Find the expression of the corresponding contribution to the Euler-Lagrange equation in terms of the skyrmion profile $F(r)$.

Problem 10

Consider the SU(2) rotating skyrmion

$$U = A(t) U_h(\vec{r})\, A^\dagger(t)$$

a) Find the contribution of $\mathcal{L}_{SK}^{(2)}$ to the skymion moment of inertia
b) Repeat a) for the axial and vector currents

Problem 11

Using the expression for the axial current found in problem 10) calculate its matrix element in the nucleon and delta states.