25 Annual Hampton University Graduate Studies Program

June 1 - 18, 2010

Lattice QCD

Hadron Structure and Interactions

Kostas Orginos

Outline

- Spectroscopy
- Hadron Structure
- Form Factors
- Moments of parton distribution functions
- Renormalization
- Recent results

Spectrum Calculations

Baryon

Meson

Calculate two point functions

$$\langle q_1 q_2 q_3(y,t)\bar{q}_1\bar{q}_2\bar{q}_3(x,0)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}[U]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\;\; q_1 q_2 q_3(y,t)\bar{q}_1\bar{q}_2\bar{q}_3(x,0)\;\; e^{-\bar{\psi}D(U)\psi - S_g(U)}$$

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The quark combinations are chosen so that the desired quantum numbers are selected

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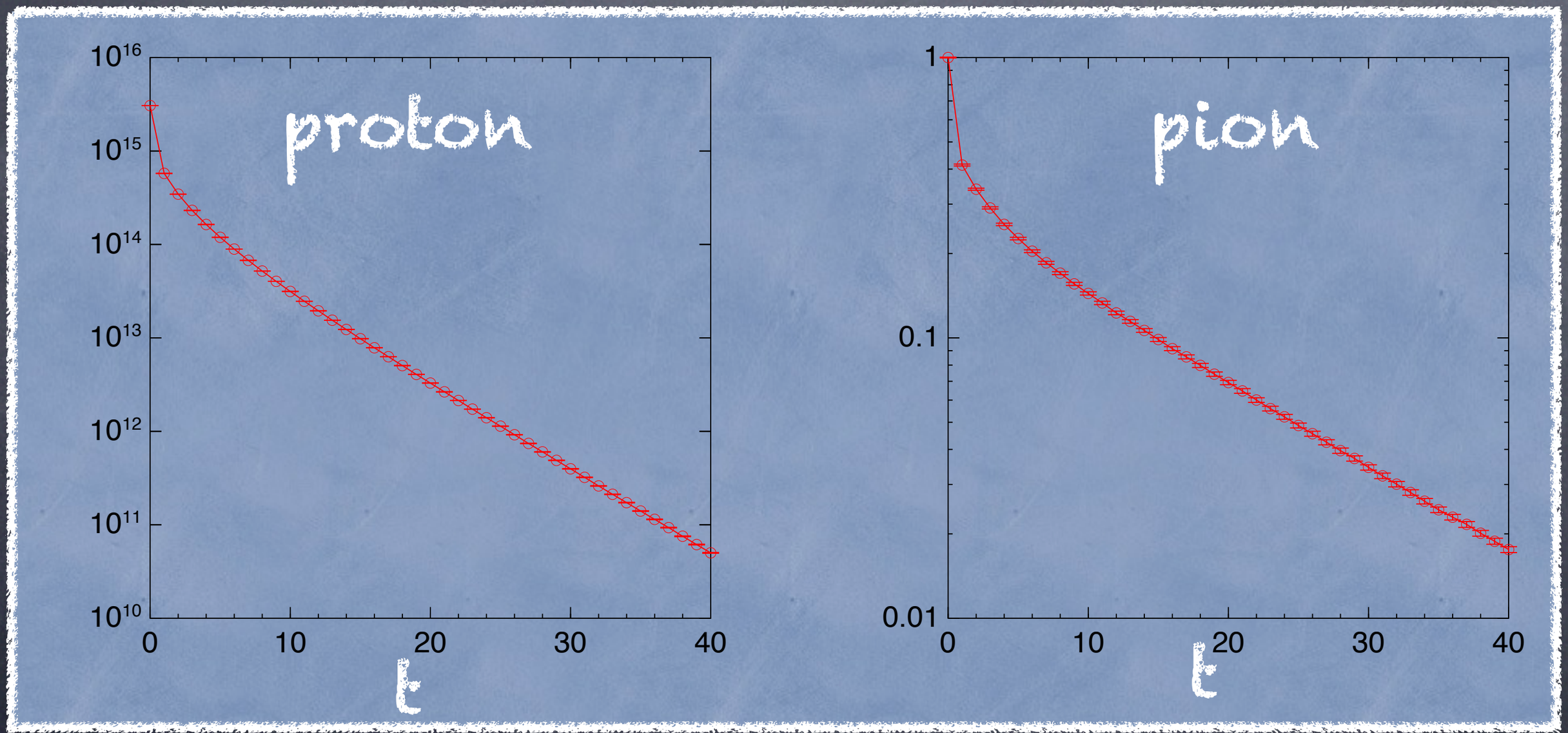
Lines in correspond to quark propagators

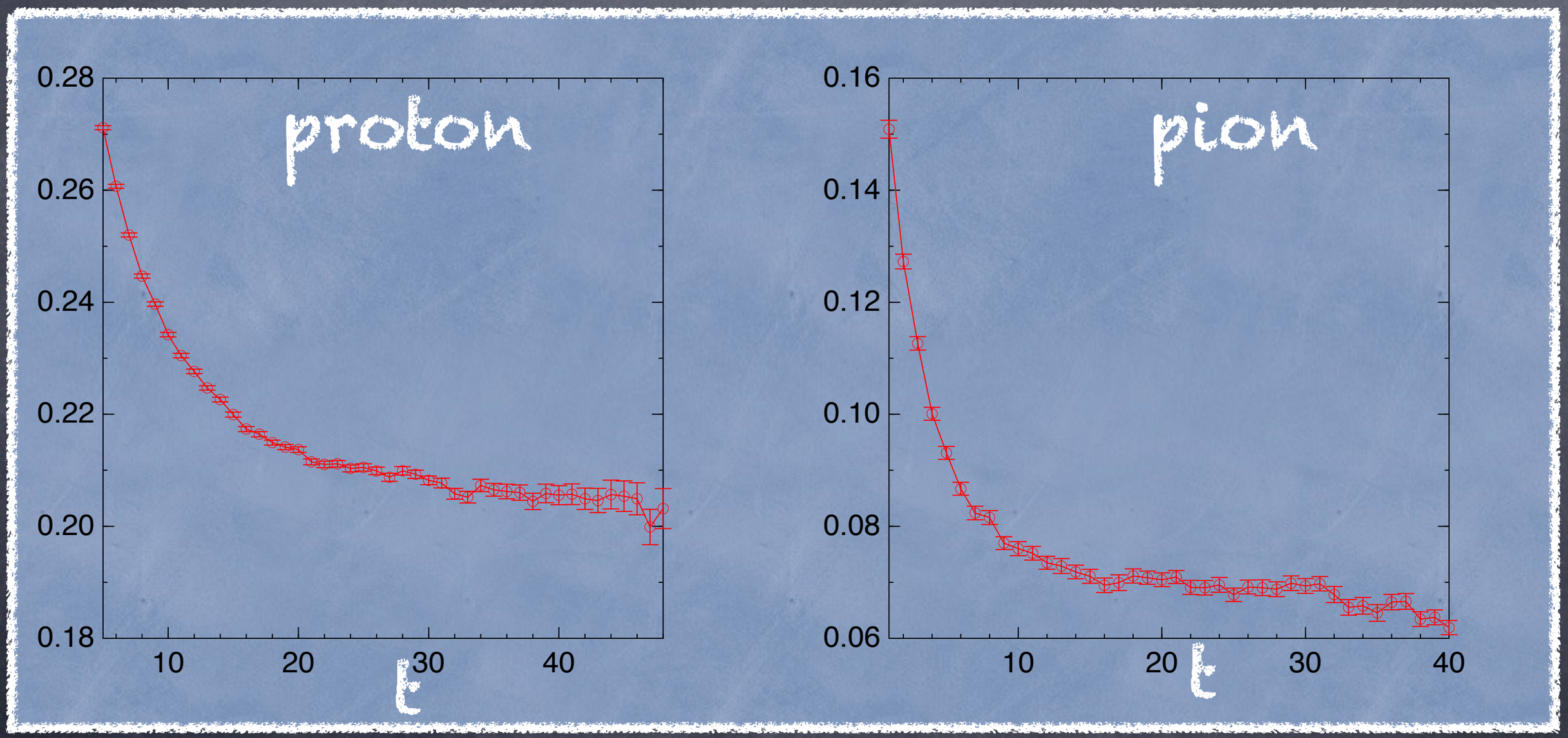
$$\langle q(x)\bar{q}(y)\rangle = D^{-1}(U)\Big|_{x,y}$$

This computation dominates the calculation and makes it hard to compute with physical quark masses

Two point functions $C(t) \approx Z_0 e^{-m_0 t} + Z_1 e^{-m_1 t} + \cdots$



effective mass: $$m_{eff}(t) = ln\left(\frac{C(t)}{C(t+1)}\right)$$



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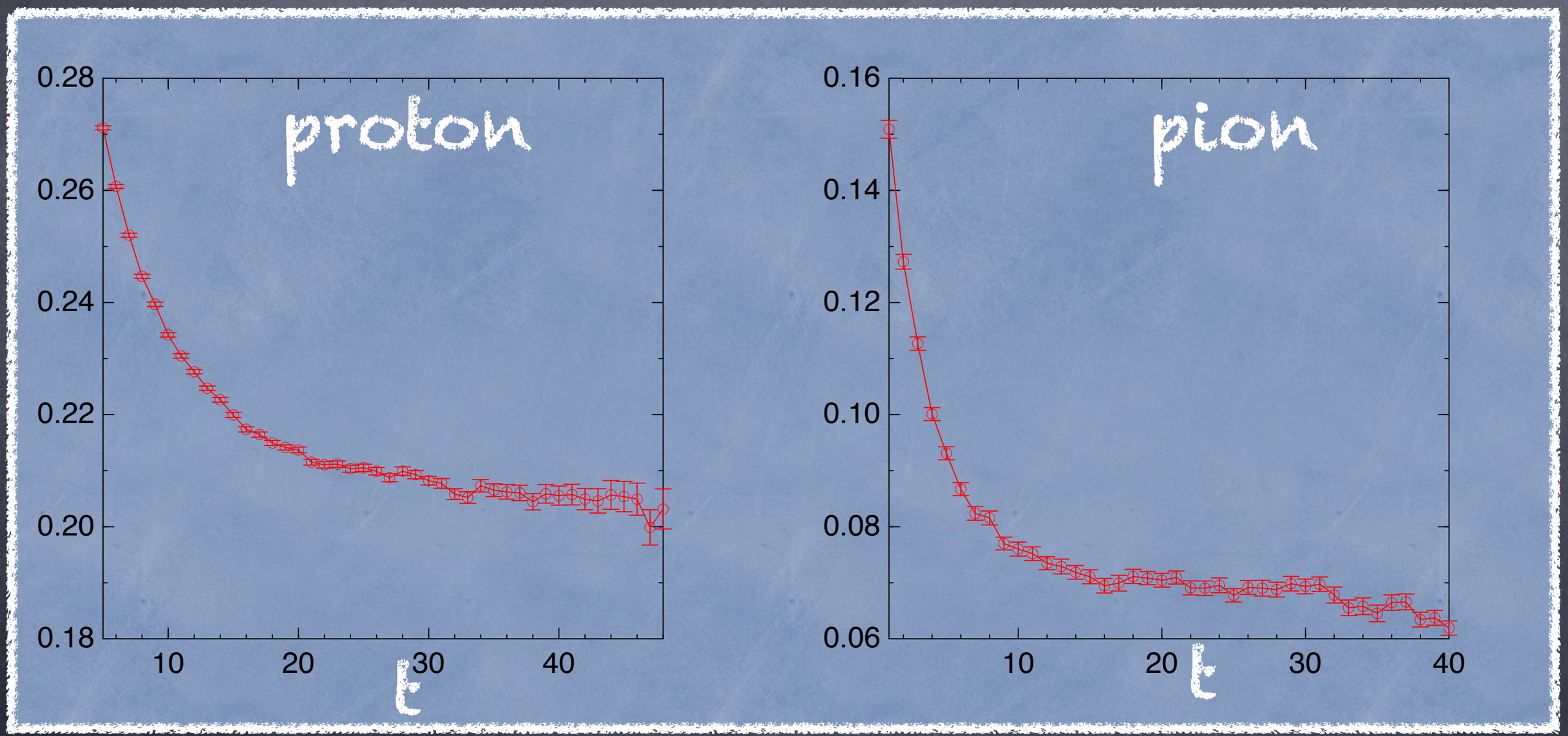


more sophisticated fitting methods are typically used

Lattice QCD Calculation

- Choose an action
  - 2+1 flavors of quarks: up down and strange
- Choose parameters
  - Bare gauge coupling the two bare quark masses
  - These correspond to the physical scales of the pion mass the kaon mass and the scale Λ_{qcd} that determines the "running" of the strong coupling constant
  - Lattice volumes
  - Lattice spacings to be used for continuum extrapolation

Spectrum-Confinement

Chiral Symmetry

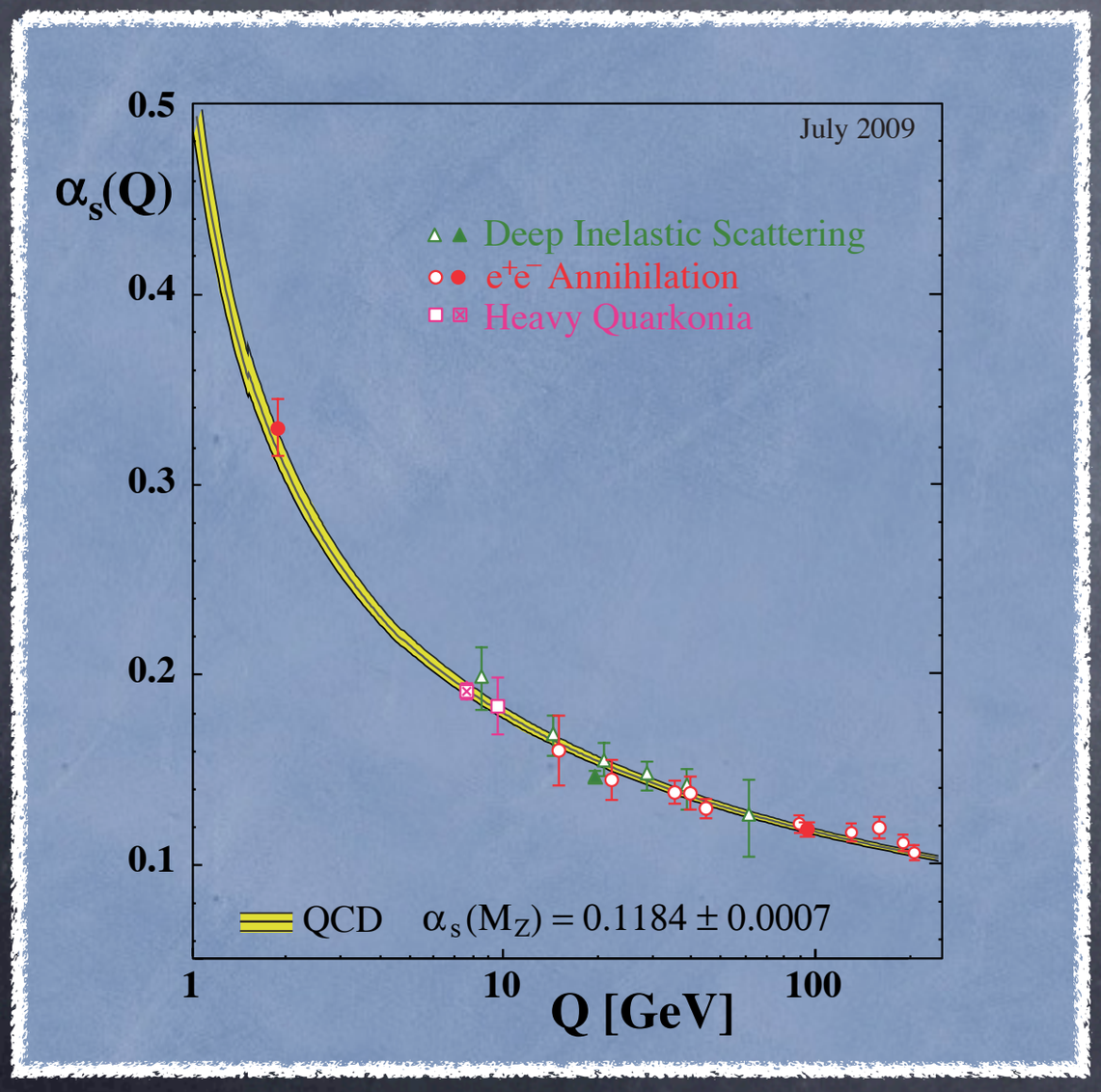
$SU(2)_L \times SU(2)_R$

$\downarrow$ $\langle \bar{q}q \rangle$

$SU(2)$

Δ	1232 MeV
N	940 MeV
ρ	770 MeV
K	490 MeV
π	140 MeV
	0 MeV

Asymptotic freedom



- Coupling constant changes with scale
- Coupling constant becomes weak at high energies
- Characteristic scale arises Λ_{qcd}
- $\alpha_s (\Lambda_{qcd}) \sim 1$

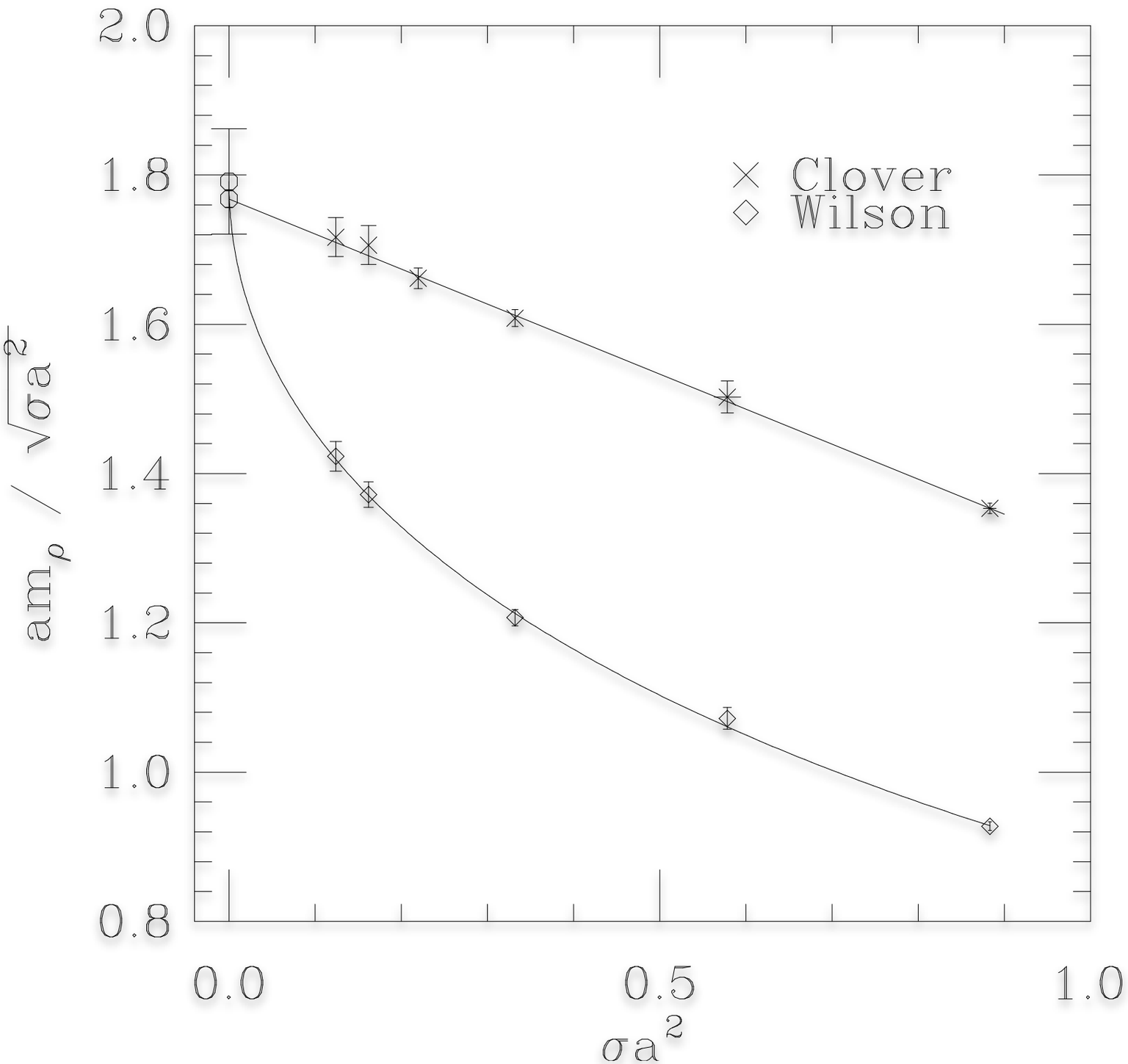
Lattice Action

K.G. Wilson Phys. Rev. D10 2445, 1974

Wilson action

$$S_f = \bar{\chi}\left(\sum_\mu \gamma_\mu \nabla_\mu(U) \; - \; r\,\Delta(U) + \; m\right)\chi$$

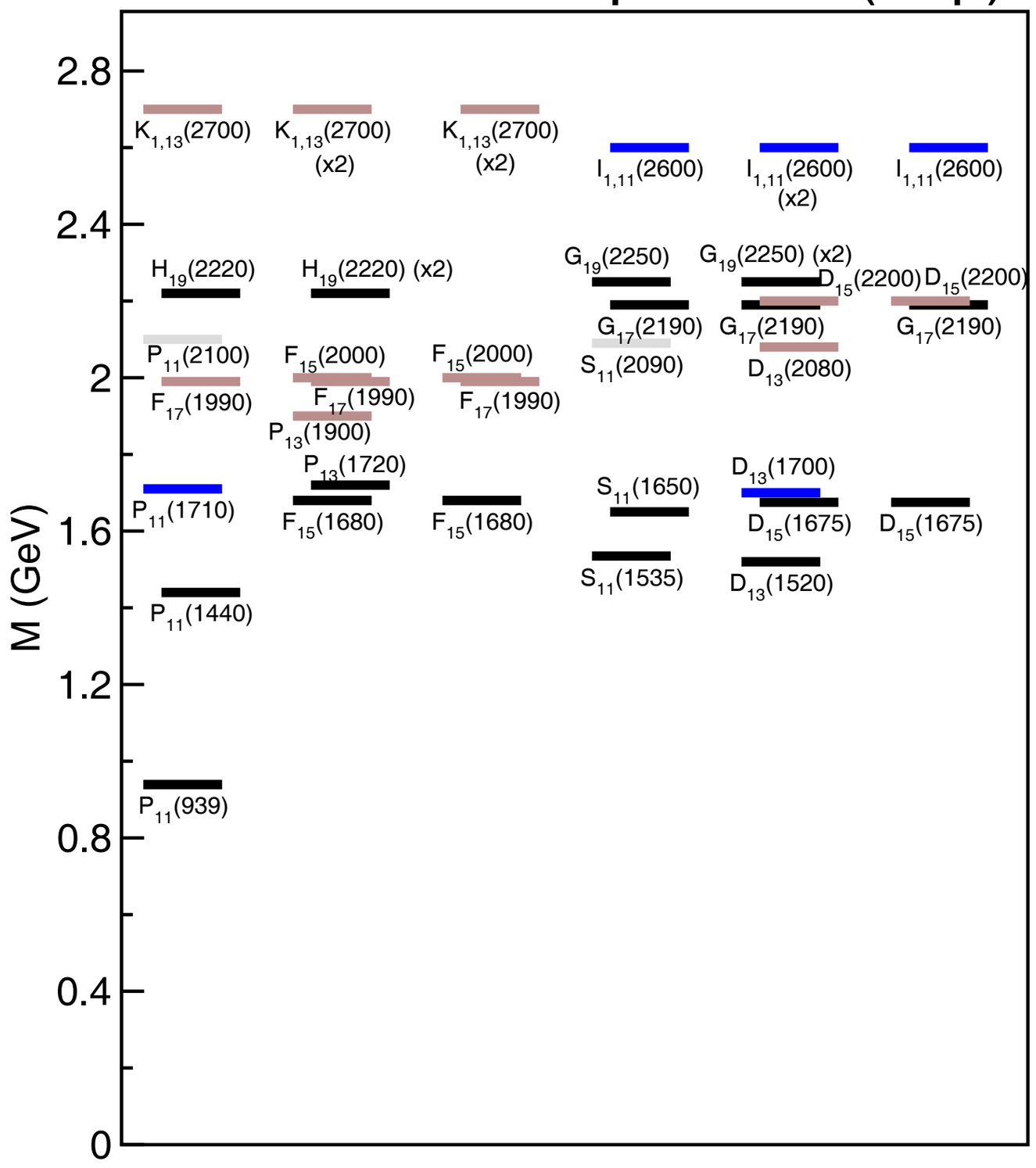
$$S_g = \frac{\beta}{3}\sum_n \sum_{\mu<\nu} Re\,\mathrm{Tr}\,(\mathbf{1} - P_{\mu\nu}(n))$$



Edwards et.al. '96

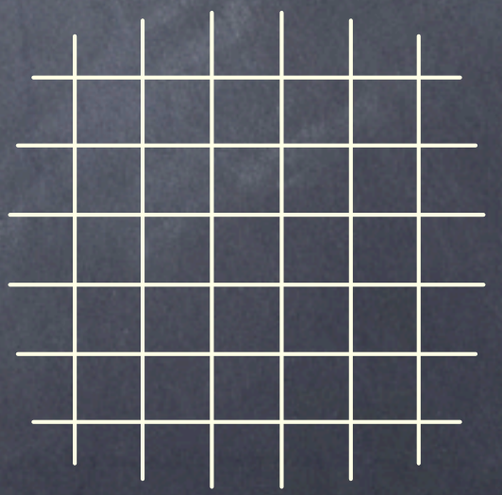
Scaling

Nucleon Mass Spectrum (Exp)

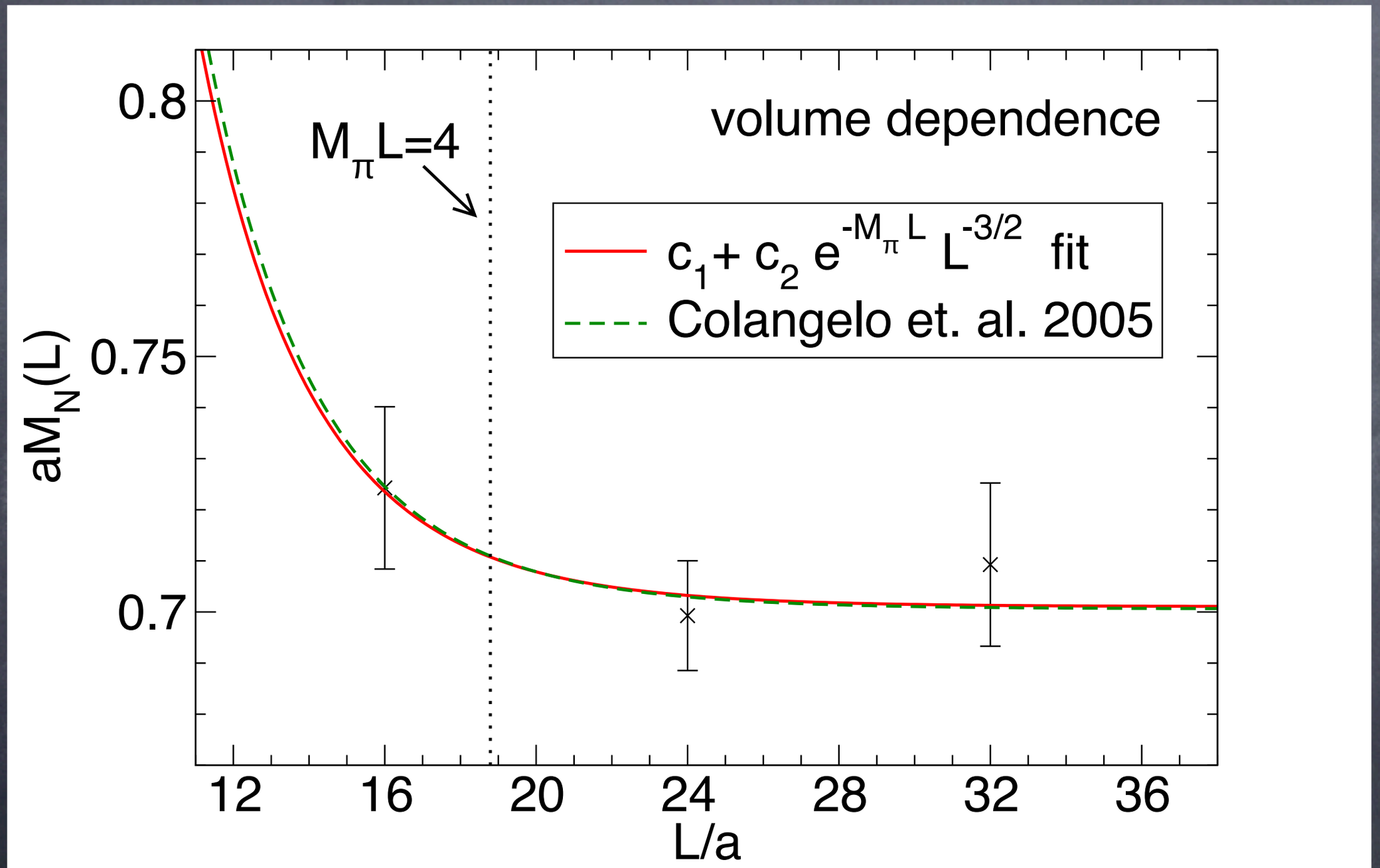


However...

- In lattice QCD we can easily compute the spectrum of the particles that do not decay strongly
- Simple resonances such as the ρ meson and the Δ baryon can be accessed
- Resonances with multiple decay channels are challenging
- The lattice has a cubic symmetry as opposed to full O(3) rotational symmetry
- Angular momentum is not a good quantum number on the lattice

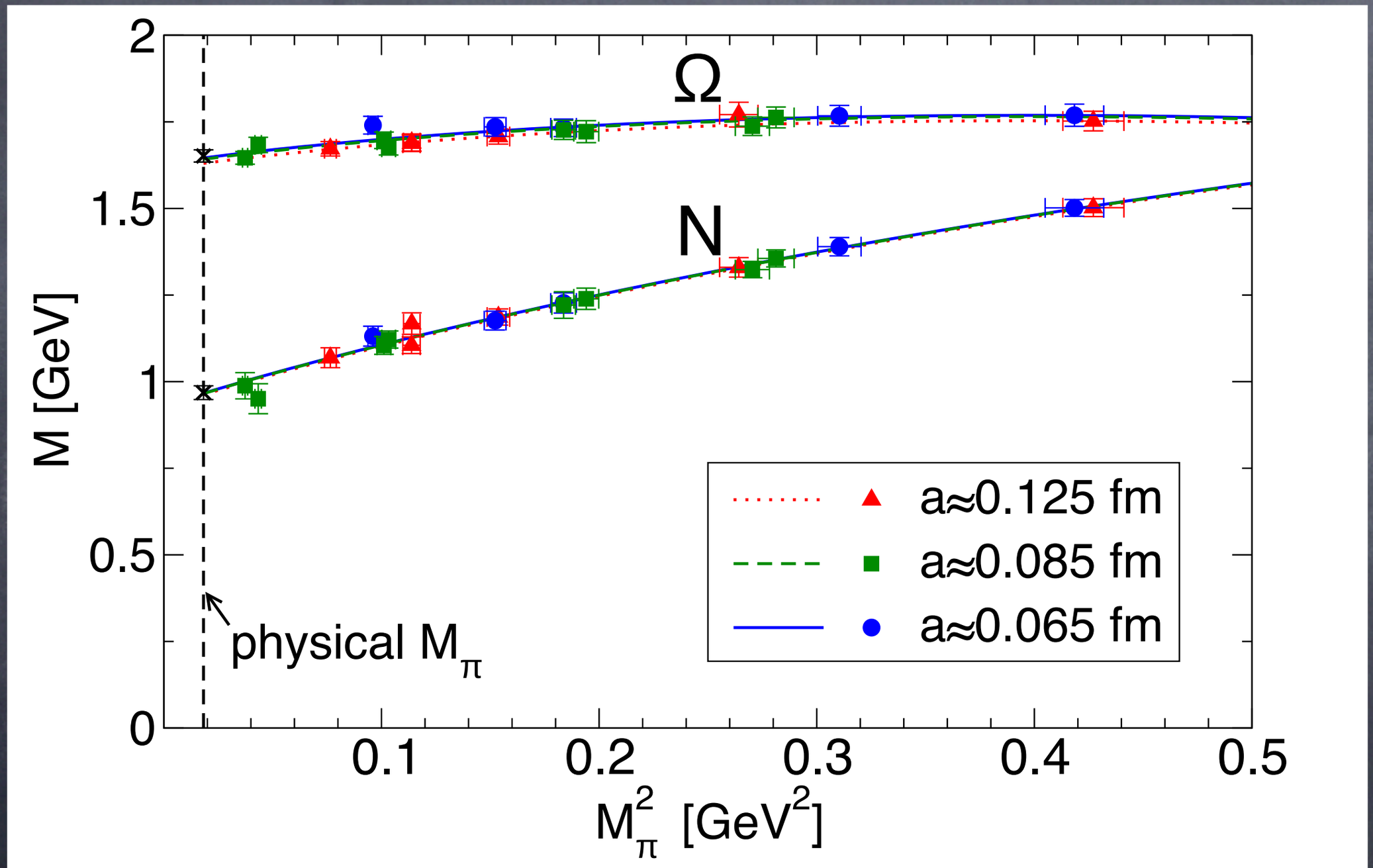


Finite volume corrections



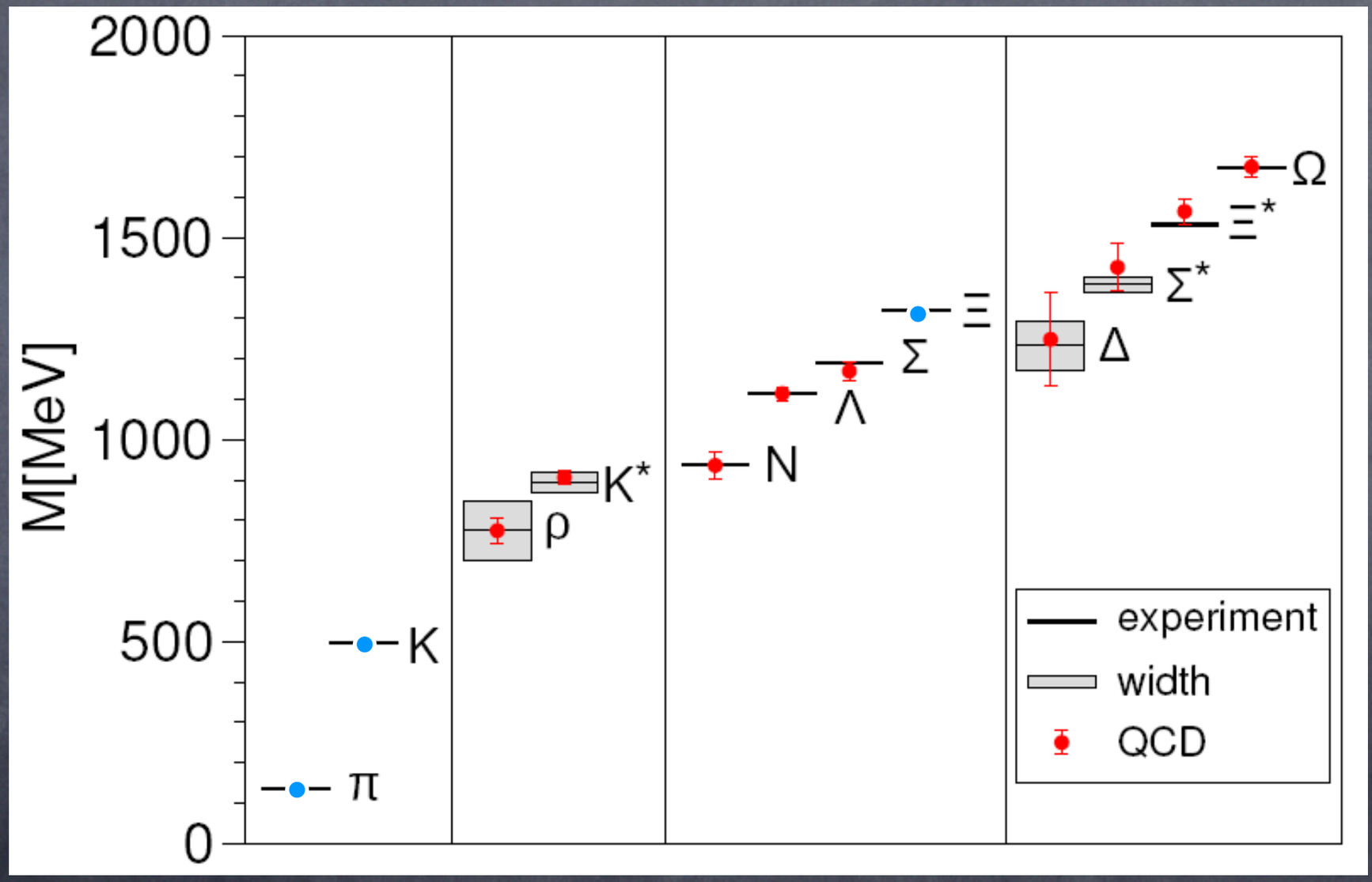
BMW collaboration, Science 322, 1224 (2008)

quark mass and continuum extrapolation



BMW collaboration, Science 322, 1224 (2008)

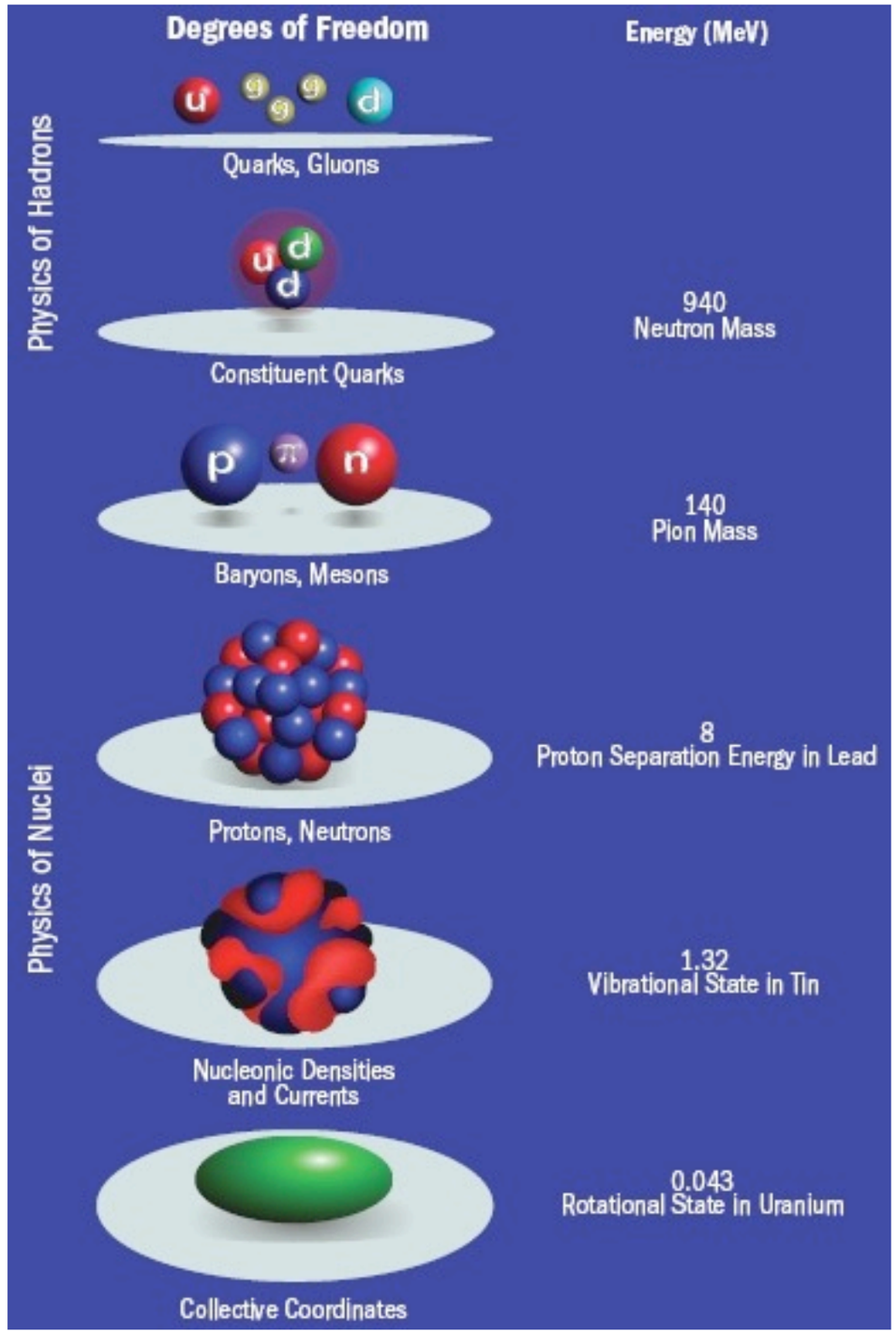
BMW spectrum results



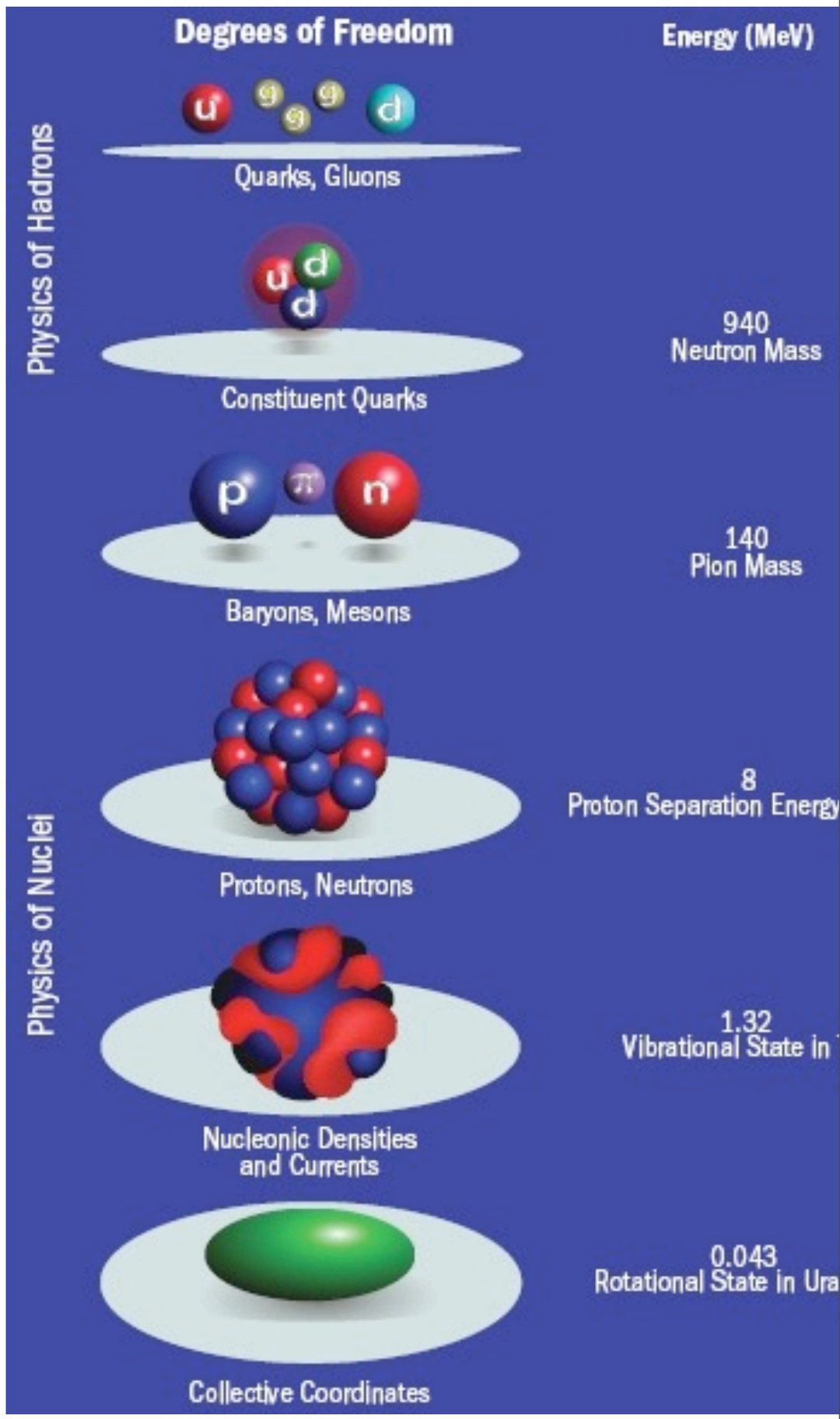
BMW collaboration, Science 322, 1224 (2008)

Hadron Structure and Interactions

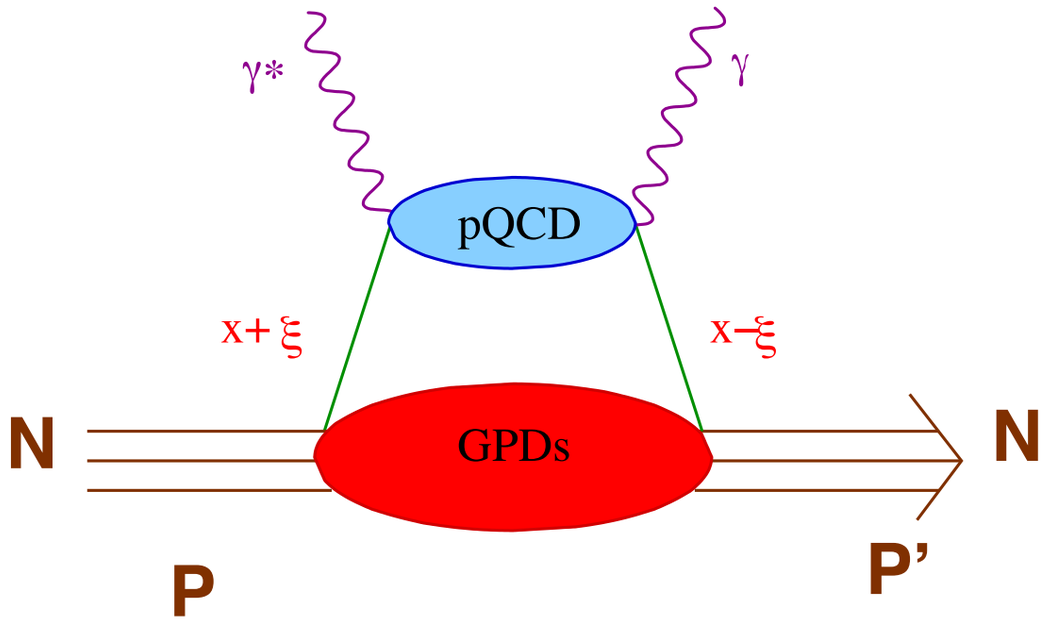
- Hadron Structure
  - Moments of structure functions
  - Form factors
  - Generalized Parton Distributions
  - Proton spin
- Hadronic interactions
  - Scattering lengths and phase shifts
  - Nuclear physics



Hadron Structure



Generalized Parton Distributions



$$\mathcal{O}_\Gamma(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \overline{q}(\frac{-\lambda n}{2}) \mathbf{\Gamma} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} q(\frac{\lambda n}{2})$$

$$\Gamma = \not{n} \text{ or } \Gamma = \not{n}\gamma_5 \text{ or } \Gamma = n_\mu \sigma^{\mu\nu}\gamma_5$$

$$\Delta = P' - P \qquad \xi = -n\cdot\Delta/2 \qquad t = \Delta^2$$

Vector: → $H(x,\xi,t)$ $E(x,\xi,t)$

Axial Vector: → $\tilde{H}(x,\xi,t)$ $\tilde{E}(x,\xi,t)$

Tensor: → $H_T(x,\xi,t)$ $E_T(x,\xi,t)$ $\tilde{H}_T(x,\xi,t)$ $\tilde{E}_T(x,\xi,t)$

GPDs: Unified picture of hadronic structure

Forward limit: t=0

$$H(x,0,0) = q(x)$$

$$\tilde{H}(x,0,0) = \Delta q(x)$$

$$H_T(x,0,0) = \delta q(x)$$

Local limit

Vector	$\int dx H(x,\xi,t) = F_1(t)$	$\int dx E(x,\xi,t) = F_2(t)$
Axial Vector	$\int dx \tilde{H}(x,\xi,t) = g_A(t)$	$\int dx \tilde{E}(x,\xi,t) = g_P(t)$
Tensor	$\int dx H_T(x,\xi,t) = g_T(t)$	

Moments of GPDs

Operator Product Expansion

Off forward Matrix elements of local operators $\langle P,S|\mathcal{O}|P',S'\rangle$

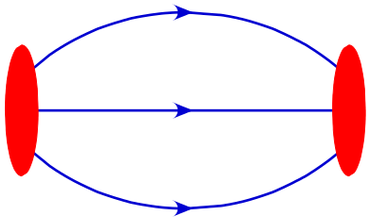
Unpolarized $$\mathcal{O}^{q}_{\{\mu_1\mu_2\cdots\mu_n\}} = \overline{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - trace\right]q$$

Polarized $$\mathcal{O}^{5q}_{\{\mu_1\mu_2\cdots\mu_n\}} = \overline{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_5\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - trace\right]q$$

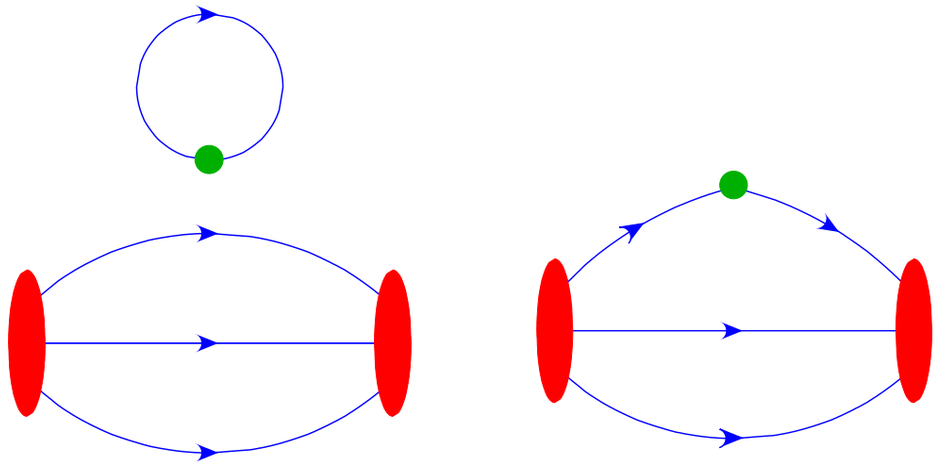
Transversity $$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \overline{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - traces]q$$

- Generalized form factors: $A_{nk}(t)$ $B_{nk}(t)$ $C_{nk}(t)$
- Moments of GPDs are polynomials in ξ with coefficients the generalized form factors.
- Breaking of rotational symmetry: Mixing with lower dimensional operators
  - Only first few moments can be computed on the lattice
- Perturbative renormalization has been used extensively
- Non-perturbative (Rome-Southampton MOM) is used

Lattice Technology

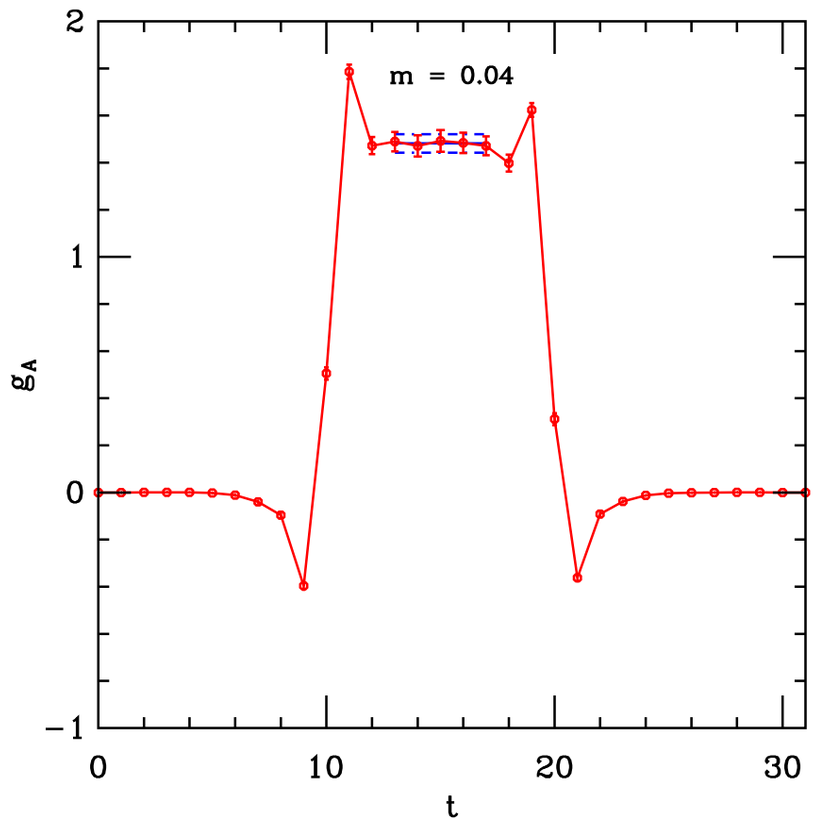


$C_{2pt}(\vec{p},t) = \langle J_{\vec{p}}(t) J(0) \rangle$

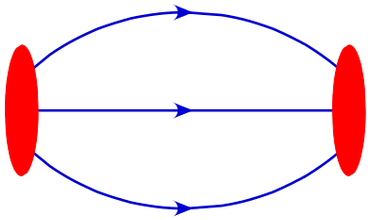


$C_{3pt}(\vec{p},\vec{q};t,\tau) = \langle J_{\vec{p}}(t) \mathcal{O}(\vec{q},\tau) J(0) \rangle$

- Typical matrix element computation
  - Ratios of two and three point functions
- Disconnected diagrams usually ignored
- Isovector quantities are easier
- Dynamical fermions (2 and 2+1 flavors) are now the standard



Lattice Technology



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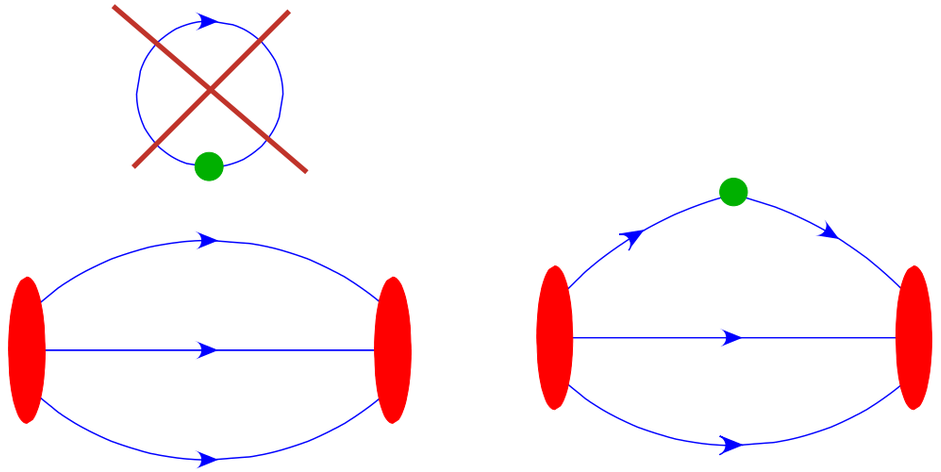

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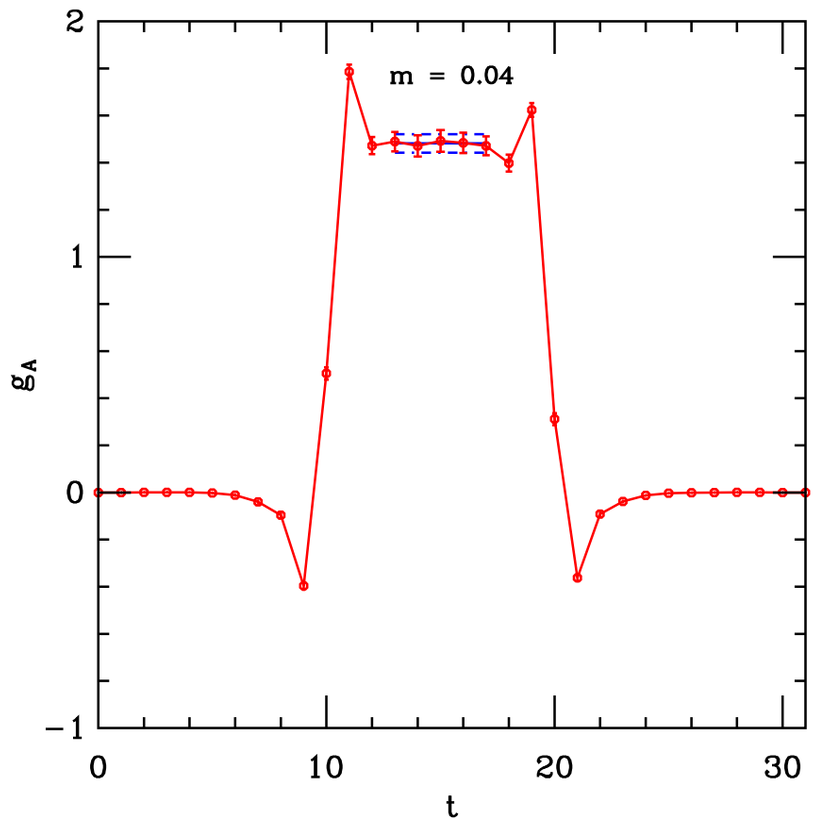

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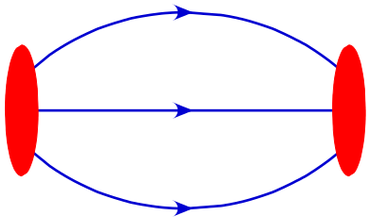

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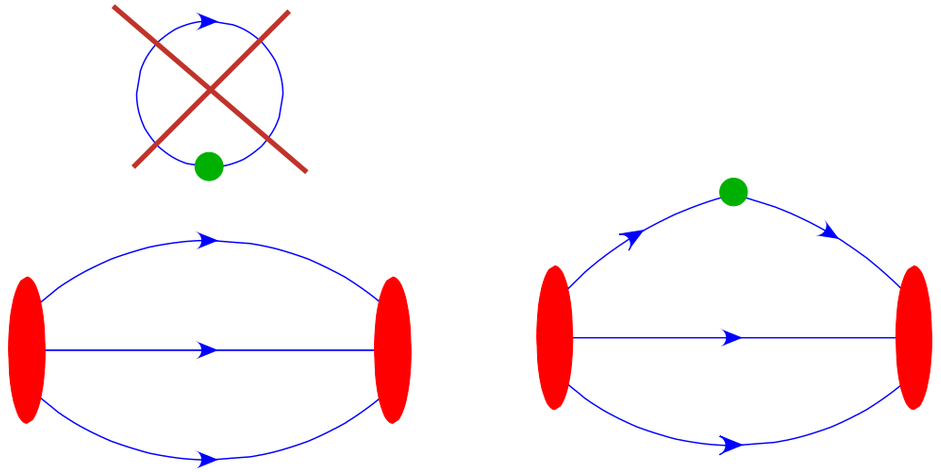

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  Recent activity [Deka Lat '06-'07]
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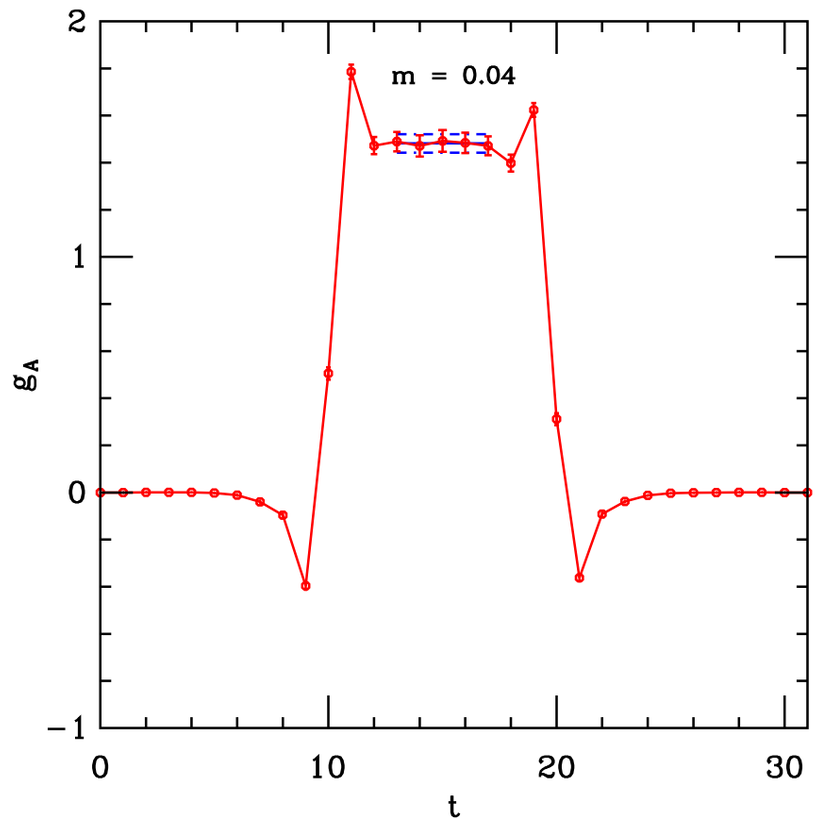


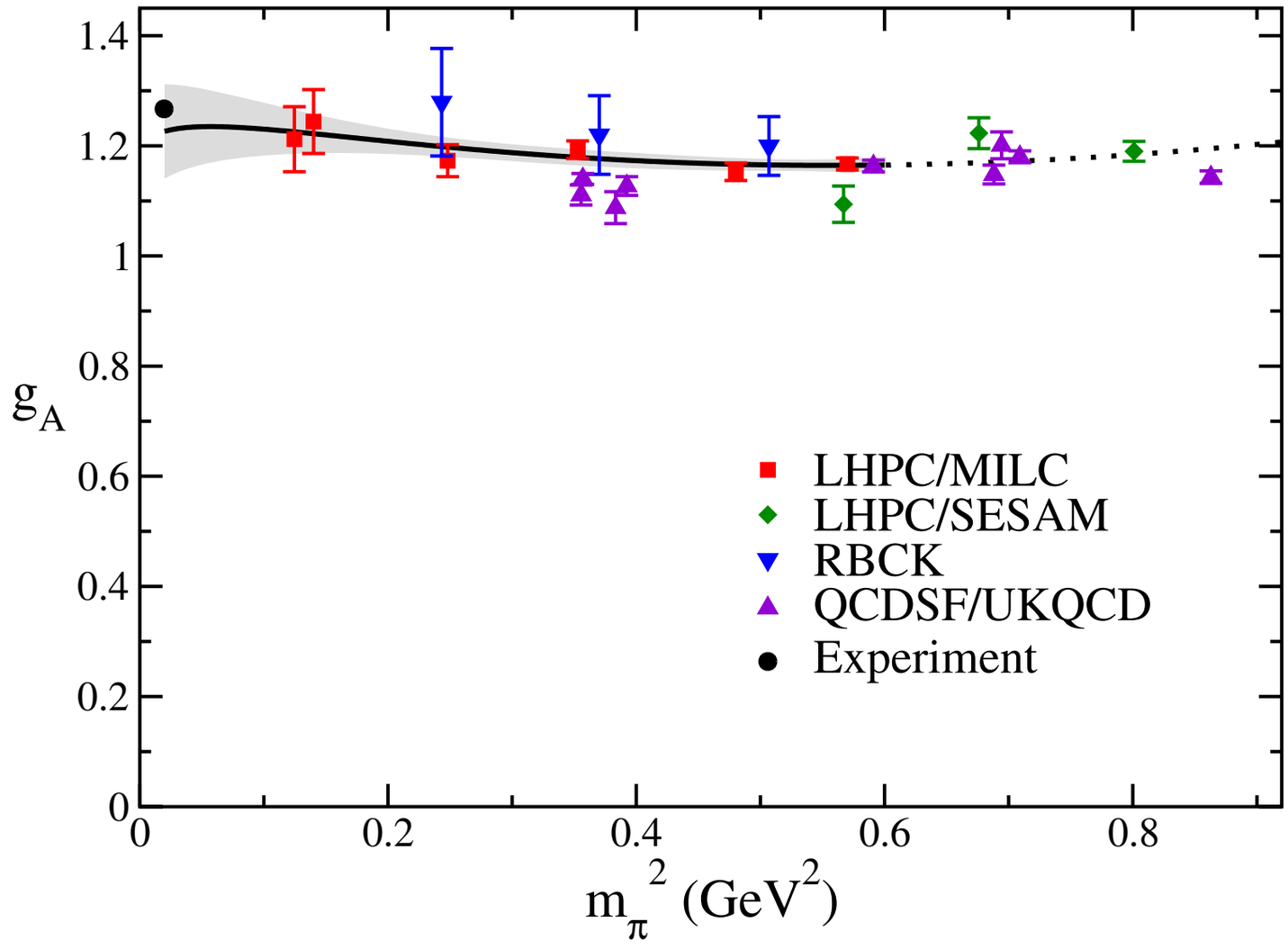
The Axial coupling

- Accurately known (neutron beta decay)

$$g_A / g_V = 1.2695(29) \quad \text{[PDG2006]}$$

- Lattice calculations had trouble computing it
- Charge symmetry breaking effects not taken into account
- ChiPT results:
  - Finite volume: [Bean, Savage '04]
  - Small Scale expansion: [T. R. Hemmert, M. Procura, and W. Weise '03]
  - Partial NNLO: [Bernard, Veronique and Meissner' 06]

Axial Coupling



- Large volumes

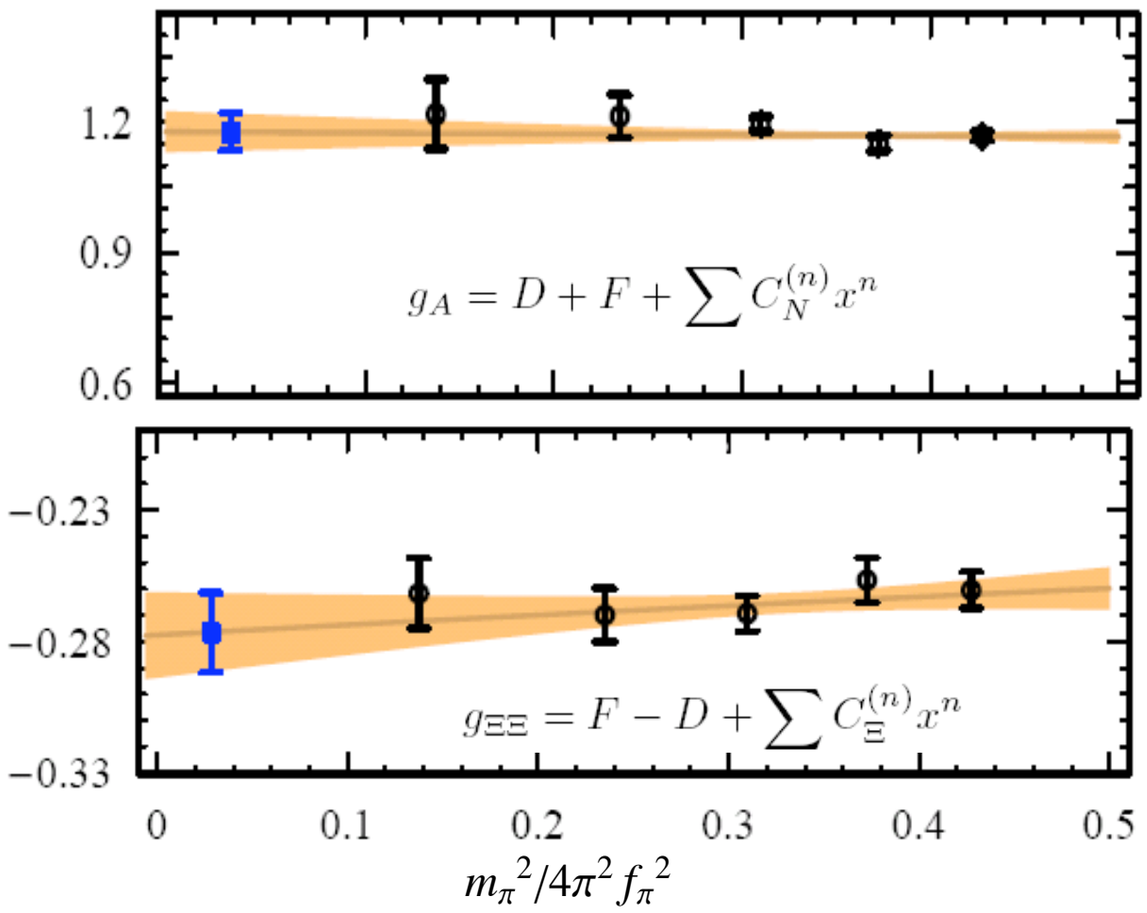
Cost: ~ 1Tflop-year

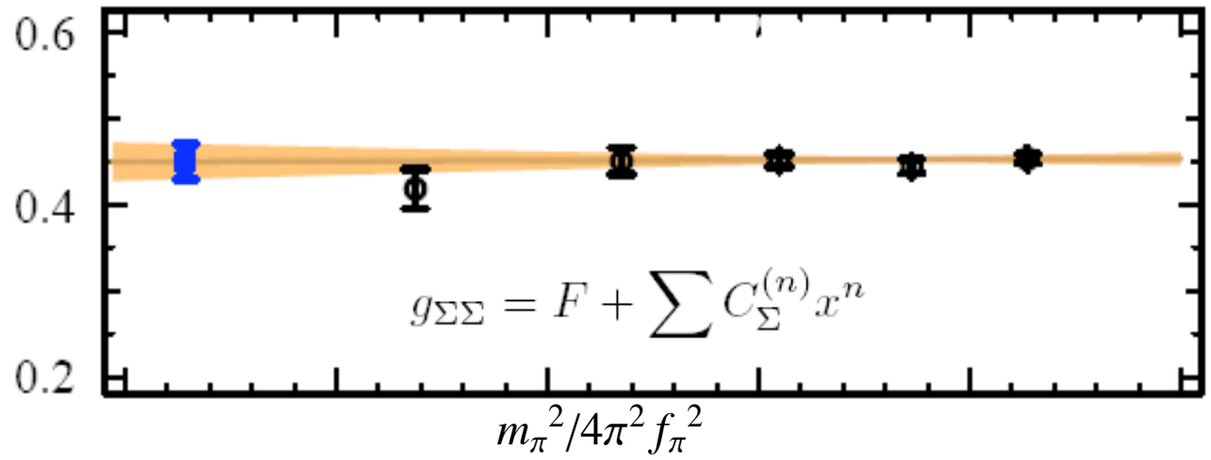
- Lighter pion masses

- $g_A(m_\pi=140\text{MeV}) =$ 1.23(8)

Octet Axial couplings

H-W. Lin, KO arXiv:0712.1214





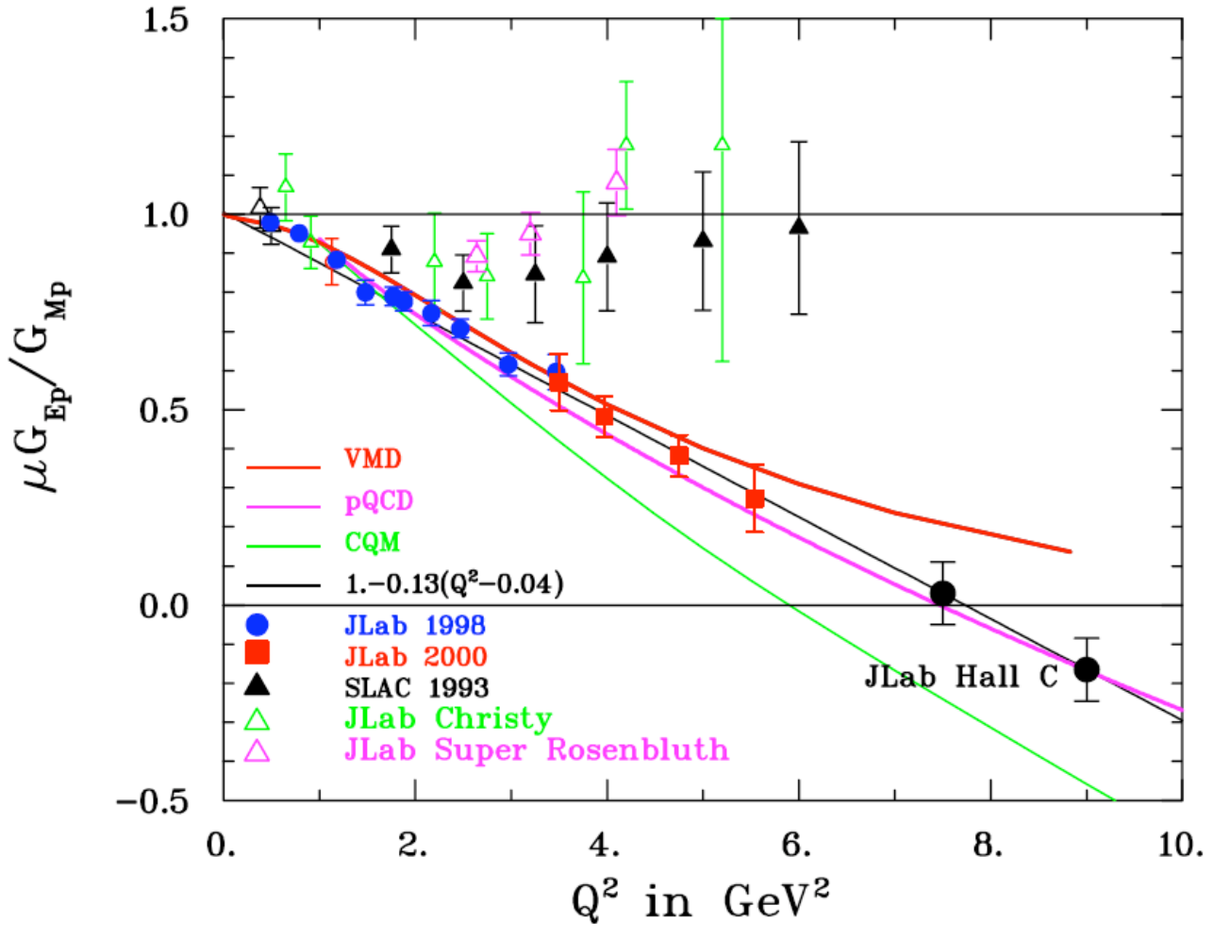
$$
\begin{aligned}
g_A &= 1.18(4)_{\rm stat}(6)_{\rm syst} \\
g_{\Sigma\Sigma} &= 0.450(21)_{\rm stat}(27)_{\rm syst} \\
g_{\Xi\Xi} &= -0.277(15)_{\rm stat}(19)_{\rm syst} \\
D &= 0.715(6)_{\rm stat}(29)_{\rm syst} \\
F &= 0.453(5)_{\rm stat}(19)_{\rm syst}
\end{aligned}
$$

- Large Nc: $0.26<g_{\Xi\Xi}<0.30$ ChiPT: $0.18<g_{\Xi\Xi}<0.36$
- ChiPT: $0.30<g_{\Sigma\Sigma}<0.55$
- Simultaneous HBChiPT fit fails do describe the data
- Use simple SU(3) inspired chiral form

$x = (m_K^2 - m_\pi^2)/(4\pi^2 f_\pi^2)$
is the SU(3) breaking parameter

Proton ElectroMagnetic Form Factors

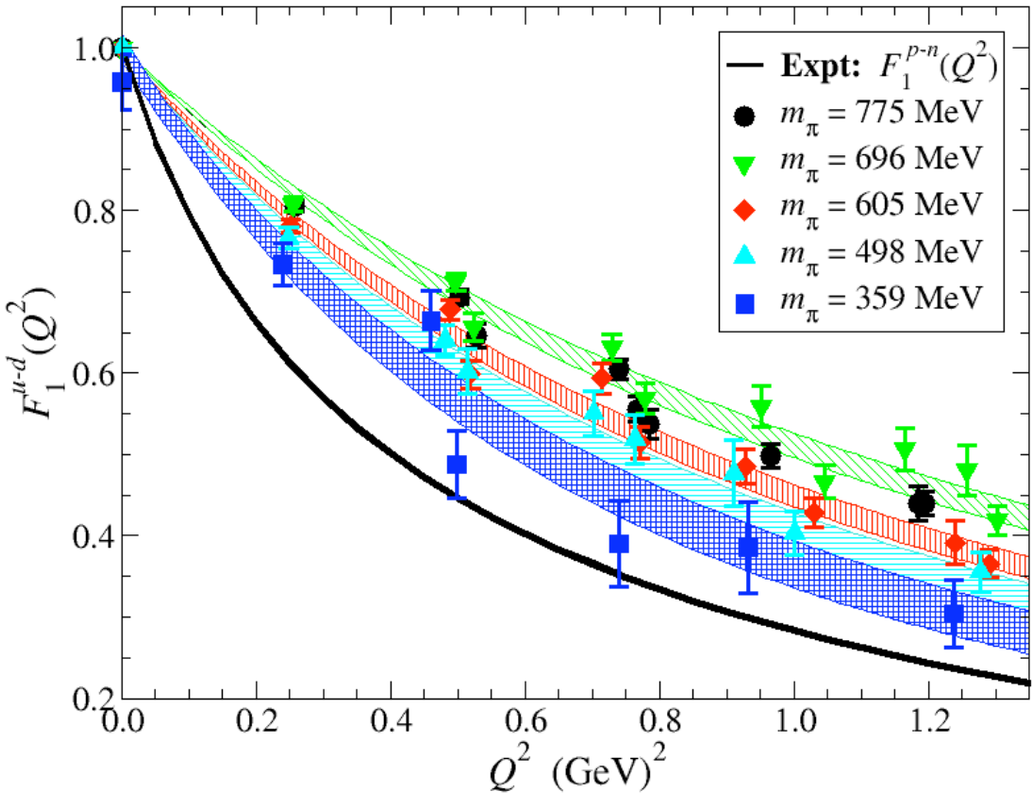
- Rosenbluth separation disagrees with Polarization transfer
- Two photon effects
- What does the lattice predict for the ratio?

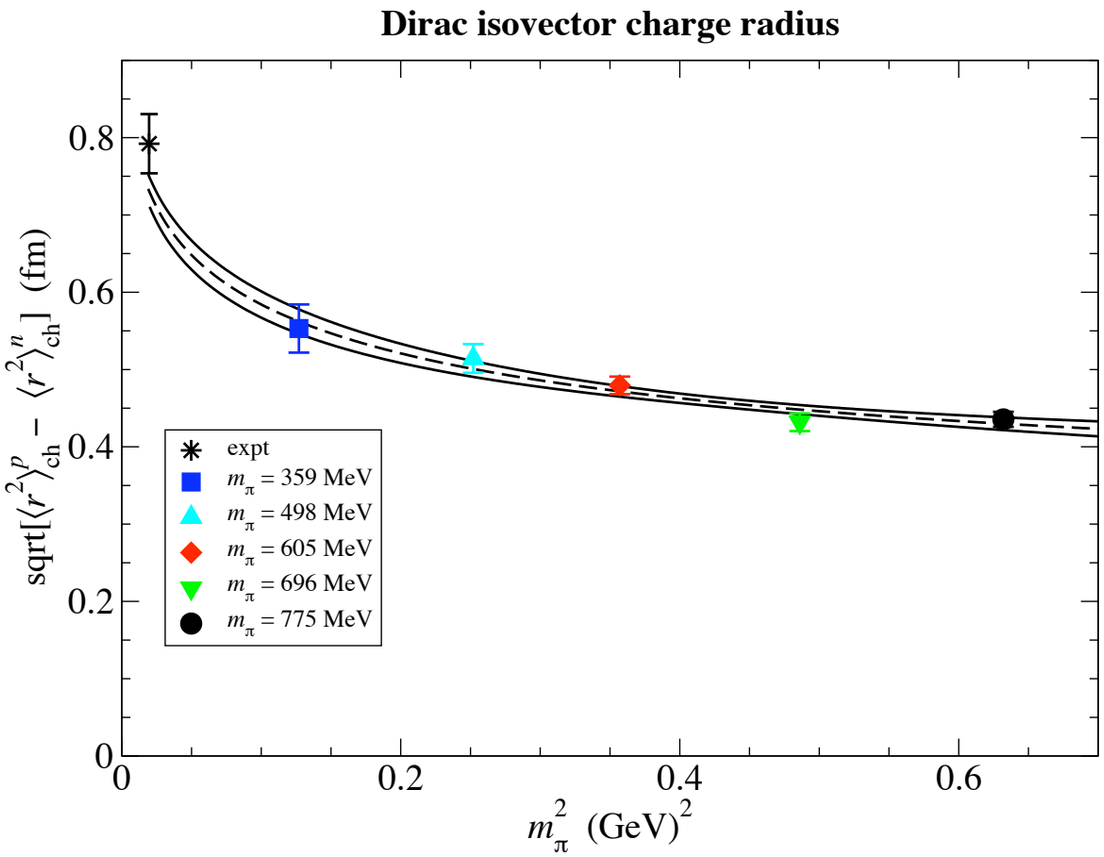


Perdristat: Jlab Users Meeting 2005

Vector Isovector form factor

[LHPC]





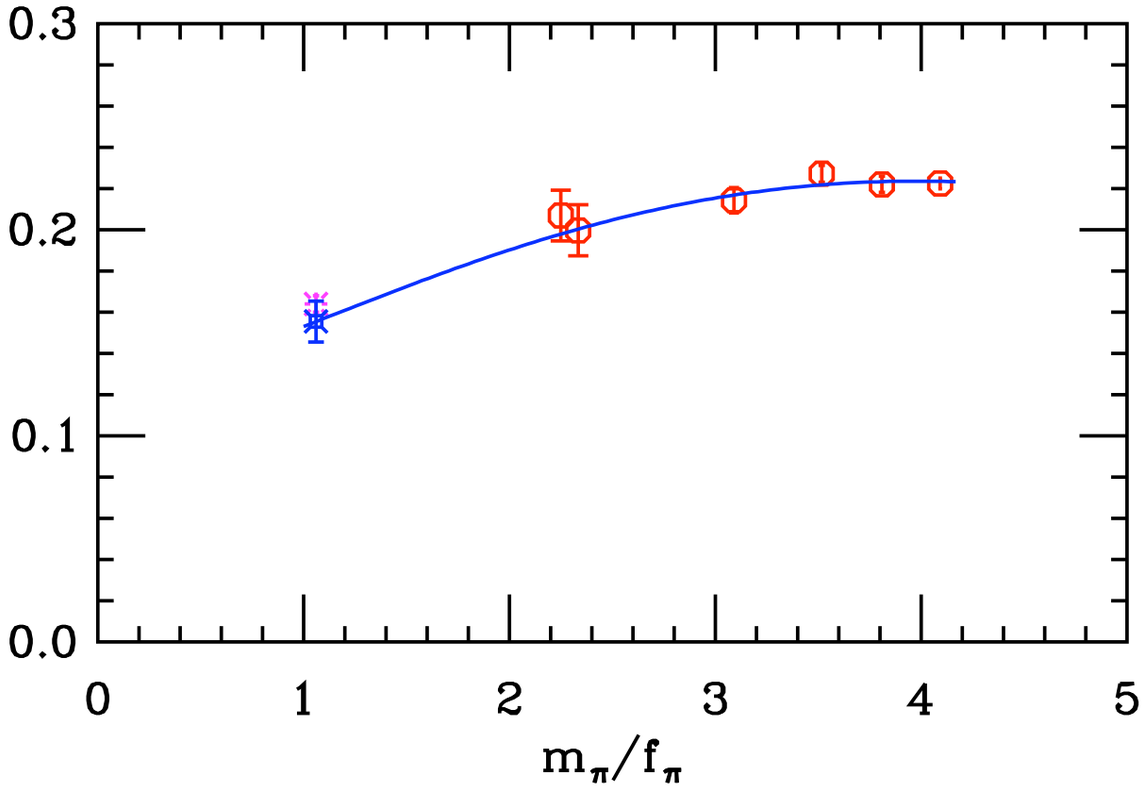
- Light pion masses: Lattice approaches experiment
- Chiral fit using leading order ChiPT [Leinweber et.al. '01]
- Nucleon size grows as quarks become lighter

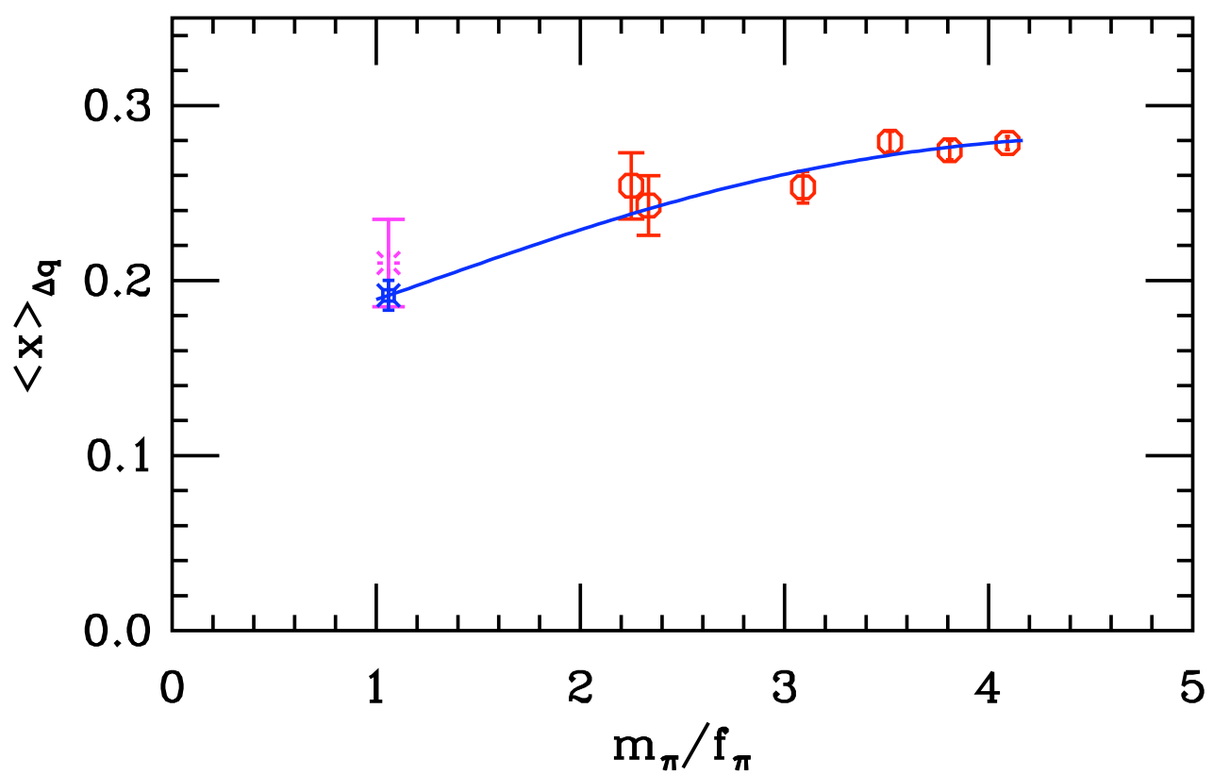
[G. T. Fleming]

First Moments of Structure functions [LHPC]

Domain Wall Valence, Staggered Sea (MILC)

$\mu = 2$ GeV





- Hint of curvature just below 400MeV [KO Lattice '06]
- Isovector moments only
- Fit to leading ChiPT keeping the mass dependence of g_A
- Perturbative renormalization [Bistrovic '05 (thesis)]

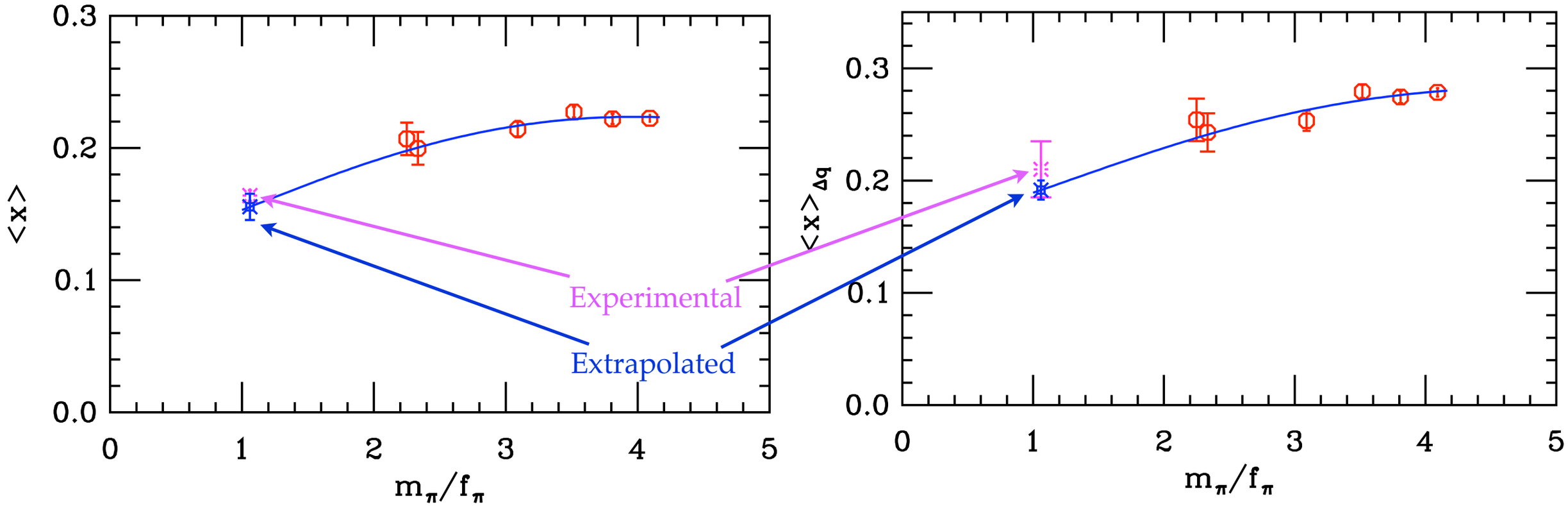
$$\langle x \rangle_{u-d} = a\left[1 - \frac{3g_A^2+1}{8\pi^2}\left(\frac{m_\pi^2}{f_\pi^2}\right)\ln\left(\frac{m_\pi^2}{f_\pi^2}\right)\right] + c\frac{m_\pi^2}{f_\pi^2}$$

$$\langle x \rangle_{\Delta u-\Delta d} = a'\left[1 - \frac{2g_A^2+1}{8\pi^2}\left(\frac{m_\pi^2}{f_\pi^2}\right)\ln\left(\frac{m_\pi^2}{f_\pi^2}\right)\right] + c'\frac{m_\pi^2}{f_\pi^2}$$

First Moments of Structure functions [LHPC]

Domain Wall Valence, Staggered Sea (MILC)

$\mu = 2$ GeV



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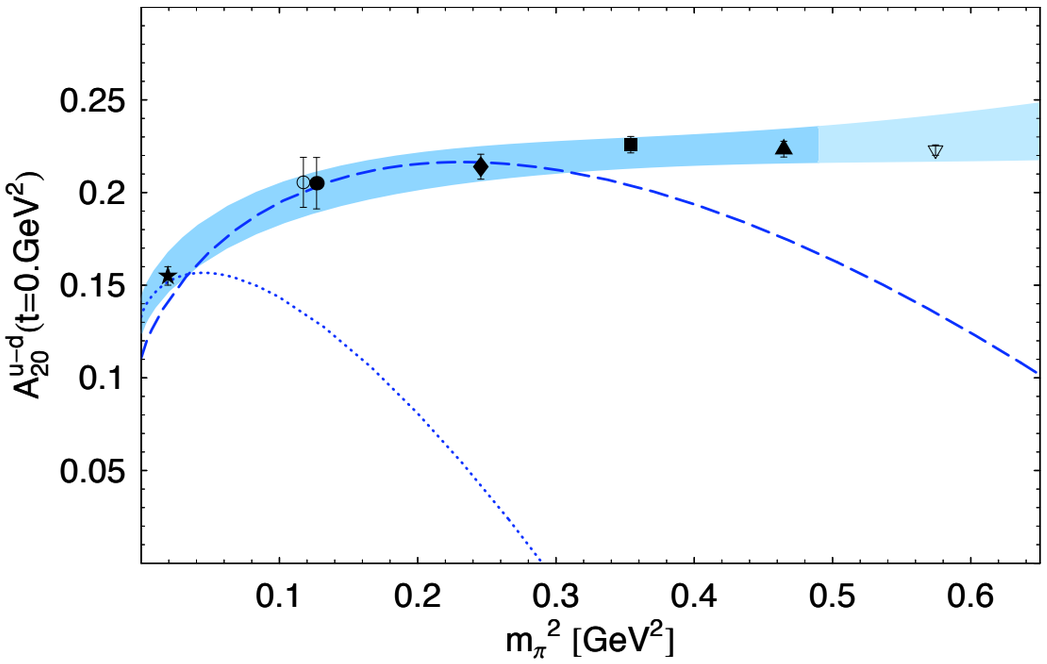
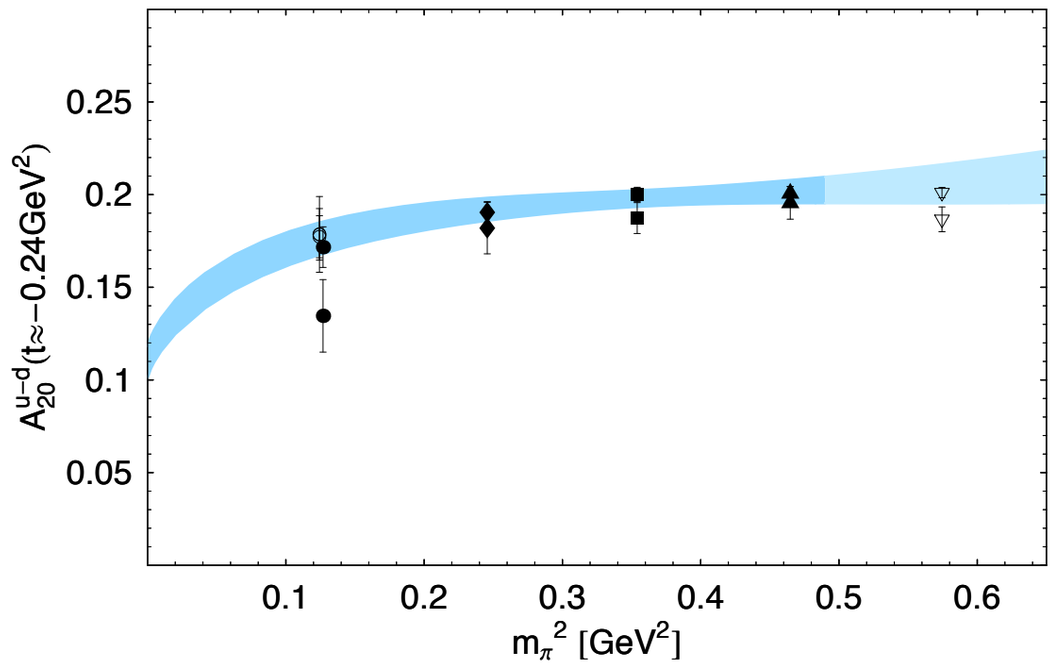
Generalized Form factor fits

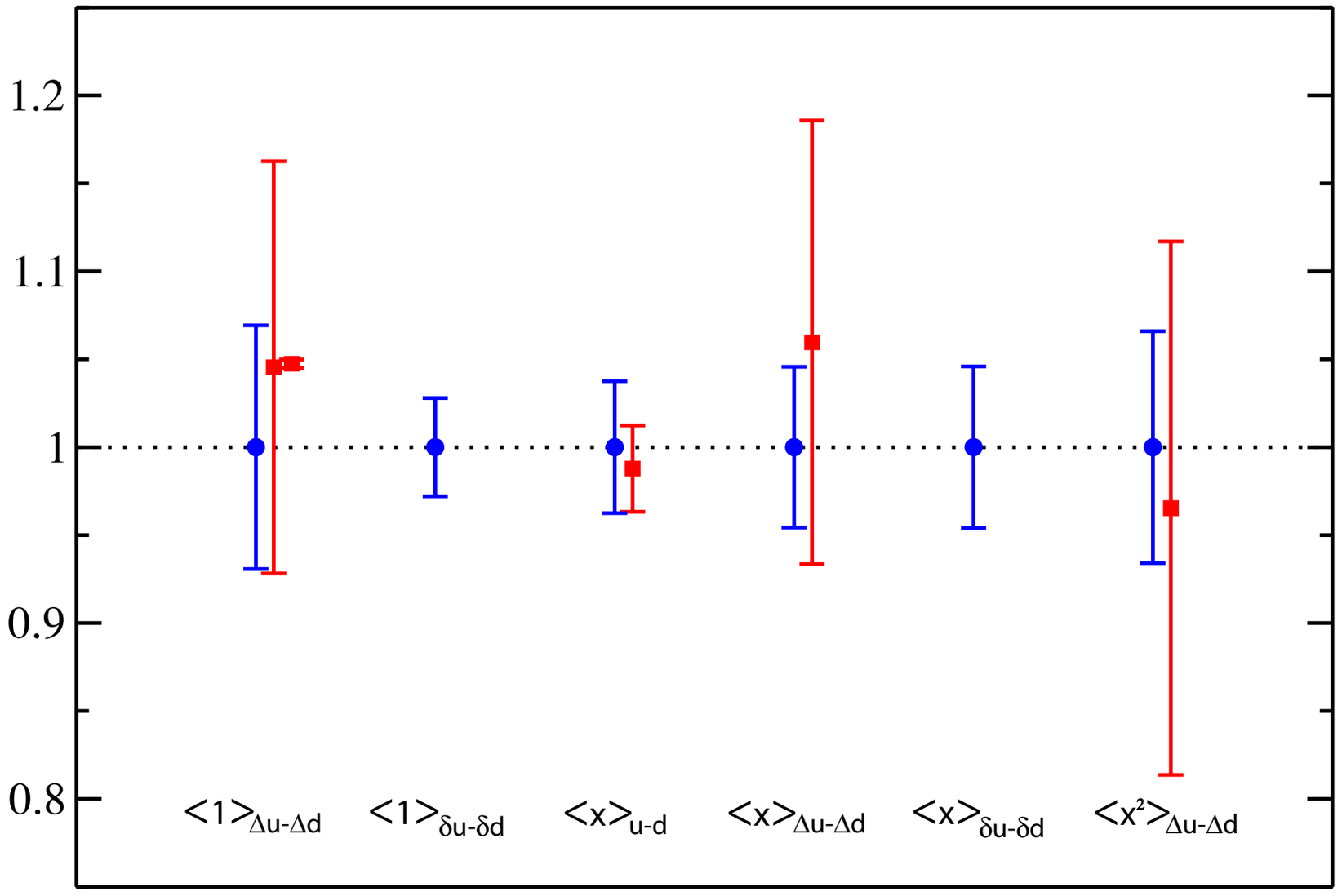
LHPC: Phys. Rev. D 77, 094502 (2008)





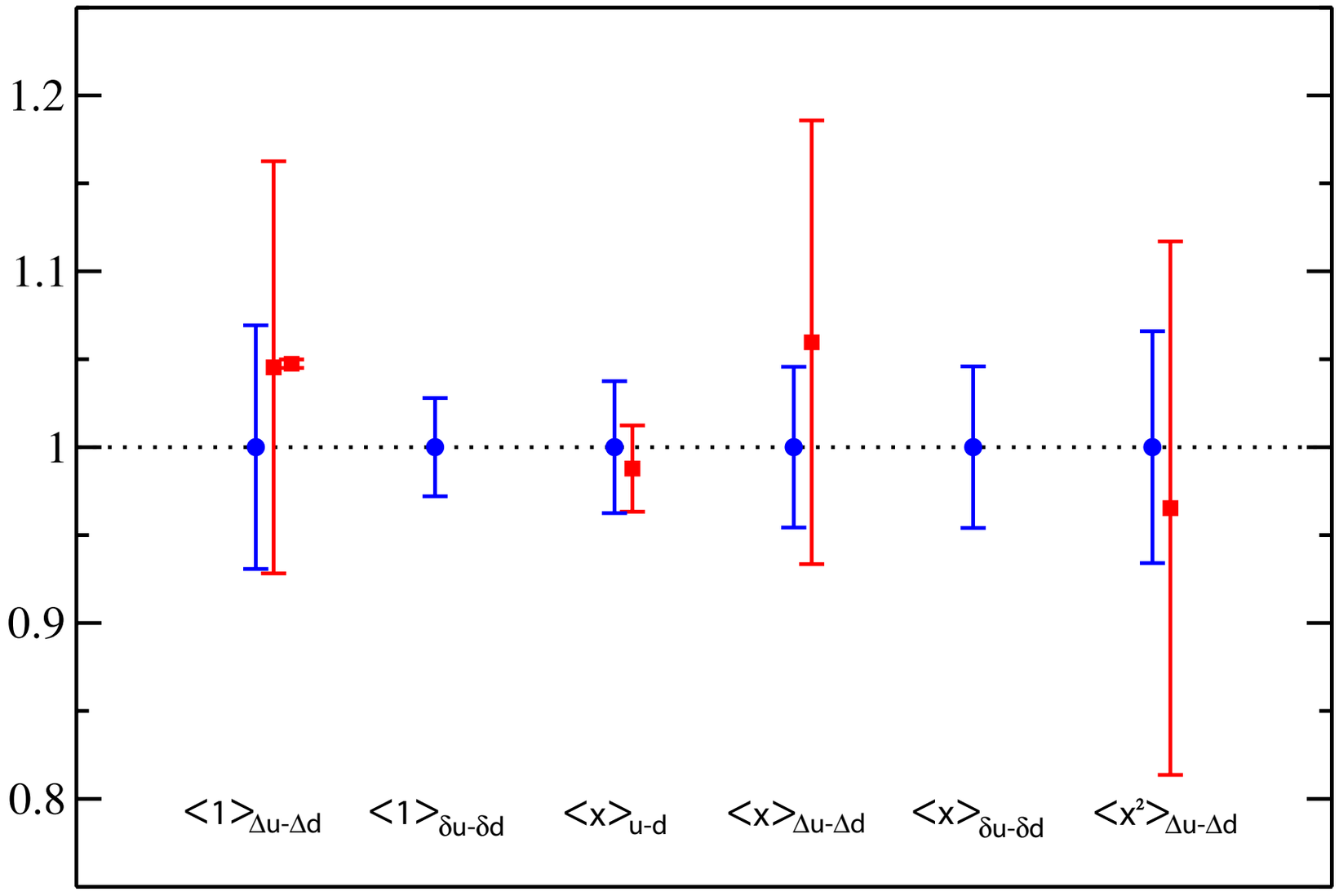
- Chiral extrapolations using CBChiPT [M. Dorati et.al. Nucl. Phys. A798, 96 (2008)]
- Results consistent with improved HBChiPT

Moments of Structure functions
Experiment vs LQCD



[Renner Lattice '06]

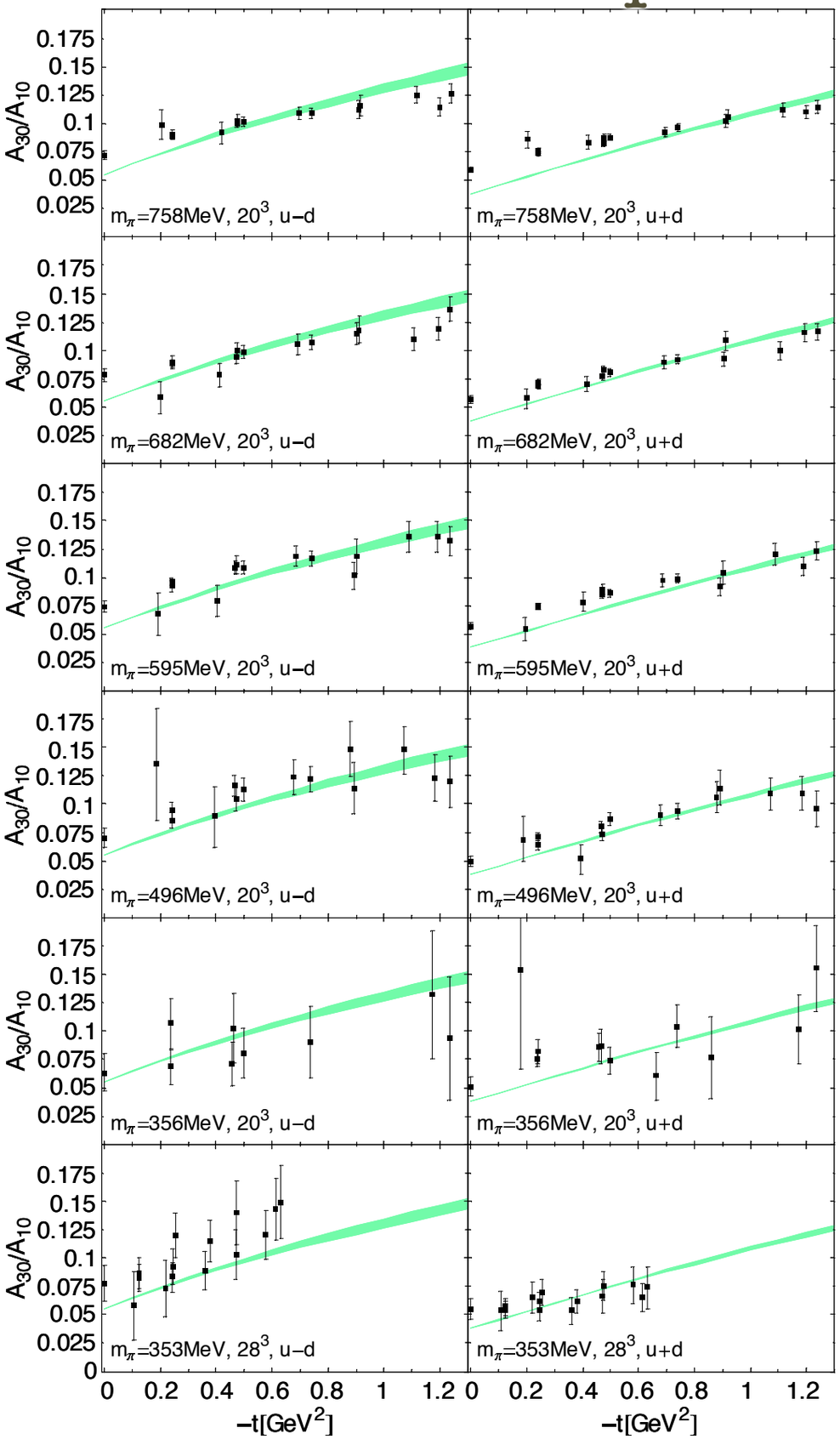
Moments of Structure functions
Experiment vs LQCD



Need to include the Delta in the fits [Arndt and Savage, '02]

[Renner Lattice '06]

Comparison to phenomenology



Unpolarized

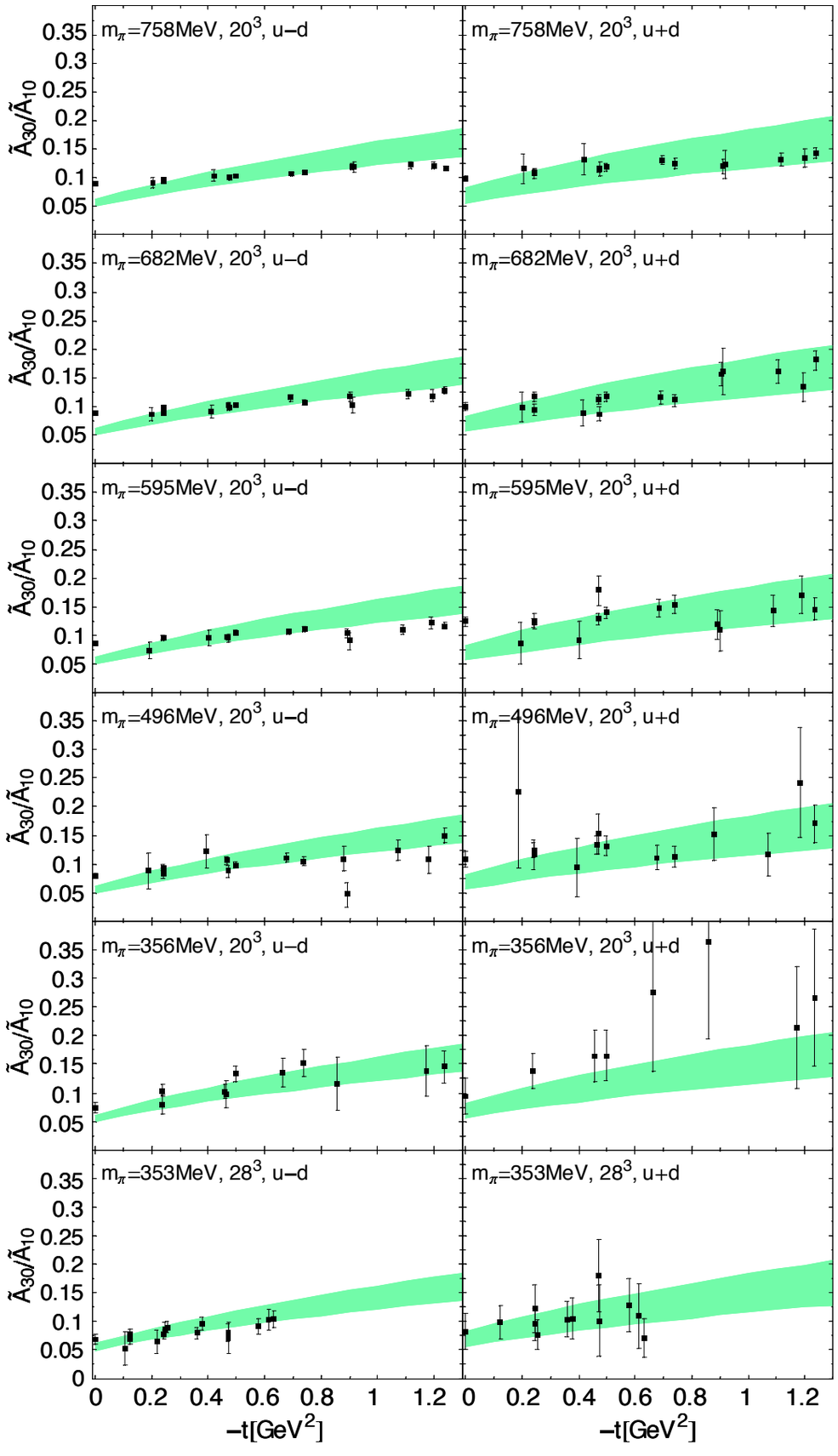
- Green band is a phenomenological parameterization using Form factor data, CTEQ patron distributions, and Regge Anzatz as input

  [Deihl et.al. hep-ph/048173]

- As pion mass becomes smaller agreement is better.

LHPC: Phys. Rev. D 77, 094502 (2008)
hep-lat/0705.4295

Comparison to phenomenology



Polarized

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The Proton Spin

- What is the contribution of the quark spin to the spin of the proton?
  - EMC (1988) $Q^2 = 10\ GeV^2$ $\Delta\Sigma = 0.00(24)$
  - SMC (1998) $Q^2 = 5\ GeV^2$ $\Delta\Sigma = 0.13(17)$
- Quarks contribute nothing to the spin of the proton!
- $\Delta\Sigma$ should be 1 if all the proton spin is due to the spin of the quarks
- Spin crisis

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

The Proton Spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_z \quad \text{or} \quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_z^q + J_z^g$$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

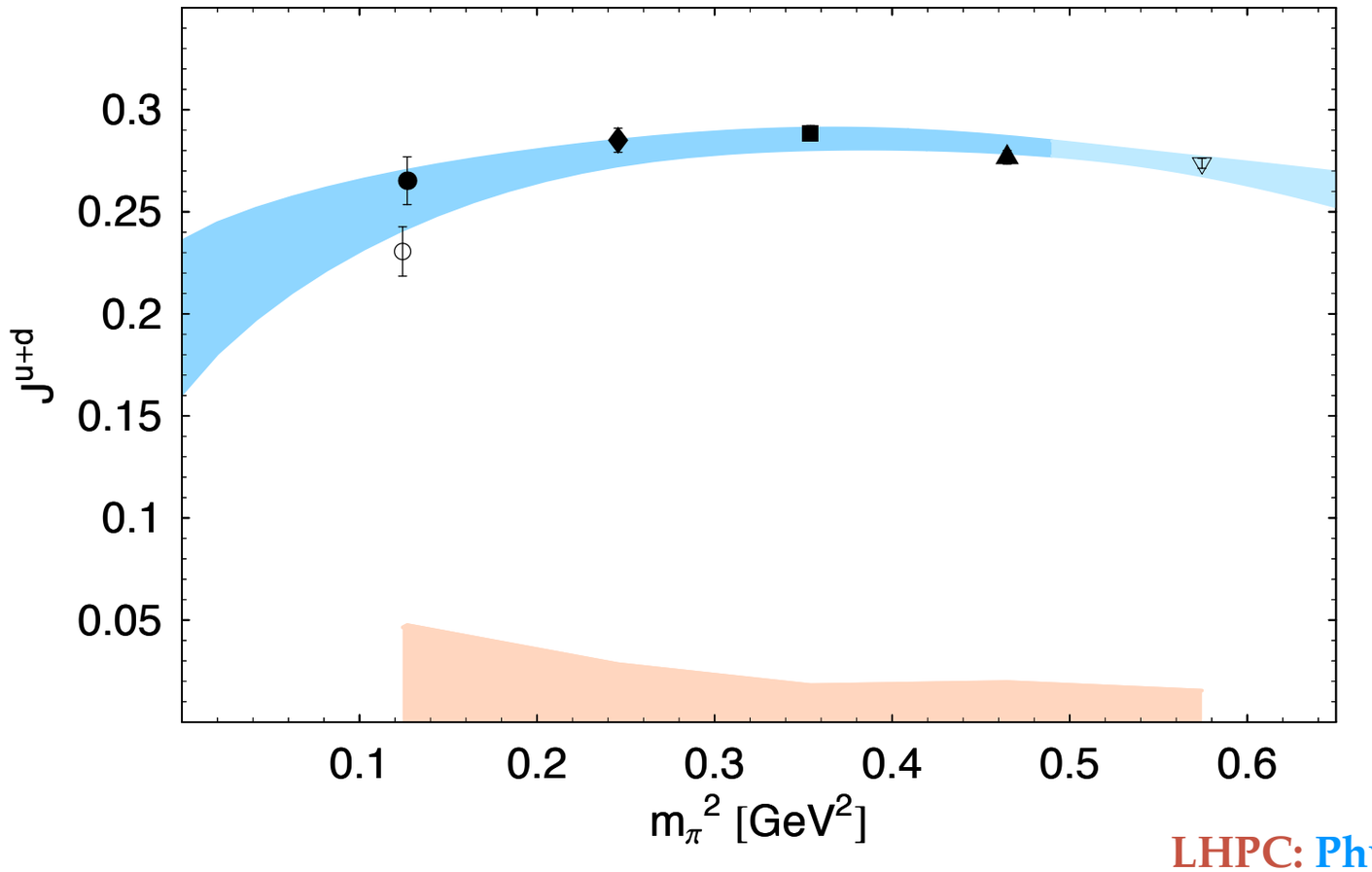
- The quark angular momentum can be computed on the lattice
- $$J_z^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$$ [Ji,'98]
- A and B are the generalized form factors of

$$\mathcal{O}_{\mu\nu} = \bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\, q = T_{\mu\nu}$$

[Mathur et. al. '00] [LHPC: Hagler et. al. '03] [QCDSF: Hagler et. al. '03]

Proton SPIN

$\mu = 2$ GeV



$J^{u+d} = 0.212(32)$

LHPC: Phys. Rev. D 77, 094502 (2008)
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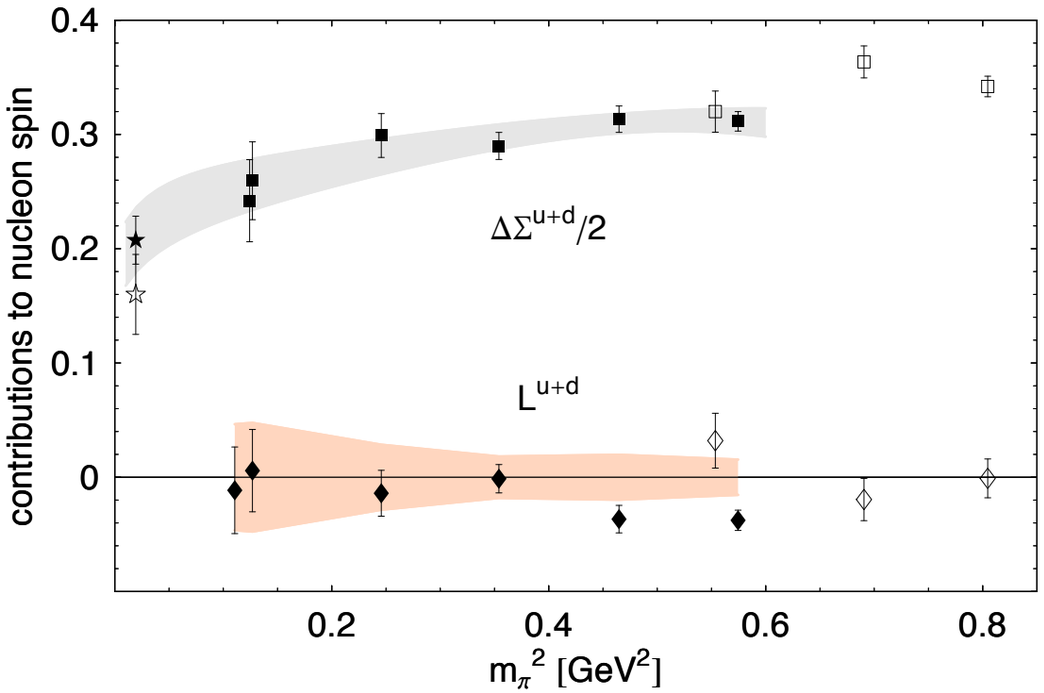
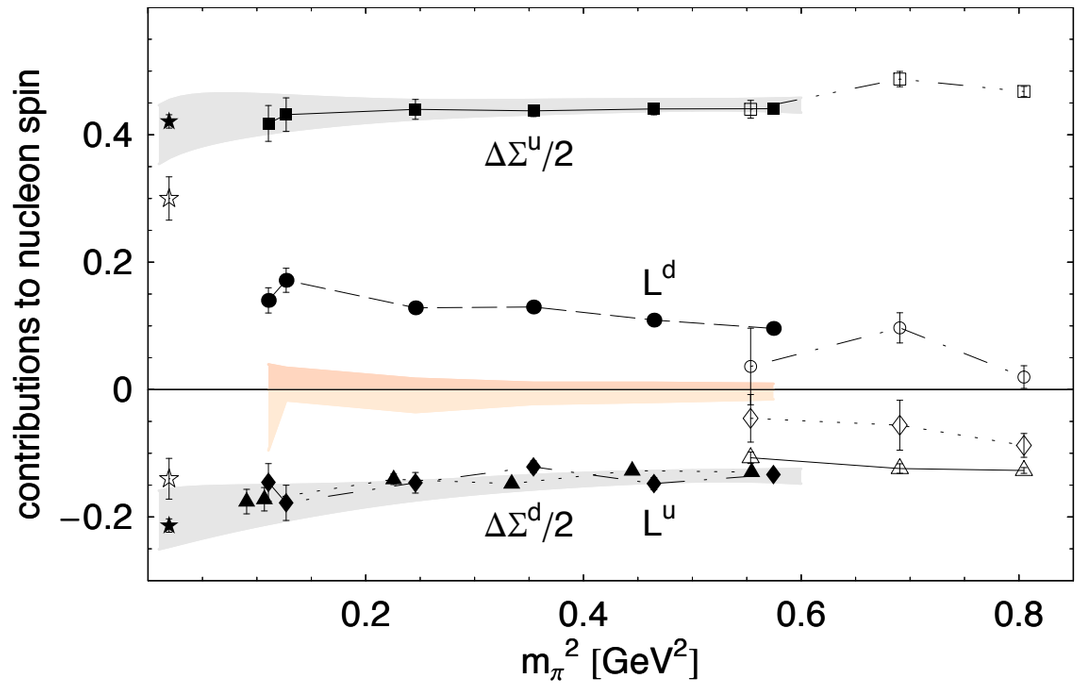
- Quark orbital angular momentum almost zero
- Disconnected diagrams are missing

Proton SPIN

$J^{u+d} = 0.212(32)$

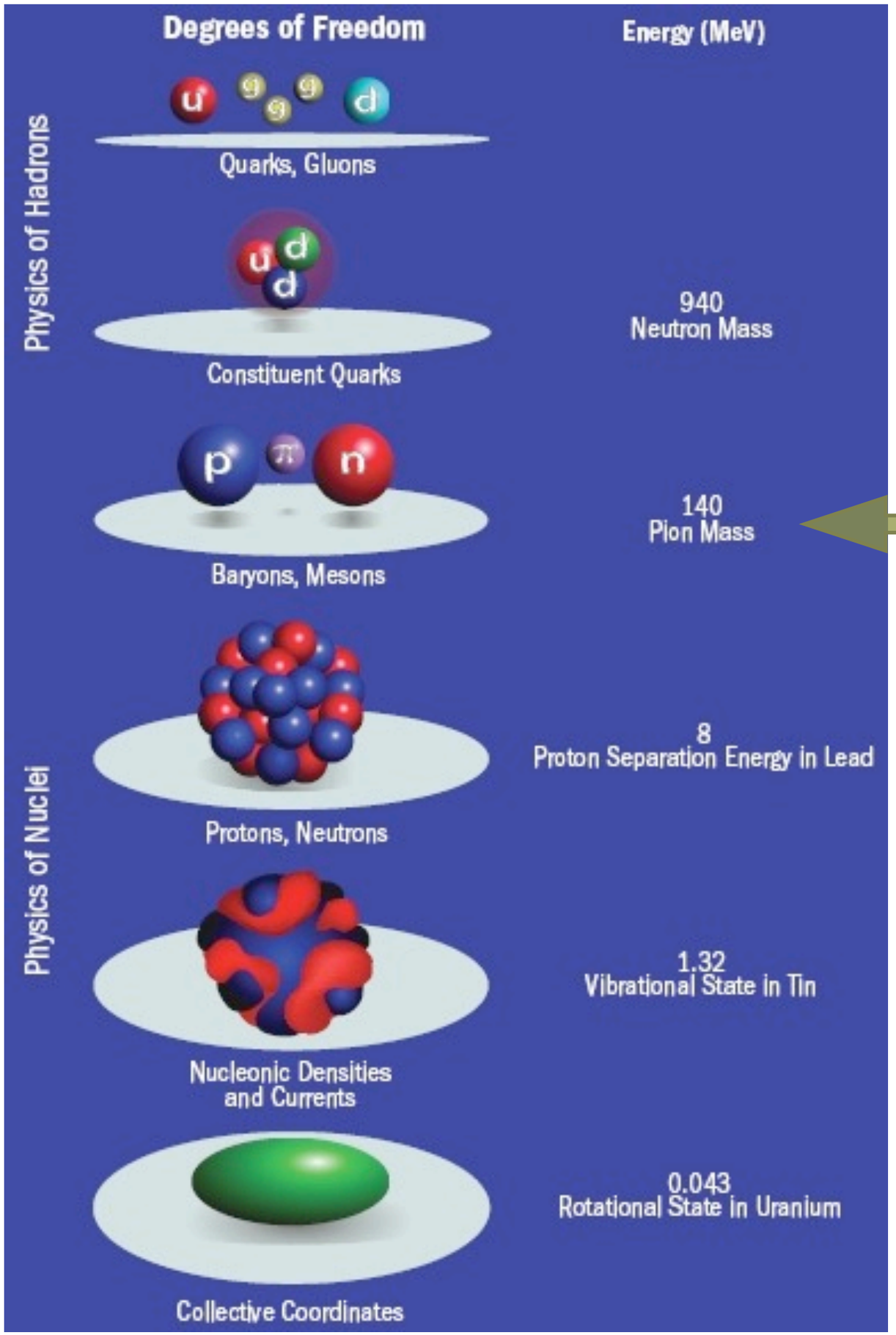




$\mu = 2$ GeV

LHPC: Phys. Rev. D 77, 094502 (2008)
hep-lat/0705.4295

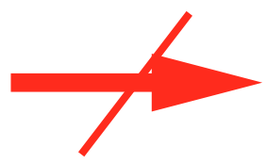
- Quark orbital angular momentum almost zero due to cancelation of the up and down contributions
- Disconnected diagrams are missing
- HERMES 2007 (star)



Hadron Interactions

Hadron Scattering

- Scattering processes from Lattice QCD are not straight forward
- Miani-Testa no-go theorem (‘90) [and C. Michael ‘89]
- Infinite Volume:

  Euclidean  Minkowski

- Finite volume: discrete spectrum
  - Avoids Miani-Testa no-go theorem [M. Luscher]

Scattering in One dimenssion

In center of mass coordinates

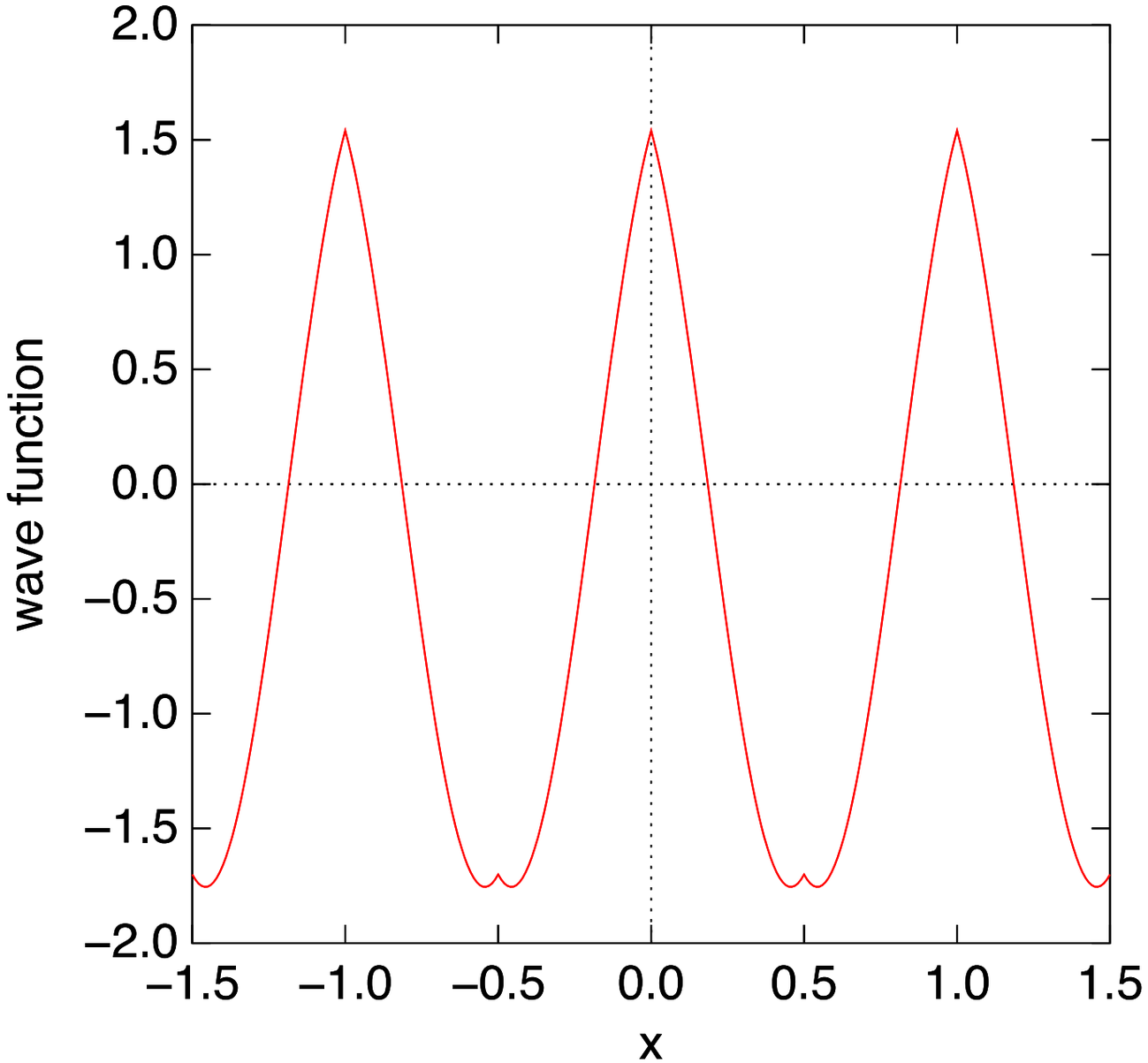
$$-\frac{1}{m}\frac{\partial^2\Psi}{\partial x^2} + c(k)\delta(x)\Psi = E\Psi$$

$$\Psi = A\left(e^{-ik|x|} + e^{ik|x|+2i\delta(k)}\right)$$

$$E = \frac{k^2}{m}$$

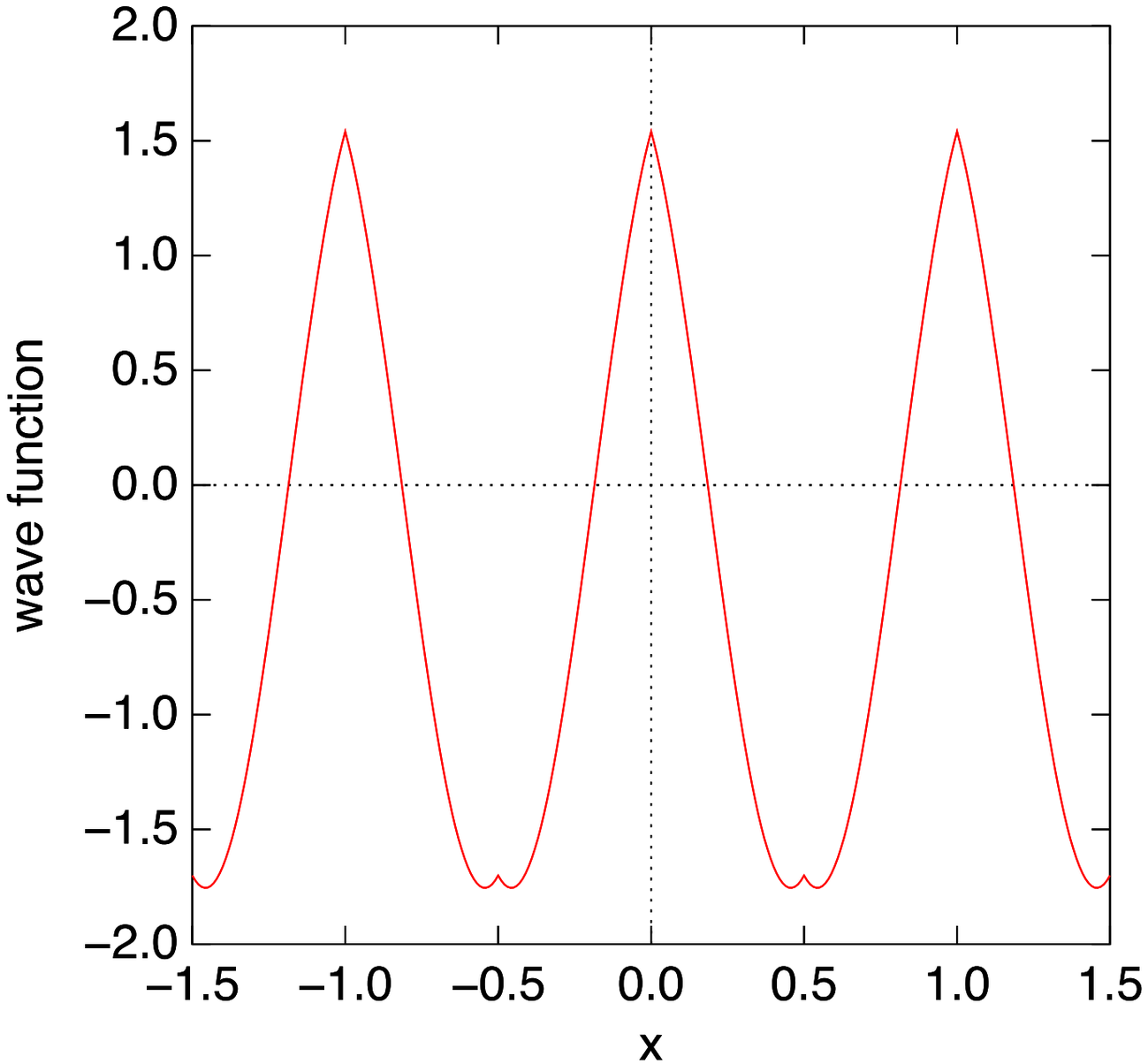
- Wave functions are almost plane waves
- Finite length with periodic boundary conditions
- Wave function needs to be periodic and even under x ---> -x (symmetric under particle exchange)

Scattering in One dimension



$$c(k) = -\frac{1}{mk}\tan\delta(k)$$

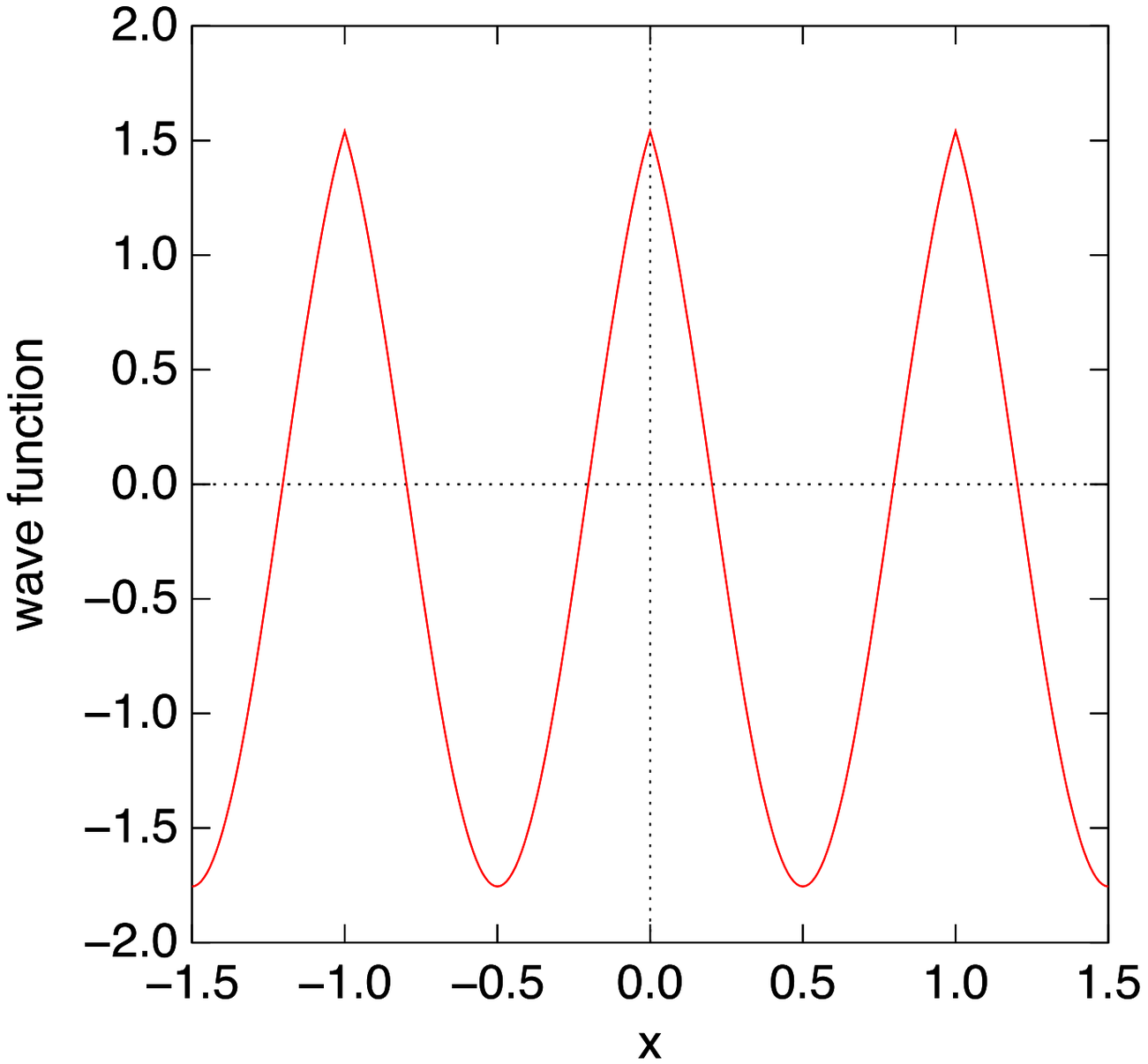
Scattering in One dimension



$$c(k) = -\frac{1}{mk}\tan\delta(k)$$

$$kL + 2\delta = 2n\pi$$

Scattering in One dimension



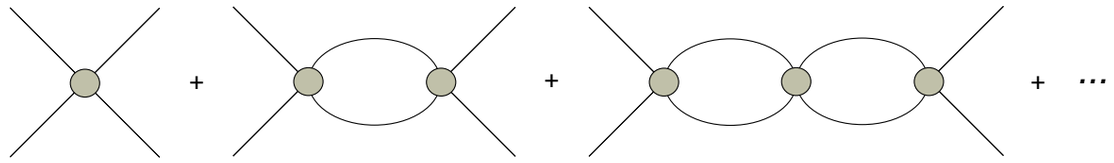
$$c(k) = -\frac{1}{mk}\tan\delta(k)$$

$$kL + 2\delta = 2n\pi$$

SCATTERING ON THE LATTICE

Luscher

Scattering amplitude:

$A(p) =$  $+ \cdots$

$$A(p) = \frac{4\pi}{m} \frac{1}{p\,cot\delta \;-\; i\,p}$$

At finite volume one can show: $\Delta E_n \;\equiv\; E_n \;-\; 2m \;=\; 2\sqrt{p_n^2 \;+\; m^2} \;-\; 2m$

p_n solutions of:

$$p\cot\delta(p) \;=\; \frac{1}{\pi L}\,\mathbf{S}\left(\frac{p^2L^2}{4\pi^2}\right) \qquad \mathbf{S}(\eta) \;\equiv\; \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2-\eta} \;-\; 4\pi\Lambda$$

Effective range expansion:

$$p\cot\delta(p) = \frac{1}{a} + \frac{1}{2}rp^2 + ...$$

a is the scattering length

Luscher Formula

Energy level shift in finite volume:

$$\Delta E_n \equiv E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$$

p_n solutions of:

$$p\cot\delta(p) = \frac{1}{\pi L}\mathbf{S}\left(\frac{p^2L^2}{4\pi^2}\right) \qquad \mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2-\eta} - 4\pi\Lambda$$

$$p_n\cot\delta(p_n) = \frac{1}{a} + \cdots \qquad \frac{1}{a} = \frac{1}{\pi L}S\left(\frac{p_0^2L^2}{4\pi^2}\right) + \cdots$$

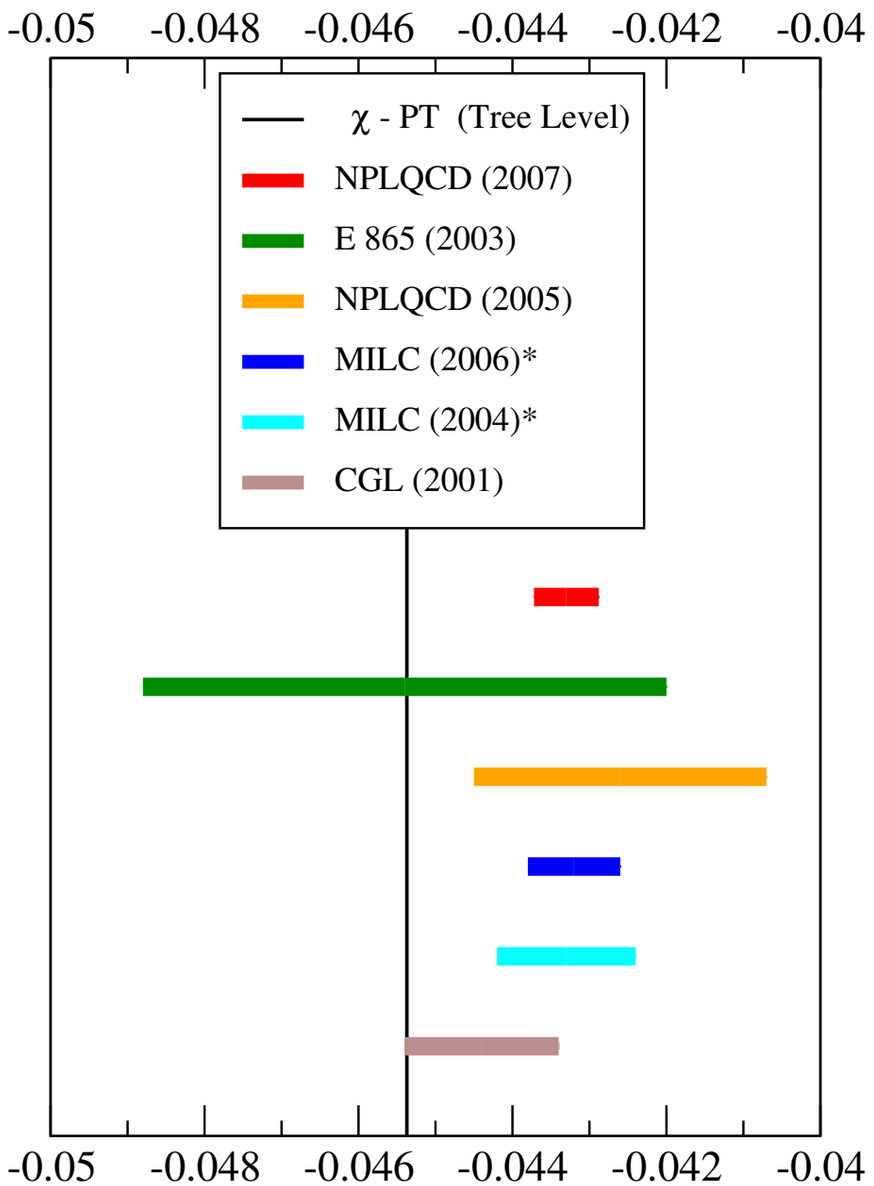
Expansion at p~0 :

$$\Delta E_0 = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

a is the scattering length

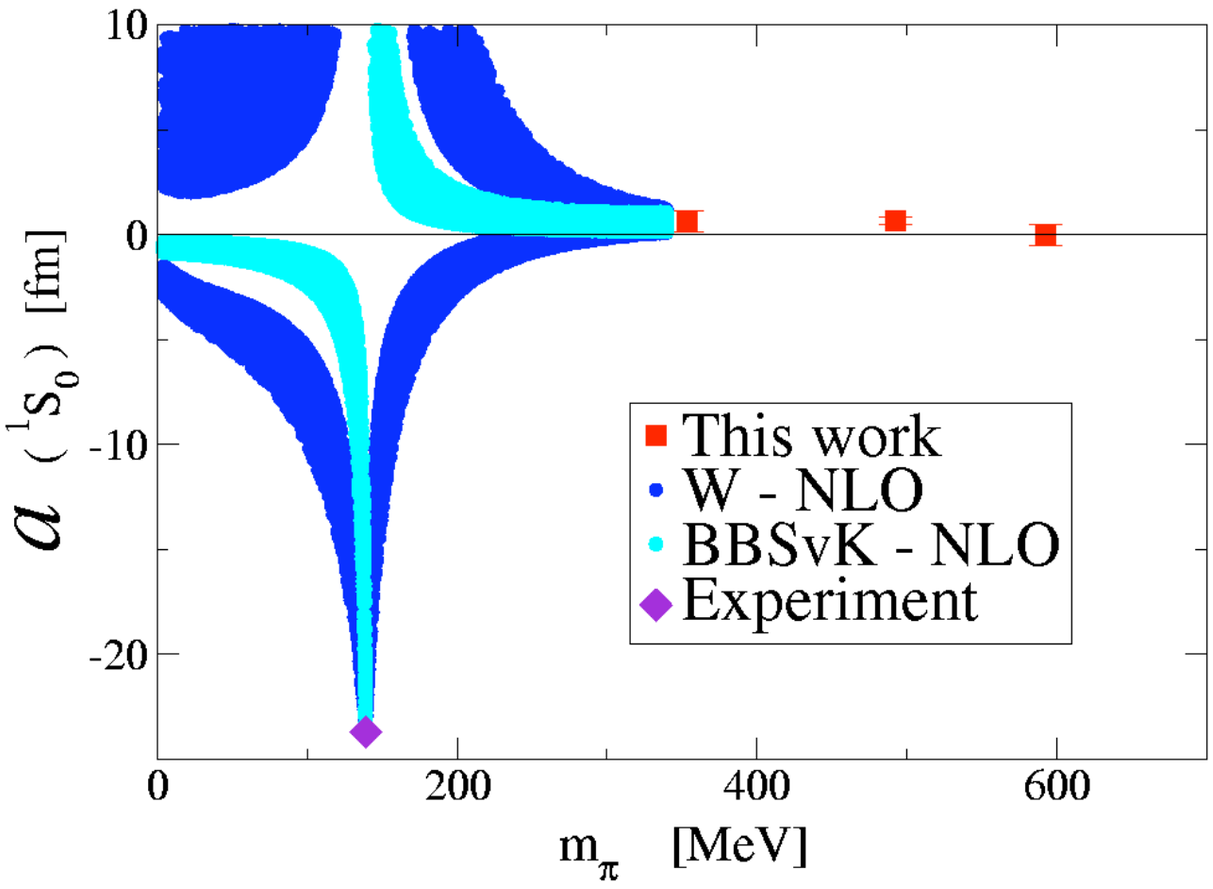
c_1 and c_2 are universal constants

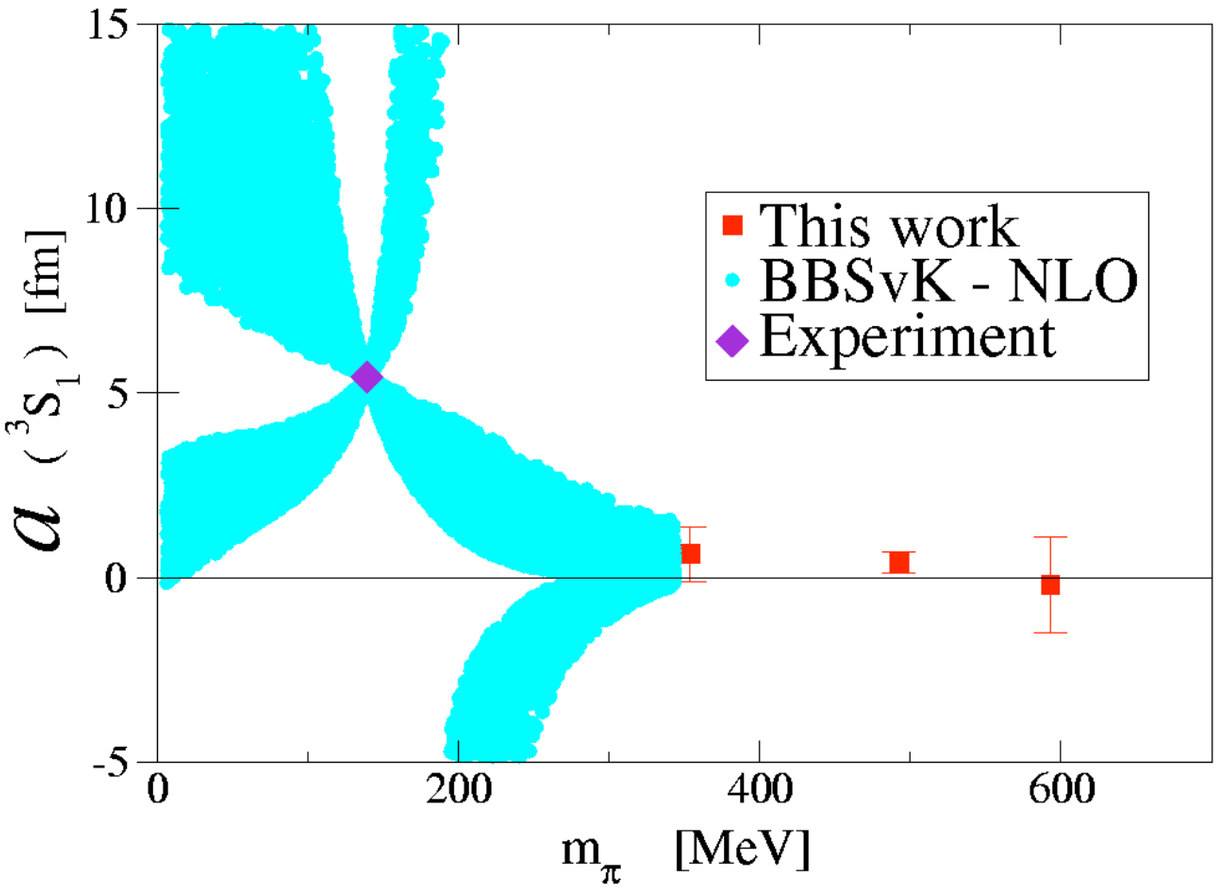
World data π-π scattering



Nucleon-Nucleon

NPLQCD: **Phys.Rev.Lett.97 2006**





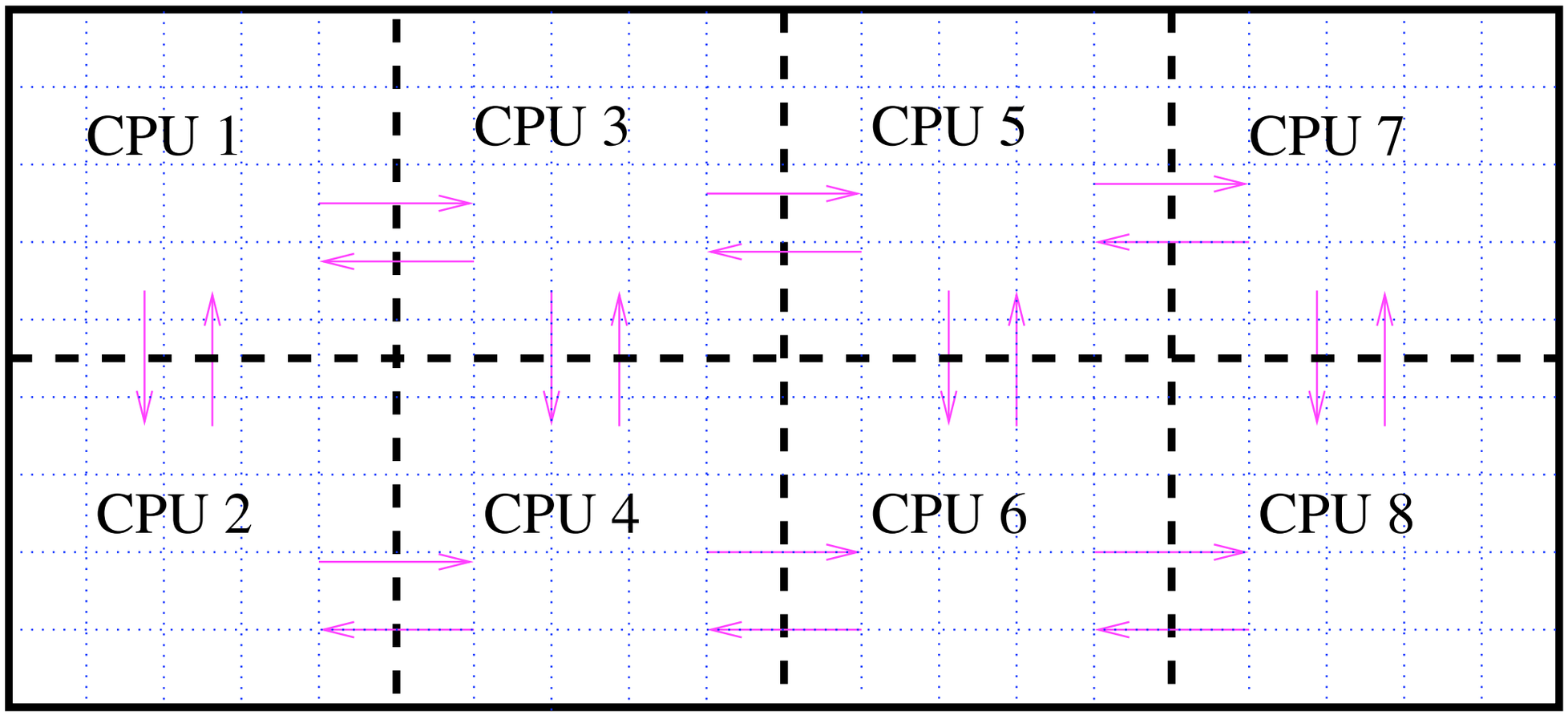
BBSvK: Beane Bedaque Savage van Kolck ‘02

W: Weinberg ‘90;Weingberg ‘91; Ordonez et.al ‘95

Fukugita et al. ‘95: Quenched heavy pions

The computers

QCD on a parallel computer



Special purpose machines

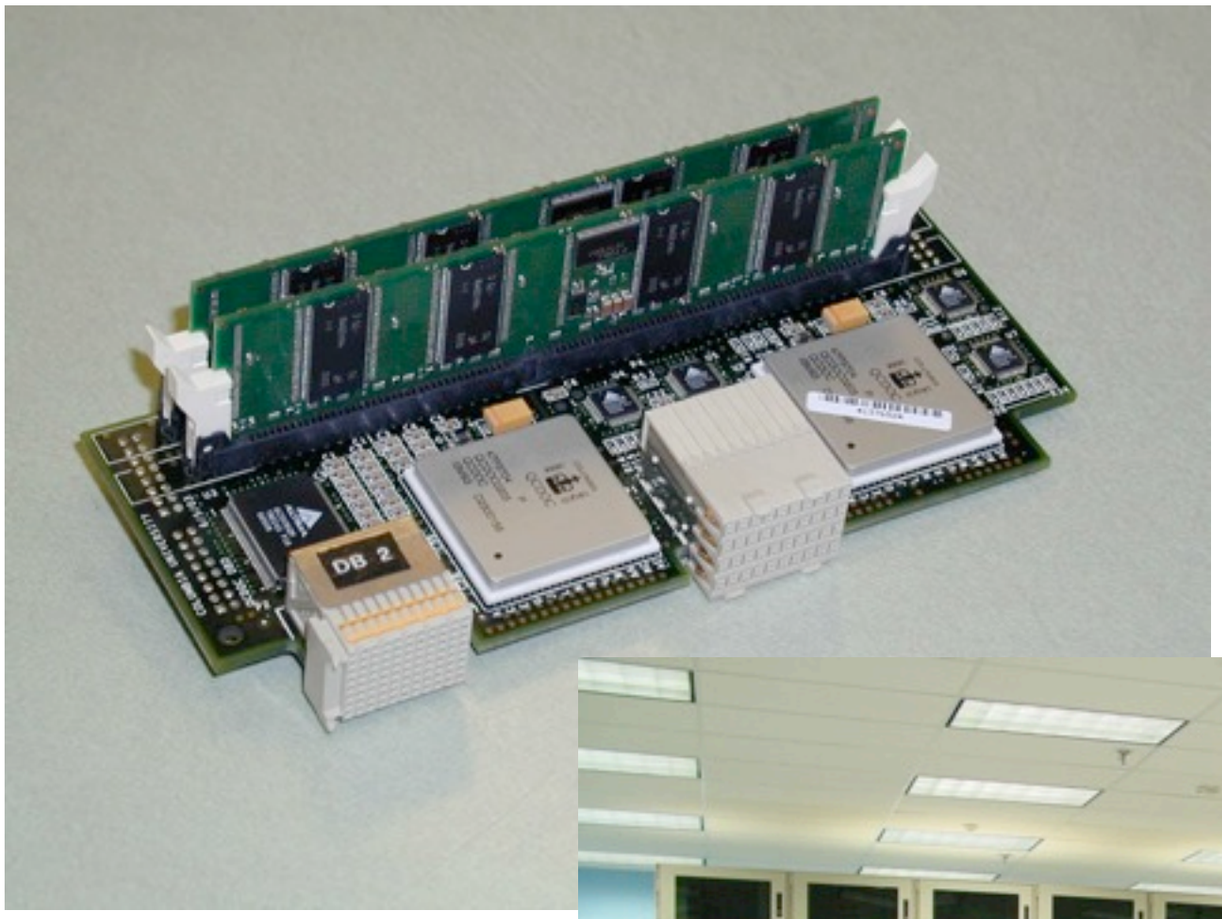
JLab cluster

DELL™

intel Xeon inside™

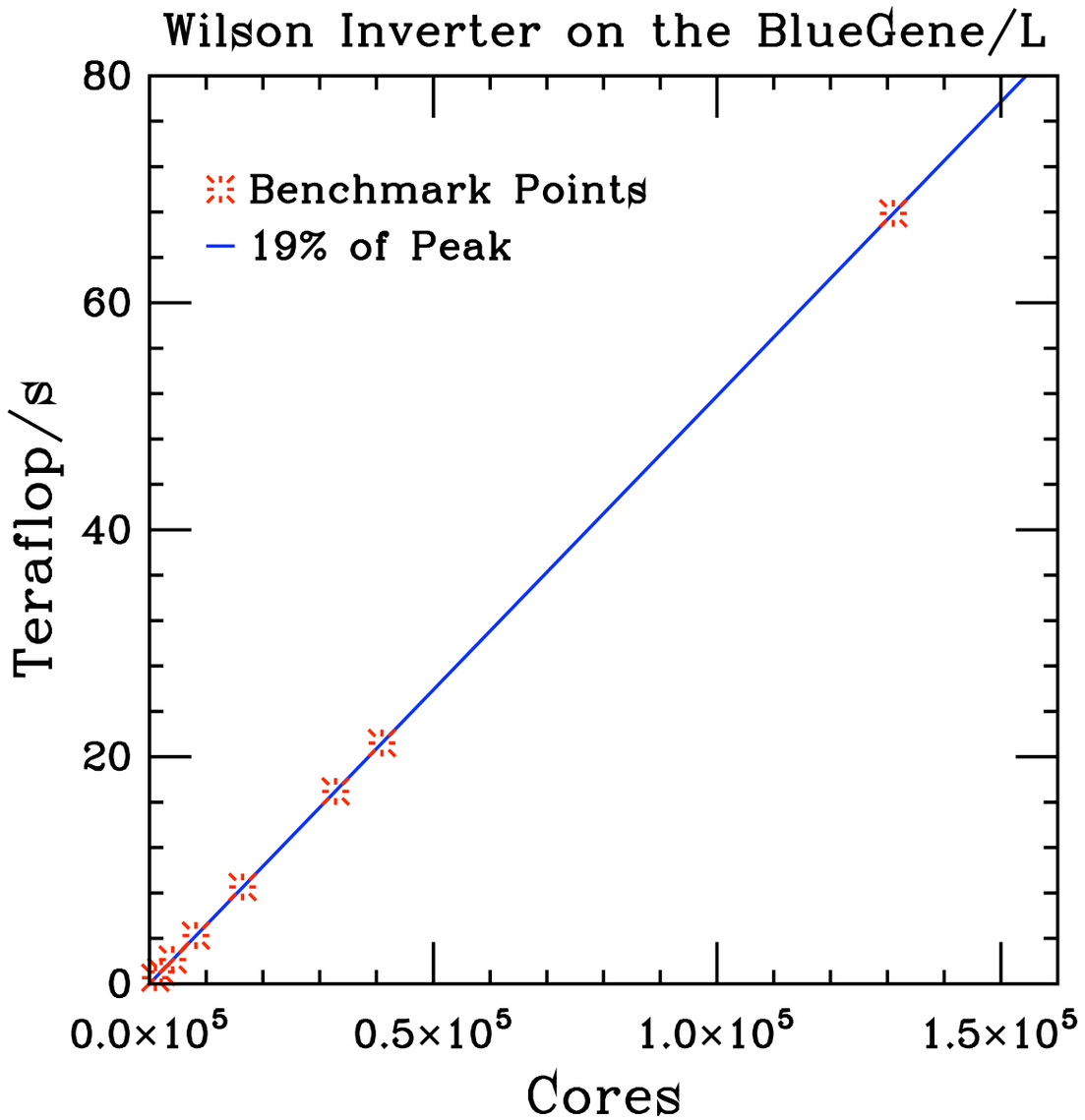
Infiniband high speed low latency network

QCD On a Chip (QCDOC)



10 Tflop peak

Commercial SuperComputers



Weak scaling: Up to **131,072 cores!** at LLNL
(Vranas et.al. 2006 Gordon Bell award

IBM Blue Gene L

The END