Nuclear Physics with Electromagnetic Probes

Lawrence Weinstein
Old Dominion University
Norfolk, VA

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Hampton University Graduate School
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Course Outline

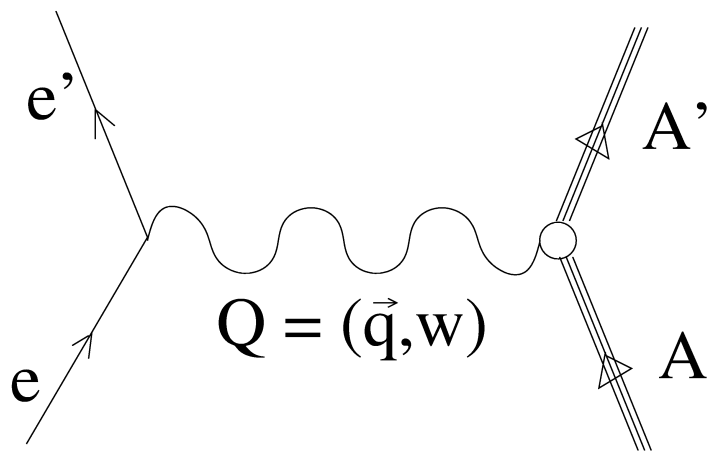
- Introduction to EM probes.
  - Electron and photon interactions with nuclei.
  - Beams and detectors.
- Nuclear Physics with electrons and photons.
  - Elastic electron-nucleus scattering.
  - Charge, matter and momentum distributions.
  - Single nucleons
  - Correlated nucleon pairs.
  - Nucleon modification in nuclei
  - Hadronization in nuclei
  - Nuclear transparency

Why use electrons and photons?

- Probe structure understood (point particles)
- Electromagnetic interaction understood (QED)
- Interaction is weak (α = 1/137)
  - Perturbation theory works!
    - First Born Approx / one photon exchange
  - Probe interacts only once
  - Study the entire nuclear volume

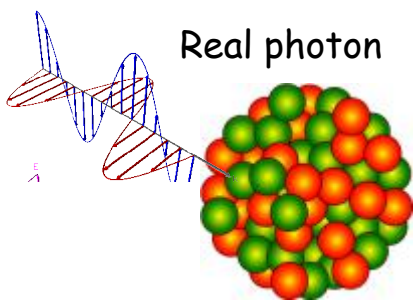
BUT:

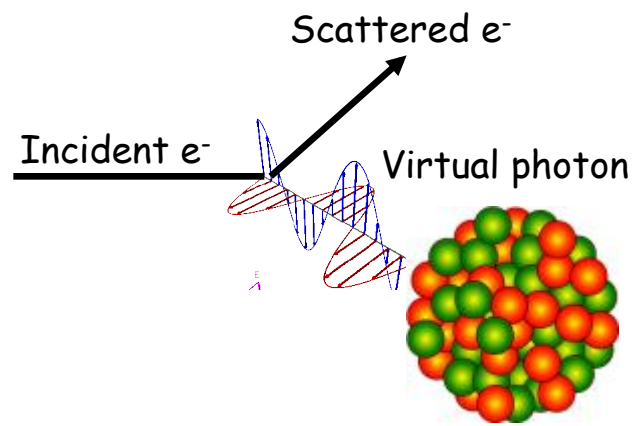
- Cross sections are small
- Electrons radiate



It's all photons!

- An electron interacts with a nucleus by exchanging a single* virtual photon.



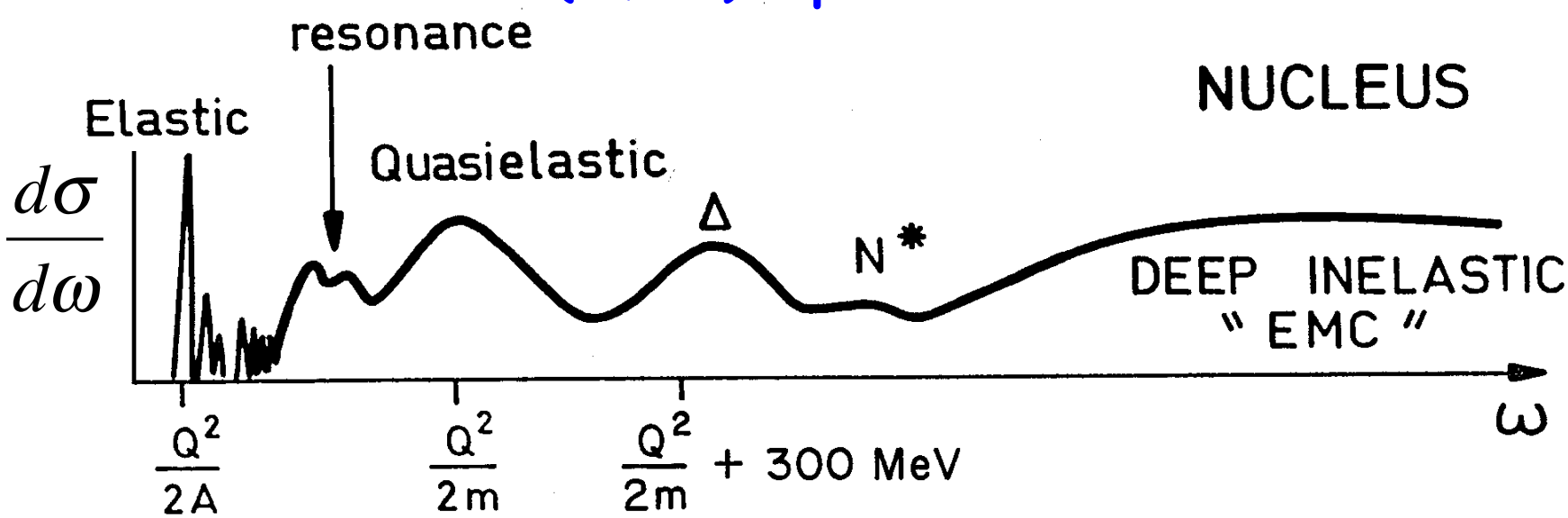


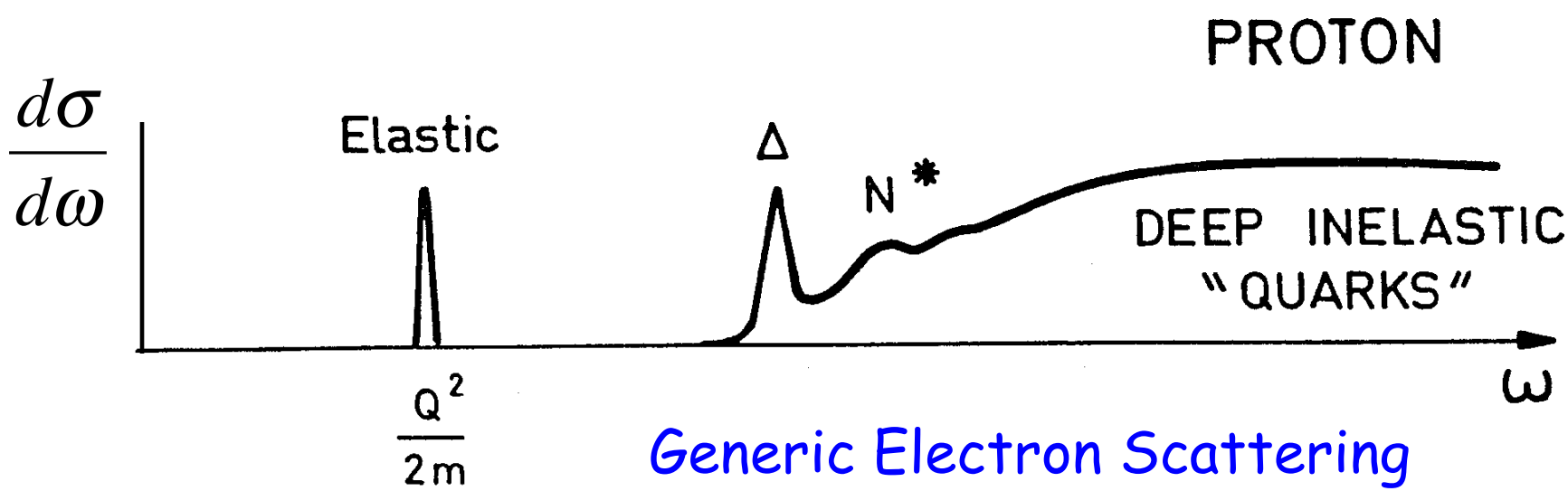
Real photon:
Momentum q = energy ν
Mass = $Q^2 = |\boldsymbol{q}|^2 - \nu^2 = 0$

Virtual photon:
Momentum $q >$ energy ν
$Q^2 = - q_\mu q^\mu = |\boldsymbol{q}|^2 - \nu^2 > 0$
Virtual photon has mass!

(ν and ω are both used for energy transfer)

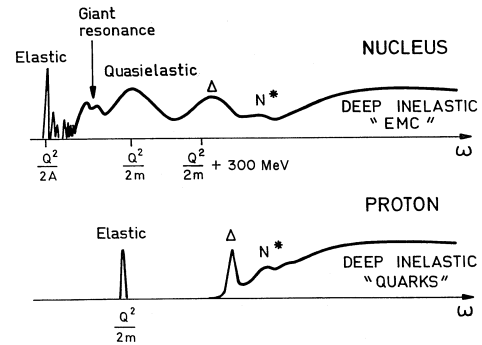
(e,e') spectrum





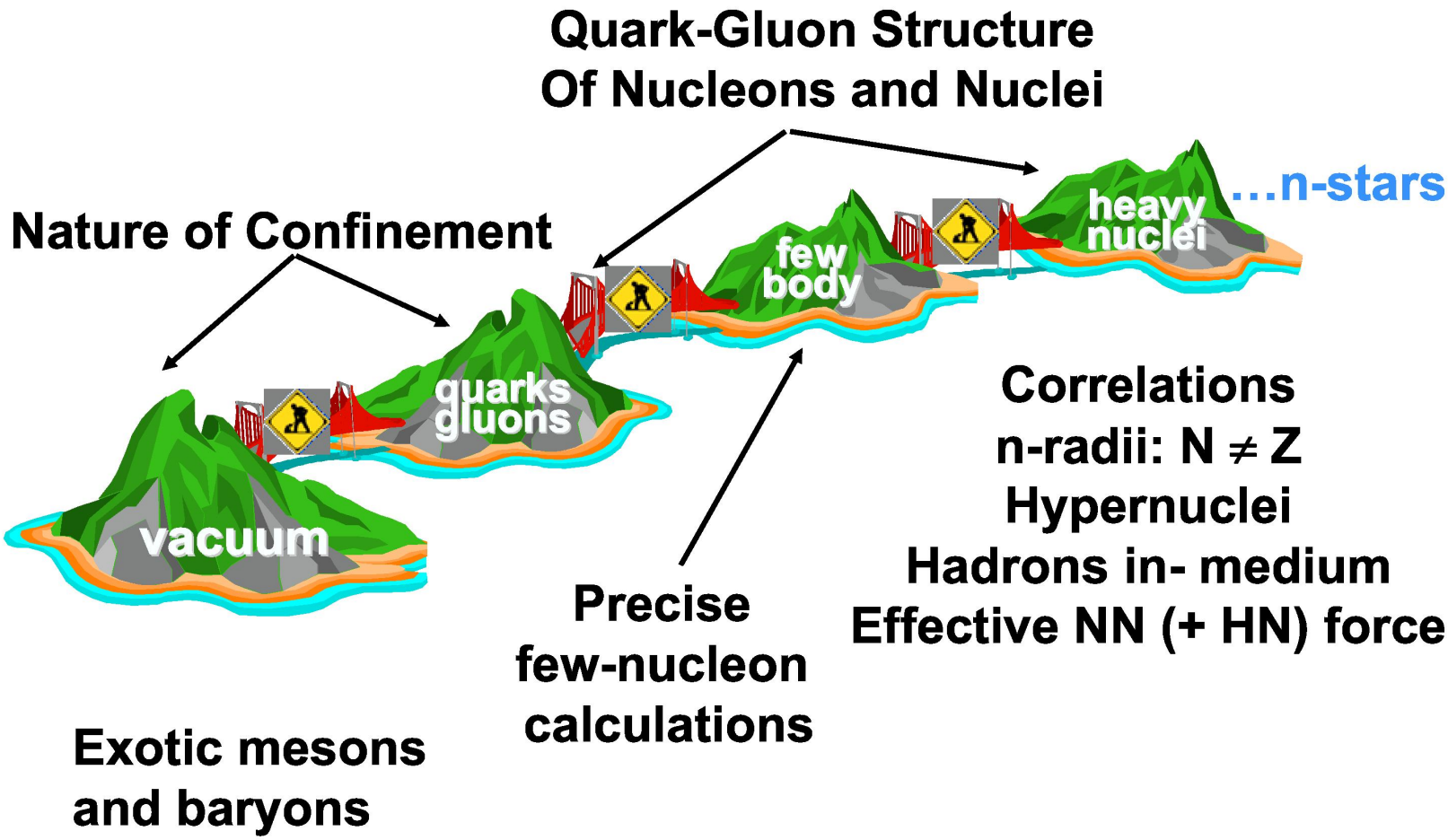
Generic Electron Scattering
at fixed momentum transfer

Experimental goals:



- Elastic scattering
  - structure of the nucleus
    - Form factors, charge distributions, spin dependent FF
- Quasielastic (QE) scattering
  - Shell structure
    - Momentum distributions
    - Occupancies
  - Short Range Correlated nucleon pairs
  - Nuclear transparency and color transparency
- Deep Inelastic Scattering (DIS)
  - The EMC Effect and Nucleon modification in nuclei
  - Quark hadronization in nuclei

Program central to all of nuclear science



Energy vs length

Select spatial resolution and excitation energy independently

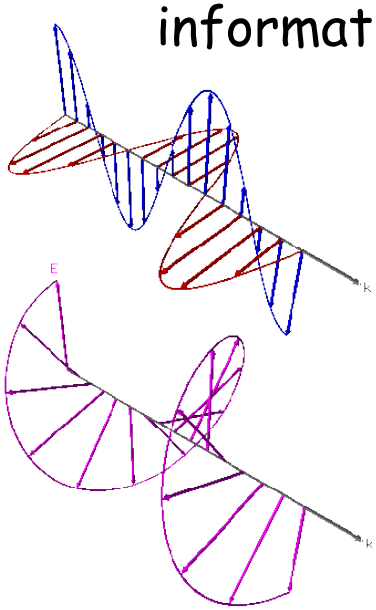
- Photon energy ν determines excitation energy
- Photon momentum q determines spatial resolution: $\lambda \approx \frac{\hbar}{q}$

Three cases:

- Low q
  - Photon wavelength λ larger than the nucleon size (R_p)
- Medium q: $0.2 < q < 1$ GeV/c
  - $\lambda \sim R_p$
  - Nucleons resolvable
- High q: $q > 1$ GeV/c
  - $\lambda < R_p$
  - Nucleon structure resolvable

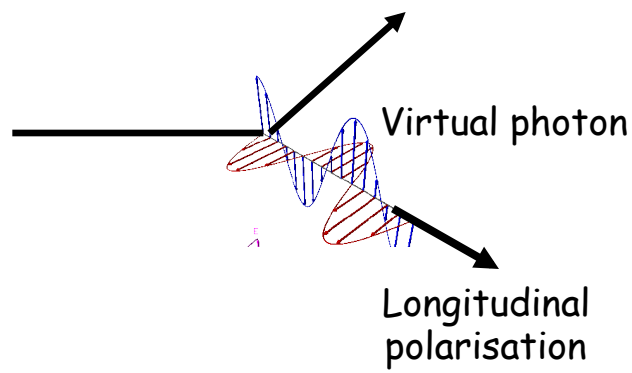
Types of photon Polarisation

- Both real and virtual photons can have polarisation
  - Real photons: transverse pol only
  - Virtual photons: longitudinal or transverse
- Determining azimuthal distribution of reaction products around these polarisation directions gives powerful information (see later)



Linear polarisation
(Electric field vector oscillates in a plane)

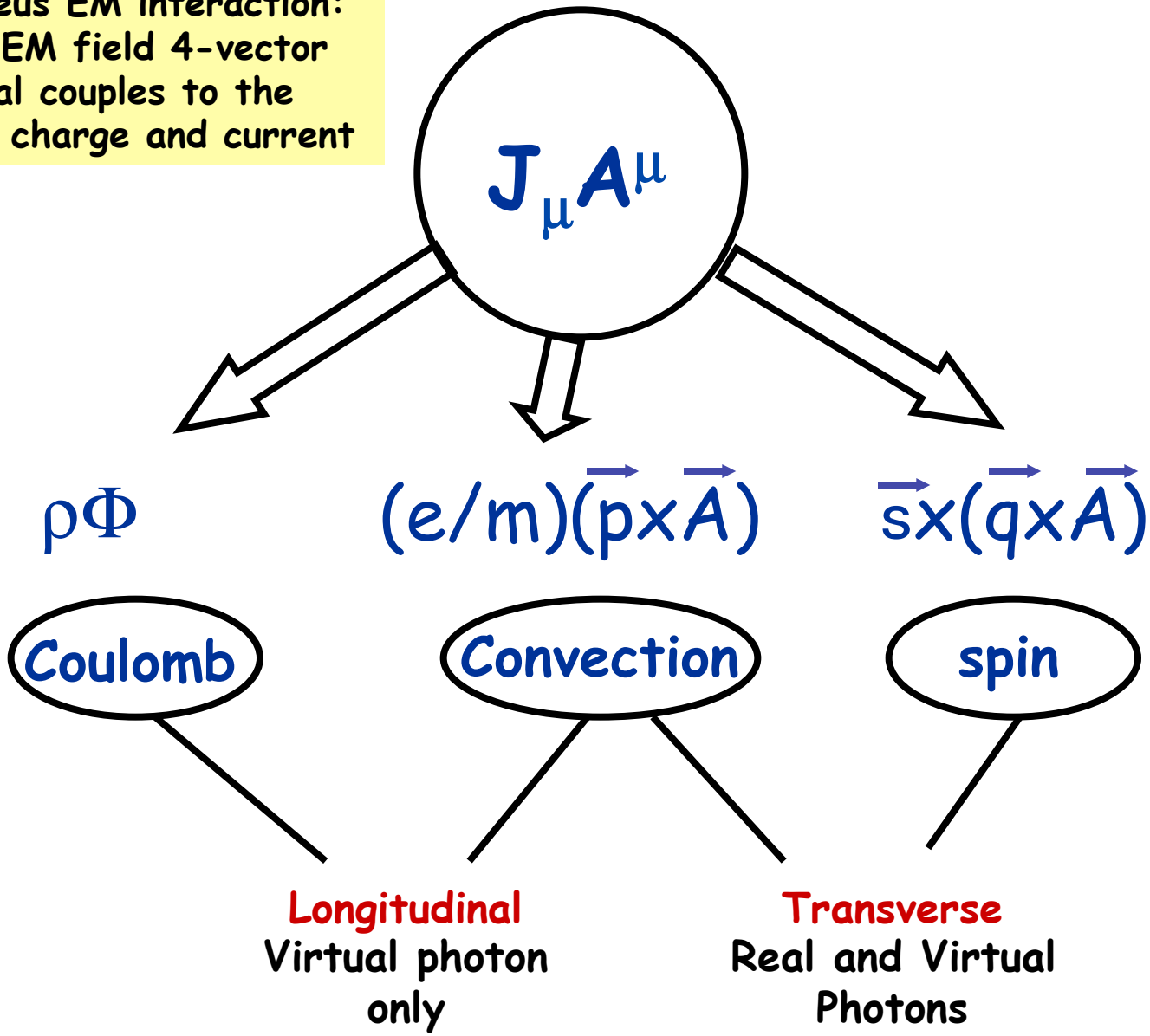
Circular polarisation
(Electric field rotates clockwise or anticlockwise)



Virtual photons can also have a longitudinal polarisation component - related to coulomb (charge) scattering

Real and virtual photon coupling

Photon-nucleus EM interaction:
Photon EM field 4-vector potential couples to the nuclear charge and current



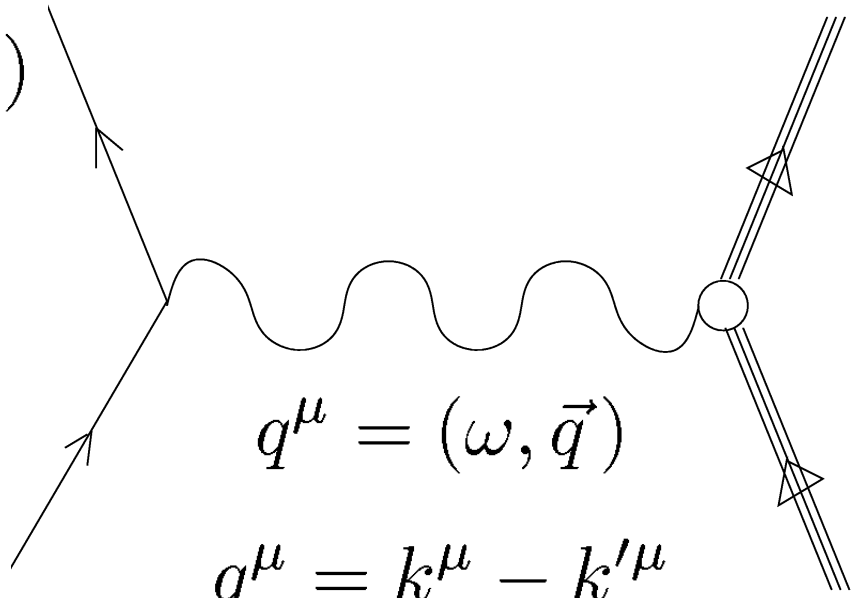
Inclusive electron scattering (e,e')

$$k'^{\mu} = (E', \vec{k}\,')$$

$$p'^{\mu} = (E_p, \vec{p}_p)$$

(not detected)

Lab frame kinematics



$$q^{\mu} = (\omega, \vec{q})$$

$$k^{\mu} = (E, \vec{k}\,)$$

$$q^{\mu} = k^{\mu} - k'^{\mu}$$

$$p^{\mu} = (M, \vec{0}\,)$$

Invariants:

$$p^{\mu} p_{\mu} = M^2$$

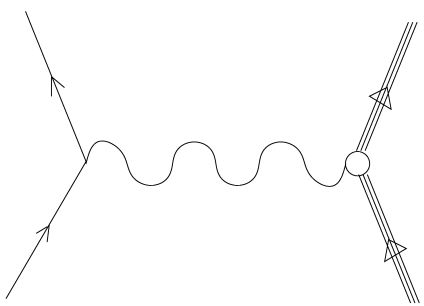
$$p_{\mu} q^{\mu} = M\omega$$

$$Q^2 = -q^{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2$$

$$W^2 = (q^{\mu} + p^{\mu})^2 = p'_{\mu} p'^{\mu}$$

Formalism

- Inclusive scattering:
  - measure scattering angle θ_e and energy E'_e ($\omega = E_e - E'_e$) and the cross section $d\sigma / d\Omega_e d\omega$
- One photon exchange:



$$\mathcal{M}_n = \frac{e^2}{q^2}\bar{u}(k')\gamma_\mu u(k) < n|J_\mu(0)|p, S >$$

$$\begin{aligned} d\sigma &= \frac{1}{4M\vec{k}}\sum_n |\mathcal{M}_n|^2 (2\pi)^4 \delta^4(p+q-p')\frac{d^3k'}{(2\pi)^3 2E'} \\ &= \frac{|\vec{k}'|}{ME}\frac{\alpha^2}{Q^4} L^{\mu\nu} H_{\mu\nu} d\omega d\Omega \end{aligned}$$

$L^{\mu\nu}$ and $H_{\mu\nu}$ are the lepton and hadron tensors

More Formalism

Lepton tensor known (QED):

$$L_{\mu\nu} = \sum_{s'} (\bar{u}(k',s')\gamma_\mu u(k,s))^* (\bar{u}(k',s')\gamma_\nu u(k,s))$$
$$= 2(k_\mu k'_\nu + k_\nu k'_\mu) + q^2 g_{\mu\nu} + 2im_l \epsilon_{\mu\nu\alpha\beta} q^\alpha s_l^\beta$$

Spin dependent

Hadron tensor unknown:

$$H_{\mu\nu} = \frac{1}{4\pi} \sum_n < p,S|\hat{J}_\mu(0)|n >< n|\hat{J}_\nu(0)|p,S > (2\pi)^4 \delta^4(p+q-p')$$
$$= \frac{1}{4\pi} \int d^4\xi \exp(iq\cdot\xi) < p,S|[\hat{J}_\mu(\xi), \hat{J}_\nu(0)]\,|p,S >$$

Nuclear current operators

Two approaches:

1. Calculate J_μ from models or
2. Create most general hadron tensor from tensors (bilinear Lorentz vectors) $A^i_{\mu\nu} = a_\mu b_\nu$ and Lorentz scalers $S_i = a_\mu b^\mu$: $H_{\mu\nu} = \Sigma A^i_{\mu\nu} F^i(S_1, S_2, \ldots)$

Most general hadron tensor:

EM current conservation ➔ $q^\mu H_{\mu\nu} = H_{\mu\nu} q^\nu = 0$

parity conservation ➔ $H_{\mu\nu}(p,q) = H_{\mu\nu}(\tilde{p},\tilde{q})$

$\tilde{p} = (p_0, -\vec{p}), \tilde{q} = (q_0, -\vec{q})$

Time reversal symmetry ➔ $H_{\mu\nu}(p,q) = H^*_{\mu\nu}(\tilde{p},\tilde{q})$

Available independent four vectors for (e,e'):

- target momentum p_μ
- photon momentum q_μ
- target polarization S_μ

$$H^{nospin}_{\mu\nu}(p,q) = F_1\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) + F_2\frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)$$

$$H^{spin}_{\mu\nu}(p,q) = \frac{M}{p\cdot q}\left[\epsilon_{\mu\nu\alpha\beta}q^\alpha S^\beta(g_1+g_2) - \epsilon_{\mu\nu\alpha\beta}q^\alpha p^\beta g_2\right]$$

Spin dependent

Four independent structure functions

- Spin **independent**: $F_1(Q^2,x)$ and $F_2(Q^2,x)$
- Spin **dependent**: $g_1(Q^2,x)$ and $g_2(Q^2,x)$

(arguments also written as (Q^2,ω) or (Q^2,ν))

$$x = \frac{Q^2}{2p\cdot q} = \frac{Q^2}{2M\nu}$$

Elastic cross section ($p'^2 = m^2$)

Recoil factor Form factors

$$\frac{d\sigma}{d\Omega} = \sigma_M \frac{E'}{E}\left\{\left[F_1^2(Q^2) + \frac{Q^2}{4M^2}\kappa^2 F_2^2(Q^2)\right] + \frac{Q^2}{2M^2}[F_1(Q^2) + \kappa F_2(Q^2)]^2 \tan^2\frac{\theta}{2}\right\}$$

$$= \sigma_M \frac{E'}{E}\left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau\tan^2\frac{\theta}{2} G_M^2(Q^2)\right]$$

$$= \sigma_M \frac{E'}{E}\left[\frac{Q^4}{\vec{q}^4} R_L(Q^2) + \left(\frac{Q^2}{2\vec{q}^2} + \tan^2\frac{\theta}{2}\right) R_T(Q^2)\right]$$

Mott cross section

$$\sigma_M = \frac{\alpha^2\cos^2\left(\frac{\theta_e}{2}\right)}{4E^2\sin^4\left(\frac{\theta_e}{2}\right)}$$

F_1, F_2: Dirac and Pauli form factors

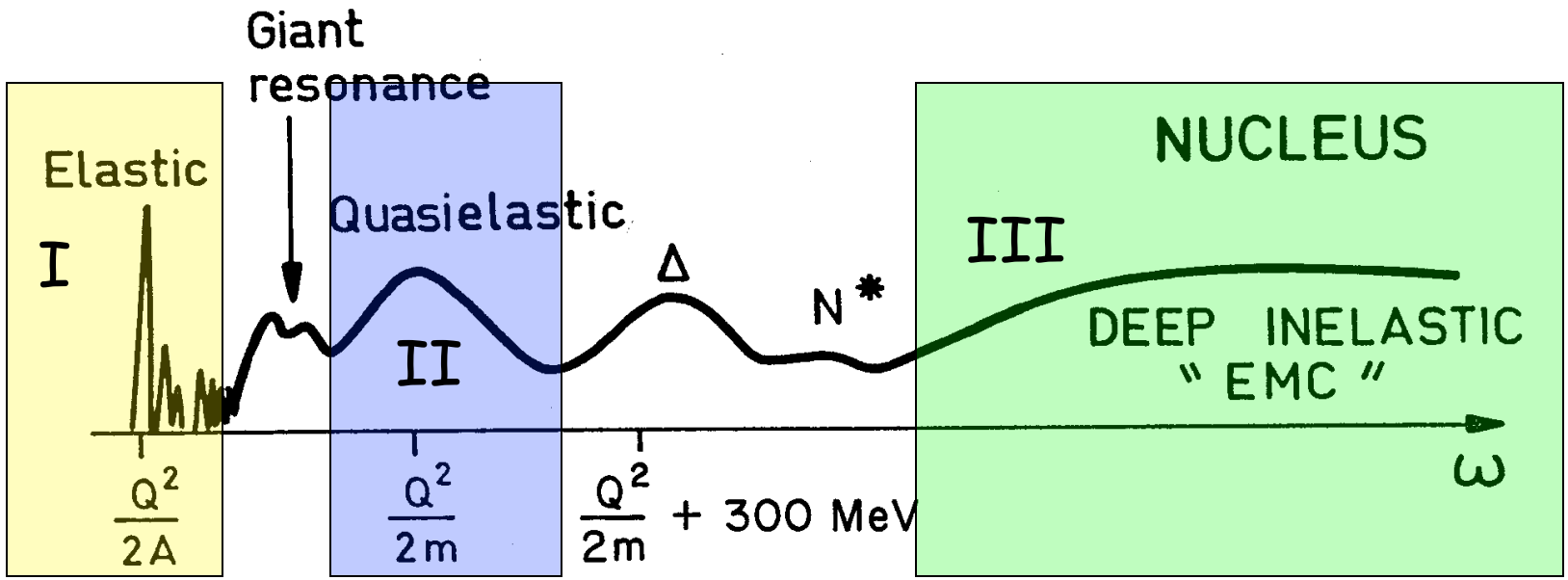
G_E, G_M: Sachs form factors (electric and magnetic)

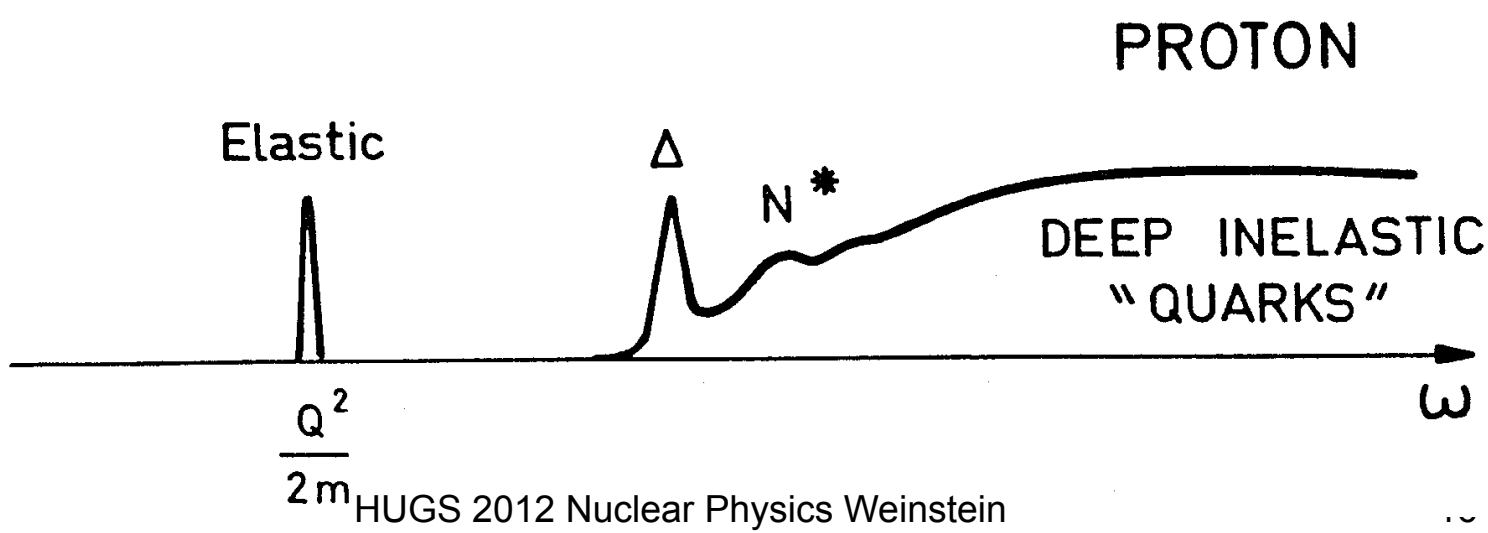
$G_E(Q^2) = F_1(Q^2) - \tau\varkappa F_2(Q^2)$ $\tau = Q^2/4M^2$

$G_M(Q^2) = F_1(Q^2) + \varkappa F_2(Q^2)$ $\varkappa$ = anomalous magnetic moment

R_L, R_T: Longitudinal and transverse response fn

Electron-nucleus interactions

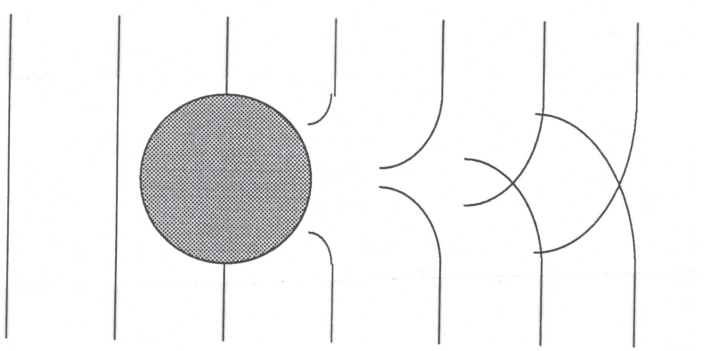




Electrons as Waves

Scattering process is quantum mechanical

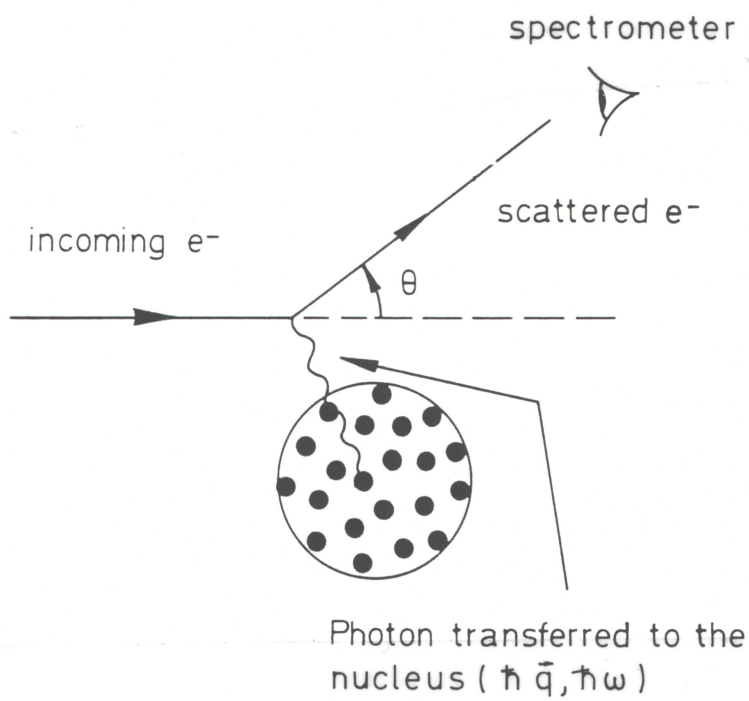
De broglie wavelength: $\lambda = \frac{h}{p}$

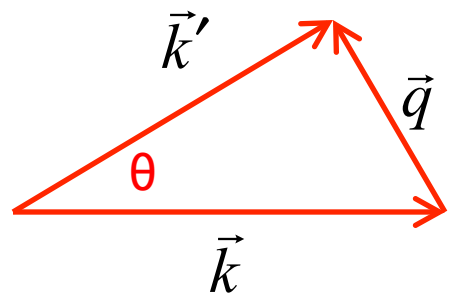


Electron energy: $E_e \approx pc$

λ resolving "scale": $\lambda = \frac{2\pi(197\ \mathrm{MeV} \cdot \mathrm{fm})}{E_e}$

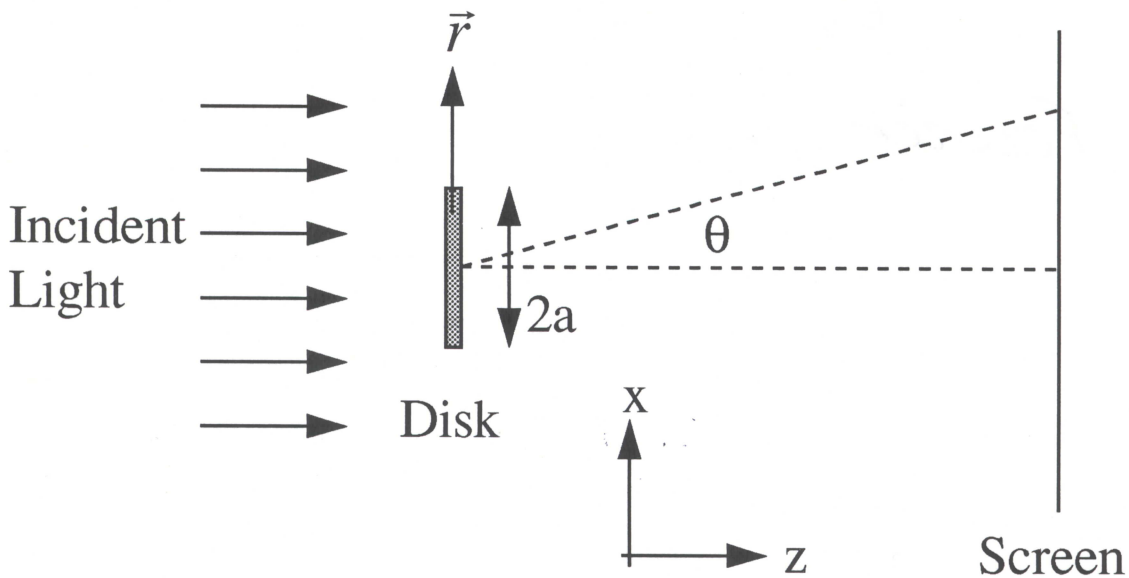
Analogy between elastic electron scattering and diffraction





Simple analogy for elastic electron scattering….

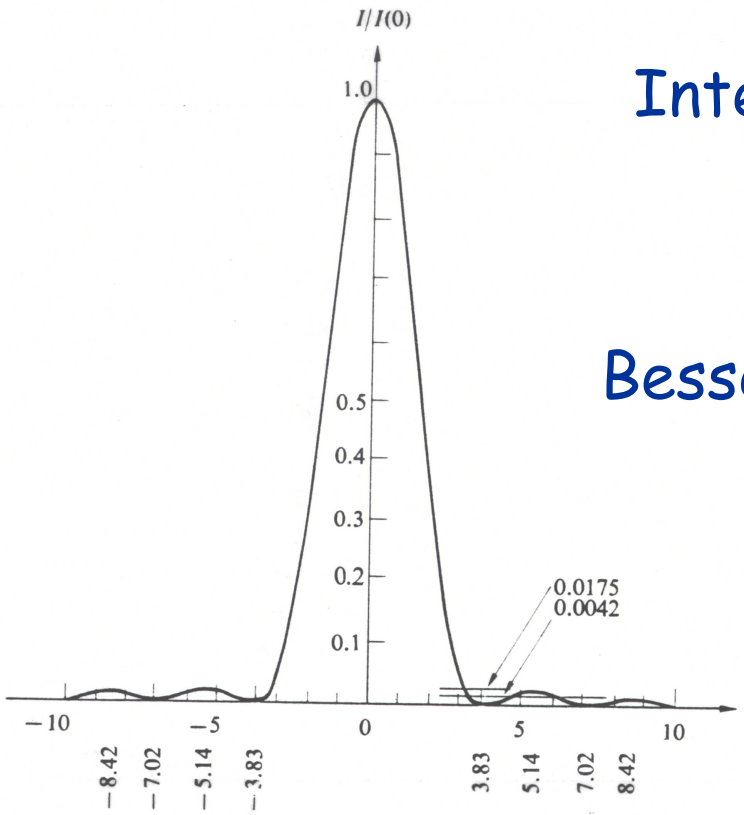
Classical Fraunhofer Diffraction



Amplitude of wave at screen:

$$\Phi \propto \int_0^a \int_0^{2\pi} \exp(ibr\cos\phi)\, r\, d\phi\, dr$$

Classical Fraunhofer Diffraction



Intensity: $\Phi^2 \propto \left(\frac{J_1((2\pi a/\lambda)\sin\theta)}{\sin\theta} \right)^2$

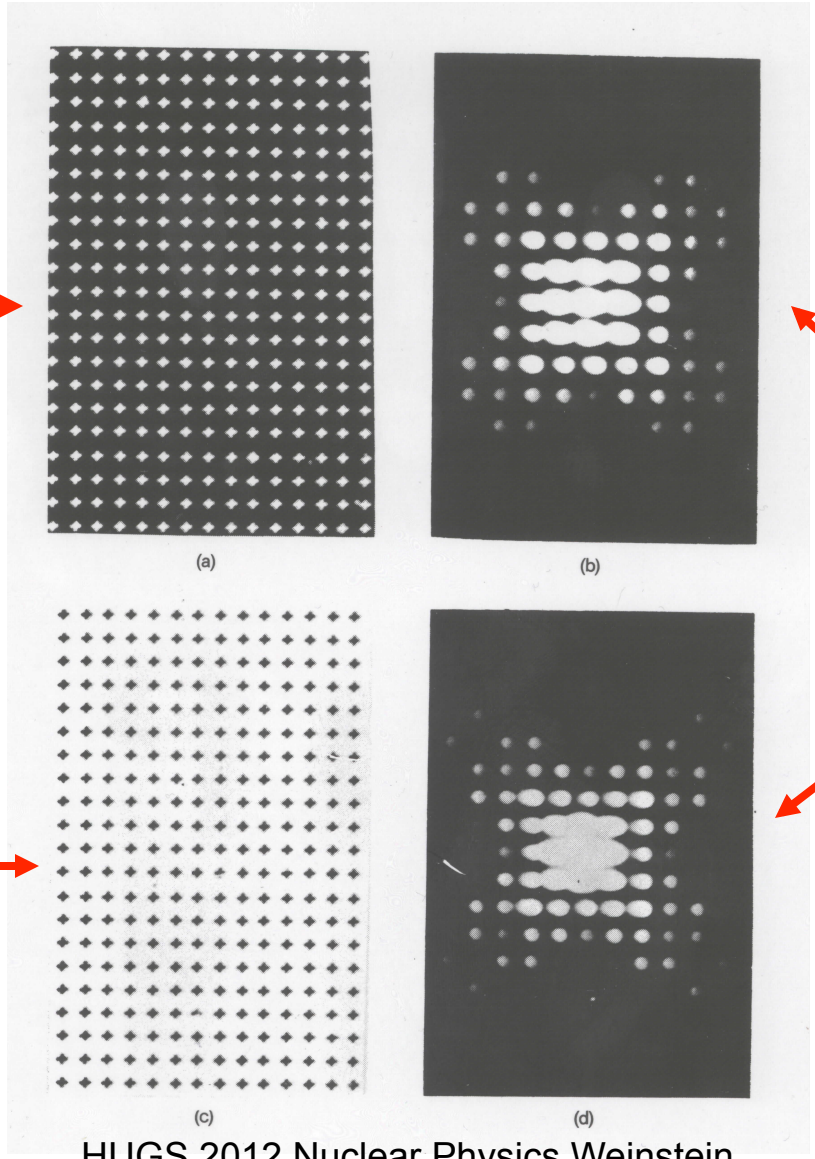
Minima occur at zeroes of Bessel function. 1st zero: x = 3.8317

...some algebra...

Hence $2a \approx \frac{1.22\lambda}{\sin\theta}$

Excursion: Babinet's principle

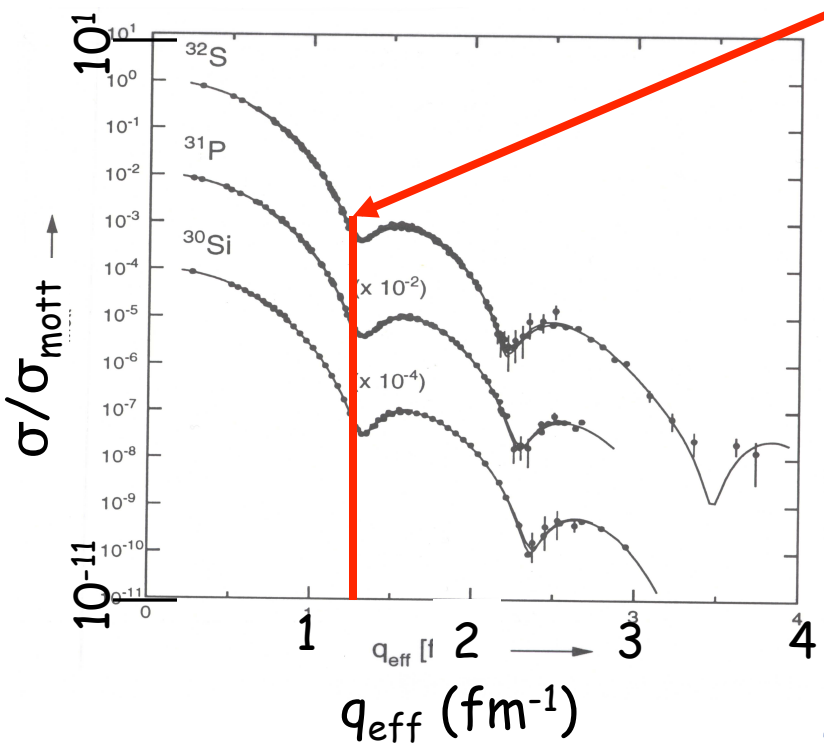
Screen with apertures



Patterns appear the same

Complementary screen

Example: $^{30}Si(e,e')$



1st minimum = 1.3 fm^{-1}
➔ $\theta = 32.8^o$

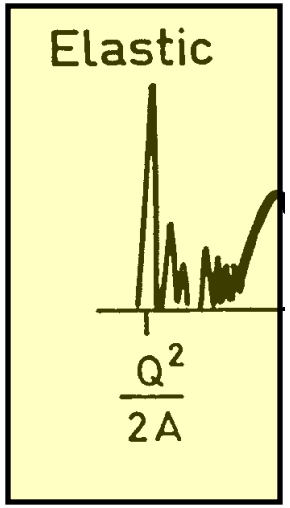
Electron energy = 454.3 MeV
➔ $\lambda = 2.73$ fm

Calculated radius = 3.07 fm

Measured rms radius = 3.19 fm
(from fit to entire curve)

(1 fm^{-1} = 197 MeV/c)

I. Elastic Electron Scattering from Nuclei (done formally)

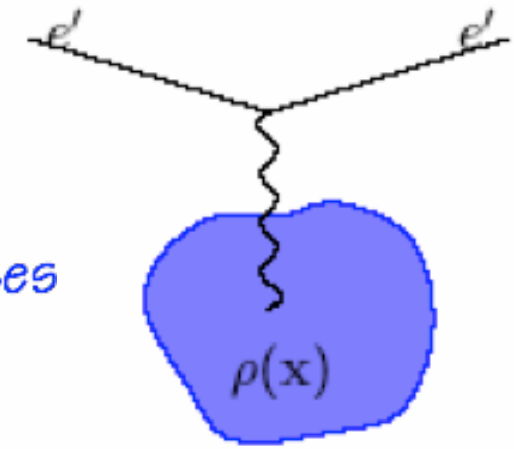


Fermi's Golden Rule

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar} \mid M_{fi} \mid^2 D_f$$

M_{fi}: scattering amplitude

D_f: density of the final states (or phase factor)



$$M_{fi} = \int \Psi_f^* V(x) \Psi_i d^3x$$

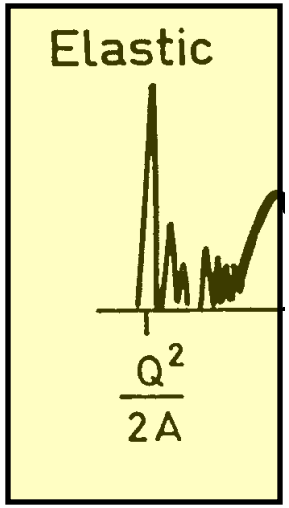
$$= \int e^{-k_f \cdot x} V(x) e^{-k_i \cdot x} d^3x$$

$$= \int e^{iq \cdot x} V(x) d^3x$$

Plane wave approximation for incoming and outgoing electrons

Born approximation (interact only once)

I. Elastic Electron Scattering from (spin-0) Nuclei



Form Factor and Charge Distribution

Using Coulomb potential from a charge distribution, ρ(x),

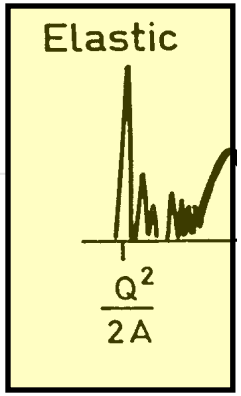
$$V(x) = -\frac{Ze^2}{4\pi\epsilon_o}\int \frac{\rho(x')}{|x-x'|}d^3x'$$

$$\begin{aligned} M_{fi} &= -\frac{Ze^2}{4\pi\epsilon_o}\int e^{iq\cdot x}\int \frac{\rho(x')}{|x-x'|}d^3x'd^3x \\ &= -\frac{Ze^2}{4\pi\epsilon_o}\int e^{iqR}\left[\int \frac{e^{iq\cdot x'}\rho(x')}{|R|}d^3x'\right]d^3R \\ &= -\frac{Ze^2}{4\pi\epsilon_o}\int \frac{e^{iqR}}{R}d^3R\int e^{iq\cdot x'}\rho(x')d^3x' \end{aligned}$$

$$F(q) = \int e^{iq\cdot x'}\rho(x')d^3x'$$

Charge form factor F(q)
is the Fourrier transform
of the charge distribution ρ(x)

I. Elastic Electron Scattering from (spin-0) Nuclei



Form factor and cross section

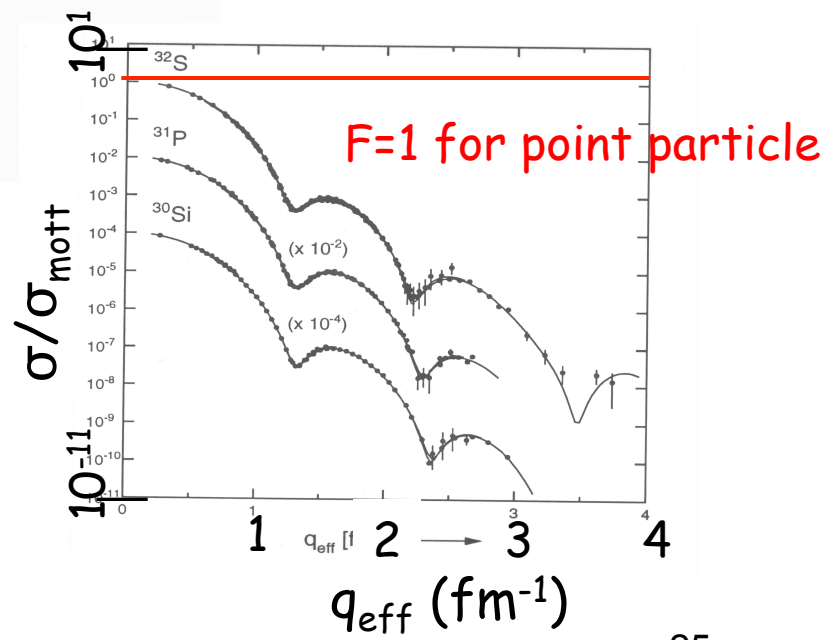
- For point-like particle, $\rho(\mathbf{x}') = \delta(\mathbf{x}')$ and $F(\mathbf{q}) = 1 \rightarrow$ Rutherford-*like* scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} \equiv \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 \cos^2\frac{\theta}{2}}{4E^2 \sin^4\frac{\theta}{2}}$$

$\cos^2$ term only difference
From Rutherford formula
Arises from Dirac theory
For spin $\frac{1}{2}$ particle

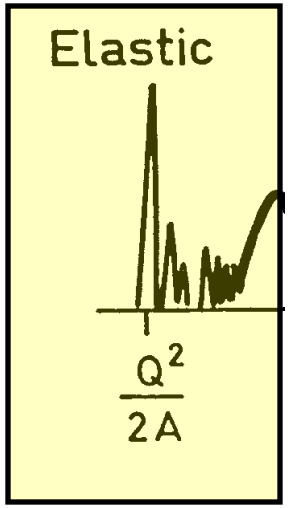
- Scattering from a charge distribution

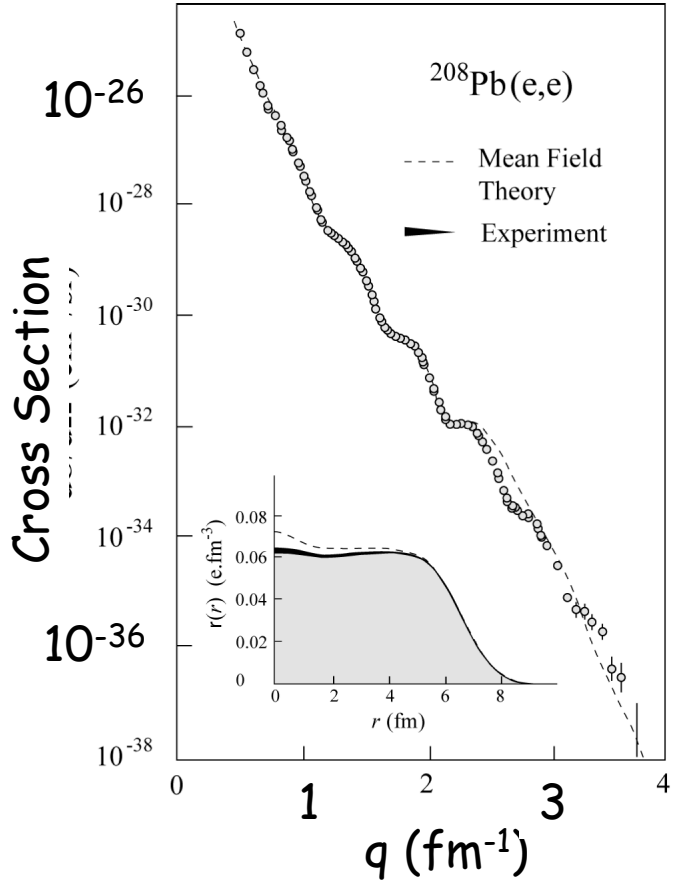
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2$$

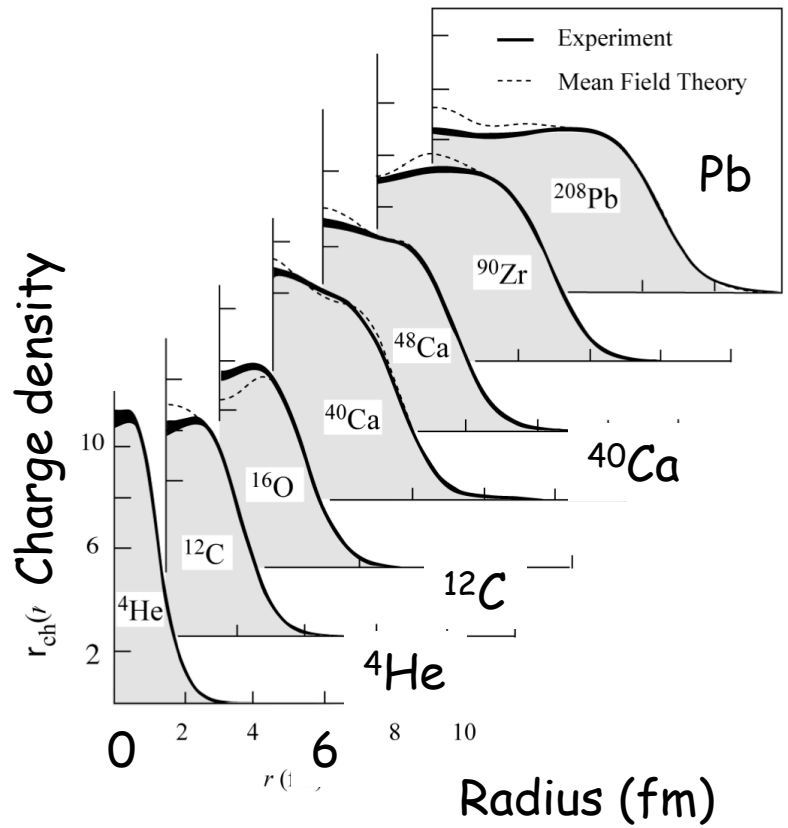


1 fm^{-1} = 197 MeV/c

I. Elastic (e,e') Scattering ⇒ charge distributions







Elastic electron scattering measured for many nuclei over a wide range of Q2(mainly at Saclay in the 1970s)

Measured charge distributions agree well with mean field theory calculations.

Lead ^{208}Pb Radius Experiment : PREX

Pb(e,e') Elastic Scattering

Parity Violating Asymmetry

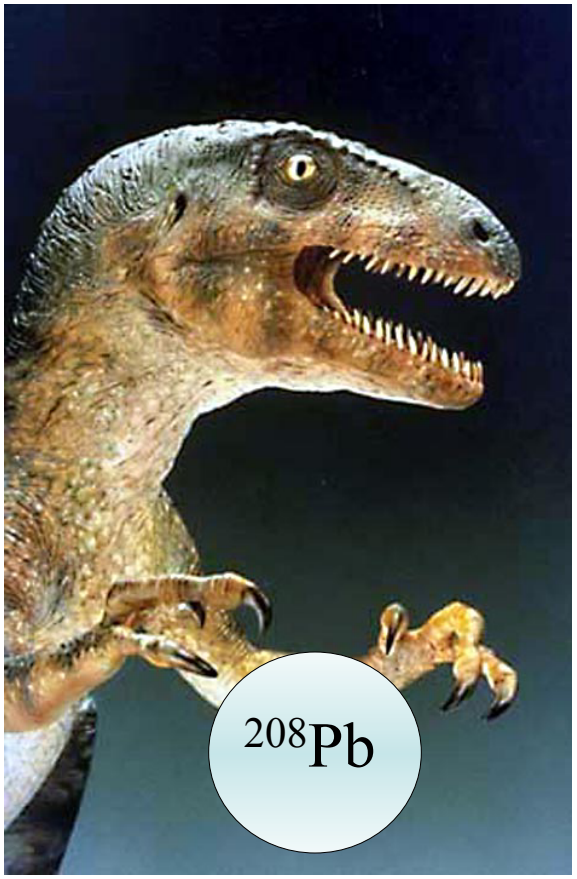
E_{beam} = 1 GeV

$\theta_e = 5^o$

I_{beam} = 60 μA

Spokespersons

- Paul Souder
- Krishna Kumar
- Guido Urciuoli
- Robert Michaels

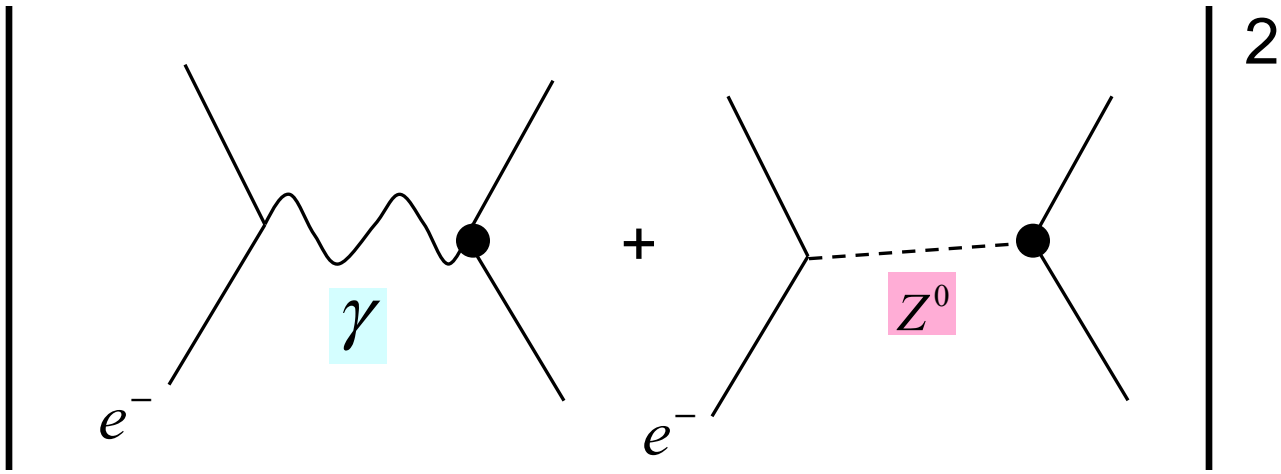


Slides courtesy of R. Michaels

Parity Violating Asymmetry

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \sim 10^{-6}$$

$\sigma \approx$



Applications of PV at Jefferson Lab

- Nucleon Structure (strangeness) -- HAPPEX / G0
- Standard Model Tests ($\sin^2\theta_W$) -- e.g. Qweak
- *Nuclear Structure (neutron density) : PREX*

Lead ^{208}Pb Radius Experiment : PREX

Z^0 of Weak Interaction:

Clean Probe Couples Mainly to Neutrons

Left-Right Cross section asymmetry (simplified):

$F_W(Q^2)$: ^{208}Pb Weak Form Factor

$$A = \frac{\left(\frac{d\sigma}{d\Omega}\right)_R - \left(\frac{d\sigma}{d\Omega}\right)_L}{\left(\frac{d\sigma}{d\Omega}\right)_R + \left(\frac{d\sigma}{d\Omega}\right)_L} = \frac{G_F Q^2}{2\pi\alpha\sqrt{2}}\left[\underbrace{1 - 4\sin^2\theta_W}_{\approx 0} - \frac{F_W(Q^2)}{F_P(Q^2)}\right]$$

$F_p(Q^2)$: ^{208}Pb Charge Form Factor

w/ Coulomb distortions:

$$\frac{dA}{A} = 3\% \quad \rightarrow \quad \frac{dR_n}{R_n} = 1\%$$

R_n = neutron matter radius

(T.W. Donnelly, J. Dubach, I Sick)

(C. Horowitz)

Clean Probe Couples Mainly to Neutrons

	proton	neutron
Electric charge	1	0
Weak charge	0.08	1

PREX & Neutron Stars

(C.J. Horowitz, J. Piekarewicz)

Crab Pulsar

R_n calibrates EOS of Neutron Rich Matter

→ Crust Thickness

→ Explain Glitches in Pulsar Frequency ?

Combine PREX R_n with Obs. Neutron Star Radii

→ Phase Transition to "Exotic" Core ?

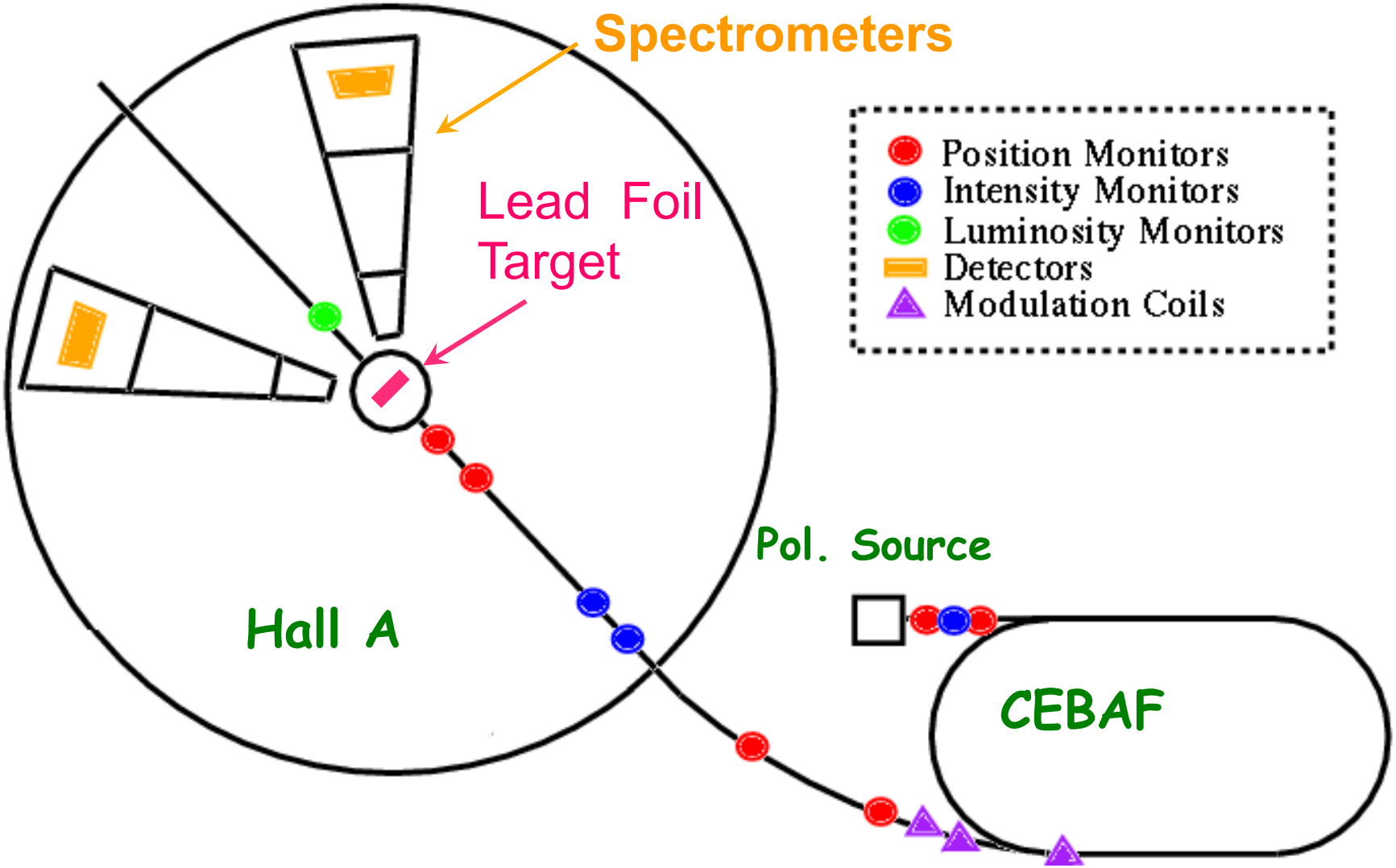
→ **Strange star ? Quark Star ?**

Some Neutron Stars seem too Cold

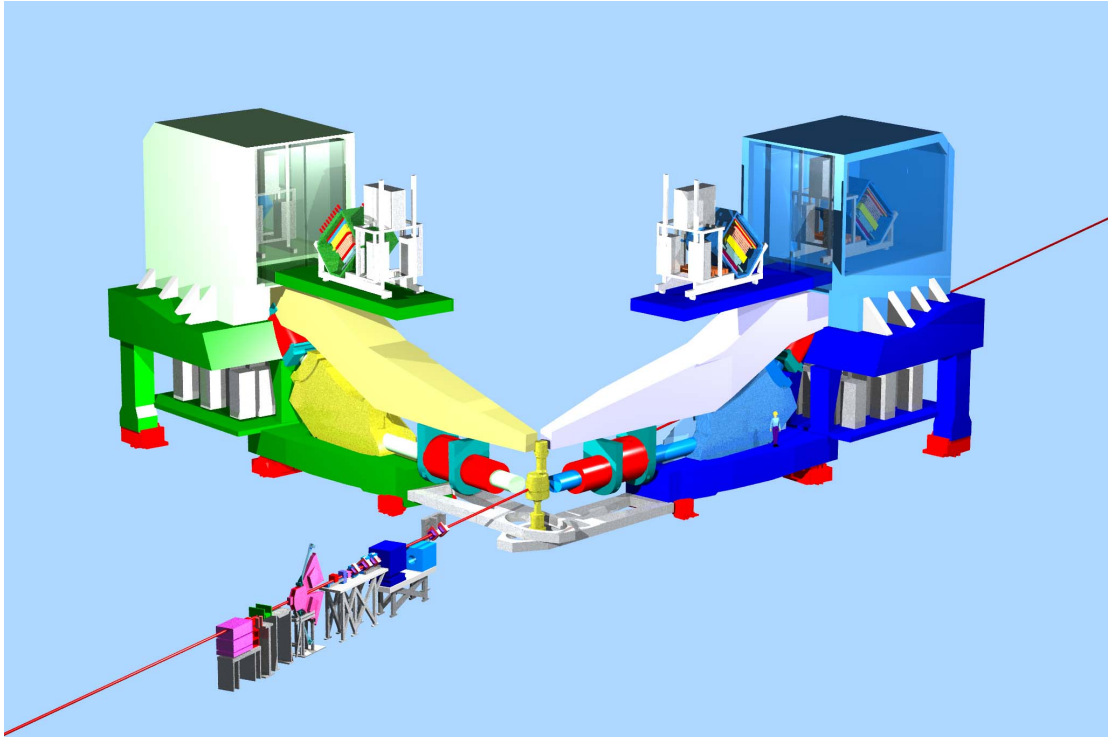
→ Cooling by neutrino emission (URCA)

→ $R_n - R_p > 0.2$ fm → URCA probable, else not

PREX in Hall A at JLab

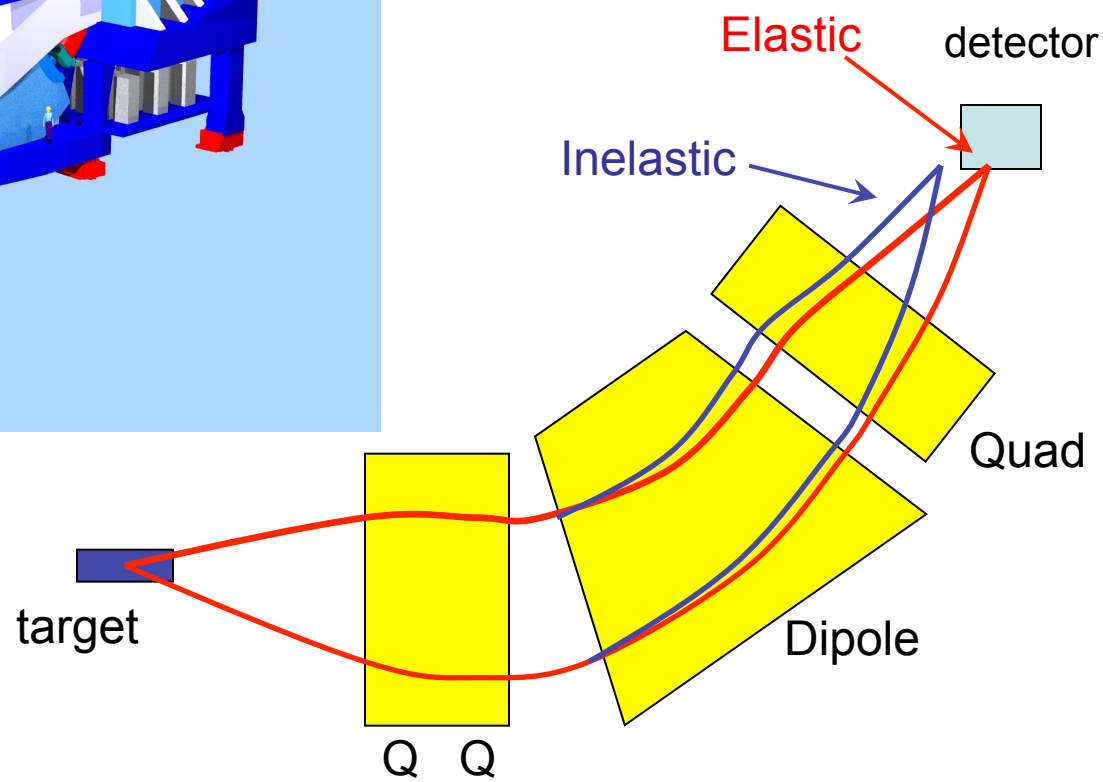


High Resolution Spectrometers

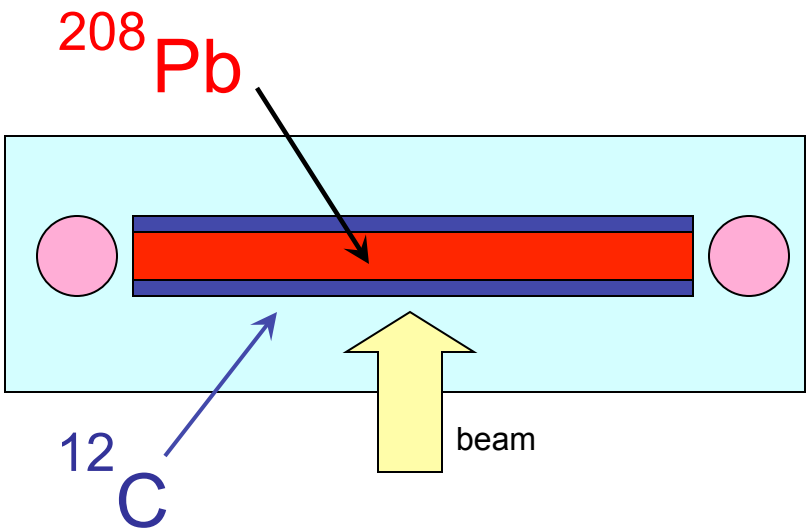


Spectrometer Concept:
Resolve Elastic

Pb 1st excited state at 2.6 MeV

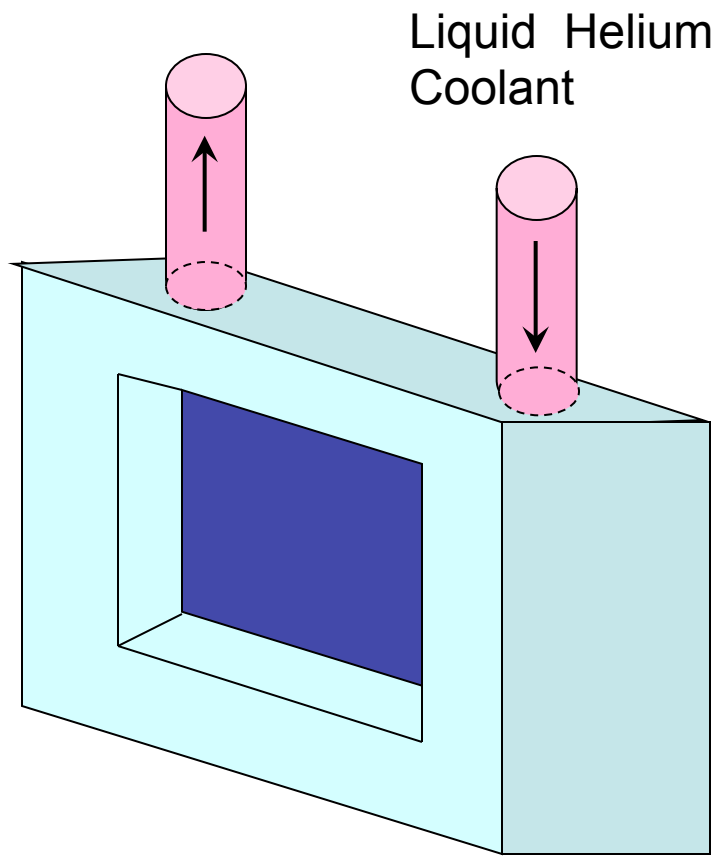


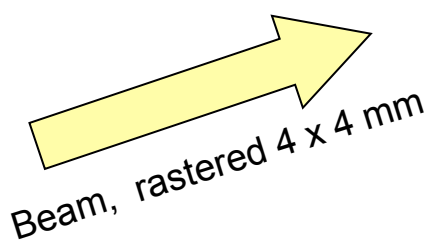
Lead Target - melts easily



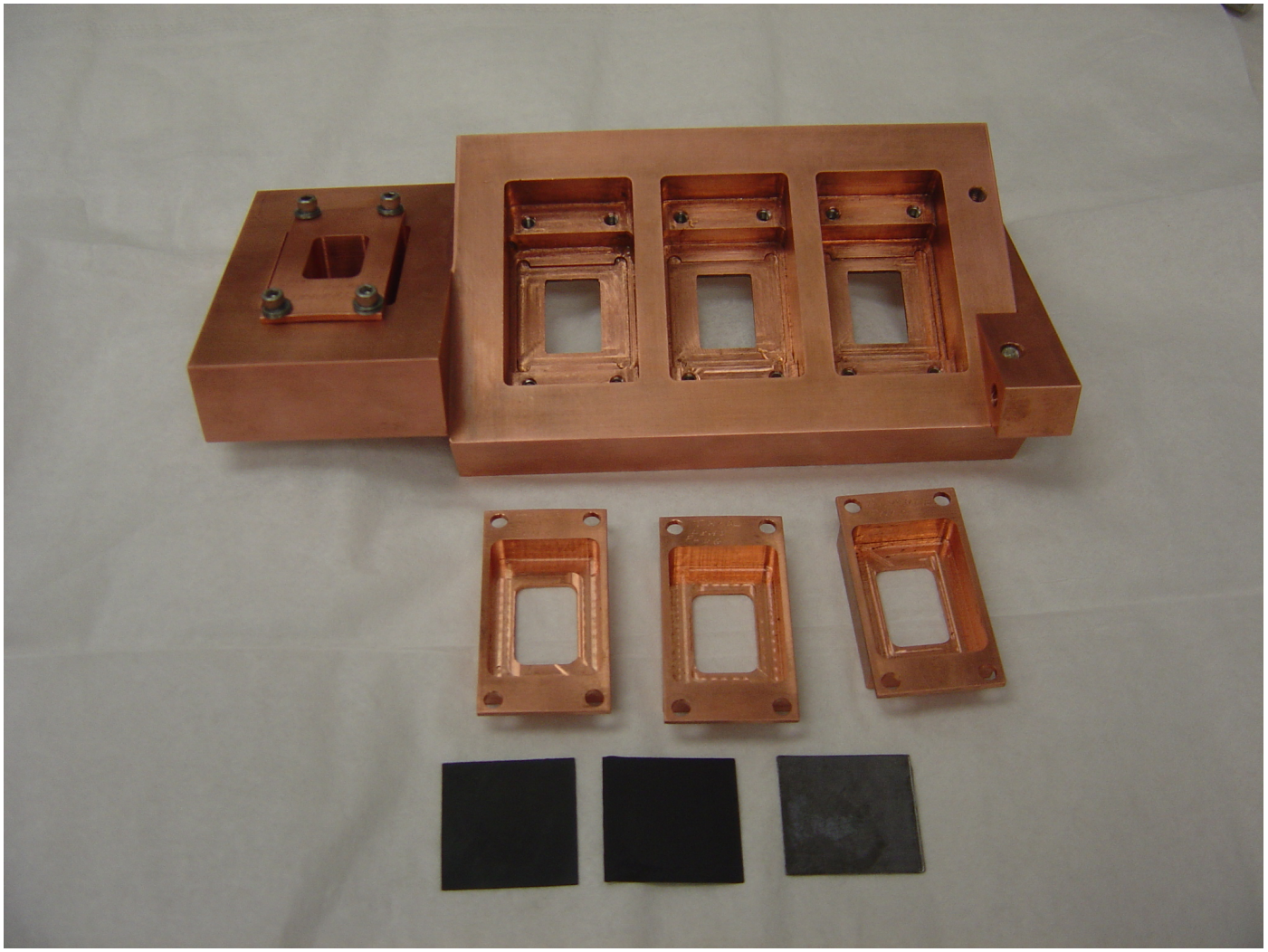
Diamond Backing:

- High Thermal Conductivity
- Negligible Systematics



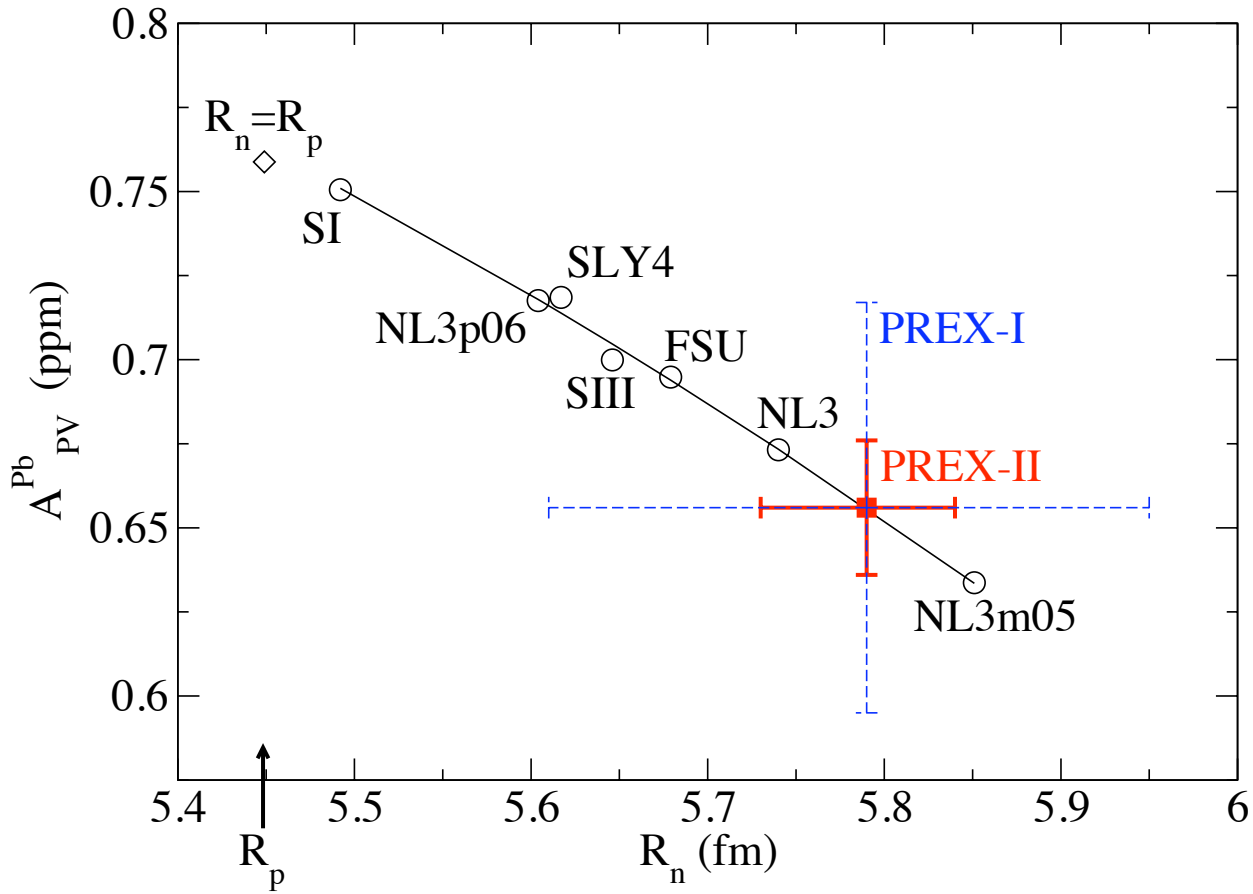


Target Assembly



PRex Results

APV = 656 ± 60 (stat) ± 14 (syst) ppb



Look for PRex II, coming soon to a lab near you!

Deuteron Elastic Scattering

- Only bound 2 nucleon system
- Prime testing ground for models: where is the limit for the description in terms of nucleons and mesons
- Questions:
  - How does the photon interact with a BOUND nucleon?
  - Effects of meson interactions?
  - Final State Interactions?
  - Deuteron wave function at small distance?
  - Where do quarks come in?

Review Articles:

- M.Garcon and J.W. van Orden Adv.Nucl.Phys.26(2001)293
- R. Gilman and F. Gross, J. Phys. G: Nucl. Part. Phys. **28** (2002) R37–R116
- R.J.Holt and R. Gilman http://arxiv.org/abs/1205.5827v1

Elastic D(e,e')

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott}\left[A(Q^2) + B(Q^2)\tan^2(\theta/2)\right]$$

$$A = G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 G_Q^2$$

$$B = \frac{4}{3}\eta(1+\eta)G_M^2$$

G_C Charge form factor

G_M Magnetic form factor

G_Q Quadrupole form factor

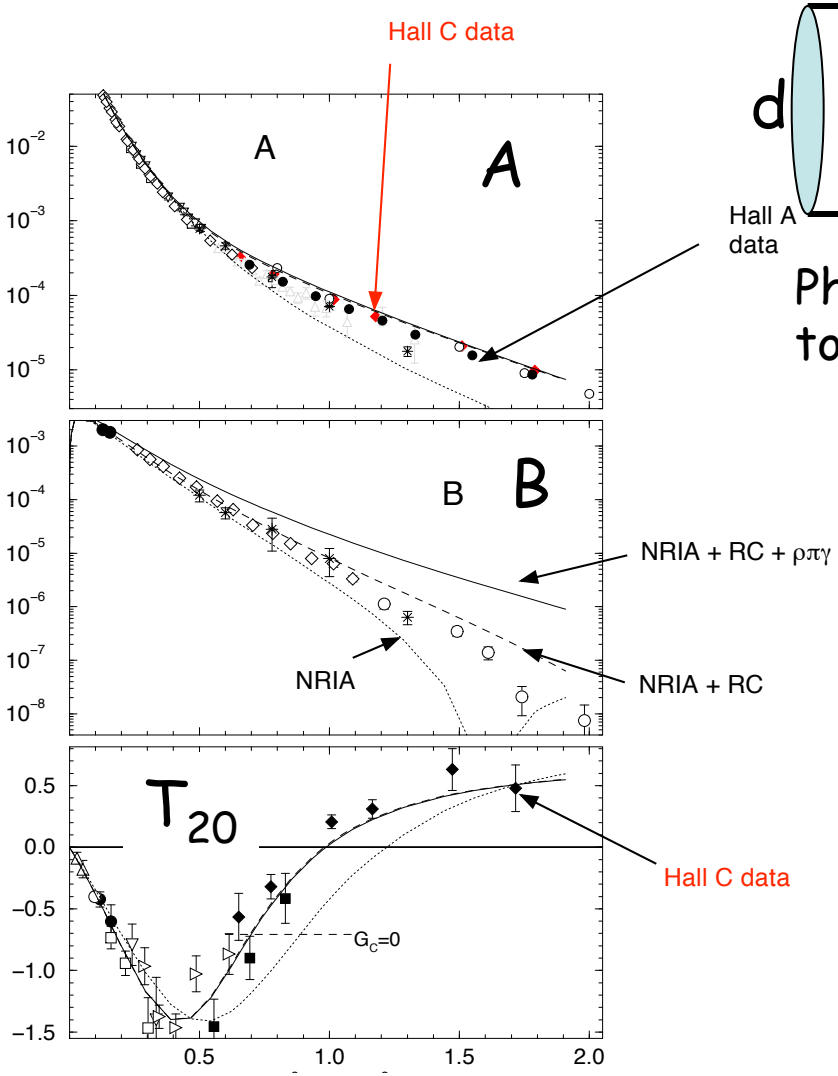
$$T_{20} = -\frac{\frac{8}{9}\eta^2 G_Q^2 + \frac{8}{3}\eta G_C G_Q + \frac{2}{3}\eta G_M^2\left[\frac{1}{2} + (1+\eta)\tan^2(\theta/2)\right]}{\sqrt{2}\left[A + B\ \tan^2(\theta/2)\right]}.$$

$$\eta = Q^2/4M_D^2$$

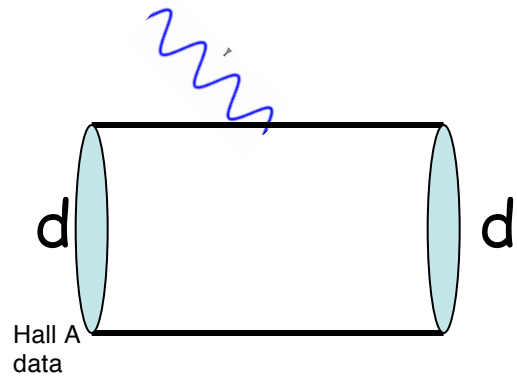
Note: A and B are linear combinations of F_1 and F_2 or of R_L and R_T.

To measure the tensor polarization T20, use: $\overleftrightarrow{d}(e,e')d$

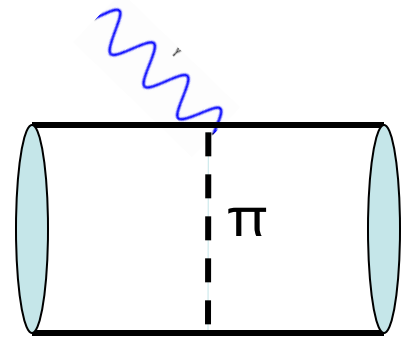
$d(e,e'\overleftrightarrow{d})$



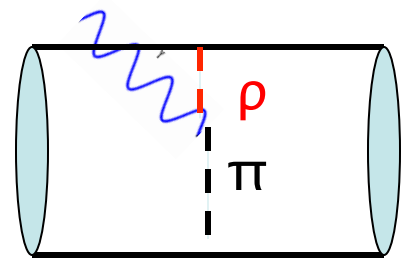
Calculation: H.Arenh PRC 61, 034002



Photon couples to nucleon



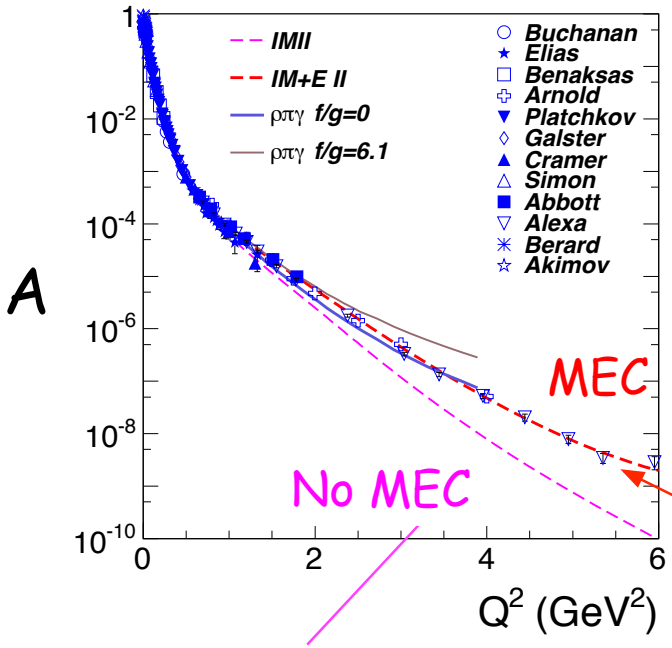
Meson Exchange Current

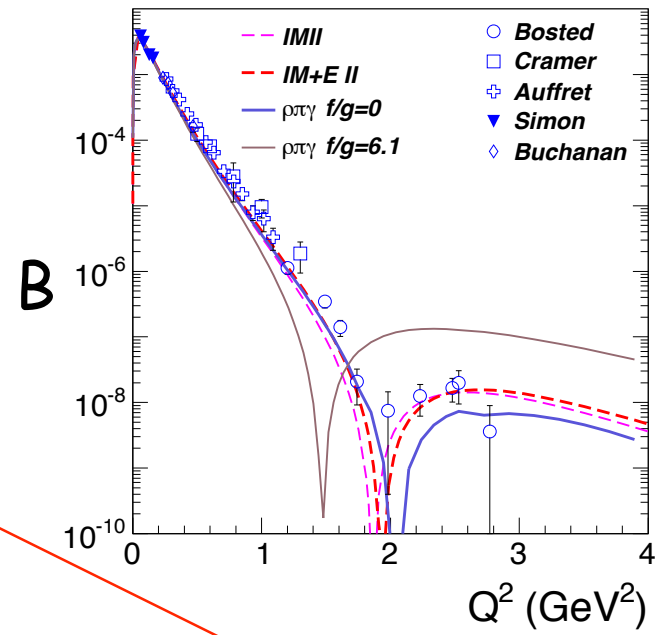


ρπγ Meson Exchange Current

Experiments:

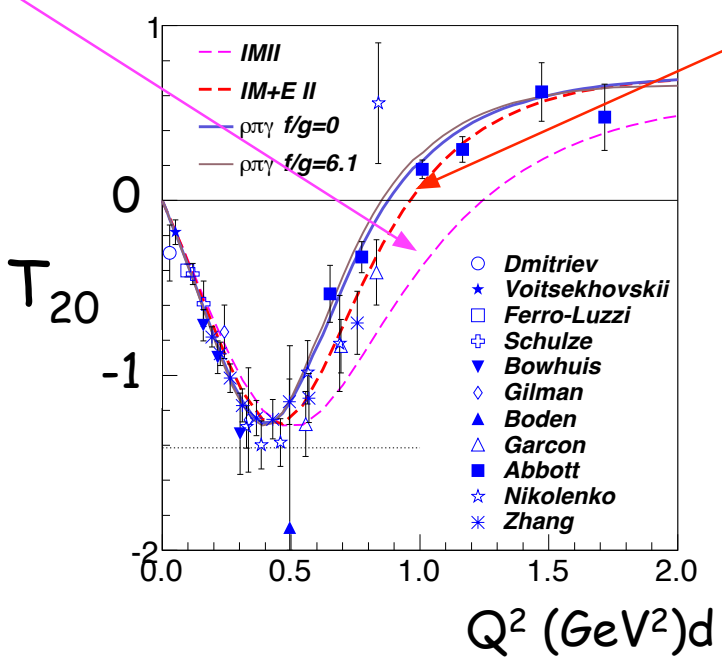
- Hall A : L.C.Alexa et al. PRL 82 (1999) 1374
- Hall C: D.Abbott et al. PRL 82 (1999) 1379
- Hall C: D.Abbott et al. PRL 84 (2000) 5053





no MEC contributions

with MEC



Rel. Calculations in Hamiltonian dynamics:

IMII and IM+EII

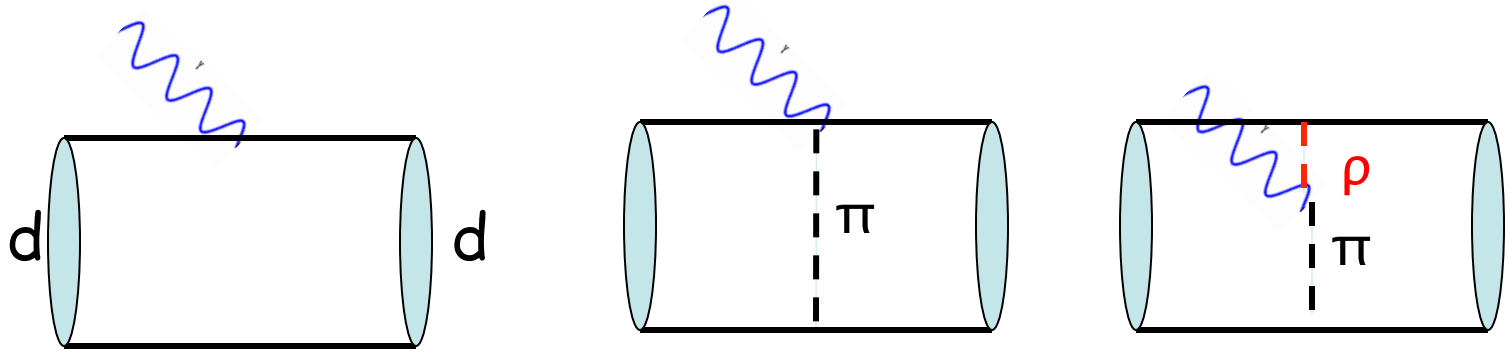
Y.Huang and W.N.Polyzou PRC80 (2009) 025503

Rel. Calculations in propagator dynamics: Need ρπγ diagram – not well constrained

D.R.Phillips et al. PRC72 (2005) 014006)

Deuteron Elastic Scattering Summary:

- Non-Relativistic models fail at the highest Q^2
- Relativistic models successfully describe Deuteron form factors
- MEC contributions are very important
- ρπγ exchange current important and not well constrained



Photon couples to nucleon

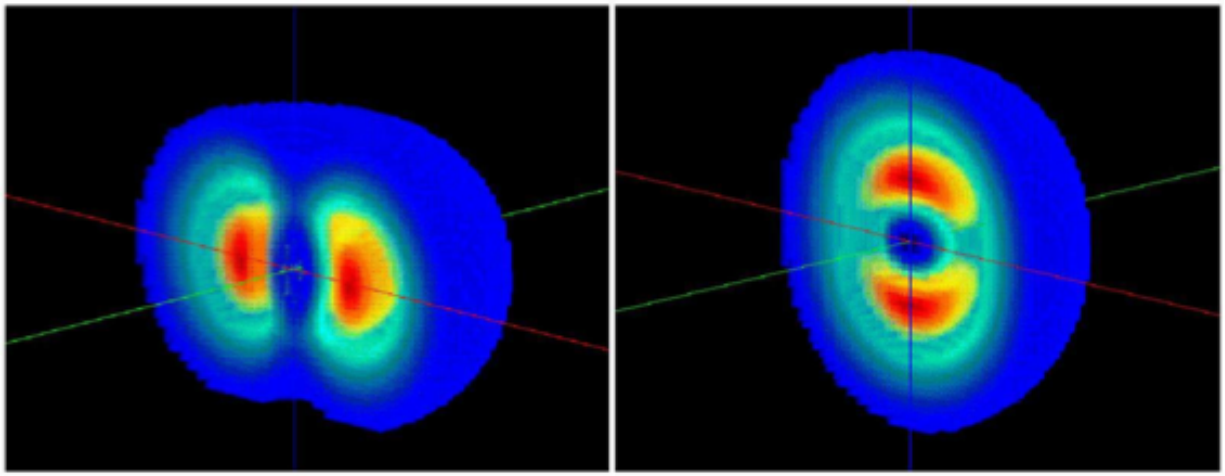
Meson Exchange Current

ρπγ Meson Exchange Current

Elastic Scattering Summary

- Beautiful measurements of the nuclear charge distribution
  - And first measurements of the nuclear matter radius
- Beautiful measurements of the deuteron
  - A, B and Tensor polarization
  - Allows separation of charge, magnetic and quadrupole form factors

m=0



m=±1